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By

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Engel's Law and Cointegration

Abstract

In cross sectional data, it is widely observed that a higher share of total expenditure goes to food for poorer households than is the case for richer households. A time series counterpart of this observation, Engel's law, is that the expenditure share on food declines as the economy grows. The purpose of the present paper is to test if the addilog utility function proposed by Houthakker (1960) can explain both of these cross sectional and time series observations simultaneously. Ogaki and Park's (1989) cointegration approach is used to estimate preference parameters governing income elasticities. Information in stochastic and deterministic trends in time series data is exploited in this approach. For most of the pairs of goods examined in the present paper, the ratio of income elasticities implied by our estimates of the preference parameters is close to Houthakker and Taylor's (1970) estimates from cross-sectional data. Thus the estimated addilog utility function is consistent with both observations.

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1. Introduction

In cross sectional data, it is widely observed that a higher share of total expenditure goes to food for poorer households than is the case for richer households (see, e.g., Houthakker [1957] and Prais and Houthakker [1971]). A time series counterpart of this observation, Engel's law, is that the expenditure share on food declines as the economy grows. The purpose of the present paper is to test if the addilog utility function proposed by Houthakker (1960) can explain both of these cross sectional and time series observations simultaneously. Ogaki and Park's (1989) cointegration approach is used to estimate preference parameters governing income elasticities. Information in stochastic and deterministic trends in time series data is exploited in this approach. For most of the pairs of goods examined in the present paper, the ratio of income elasticities implied by our estimates of the preference parameters is close to Houthakker and Taylor's (1970) estimates from cross-sectional data. Thus the estimated addilog utility function is consistent with both observations.

If the relative of food and automobiles does not exhibit a trend and if food consumption of automobiles grow faster than consumption of food as the economy grows, economists will infer that the income elasticity for food is lower than that automobiles. Thus stochastic and deterministic time trends in consumption expenditures and prices contain identifying information about preference parameters governing income elasticities. To formalize this idea, a restriction will be derived on trends of economic variables from the first order condition that equates the relative price with the marginal rate of substitution. This restriction states that consumption of a good with higher income elasticity should grow faster than consumption of a good with lower income elasticity, in the long run, after correcting for the effect of the trend in the relative price. In the terminology of Engle and Granger (1987), the time series of the logarithm of the relative price and two time series of the logarithms of consumption are cointegrated. This restriction forms the basis of the cointegration approach.

Ogaki and Park (1989) showed that the cointegration approach allows for liquidity constraints, aggregation over heterogeneous consumers, unknown preference parameters, and a general form of time-nonseparability in preferences. The present paper shows that the cointegration approach also allows for nonseparability across goods as long as time separability is assumed. As an identifying assumption for the relative risk aversion coefficient for nondurable consumption, Ogaki and Park maintained the hypothesis of separability across goods.

The econometric procedures used in the present paper are the same as those used by Ogaki and Park (1989). These procedures incorporate recent improvements to standard econometric procedures for cointegrated systems. First, our estimators of cointegrating vectors have asymptotic distributions that can be essentially considered as normal distributions. In the standard procedures, the estimators of cointegrating vectors have nonstandard distributions. Second, we test the first order condition that implies cointegration by testing the null of cointegration rather than the null of no cointegration. Third, we utilize what Ogaki and Park call the deterministic cointegration restriction for estimation and testing. This restriction has been neglected in the standard procedures. Fourth, and finally, we utilize information in the long run correlation of the disturbances of regressions by the Seemingly Unrelated Canonical Cointegrating Regressions (*SUCCR*) procedure.

2. *The Cointegration Approach*

The Stationarity Restriction

The Addilog Utility Function

Consider an economy with n goods. Suppose that a representative consumer maximizes the lifetime utility function

$$(2.1) \qquad U = E_0[\sum_{t=0}^{\infty} \beta^t u(t)]$$

at period 0, where $E_t(\cdot)$ denotes expectations conditional on the information available at period t. The intra-period utility function over n goods is assumed to be of a monotone transformation of the addilog utility function:

$$(2.2) \qquad u(t)= f_t(\sum_{i=1}^{n} \sigma_i(t) \frac{C_i(t)^{1-\alpha_i} -1}{1-\alpha_i}).$$

where $\alpha_i>0$ for $i=1,2,\ldots,n$. When $\alpha_i = 1$, we interpret $C_i(t)^{1-\alpha_i}/(1-\alpha_i)$ to be $\log(C_i(t))$. Here $C_i(t)$ is the real consumption expenditure on the i-th good at period t. Nonseparability across goods are allowed by an arbitrary monotone transformation f_t with $f_t'>0$. This formulation is general enough to include the nonseparable utility function used by Mankiw, Rotemberg, and Summers (1985) for consumption and leisure as a special case. The stochastic process $\{\sigma_1(t), \sigma_2(t)\}$, which is assumed to be (strictly) stationary, represents preference shocks. A special case is that $\sigma_i(t)$ is constant over time for i=1,2. The representative consumer is assumed to be endowed with $C_i^e(t)$ units of the i-th good.

