

International Borrowing and Time-Consistent Fiscal Policy

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Abstract

The paper discusses optimality and time consistency of fiscal policy in an open economy without money and capital. For a large open economy it is shown that the optimal policy under commitment can be made time consistent under discretion, if each government chooses an adequate maturity structure for the national and international debt. For a small open economy the optimal policy under commitment cannot be made time-consistent. The general result is that each government requires at least as many "effective" debt instruments as the number of (explicit and implicit) tax rates chosen, in order to ensure time-consistent policy of its successor.

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## Introduction

The influential papers by Kydland and Prescott (1977) and Calvo (1978) were the first to seriously analyze the time-consistency problem of government policy. Their finding, which showed to apply to most dynamic problems with forward looking agents, was the following: Suppose the government chooses an optimal policy at some date by maximizing its objective function. If the government could commit itself for its entire horizon the optimum policy would obviously be carried out as originally planned. Suppose, however, that the government cannot commit itself to a particular policy, but instead chooses its policy under discretion - that is it maximizes the same welfare function at each later date. Then, in general, it will not choose the policy that was optimal under commitment, which policy hence is time-inconsistent under discretion. Further, the optimal time consistent policy under discretion, when the public bases its action on an understanding of the government's objectives, generally gives a lower welfare level than does the optimal policy under commitment. These findings have led some writers to advocate fixed rules for policy (assuming thereby implicitly that credible commitment to such rules are in fact possible).

A recent study by Lucas and Stokey (1983) sheds new light on these issues. The (major part of their) paper deals with the following problem: Assume a given path of government spending that can be financed either by distortionary taxes on labor income, or by borrowing. As is well-known the optimal fiscal policy under commitment, in the sense of maximizing the representative citizen's welfare, involves smoothing the tax distortion over time. Assume now that the government in each period chooses its tax policy under discretion, but must honor the outstanding government debt. Lucas and

Stokey show that the optimal policy under commitment can actually be made time-consistent under discretion, provided there exists debt of sufficiently rich maturity (and contingency in the case of uncertainty). Then, each government can induce its successor to continue following the optimal policy by a unique restructuring of the public debt.

This result is important for two reasons. First, it shows, contrary to previous beliefs, that it may not be necessary to precommit the government's policy instruments by a fixed rule to avoid welfare losses of discretion.<sup>1</sup> Second, in the words of Fischer (1983), it provides an example of an anti-Modigliani-Miller theorem, where the maturity structure of the debt matters.

In this paper we extend Lucas and Stokey's analysis of optimal fiscal policy to open economics, thereby relaxing the assumption that all debt is a "debt to ourselves". We look both at a small open economy and at an economy large enough to affect the intertemporal terms of trade at which it can borrow and lend. We ask whether the policy under commitment still can be made time consistent and, if so whether that requires any management of the maturity structure of the domestic and foreign debt.

Section II presents our model, a deterministic open-economy version of Lucas and Stokey's intertemporal general equilibrium model. In the first part of the paper we assume that only the government, but not the private sector, can engage in foreign borrowing and lending. We derive the optimal fiscal policy under commitment and discuss the properties of the resulting allocation.

In section III we show that time-consistency requires the government's domestic debt obligations to be restructured over time and we discuss the properties of the unique necessary restructuring scheme. That section also provides an alternative interpretation and some extensions of Lucas and

Stokey's results; for this see also Persson and Svensson (1984).

Section IV discusses the requirements on the government's (and the country's) foreign debt. In the large economy, there is a unique maturity structure of the foreign debt that sustains the optimal policy. In a small economy, on the other hand, the optimal policy under commitment cannot be made time-consistent under discretion.

In Section V we relax the assumption that private agents can not trade on the international capital market. Again time-consistency can be supported in the large economy (although here the supporting maturity structure is non-unique), but fails in the small economy.

Section VI offers some concluding remarks.

## II. Optimal policy under commitment

We look at an open-economy, one-good, deterministic version of the model in Lucas and Stokey (1983). Many goods and uncertainty can be introduced, at the cost of considerable complexity, but this adds nothing essential to the time-consistency discussion. We start by considering an economy where the government, but not the private sector, can trade on the international capital market.

The economy's production technology is linear: one unit of labor results in one unit of output. There is one representative consumer with an endowment of labor, normalized to unity, in each period. Private consumption of goods and leisure in period  $t$  is  $c_t$  and  $x_t$ , while the given amount of government consumption is  $g_t$ . With imports  $m_t$ , the economy's resource constraint is

thus:

$$(1) \quad c_t + g_t + x_t \leq 1 + m_t, \quad t=0,1,2,\dots$$

Our representative consumer has additively separable preferences

$$(2) \quad \sum_0^{\infty} \beta^t U(c_t, x_t), \quad 0 < \beta < 1,$$

where  $U(\cdot)$  is strictly concave in  $c_t$  and  $x_t$ .

There are proportional taxes on labor income and  $\tau_t$  denotes the tax rate in period  $t$ . At the outset, government debt obligations to the consumer are predetermined and described by the vector  ${}_0^b = ({}_0^b, {}_0^b, \dots)$ , where  ${}_0^b_t$  is the consumer's claim to goods in period  $t$  -- the sum of interest and repayment of maturing domestic debt, or total debt service. Denoting the domestic present value prices (interest rate factors) by  $p = (p_0, p_1, \dots)$ , we can express the consumer's intertemporal budget constraint as

$$(3) \quad \sum_0^{\infty} p_t c_t - \sum_0^{\infty} p_t (1-\tau_t)(1-x_t) \leq \sum_0^{\infty} p_t {}_0^b_t.$$

The consumer maximizes (2) subject to (3), which gives the first-order conditions:

$$(4a) \quad (1-\tau_t)U_c(c_t, x_t) = U_x(c_t, x_t), \quad t = 0,1,2,\dots, \text{ and}$$

$$(4b) \quad \beta^t U_c(c_t, x_t) = p_t, \quad t = 0,1,2,\dots,$$

where we have normalized the present value prices by using units of utility as the numeraire.

The government's given sequence of consumption  $g = (g_0, g_1, \dots)$ , is financed either with the proportional taxes on labor or by borrowing. The borrowing can be done at home or abroad. On the domestic capital market the government trades at prices  $p$ , while at the international market the present value prices are  $p^* = (p_0^*, p_1^*, \dots)$ .

