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In the literature on international trade theory it is not unusual to find pioneers and developers of key ideas rewarded by having their names identified with the basic results. Thus we have Heckscher–Ohlin Theory, the Rybczynski Theorem, the Leontief Paradox, and, perhaps the most frequently cited of all, the Stolper–Samuelson Theorem. This latter result, promulgated in the key 1941 article by Wolfgang Stolper and Paul Samuelson, established for a two–factor, two–commodity setting that a tariff (or any other impediment) which caused the domestic price of importables to rise would, if these commodities were produced locally by labor–intensive techniques, result in an unambiguous real wage increase and a corresponding reduction in the real return earned by the other productive factor. Some years later a search was launched for conditions which would replicate this result in cases of many commodities and factors, and of the key names associated with this work those of John Chipman (1969), Murray Kemp and Leon Wegge (1969), and Yasuo Uekawa (1971) are particularly noteworthy. Some of their results are highly technical, and our purpose in this paper is to utilize a recent technique for the 3 x 3 case developed by Edward Leamer (1987) in order to illustrate the conditions which Kemp and Wegge (1969) emphasized as necessary for the strong form of the Stolper–Samuelson Theorem to hold, as well as the special case which we call the "Produced Mobile Factor" structure. This latter restriction on production structure, first introduced by Jones and Marjit (1985), is sufficient to ensure strong Stolper–Samuelson results in the general n x n case, and derives its name from the close analogy to the specific–factors model which highlights the use of n specific factors and a single mobile factor. In the "Produced Mobile Factor" (PMF) structure this mobile factor is itself produced by the n "specific" factors.

The first section of the paper illustrates how the Leamer triangle can be used to identify those production structures which satisfy the strong Stolper–Samuelson condition in the 3 x 3 case. In the subsequent section we illustrate the further restrictions imposed by the PMF structure. This discussion emphasizes that the Stolper–Samuelson result requires not only the basic asymmetry which distinguishes the factor used intensively in any industry from the set of other factors, but also a basic symmetry or balance in the use of the unintensive factors. A striking reflection of this latter requirement is the result that strong Stolper–Samuelson properties cannot obtain if there exists any industry which does not use positive amounts of every productive factor. In the final section of our paper we discuss in more general terms the meaning and significance of the Stolper–Samuelson property.

I. The Leamer Triangle

The use of two–dimensional representations of production structures involving three productive factors can be found in an early contribution by Lionel McKenzie (1955) on the subject of factor–price equalization. More recently, Ed Leamer (1987) has revived the technique to discuss paths of development in a three–factor, many–commodity setting. As Leamer shows, this device allows for simple representation of the change in outputs required to absorb any particular change in factor endowments. By Samuelson's (1953) reciprocity conditions, these results can then be translated into the effect of commodity price changes on factor prices.

Figure 1 illustrates the technique. The vertices of the simplex correspond to the axes for each factor and the vertices of the internal triangle represent, through barycentric co–ordinates, the techniques used in industries 1, 2, and 3. Thus dotted line hg, drawn parallel to the 2'–3' edge of the simplex, shows alternative bundles of inputs of factors 2 and 3 for a given input of factor 1. The ratio 1g/h1 represents the ratio of factor 2 used to produce commodity 1 relative to the quantity of factor 3 used to produce commodity 1. (The quantity of factor 3 used to produce commodity 1 is h'1, while that of factor 2 to produce commodity 1 is 1g', where barycentric co–ordinates are measured by perpendicular distances to the axis opposite to that factor's vertex. But 1g'/h'1 equals 1g/h1. Points 1, 2, and 3 correspond to a given set of factor prices). All points on ray 1'–1–a from factor–1 vertex would reveal the same proportions of factor 2 to factor 3 as does point 1.

Consider a point such as e. within the production triangle Any increase in the economy's factor endowment base in the proportions shown by e can be absorbed at unchanged factor prices by a positive expansion of all three industries; point e is a positive convex combination of the three input requirements. Of more direct relevance for Stolper–Samuelson types of properties is the manner in which industry outputs must change in order to absorb an increase in the quantity of one factor alone. Thus suppose only factor 1 increases. Since point 1' does not lie within the 1–2–3 triangle, all outputs cannot expand. Draw the ray from vertex 1' through point 1, intersecting the 2–3 side of the triangle at point a. Since a is a positive convex combination of points 2 and 3, it represents an activity that consists roughly of 60% of activity 3 and 40% of activity 2 Points 1', 1 and a are co–linear, but 1' does not lie between 1 and a. Instead, 1' is a linear combination of 1 and a with a negative weight attached to a. Translated: An increase in the endowment of factor 1 would be absorbed by an increase in the first industry's output and a reduction in outputs of the second and third industries. By reciprocity this pattern implies that a rise in the return to factor

1 would be induced by a rise in the price of commodity 1 or by a fall in the price of commodity 2 or of commodity 3.

An application of this logic to the ray 2'–2–b from factor 2's vertex in Figure 1 similarly reveals that an increase in the endowment of factor 2 could be absorbed by an increase in industry 2's output and a reduction in the activity marked by point b, i.e. a fall both in output of industries 1 and 3. However, a pure increase in the endowment of factor 3 cannot be absorbed by an increase in x_3 with reductions in x_1 and x_2. By construction, an increase in the endowment of factor 3 could be balanced by a reduction in x_2 and an increase in the activity marked c – which entails an increase in x_3 but a (slight) increase as well in x_1. This latter effect is, by reciprocity, equivalent to the result that a pure increase in the price of the first commodity would raise (slightly) the return to the third factor – a violation of the strong Stolper–Samuelson result.