Let $P_i(t)$ be the price of the ith good. We take the first good as a numeraire for each period: $P_1(t)\equiv 1$. Let $I_{A(t)}$ be the indicator function for a set $A(t)$ in the information available at period t. Let $\{\eta(t):0\leq t<\infty\}$ be the price process such that $E_0\{\sum_{t=0}^{\infty}\beta^t\eta(t)I_{A(t)}\}$ is the price of a contingent claim to one unit of the numeraire good at each period t for $t\geq 0$ if $A(t)$ happens in terms of the numeraire good at period 0 (see Hansen [1987] for a similar treatment of the notion of the price process). Then the budget constraint is

$$(2.3)\qquad E_0\{\sum_{t=0}^{\infty}\eta(t)\sum_{i=1}^{n}P_i(t)C_i(t)\}\leq W(0),$$

where $W(0)$ is the value of the wealth the consumer has at period 0 in terms of the numeraire good at period 0.

The consumer is assumed to maximizes (2.1) subject to (2.3). In an equilibrium, $C_i(t)=C_i^*(t)$ must be satisfied. Let $I(t)=\sum_{i=1}^{n}P_i(t)C_i^*(t)$ be the total consumption expenditure at period t. For $\{C_i^*(t):t\geq 0\}$ to be optimal, $C_i^*(t)$ must solve the optimization problem to maximize $u(t)$ subject to the intra-period budget constraint $I(t)=\sum_{i=1}^{n}P_i(t)C_i(t)$. The first order necessary conditions for this intra-period optimization problem include

$$(2.4)\qquad f_t'\sigma_i(t)\{C_i^*(t)^{-\alpha_i}\} = \lambda(t)P_i(t),$$

where $\lambda(t)$ is the Lagrange multiplier for the intra-period budget constraint. Taking the ratio of (2.4) yields

$$(2.5)\qquad P_i(t) = \frac{\sigma_i(t)\{C_i^*(t)^{-\alpha_i}\}}{\sigma_1(t)\{C_1^*(t)^{-\alpha_1}\}}$$

for $i=2,\ldots,n$. Since the first good is the numeraire, $P_i(t)=P_i(t)/P_1(t)$ is the relative price between the i-th good and the first good.

Taking the natural logarithm of both side of (2.5) yields

$$(2.6) \qquad p_i(t) - \alpha_1 c_1^*(t) + \alpha_i c_i^*(t) = log(\sigma_i(t)/\sigma_1(t))$$

where $p_i(t)=log(P_i(t))$, $c_i^*(t)=log(C_i^*(t))$. Thus the first order condition (2.5) implies a restriction that $p_i(t) - \alpha_1 c_1^m(t) + \alpha_i c_i^m(t)$ be stationary for $i=2,\ldots,n$. We shall call this restriction the stationarity restriction.

The Cobb-Douglas Utility Function

Ogaki and Park (1989) assumed additive separability across goods in a similar model and showed that the stationary restriction is robust to time-nonseparability of preferences. In general, the stationary restriction is not robust to time-nonseparability when the additive separability across goods is not assumed. There is an important special case where time-nonseparability and nonseparability across goods can be allowed simultaneously. We will discuss this special case.

Replacing (2.2), let us consider a version of the Cobb-Douglas utility function:

$$(2.2') \qquad u(t)= \sigma_0(t)(\prod_{i=1}^{n} S_i(t)^{\sigma_i})$$

where $\sigma_i>0$ for $i=1,\ldots,n$ and $\sigma_0(t)$ is a stationary preference shock. Here $S_i(t)$ is service flow from consumption purchases of the i-th good ($i=1,\ldots,n$). Purchases of consumption goods and service flows are related by

$$(2.7) \qquad S_i(t) = A^i(L)C_i(t) = a_0^i C_i(t) + a_1^i C_i(t-1) + a_2^i C_i(t-2)+ \ldots,$$

