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An important and apparently robust empirical feature of cyclical fluctuations is that increases in unemployment seem often to be associated with "unusually large sectoral shifts" in the composition of employment [Lilien (1982), p. 779]. Lilien's (1982) observation, which is based on U.S. data, is confirmed by Rogerson (1986), who examines a variety of European economies and reaches the same conclusion. To date the leading explanation for this phenomenon is the one proposed by Lilien (1982): changes in labor demand across sectors (which could occur for a variety of reasons) require the inter–sectoral relocation of workers. However such relocation involves job search and search related unemployment. Thus there will be higher unemployment when large sectoral shifts in the composition of employment occur. This intuition is formalized by Rogerson (1986, 1987) and Hamilton (1988).

However, there are some reasons to want to explore other explanations for the Lilien/Rogerson observations. One is that it is by no means obvious that a search theoretic explanation of these observations is consistent with evidence provided by panel data sets. For instance, according to Murphy and Topel (1987, p. 16), "industry changes account for a minor, and virtually constant, amount of total unemployment," and the "evidence is that cyclical and secular changes in unemployment are overwhelmingly accounted for by varying incidence among persons who do not change industry." This makes it natural to seek explanations for the Lilien/Rogerson observation that allow unemployment to vary cyclically among groups of workers who remain attached to the same labor market sector.

Secondly, there have been few attempts to produce a model consistent with the Lilien/Rogerson findings, and that is also consistent with a variety of other cyclical observations at the same time. As an example, Lilien (1982) also reported finding a Phillips curve: in particular, that (unanticipated) increases in the money growth rate had a significant effect in reducing unemployment (while controlling for the sectoral dispersion of employment). However, there appears to be no existing model designed to explain the Lilien/Rogerson observation and that incorporates monetary considerations. A model intended to do so should also be consistent with the finding of Cooley and Hansen (1989) that, cross–sectionally, economies with high rates of secular money growth have high rates of unemployment.

Moreover, explanations for cyclical fluctuations that rely on changing the sectoral composition of employment should presumably be consistent with other aspects of observed inter–sectoral behavior over the cycle. For instance, it appears to be well–established that sectoral wage dispersions decline at cyclical peaks [Reder (1962), Krueger and Summers (1987)]. It seems natural to try to confront observations on the cyclical behavior of sectoral employment and relative wage rates simultaneously. Also, one might want to investigate whether it is possible for changes in the sectoral composition of employment to drive employment and hours fluctuations in a way that is consistent with the finding of Christiano and Eichenbaum (1988) that average productivity and hours are negatively correlated.

The present paper proposes an alternative model of the sectoral composition of employment and cyclical fluctuations, which is based on the following idea. At each date there is a heterogeneous work force producing multiple consumption goods. Workers differ in their ability to produce different goods, with ability being privately observed by workers. Then employers in different sectors will want to infer the abilities of different workers. To do so, they will use different dimensions of the employment relationship (wages and hours) to induce workers to sort themselves according to relative productivity. In general, hours restrictions (unemployment) in certain sectors will be an important component of this self–selection mechanism. Thus unemployment results from the assumption of private information. Moreover, this particular self–selection mechanism will be most significant when wages differ most across sectors, so that wage dispersions and unemployment can easily be negatively correlated. Finally, the model is completed by postulating a randomly fluctuating demand for different products. This will require that sectoral employment shares also fluctuate, so the model can be used to investigate when high unemployment is associated with declining employment shares for high wage sectors [Lilien (1982), p. 779].

In proposing this model, the paper adopts a formulation of self–selection into sectors by workers which is essentially a simple version of the Roy (1951) model [as formalized by Heckman and Honore (1989)], with the additional feature of private information to generate unemployment.[1] Thus the analysis follows standard models of the allocation of workers to sectors of employment. The underlying idea draws heavily on Williamson (1988) as well, which also considers a sorting model of sectoral employment composition and cyclical fluctuations. However, in Williamson (1988) incentive constraints are "just binding" in equilibrium. Thus unemployment does not function as a self–selection mechanism, but instead arises because labor is indivisible. Here labor is perfectly divisible and freely mobile, and unemployment arises as a device for inducing appropriate self–selection in the presence of informational asymmetries.

The results obtained are as follows. First, unemployment is observed even if workers can relocate instantaneously and labor is perfectly divisible. [In Rogerson (1987), Hamilton (1988), and Williamson (1988), unemployment is generated either by the assumption that it takes time to change sectors, or the assumption that labor is indivisible.] A Phillips curve relation is also observed: unemployment rates are low when expected inflation is high (temporarily). However, secular increases in inflation reduce employment unambiguously. Depending on parameters, the model is consistent with the Lilien/Rogerson observation that unemployment is high when the high wage sector's share of total employment declines. The model is also consistent with wage dispersions that decline at cyclical peaks, and with a negative correlation between average productivity and hours. Finally, as shocks to the composition of goods demand become more persistent (so that changes in the sectoral composition of employment occur less often), the amplitude of cyclical fluctuations increases. (Thus changes are more severe when they do occur.)

The paper proceeds as follows. Section I lays out the physical characteristics of the environment and discusses trading arrangements. Section II describes an equilibrium when information is perfect. In section III an equilibrium is derived under the assumption of private information, and existence issues are considered. Section IV considers some properties of this equilibrium, and section V discusses possible extensions. Section VI concludes, and discusses some features of the model specification.

I. The Model

The economy consists of an infinite sequence of two period lived, overlapping generations. Each generation is identical in size and composition, and contains a continuum of agents. Throughout periods are indexed by t=1,2,....

At each date there exist two generations, then, one young and one old. For simplicity, it is assumed that agents supply labor when young, and are retired and engage in all consumption when old.[2]

There are two non–storable consumption goods at each date, denoted by x and y. It is assumed that the technologies for producing these goods are owned by old (retired) agents, who are also referred to as firms. Any young worker employed by a producer of x can produce one unit of x per unit time. However, workers differ in their ability to produce y. In particular, workers are divided into two types, indexed by $i \in \{1,2\}$. A type i worker when young can produce π_i units of y, with $\pi_2 > \pi_1$. Worker type is private information, and a fraction $\theta \in (0,1)$ of young workers are of type 1.