With this example in mind, we proceed now in a constructive way to isolate those techniques that could be adopted in one industry given a set of techniques for the other two industries that satisfies the strong Stolper–Samuelson condition. To aid in our construction we cite a result due to Kemp and Wegge (1969). Let the general input/output matrix for the n x n case be shown by A:

$$(1) \quad A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1i} & \cdots & a_{1j} & \cdots & a_{1n} \\ \vdots & \vdots & & \vdots & & \vdots & & \vdots \\ a_{i1} & a_{i2} & \cdots & a_{ii} & \cdots & a_{ij} & \cdots & a_{in} \\ \cdot & \cdot & & \cdot & & \cdot & & \cdot \\ a_{k1} & a_{k2} & \cdots & a_{ki} & \cdots & a_{kj} & \cdots & a_{kn} \\ \vdots & \vdots & & \vdots & & \vdots & & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{ni} & \cdots & a_{nj} & \cdots & a_{nn} \end{bmatrix}$$

Consider the element in the i^{th} row and column, a_{ii}, and the relative intensity with which factor i is used in industry i compared with some other factor, k, used in i. This intensity ratio is a_{ii}/a_{ki}. The Kemp–Wegge result is that if the strong form of the Stolper–Samuelson Theorem holds, this ratio exceeds the comparable ratio a_{ij}/a_{kj} for all other industries $j \neq i$, and, furthermore, such a comparison holds for all factors i and $k \neq i$. That is, if the A matrix of input–output coefficients has an inverse with a positive diagonal and negative off–diagonal elements, the original A matrix must be characterized by the strong factor intensity rankings shown by (2):

$$(2) \qquad \frac{a_{ii}}{a_{ki}} > \frac{a_{ij}}{a_{kj}} \qquad \forall \ k \neq i, \ j \neq i$$

Although Kemp and Wegge illustrate by means of a counter example that conditions (2) are in general not sufficient to guarantee the strong form of the Stolper–Samuelson condition, these inequalities are indeed necessary and sufficient if $n \leq 3$. We now use this result in constructing Figure 2.

Point 1 in Figure 2 is selected arbitrarily to illustrate the techniques used in the first industry. Draw rays from 1' and 3' through point 1. These rays define the triangular area 1–3'–b, and in order that condition (2) be satisfied, the technique vector for industry 3 must be located within this triangle, such as at point 3. Ray 1'–3 (not drawn) is flatter than ray 1'–1; this implies that the ratio of factor 3 to factor 2 used in industry 3 exceeds that in industry 1. Ray 3'–3 is steeper than ray 3'–1; the ratio of factor 1 to factor 2 utilized in industry 1 exceeds that used in industry 3. Given points 1 and 3, draw rays 2'–1 and 2'–3 from factor 2's vertex. The shaded area in Figure 2 illustrates the region within which techniques for industry 2 could lie in order that condition (2) is satisfied and the strong form of the Stolper–Samuelson Theorem holds. That point 2 must lie above rays 1'–1 and 3'–3 shows, respectively, that x_2 must use a higher ratio of factor 2 to factor 3 than does industry 1 and a higher ratio of factor 2 to factor 1 than does industry 3. In this sense a_{22} must be sufficiently high. But the shaded area also reveals that the coefficient for the unintensively used factors in industry 2, factors 1 and 3, must neither be too high nor too low. That is, ray 2'–2 must reveal a lower factor 1/factor 3 ratio than in industry 1, and a higher ratio than in industry 3.

As can easily be verified, the activities triangle, (1 2 3) satisfies for all factors the criteria illustrated in Figure 1 just for points 1 and 2 (whereby an increase in factor 1 or 2 causes the output of industry 1 or 2 (respectively) to rise and the other two outputs to fall).

II The Produced Mobile Factor Structure

The specific factor model (see Jones (1971) and Samuelson (1971)) in its n–commodity version posits that each commodity, j, is produced with only two productive factors: factor j used in no other commodity, and a mobile factor, M, used in all final commodities. The suggestion made in Jones and Marjit (1985) is that such a model can be converted into a special version of an n x n model if the mobile factor, M, is itself produced by a combination of all the "specific" factors. In this manner each commodity requires for its production all factors – all but one of them only indirectly via their contribution to the produced mobile factor. Such a production structure guarantees strong Stolper–Samuelson properties.

The proof provided in Jones and Marjit utilized the entire set of competitive profit conditions and can readily be sketched out here. Suppose only p_1 rises. Commodities 2, ... , n are each produced with the mobile factor and one "specific" factor, so that with $p_2, \dots, p_n$ all fixed the return to the mobile factor must move in a direction opposite to that of $w_2, \dots, w_n$. Since factors 1, ... , n in turn produce mobile factor M, factor 1's return must move in the same direction as that of factor M but by a relatively greater amount. Between them the changes in w_1 and w_M must trap the change in p_1 so that w_1 rises by relatively more than p_1 and $w_2, \dots, w_n$ fall.

In terms of the matrix of input–output coefficients, A, each diagonal term, a_{ii}, is the sum of the direct application of factor i to produce a unit of commodity i, which we call c_{ii}, and an indirect application of factor i via its use in M and subsequent use of M to produce i. Call this indirect coefficient b_{ii}, equal to the product of a_{iM} and a_{Mi}. More generally, each factor i is indirectly involved in the production of every good j via

$$(3) \qquad b_{ij} \equiv a_{iM} a_{Mj}$$

The n x n matrix of input–output coefficients, A, is thus the sum of a positive diagonal matrix, C, and a positive matrix, B, that has rank unity since all its columns are proportional to the vector whereby the produced mobile factor is assembled, $\left[a_{1M}, a_{2M}, \dots, a_{nM}\right]'$. (Alternatively put, all rows are proportional to the use of the mobile factor in each industry, $\left[a_{M1}, a_{M2}, \dots, a_{Mn}\right]$.)

