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Economic Research

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Working Paper No. 213
January 1990

University of

Rochester

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GENERAL PROPERTIES OF EQUILIBRIUM BEHAVIOR IN A CLASS OF ASYMMETRIC INFORMATION GAMES*

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January 1990

*Financial support from the National Science Foundation and the Sloan Foundation is gratefully acknowledged.

GENERAL PROPERTIES OF EQUILIBRIUM BEHAVIOR IN A CLASS OF ASYMMETRIC INFORMATION GAMES

1. Introduction

Game–theoretic analysis has become a common if not wholly accepted vehicle for generating predictions about economic behavior, especially in the field of industrial organization or "business strategy" [see the recent public debate between Fisher (1989) and Shapiro (1989) on game theory–induced advances in I.O., with the latter providing a survey of some of game theory's "greatest hits"]. Of particular interest for the current endeavor is the growth in game theory models with incomplete information, where these models relax the at–times implausible assumption that all participants possess all of the available payoff–relevant data. Incomplete information environments allow for the emergence of reputation–building, limit pricing, etc. as equilibrium phenomena, and in so doing highlight and endogenize the role of information transmission and control in microeconomic theory.

One of the often–heard complaints about game theoretic analysis, in fact, one of Fisher's, is that just about anything can happen with the right choice of game forms, in that the particulars of the game form analyzed can have a disproportionate influence on the equilibrium predictions relative to the underlying structure of the problem. Thus the predictions can lack generality, in that they may only hold for a subset of possible strategic scenarios. In additional shortfall of incomplete information games is the common existence of a multitude of equilibria, thereby seeming to require a selection from the set of equilibria in order to make any meaningful statements about the model.

In this paper we analyze a class of 2–player games with 1–sided incomplete information, where these games share a common structure or environment (up to labellings). For concreteness we describe this environment in a simple game involving a firm contemplating the takeover of another firm; however subsequently we argue how the structure of this model is equivalent to a variety of models currently in the literature. We derive results concerning equilibrium behavior in the takeover game, where these results are "game–free", in that the properties hold in any equilibrium of any game with the specified environment. Hence we avoid the criticisms mentioned above, in that the results are insensitive to the particulars of the game form or to the selection of a subset of equilibria from the game.

The results in this paper are derived from constraints on the informed player's behavior known as <u>incentive compatibility</u> constraints, where these constraints are a necessary feature of equilibrium behavior given the specified environment. While such constraints are commonplace and explicit in principal–agent models with adverse selection [eg. McAfee and McMillan (1987)], the typical game theory model has these constraints present implicitly as part of the equilibrium concept. What we attempt to do here is to separate out the effects on equilibrium behavior due solely to the nature of the informational asymmetry and the preferences, effects which show up in the incentive compatibility constraints, at the expense of the effects due to the specifics of the game form.

What should become apparent upon inspection of the results is that, while the incentive compatibility approach is a useful tool in generating a particular class of results, other types of results, eg. the effect on equilibrium behavior of the uninformed player's beliefs about the opponent's information, will remain a function of the game form selected. Therefore the incentive compatibility approach should not be viewed as a substitute for the more specific game form approach, but rather as a compliment, in that it distinguishes those results which hold "globally" in the space of game forms from those which may hold only "locally".

A final point is that the theoretical results derived in section 3 are not particularly novel, in that they can be deduced or inferred from previous work on, eg., bargaining models with incomplete information; indeed, Banks (1989a) derives almost identical results as those here from a bargaining model in a political science context. Rather, the goal in the current paper is to show the applicability of the results to a wide array of economic interactions. By establishing the features these interactions have in common, one can hopefully begin to understand the role of asymmetric information in economic behavior at a more general level.