Let x and y denote the quantities of good x and y consumed by an agent in old age, and let $L \in [0,1]$ denote labor supply in youth. Then the preferences of a type i worker are given by

$$(1) \qquad u_i(x,y,L;s) = x^{\alpha(s)}y^{[1-\alpha(s)]} - \beta_i L; \ \beta_i \geq 0,$$

where s is a random variable realized at the beginning of old age. Thus s represents shocks to the composition of goods demand, and is common to all old agents. It is assumed that s evolves according to a two–state Markov chain, that $\alpha(s) \in (0,1)\ \forall$ s, and that $\alpha(1) > \alpha(2)$.

Letting s denote the current period state and s' "next period's" state, $q(s) = \text{prob}[s' = 1: s]$; i.e., $q(s)$ is the probability that $s' = 1$ given that the current state is s.[3]

In addition to the two goods and labor, the initial old are assumed to be endowed with a per capita stock of $M_0 > 0$ units of fiat money. There is also a government, which purchases both goods in each period and pays for these purchases by printing money. The per capita money stock evolves according to $M_{t+1} = (1 + \sigma)M_t$, with $\sigma \geq 0$ given and constant.

Good x is chosen as the numeraire at each date. Let $p_y(s)$ denote the relative price of y in terms of x if the current period state is s, let $p_t(s)$ be the dollar price of good x at t in state s, let $w_x = 1$ be the real wage paid to workers engaged in production of x, and let $w_{yi}(s)$ be the real wage paid to type i workers engaged in production of y in state s, where all real wages are measured in units of x. If employers engaged in production of y can infer type, zero profits requires that $w_{yi}(s) = p_y(s)\pi_i$; $i = 1,2$.

It will be necessary to make several assumptions on parameter values, which are now stated. First, it is assumed that

$$\text{(A.1)} \qquad \beta_2 > \beta_1 \geq \beta_2(\pi_1/\pi_2).$$

The first inequality in (A.1) is necessary for self–selection to occur; the second is a simplifying assumption. The assumption that $\beta_2 > \beta_1$ states that type 2 workers have a higher opportunity cost of leisure than type 1 workers. Under the assumption that $\pi_2 > \pi_1$, and if $p_y(s) > 1/\pi_2$, type 2 workers are more productive than type 1 workers in the marketplace. If the alternative to market production is home production, for instance, then it seems natural to assume that workers who are more productive in the marketplace also have a higher opportunity cost of leisure.

Second, it is assumed that

$$\text{(A.2)} \qquad \theta[1-\alpha(s)] > \alpha(s)(1-\theta)\pi_1/\pi_2 \ \forall s.$$

(A.2) implies that incentive constraints always bind in the sequel, and that $p_y(s) > 1/\pi_2$. [Consequences of relaxing (A.2) are discussed in section V.] Third, it is assumed that

$$(1+\sigma)\pi_1(\beta_2 - \theta\beta_1)/(1-\theta)\pi_2 \geq q(s)\psi(1)\pi_1^{[1-\alpha(s)]} + [1-q(s)]\psi(2)\pi_1^{[1-\alpha(2)]} > (1+\sigma)\beta_1 \;\; \forall s \tag{A.3}$$

where

$$\psi(s) = \alpha(s)^{\alpha(s)}[1-\alpha(s)]^{[1-\alpha(s)]}.$$

The first inequality in (A.3) guarantees the existence of an equilibrium in pure strategies for firms, and will hold for θ sufficiently near one. The second inequality in (A.3) guarantees positive participation by all young agents in labor markets.

Finally, in the analysis it is assumed that there are no markets in state contingent claims. If it is assumed that the current period state is realized at each date before young agents are born, this assumption is innocuous. In particular, it is easy to check that, in the equilibrium derived below, all old agents have identical marginal rates of substitution across states.

A. Goods Demand

To begin, consider the goods demand of a representative old agent at t. This agent worked when young, and accumulated real balanced to be spent when old. Let z(s) denote the inherited level of real balances if the current period state is s. Then, taking z(s) as given, an old agent chooses x and y to maximize $x^{\alpha(s)}y^{[1-\alpha(s)]}$ subject to $x + p_y(s)y \leq z(s)$. The solution to this problem sets $x = \alpha(s)z(s)$ and $y = [1 - \alpha(s)]z(s)/p_y(s)$. Therefore the utility of an old agent in state s who inherits real balances z(s) is

$$[\alpha(s)z(s)]^{\alpha(s)}\{[1-\alpha(s)]z(s)/p_y(s)\}^{[1-\alpha(s)]} = \psi(s)p_y(s)^{-[1-\alpha(s)]}z(s).$$

Of course in the aggregate, real balances held by old agents at t sum to $M_{t-1}/p_t(s)$, since the old at t hold the entire t−1 money supply. Thus in state s old agents demand $\alpha(s)M_{t-1}/p_t(s)$ units of good x, and $[1-\alpha(s)]M_{t-1}/p_t(s)p_y(s)$ units of good y.

Let x_t^g and y_t^g denote government purchases of x and y at t. The government budget constraint requires that

$$x_t^g + p_y(s)y_t^g = (M_t - M_{t-1})/p_t(s) = \sigma M_{t-1}/p_t(s).$$

For simplicity, it is assumed that the government allocates its expenditures between x and y in the same way as private agents do at each date. Thus $x_t^g = \alpha(s)\sigma M_{t-1}/p_t(s)$ and $y_t^g = [1-\alpha(s)]\sigma M_{t-1}/p_t(s)p_y(s)$. Then total demand for x by old agents and the government at t is $\alpha(s)(M_{t-1} + \sigma M_{t-1})/p_t(s) = \alpha(s)M_t/p_t(s)$, and total demand for y is $[1-\alpha(s)]M_t/p_t(s)p_y(s)$.

B. Goods Market Equilibrium

Let $L_x(s)$ denote the labor supply of workers who produce x in state s, and let $L_{yi}(s)$ be the hours worked in state s by type i agents engaged in the production of y. Finally, let $\lambda(s)$ denote the fraction of type 1 workers engaged in the production of x. Then the supply of x in state s is $\theta\lambda(s)L_x(s)$, while the supply of y is $\theta[1-\lambda(s)]\pi_1L_{y1}(s) + (1-\theta)\pi_2L_{y2}(s)$.[4] Therefore goods market clearing requires that, $\forall$s, t,

(2) $\alpha(s)M_t/p_t(s) = \theta\lambda(s)L_x(s)$

(3) $[1-\alpha(s)]M_t/p_t(s)p_y(s) = \theta[1-\lambda(s)]\pi_1L_{y1}(s) + (1-\theta)\pi_2L_{y2}(s).$

C. Labor Supply Behavior

Consider the expected utility of a young worker who supplies L units of labor and earns the wage rate w at t. This agent accumulates real balances equal to wL in order to consume when old. Then, at t+1, the inherited real balances of this individual are equal to $wL[p_t(s)/p_{t+1}(s')]$ if the time t state is s and the time t+1 state is s′. Therefore the utility of this agent in state s′ at t+1 is $wL[p_t(s)/p_{t+1}(s')]\psi(s')p_y(s')^{-[1-\alpha(s')]}-\beta_i L$. As of t, his expected utility from supplying L units of labor at the wage rate w is

$$\text{(4)} \qquad \{E_t[p_t(s)/p_{t+1}(s')]\psi(s')p_y(s')^{-[1-\alpha(s')]}w - \beta_i\}L,$$

where E_t denotes an expectation conditional on time t information. Thus agents of type i evaluate employment contracts (values of w and L) using the expression in (4). When they do so, of course, they take the stochastic processes $p_t(s)$ and $p_y(s)$ as given.