Let us now define government cash-flow, namely the excess of tax income over total government outlays in a particular period; this concept will turn out to be extremely useful later on. Total cash-flow of the government (at  $t = 0$ ) in period  $t$ ,  $y_{0t} + z_{0t}$ , is thus given by

$$(5a) \quad y_{0t} + z_{0t} = \tau_t(1-x_t) - g_t - {}_0b_t - {}_0b_t^*, \quad t = 0,1,2,\dots,$$

where  ${}_0b_t^*$  is the claims on goods from the rest of the world in period  $t$  -- the government's foreign debt service. We note that government cash-flow is not in general equal to the budget surplus,<sup>2</sup> nor is it equal to net government savings - see Persson and Svensson (1984) for further comments.

Total government cash-flow has two components: foreign cash-flow

$$(5b) \quad z_{0t} = -m_t - {}_0b_t^*, \quad t = 0,1,2,\dots,$$

that is net exports minus foreign debt service, and domestic cash-flow, which from (5a) and (5b) is

$$(5c) \quad y_{0t} = \tau_t(1-x_t) + m_t - g_t - {}_0b_t, \quad t = 0,1,2,\dots$$

With these in hand we may write the intertemporal budget constraints faced by the time 0-government as

$$(5d) \quad \sum_0^\infty p_t^* z_{0t} \geq 0 \text{ and}$$

$$(5e) \quad \sum_0^\infty p_t y_{0t} \geq 0.$$

We also have to describe the behavior of the world, which we refer to as the foreign country. The foreign country looks like the home country although we make a few simplifying assumptions. First, we assume that foreign consumption of leisure,  $x^*$ , is fixed and constant over time. Then we write the foreign representative consumer's utility as

$$(6) \quad \sum_0^\infty \beta^{*t} U^*(c_t^*), \quad 0 < \beta^* < 1.$$

Second, we abstract from all foreign government activity. Since foreign imports are the negative of home imports, we can then write the foreign



resource constraint as

$$(7) \quad c_t^* \leq 1 - x_t^* - m_t, \quad t = 0, 1, 2, \dots$$

The foreign country's intertemporal budget constraint may be expressed as

$$(8a) \quad \sum_0^\infty p_t^*(m_t + b_t^*) \geq 0.$$

Maximizing (6) subject to (7) and (8a), we get the relation

$$(8b) \quad p_t^* = \beta^t U_c^*(1 - x_t^* - m_t) \equiv P_t^*(m_t), \quad t = 0, 1, 2, \dots,$$

where we have invoked the same normalization as for the home country. These conditions define the foreign country's demand-price functions  $P_t^*(m_t)$ .

Equations (8) (with (8a) fulfilled with equality) summarizes what we need to know about the foreign country's behavior.

In equilibrium any decisions of the government must be consistent with private maximizing behavior at home and abroad. Using (4) and (8) we may therefore rewrite (5) as

$$(9a) \quad U_c(c_t, x_t)(1 - x_t - g_t - y_{0t} - b_t + m_t) + U_x(c_t, x_t)(x_t - 1) = 0,$$

$$(9b) \quad \sum_0^\infty P_t^*(m_t)(-m_t - b_t^*) \geq 0, \text{ and}$$

$$(9c) \quad \sum_0^\infty \beta^t U_c(c_t, x_t)y_{0t} \geq 0.$$

The government's optimum tax problem can be formulated as choosing  $c_t$ ,  $x_t$ , and  $m_t$ , so as to maximize the representative consumer's welfare given by (2), subject to the constraints (9), and the economy's resource constraint (1). The first-order conditions to this problem are

$$(10a) \quad \beta^t U_{ct} + \lambda_0 \beta^t [U_{cct}(1 - \tau_t)(1 - x_t) + U_{xct}(x_t - 1)] \\ + \lambda_0 \beta^t U_{cct} y_{0t} - \beta^t \mu_{0t} = 0, \quad t = 0, 1, 2, \dots,$$

$$(10b) \quad \beta^t U_{xt} + \beta^t \lambda_0 [U_{xt} - U_{ct} + U_{cxt}(1 - \tau_t)(1 - x_t) + U_{xxt}(x_t - 1)] \\ + \lambda_0 \beta^t U_{cxt} y_{0t} - \beta^t \mu_{0t} = 0, \quad t = 0, 1, 2, \dots, \text{ and}$$

$$(10c) \quad \lambda_0 \beta^t U_{ct} - \gamma_0 (p_t^* - p_{tm}^* z_{0t}) + \beta^t \mu_{0t} = 0, \quad t = 0, 1, 2, \dots,$$

where we have employed the shorthand  $U_{ct} = U_c(c_t, x_t)$ , etc., where  $\mu_{0t}$ ,  $\lambda_0$  and  $\gamma_0$  are the Lagrange multipliers associated with (1), (9a) and (9b), and where we have used that  $1 - x_t - g_t - y_{0t} - b_t + m_t = (1 - \tau_t)(1 - x_t)$ . We assume that the first-order conditions are not only necessary but sufficient for an interior unique solution. However, as is well known from the literature on optimal taxation, this is not an innocuous assumption (see Diamond and Mirrlees (1971), for instance).

To understand the government's trade-offs, let us look at the welfare effect of an arbitrary change in the allocation in period  $t$ , which should of course be zero in optimum. To do this multiply (10a), (10b) and (10c) by  $dc_t$ ,  $dx_t$  and  $dm_t$ , respectively, and add the resulting equations to get<sup>3</sup>

$$(11) \quad \beta^t (U_{ct} dc_t + U_{xt} dx_t) + \lambda_0 p_t d[\tau_t (1 - x_t)] + \lambda_0 y_{0t} dp_t \\ + \lambda_0 p_t dm_t - \gamma_0 (p_t^* - p_{tm}^* z_{0t}) dm_t \\ - \beta^t \mu_{0t} (dc_t + dx_t - dm_t) = 0, \quad t = 0, 1, 2, \dots$$

Each of the terms in (11) has clearcut interpretation. First, we have a direct effect on utility of the change in consumption and leisure,  $\beta^t dU_t = \beta^t (U_{ct} dc_t + U_{xt} dx_t)$ . While this would be zero with lump-sum taxes, it is non-zero here due to the tax distortion.

The changes in  $c_t$  and  $x_t$  also change tax revenue in period  $t$  by  $d[\tau_t (1 - x_t)]$ . The effect on utility of the tax change is equal to  $\lambda_0 p_t d[\tau_t (1 - x_t)]$ ; the second term in (11). The multiplier in the government's domestic wealth constraint  $\lambda_0$  measures the distortionary effect of proportional taxation, more precisely the marginal effect of switching from proportional to lump-sum taxes, at constant government expenditure. It is thus equal to the "cost of public funds" minus one, but for brevity we refer

to  $\lambda$  as the cost of public funds in the sequel.

