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ABSTRACT: This paper develops a general equilibrium model to generate endogenously a linear city. Rather than assuming interaction externalities or increasing returns in production, interregional transactions costs are considered. To rectify a deficiency associated with the standard continuum framework, we modify the concept of social optimum. Such a modified social optimum exists under appropriate assumptions and exhibits a concentrated market structure when there are high market set-up costs, thus yielding a unitary central business district. In an economy without trust, a standard monocentric city may emerge. The social optimum generating a monocentric city has a competitive equilibrium realization under proper assumptions.

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*Department of Economics, University of Rochester, Rochester, NY 14627.

**Department of Economics, The Pennsylvania State University, University Park, PA 16802.

1. Introduction

This paper generates a central business district endogenously through the considerations of interregional transactions costs: the utility cost of travel time and the set-up cost of marketplaces. These costs do *not* generate a failure of the welfare theorems. In contrast with the existing literature, neither interaction externalities nor increasing returns in production are required. To resolve problems associated with the continuum setup, we modify the concept of social optimum and spatial equilibrium using a finite (or discrete) approximation. We then provide a set of sufficient conditions to generate an optimal monocentric city with an endogenous central business district (CBD) to which every economic agent travels. Finally, we show that the social optimum resulting in a monocentric city has a competitive price realization.

Monocentric city models have been the most favored setup of regional scientists and urban economists for years [for example, see Alonso (1964), Beckmann (1969), Mirrlees (1972) and two survey papers by Wheaton (1979) and Fujita (1986a).] Except for Alonso's seminar work, a continuum of locations is usually considered together with a continuum of agents. The size of city can be either exogenously given [cf. Alonso (1964)] or endogenously determined [cf. Mills (1967)]. Given an exogenous CBD, an optimal town can be constructed having a competitive realization [cf. Mirrlees (1972)].[1] This nice property enables us to use the framework to investigate numerous urban economic issues of importance.

In this paper, we follow the structure of the continuum models in order to avoid the complexity associated with finite setups. Yet, a modification is imposed so that we can restore classical existence and welfare properties. Under this framework, we study an important (but usually ignored) driving force of the formation of a city – the interregional transaction technology. In his pivotal work, Mills (1967) discussed how market imperfections or increasing return production technologies may induce city agglomeration. For instance, he constructed an imperfect competition model in an urbanized economy where equilibria exist because individual firms face sufficiently inelastic demands. Due to spatial advantages, firms would restrict their size and then operate in increasing-returns-to-scale regions. This generates a factory town with a single, monopolistic firm. The city center is in effect the location chosen by that firm and residential location is determined as in standard monocentric city models [see also Papageorgiou and Thisse (1985) and Fujita (1986b) for further discussion]. More formally, Baesemann (1977) verified convergence to a Nash equilibrium under the standard assumption of concavity and Cournot location-choice behavior; the payoff maximizing behavior would result in an agglomerative process in the sense that consumers (traders) would form and maintain a single point market in the steady state.[2] Ambiguities remaining in that model include the presence of a discontinuity in admissible consumption due to the mismatch of interregional trading with respect to the travel distance for each trader and the exogeneity of land uses and residential locations. In Papageorgiou and Smith (1983), agglomeration was induced by (nonprice) positive interaction externalities among consumers. Since the uniform population distribution is a steady-state, an agglomerative process emerges only if this uniform steady-state is unstable. Other work along these lines are discussed in a comprehensive survey by Fujita (1988). However, in this class of spatial models, the formation of the CBD remains indeterminate without assigning a "tentative center" or a "factory site" a priori.[3]

In the present work, the concentration of transaction activities determines the endogenous city center (i.e. marketplace).[4] Regarding the analysis of an endogenous CBD, it is worth noting that within the general equilibrium framework, a "spatial impossibility theorem" is usually obtained [cf. Starrett (1978)]: In a closed, homogeneous spatial economy with perfect markets for all commodities at all locations and without relocation cost, there is no competitive equilibrium with a positive aggregate transportation cost [see Fujita (1986a) for a discussion].[5] Hence, neither transaction concentration nor population agglomeration can result in the Starrett economy. This theorem tells us that to introduce spatial heterogeneities, increasing return technologies or market imperfections are necessary to generate a unitary CBD with concentrated transactions. Examples using models with increasing returns are Starrett (1974, 1978), in which an agglomerative city or a factory town is generated. Through the consideration of a market imperfection, Stuart (1970) built up a search model to determine the "spatial organization of trading" (i.e. marketplace) for economic agents. Notice that this class of models adopts a partial equilibrium framework in which demand is exogenously given. Consequently, it is senseless to discuss its resulting welfare properties.[6] In Wang (1989), due to spatial heterogeneity of preferences and endowments, the optimal marketplace in which a finite number of spatially separated consumers transact was endogenously determined and had a competitive realization. Nevertheless, residential location of consumers in a continuum model with an endogenous marketplace remains open.

In this paper, we develop a general equilibrium model of spatially competitive comsumers with regionally heterogeneous endowments in a linear city under a continuum setting following Solow and Vickrey (1971). A city with concentrated transaction activities is endogenously formed due to the presence of interregional transactions costs: the utility cost of travel time and the set-up cost of marketplaces. In contrast to Papageorgiou and Smith (1983) and Starrett (1974), interaction externalities among consumers or increasing returns of the production technology are not imposed. In contrast to Baesemann (1977) and Starrett (1974), a continuum of consumers is considered in order to capture the standard monocentric city structure. In contrast to Mills (1967) and Stuart (1970), a general equilibrium framework is employed. In contrast to all the previous work, both market location and residential location are endogeneously and simultaneously determined.