D. Firm Behavior

It is assumed that firms engaged in the production of x announce contracts consisting of a real wage rate w_x and an employment level $L_x(s)$. Similarly, firms engaged in producing y announce contracts for type i agents consisting of pairs $[w_{yi}(s), L_{yi}(s)]$ in state s. (Announcements are made after s is realized.) In making their announcements firms take the announcements of all other firms as given.

II. Equilibrium: Full Information

An equilibrium in now briefly described under the assumption that each worker's type is publicly known. In this case an equilibrium is a set of stochastic processes $p_t(s)$, $p_y(s)$, $\lambda(s)$, and a set of contracts $[w_x, L_x(s)]$ and $[w_{yi}(s), L_{yi}(s)]$ such that (i) equations (2) and (3) hold, and (ii) no firm has an incentive to offer an alternative set of contracts in the presence of the contracts $[w_x(s), L_x(s)]$ and $[w_{yi}(s), L_{yi}(s)]$.

Under the assumption of full information, competition among firms implies that all firms earn zero profits; i.e., $w_x = 1$, $w_{yi}(s) = p_y(s)\pi_i$. Moreover, competition among firms for workers implies that $L_x(s)$ and $L_{y1}(s)$ must be chosen to maximize

$$\{E_t[p_t(s)/p_{t+1}(s')]\psi(s')p_y(s')^{-[1-\alpha(s')]} - \beta_1\}L$$

and

$$\{E_t[p_t(s)/p_{t+1}(s')]\psi(s')p_y(s')^{-[1-\alpha(s')]}p_y(s)\pi_1 - \beta_1\}L$$

respectively. Define

$$\Phi(s) \equiv E_t\psi(s')p_y(s')^{-[1-\alpha(s')]}[p_t(s)/p_{t+1}(s')].$$

Then $L_x(s) = 1$ if $\Phi(s) > \beta_1$, and $L_{y1}(s) = 1$ if $\Phi(s)\pi_1 p_y(s) > \beta_1$. And apparently, if $\Phi(s) > 1$ and $\lambda(s) \in (0,1)$,

$$(5) \qquad \pi_1 p_y(s) = 1$$

must hold. Similarly, $L_{y2}(s)$ must be chosen to maximize $[\Phi(s)p_y(s)\pi_2 - \beta_2]L$. If (5) is satisfied and $\Phi(s) > \beta_1$, then (A.1) implies that $\Phi(s)p_y(s)\pi_2 = \Phi(s)\pi_2/\pi_1 > \beta_2$, so $L_{y2}(s) = 1$ as well.

Provisionally assuming that $\Phi(s) > \beta_1$ and $\lambda(s) \in (0,1)\ \forall$ s, an equilibrium is now derived. In this case (5) holds, so that (2) and (3) reduce to

$$(2') \qquad \alpha(s)M_t/p_t(s) = \theta\lambda(s)$$

(3′) $[1-\alpha(s)]M_t/p_t(s) = \theta[1-\lambda(s)] + (1-\theta)\pi_2/\pi_1.$

Solving (2′) and (3′) for $\lambda(s)$ yields

(6) $\lambda(s) = \alpha(s)[1 + (1-\theta)\pi_2/\theta\pi_1],$

and $\lambda(s) \in (0,1)$, by assumption (A.2). Moreover, using (2′) for t and t+1 gives

$$p_t(s)/p_{t+1}(s') = \left[\frac{\alpha(s)}{\lambda(s)}\right]\left[\frac{\lambda(s')}{\alpha(s')}\right](M_t/M_{t+1}) = (1+\sigma)^{-1}$$

where the latter equality follows from (6). Thus

$$\Phi(s) = (1+\sigma)^{-1}E_t\psi(s')\pi_1^{[1-\alpha(s')]} = (1+\sigma)^{-1}\{q(s)\psi(1)\pi_1^{[1-\alpha(1)]}$$

$$+ [1-q(s)]\psi(2)\pi_1^{[1-\alpha(2)]}\} > \beta_1,$$

where the inequality follows from (A.3). Therefore, under full information there is an equilibrium with $L_x(s) = L_{y1}(s) = L_{y2}(s) = 1\ \forall\ s$.

III. Equilibrium: Private Information

A. Contracts

When worker type is private information, producers of y can either attempt to induce self–selection (offer contracts such that $[w_{y1}(s), L_{y1}(s)] \neq [w_{y2}(s), L_{y2}(s)]$) or not. If self–selection is to occur contract announcements must be incentive compatible, or satisfy the conditions

(7) $[\Phi(s)w_{y1}(s) - \beta_1]L_{y1}(s) \geq [\Phi(s)w_{y2}(s) - \beta_1]L_{y2}(s)$

(8) $[\Phi(s)w_{y2}(s) - \beta_2]L_{y2}(s) \geq [\Phi(s)w_{y1}(s) - \beta_1]L_{y1}(s)$

$\forall s$. And, of course, if $\lambda(s) \in (0,1)$,

(9) $[\Phi(s)w_x - \beta_1]L_x(s) = [\Phi(s)w_{y1}(s) - \beta_1]L_{y1}(s)$

must hold. Finally, following Rothschild and Stiglitz (1976), an additional restriction is imposed on announced contracts: each contract is required to earn non–negative profits given the workers it attracts. Then, if self–selection occurs, $w_{yi}(s) \leq p_y(s)\pi_i$ must hold; i=1,2.