Next, the changes in consumption and leisure in period  $t$  imply a change in that period's domestic present value price (interest rate factor)  $dp_t$ . As a consequence, government domestic net wealth  $\sum_0^\infty p_t y_{0t}$  changes by  $y_{0t} dp_t$ . In effect, then, a wealth revaluation (synonymous with a debt devaluation) is identical to a switch to lump-sum taxes. Consequently its effect on utility is  $\lambda_0 y_{0t} dp_t$ ; the third term in (11).

An increase in imports  $dm_t$  at given tax rates and government spending increases domestic cash-flow and decreases foreign cash-flow by the same amount. The increase in domestic cash-flow, by itself increases domestic wealth and hence utility by  $\lambda_0 p_t dm_t$ ; the fourth term (11). (Conversely, the decrease in foreign cash-flow decreases utility; this appears in the next term.)

The fifth term reflects the cost of imports, or equivalently of foreign borrowing. The bracketed expression is the country's true international shadow price of goods in period  $t$ , which is not equal to  $p_t^*$  since a change in imports alters intertemporal prices and hence the country's "foreign net wealth" by  $z_{0t} dp_t^* = z_{0t} P_{tm}^* dm_t$ . The expression  $p_t^* - P_{tm}^* z_{0t}$  can thus be thought of as an "effective price" of imports. This effective price is multiplied by the Lagrange multiplier on the government's foreign wealth constraint,  $\gamma_0$ .

Finally, the sixth term measures the resource cost of changing the allocation in period  $t$ . For all reallocations that obey the economy's resource constraint (1), this term is zero, of course.

In an optimum, then, the government essentially trades off the tax distortion and the cost of imports (the first and the fifth term) against the comprehensive effect on government wealth (the sum of the second through fourth terms) in such a way that further reallocations cannot increase

welfare. We may now describe where the time-consistency problems arise. Obviously the trade-offs we have just described depend on  $\lambda_0$  and  $\gamma_0$ . Since these multipliers will not stay constant over time (see further below), neither will the trade-offs, and successive governments will have incentives to chose different allocations, *ceteris paribus*.

Before we show how these problems can be resolved, let us discuss briefly the characteristics of the optimal tax policy. It can readily be verified from (10a) and (10c) that in general  $p_t = \beta^t U_{ct}$  is not proportional to  $p_t^*$ ; that is, it is not optimal to set domestic and foreign prices (interest rates) equal. This is true even for an economy small enough to take world prices as given (so that  $P_{tm}^* = 0$  in (10c)). The wedge arises because taxes are distortionary, and because it is possible to change the real value of the domestic debt. If the economy is large enough to affect  $p_t^*$  -- its intertemporal terms of trade -- there will be an additional "optimal intertemporal tariff", implicitly defined in the optimal allocation.

The optimal tax policy will in general smooth both labor supply and consumption. In a closed economy an optimal pattern of taxes should smooth out the tax distortions over time, which involves government borrowing (lending) in periods with high (low) government consumption; see the discussion and examples in Lucas and Stokey (1983). Essentially, this result comes about because it is desirable to stabilize the marginal rate of substitution between consumption and leisure  $U_{xt}/U_{ct}$  and keep it as close as possible to the marginal rate of transformation, which is 1 by definition.

In an economy that can trade at the world capital market, the optimum policy involves further smoothing of consumption and leisure than in a closed economy where the resource constraint forces government consumption to

completely "crowd out" private consumption and leisure. From another angle, when government consumption is low there is a high level of resources left for private consumption. In such periods the country has a "comparative advantage" and engages in net exports. Conversely, there are net imports in periods when government consumption is high. The smaller the country is in relation to the world economy, the more it can smooth out the adjustments.

The implication is that it is optimal to borrow abroad in periods with positive government borrowing, yielding a positive correlation between the deficits in the government budget and the country's current account. This result is more general than the present model and would hold also if there were fluctuations in the productivity of the economy's endowment of labor.<sup>4</sup>

### III. Domestic debt and time consistency under discretion

If policy could be committed at  $t = 0$ , the optimal tax policy discussed in the previous section would be followed by all future governments at  $t = 1, 2, \dots$ . If each of these governments sets its own tax rates under discretion, there is a particular restructuring scheme of the domestic and foreign debt obligations that gives incentives to follow the same policy; i.e. that makes the optimal policy under commitment time-consistent under discretion.

The government at time 0 must retire or issue some net debt according to whether its total cash-flow in period 0,  $y_{00} + z_{00}$ , is positive or negative. This government inherited the domestic and foreign debt obligations  $({}_0b_0, {}_0b_1, \dots)$  and  $({}_0b_0^*, {}_0b_1^*, \dots)$  and chooses now its turn new debt structures  ${}_1b = ({}_1b_1, {}_1b_2, \dots)$ ,  ${}_1b^* = ({}_1b_1^*, {}_1b_2^*, \dots)$  that the government at  $t = 1$  will inherit. For solvency vis-a-vis the foreign country, the value of the net issue of foreign debt must satisfy

$$(12a) \quad \sum_1^{\infty} p_t^* (b_t^* - b_0^*) / p_0^* = -z_{00}^*.$$

Analogously, the net issue of domestic debt must fulfill

$$(12b) \quad \sum_1^{\infty} p_t (b_t - b_0) / p_0 = -y_{00}.$$

If the government at  $t = 1$  were committed to set future taxes as the optimal policy at  $t = 0$  prescribed, any debt structure satisfying (12) would do. How then should the debt be restructured to make the  $t = 1$  government choose the same policy under discretion? That problem turns out to be recursive, and in this section we describe how the government's domestic debt should be restructured. The management of foreign debt that maintains time consistency is treated in the next section.