(4) $A = B + C$

$$\text{Where } C = \begin{bmatrix} c_{11} & & O \\ & \ddots & \\ O & & c_{nn} \end{bmatrix}$$

$$B = \begin{bmatrix} (a_{1M}\, a_{M1}) & (a_{1M}\, a_{M2}) & \cdots & (a_{1M}\, a_{Mn}) \\ (a_{2M}\, a_{M1}) & (a_{2M}\, a_{M2}) & \cdots & (a_{2M}\, a_{Mn}) \\ \vdots & \vdots & & \\ (a_{nM}\, a_{M1}) & (a_{nM}\, a_{M2}) & \cdots & (a_{nM}\, a_{Mn}) \end{bmatrix}$$

Our result can be rephrased as a statement that the inverse of a matrix which is the sum of a positive diagonal matrix and a positive matrix of rank one must exhibit a positive diagonal and negative off–diagonal elements. The produced mobile factor (PMF) structure must exhibit strong Stolper–Samuelson properties.

We turn, now, to the relationship between this structure and the conditions (2) established by Kemp and Wegge as generally necessary for productive structures satisfying the Stolper–Samuelson conditions. Clearly (2) is satisfied since it entails

(5a) $$\frac{c_{ii} + a_{iM}a_{Mi}}{a_{kM}\, a_{Mi}} > \frac{a_{iM}}{a_{kM}}$$

and

(5b) $$\frac{c_{ii} + a_{iM}\, a_{Mi}}{a_{kM}\, a_{Mi}} > \frac{a_{iM}\, a_{Mk}}{c_{kk} + a_{kM}\, a_{Mk}}$$

Of special relevance for all cases where $n \geq 4$ is that the ratio of corresponding off–diagonal terms, $\frac{a_{ij}}{a_{kj}}$, $j \neq i$, k are all identical in the PMF structure:

$$\frac{a_{ij}}{a_{kj}} = \frac{a_{iM}}{a_{kM}} \quad \forall \; j \neq i \text{ or } k \tag{6}$$

In the more general Kemp–Wegge structure, each of these ratios can be different – they must only all fall short of a_{ii}/a_{ki}. For the case of $n = 3$, in which we are using the Leamer triangle for illustration, these extra constraints on the PMF structure do not appear. That is, a pair of ratios of strictly off–diagonal terms requires the matrix to have at least four columns and rows. This helps to explain why, when $n \geq 4$, the Kemp–Wegge conditions are no longer sufficient to guarantee the strong Stolper–Samuelson properties whereas the PMF structure is sufficient.

Of particular use in applying the Leamer triangle apparatus to the PMF structure is the general relationship which exists among products of the elements in the indirect B matrix. Let j(i) be an assignment of commodities, j, to factors, i, such that each commodity is assigned to one and only one factor. Then for all such assignments,

$$\prod_i b_{ij(i)} = \prod_i a_{iM} \prod_j a_{Mj} \tag{7}$$

That is, every element in the expansion of the determinant of B is identical to every other. Focus, now, on products which contain only non–diagonal elements; in the 3 x 3 case which we are illustrating with the Leamer triangle there are only two such products, $b_{12}\, b_{23}\, b_{31}$ and $b_{13}\, b_{21}\, b_{32}$. These translate into the off–diagonal elements of the direct plus indirect A matrix of input–output coefficients to yield:

$$\frac{a_{21}}{a_{31}} \cdot \frac{a_{32}}{a_{12}} \cdot \frac{a_{13}}{a_{23}} = 1 \tag{8}$$

Condition (8) is a requirement that factor intensities for unintensive factors achieve a kind of balance over all commodities. As we now prove, condition (8) implies that the rays from each factor vertex passing through the point depicting the activity vector for the industry in which that factor is used intensively all must have a common intersection. In Figure 2 this implies that given points 1 and 3, and intersection–point c of rays 1'–1 and 3'–3, the activity vector for commodity 2 must, in the PMF case, lie somewhere on the straight line chord 2'–c.

Figure 3 provides the ingredients for the proof that if rays 1'–1, 2'–2, and 3'–3 all meet at a common point, Q, condition (8) is satisfied. The ratio of the unintensive factors in the first industry, a_{21}/a_{31}, equals the ratio a/b in Figure 3, where the three solid lines through point Q have each been drawn parallel to an axis. In similar fashion, a_{32}/a_{12} equals f/e and a_{13}/a_{23} equals c/d. But each of the three triangles meeting at Q are equilateral, so that a = d, b = f, and c = e, establishing condition (8).

More can be said. The Produced Mobile Factor structure requires the kind of balance among ratios of factors unintensively used in each industry shown by equation (8). This implies that all factor–intensity rays for the unintensive factors intersect at a common point, Q. As in Figure 3, the point depicting the input vector for each activity lies on the vector from the vertex of the intensive factor to Q, but closer to the vertex than does Q. The greater is the ratio a_{ii}/a_{Mi}, the closer will the industry point along the i'–i ray lie to the factor vertex. Thus in the PMF structure quite distorted looking activity triangles may emerge (e.g. the triangle in Figure 3 connecting points 1, 2, and 3), but there must be the balance among the unintensive factors shown by relationship (8).