An equilibrium is now a set of stochastic processes $p_t(s)$, $p_y(s)$, and $\lambda(s)$, and a set of announced contracts such that (i) equations (2) and (3) hold; (ii) no firm has an incentive to offer an alternative set of contracts, with any offers subject to (7), (8), and the non–negative profit condition; and (iii) workers choose their most preferred contract from among the set of announced contracts.

The arguments given by Rothschild and Stiglitz (1976) can be repeated exactly to establish that (a) self–selection must occur in any non–trivial equilibrium, and (b) equilibrium profits are zero. Then, in any non–trivial equilibrium, $w_x = 1$ and $w_{yi}(s) = p_y(s)\pi_i$. Furthermore, competition among firms for workers implies that $L_x(s)$ and $L_{y1}(s)$ must maximize $[\Phi(s) - \beta_1]L$ and $[\Phi(s)\pi_1 p_y(s) - \beta_1]L$ respectively. Then, if $\Phi(s) > \beta_1$, $L_x(s) = 1$, and if $\lambda(s) \in (0,1)$, (5) must hold as well while $L_{y1}(s) = 1$.

Finally, if a set of Nash equilibrium contracts exists in pure strategies, $L_{y2}(s)$ must be chosen to maximize the expected utility of type 2 workers given that $w_{y2}(s) = p_y(s)\pi_2 = \pi_2/\pi_1$ [by (5)], given the contract $[w_{y1}(s), L_{y1}(s)] = (1,1)$, and subject to the self–selection constraint (7). Clearly if $\Phi(s) > \beta_1$ (7) must be binding, so $L_{y2}(s)$ is chosen to maximize $[\Phi(s)\pi_2/\pi_1 - \beta_2]L$ subject to

(10) $[\Phi(s)\pi_2/\pi_1 - \beta_1]L_{y2}(s) = \Phi(s) - \beta_1.$

Since $\beta_2\pi_1/\pi_2 \leq \beta_1 < \Phi(s)$, the solution is given by (10′):[5]

$$(10')\qquad L_{y2}(s) = \frac{\Phi(s) - \beta_1}{\Phi(s)\pi_2/\pi_1 - \beta_1} < 1.$$

As in Rothschild–Stiglitz (1976), it may be the case that no Nash equilibrium in pure strategies exists. A candidate pure strategy equilibrium is now derived, and it will then be demonstrated that (A.3) guarantees that firms have no incentive to offer any alternative contracts.

B. General Equilibrium

Substituting $L_x(s) = 1$ into (2) and $L_{y1}(s) = 1$ along with (5) into (3) gives the market clearing conditions

$$(11)\qquad \alpha(s)M_t/p_t(s) = \theta\lambda(s)$$

$$(12)\qquad [1-\alpha(s)]M_t/p_t(s) = \theta[1-\lambda(s)] + (1-\theta)(\pi_2/\pi_1)L_{y2}(s).$$

Solving (11) and (12) for $\lambda(s)$ yields

$$(13)\qquad \lambda(s) = \alpha(s)[1 + (1-\theta)\pi_2 L_{y2}(s)/\theta\pi_1] < 1,$$

where the inequality follows from (A.2) and $L_{y2}(s) < 1$. Moreover, using (11) for t and t+1 gives

$$(14)\qquad p_t(s)/p_{t+1}(s') = \left(\frac{1}{1+\sigma}\right)\left[\frac{\alpha(s)}{\lambda(s)}\right]\left[\frac{\lambda(s')}{\alpha(s')}\right] = \left(\frac{1}{1+\sigma}\right)\left[\frac{\theta + (1-\theta)(\pi_2/\pi_1)L_{y2}(s')}{\theta + (1-\theta)(\pi_2/\pi_1)L_{y2}(s)}\right] \ \forall\ s,s'.$$

Then, since,

$$(15) \quad \Phi(s) \equiv E_t\psi(s')\pi_1^{[1-\alpha(s')]}[p_t(s)/p_{t+1}(s')],$$

substitution of (10′) and (14) into (15) gives (upon rearranging terms)

$$(16) \quad \Phi(s)\{\theta + (1-\theta)(\frac{\pi_2}{\pi_1})[\frac{\Phi(s) - \beta_1}{\Phi(s)\pi_2/\pi_1 - \beta_1}]\} = (1+\sigma)^{-1}E_t\psi(s')\pi_1^{[1-\alpha(s')]}\{\theta$$

$$+(1-\theta)(\frac{\pi_2}{\pi_1})[\frac{\Phi(s') - \beta_1}{\Phi(s')\pi_2/\pi_1 - \beta_1}]\} \ \forall s.$$

Defining the function F by

$$F(x) \equiv \theta + (1-\theta)(\pi_2/\pi_1)(x - \beta_1)/[(\pi_2/\pi_1)x - \beta_1],$$

(16) can be written as

$$(16') \quad \Phi(s)F[\Phi(s)] = (1+\sigma)^{-1}E_t\psi(s')\pi_1^{[1-\alpha(s')]}F[\Phi(s')] \ \forall \ s.$$

Thus $\Phi(1)$ and $\Phi(2)$ satisfy

$$(17) \quad (1+\sigma)\Phi(1) = q(1)\psi(1)\pi_1^{[1-\alpha(1)]} + [1-q(1)]\psi(2)\pi_1^{[1-\alpha(2)]}F[\Phi(2)]/F[\Phi(1)]$$

and

$$(18) \quad (1+\sigma)\Phi(2) = q(2)\psi(1)\pi_1^{[1-\alpha(1)]}F[\Phi(1)]/F[\Phi(2)] + [1-q(2)]\psi(2)\pi_1^{[1-\alpha(2)]}.$$

Equations (17) and (18) are depicted diagrammatically in Figure 1. From assumption (A.3), when $\Phi(1) = \beta_1$, the value of $\Phi(2)$ given by (17) is less than β_1, while the value of $\Phi(2)$ given by (18) exceeds β_1. In addition, the slopes of the loci defined by (17) and (18) are given by

$$(19) \quad \left.\frac{d\Phi(2)}{d\Phi(1)}\right|_{(17)} = \frac{F[\Phi(2)]F'[\Phi(1)]}{F[\Phi(1)]F'[\Phi(2)]} + \frac{(1+\sigma)F[\Phi(1)]}{[1-q(1)]\psi(2)F'[\Phi(2)]\pi_1^{[1-\alpha(2)]}}$$

$$(20) \quad \left.\frac{d\Phi(2)}{d\Phi(1)}\right|_{(18)} = \left\{\frac{F[\Phi(1)]F'[\Phi(2)]}{F[\Phi(2)]F'[\Phi(1)]} + \frac{(1+\sigma)F[\Phi(2)]}{q(2)\psi(1)F'[\Phi(1)]\pi_1^{[1-\alpha(1)]}}\right\}^{-1}$$