We have already seen that one source of the time-consistency problem is that  $\lambda$ , the cost of public funds, is not constant over time. Let us therefore first derive explicit expressions for  $\lambda_0$  and  $\lambda_1$ . As a first step, let us subtract the first-order condition for  $c_t$  and  $x_t$ , (10a) and (10b), and resulting expression by  $(U_{cct} - U_{cxt})$  to obtain

$$(13a) \quad \lambda_0 y_{0t} + \lambda_0 A_t = B_t, \quad t = 0, 1, 2, \dots,$$

where  $A_t$  and  $B_t$  are given by

$$(13b) \quad A_t = A(c_t, x_t) \\ = [U_{ct} - U_{xt} + (U_{cct} - U_{cxt})(1 - \tau_t)(1 - x_t) + (U_{xct} - U_{xxt})(x_t - 1)] / \\ (U_{cct} - U_{cxt}) \text{ and}$$

$$(13c) \quad B_t = B(c_t, x_t) = -(U_{ct} - U_{xt}) / (U_{cct} - U_{cxt}),$$

and where  $B_t$  is positive as long as taxes are positive and consumption and leisure are both normal.<sup>5</sup> Multiplying (13a) by  $p_t$ , adding for  $t = 0, 1, 2, \dots$ , and using (5a) with equality, we get

$$(14) \quad \lambda_0 = \sum_0^{\infty} p_t B_t / \sum_0^{\infty} p_t A_t.$$

Now,  $B_t$  measures the wedge between marginal rates of transformation and substitution in period  $t$  caused by the distortionary taxes, while it can be shown that  $A_t$  is the derivative of government tax income with respect to  $c_t$  and  $x_t$  (for the government at constant intertemporal prices).<sup>6</sup> Therefore  $\lambda_0$  is indeed a natural measure of the (excess) cost of public funds in the economy (for the government at  $t = 0$ ).

Let us turn to the decision problem of the government at  $t = 1$ . It maximizes  $\sum_1^\infty \beta^t U(c_t, x_t)$  subject to (1) and the  $t = 1$  analog to (9), which yields first-order conditions for  $c_t, x_t$  and  $m_t$  on the same form as in (10).

$$(15a) \quad \beta^t U_{c_t} + \lambda_1 \beta^t [U_{cct}(1-\tau_t)(1-x_t) + U_{xct}(x_t-1)] \\ + \lambda_1 \beta^t U_{cct} y_{1t} - \beta^t \mu_{1t} = 0, \quad t = 1, 2, \dots,$$

$$(15b) \quad \beta^t U_{x_t} + \beta^t \lambda_1 [U_{x_t} - U_{c_t} + U_{cxt}(1-\tau_t)(1-x_t) + U_{xxt}(x_t-1)] \\ + \lambda_1 \beta^t U_{cxt} y_{1t} - \beta^t \mu_{1t} = 0, \quad t = 1, 2, \dots, \text{ and}$$

$$(15c) \quad \lambda_1 \beta^t U_{c_t} - \gamma_1 (p_t^* - p_{tm}^* z_{1t}) + \beta^t \mu_{1t} = 0, \quad t = 1, 2, \dots$$

Here  $\lambda_1$ ,  $\mu_{1t}$  and  $\gamma_1$  are the Lagrange multipliers of the analogs to (9a), (1) and (9b) for the government at  $t = 1$ , and the domestic and foreign cash-flow of the government at  $t = 1$ ,  $y_{1t}$  and  $z_{1t}$  fulfill

$$(16a) \quad y_{1t} = \tau_t(1-x_t) + m_t - g_t - {}_1b_t, \quad t = 1, 2, \dots, \text{ and}$$

$$(16b) \quad z_{1t} = -m_t - {}_1b_t^*, \quad t = 1, 2, \dots,$$

where  ${}_1b_t$  and  ${}_1b_t^*$  are home and foreign debt inherited from the government at  $t = 0$ . As above, the first order conditions for  $c_t$  and  $x_t$ , (15a) and (15b) can be subtracted and manipulated to yield

$$(17) \quad \lambda_1 y_{1t} + \lambda_1 A_t = B_t, \quad t = 1, 2, \dots$$

We may solve for solve for the new cost of public funds,  $\lambda_1$ , as

$$(18) \quad \lambda_1 = \Sigma_1^{\infty} p_t B_t / \Sigma_1^{\infty} p_t A_t.$$

Using this, together with (14) and (13a) we can determine the change in the cost of public funds

$$(19) \quad \lambda_1 - \lambda_0 = -\lambda_0 p_0 y_{00} / \Sigma_1^{\infty} p_t A_t.$$

So we may deduce

$$(20) \quad \lambda_1 \gtrless \lambda_0 \text{ if and only if } y_{00} \lesseqgtr 0.$$

In other words if the domestic cash flow at  $t = 0$  is positive (negative) the cost of public funds goes down (up), which certainly makes sense.

The next step in the argument is to show that the government at  $t = 1$  can be induced to choose the same  $c_t$  and  $x_t$  as did the government at  $t = 0$  despite the fact that  $\lambda_1$  and  $\lambda_0$  are different. This is obviously necessary for time-consistency (but not sufficient since the government at  $t = 1$  must also choose the same  $m_t$ ; see further below). Note that equations (13) and (17) compactly summarize the first-order conditions for  $c_t$  and  $x_t$  for the two governments. Since  $A_t$  and  $B_t$  depend only on  $c_t$  and  $x_t$  they too must be the same if the optimal allocation is to coincide. Combining the two equations, we see that if  $y_{1t}$  and  $y_{0t}$  fulfill

$$(21) \quad \lambda_1 y_{1t} = \lambda_1 y_{0t} - (\lambda_1 - \lambda_0) A_t, \quad t = 1, 2, \dots,$$

the same  $c_t$  and  $x_t$  are in fact optimal. Using (5) and (16), we see that this requires the government at  $t = 0$  to restructure its domestic debt according to

$$(22) \quad {}_1 b_t - {}_0 b_t = -(y_{1t} - y_{0t}) = (1 - \lambda_0 / \lambda_1)(y_{0t} + A_t), \quad t = 1, 2, \dots.$$

It is easy to show that this restructuring scheme obeys (12b) the domestic solvency condition.<sup>7</sup> To gain some further understanding of what the scheme involves note that from (13) and (17)

$$(23) \quad \lambda_0 (y_{0t} + A_t) = \lambda_1 (y_{1t} + A_t) = B_t > 0, \quad t = 1, 2, \dots,$$



so that

$$(24) \quad {}_1b_t \gtrless {}_0b_t \text{ and } y_{1t} \lesseqgtr y_{0t} \text{ if and only if } \lambda_1 \gtrless \lambda_0 \quad t = 1, 2, \dots$$

From (24) and (20) a positive (negative) cash flow in period 0 should thus be used to buy up (sell) some debt of all maturities - and (22) states the precise way this should be done.

The intuition for this scheme is as follows. As long as  $y_{0t}$  is non-zero the government at  $t=1$  faces a different cost of public funds, and hence a different trade-off between tax distortions and wealth effects in each period ceteris paribus, than did the government at  $t=0$ . But the debt restructuring changes the domestic cash-flows and hence the "base" for the wealth revaluations (debt devaluations) that follow upon changes in domestic interest rates - cf. the third term in (11). By revising the maturity structure of the domestic debt as in (22) the  $t=0$  government can indeed give its successor an unchanged trade-off in each period when choosing  $c_t$  and  $x_t$  (and hence implicitly the tax rates).