where $C_i(t)$ is the real consumption expenditure for the ith good at period t. We assume that the representative consumer is endowed with $C_i^*(t)$ units of the i-th good at period t. We denote values of $S_i(t)$ obtained when $C_i(\tau)=C_i^*(\tau)$ for all $\tau\le t$ by $S_i^*(t)$. We assume that $A^i(z)$ satisfies the condition that the life time utility U evaluated at $S_i(t)=S_i^*(t)$ is finite. This type of specification for time-nonseparability has been used by Hayashi (1985), Eichenbaum, Hansen, and Singleton (1988), Eichenbaum and Hansen (1987), and Heaton (1988) among others. Note that the purchase of one unit of the ith good at period t increases $S_i(\tau)$ by $a^i_{\tau-t}$ units for $\tau\ge t$. Note that (2.2′) is a special case of (2.2) with $f_t(\cdot)=\sigma_0(t)exp(\cdot)$, $\alpha_i=1$, and $\sigma_i(t)=\sigma_i$ for $i=1,\dots,n$ if preferences are time-nonseparable ($S_i(t)=C_i(t)$).

The first order necessary conditions include

$$(2.8)\qquad P_i(t) = \frac{\partial U/\partial C_i(t)}{\partial U/\partial C_1(t)} = \frac{E_t[\sum_{\tau=0}^{\infty}\beta^\tau \partial u(t+\tau)/\partial C_i(t)]}{E_t[\sum_{\tau=0}^{\infty}\beta^\tau \partial u(t+\tau)/\partial C_1(t)]}$$

$$= \frac{E_t[\sum_{\tau=0}^{\infty}\beta^\tau \sigma_0(t+\tau)a^i_\tau \sigma_i u(t+\tau)/S_i(t+\tau)]}{E_t[\sum_{\tau=0}^{\infty}\beta^\tau \sigma_0(t+\tau)a^1_\tau \sigma_1 u(t+\tau)/S_1(t+\tau)]}$$

We take $C_i(t)=C_i^*(t)$ for $i=1,2$ and $-\infty<t<\infty$ as an equilibrium condition. In an equilibrium, relation (2.8) is satisfied with the equilibrium values of $S_i(t)$, $S_i^*(t)$.

We need the following assumption to insure that the ratio of $S_i(t)$ and $C_i(t)$ to be stationary.

Assumption 1: The stochastic process $\{C_i^*(t)/C_i^*(t-1):-\infty<t<\infty\}$ is stationary for $i=1,\dots,n$.

Under Assumption 1, the process $\{C_i^*(t+\tau)/C_i^*(t):-\infty<t<\infty\}$ is also stationary for any fixed integer τ because $C_i^*(t+\tau)/C_i^*(t) = \{C_i^*(t+\tau)/C_i^*(t+\tau-1)\}\{C_i^*(t+\tau-1)/C_i^*(t+\tau-2)\}\cdots\{C_i^*(t+1)/C_i^*(t)\}$. It follows that the process $\{S_i^*(t+\tau)/C_i^*(t):-\infty<t<\infty\}$ is also stationary for any fixed τ because the right hand side of

$$(2.9) \qquad S_i^*(t+\tau)/C_i^*(t) = a_0^i C_i^*(t+\tau)/C_i^*(t) + a_1^i C_i^*(t+\tau-1)/C_i^*(t) + a_2^i C_i^*(t+\tau-2)/C_i^*(t) + \ldots$$

is stationary. It follows that $\{S_i^*(t+\tau)/S_i^*(t):-\infty<t<\infty\}$ and $\{u(t+\tau)/u(t):-\infty<t<\infty\}$ in an equilibrium is stationary for any τ. We also make an extra assumption that the growth rates of consumption are jointly stationary with the state variables on which the conditional expectations are based. Then $\{P_i(t)C_i^*(t)/C_1^*(t)]:-\infty<t<\infty\}$ is stationary because the right hand side of

$$(2.10) \qquad \frac{P_2(t)C_i^*(t)}{C_1^*(t)} = \frac{E_t[\sum_{\tau=0}^{\infty}\beta^\tau\sigma_0(t+\tau)a_\tau^i\sigma_i\{u(t+\tau)/u(t)\}\{C_i^*(t)/S_i^*(t+\tau)\}]}{E_t[\sum_{\tau=0}^{\infty}\beta^\tau\sigma_0(t+\tau)a_\tau^1\sigma_1\{u(t+\tau)/u(t)\}\{C_1^*(t)/S_1^*(t+\tau)\}]}$$

is stationary. Taking the natural logarithm of the left hand side, we conclude that $p_i(t) - c_1^*(t) + c_i^*(t)$ is stationary for $i=2,\ldots,n$. Thus the stationarity restriction is satisfied with $\alpha_i=1$ for $i=1,..,n$ in this case.