Then, since $F'(x) > 0$,

$$\left.\frac{d\Phi(2)}{d\Phi(1)}\right|_{(17)} > \left.\frac{d\Phi(2)}{d\Phi(1)}\right|_{(18)} > 0.$$

$\forall\ \Phi(1) \geq \beta_1$. Finally, (17) intersects the 45° line at the value $\hat{\Phi}(1)$, defined by

$$(21) \quad \hat{\Phi}(1) = (1+\sigma)^{-1}\{q(1)\psi(1)\pi_1^{[1-\alpha(1)]} + [1-q(1)]\psi(2)\pi_1^{[1-\alpha(2)]}\}$$

while (18) intersects the 45° line at

$$(22) \quad \tilde{\Phi}(1) = (1+\sigma)^{-1}\{q(2)\psi(1)\pi_1^{[1-\alpha(1)]} + [1-q(2)]\psi(2)\pi_1^{[1-\alpha(2)]}\}.$$

There are now two situations to consider. In the first, $\hat{\Phi}(1) > \tilde{\Phi}(1)$. Then (18) is depicted by the solid locus in figure 1. In this case $\Phi(1) > \Phi(2) > \beta_1$ obtains. In the second, $\tilde{\Phi}(1) > \hat{\Phi}(1)$, and (18) is depicted by the dashed locus in figure 1. In the latter case $\Phi(2) > \Phi(1) > \beta_1$.[6] From (21) and (22), $\hat{\Phi}(1) > \tilde{\Phi}(1)$ holds iff

(23) $[q(1)-q(2)]\psi(1)\pi_1^{[1-\alpha(1)]} > [q(1)-q(2)]\psi(2)\pi_1^{[1-\alpha(2)]}$

Clearly solutions to (17) and (18) exist, are unique, and satisfy $\Phi(s) > \beta_1$ $\forall$s. Having obtained these solutions, employment of type 2 workers satisfies

(24) $L_{y2}(s) = \{F[\Phi(s)]-\theta\}\pi_1/\pi_2(1-\theta)$

$\forall$s. Then, from (13),

$$\lambda(s) = \alpha(s)F[\Phi(s)]/\theta < 1,$$

while (5) gives $p_y(s) = 1/\pi_1$ $\forall$s. Total per capita output in state s, measured in units of x, is $\theta\lambda(s) + \theta[1-\lambda(s)] + (1-\theta)\pi_2 L_{y2}(s)/\pi_1 = F[\Phi(s)]$, while

(25) $p_t(s)/p_{t+1}(s') = (1+\sigma)^{-1}F[\Phi(s')]/F[\Phi(s)]$

$\forall$s, s′, t.

C. Equilibrium in Pure Strategies

The values just derived constitute an equilibrium so long as, given $\Phi(s)$ and $p_y(s)$, no firm has an incentive to announce an alternative set of contracts (with any announcements subject to (7), (8), and the non–negative profit condition). By construction, no firm can profitably attract type 1 workers alone, and any profitable contract that attracts type 2 workers must also attract type 1 workers. Therefore, if any firm has an incentive to offer an alternative contract, the contract must pool type 1 and 2 workers in their population proportions. The most preferred pooling contract for type 2 workers that earns non–negative profits must set $w_y(s) = p_y(s)[\theta\pi_1 + (1-\theta)\pi_2] = [\theta\pi_1 + (1-\theta)\pi_2]/\pi_1$, and must set L to maximize $\{\Phi(s)[\theta\pi_1 + (1-\theta)\pi_2]/\pi_1 - \beta_2\}L$. Then there is no profitable pooling contract that attracts type 2 workers iff

$$(26)\quad [\Phi(s)p_y(s)\pi_2 - \beta_2]L_{y2}(s) = [\Phi(s)\pi_2/\pi_1 - \beta_2][\Phi(s)-\beta_1]/[\Phi(s)\pi_2/\pi_1 - \beta_1]$$
$$\geq \Phi(s)[\theta\pi_1 + (1-\theta)\pi_2]/\pi_1 - \beta_2 \ \forall \ s.$$

Rearranging terms in (26) yields the equivalent condition

$$(27)\quad (\beta_2 - \theta\beta_1)\pi_1/\pi_2(1-\theta) \geq \Phi(s) \ \forall \ s.$$

As is clear from figure 1, $\Phi(s) < \max[\hat{\Phi}(1), \tilde{\Phi}(1)] \ \forall$ s. Moreover, (A.3) implies that $(\beta_2-\theta\beta_1)\pi_1/\pi_2(1-\theta) \geq \max[\hat{\Phi}(1), \tilde{\Phi}(1)]$. Thus (27) holds, so that the contracts derived in section A do constitute Nash equilibrium contracts in pure strategies.

IV. Properties of Equilibrium

In the equilibrium of section III, $\Phi(s) > \beta_2\pi_1/\pi_2$ holds $\forall$ s. This, in turn, implies that at the wage rate $w_{y2}(s) = p_y(s)\pi_2 = \pi_2/\pi_1$, type 2 workers would like to set $L_{y2}(s) = 1$. Then the unemployment rate in state s is (since all unemployment is confined to type 2 workers) $(1-\theta)[1-L_{y2}(s)]$, which is inversely related to total output in state s, $F[\Phi(s)]$. This section takes up the issue of how output, or unemployment, is related to the inflation rate, the share of the y good sector in total employment, and to average productivity. In addition, it is possible to investigate how changes in the probability distribution of demand shocks affect the cyclical behavior of output or unemployment.

A. The Phillips Curve

For a given rate of money growth, σ, the gross rate of inflation is $p_{t+1}(s')/p_t(s) = (1+\sigma)F[\Phi(s)]/F[\Phi(s')]$. Thus when current period output $F[\Phi(s)]$ is high, the expected inflation rate will be high as well, or the equilibrium displays a Phillips curve. However, this Phillips curve relation cannot be exploited by varying the growth rate of the money supply. In particular, figure 2 depicts the consequences of increasing σ. Clearly, an increase in σ shifts the locus (17) up at each $\Phi(1)$, while shifting the locus (18) to the right at each $\Phi(2)$. The result is a decline in the equilibrium values of both $\Phi(1)$ and $\Phi(2)$, and consequently in output in all states. Therefore the analysis is consistent both with Lilien's (1982) observation of a "short–run Phillips curve," and with Cooley and Hansen's (1989) finding that high secular inflation rates tend to be associated with high rates of unemployment.