2. The model

Consider the following scenario: firm 1 is a producer of a good, and firm 2 is considering taking over or buying out firm 1. Through some process the firms decide whether firm 2 takes over firm 1 and, conditional on a takeover, a purchase price; thus an outcome from the process consists of a probability of takeover $p \in [0,1]$ and a price $x \in \mathbb{R}$. Firm 1 possesses private information concerning its value: let $t \in [\underline{t},\overline{t}] \equiv T \subset \mathbb{R}_+$ denote the value of firm 1, and let $f(\ )$ be firm 2's common knowledge prior belief concerning t, where $f(t) > 0$ iff $t \in T$. Let $U_i(t,p,x)$ denote the expected utility to firm i from the outcome (p,x) given t is the value of firm 1.

We model the process generating the outcome (p,x) as a Bayesian game within which the firms interact [Harsanyi (1967–8), Myerson (1985)]. Let D_i denote the set of decisions available to firm i, where a decision d_i describes the actions taken by player i in every contingency, and let $G : D_1 \times D_2 \longrightarrow [0,1] \times \mathbb{R}$ be the outcome function. Hence associated with any pair of decisions (d_1,d_2) is an outcome $G(d_1,d_2)$ giving the probability of a takeover and, conditional on a takeover, the purchase price. Firm 1 is aware of the value t prior to the choice of a decision; thus a (pure) strategy for firm 1 in the Bayesian game is a function $\sigma_1 : T \longrightarrow D_i$, where $\sigma_1(t)$ is the decision by firm 1 if the value of the firm is t. Firm 2 possesses no private information, so a (pure) strategy for firm 2 is simply a selection $\sigma_2 \in D_2$. Therefore in the game defined by (D_1,D_2,G), $U_i(t,G(\sigma_1(t),\sigma_2))$ describes the expected utility of firm i generated by the strategy profile (σ_1,σ_2) when firm 1's value is t. A <u>Bayesian equilibrium</u> is then a pair of strategies (σ_1^*,σ_2^*) such that, i) for all $t \in T$, $\sigma_1^*(t)$ maximizes $U_1(t,G(d_1,\sigma_2^*))$, and ii) σ_2^* maximizes $\int U_2(t,G(\sigma_1^*(t),d_2))f(t)dt$.

Rather than positing a particular game form (D_1,D_2,G), and then deriving properties of the resulting Bayesian equilibria, we make use of the incomplete information environment to derive general results concerning equilibrium behavior associated with any game form. We do this by appealing to the notion of <u>incentive compatibility</u> constraints [d'Aspremont and Gerard–Varet (1979)], and by noting that such constraints are a necessary feature of optimizing behavior in incomplete information environments.

Given any game form (D_1,D_2,G), a pair of strategies (σ_1,σ_2) induces a pair of mappings

$p : T \to [0,1]$,

$x : T \to \mathbb{R}$,

describing the probability of takeover and the purchase price as a function of the value of the firm t, by $(p(t),x(t)) \equiv G(\sigma_1(t),\sigma_2)$. Let

$$\Omega = \{(p,x) : p : T \rightarrow [0,1],\ x : T \rightarrow \mathbb{R}\}$$

denote the set of all possible outcome functions generated by game forms (D_1,D_2,G). If the pair (σ_1,σ_2) constitutes a Bayesian equilibrium, then it must be that for all $t \in T$ $\sigma_1(t)$ is a (weakly) better response than $\sigma_1(t')$ for all $t' \in T$, since otherwise firm 1 could adopt the alternative strategy σ_1', defined as $\sigma_1'(\hat{t}) = \sigma_1(\hat{t})\ \forall\ \hat{t} \neq t$ and $\sigma_1'(t) = \sigma_1(t')$, receive the same expected utility for all $\hat{t} \neq t$ and receive a strictly higher payoff for t; but this violates the definition of Bayesian equilibrium. Hence a necessary condition for equilibrium behavior is that the functions (p,x) induced by the strategy pair be incentive compatible.