When location-dependent variables (such as leisure and geographical factors) enter utilities, this class of (double) continuum models is subject to some problems, including in particular the non-existence of optima and the failure of the second welfare theorem (see discussion in section 3 below). To rectify this deficiency, we modify the concept of social optimum by using a *μ-optimum* which is in effect an approximation of a social optimum under the standard continuum setup. It is used to resolve a problem with sets of measure zero in a continuum economy. When the utility function is separable in consumption and locational-dependent leisure, the modified social optimum exists under standard assumptions. The strict concavity of preference for leisure generally leads to a dispersed market structure. When a substantially high set-up cost of marketplaces is considered, the optimal market structure is concentrated and a unitary CBD is thus obtained. In the presence of a queueing time cost, this CBD may locate at the geographical center. Contrary to standard beliefs, a CBD is endogenously generated without assuming positive interaction externalities or increasing returns in production and travel/transportation technologies. The transactions cost technologies imposed here can be regarded as resulting from an increasing-return technology of "market transactions" which enables us to generate a standard monocentric city. More importantly, the modified social optimum which yields a monocentric city has a competitive price realization under proper assumptions. Thus, we may avoid the computational complexity associated with spatial equilibria by focusing merely on social planner's modified optimization problem.

The remainder of this paper is organized as follows. In section 2, we present a spatial model under a modified continuum framework. We next modify the concept of social optimality and prove the existence of a modified social optimum in section 3. By introducing endowment heterogeneity and interregional transactions costs, section 4 discusses the formation of a city with transaction concentration. We then prove that there exists a competitive realization of a concentrated social optimum in section 5. Section 6 concludes the paper.

2. The Model

In this paper, a pure-exchange spatial economy is considered in which *region-specific* commodity endowments can be collected costlessly. The landscape is bounded, homogeneous and linear. It is specified as a one-dimensional closed interval, $Z = [-\frac{3}{2}, \frac{3}{2}]$. To allow intermediate transactions, we consider 3 types of a continuum of consumers (or households), indexed by a superscript h, $h \in H = \{1, 2, 3\}$. Consumers are assumed to be uniformly distributed over the landscape at the start and the population of each type of consumers, denoted by N^h, $h \in H$, is fixed, where $\sum_{h \in H} N^h = N$. For simplicity, let type 1, 2 and 3 agents locate over 3 unit intervals of the linear landscape: $[-\frac{3}{2}, -\frac{1}{2}]$, $[-\frac{1}{2}, \frac{1}{2}]$ and $[\frac{1}{2}, \frac{3}{2}]$, referred to as regions Z^1, Z^2 and Z^3, respectively. Notice that regions Z^1 and Z^3 are parallel to the suburb zone and Z^2

is analogous to the center zone in de Palma and Papageorgiou (1988).[7]

Each of the type-h consumers can collect a region-specific mobile commodity endowment, e_h ($0 \ll e_h < \infty$ for all $h \in H$), and is endowed with one unit of time that can be devoted to travel or leisure. All members of a type of consumers have an identical preference order, represented by a utility function over consumption of mobile commodities, land and leisure. For simplicity, we assume that there is no relocation cost nor physical transportation cost. That is, all consumers are free to choose location and free to carry goods across locations except for a utility cost of travel time (for making the round trip from home to marketplaces). To avoid the nonexistence problem pointed out by Bewley (1981), we further assume that everyone merely has the region-specific "knowledge" to collect his/her home good so that no one has the incentive to change his/her habitation region;[8] however, he/she can relocate within the region. Thus a type-h consumer can be called a good-e_h collector or a region-Z^h resident. To close the model, we introduce an absentee landlord who is endowed with the whole landscape Z (with density 1 at each location) but who consumes mobile commodities only. Since there is no relocation cost, only the final location index, z, is relevant to our analysis. Thus a representative consumer can be indicated by (h, z). Because a consumer cannot move outside of the region in which he/she initially resides, one would know (h, z) given z. Specifically, a consumer is type-h if he/she finally resides in region Z^h. This enables us to simply denote a (h, z) consumer's consumption of good i ($i = 1, 2, 3$), land at z and leisure by $c_i(z)$, $q(z)$ and $\ell(z)$, respectively. His/her utility function is assumed to take the following additive form: $U^h = u^h(c_1, c_2, c_3, q) + v^h(\ell)$. The absentee landlord's preference can be represented by $U^L = u^L(c_1^L, c_2^L, c_3^L)$, where superscript L indicates the landlord.

The density of (h, z)-consumers is denoted by $n(z)$. Since there is N^h measure of (h, z)-consumers distributed uniformly initially over a unit interval, we have $\int_{Z^h} n(z)dz = N^h$. We note that any single agent in a continuum economy has a zero-measure. This, in effect, creates some in obtaining a well-defined social optimum (see a discussion in the section 3 below).

Finally, we assume:

A.1. (Well-behaved preferences) u^h, v^h *and* u^L *are strictly increasing, positive, twice continuously differentiable and strictly concave; there is a* $\bar{c}_i > 0, \bar{c}_i \leq \frac{N^i e_i}{N}$, *such that, for all* $\{c_i\}$ *with* $\prod_{i \in H} c_i = 0$, $u^h(\{\bar{c}_i\}, q) > u^h(\{c_i\}, q)$.

The last condition in A.1 assures the necessity of all varieties of consumption goods, thus ruling out the autarkic allocation. This condition is weaker than the standard Inada condition used in the literature [for example, see de Palma and Papageorgiou (1988)]. In Scotchmer (1985), the necessity of land was assumed (see her assumption A.2: $U^h = 0 \;\; iff \;\; q^h = 0$). Due to the separability of U^h in goods/land and leisure, we cannot make a corresponding assumption here.