B. <u>The Sectoral Composition of Employment</u>

Lilien (1982) and Rogerson (1986) observed that large increases in unemployment rates are generally accompanied by a decline in manufacturing's share of total employment in favor of lower wage sectors. Here the x good sector is the low (average) wage sector. Its share of total employment is

$$\frac{\theta\lambda(s)}{\theta+(1-\theta)L_{y2}(s)} = \frac{\alpha(s)F[\Phi(s)]}{(\pi_1/\pi_2)F[\Phi(s)] + \theta(\pi_2-\pi_1)/\pi_2}$$

Then sector x's employment share is highest in state 1 iff

$$\frac{\alpha(1)F[\Phi(1)]}{\pi_1F[\Phi(1)] + \theta(\pi_2-\pi_1)} > \frac{\alpha(2)F[\Phi(2)]}{\pi_1F[\Phi(2)] + \theta(\pi_2-\pi_1)} \tag{28}$$

Rearranging terms in (28) gives

$$\alpha(1) - \alpha(2) > (\theta/\pi_1)(\pi_2-\pi_1)\{\alpha(2)/F[\Phi(1)] - \alpha(1)/F[\Phi(2)]\}, \tag{28$'$}$$

where it will be recalled that $\alpha(1) > \alpha(2)$. There are now two cases to consider.

Case 1: $\hat{\Phi}(1) > \tilde{\Phi}(1)$. In this case $\Phi(1) > \Phi(2)$. Then, since F is increasing, (28′) holds. Moreover, in this case $F[\Phi(1)] > F[\Phi(2)]$, so output is high when the x good sector has a high employment share. Also, in this case the unemployment rate is lowest in state 1, so high unemployment is not associated with the low wage sector having a relatively large employment share.

Case 2: $\hat{\Phi}(1) < \tilde{\Phi}(1)$. In this case $\Phi(2) > \Phi(1)$, so unemployment is highest in state 1. This occurs when sector x has a relatively high employment share iff (28′) holds. A sufficient condition for (28′) to be satisfied is that $\alpha(1)/\alpha(2) \geq F[\Phi(2)]/F[\Phi(1)]$. However, for given values of $\alpha(1)$, $\alpha(2)$, $q(1)$, and $q(2)$, $\Phi(1)$ can be made as close to $\Phi(2)$ as desired. In particular, $\hat{\Phi}(1) < \Phi(1) < \Phi(2) < \tilde{\Phi}(1)$, and from (23), $\hat{\Phi}(1)$ and $\tilde{\Phi}(1)$ can be made arbitrarily close by choice of π_1 for any other set of parameter values. Thus parameters can be chosen so that (28′) holds, and declines in employment are associated with an increasing employment share for the low wage sector.

C. Productivity and Hours

Average productivity is total output divided by total hours:

$$\frac{F[\Phi(s)]}{\theta + (1-\theta)L_{y2}(s)} = \frac{\theta + (1-\theta)(\pi_2/\pi_1)L_{y2}(s)}{\theta + (1-\theta)L_{y2}(s)}$$

Evidently average productivity must increase as $L_{y2}(s)$ (or total hours) increases.

The actual empirical relationship in U.S. data between average productivity and hours is a matter of some controversy, although Christiano and Eichenbaum (1988) argue that these series are negatively correlated. When incentive constraints bind in all states, average productivity and hours must be positively correlated here, although this correlation can be made quite small. Section V describes a modification of the model that allows average productivity and hours to be negative correlated.

D. Serial Correlation of Disturbances

The consequences of increasing q(1) (increasing persistence) are now considered. The effects of decreasing q(2) are qualitatively similar, and are not formally analyzed here.

The effects of increasing q(1) are depicted in figure 3 under the assumption that $\hat{\Phi}(1) > \tilde{\Phi}(1)$ [and hence that $\Phi(1) > \Phi(2)$]. Clearly changes in q(1) do not affect the locus defined by (18), while if $\hat{\Phi}(1) > \tilde{\Phi}(1)$, increases in q(1) shift the locus defined by (17) down and to the right. Therefore increases in q(1) increase the equilibrium values of $\Phi(1)$ and $\Phi(2)$. Moreover, from (18), $\Phi(2)$ increases iff $F[\Phi(1)]/F[\Phi(2)]$ does as well. Since $F[\Phi(1)] > F[\Phi(2)]$, the dispersion of output must rise, or in other words, the amplitude of output fluctuations is greater as shocks are more persistent. The same result obtains if $\Phi(2) > \Phi(1)$. Thus when changes in demand become less probable, these changes have greater effects when they do occur.

V. Extensions

The model in its current form cannot capture at least two of the features discussed in the introduction. First, as has been seen, average productivity and hours must be positively correlated under the assumptions made to date. And, in addition, since $p_y(s) = 1/\pi_1$, $w_x = 1$, $w_{y1}(s) = 1$, and $w_{y2}(s) = \pi_2/\pi_1$, or in other words, wage dispersions do not decline at cyclical peaks. A modification of the model is now described which allows average productivity to be countercyclical, and which allows wage dispersions to decline at cyclical peaks.

Suppose that assumption (A.2) is relaxed (fails to hold) when s=1, but does hold when s=2. Then in state 1 all type 1 workers and some type 2 workers are employed in production of x, while in state 2 all type 2 workers and some type 1 workers are employed in production of y. The first case can obtain only if $w_{y2}(1) = p_y(1)\pi_2 = w_x = 1$, while the second can obtain only if $w_{y1}(2) = p_y(2)\pi_1 = w_x = 1$. Then in state 1 all workers receive the same wage rate. Of course in this case there is no incentive problem, so unemployment is zero. When s=2, the average (hours weighted) wage in the y good sector is

$$\frac{\theta[1-\lambda(2)] + (1-\theta)L_{y2}(2)(\pi_2/\pi_1)}{\theta[1-\lambda(2)] + (1-\theta)L_{y2}(2)} > 1 = w_x.$$

In addition, incentive constraints bind, so $L_{y2}(2) < 1$. Thus wage dispersions are greatest across sectors when unemployment is high. Moreover, clearly average productivity (measured in units of x) is one when s=1, while when s=2 average productivity exceeds one. Since total hours are lowest in state 2, average product and hours are negatively correlated.