Definition. The pair $(p,x) \in \Omega$ is incentive compatible iff for all $t,t' \in T$,

$$U_1(t,p(t),x(t)) \geq U_1(t,p(t'),x(t')). \tag{1}$$

It is easily seen that incentive compatibility is also sufficient, ie. any incentive compatible pair (p,x) is the induced outcome from some Bayesian equilibrium of some game. To see this let $D_1 = T$, D_2 be arbitrary, and for all $d_2 \in D_2$ define the outcome function G as $G(t,d_2) = (p(t),x(t))$. Then if (p,x) is incentive compatible the strategy pair (σ_1,σ_2) where $\sigma_1(t) = t\ \forall\ t \in T$ is a Bayesian equilibrium and induces the outcome (p,x). Combining these two features gives us an application of the <u>revelation principle</u> [Dasgupta, Hammond, and Maskin (1979), Myerson (1979), Rosenthal (1978)]: for any Bayesian equilibrium (σ_1,σ_2) of any game form (D_1,D_2,G), there exists a "direct" game form (T,D_2,G') where (σ_1',σ_2), $\sigma_1'(t) = t\ \forall\ t \in T$, is a Bayesian equilibrium, and where the induced outcomes coincide. Hence without loss of generality our analysis of equilibrium behavior in arbitrary game forms (D_1,D_2,G) will proceed by characterizing the set of outcomes in Ω that are incentive compatible.

Note that since only firm 1 possesses private information, the incentive compatibility constraints are only manifested through 1's payoffs. Thus incentive compatibility is actually a feature of the best–response function of firm 1 since (as seen in the "direct" game above) the choice of firm 2's strategy is somewhat arbitrary. If on the other hand firm 2 possessed private information, an analogous set of constraints would hold for 2 as well [cf. Myerson and Satterthwaite (1983) for a model of the bargaining problem with two–sided incomplete information].

Suppose firm 1 is risk–neutral with respect to the value of the firm and the purchase price [Proposition 1 below does not depend on risk–neutrality, and dropping this assumption just makes the remaining Propositions needlessly cumbersome]. Then we can write $U_1(t,p,x)$ as

$$U_1(t,p,x) = p(t)x(t) + [1-p(t)]t. \tag{2}$$

In any strategy pair and game inducing the pair (p,x) firm 1's choice of decision is made contingent on the firm's value, so that whereas in complete information games von Neumann–Morgenstern utility representations are unique only up to positive linear transformations, here we can make such transformations "type–by–type" and preserve the optimality of any strategy. Hence a <u>utility–equivalent transformation</u> [Myerson (1985)] of (2), generated by simply subtracting t for each t, is

$$U(t,p,x) = p(t)[x(t) - t]. \tag{3}$$

Since (3) is somewhat simpler than (2) we will use (3) as our representation of firm 1's preferences in the analysis to follow. For all $t,t' \in T$ define

$$U(t,t',p,x) \equiv U_1(t,p(t'),x(t')) = p(t')[x(t') - t]. \tag{4}$$

Then incentive compatibility says that for all $t,t' \in T$, $U(t,p,x) \geq U(t,t',p,x)$, or

$$p(t)[x(t) - t] \geq p(t')[x(t') - t] \tag{5}$$

$$p(t')[x(t') - t'] \geq p(t)[x(t) - t']. \tag{6}$$

Proposition 1 below is derived from simple manipulations of inequalities (5) and (6), whereas Propositions 2 and 3 are derived from Proposition 1 and by noting that (5) holds with equality when $t' = t$.

In addition to incentive compatibility, we add the following constraint: in any "reasonable" game we would imagine that firm 1 never has to accept a purchase price less than the value of the firm, since there undoubtedly exists a strategy which insures a failure of a takeover, eg. always demanding everything. Therefore in any Bayesian equilibrium the induced purchase price $x(t)$ must be greater than or equal to t; this condition is known as (interim) individual rationality. Let $\Omega^* \subseteq \Omega$ denote the set of all individually rational and incentive compatible outcome functions.