The point of intersection (Q) of the three rays passing through the activity vectors represents the technique used to produce the "mobile" factor, $[a_{1M}, \ldots, a_{nM}]'$, normalized such that $\sum_i a_{iM} = 1$ (so that it lies in the triangular simplex). Each activity represents a positive convex combination of the origin representing the factor used intensively in that industry and point Q.

Thus:

$$\text{(9)} \qquad \begin{bmatrix} a_{1j} \\ a_{2j} \\ \vdots \\ a_{jj} \\ \vdots \\ a_{nj} \end{bmatrix} = c_{jj} \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 1 \\ \vdots \\ 0 \end{bmatrix} + (1-c_{jj}) \begin{bmatrix} a_{1M} \\ a_{2M} \\ \vdots \\ a_{jM} \\ \vdots \\ a_{nM} \end{bmatrix}$$

In these normalized terms the weight attached to point Q in the j^{th} activity, a_{Mj}, is just $(1-c_{jj})$.

The Kemp–Wegge condition is both necessary and sufficient for the Stolper–Samuelson result in the n = 3 case. To illustrate, we use the Leamer triangle in Figure 4 to construct a set of activities satisfying the Kemp–Wegge conditions. Three arbitrary rays have been drawn from each factor origin such that they do not intersect at a common point, as in the PMF structure. Instead, the shaded triangle $E_1E_2E_3$ is formed by these rays. Now let the industry activities be shown by points 1, 2, 3, where the i^{th} activity vector lies on the ray from the i^{th} origin closer to the origin than corner point E_i. As can easily be checked, the industry triangle 1–2–3 satisifies the strong Stolper–Samuelson condition. (Figure 4 can also be used to illustrate that activities (1", 2", 3"), such that each activity lies on the ray from the factor's origin, but on the other side of the shaded triangle, satisifies the conditions of the Inada case (1971). This latter case is identified with a Metzler matrix whereby the inverse of the activity matrix exhibits a negative diagonal and positive off–diagonal elements.)

The difficulty with the Kemp–Wegge conditions in higher dimensions is that in general the n rays would not define a convex subspace in the n–1 dimensional tetrahedron, so that this procedure breaks down. By contrast, in the PMF structure rays from each origin all intersect at a common point which exhibits the composition of inputs required to produce the "mobile" factor.

The strong Stolper–Samuelson property is violated if there exists some factor not used at all in some industry. For example, suppose a_{21} is zero. In Figure 1 this would imply that the activity vector for industry 1 lies on the 1'–3' edge, e.g. at point g. Clearly if the activity vectors 2 and 3 lie strictly in the interior of the simplex, the Stolper–Samuelson properties fail. In particular, an increase in the endowment of factor 1 would require an actual increase in the output of commodity 2. In terms of the construction in Figure 3, the intensity rays through each vertex could meet at a common point, Q, only if a_{31} were also to equal zero. This is the case illustrated in Figure 5. In such a case an increase in the endowment of factor 1 could be absorbed by an increase in the output of commodity 1 and a reduction in commodity 3's output, with no change required in the output of commodity 2. Alternatively phrased, an increase in the price of the second commodity would leave unchanged the returns to factors 1 and 3. Thus a zero in the original 3 x 3 A matrix, balanced by another zero (so that one factor is used only in the industry sharing its number) can lead to an inverse with a pair of zeroes in the off-diagonal elements, with no off-diagonal element actually positive. This would represent a borderline case of the Produced Mobile Factor Structure.

III. Concluding Remarks

As Uekawa (1971, 1979) and others have demonstrated, severe restrictions are imposed on the production structure of an n x n no-joint-production economy in order that it satisfy the strong Stolper-Samuelson property whereby an increase in any single commodity price causes one factor's return to rise by a greater relative amount and all other factor returns to fall. The device of showing factor intensities in a 3 x 3 case by means of a two dimensional simplex – what we have termed the Leamer triangle – has allowed us to illustrate the restrictions placed on input proportions for two commodities given arbitrary proportions for the first ~~two~~ commodity. This illustration reveals the condition developed by Kemp and Wegge involving binary comparisons of factor proportions, a necessary condition for any Stolper-Samuelson structure which, in the 3 x 3 case, is also sufficient.

The Produced Mobile Factor (PMF) structure provides a case sufficient to guarantee Stolper-Samuelson properties for the general n-dimensional case. The PMF structure posits that each industry uses a particular factor more intensively than any others, and its use of unintensive factors can be viewed as the use of an intermediate input which is produced by all the n primary factors in the system. Such a structure leads to a special restriction on the ratios in which unintensive factors are used, and a stronger restriction than that of Kemp and Wegge. It is a structure which serves to emphasize the balance required among each sector's techniques involving unintensive factors, as well as the asymmetry between intensive and unintensive factors. The strongest example of the PMF structure highlights this feature; it would be the case in which the distributive share of factor $\underline{i}$ in industry $\underline{i}$ is the same for all $\underline{i}$, and exceeds the value of the share of factor $\underline{i}$ in all industries, $j \neq i$, with such off–diagonal shares being equal throughout the economy. The Leamer triangle would be a shrunken replica of the simplex itself. The more general PMF structure reveals how far this triangle can be stretched and reshaped without sacrificing the strong Stolper–Samuelson properties.