Exactly the same reasoning can be applied to any pair of governments. A complete description of the sequence of debt restructurings that are necessary for time consistency is therefore obtained just by changing the subscripts 0 and 1 to  $s$  and  $s+1$ , respectively, in equations (13) through (24).

If the economy were closed, these necessary conditions would also be sufficient for time consistency of the optimal policy. Infact, the restructuring scheme we have derived is exactly the scheme derived in the closed-economy model of Lucas and Stokey (1983). However, our derivation is different and we also provide some extensions and interpretation of their results. (For a further comparison with Lucas and Stokey we refer the reader to Persson and Svensson (1984)).

Since we deal with an open economy, however, we must also find out whether succeeding governments have incentives to continue choosing the same  $m_t$  for  $t = 1, 2, \dots$ . If so, the optimal policy is indeed time-consistent.

#### IV. Foreign debt

It remains to examine when the government at  $t=1$  has incentive to choose the same import levels  $m_t$ ,  $t=1, 2, \dots$ , as the government at  $t=0$ . We first analyze the case of a country large enough to affect world market prices.

We see from (10c) that choosing an optimal path for imports at  $t=0$  at  $t=0$  involves equating the social marginal benefit of imports  $\beta^t \mu_{0t} + \lambda_0 \beta^t U_{ct}$  with their social cost  $\gamma_0 (p_t^* - P_{tm}^* z_{0t})$  at each  $t=0, 1, 2, \dots$ . The analogous first-order condition at  $t=1$  is given by (15c). We have already seen that  $\lambda_1$  in general differs from  $\lambda_0$ . Furthermore  $\mu_{1t}$  and  $\mu_{0t}$  are in general different, as well as  $\gamma_1$  and  $\gamma_0$ . Clearly, a time-consistency problem, similar to that in the previous section, exists. Subtracting (10c) from (15c) and using (10a) and (10b) to eliminate  $\mu_{0t}$  and  $\mu_{1t}$ , we get the following conditions for  $c_t, x_t$  and  $m_t$  being consistent with optimum:

$$(25a) \quad P_t^*(m_t)(\lambda_1 - \lambda_0) - P_{tm}^*(m_t)(\gamma_1 z_{1t} - \gamma_0 z_{0t}) = \\ D_t(c_t, x_t)(\lambda_1 - \lambda_0), \quad t = 1, 2, \dots,$$

where  $D_t(c_t, x_t)$  is given by

$$(25b) \quad D_t(c_t, x_t) \equiv \beta^t [U_{ct} + U_{cct}(1-\tau_t)(1-x_t) + U_{xct}(x_t-1) - U_{cct}A(c_t, x_t)]$$

and  $A(c_t, x_t)$  is given by (13b). It can be shown that  $D_t(c_t, x_t)$  is positive for all  $t$  if goods and leisure are normal goods.<sup>8</sup>

Note, however, that there is a unique  $z_{1t}$  for  $t = 1, 2, \dots$  that satisfies (25). Thus, in analogy with our argument in the previous section, we should be able to find a restructuring scheme for the foreign debt that gives the government at  $t = 1$  a sequence of unchanged trade-offs between costs and benefits of imports. The optimum policy under commitment can therefore be made time consistent, or incentive compatible under discretion.

Let us try to characterize the required restructuring scheme. We already know how  $\lambda_1$  relates to  $\lambda_0$ , but we need to know what changes  $\gamma$  over time. By multiplying (10c) and (15c) by  $p_t^*$ , adding for all time periods and using the solvency conditions<sup>9</sup>  $\sum_0^\infty p_t^* z_{0t} = 0$ , and  $\sum_1^\infty p_t^* z_{1t} = 0$ , we can solve explicitly for  $\gamma_1 - \gamma_0$  as

$$(26) \quad \gamma_1 - \gamma_0 = \gamma_0 p_0^* z_{00} / \sum_1^\infty (p_t^{*2} / p_{tm}^*) + (\lambda_1 - \lambda_0) \sum_1^\infty (p_t^* D_t / p_{tm}^*) / \sum_1^\infty (p_t^{*2} / p_{tm}^*).$$

We see that  $\gamma_1$  tends to exceed (fall short of)  $\gamma_0$  if  $z_{00}$  is positive (negative). However,  $\gamma$  is also influenced by the cost of public funds, in that  $\gamma_1$  is higher (lower) than  $\gamma_0$  if  $\lambda_1$  is higher (lower) than  $\lambda_0$ . The intuition here is that a higher cost of public funds raises the cost of foreign borrowing (i.e. imports), since eventually it will have to be repaid by raising more (distortionary) taxes.

The situation is thus fairly complicated. We may nevertheless derive how the foreign debt should be restructured, using (5b), (16b), (19), (25), (26) and some algebra; we get

$$(27) \quad {}_1b_t^* - {}_0b_t^* = -(z_{1t} - z_{0t}) = - (p_t^* / p_{tm}^* - z_{0t}) [\gamma_0 / \gamma_1 \sum_1^\infty (p_t^{*2} / p_{tm}^*)] p_0^* z_{00} \\ + \lambda_0 [\gamma_1 \sum_1^\infty p_t^* A_t]^{-1} [ (p_t^* / p_{tm}^* - z_{0t}) \sum_1^\infty (p_t^* D_t / p_{tm}^*) / \sum_1^\infty (p_t^{*2} / p_{tm}^*) - D_t ] p_0 y_{00} \\ t = 1, 2, \dots .$$

Suppose, we have a situation where  $y_{00} = 0$ ; the domestic cash-flow is zero at  $t = 0$ . Then we see, from (27) that the government at  $t = 0$  should use a positive (negative) cash-flow to buy up (sell) some foreign debt of all maturities. To see why, recall that when  $y_{00} = 0$ ,  $\lambda_0 = \lambda_1$ , and so are  $\mu_{0t} = \mu_{1t}$ . It follows that the social benefits of imports do not change from  $t = 0$  to  $t = 1$ , that is  $\lambda_0 \beta^t U_{ct} + \beta^t \mu_{0t} = \lambda_1 \beta^t U_{ct} + \beta^t \mu_{1t}$ . For the same  $m_t$  to be optimal the cost of imports should also stay constant, in other words we need  $\gamma_0 (p_t^* - P_t^* z_{tm 0t}) = \gamma_1 (p_t^* - P_t^* z_{tm 1t})$ . Since  $z_{00} > 0$  leads to  $\gamma_1 > \gamma_0$  from (26), that means the effective price of imports faced by the  $t = 1$  government has to be lower in every period. This is precisely what happens if  $z_{1t} > z_{0t}$  as the debt is restructured according to (27).