VI. Conclusions

The model used to analyze cyclical fluctuations and the sectoral composition of employment has been essentially the Roy (1951) model, to which private information has been added to generate unemployment, and in which workers are assumed to care about hours (to enable sorting to occur). The Roy (1951) model is a quite standard model of the allocation of labor to sectors of employment [see Heckman and Sedlacek (1985), or Heckman and Honore (1989)]. The observations of Lilien (1982) and Rogerson (1986) are often treated as if they require non–standard features in order to explain them; for instance limited mobility [Rogerson (1987), Hamilton (1988)] or indivisibilities [Williamson (1988)]. However, the Lilien/Rogerson findings are easily confronted by standard self–selection models of labor markets, with private information introduced as well. In addition, these findings can be confronted in ways that are consistent with observations on the cyclical behavior of wage dispersions, the cyclical behavior of productivity and hours, and with Phillips curve observations.

The Lilien/Rogerson observations strongly suggest that the sectoral composition of employment is an important aspect of business cycles. If one accepts that sectoral wage distributions are also important in understanding the sectoral composition of employment, this further suggests that relative wages are important in cyclical fluctuations. The importance of relative wages is, of course, emphasized by Keynes (1936), Dunlop (1950), and Solow (1980).

In the model of this paper, differences in relative wages create incentive problems that must be resolved by the use of hours restrictions. As relative wages vary cyclically (see section V), more or less severe hours restrictions will need to be used to resolve incentive problems. Thus, as argued by Keynes, Dunlop, and Solow, the behavior of relative wages is very important in determining macroeconomic behavior in this model. Moreover, it has been seen that the model can be structured in such a way that all cyclical variation in unemployment is accounted for by type 2 workers (who in sections I–IV are always attached to the y good sector). Therefore the analysis is consistent with the observation of Murphy and Topel (1987) that cyclical fluctuations in unemployment are largely accounted for by workers who are attached to one industry.

It seems appropriate to conclude by commenting on some features of the model specification. One feature that merits comment is the use of an overlapping generations model, in which a period is naturally thought of as being fairly long, to study business cycle issues. However, in order to consider monetary issues, an infinite horizon model is required. At the same time, in order to consider private information while avoiding complications due to multi–period incentive problems, it is natural to have workers be in the work force only once. Together these two factors dictate the use of a two period lived, overlapping generations model.

A second feature of the analysis requiring comment is that unemployment is confined to type 2 workers, who are high productivity workers. This may appear to be counterfactual. However, it would be easy to generalize the analysis to avoid this implication. For instance, additional worker types could be added which, in equilibrium, are more productive than type 2 workers. If more heterogeneity in preferences is allowed for, incentive constraints need not bind between these types and workers of type 1 or 2, so that these additional types would experience no unemployment. Then unemployment would be confined to intermediate types (type 2), which is consistent with the casual observation that jobs requiring the least productive workers seem always to be available (i.e., that type 1 workers do not experience involuntary unemployment). Since doing this would not alter the basic point, only the two type specification has been formally considered.

Finally, in the analysis above shocks occurred only to the composition of goods demand. However, it is not difficult to also allow productivity shocks, or to allow for more dynamics by introducing capital. Since these issues are considered in Smith (1989a, b), and since they are not important to the issue under consideration in the present paper, these complications have been avoided here.

Footnotes

1. In the Roy (1951) model workers care only about income, whereas here they care about income and hours, however.

   The importance of privately observed differences for some of the issues under discussion is suggested by Krueger and Summers (1987), who argue (p. 19) that observable differences across workers in different industries do very little to account for measured inter–industry wage differentials. Nor are such differentials easily explained as compensating for unemployment risk or other job characteristics [Abowd and Ashenfelter (1981), Krueger–Summers (1987), p. 39]. It therefore seems natural to model them as arising from unobservable differences.

2. This assumption is not central to any results. However, it allows savings behavior to be effectively ignored in what follows. Since savings behavior is tangential to the issues under consideration, it seems best to adopt the simplest formulation possible.

3. The analysis is easily conducted under the assumption that s is a continuous random variable. However, the two state formulation allows a sharper characterization of the pattern of cyclical fluctuations. Rogerson (1987), Hamilton (1988), and Williamson (1988) all adopt the assumption that random variables are discrete.

   A comment is also in order about the preference specification of equation (1). While a very specific form is assumed for preferences, this appears to be standard in the literature on sectoral employment and the cycle. Rogerson (1987) assumes that goods are perfect substitutes, Williamson (1988) assumes that only a subset of goods are consumed at any date, and Hamilton assumes CES preferences (but disallows savings by assumption). Finally, allowing for shocks to preferences is an easy way to allow changes in the composition of demand to induce changes in the derived sectoral demand for labor. Common preference shocks are considered in real business cycle settings by Bencivenga (1988) and Parkin (1988).

4. As will become apparent, assumption (A.2) implies that all type 2 workers and some type 1 workers are employed in the production of y.

5. It is easy to verify that the values $L_{y2}(s)$ given by (10′) satisfy the incentive compatibility conditions (8) $\forall$s.

6. The possibility that $\hat{\Phi}(1) = \tilde{\Phi}(1)$ is ignored, since in this case $\Phi(1) = \Phi(2)$, and no cyclical fluctuations will be observed.