3. The Modification of the Concept of Social Optimum

First, consider a traditional equally weighted utilitarian social welfare function: $SW = \sum_{h \in H} \int_{Z^h} n(z)[u^h(c_1(z), c_2(z), c_3(z), q(z)) + v(\ell(z))]dz + u^L(\{\theta_i N^i e_i\})$. As opposed to Mirrlees (1972), location enters into the individual utility function directly through the travel-leisure tradeoff. Moreover, the location of the marketplace is not given a priori; the formation of regional business districts and intermediate transactions are both endogenous in this paper. A social optimum is simply a maximum of SW subject to the feasibility constraints. Surprisingly, an example with no social optimum can easily be found. Since travelling is a pure social cost to the economy, a social optimum must minimize the aggregate travel time. With intermediate transactions allowed, the social planner has to arrange a smallest set of intermediaries from each region Z^h to make transactions for all residents in the region. If a finite model is considered, the minimal set of intermediaries becomes a singleton (i.e. only one economic agent serves as an intermediary). Under a continuum setup, such a minimal set cannot be found. Suppose that the minimal set of intermediaries has positive measure. Then its correspondent allocation must be associated with positive travel costs and hence is strictly social welfare dominated by an allocation with an intermediary set of zero measure. However, once the minimal set of intermediaries has zero-measure, the set of mobile good transactions has zero measure. Thus an autarkic allocation results. Now consider an allocation that allows transactions among consumers from different sides of either of the two regional borders $-\frac{1}{2}$ and $\frac{1}{2}$, say for example from $\frac{1}{2}-\epsilon$ and $\frac{1}{2}+\epsilon$. By A.1, this allocation dominates the autarkic one if ϵ is small enough so that the utility gain from trading outweighes the disutility of travelling. Therefore, a social welfare maximizing set of intermediaries does not exist and hence a social optimum does not exist. This difficulty can be referred to as a problem of *transaction discountinuity.*

To have a well-defined concept of social optimum, a positive measure of agents must be considered. In this paper, we use an i-partition[9] consisting of a finite number of *closed* intervals of equal size, to replace the zero-measure single point in the original continuum economy. More specifically, denote by $B_\epsilon(z)$ a closed interval with radius ϵ around z. This closed interval is an element of an i-partition. Without loss of generality, we restrict attention to cases in which each Z^h contains an even number of elements of an i-partition by properly choosing ϵ (for $\epsilon \le \frac{1}{4}$). Therefore, rather than formulating a standard measure space of agents,[10] we construct a subset of a standard measure space to restore general equilibrium properties, notably the existence of social optima. Let S^h be the set of the center points of closed interval B_ϵ's which cover the region Z^h.[11] A set of consumers of positive measure can thus be represented by $(h, B_\epsilon(s^h))$, consisting of all the consumers (h,z), where $z \in B_\epsilon(s^h)$. For notational simplification, write $B_\epsilon(s^h)$ as B and a representative group of consumers, (h,B), can then be denoted by B. By construction, the measure of such group of consumers is $N(B) = \int_B n(z)dz = 2\epsilon N^h$; it is positive for all $n(z) > 0$ and for all $\epsilon > 0$. The associated *mean* consumption bundle for consumer group B is thus $(\{c_i(B)\}, q(B), \ell(B))$, where $f(B) = \int_B n(z)f(z)dz/N(B)$, $f = c_i, q, \ell$.

To study the market structure, call m the marketplace located at $m \in Z$ and $J(m)$ the set of transactors at market m. The set of marketplaces in which B transacts is defined as $M(B) = \{m \in Z \mid z \in B_\epsilon(s^h), (h,z) \in J(m)\} \in \bar{Z}\backslash\emptyset$, where \ denotes set substration and $\bar{Z}$ denotes the σ-algebra of Borel subsets of Z. Two types of market structure can be characterized, concentrated and dispersed market structures. A market structure is said to be ***concentrated*** if all the transactors transact within a closed interval (or a "block") $B_\epsilon(s)$, for $\epsilon > 0$ and $s \in \bigcup_{h \in H} S^h$; otherwise, it is said to be *dispersed.* To construct the distance measure, we propose a standard *Hausdorff metric* since both the middle point distance and the minimum distance (which are frequently used in the urban economics literature) are not well-defined metrics.[12] Let $\mathcal{H}(F_1, F_2)$ be the Hausdorff metric on nonempty, closed subsets, F_1 and F_2, of the compact space of landscape, Z.[13] In a linear-city model, the Hausdorff metric is defined as the maximum of the distance from the outermost point of a closed interval K_1 to the innermost point of another closed interval K_2 and the distance from the innermost point of K_1 to the outermost point of K_2. A nice property of the Hausdorff metric is the compactness of its topology, which is essential in proving existence theorems. We now express B's mean one-way travel distance to all the relevant markets as $\delta(B) = \mathcal{H}(B, M(B))$. This distance measure enables us to eliminate the problem of double-counting for travel distance in models using the middle point distance measure.

Let $N^T(B)$ denote the measure of consumers B serving as transactors; it is an endogenously determined fraction of $N(B)$. Thus the aggregate round-trip travel distance for the consumer group B is $2N^T(B)\delta(B)$. For the sake of simplicity, we assume that the travel technology exhibits constant-returns-to-scale with a normalized travel time-distance coefficient $\frac{1}{6}$; this enables every consumer to travel to anywhere along the linear city given a unity time endowment. Denote by $t(B)$ the representative consumer group's mean travel time, $t(B) = \frac{1}{3}I(h)\delta(B)$, where $I(h)$ equals one if h is a transactor and zero otherwise. The time constraint is thus $t(B) + \ell(B) = 1$, which can be rewritten as

$$\frac{1}{3}N^T(B)\delta(B) + N(B)\ell(B) = N(B). \tag{1}$$

The population identity requires

$$\sum_{s^h \in S^h} N(B) = N^h. \tag{2}$$

Material balance conditions for land and goods are

$$N(B)q(B) = 2\epsilon \quad \text{for all } s^h \in S^h, h \in H \tag{3}$$

$$\sum_{h\in H}\sum_{s^h\in S^h} N(B)c_i(B) = (1-\theta_i)N^i e_i, \tag{4}$$

where $\theta_i = \frac{c_i^L}{N^i e_i}$ indicates the fraction of good i consumed by the landlord. This completes the description of the structure of the "finite" modified continuum model in the absence of any friction (transactions or informational costs).