In concluding it is worth remarking that the basic question raised by Stolper and Samuelson in their original article can be reformulated in such a fashion that much more general structures are permissible. As queried in Jones (1985, 1987), what conditions suffice to guarantee that any pre–selected factor can have its real return unambiguously raised by a simple alteration in relative commodity prices? The answer is that this can always be done in production structures in which joint production is ruled out (and each commodity requires at least a pair of inputs) and in which the number of commodities is at least as great as the number of factors. Unlike strong Stolper–Samuelson structures, however, the required change in relative commodity prices may benefit more than one factor and may require more than one commodity to experience a price rise. From the standpoint of political economy, these extra caveats may not be troublesome. A general interpretation of Stolper–Samuelson results is that factoral income distribution is severely affected by relative commodity price changes, and this is a property that rests fundamentally on the assumption that many factors combine to produce individual commodities in separate activities. Much stricter requirements must be imposed in order to get the uniformities characterizing the strong Stolper–Samuelson Theorem.

REFERENCES

Chipman, John (1969): "Factor Price Equalization nd the Stolper–Samuelson Theorem", International Economic Review, 10:399–406.

Inada, Ken–Ichi (1971): "The Production Coefficient Matrix and the Stolper–Samuelson Condition", Econometrica, 39:219–40.

Jones, Ronald (1971): "A Three–Factor Model in Theory, Trade and History", ch. 1 in J.N. Bhagwati et. al, Trade, Balance of Payments, and Growth (North Holland, Amsterdam).

Jones, Ronald (1985): "Relative Prices and Factor Rewards: A Reinterpretation", Economics Letters, 19, 47–49.

Jones, Ronald (1987): "Heckshe–Ohlin Trade Theory", in Eatwell, et. al., eds, The New Palgrave, pp. 620–627.

Jones, Ronald and Sugata Marjit (1985): " A simple Production Model with Stolper–Samuelson Properties", International Economic Review, 26:565–567.

Kemp, Murray and Leon Wegge (1969): "On the Relation between Commodity Prices and Factor Rewards", International Economic Review, 10:407–413.

Leamer, Edward (1987): "Paths of Development in the Three–Factor, n – Good Equilibrium Model", Journal of Political Economy, 95, 961–999.

McKenzie, Lionel (1955): "Equality of Factor Prices in World Trade", Econometrica, 23:239–257.

Samuelson, Paul A. (1953): "Prices of Factors and Goods in General Equilibrium", Review of Economic Studies, 21:1–20.

Samuelson, Paul (1971): "Ohlin was Right", Swedish Journal of Economics, 73:365–384.

Stolper, W. and P.A. Samuelson (1941): "Protection and Real Wages", Review of Economic Studies, 9:58–73.

Uekawa, Yasuo (1971): "Generalization of the Stolper–Samuelson Theorem", Econometrica, 39:197–213.

Uekawa, Yasuo (1979): " On the Concepts of Factor Intensities and the Relation Between Commodity Prices and Factor Rewards", in Green and Scheinkman, eds, General Equilibrium Growth, and Trade (Academic Press, New York), 333–346.