In the next section we characterize elements of the set Ω^*, with the confidence that any results hold generally for any Bayesian equilibria in any takeover game with the above incomplete information environment. However it is also the case that the "reduced form" expressions in (2) and (3) describing firm 1's payoffs, and from which all of the results follow, are equivalent to those found in many applications other than the takeover game outlined here, and therefore the results found in the next section will hold in a variety of asymmetric information games. For instance, in bilateral bargaining games with 1–sided incomplete information firm 1 would be a seller with private information about the value of the object to him or, with an appropriate transformation, a buyer with privately known value. Then x denotes an agreed–upon price, and p denotes either a probability of trade or, if bargaining occurs over time, a discount multiple $p = e^{-r\tau}$, where τ is the time until trade occurs and r is the discount rate. For example, Sobel and Takahashi (1983) and Fudenberg, Levine, and Tirole (1985) analyze games where at discrete time intervals the uninformed player makes an offer, which the informed player either accepts or rejects, while Ausubel and Deneckere (1989a) study a bargaining game with alternating offers. However either scenario gives rise to the same reduced form expression for the expected utility of the informed player; therefore the results in the next section will hold regardless of whether the model has alternating offers, or has one side (informed <u>or</u> uninformed) making all the offers.

Other economics applications include the following:

i) In the Reinganum–Wilde (1986a) model of pre–trial bargaining, let firm 1 be the plaintiff in a civil suit, firm 2 be the defendant, and t be the amount transferred from the defendant to the plaintiff if a trial occurs net of the plaintiff's legal costs, where t measures the extent of the plaintiff's damages from some accident for which the defendant is liable; the assumption then is that this level of damages is known only to the plaintiff. The plaintiff makes a take–it–or–leave–it settlement demand, which the defendant either accepts, thereby averting a trial, or rejects. Then we can let x denote the plaintiff's settlement demand and p the defendant's chosen probability of trial. Alternatively, in Bebchuk (1984) the <u>defendant</u> makes the take–it–or–leave–it offer, which the plaintiff either accepts or rejects. In either case, however, (2) describes the preferences of the plaintiff, ie. the informed player, in the pre–trial bargaining game.

ii) In a stylized, continuous–type version of the Besanko–Spulber (1989a) model of anti–trust regulation, let 1 be a set of homogeneous firms deciding whether or not to collude, 2 be the antitrust authority, and suppose the firms' marginal cost is private information. Firms collusively set quantities and, upon observing the quantities chosen, the authority decides whether to sue the firms for antitrust violations. If a suit occurs the court is assumed to identify with certainty the firms' marginal cost, and then impose Cournot behavior; otherwise the market clears at the collusive quantities. Let t measure normal Cournot profits as a function of the firms' marginal cost, x be the profits generated by the collusive quantities, and p be the probability the regulator does not file a suit. Then (2) captures the firms' joint expected utility over outcomes.

iii) In the Besanko–Spulber (1989b) model of private anti–trust suits and treble damages, let now firm 2 be a representative consumer, t denote constant marginal cost, x the collusive price chosen by the homogeneous firms, and q the quantity of the good chosen by the consumer. Assuming legal costs are zero, after purchasing q units at price x, the consumer always brings a suit to recover damages due to collusion. Let β denote the probability the court finds the firms guilty of collusion, and $d > 0$ a damage multiple, where it is assumed $\beta d < 1$. The expected utility of the firms is then $q(x - t)(1 - \beta d)$; thus, setting $p = q(1 - \beta d)$, we have (3).

v) In the Banks (1988) version of the Baron–Besanko (1984) regulatory auditing model, let 1 denote a monopolist, 2 the regulatory authority, and t denote a monopolist's constant marginal cost. The monopolist chooses a price x, after which the authority decides to audit; auditing implies marginal cost pricing and hence zero profits, while no audit implies the market clears at the chosen price. Letting p denote the market clearing quantity at the chosen price times the probability of no audit, and we have (3).

v) In the Reinganum–Wilde (1986b) model of tax compliance, let 1 be a taxpayer, 2 the IRS, and let the taxpayer have privately–known income y. The taxpayer reports an income z, and the IRS decides whether to audit. If the IRS does not audit, the taxpayer pays a tax of αz, while if he is audited he pays $\alpha y + r$, where r is some fixed penalty [Reinganum and Wilde (1986b) assume a penalty proportional to y–z, but the fixed penalty makes the analogy more transparent]. Then if p is the probability of no audit, the taxpayer's expected utility is $p[y - \alpha z] + (1-p)[y - \alpha y - r]$, which by a utility–equivalent transformation becomes $p[\alpha y - \alpha z + r]$. Defining $t = -\alpha y$ and $x = -\alpha z + r$, and we have (3) for the taxpayer's preferences.