We are now ready to formalize the modification of the concept of social optimum. A *μ-allocation* is a list of positive measurable functions, specifying quantities of mobile goods, land and leisure consumed by each group of consumers (B), the associated population measure of each group, the measure of transactors for each group, and fractions of goods consumed by the landlord: $(\{c_i(B)\}, q(B), \ell(B), N(B), N^T(B); \{\theta_i\})$ with $N(B) \geq \mu \gg 0$ and $N^T(B) \geq \mu \gg 0$ whenever $N^T(B) > 0$. A μ-allocation is *feasible* if it satisfies the identity (2) and constraints (1), (3) and (4) with inequalities $(\leq)$. A μ-allocation is said to be *transactively feasible* if it is feasible and $0 \leq N^T(B) \leq N(B)$ for all $(h, B_\epsilon(s^h), B)$ with $N^T(B) \geq \mu$ for at least one B. Next define an (equally weighted) utilitarian social welfare function: $SW = SW^H + u^L(\{\theta_i N^i e_i\}_{i\in H})$ where $SW^H = \sum_{h\in H}\sum_{s^h\in S^h} N(B)[u^h(\{c_i(B)\}, q(B)) + v^h(B)]$, representing the aggregate social welfare of households. A *μ-optimum* is defined as a transactively feasible μ-allocation which maximizes the social welfare SW among all transactively feasible μ-allocations. By manipulation,[14] we can rewrite SW as:

$$SW = SW^A + SW^B, \tag{5}$$

where

$$SW^A = \sum_{h\in H}\sum_{s^h\in S^h} N(B)[u^h(\{c_i(B)\}, q(B)) + v^h(1)] \; + \; u^L(\{\theta_i N^i e_i\})$$

$$SW^B = \sum_{h\in H}\sum_{s^h\in S^h} N^T(B)[v^h(1 - \frac{1}{3}\delta(B)) - v^h(1)].$$

Since the utility function is separable in goods/land and leisure, transaction/market structures merely affect $\delta(B)$ and thus SW^B. As a consequence, the optimal $(N^{T*}(B), \delta^*(B), \ell^*(B))$ is determined independent of optimal values of $(\{c_i^*(B)\}, q^*(B), N^*(B), \{\theta_i^*\})$.

REMARKS. We note that transactive feasibility is crucial in obtaining a well-behaved μ-optimum. $N^T(B) \leq N(B)$ is apparently required under the population identity. When $N^T(B) = 0$ for all B, there can be no interregional transactions and hence the economy has to be autarkic. Such a transaction discontinuity problem is eliminated in the present work by imposing transactive feasibility and a modified intermediation transactions structure defined by the μ-allocation, together with assumption A.1 (which rules out any autarkic equilibrium). Mathematically, this μ-modification is used to resolve the problem with sets of measure zero in a continuum setup, which would cause the nonexistence of (equilibria and) optima as well as the failure of welfare theorems. In models with a finite or countably infinite number of agents this μ-modification is not needed. In order to facilitate comparison with the standard urban economic literature and to avoid the analytic complexity associated with finite models, the above modified continuum framework serves the purpose. Intuitively, μ can be regarded as "1-person" in a finite model: $N(B) \geq 1$ means that there is at least one resident occupying the area of land at each subset of landscape (which assures there is no idle land), while $N^T(B) \geq 1$ whenever $N^T(B) > 0$ implies that at least one transactor (or truck-driver) is required for any interregional transactions. Notice that this construction is not an exact finite-approximation of standard continuum models. It generates results capturing the spirit of the monocentric city framework qualitatively. Moreover, the mathematics required is far less technical than that used in standard finite models.

The existence of the aforementioned modified social optimum is demonstrated next.

THEOREM 1 *(Existence of social optima). Under assumption A.1, for each $\mu > 0$, there is a μ-optimum.*

Proof. The proof of the existence of a μ-optimal $(\{c_i^*(B)\}, q^*(B), N^*(B), \{\theta_i^*\})$ follows directly from the fact that one is maximizing a continuous objective function, SW^A, subject to a compact feasible set. Next notice that SW^B is decreasing in $n^T(B)$ whenever there is a minimum measure of transactors μ over the whole habitation area Z. But $N^{T*}(B) = 0 \ \forall \ B$ would imply $c_i^*(B) = 0 \ \forall \ h \neq i$, which can not be optimal under A.1. Thus the optimal measure of transactors is the one attaining the minimum, μ, across all locations. Q.E.D.

A candidate for μ-optimal transaction structures is that a group of left-hand side transactors of measure μ and a group of right-hand side transactors of measure μ travel from each border to an "innermost marketplace", say at the geographical center ($z = 0$), and travel back home after exchanges. Candidates for the set of "representative transactors" of each side of the linear city are as follows:

(i) it merely consists of measure μ consumers from each of the outermost 2 subsets, $B_\epsilon(s^1_{min})$ and $B_\epsilon(s^3_{max})$ and each subset of transactors makes a round-trip from their home to $z = 0$;

(ii) it consists of measure μ consumers from each subset $\{B_\epsilon(s^h)\}_{s^h \in S^h, h \in H}$; each of the innermost subsets of transactors makes a round-trip from their home to $z = 0$ but each of other subsets of transactors makes a round-trip from their home to the adjacent subset toward the center;

(iii) it can be any case in between (i) and (ii); for instance, it consists of measure μ consumers from each of the outermost 4 subsets, $B_\epsilon(s^1_{min})$, $B_\epsilon(s^1_{min} + 2\epsilon)$, $B_\epsilon(s^3_{max} - 2\epsilon)$ and $B_\epsilon(s^3_{max})$; each of the outermost subsets of transactors makes a round-trip from their home to the adjacent subset toward the center but each of the second outermost subsets of transactors makes a round-trip from their home to $z = 0$.