Income Elasticities

In this subsection, we show that the curvature parameters, α_i's, in (2.2) govern income elasticities in the case of nonhomothetic preferences. Comparing this case with the case of the Cobb-Douglas utility function, we provide an intuitive explanation for the stationary restriction.

From (2.4), $C_i(t)=\{f_t\sigma_i(t)\}^{1/\alpha_i}\lambda(t)^{-1/\alpha_i}P_i(t)^{-1/\alpha_i}$, and hence $\partial C_i(t)/\partial\lambda = (-1/\alpha_i)C_i(t)/\lambda(t)$. It follows that $\partial C_i(t)/\partial I(t) = \{\partial C_i(t)/\partial\lambda(t)\}\{(\partial\lambda(t)/\partial I(t)\} = (-1/\alpha_i)C_i(t)\{\partial\lambda(t)/\partial I(t)\}$. Let $e_i(t) = \{\partial C_i(t)/\partial I(t)\}\{I(t)/C_i(t)\}$ be the income elasticity for the i-th good. Then $e_i(t) = (1/\alpha_i)\kappa(t)$ where $\kappa(t)$ does not depend on i. Hence the ratio of the income elasticities of the i-th good and the j-th good, $e_i(t)/e_j(t)$, is α_j/α_i. Thus the i-th good is more income elastic than the j-th good if $\alpha_i<\alpha_j$.

To develop intuition for the stationarity restriction, imagine that the relative price $p_2(t)$ is stationary for simplicity. If preferences for the first two goods can be represented by the Cobb-Douglas utility function, then $p_2(t)-c_1(t)+c_2(t)$ does not possess any time trends, and therefore $c_1(t)$ and $c_2(t)$ must grow at the same rate in the long run. The addilog utility function implies that $p_2(t)-\alpha_1c_1(t)+\alpha_2c_2(t)$ is stationary. When $\alpha_1>\alpha_2$, the first good has a lower income elasticity than the second good and consumption for the first good can grow at a slower rate than the second good in the long run.

If at least one of the consumption series is difference stationary, then the parameters α_1 and α_2 are identified by information in trends and can be estimated by cointegrating regressions as discussed below. This is because trends of the relative price and consumption contains information about income elasticities. For example, if consumption of food is growing at a slower rate than consumption of transportation after correcting for the effect of the relative price, we can infer that the income elasticity for food has a lower income

Implications of the Stationarity Restriction

In this subsection, implications of the stationarity restriction are discussed. We focus on the relation between the first and second goods. The restriction on trend properties of the variables from the demand side is summarized by the stationarity restriction. For supply side, we need to require that at least one of the endowment process is difference stationary for identification of preference parameters.

Ogaki and Park's (1989) notions of the stochastic cointegration and the deterministic cointegration are useful when economic variables of interest are modeled as difference stationary with drift. The present paper focuses on integrated of order one processes and trend stationary processes around a linear deterministic trend. Suppose that the components of a vector series $Z(t)$ are difference stationary with drift. If a linear combination of $Z(t)$, $\gamma' Z(t)$ is trend stationary, the components of $Z(t)$ are said to be cointegrated with a cointegrating vector γ. It is often convenient to normalize the first element of γ to be one, so that $\gamma=[1,\gamma_x]$, and to call γ_x the normalized cointegrating vector. If $\gamma' Z(t)$ is stationary, then the cointegrating vector γ eliminates the deterministic trends as well as the stochastic trends. This restriction is called the deterministic cointegration restriction. If $Z(t)$ consists of difference stationary and/or trend stationary series, and if $\gamma' Z(t)$ does not have any deterministic trend, then the components of $Z(t)$ are said to be cotrending with a cotrending vector γ. The deterministic cointegration restriction requires the cointegrating vector to be a cotrending vector. We will see that Assumption 3 leads to a stochastic cointegrated difference stationary series that does not satisfy the deterministic cointegration restriction.

First, we consider the case where both the logarithm of the endowment of the first good and that of the second good are difference stationary:

Assumption 2a: The process $\{c_i^*(t):t\geq 0\}$ is difference stationary for $i=1,2$.