vi) In the Banks (1989b,c) model of political agenda control, let firm 1 be the bureaucrat, firm 2 the (median) voter, t be the status quo outcome, x the outcome proposed by 1, and p the probability the voter votes for the proposed outcome. For example, the bureaucrat may be a highway authority, and may possess better information about the current state of the roads. Assuming the bureaucrat prefers higher outcomes to lower and is risk–neutral, (2) describes the bureaucrat's preferences.

All of these models are in some sense "reduced–form–equivalent" from the perspective of the informed player, in that the functional form of this player's expected utility over the space of outcomes is the same. In this sense then (2) and (3) will capture the influence of the informational asymmetry on the equilibrium outcomes by identifying the differential incentives for the informed player to accept certain outcomes over others. Therefore, since all of the results in the next section are derived from these reduced form expressions, the results are common features of equilibrium behavior in <u>all</u> of these models. More generally, the results will be common features for <u>any</u> game form describing how the players interact, as long as the informed player's preferences over outcomes are as specified in either (2) or (3). Therefore, in speculating on extensions of these models to other, possibly more realistic game forms, one should consider whether the results of interest are related to those in the next section. If this is so then such extensions would be meaningless, since the same results will hold irrespective of the game form.

In addition, the Besanko–Spulber (1989a) and Baron–Besanko (1984) models employ a principal–agent framework in which the uninformed player, eg. the antitrust authority in Besanko–Spulber (1989a), has the ability to commit to a probability schedule prior the firms' choice of quantities, whereas the remaining models adopt a sequential rationality approach. Yet the reduced form expression for the informed player remains the same regardless of the opponent's ability to commit, implying that the results derived below are not a function of whether the underlying model takes a principal–agent or a sequential rationality viewpoint.

Finally, it is also the case that the results in the next section are related to equilibrium behavior in certain types of <u>complete</u> information games as well. This follows from the well–known equivalence of bargaining problems with 1–sided incomplete information and durable goods monopoly models [cf. Ausubel and Deneckere (1989b)]. Suppose there exists a continuum of infinitely–lived consumers indexed by $q \in [0,1]$; let $f(q)$ denote the reservation price of a type q consumer, where $f(\ )$ is strictly increasing. At discrete time intervals the monopolist announces the current price, which the consumers can either accept or not. If a type q consumer purchases the good at time τ for the price π, then $e^{-r\tau}[f(q) - \pi]$ denotes the consumer's payoff. Thus, setting $t = -f(q)$, $x = -\pi$, and $p = e^{-r\tau}$, and we get (3). Further, "The same mathematical model may depict either a continuum of actual consumers with different valuations or a single buyer with a continuum of possible valuations" [Ausubel and Deneckere (1989b:512)]. Therefore the results in the next section can in addition tell us something about consumer behavior faced with a durable goods monopolist.

3. Results

To simplify on notation we assume below that for all $t \in T$ the probability of a takeover, $p(t)$, is strictly positive; otherwise the associated purchase price $x(t)$ is meaningless and many of the statements would carry the qualifier "for all t such that $p(t) > 0$,..." Proposition 1 shows how the monotonicity of (3) with respect to the value t leads to a monotonicity in the equilibrium outcomes.

Proposition 1. If $(p,x) \in \Omega^*$, then

(i) $p(\ )$ is weakly decreasing on T;

(ii) $x(\ )$ is weakly increasing on T.