Case (i) has the (completely) concentrated market structure, while case (ii) yields the (completely) dispersed market structure. To study the optimal market structure, we have:

THEOREM 2 *(Characterization of social optima). Under assumption A.1, any μ-optimal market structure is dispersed.*

Proof. We first compare case (i) with case (iii). Let $\delta_c = \mathcal{H}(B_\epsilon(s^3_{max}), B_\epsilon(s^3_{max} - 2\epsilon))$, and

$\delta_j = \mathcal{H}(B_\epsilon(s^3_{max} - 2(j-1)\epsilon), [0, 1.5 - 2j\epsilon])$, for $j \geq 1$. It is apparent that $\delta_{j+1} = \delta_j - \delta_c$ and $\delta_j \geq \delta_c \;\; \forall\, j$. The transaction structure for case (i) represented by $\{(n^{T^1}(s), \delta^1(s)\}$, $s \in \bigcup_h S^h$, is

$$n^{T^1}(s) = \begin{cases} \mu, & \text{for } s = s^1_{min}, s^3_{max} \\ 0, & \text{otherwise,} \end{cases}$$

$$\delta^1(s) = \begin{cases} \delta_1, & \text{for } s = s^1_{min}, s^3_{max} \\ 0, & \text{otherwise.} \end{cases}$$

Similarly, that for case (iii) is

$$n^{T^2}(s) = \begin{cases} \mu, & \text{for } s = s^1_{min}, s^1_{min} + 2\epsilon, s^3_{max} - 2\epsilon, s^3_{max} \\ 0, & \text{otherwise,} \end{cases}$$

$$\delta^2(s) = \begin{cases} \delta_c, & \text{for } s = s^1_{min}, s^3_{max} \\ \delta_2, & \text{for } s^1_{min} + 2\epsilon, s^3_{max} - 2\epsilon \\ 0, & \text{otherwise.} \end{cases}$$

Define $SW^B(1)$ and $SW^B(2)$ as the associated values of social welfare SW^B given the above transaction structures respectively. Notice that $[(1 - \frac{1}{3}\delta_2) - (1 - \frac{1}{3}\delta_c - \frac{1}{3}\delta_2)] = [1 - (1 - \frac{1}{3}\delta_c)] = \frac{1}{3}\delta_c$. By strict concavity of v^h, it can be shown that

$$\begin{aligned} & SW^B(2) - SW^B(1) \\ = & \sum_{h=1,3} \mu \left[v^h(1 - \frac{1}{3}\delta_c) + v^h(1 - \frac{1}{3}\delta_2) - v^h(1 - \frac{1}{3}\delta_c - \frac{1}{3}\delta_2) \right] \\ = & \sum_{h=1,3} \mu \left\{ \left[v^h(1 - \frac{1}{3}\delta_2) - v^h(1 - \frac{1}{3}\delta_c - \frac{1}{3}\delta_2) \right] - \left[v^h(1) - v^h(1 - \frac{1}{3}\delta_c) \right] + v^h(1) \right\} \\ > & \; 0. \end{aligned}$$

Repeating the same procedure to compare $SW^B(j+1)$ with $SW^B(j)$, one asserts the claim that (ii) is the best and a μ-optimum is thus dispersed satisfying

$$N^{T*}(B) = \mu \quad \text{for all } B \tag{6}$$

$$\delta^*(B) = \begin{cases} \delta_0, & \text{for } s = -\epsilon \text{ or } \epsilon; \\ \delta_c, & \text{otherwise,} \end{cases} \tag{7}$$

where $\delta_0 = \mathcal{H}(B_\epsilon(\epsilon), \{0\})$. Q.E.D.

Therefore, in the absence of interregional transactions costs, a dispersed market structure would result and a continuum regional business districts would form. Note that the Slater condition is satisfied under positive endowments and A.1. Straightforward application of the Kuhn-Tucker theorem for the social planner's concave programming problem yields

$$\frac{\partial u^h}{\partial c_i} = \frac{1}{N^i e_i}\left(\frac{\partial u^L}{\partial c_i^L}\right) \tag{8}$$

$$(u^h - \sum_{i \in H} c_i^* \frac{\partial u^h}{\partial c_i} - q^* \frac{\partial u^h}{\partial q}) + v^h(1) = \eta, \tag{9}$$

where η is the Lagrangian multiplier associated with (2). These no-arbitrage conditions together with (2), (3) and (4) determine a μ-optimal allocation $(\{c_i^*(B)\}, q^*(B), N^*(B), \{\theta_i^*\})$ (and the shadow value of population, η). There are in general a continuum of μ-optima as the "final marketplace" where two side transactors meet may not be at the geographical center $z = 0$. To obtain results consistent with those in the standard monocentric city literature, we consider a queueing technology:

A.2. (Queueing technology) *The queueing time spent by each transactor reduces his/her leisure.*

All transactors have the same time endowment and the same travel technology. This queueing technology plus a strictly increasing utility function in leisure time implies that two edge transactors have to meet at the geographical center $z = 0$. We now obtain a unique μ-optimum which forms a continuum of dispersed markets:

$$M^* = \{M^*(s) \mid M^*(s) = B_\epsilon(s), s \in \bigcup_{h \in H} (S^h \backslash \{s^1_{min}, s^3_{max}\})\} \bigcup \{0\}.$$

Thus, a unitary CBD cannot be generated unless it is enforced by a social planner [cf. Boruchov and Hochman (1977)] or by a prelocated firm [cf. Mills (1967)].

4. Formation of a Monocentric City

Given the queueing technology described above, a *CBD* is now formally defined as a closed interval $B_\varsigma(0)$ ($\varsigma > 0$) or a point $\{0\}$ in which every set of representative transactors from each group of consumers, (h, B), transacts. In words, a CBD is a single marketplace having concentrated transactions. To generate a unitary CBD, we introduce a natural cost of transactions – a market set-up cost: for instance, the cost of building a shelter, a loading dock and a parking lot.