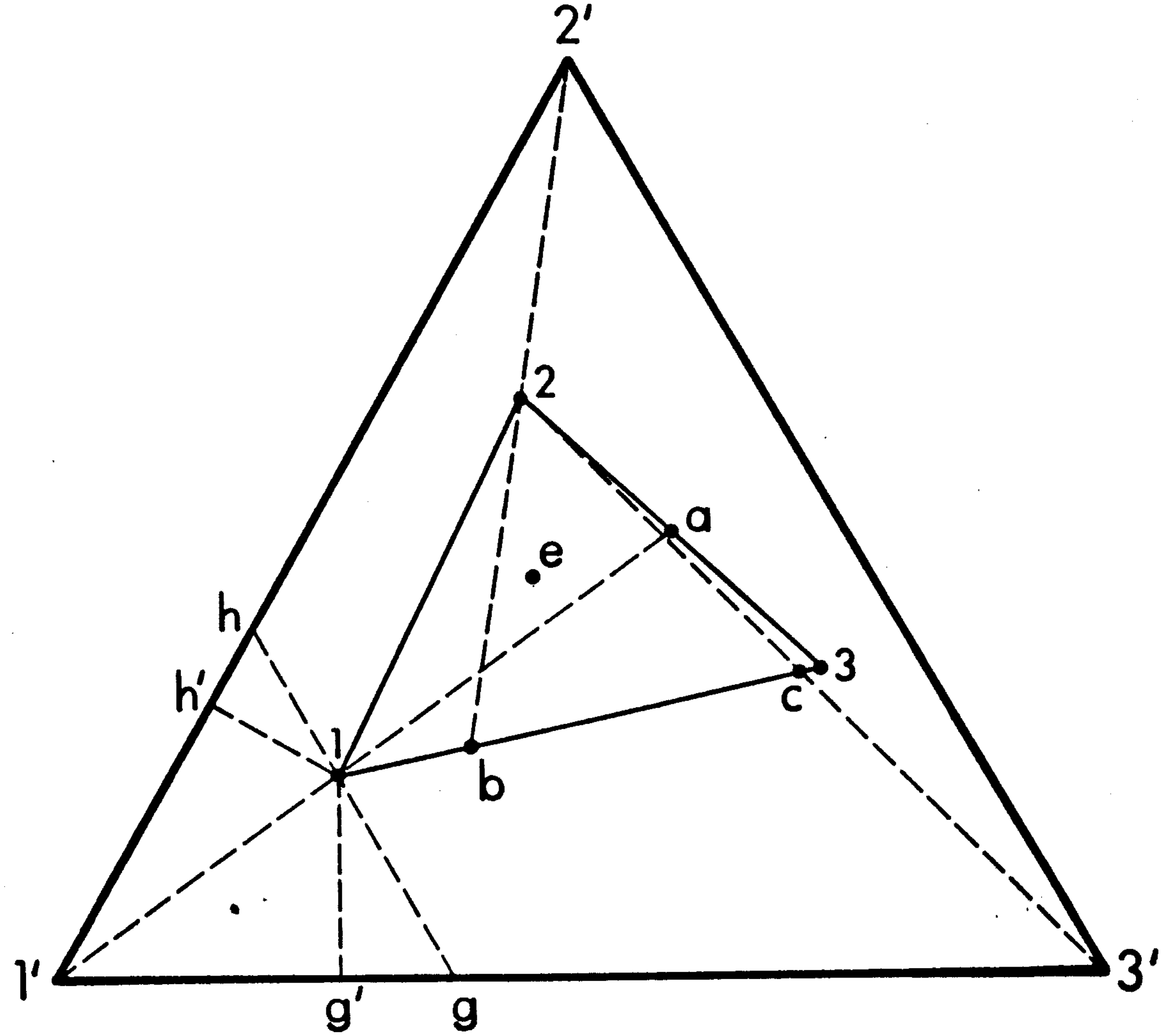

Figure 1

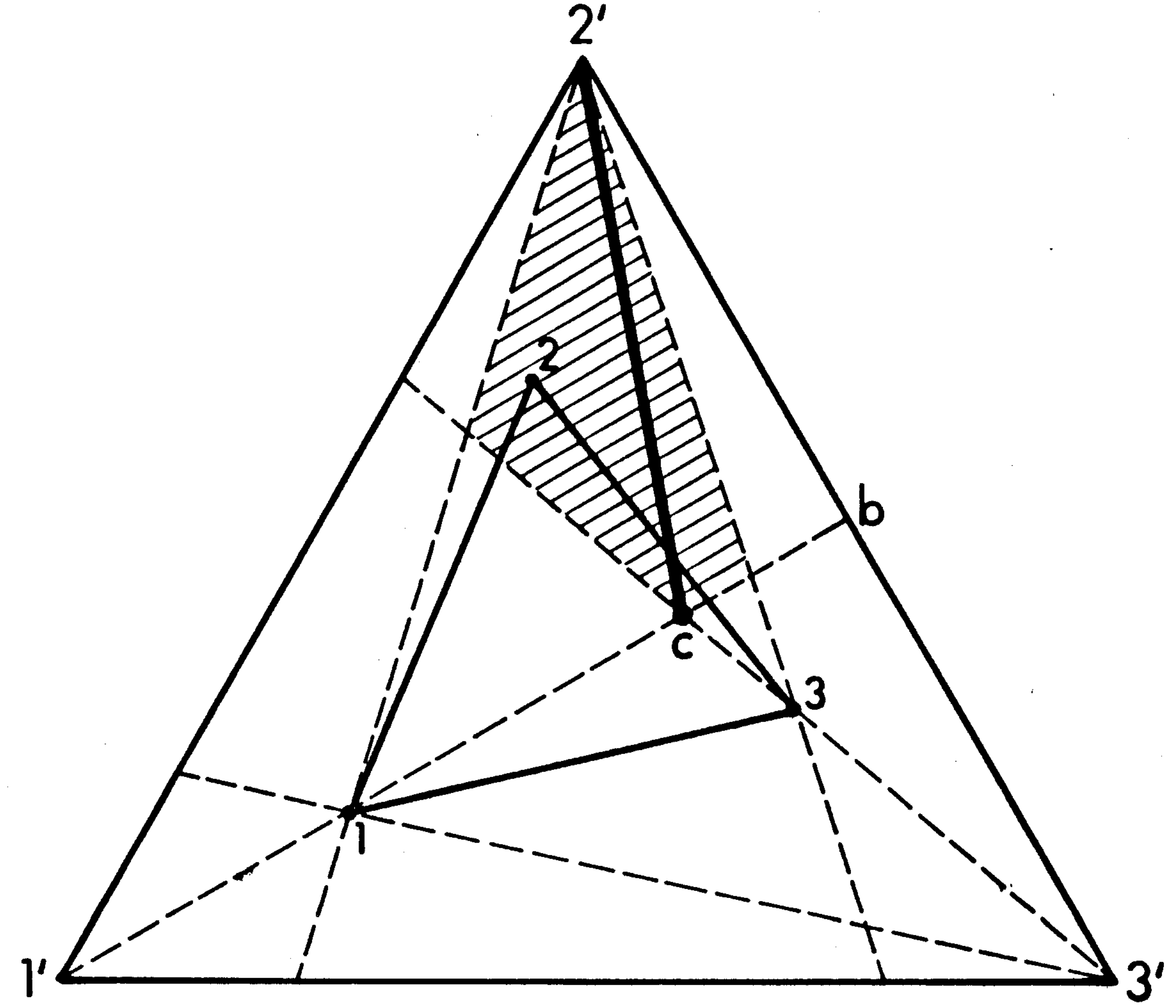