Proof. Let $t' > t$, and let $p = p(t)$, $p' = p(t')$, $x = x(t)$, $x' = x(t')$. (i) subtracting the RHS of (6) from the LHS of (5), and the LHS of (6) from the RHS of (5) gives

$$p[t' - t] \geq p'[t' - t], \tag{7}$$

implying $p \geq p'$. (ii) By (i) and the above assumption, $p \geq p' > 0$. This plus individual rationality gives

$$p[x' - t'] \geq p'[x' - t']. \tag{8}$$

Combining (8) and (6) gives

$$p[x' - t'] \geq p[x - t']. \tag{9}$$

Dividing by p then implies $x' \geq x$. QED

Proposition 1 identifies an "equilibrium" selection bias in the predicted outcomes of the takeover game: firms with a lower value are more likely to be taken over. Thus the distribution of values for firms observed to be taken over is not the same as the prior, but rather reflects the differential incentives for low value firms to acquiesce to such a takeover. In addition, conditional on a takeover the firm's purchase price is lower for lower value firms. Therefore in equilibrium there will exist a trade–off between achieving a higher price and realizing this price less often. Further, it is immediate that wherever x( ) is strictly increasing p( ) must be strictly decreasing, and vice versa.

Obviously, analogous results hold in the other "payoff–equivalent" models as well. For instance, we can conclude that,

i) plaintiffs with greater damages are more likely to end up in court, but receive a higher settlement if a trial is averted;

ii) firms with higher Cournot profits are more likely to be sued for anti–trust violations, but receive greater profits if not sued;

iii) taxpayers with higher incomes report higher incomes, but are still more likely to be audited by the IRS; and

iv) consumers with a higher valuation for a monopolist's durable good purchase the good sooner but at a higher price.

Again, the importance of these results is not their existence but their generality; namely, they will hold regardless of any assumptions as to how the players interact.

Monotonicity of the functions p( ) and x( ) implies differentiability almost everywhere [Royden (1968)], and therefore U(t) and U(t,t′) will be differentiable almost everywhere as well. Now as noted in Section 2 the incentive compatibility constraint $U(t) \geq U(t,t')$ holds with equality at $t' = t$. Therefore, wherever U(t,t′) is differentiable, it must satisfy

$$\left.\frac{\partial U(t,t')}{\partial t'}\right|_{t'=t} = 0, \tag{10}$$

or

$$\frac{\partial p}{\partial t}\,[x(t) - t] + \frac{\partial x}{\partial t}\,p(t) = 0, \tag{11}$$

since otherwise t could mimic a type t′ arbitrarily close t and receive a strictly higher payoff. We can think of (11) as a "local" incentive compatibility constraint, where this constraint is a necessary condition for an incentive compatible outcome.

Proposition 2. If $(x,p) \in \Omega^*$, then U( ) is strictly decreasing and continuous on T, and where $\partial U/\partial t$ exists it satisfies

$$\frac{\partial U}{\partial t} = - p(t). \tag{12}$$

Proof. If $U(t') \geq U(t)$, then

$$p'[x' - t'] \geq p[x - t]. \tag{13}$$

If $t' > t$ and $p' > 0$, then

$$p'[x' - t] > p'[x' - t']. \tag{14}$$

Combining these gives

$$p'[x' - t] > p[x - t], \tag{15}$$

which contradicts (5). Continuity follows from an obvious argument.

To see (12) holds, differentiate U(t) with respect to t:

$$\frac{\partial U(t)}{\partial t} = \frac{\partial p}{\partial t}[x(t) - t] + p(t)[\frac{\partial x}{\partial t} - 1]. \tag{16}$$

Substituting (11) into (16) gives (12). QED.

Equation (12) is simply the "envelope theorem" in another context: given the functions (p,x), each type is maximizing $U(t,t')$ over the choice of t'; if (p,x) is incentive compatible, ie. derived from equilibrium behavior, then for all types the optimal choice is $t' = t$. Local incentive compatibility then implies that the indirect effects of varying the "parameter" t on U( ) through the functions (p,x) vanish, leaving simply the direct effect. Hence the marginal increase in firm 1's equilibrium expected utility as the true value of the firm increases is simply equal to minus the probability of a takeover.