Assumption 2b: The processes $\{c_1^*(t):t\geq 0\}$ and $\{c_2^*(t):t\geq 0\}$ are not stochastically cointegrated.

Let $C_i^m(t)$ be measured consumption and $\xi_i(t)=\{C_i^m(t)-C_i^*(t)\}/C_i^*(t)$ be the ratio of the measurement error and consumption.[1] We assume that $\xi_i(t)$ is stationary. Note that this assumption, together with Assumption 2, implies that the log of measurement error, $log[C(t)-C_i^m(t)]_i^*$ is difference stationary with stochastic trends. Taking the log of both sides of $C_i^m(t)=[1+\xi_i(t)]C_i^*(t)$, we obtain $c_i^m(t)=c_i^*(t)+log[1+\xi_i(t)]$ where $c_i^m(t)=log[C_i^m(t)]$. It should be noted that $log[1+\xi_i(t)]$ is stationary. Then $c_i^m(t)$ is the sum of a difference stationary and stationary processes and therefore is difference stationary. Similarly, let $P_2^m(t)$ be the measured relative price and assume that $\xi_0(t)=\{P_2^m(t)-P_2(t)\}/P_2(t)$ is stationary. Since $p_i^m(t) - \alpha_1 c_1^m(t) + \alpha_i c_i^m(t) = \{p_i(t) - \alpha_1 c_1^*(t) + \alpha_i c_i^*(t)\} + + \{log[1+\xi_0(t)] - \alpha_1 log[1+\xi_1(t)] + \alpha_i log[1+\xi_2(t)\}$, the stationarity restriction implies that $p_2^m(t) - \alpha_1 c_1^m(t) + \alpha_2 c_2^m(t)$ is stationary.

Let $y(t) = p_2^m(t)$ and $X(t)= [c_1^m(t),\ c_2^m(t)]'$, using the notation introduced above. Assumption 2a implies Assumption 1 in Section 2, and hence $y(t) - \gamma_x'X(t)$ is stationary with $\gamma_x=[\alpha_1,\ -\alpha_2]'$. Assumption 2b is equivalent to an assumption that there is no 2-dimensional vector γ_x such that $\gamma_x'X(t)$ is trend stationary. Assumption 2b requires that two endowment series possess different stochastic trends. Since $y(t) - \gamma_x'X(t)$ is

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[1]We thank Adrian Pagan and Edward Prescott for helpful discussions about the formulation of measurement errors.

stationary with $\gamma_x=[\alpha_1, -\alpha_2]'$, this implies that $y(t)$, which is the sum of a difference stationary $\gamma_x' X(t)$ and a stationary process, is difference stationary. Thus the stationarity restriction implies that (i) $p_2^m(t)$ is difference stationary, (ii) $p_2^m(t)$ and $[c_1^m(t), c_2^m(t)]'$ are stochastically cointegrated with a normalized cointegrating vector $[\alpha_1, -\alpha_2]'$, and (iii) the deterministic cointegration restriction is satisfied, under Assumption 2.

Second, we consider the case where the log of the endowment of the first good is difference stationary and that of the second good is trend stationary:

Assumption 3: The process $\{c_1^*(t): t \geq 0\}$ is difference stationary, and the process $\{c_2^*(t): t \geq 0\}$ is trend stationary.

Assumption 3 implies Assumption 1. In this case, we let $y(t) = p_2^m(t)$, $X(t) = c_1^m(t)$, and $z(t) = c_2^m(t)$ to apply the argument in the last subsection. The stationarity restriction implies that (i) $p_2^m(t)$ is difference stationary, (ii) $p_2^m(t)$ and $c_1^m(t)$ are stochastically cointegrated with a normalized cointegrating vector $\gamma_x = \alpha_1$, and (iii) $p_2^m(t)$ and $[c_1^m(t), c_2^m(t)]$ are cotrended with a normalized cotrending vector $[\gamma_x, \gamma_Z]' = [\alpha_z, -\alpha_2]'$.