When  $y_{00}$  is non-zero, so that  $\lambda_1 \neq \lambda_0$ , the situation becomes much more complex. This is apparent from (27) since the term multiplying  $p_0 y_{00}$  can have either sign. Thus we do not get such a simple unambiguous rule for the restructuring scheme as when  $y_{00} = 0$ , unless  $p_0 y_{00}$  is sufficiently small relative to  $p_0^* z_{00}$  so that the "pure" effect of  $p_0^* z_{00}$  in (27) dominates.

Having shown how time-consistency can be supported in a large economy, let us turn to the case of a small country which cannot affect the world market interest rates - present value price  $p_t^*$ . In other words, we assume that the import-derivative of the foreign demand-price functions is zero

$$(28) \quad P_{tm}^* (m_t) = 0 \quad t = 0, 1, 2, \dots$$

This modifies the first-order conditions (10) and (15c) to

$$(29) \quad \lambda_0 \beta^t U_{ct} + \beta^t \mu_{0t} = \gamma_0 p_t^* \quad t = 0, 1, 2, \dots,$$

and

$$(30) \quad \lambda_1 \beta^t U_{ct} + \beta^t \mu_{1t} = \gamma_1 p_t^* \quad t = 1, 2, \dots$$

Can the optimal policy chosen at  $t = 0$  be maintained in this case? The answer to this question, maybe somewhat surprisingly, turns out to be no. At  $t = 0$  (29) will be satisfied for some  $\lambda_0, \gamma_0$  and  $\mu_{0t}, t = 0, 1, \dots$ . At  $t = 1$  we know that the government is faced with new multipliers  $\lambda_1, \gamma_1$  and  $\mu_{1t}, t = 1, 2, \dots$ . But there is nothing that guarantees that (30) will be satisfied at these new values. To see this clearer, subtract (29) from (30) and eliminate  $\mu_{1t} - \mu_{0t}$  to get the small-economy analog to (25a), namely the conditions for the same paths for  $c_t, x_t$  and  $m_t$  to be optimal at  $t = 1$ ,

$$(31) \quad p_t^*(\gamma_1 - \gamma_0) = D(c_t, x_t)(\lambda_1 - \lambda_0), \quad t = 1, 2, \dots$$

In the large economy the  $t = 0$  government could use the maturity structure of the foreign debt as instruments to maintain the trade-off between costs and benefits of imports at each  $t$  for its successor by picking the one value of  $z_{1t}$  that satisfied (25a). In the small economy this is no longer possible. Although the  $t = 0$  government can still restructure the foreign debt as it wishes, this is a useless instrument to make (31) hold for  $t = 1, 2, \dots$  and hence maintain time-consistency in the small economy case. Because the small economy cannot affect the world prices, no foreign cash-flow terms enter in the appropriate conditions given by (31). Consequently, we conclude that the optimal policy under commitment can not be made time-consistent under discretion.

This shows that the argument in Lucas and Stokey (1983), and in Persson and Svensson (1984), that it is the governments ability to manipulate the intertemporal prices that gives rise to a time-consistency problem in this class of models, is mistaken. On the contrary, it seems that it is precisely this ability which makes it possible to resolve the problem! One way to understand this is the following. Consider a finite horizon  $T$ , that is  $t = 1, 2, \dots, T$  (the argument holds when  $T$  approaches infinity). In the

closed economy case we have a set of  $T$  first-order conditions - the combined conditions for  $c$  and  $x$  - corresponding to the path of  $T$  tax rates that are the direct choice variables of each government. Furthermore, each government can, by manipulating the debt structure, influence the  $T$  cash-flows inherited by its successor. Hence, each government has  $T$  instruments to affect the  $T$  choice variables of its successor.

In the large open economy each government has in effect  $2T$  choice variables. These are the  $T$  tax rates, and the  $T$  implicit interest taxes given by the difference between home and foreign interest rates (since the difference between world intertemporal prices and domestic intertemporal prices can be interpreted as implicit taxes). Furthermore, each government has  $2T$  instruments to influence its successor, namely the debt structure of both domestic and foreign debt. In the small open economy, however, each government has still  $2T$  choice variables (tax rates and implicit interest taxes), but only  $T$  instruments to influence its successor, since world interest rates cannot be influenced and foreign cash-flows do not matter. The general principle that seems to emerge is thus that to use a debt restructuring scheme to support a time-consistent optimum tax policy one needs as many debt instruments as there are choice variables in the optimal taxation problem.

If the optimal policy under commitment is not time consistent under discretion, what can we say about the optimal time-consistent policy? To solve for that policy our previous method doesn't work. Instead, we should set the problem up as a dynamic programming problem, imposing the condition that each government takes as given the optimum policy of its successor. This is outside the scope of the present paper, however. But we can say one thing. If time-consistency is imposed on the optimal taxation problem as an

additional constraint, this must imply welfare losses relative to the optimal policy under commitment. Thus we are back in the rules-discretion dilemma.

#### V. Private capital mobility

Let us now relax the capital controls and allow free private foreign borrowing and lending. With no taxes on international capital flows, home present value prices must be proportional to international prices, that is

$$(32) \quad p_t = \beta^t U_c(c_t, x_t) = \alpha P_t^*(m_t), \quad t = 0, 1, \dots,$$

for some  $\alpha > 0$ . This expression will be added as an extra constraint to the government's optimal tax problem. We may therefore suspect already at this stage that allowing private capital mobility will deteriorate welfare; see further below. Indeed, imposing (32) is equivalent to remove the implicit taxation of foreign borrowing, which should result in the remaining income taxes being more distortionary.