And, as before, analogous statements hold for the payoff–equivalent models, eg. the marginal increase in an individual's expected full tax payment (which includes the possibility of a fine following an audit) as a function of an increase in true tax liability is simply equal to the probability the individual is not audited.

This envelope theorem result is common in optimal contracts in a principal–agent framework with adverse selection [eg. McAfee and Mcmillan (1987)], and also turns out to be useful in applying various equilibrium refinements in particular signaling games [eg. Banks (1989c)]. The reason for the former is immediate, since by the revelation principle the principal can without loss of generality be restricted to "direct" contracts where the agent simply reports his type; the above incentive compatibility conditions are then constraints on the principal's optimization problem. With regard to the latter, most of the refinements, eg. universal divinity [Banks and Sobel (1987)], measure the potential gains from a type deviating to a previously unsent signal relative to the type's equilibrium payoff; the criterion of universal divinity then places weight only on those types most likely to deviate. For example, in the Reinganum–Wilde (1986a) model of pre–trial bargaining outlined in Section 2, let $U(t)$ denote the plaintiff's expected utility in some equilibrium and let x' be an out–of–equilibrium settlement demand. Then if $x' > t$, $\theta(t,x) \equiv U(t)/(x' - t)$ describes the probability of accepting the demand x' which makes a type t plaintiff indifferent between staying along the equilibrium path and deviating to x'. Universal divinity then says that the belief at x' should place positive probability on t' only if $t' \in \underset{t}{\operatorname{argmin}}\ \theta(t,x')$. Since by Proposition 2 above $U(\ )$ is differentiable almost everywhere and continuous, so will be $\theta(\ ,x')$, so that solving for $\underset{t}{\operatorname{argmin}}\ \theta(t,x')$ involves inspection of $\partial\theta/\partial t$. But from Proposition 2 this is simply

$$\frac{\partial\theta}{\partial t} = \frac{-p(t)(x' - t) + p(t)(x(t) - t)}{(x' - t)^2}, \tag{17}$$

so that $\mathrm{sgn}\{\partial\theta/\partial t\} = \mathrm{sgn}\{x(t) - x'\}$. Hence the determination of the type most likely to deviate involves a simple inspection of the equilibrium demands. For example, if in the equilibrium under consideration $x(\ )$ has a jump discontinuity at $\hat{t}$, and $x' \in (x(\hat{t}-\epsilon),x(\hat{t}+\epsilon))$, then $\theta(\ )$ is decreasing (resp. increasing) for t below (resp. above) $\hat{t}$. By continuity, then, $\hat{t} = \operatorname{argmin}_t \theta(t,x')$, and universal divinity requires the defendant to believe the plaintiff's type is $\hat{t}$ at the out–of–equilibrium demand x'.

The next result shows that local incentive compatibility, $p(\ )$ decreasing, and $U(\ )$ continuous and differentiable completely characterize the notion of "global" incentive compatibility in this model.

Proposition 3. If $(x,p) \in \Omega$ is such that $p(\ )$ is weakly increasing, $U(\ )$ is differentiable almost everywhere and continuous, and local incentive compatibility (11) holds, then (p,x) is incentive compatible.

Proof. Rewrite $U(t,t')$ as

$$U(t,t') = U(t') - p(t')[t - t']. \tag{18}$$

Since (18) is an identity in t', the derivatives of both sides are equal, implying

$$\frac{\partial U(t,t')}{\partial t'} = \frac{\partial U}{\partial t'} - \frac{\partial p}{\partial t'}[t - t'] + p(t'). \tag{19}$$

From Proposition 2 local incentive compatibility implies (12); plugging this into (19) gives

$$\frac{\partial U(t,t')}{\partial t'} = -\frac{\partial p}{\partial t'}[t - t']. \tag{20}$$

Thus where $\partial U(t,t')/\partial t$ exists p( ) decreasing implies $\text{sgn}\{\partial U(t,t')/\partial t'\} = \text{sgn}\{t - t'\}$, so $U(t,t')$ is increasing in t' on $[\underline{t},t)$ and decreasing on $(t,\bar{t}]$. This plus the continuity of U( ), which implies the continuity of $U(t,t')$ at $t' = t$, implies $t = \underset{t'}{\text{argmax}}\, U(t,t')$. QED.