Figure 2

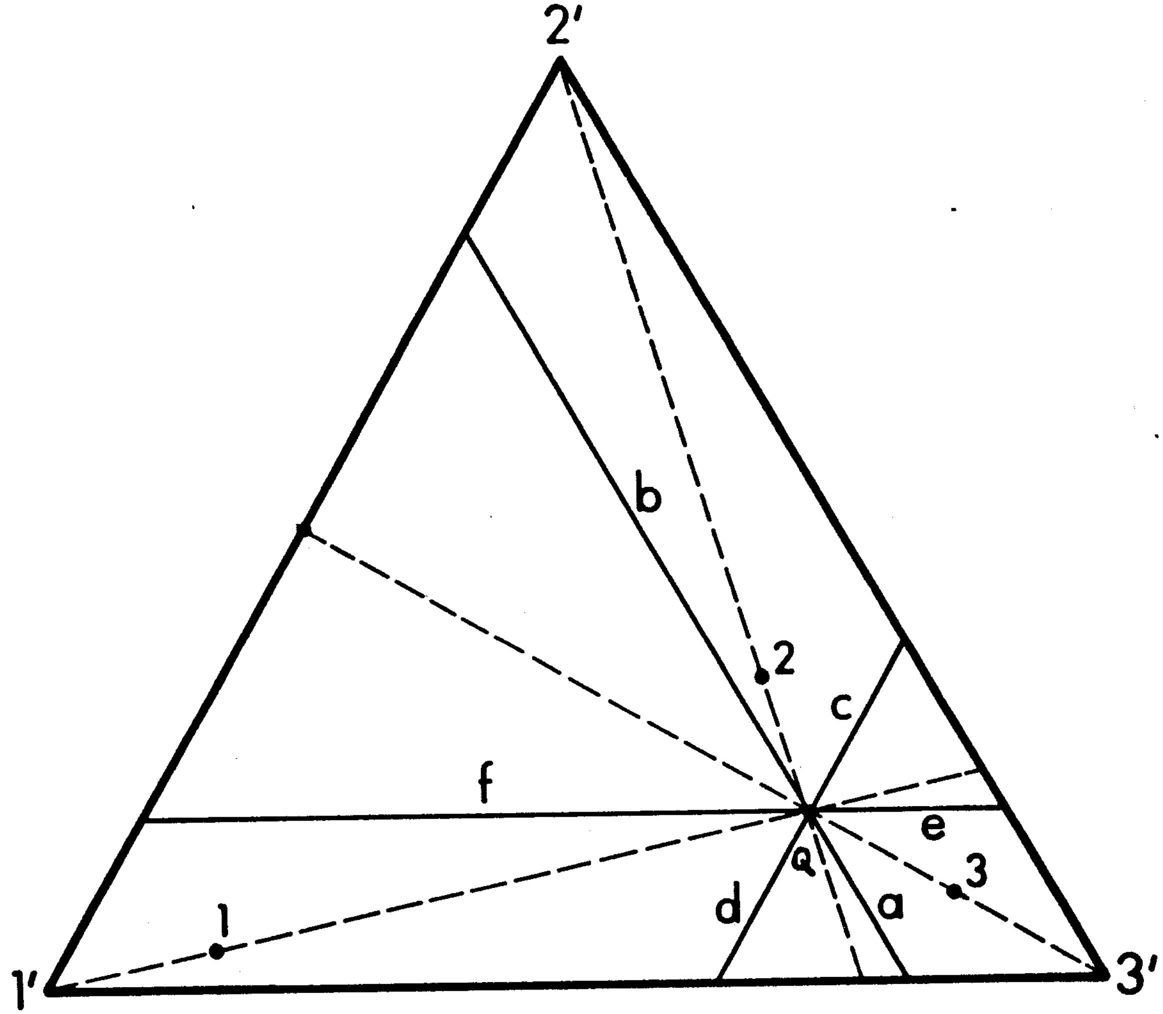

Figure 3