We must also distinguish government foreign cash-flows and import,  $\tilde{z}_{0t}$  and  $\tilde{m}_t$ , from the economy's total foreign cash-flows and import,  $z_{0t}$  and  $m_t$ . Also, we should distinguish government foreign debt  $\tilde{b}_t^*$  from total foreign debt  $b_t^*$ , the difference  $b_t^* - \tilde{b}_t^*$  being private foreign debt, we have the identities

$$(33a) \quad z_{0t} = -m_t - b_t^*, \quad t = 0, 1, 2, \dots, \text{ and}$$

$$(33b) \quad \tilde{z}_{0t} = -\tilde{m}_t - \tilde{b}_t^*, \quad t = 0, 1, 2, \dots$$

The constraints (9) can be replaced by

$$(34a) \quad \sum_0^\infty \beta^t U_{ct}(y_{0t} + \tilde{z}_{0t}) \geq 0,$$

$$(34b) \quad \sum_0^\infty P_t^*(m_t)(-m_t - b_t^*) \geq 0, \text{ and}$$

$$(34c) \quad U_{ct}(1 - x_t - g_t - b_t - y_{0t} - \tilde{b}_t^* - \tilde{z}_{0t}) + U_{xt}(x_t - 1) = 0.$$

The second is the economy's wealth constraint relative to the foreign country. The third is the constraint expressing private maximizing behavior which follows since total government cash-flow fulfills

$$(35) \quad y_{0t} + \tilde{z}_{0t} = \tau_t(1-x_t) - g_t - {}_0b_t - {}_0\tilde{b}_t^*$$

and thus

$$(36) \quad (1-\tau_t)(1-x_t) = 1 - x_t - g_t - {}_0b_t - y_{0t} - {}_0\tilde{b}_t^* - \tilde{z}_{0t},$$

which can be substituted into (4a) to give (34c).

Carrying out the maximization at  $t = 0$ , subject to (32) and (34), one gets first-order conditions analogous to (10) which we do not display here. As before, we assume that these are sufficient and that the optimal solution is unique with respect to  $c_t$ ,  $x_t$ ,  $m_t$ , and  $\mu_{0t}$ ,  $\lambda_0$ , and  $\gamma_0$  -- the multipliers in (1), (34a), and (34b). However, one may easily check that the solution need not be unique with respect to  $y_{0t}$ , and  $\tilde{z}_{0t}$ . This follows since only the sum  $y_{0t} + \tilde{z}_{0t}$  enters in the constraints (34a) and (34c). Indeed, total government cash-flow as given in (35) is unique, but not the government's private and foreign cash-flows separately.

Similarly, the economy's total foreign cash-flow  $z_{0t}$  defined in (33a) is unique, but not the private and government foreign cash-flows separately. Intuitively, with the same prices at home and abroad, what is relevant for the revaluation effects on government and foreign wealth due to changes in these prices are the total cash-flows, but not their composition.

Can the optimal policy under commitment be sustained in the large economy case? Let us apply our earlier argument in terms of choice variables and instruments. Note then that with private capital mobility there is effectively only  $T$  choice variables for the government. While earlier only two out of  $c_t$ ,  $x_t$  and  $m_t$  could be chosen independently because of



the resource constraint, we have now added the extra constraint (32) at each  $t$ , so that each government can only choose one of  $c_t$ ,  $x_t$  and  $m_t$  independently. (Alternatively; the government chooses only  $\tau_t$  and not the implicit tax on capital flows it chose with capital controls.) The  $t = 1$  government thus has a set of  $T$  independent first-order conditions. But total government cash-flow  $y_{1t} + \tilde{z}_{1t}$  as well as total foreign cash-flow  $z_{1t}$  enter these conditions.<sup>10</sup> The  $t = 0$  government can control both these when restructuring the government debt, and has in fact  $2T$  instruments to give appropriate incentives to its successor. Clearly, time-consistency can be maintained in the large economy, and the required restructuring scheme is, unlike before, not unique.

Before moving on to the small economy, let us discuss the welfare properties of the optimal allocation. We have seen that government foreign cash-flow  $\tilde{z}_{0t}$  is not unique. Then it can be chosen equal to  $z_{0t}$ , which means that private foreign cash-flow is zero, while maintaining the same level of utility. A restriction to zero private foreign cash-flows is thus not binding. This in turn suggests that we can get the same solution if we start in the situation when private international borrowing is forbidden and home (relative) prices are not restricted to equal foreign (relative) prices, and then add the constraint that home and foreign prices are equal. Clearly, this implies that utility cannot be higher with private international borrowing, and if the constraints (32) are binding, utility is actually lower. In the distorted world we are considering, it is better to forbid private international borrowing, separate the home and foreign credit market, and allow home interest rates to differ from world interest rates.

What about the small economy with private capital mobility? It is fairly straightforward to show that the  $t = 1$  multipliers  $\lambda_1$  and  $\gamma_1$  are in general different than the corresponding  $t = 0$  multipliers.<sup>11</sup> Therefore the government at  $t = 1$  has incentive to choose a different path of tax rates than the government at  $t = 0$ . To prevent this the government at  $t = 0$  need to influence the first-order conditions for the government at  $t = 1$ . Since all prices are given in the small economy with free capital mobility, no cash-flow terms enter the first-order conditions, however. Therefore the  $t = 0$  government has no instruments whatsoever to affect the government at  $t = 1$ . As in the case without private capital mobility, the optimum policy under commitment is not time-consistent under discretion.

#### VI. Concluding remarks

Optimal fiscal policy in an open economy, apart from smoothing out the tax distortions associated with financing a given sequence of government consumption over time, also smooths out private consumption of goods and leisure by borrowing (lending) on the international capital market in periods of high (low) government consumption.

The bulk of the paper dealt with if and how the optimal policy under commitment can be made time consistent under discretion. A necessary condition is that there exists debt instruments of sufficiently rich maturity. In the case when the government, but not the private sector, is allowed to trade at the international capital market, and the economy is large enough to influence world interest rates, there are unique restructuring schemes for the government's domestic and foreign debt that give succeeding governments incentives to continue following the optimal policy. In the small economy case such a scheme does not exist, however.

When there is free private capital mobility, the time consistency results are similar. For a large economy there is a non-unique maturity structure of total government debt and/or total foreign debt that supports the optimal policy under commitment. Again, that policy cannot be made time consistent in a small economy.

The intuition for these results is as follows. The time-consistency problems arises because parameters in the optimal tax problem, like the cost of public funds, the marginal cost of foreign borrowing, do not stay constant over time. Successive governments then face different trade-offs for their choice variables *ceteris paribus*. Manipulating the maturity structure of the debt can be used to restore the trade-offs and hence optimality of unchanged paths for the choice variables, provided the economy is closed or large enough to affect world prices so that cash-flow terms enter the first-order conditions. In the small open economy this is not the case, which is why time consistency fails. A general principle seems to emerge. In order to use a debt restructuring scheme to support an optimal policy it is necessary that each government has at least as many effective debt instruments as the choice variables (explicit and implicit tax rates) of its successor. Here "effective" debt instruments are ones that enter the first-order conditions for the optimal taxation problem, which requires that relevant interest rates can be influenced by the governments. We hope to return to a more general treatment of this problem in future work.