Therefore Propositions 1,2, and 3 show that incentive compatibility is equivalent to local incentive compatibility, p( ) decreasing, and U( ) continuous and differentiable, thus justifying a qualified "first-order approach" to the characterization of equilibrium behavior in these models.

From the envelope theorem result we can characterize the "equilibrium" relationship between the functions p( ) and x( ), in that given p( ) we can solve for the function x( ) such that $(p,x) \in \Omega^*$. Differentiating both sides of (12) we get

$$U(t) = U(\underline{t}) - \int_{\underline{t}}^{t} p(\hat{t})d\hat{t}. \tag{21}$$

solving for x( ) as a function of p( ) we get

$$x(t) = t + \frac{U(\underline{t}) - \int_{\underline{t}}^{t} p(\hat{t})d\hat{t}}{p(t)}. \tag{22}$$

Thus given a decreasing function $p : T \rightarrow [0,1]$ we can solve (22) (up to a constant) for the function $x : T \rightarrow \mathbb{R}$ such that (x,p) is incentive compatible. From Proposition 2, individual rationality reduces to $x(\bar{t}) \geq \bar{t}$ as long as (p,x) is incentive compatible, and for types where p( ) is not differentiable the value of x( ) follows from the requirement of U( ) continuous. Therefore adding these constraints to (22) and we can solve for the function x which "rationalizes" p, in that $(x,p) \in \Omega^*$. [of course the reverse process works as well, namely, given x we can solve for p.] Thus, whereas for a specified game form one can solve for both x( ) and p( ) through the computation of equilibrium behavior, without a game form one get get "halfway" there, in the sense of being able to solve for one function given the other.

Incentive compatibility also tells us something about whether in equilibrium p( ) decreases faster than x( ) increases. From (11) we see that local incentive compatibility and p( ) decreasing implies

$$\frac{\partial x}{\partial t} p(t) + \frac{\partial p}{\partial t} x(t) = \frac{\partial p}{\partial t} t < 0, \tag{23}$$

implying (since $T \subset \mathbb{R}_+$)

$$\frac{\partial x}{\partial t} \frac{t}{x(t)} < \left| \frac{\partial p}{\partial t} \frac{t}{p(t)} \right| . \tag{24}$$

The LHS is simply the elasticity ϵ_X of x( ) with respect to the value t, while the RHS is ϵ_p, the elasticity of p( ). Thus we have shown:

Proposition 4. If $(x,p) \in \Omega^*$, then $\epsilon_x < \epsilon_p$, ie. a one percent increase in the firm's value leads to a greater percentage decrease in the probability of a takeover than the percentage increase in the purchase price.

4. Conclusion

This paper has examined the effect of asymmetric information on equilibrium behavior in a class of economic games. We have seen how the monotonicity properties of the informed player's expected utility lead invariably to a monotonicity in the equilibrium outcomes, thus generating a selection bias in the observed prevalence of outcomes as a function of the private information. In addition, we established an envelope theorem result for the informed player's expected utility, and characterized the "equilibrium" relationship between (in the takeover game) the probability of takeover and the price of a takeover.

However, as mentioned in the Introduction, the main conclusion of this paper is not that one can construct a game form which will generate these results as equilibrium predictions. Rather, it is that certain types of results hold regardless of the game form, and for these results therefore it is not necessary to specify in detail how the players interact. The results based on incentive compatibility identify features of equilibrium behavior that are functions of the underlying structure of the problem, not of the process through which the players resolve the relevant issues.

Extending the structure of the problem in certain ways, such as relaxing risk–neutrality, will not substantially effect the results. Other, more meaningful extensions, include deriving similar results for other classes of preferences, and identifying games generating these preferences, and allowing more players to possess private information. These will hopefully be the objects of analysis in future research.

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