Call $\Lambda^A(M)$ the number of interregional marketplaces and $G[\Lambda^A(M)]$ the total social welfare cost of forming marketplaces described by M in terms of transferable utils.[15] As an example, the aforementioned dispersed market structure has $\Lambda^A(M^*) = \epsilon^{-1} - 1$. In order to simplify the analysis, we assume that the set-up cost function takes the following constant returns to scale form:

A.3. (Transaction technology) *The set-up cost of each marketplace is a positive constant so that* $G[\Lambda^A(M)] = g\Lambda^A(M)$, $0 < g \ll \infty$.

Assumption A.3 enables us to rewrite the social welfare function (5) as:

$$SW[g, \Lambda^A(M)] = SW^A + SW^B - g\Lambda^A(M). \tag{10}$$

To generate a concentrated market structure, this set-up cost of each additional marketplace (g) has to be high enough to outweigh the marginal benefit of market decentralization. To obtain this, it is necessary to assume

A.4. (Lipschitz condition) v^h *satisfies* $|v^h(\ell_1) - v^h(\ell_2)| \leq \sigma|\ell_1 - \ell_2|$ *for* $\sigma \in \mathbf{R}_+$, $\sigma < \infty$.

This condition provides an upper bound on the marginal benefit of market decentralization. If v^h is twice continuously differentiable over a compact set, A.4 is satisfied. Recall that $\delta_0 = \mathcal{H}(B_\epsilon(\epsilon), \{0\})$ and define $\delta_{max} = \mathcal{H}(B_\epsilon(s^1_{max}), \{0\})$. A sufficient condition on the unit set-up cost of marketplaces that implies the formation a unitary CBD is:

A.5. (Lower bound on unit set-up cost) $|v^h(1 - \frac{1}{3}\delta_0) - v^h(1 - \frac{1}{3}\delta_{max})| < \epsilon g$ for all h.

There exists a g satisfying A.5. Take $g = \sigma\epsilon^{-1} + \kappa$, $\kappa > 0$, and apply A.4, so $|v^h(1 - \frac{1}{3}\delta_0) - v^h(1 - \frac{1}{3}\delta_{max})| \leq \sigma|(1 - \frac{1}{3}\delta_0) - (1 - \frac{1}{3}\delta_{max})| \leq \sigma$.

THEOREM 3 *(Determination of a CBD). Under assumptions A.1-A.5, any μ-optimum has a unitary CBD.*

Proof. Notice that $2|\delta_0 - \delta_{max}|$ is the maximum reduction in travel distance for a transactor from having one additional market. Hence the maximum incremental social benefit of having one more market $B_\epsilon(m)$ is $\frac{1}{\epsilon}\max_h \mid v^h(1-\frac{1}{3}\delta_0)-v^h(1-\frac{1}{3}\delta_{max}) \mid$, which is, by assumption A.5, less than g. So to achieve an optimum, Λ^A has to be kept at minimum. $\Lambda^A(M)=0$ is, however, not optimal given A.1. Moreover, (A.7) implies the autarkic allocation is not preferrable and thus $\Lambda^A(M)=1$ is optimal. Hence $SW[g,1] > SW[g,k]$ for all $k \neq 1$, $0 \leq k \leq \epsilon^{-1}-1$.Q.E.D.

REMARKS. In fact, when each Z^h contains an even number of elements of an i-partition (B), the CBD is the center-point, $\{0\}$. That is, all transaction activities are carried out at the geographical center. Notice that even with an increasing-returns-to-scale travel or transportation technology, a unitary CBD cannot form without imposing a set-up cost of marketplaces. More precisely, an increasing-returns-to-scale travel or transportation technology results in an "intermediation economy" in which only a few transactors from certain outer subsets (instead of all subsets) carry out all transactions for everybody. Thus the optimal market structure is by all means decentralized; this is contrary to the standard conjecture in the urban economic literature [for example, see Mills (1984, p.p. 6-19) and Starrett (1974); for a discussion, see Fujita (1986a)]. In contrast, with the above set-up cost on marketplaces which is linear in numbers of markets but independent of volumes of transactions, there exist scale economies in transactions that would lead to a CBD. Finally, a simple example which generates a unitary CBD is as follows: For all h, i, and $Q > 0$, $N^h = g = 4$, $e_i = 8$, $\epsilon = \frac{1}{4}$, $u^h = c_1^{\frac{1}{3}} c_2^{\frac{1}{3}} c_3^{\frac{1}{3}} + q^{\frac{1}{2}}$, $v^h = (\frac{1}{4}+\ell)^{\frac{1}{2}}$.

In standard monocentric city models, every consumer travels to the CBD and hence there are no intermediate transactions. To obtain this as a result, we borrow from Gale (1978) the concept of an "economy without trust". Gale (1978) used this assumption to eliminate intermediate barter transactions and to motivate the use of money. An analogue can be constructed here by introducing an "information cost" for each "intraregional" market activity at each location. Call $\Lambda^B(M)$ the number of intraregional marketplaces and assume:

A.6. (Information Cost) *The information cost function takes the following form:*

$$F[\Lambda^B(M)] = g\Lambda^B(M),\ 0 < g \ll \infty.$$

Without loss of generality, we let the unit information cost be the same as the unit set-up cost of each marketplace (g). Now (10) can be rewritten as

$$SW[g, \Lambda(M)] = SW^A + SW^B - g\Lambda(M),$$

where $\Lambda = \Lambda^A + \Lambda^B$, denoting the total number of marketplaces. With intraregional transactions (information) costs, A.5 implies that every agent would be better off by travelling to the CBD instead of having an "intraregional broker" transact for him/her, which leads to

THEOREM 4 *(Formation of a monocentric city). Under assumptions A.1-A.6, a μ-optimum has a unitary CBD to which every consumer travels.*

Under assumptions A.1-A.6, a monocentric city is endogenously generated. Neither interaction externalities nor market imperfections are required.