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Figure 1

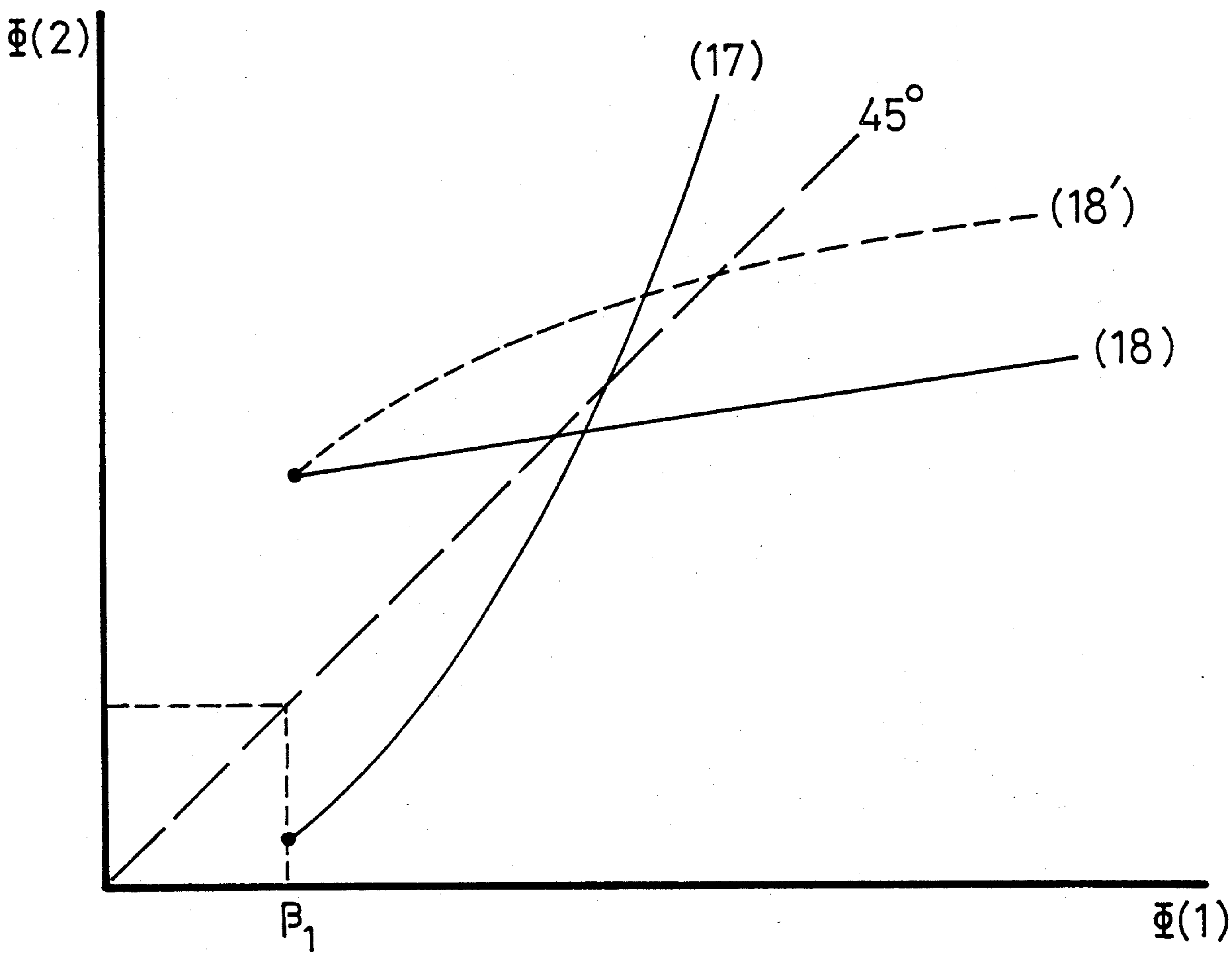

Figure 2

Increasing Money Growth Rates

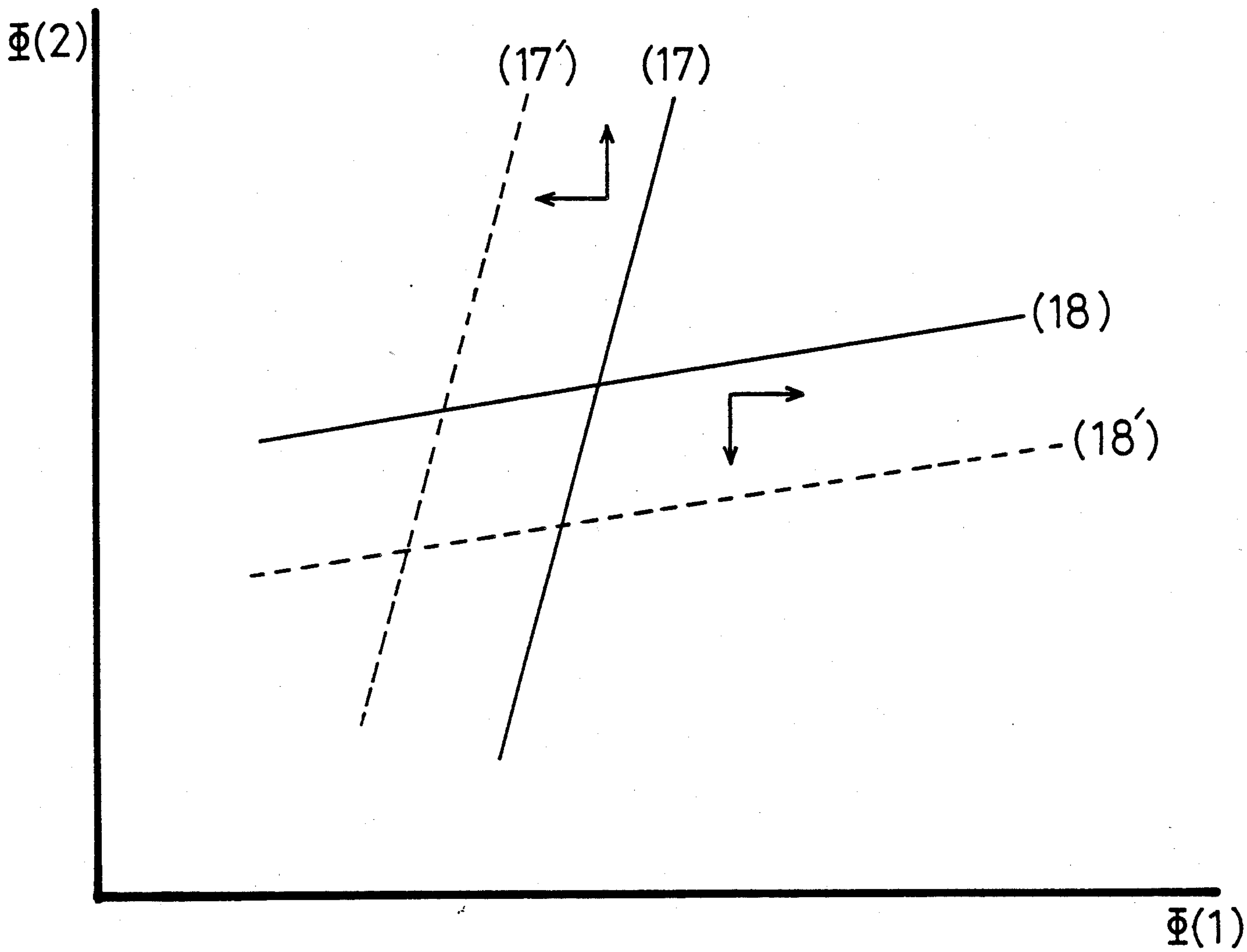

Figure 3