3. *Econometric Procedures*

Canonical Cointegrating Regressions

Let $X(t)$ be a k-dimensional difference stationary process:

$$(3.1) \qquad X(t) - X(t-1) = \mu_x + \epsilon_x(t)$$

for $t \geq 1$ where μ_x is a k-dimensional vector of real numbers where $\epsilon(t)$ is stationary with mean zero. Let $y(t)$ be a scalar difference stationary process:

$$(3.2) \qquad y(t) - y(t-1) = \mu_y + \epsilon_y(t)$$

Suppose that $y(t)$ and $X(t)$ are stochastically cointegrated with a normalized cointegrating vector γ_x, and the components of $X(t)$ are not cointegrated. Then we can apply the Canonical Cointegrating Regressions (*CCR*) procedure developed by Park (1988) to

$$(3.3) \qquad y(t) = \theta_c + \mu_c t + \gamma_x' X(t) + \epsilon_c(t)$$

If $y(t)$ and $X(t)$ satisfies the deterministic cointegration restriction, then μ_c is zero and the *CCR* is applied to

$$(3.4) \qquad y(t) = \theta_c + \gamma_x' X(t) + \epsilon_c(t).$$

This *CCR* procedure only requires to transform data before running a regression and corrects for endogeneity and serial correlation. The *CCR* estimators have asymptotic distributions that can be essentially considered as normal distributions, so that their standard errors can be interpreted in the usual way.

An important property of the *CCR* procedure is that linear restrictions can be tested by χ^2 tests which are free from nuisance parameters. We can use χ^2 tests in a regression with spurious deterministic trends added to (3.4) to test for stochastic and deterministic cointegration. For this purpose, the *CCR* procedure is applied to a regression

$$(3.5) \qquad y(t) = \theta_c + \sum_{i=1}^{q} \eta_i t^i + \gamma_x' X(t) + \epsilon_c(t) .$$

Let $H(p,q)$ denote the standard Wold statistic to test the hypothesis

$\eta_p = \eta_{p+1} = \ldots = \eta_q = 0$. Then $H(p,q)$ converges in distribution to a χ^2_{p-q} random variable under the null of cointegration. In particular, the $H(0,1)$ statistic tests the hypothesis $\mu_c = 0$ in (3.3) and thus tests the deterministic cointegrating restriction. If $y(t)$ and $X(t)$ are not stochastically cointegrated, then $\epsilon_c(t)$ is difference stationary for any vector of real numbers used as γ_x in (3.4). In this case, (3.4) is a spurious regression and $H(1,q)$ statistics diverge in probability. Hence the $H(1,q)$ tests are consistent against the alternative of no stochastic cointegration.

Let us consider a cointegrated system involving a trend stationary process. Let $z(t)$ be a trend stationary process:

$$z(t) = \theta_z + \mu_z t + \epsilon_z(t), \tag{3.6}$$

where $\epsilon_z(t)$ is stationary with zero mean and $\mu_z \neq 0$. Suppose that an economic model lead to a restriction that $y(t) - \gamma_x' X(t) - \gamma_z z(t)$ is stationary. Since $y(t) - \gamma' X(t) - \gamma_z z(t) = -\gamma_z \theta_z + (\mu_y - \gamma_x' \mu_x - \gamma_z \mu_z)t + \{y^0(t) - \gamma_x' X^0(t)\}$, this restriction implies that

$$\mu_y = \gamma_x' \mu_x + \gamma_z \mu_z \tag{3.7}$$

and that $y(t)$ and $X(t)$ are stochastically cointegrated with a normalized cointegrating vector γ_x.

For a cointegrated system with a trend stationary process, we can apply the *CCR* to a system of Seemingly Unrelated Regressions *(SUR)* consisting of (3.3) and (3.7) to estimate γ_x and γ_z as in Park and Ogaki (1989). We call this system the Seemingly Unrelated Canonical Cointegrating Regressions *(SUCCR)*. We apply the *GLS* to the system of *SUCCR*.

4. *Trend Properties of the Data*

In this section, we test empirical validity of Assumptions 2 and 3. In the first subsection, we explain the data used in this paper. In the second subsection, we report results of tests for difference stationarity and trend stationarity of time series of real consumption expenditures and relative prices. In the third subsection, we report results of tests for Assumption 2a.

The Data

Seasonally adjusted monthly data in the National Income and Product Accounts (NIPA) was used. Five goods used were food, clothing and shoes (clothing for short), housing, household operation (including furniture and household equipment and fuel oil and coal), and transportation (including motor vehicles and parts and gasoline and oil). Seasonally adjusted monthly data for the NIPA was taken from the PCE magnetic tape of the NIPA prepared by the Bureau of Economic Analysis, U.S. Department of Commerce. For measured consumption in the model, real per capita consumption expenditures were constructed by dividing personal consumption expenditures in constant 1982 dollars by the total population including armed forces overseas obtained from the CITIBASE.[2] The implicit deflator was used as the price for each consumption series. The implicit deflators for each series was constructed by dividing personal consumption expenditure in current dollars by that in constant 1982 dollars. The sample period was from February 1959

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[2] We incorporated the revision of population estimates reported in *Current Population Reports* (Series p-25, No.1036) issued in March 1989 by Bureau of the Census after the release of the version of the CITIBASE we used.

to December 1986 unless otherwise noted. Hence each time series consists of three hundred thirty five observations.