It is, of course, central to our results that the debt obligations are always honored. As a consequence, we could not allow any taxation of interest income nor of international capital flows. These assumptions of no debt repudiation are, in fact, analogous to a binding commitment of (the sequence of) governments<sup>12</sup>.

The problem arising from allowing taxation of interest income is identical to the classical problem of capital levies<sup>13</sup>. Such levies constitute a non-distortionary and hence desirable form of taxation from a myopic point of view, but are bound to induce under-accumulation once it is understood that they will be used. It is clear then that the absence of capital in the model is crucial for time consistency of the optimal policy. Our model thus, does not add anything to the time-consistency problem associated with surprises in debt and capital taxation. It may be that one must resort to reputational considerations (cf. Footnote 1) to rule out such surprises.

It has also been maintained that the absence of money is crucial for time consistency of optimal policy, since in a monetary economy governments would have short-run incentives to engage in "surprise" inflations so as to deflate the real value of their outstanding nominal debt. Indeed Lucas and Stokey (1983), as well as others like Chamley (1985), claim that time consistency in a monetary economy requires a binding commitment to a particular path of nominal prices<sup>14</sup>. It turns out, however, that one can design a debt structure such that it eliminates the incentives to engage in surprise inflation. This point is further developed in Persson, Persson and Svensson (1985), who deal with time consistency of fiscal and monetary policy in a monetary extension of the real framework used in this paper.

Finally, an important qualification to our results (in the large country case) is the neglect of policy in the rest of the world. We could easily have allowed for passive policies in the foreign country. Strategic considerations abroad, however, would lead directly to a full-fledged game-theoretic analysis of conflicting government policies.

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Footnotes

1. Another way out of the discretion dilemma, without precommitted rules for policy, might be reputational considerations in government policy-making; see Barro and Gordon (1983).
2. Only in the special case when all government debt is in the form of consols, so that  $b_t$  and  $b_t^*$  consist only of interest payments, does cash-flow correspond to the budget surplus.
3. The second term in (11) comes from the second term in (10a) and (10b) since,  $\lambda_0 p_t [(U_{xt} - U_{ct}) dx_t + (U_{cct} dc_t + U_{cxt} dx_t \tau_t)(1-x_t) + (U_{xct} dc_t + U_{xxt} dx_t)(x_t - 1)] = \lambda_0 p_t [(U_{xt} - U_{ct}) dx_t / U_{ct} - dU_{xt}(1-x_t) / U_{ct} + dU_{ct} U_{xt}(1-x_t) / (U_{ct})^2] = \lambda_0 p_t [d(\tau_t(1-x_t))]$ .
4. This is qualitatively the same result as that obtained by Razin and Svensson (1983), who look at optimal taxation in a small open economy subject to productivity shocks.
5. We have  $U_{ct} - U_{xt} = \tau_t U_{ct} > 0$ , if  $0 < \tau_t < 1$ . If goods are normal,  $(\partial/\partial c)(U_c/U_x) = (U_{xc} - U_{cx}) / (U_x)^2 < 0$  for all  $(c, x)$ , which implies  $U_{cc} - U_{cx} < 0$ .
6. Cf. Footnotes 3 and 5.
7. To see this, multiply each equation in (22) by  $p_t$ , add them together, and use (14), (18) and (19).
8. Evaluating  $D_t$  we get  $D_t = \beta_t [(U_{cc} U_x - U_{cx} U_c) + (U_{cc} U_{xx} - U_{cx}^2)(x_t - 1)] / (U_{cc} - U_{cx})$ . The first term within

brackets and the dominator are negative when goods and leisure are normal, and  $U_{cc}U_{xx} - U_{cx}^2$  is positive by the concavity of  $U(c)$ .

9. Using the solvency constraint  $\sum_1^{\infty} p_t^* z_{1t} = 0$ , already in the derivation guarantees that the restructuring scheme that we will come up with fulfills the solvency constraint (12).
10. To see this, use the resource constraint (1) to substitute for  $m_t$  in (32) and (34). Then choose  $c_t$  and  $x_t$  to maximize (2) subject to (32) and (34). Eliminating  $\pi_{0t}$ , the multiplier on (32), between the first-order conditions for  $c_t$  and  $x_t$  one gets a single condition for each  $t$  which contains both  $y_{0t} + \tilde{z}_{0t}$  and  $z_{0t}$ . A corresponding argument holds for the  $t = 1$  government, of course.
11. The first-order conditions for the government at  $t = 0$  for this case can be rewritten as

$$\pi_{0t} + \lambda_0 A_t = B_t, \quad t = 0, 1, \dots,$$

$$\pi_{0t} + \lambda_0 E_t + \gamma_0 F_t = G_t, \quad t = 0, 1, \dots, \text{ and}$$

$$\sum_0^{\infty} p_t^* \pi_{0t} = 0,$$

where  $A_t$ ,  $B_t$ ,  $E_t$ ,  $F_t$ , and  $G_t$  depend on  $(c_t, x_t)$  ( $F_t$  depends also on  $t$ ), and where  $\pi_{0t}$  is the multiplier of (32). The first-order conditions for the government at  $t = 1$  are analogous except that they apply to  $t = 1, 2, \dots$  and the summation is  $\sum_1^{\infty}$ . It is fairly straightforward to show that whenever  $\pi_{00} \neq 0$  the first-order conditions for the  $t = 1$  government does not have a solution for the same path of  $c_t, x_t$  as the solution of the  $t = 0$  government.

12. Opening up the possibility of default lends the problem of optimal foreign borrowing entirely new dimensions, as shown by the recent literature that allows for possible repudiation of foreign debt by sovereign borrowers; see Sachs (1984) for a survey of the literature. For instance, choosing the optimal maturity structure of foreign debt then also involves influencing the expectations of foreign creditors in a proper way (as discussed by Cohen and Sachs (1984)). Such considerations would obviously limit the degrees of freedom to engage in debt restructuring schemes like those we have discussed here.
13. In fact, a promise not to repudiate the foreign debt is completely isomorphic to a promise not to levy taxes on existing capital in the present model, once the foreign demand price functions are viewed as an intertemporal transformation surface. This analogy was suggested to us by Elhanan Helpman.
14. Such a commitment is essentially the same as the "honesty" constraint Aurenheimer (1974) imposed on a government maximizing revenue from money creation, namely that the price level is not allowed to jump. We owe this point to Guillermo Calvo.





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