5. Competitive Prices

In contrast to Boruchov and Hochman (1977) in which the equilibrium allocation differs from the social optimal allocation, this paper establishes the second welfare theorem to assert that a μ-optimum has a competitive price support.

The *locational equilibrium condition* requires, in equilibrium, that there is no set of positive measure of consumers having the incentive to change their residential locations and that everyone is indifferent between being a transactor and not being a transactor. Specifically, for all B,

$$U^h(B) = U_0^h, \tag{11}$$

for a constant U_0^h. A *competitive spatial equilibrium* can then be defined as an allocation satisfying: (i) each representative consumer maximizes his/her utility subject to the budget and time constraints; (ii) the landlord maximizes his/her utility subject to the total rent collection; (iii) population identity (2), material balance conditions (3) and (4) and locational equilibrium condition (11) hold.

Notice that the μ-optimum which generates a monocentric city (see theorem 4) implies that everyone acts as a transactor, which is consistent with the locational equilibrium condition (11). To study the competitive price realization of such a μ-optimum, we begin by defining the concept of spatial equilibrium. Let p_i be the price for good i and $r(B)$ be the price for land over B. Denote type-h's payment for the set-up of marketplaces by τ^h and the landlord's total rent collection of land by R. Consider a starting allocation, $(\{\{c_i^{\triangle}(B)\}, q^{\triangle}(B)\}, \{\theta_i^{\triangle}\})$ and a "price" system $(\{p_i\}, \{r(B)\}, \{\tau^h\})$, denoting prices of mobile goods, immobile goods and travel. The budget constraint of a type-h consumer residing in interval B is then

$$\sum_i p_i c_i + r(B)q + \tau^h \leq \sum_i p_i c_i^{\triangle}(B) + r(B)q^{\triangle}(B). \tag{12}$$

The landlord's budget constraint is

$$\sum_i p_i c_i^L \leq R^{\triangle} = \sum_i p_i \theta_i^{\triangle} N^i e_i. \tag{13}$$

By construction, this starting rent collection is

$$R^{\triangle} = \sum_h \sum_{s^h} r(B)q^{\triangle}(B). \tag{14}$$

Since preferences are location-dependent, the second welfare theorem may fail to hold, as pointed out by Berliant, Papageorgiou and Wang (1989). So we propose a modified equilibrium concept. A *μ-equilibrium relative to a price system* as a transactively feasible μ-allocation, $(\{\{c_i(B)\}, q(B)\}, \{\theta_i\})$, together with a nonnegative price system, $(\{p_i\}, \{r(B)\}, \{\tau^h\})$ with $r(B)$ measurable, such that: for each (h, B), the allocation maximizes both consumers' and the landlord's utilities subject to (12) and (13) respectively and conditions (11) and (14) hold. A μ-optimum is said to have a *competitive price realization* if it is a μ-equilibrium allocation relative to a price system given the μ-optimal allocation as the starting allocation.

Start from a μ-optimal allocation $(\{\{c_i^*(B)\}, q^*(B)\}, \{\theta_i^*\})$, compactly denoted by $(\cdot^*)$. Substitution of (14) into (13) to eliminate $\{c_i^L\}$ (and the landlord's optimization problem) from the above system enables us to focus merely on the consumer's problem. Normalize $p_2^* = 1$. Then a candidate set of supporting "prices" is:

$$p_i^* = \frac{\partial u^h / \partial c_i(\cdot^*)}{\partial u^h / \partial c_2(\cdot^*)} \tag{15}$$

$$r^*(\cdot) = \frac{\partial u^h / \partial q(\cdot^*)}{\partial u^h / \partial c_2(\cdot^*)}. \tag{16}$$

We are now prepared to prove the following:

THEOREM 5 *(Competitive Equilibrium Realization of μ-optima or Second Welfare Theorem). Under assumptions A.1-A.6, the μ-optimum has a competitive price realization.*

Proof. Using (9), (12), (15), (16) together with A.1, it is apparent that for any transactively feasible μ-allocation, $(\{\{c_i^h\}, \{q^h\}, \{\theta_i\}\})$,

$$\begin{aligned}
&u^h(\cdot) + v^h(\cdot) \\
<&u^{h*}(\cdot) + \sum_i u_{c_i}^{h*}(\cdot)(c_i^h - c_i^{h*}) + u_q^{h*}(\cdot)(q^h - q^{h*}) + v^h(\cdot) \\
=&\eta^* - v^h(1) + [\sum_i p_i^* c_i^h + r^*(\cdot) q^h] u_{c_2}^{h*} + v^h(\cdot) \\
\leq&\eta^* - v^h(1) + [\sum_i p_i^* c_i^{h*} + r^*(\cdot) q^{h*}] u_{c_2}^{h*} - \tau^h + v^h(\cdot) = U^A(\cdot).
\end{aligned}$$

Under A.4-A.6, it can be shown that

$$\begin{aligned}
&U^A(\cdot) \\
\leq&\eta^* - v^h(1) + [\sum_i p_i^* c_i^{h*} + r^*(\cdot) q^{h*}] u_{c_2}^{h*} + v^{h*}(\cdot) \\
\leq&u^h(\cdot^*) + v^h(\cdot^*).
\end{aligned}$$

To complete the supporting price system, take $\tau^* = \frac{g}{N}$. Q.E.D.

Theorems 4 and 5 together imply that there is a μ-optimum with a competitive price realization generating a monocentric city with a unitary CBD in which everyone transacts. This enables us to study the characteristics of a "competitive" equilibrium formation of a monocentric city with an endogenously determined CBD without externalities or market imperfections, unlike nonprice interaction models, monopolistic competition models and oligopolistic interaction models.[16] Furthermore, we can focus only on the modified social planner's problem to avoid the computational complexity associated with solving for a spatial equilibrium.