Tests for Difference and Trend Stationarity

Tests for Time Series of Consumption Expenditures

Both Assumption 2 and Assumption 3 require that $c_i^m(t)$ is stationary after first differencing (note that trend stationary processes must also satisfy this requirement.) In figures 1-5, we plot the first differences of (the logarithms) of real per capita consumption for food, clothing, housing, household operation, and transportation. These series show no apparent nonstationarity in these figures except for the series for housing in figure 3. Variance of the series for housing appears to increase substantially after the beginning of 1968 and to decrease again after the beginning of 1982. Since 1967 is one of the benchmark years used to construct monthly consumption series (see, e.g., Byrnes, Donahoe, Hook, and Parker [1979, p.24]), this seems to be caused by nonstationary measurement errors that we do not allow. Hence our empirical results related with this series should be interpreted with caution.

Our next results are concerned with discrimination between trend stationarity and difference stationarity of consumption series. Let $\{X(t)\}$ be the process of interest. We are interested in whether $X(t)$ is difference stationary or trend stationary. Consider an *OLS* regression

$$(4.1) \qquad X(t) = \sum_{i=0}^{q} \hat{\eta}_i t^i + \hat{\epsilon}(t),$$

and define $\hat{\sigma}^2 = (1/T)\sum_{t=1}^{T} \hat{\epsilon}(t)^2$. Let $F(p,q)$ denote the standard Wald test statistic in regression (4.1) for the null hypothesis $\eta_{p+1}=\eta_{p+2}=,\ldots,=\eta_q=0$. Let $J(p,q) = (1/T)F(p,q)$ and $G(p,q) = (\hat{\sigma}^2/\hat{\Omega})F(p,q)$, where $\hat{\Omega}$ is defined by (3.1) for $\hat{\epsilon}(t)$ in (4.1). Then $J(1,q)$ converges in distribution to a nondegenerate random variable under the null hypothesis that $X(t)$ is difference stationary; $G(1,q)$, to a χ^2_{q-1} random variable under the null hypothesis that $X(t)$ is trend stationary (see Park and Choi [1988]). Hence $J(1,q)$ can be used to test the null of difference stationarity against the alternative of trend stationarity. We reject the null of difference stationarity when the $J(1,q)$ statistic is smaller than critical values tabulated by Park and Choi (1988). The $G(1,q)$ statistic can be can be used to test the null of trend stationarity against the alternative of difference stationarity. We reject the null of trend stationarity when the $G(1,q)$ statistic is larger than critical values. These tests are consistent.

For the null of stationarity, we also used Z_α and Z_t test statistics of Phillips and Perron (1988) and Ouliaris, Park, and Phillips (1988) that correct Dickey and Fuller (1979) statistics for serial correlation. Critical values for Z_α and Z_t statistics are the same with those for Dickey and Fuller (1979) test statistics tabulated in Fuller (1976, p.371 and p.373) by construction. Consider an *OLS* regression

$$(4.2) \qquad X(t) = \hat{\eta}_0 + \hat{\eta}_1 t + \hat{\alpha}X(t-1) + \hat{\epsilon}(t).$$

The Z_α statistic modifies $T(\hat{\alpha}-1)$ and the Z_t statistic modifies the t statistic for the hypothesis that α is equal to one. These modifications for serial correlation involve estimation of the long run variance. For reasons suggested by Park (1989), we focused on these single unit root tests rather than the joint tests for the null hypothesis that α is equal to one and η_1 is equal to zero that were analyzed by Dickey and Fuller (1981) among others.

A serious problem about the Z_α and Z_t statistics are size distortions in small samples. Simulations reported in Phillips and Perron (1988) showed that the size distortion problem could be substantial when the stationary component of the series is small relative to the random walk component in the sense of Cochrane (1988): the probability that Z_α and Z_t tests with nominal size 5 per cent reject the null hypothesis of difference stationarity may exceed 90 per cent when the null hypothesis is true (Also see Schwert [1987] for related simulation results). Simulations by Park and Choi (1988) showed that that the size distortion problem for $J(p,q)$ tests could be much less severe and could disappear at a much faster rate as the sample size increases than that for the Z_α and Z_t tests. The size distortion problem for the Z_α and Z_t tests seems to be related with the estimation of the long run variance. It should be noted that the $J(p,q)$ tests do not require the estimation of the long run variance.On the other hand, their simulations showed that the $J(p,q)$ tests may have less power than the Z_α and Z_t tests in small samples.