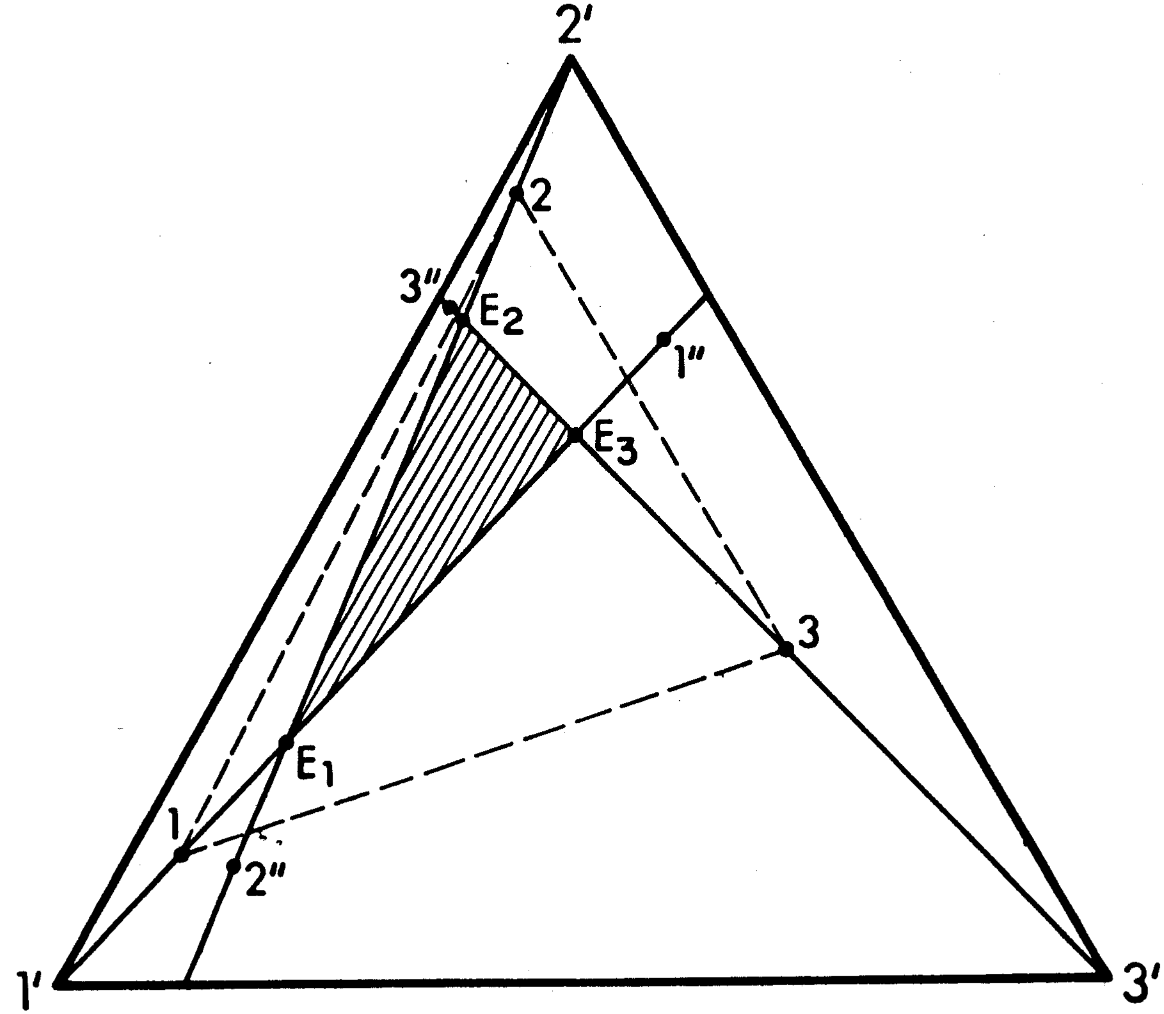

Figure 4

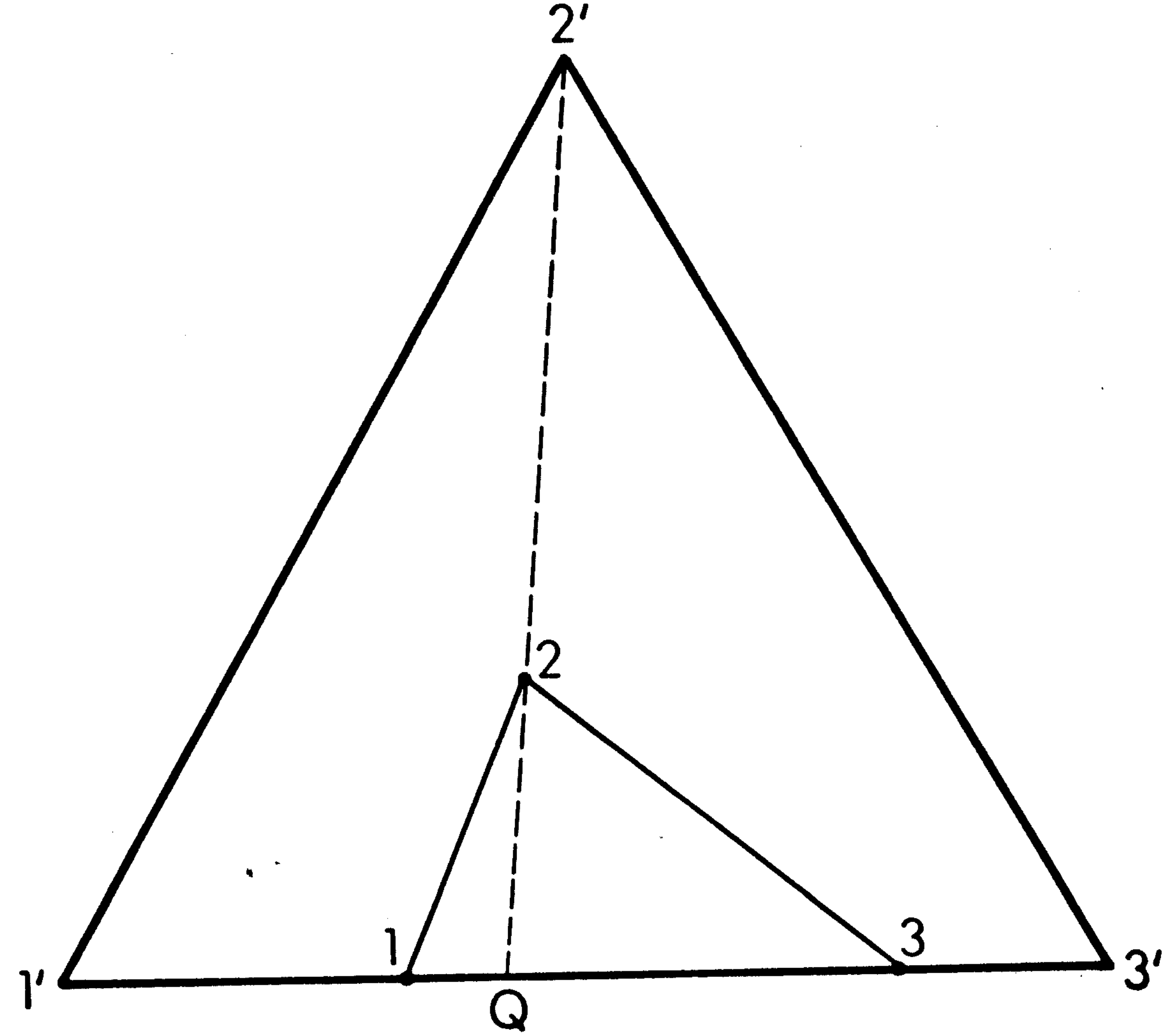

Figure 5