6. Concluding Remarks

The paper develops a general equilibrium model of spatially competitive consumers with region-heterogeneous endowments in a linear city. The endogenous formation of a city emerges through the considerations of two types of interregional transactions costs – the utility cost of travel time and the set-up cost of marketplaces. In contrast to the previous work, both the market location and the residential location are endogeneously determined. A slightly modified social optimum exists under standard assumptions. With high set-up costs, the optimal market structure is concentrated and hence yields a unitary central business district (CBD). If a queueing time cost is further introduced, the unitary CBD may locate at the geographical center. Intermediate transactions are eliminated in an economy without trust and the standard monocentric city structure emerges. Further, the social optimum which generates a monocentric city has a competitive price realization under proper assumptions. Therefore, one may avoid the computational complexity of spatial equilibrium by solving the modified social planner's optimization problem.

Although we have shown that a modified social optimum exists and has a competitive price realization, we have only proved a second welfare theorem. The existence of a competitive spatial equilibrium cannot be proved by following the classical Arrow-Debreu framework when the market structure and the location of marketplaces are endogenously determined. Even if an equilibrium exists, it might not be optimal under standard assumptions in the general equilibrium literature. These issues remain open and might be addressed in future work.

Footnotes

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1. Mirrlees defined an optimum through social welfare maximization. However, such a social optimum may not exist when location enters the utility function [see Berliant, Papageorgiou and Wang (1989)].

2. He claimed that the concentrated Nash equilibrium is more efficient than any scattered spatial state of the economy, but it may not be Pareto optimal in an intertemporal sense (although this has not been proved formally and the definition of optimality has not been fully specified).

3. By and large, there are various ways to define the CBD: (i) The concentration of job opportunities [cf. Fujita (1986b), Mills (1967) and Imai (1982)]; (ii) the concentration of market (or transaction) activities [cf. Stuart (1970) and Baesemann (1977)]; and, (iii) the concentration of population [cf. Boruchov and Hochman (1977) and Papageorgiou and Smith (1983)].

4. In urban economics, consumers generally do not all live at the marketplace; the marginal benefit of immobile goods and the marginal cost of travelling (to and from the center) determine the optimal residential location.

5. In Boruchov and Hochman (1977), a social planning program under a partial equilibrium setup is proposed to generated a city center which minimizes aggregate transportation costs. The city center was found to be associated with the maximal population density due to the circularly symmetric spatial structure of the city. However, the "social optimum" obtained there is different from the "competitive equilibrium" since the optimum has

higher population concentration. Thus the question of equilibrium formation of a city in their model remains open.

6. Intuitively, under a uniformly distributed endowment structure and standard convexity assumptions, a transportation-cost minimizer will spread out all the economic activities over all locations.

7. Notice that this space structure can be used to characterize the geographical symmetry in standard monocentric city models by letting type-1 and type-3 consumers have identical preferences and endowments.

8. For instance, a banker may prefer to live in the center region, while a farmer may prefer to live in a suburban region.

9. A collection of sets $\{X^k\}$ is said to be an *i-partition* of X if X^k's are non-empty, interior-disjoint subsets of X and $\bigcup_k X^k = X$. For instance, $\{Z^h\}_{h \in H}$ forms an i-partition of Z.

10. A standard space of agents for each type of consumers is constituted by a compact set $A \subset \mathbf{R}$, a σ-algebra $\bar{\mathcal{A}}$ of Borel subsets of A, and Lebesgue measure on $\mathbf{R}$.

11. Formally, $S^h = \{s^h \mid s^h = \underline{s}^h + 2k\epsilon, s^h < \bar{s}^h, k \in \mathbf{N}\}$, where $(\underline{s}^1, \bar{s}^1, \underline{s}^2, \bar{s}^2, \underline{s}^3, \bar{s}^3) = (-\frac{3}{2} + \epsilon, -\frac{1}{2}, -\frac{1}{2} + \epsilon, \frac{1}{2}, \frac{1}{2} + \epsilon, \frac{3}{2})$.

12. Consider two intervals $[-1, 1]$ and $[-2, 2]$; they are not identical but the middle point distance between them is zero. Next consider $[-1, 0]$ and $[0, 1]$; they are not identical but the minimum distance between them is zero. So both distances are not well-defined metrics.

13. Formally, we denote the lower and the upper hemimetrics, δ_ℓ and δ_u, as $\delta_\ell(F_1, F_2) = \inf\{a \geq 0 \mid F_1 \subset a + F_2\}$ and $\delta_u(F_1, F_2) = \inf\{a \geq 0 \mid F_2 \subset a + F_1\}$, respectively. The Hausdorff metric on nonempty, closed sets, F_1 and F_2, in Z, is defined by $\mathcal{H} = \max\{\delta_\ell(F_1, F_2), \delta_u(F_1, F_2)\}$. Let d be the Euclidean metric and (Z, d) be a compact metric space. A theorem of Hausdorff tells us the set of nonempty closed subsets of Z together with the Hausdorff metric on that set forms a compact metric space. For further discussion, the reader is referred to Hildenbrand (1974, p.p. 16-17), Klein and Thompson (1984, p. 39) and Berliant and ten Raa (1988).

14. This is derived using the fact that $nv^h = (n - n^T)v^h(1) + n^T v^h(1 - \frac{1}{3}\delta) = nv^h(1) + n^T[v^h(1 - \frac{1}{3}\delta) - v^h(1)]$.

15. This social welfare cost can be transformed to a reduction in the aggregate social income in an expenditure function using a duality theorem. It can also be regarded as a lump-sum tax on consumers imposed by the social planner.

16. To understand the main features and conclusions of these models, the reader is referred to Fujita (1988). In terms of his survey, this paper can be classified as a comparative advantage model. Unlike other comparative advantage models, our model is, however, the only one generating the location of the CBD endogenously.

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