Table 1 presents results of the $J(1,2)$, $J(1,3)$, ..., $J(1,6)$, Z_α and Z_t tests with the null of difference stationarity of the logarithm of real per capita consumption expenditures. According to $J(1,q)$ tests, there was no evidence against difference stationarity of consumption series for all the goods examined at the 5 per cent significance level. Two of the $J(1,q)$ tests, however, rejected the null of difference stationary food consumption at the 10 per cent level. The Z_α test and Z_t tests rejected the null of difference stationarity for food and household operation at the 1 per cent level and that for clothing at the 5 per cent level. For the results reported in table 1 for the Z_α and Z_t tests, we used the lag truncation number of 30 and the lag window of Parzen's estimates in estimating the long run variance. We also tried the lag truncation numbers of 10, 20, 40, 50, 60, 70, and 80. Our results for Z_α and Z_t tests were not sensitive to the choice of the lag truncation number that is greater than 30 in terms of the statistical inference based on the 1, 5, or 10 per cent levels. We found less evidence against the null of difference stationarity with smaller lag truncation numbers for the series that we rejected the null. Since results changed very much with smaller lag truncation numbers, we interpreted that the lag truncation number was not large enough.

Two interpretations are possible for these conflicting results between the $J(p,q)$ tests and the Z_α and Z_t tests for food, clothing, and household operation. A possibility is that the size distortion of Z_α and Z_t is the source of the problem. Since the time series for food, clothing, and household operation are not smooth, the random walk component of these series seem to be small relative to the stationary component. This means that the Z_α and Z_t tests may not be reliable for these time series. Even if these series have nonzero random walk components and are difference stationary, these tests may reject the null of difference stationary with high probability. Another possibility is that lower power of $J(p,q)$ tests in small samples is causing the problem when these series are trend stationary.

In table 2, we report results of the $G(1,2),\ldots, G(1,6)$ tests for the null hypothesis of trend stationarity. For estimation of the long run variance, Parzen's lag window was used and the lag truncation numbers of 20, 40, 60, 80, and 100 were tried. The lag truncation number used for the results in table 2 was 80. The $G(1,q)$ test results for all the series were stable for the lag truncation numbers of 60, 80, and 100 in terms of statistical inference based on the 1, 5, and 10 per cent significance levels. Most of the test statistics were not stable for the lag truncation numbers that were less than 60, indicating that the lag truncation numbers were not large enough. At least, one of the *G(1,q)* tests rejected the null of trend stationarity in favor of the alternative of difference stationarity at the 5 per cent level for clothing and housing. On the other hand, no test statistics were significant at the 5 per cent level for food, household operation, and transportation.

In the light of these results, we will employ the following specifications for the rest of the empirical results reported in the present paper. We will specify that the logarithm of real consumption series is difference stationary for clothing and housing. We will try both the specifications that the logarithm of consumption is difference stationary and that the logarithm of consumption is trend stationary for food, household operation, and transportation.

Tests for Time series of Relative Prices

As shown in Section 2, the stationarity restriction implies that the logarithm of the relative price is difference stationary under either Assumption 2 or Assumption 3. We now test this implication of the model.

Table 3 presents results of the $J(1,2),\ldots, J(1,6)$ and Z_α and Z_t tests for the logarithm of relative prices. No test statistics reported in table 3 were significant at the 10 per cent level. The lag truncation number used for the results of the Z_α and Z_t tests reported in table 3 was 30. No Z_α and Z_t test statistics showed evidence against the null hypothesis of difference stationary relative prices at the 10 per cent significance level.

This conclusion was robust against the choice of the lag truncation numbers of 10, 20, 30, 40, 50, 60, 70, 80, 90, and 100.

Table 4 presents results of the *G(1,2)*, ..., *G(1,6)* tests for the null of trend stationarity. The main purpose of table 4 is to give some ideas about small sample power of the *H(1,q)* tests as we will discuss in the next section. The lag truncation number used for the results in this table was 80. The results were not sensitive in terms of the statistical inference when the lag truncation numbers of 60 and 100 were tried. At least one the *G(p,q)* tests rejected the null of trend stationarity for all the relative prices with two exceptions: no test rejected the null at the 10 per cent level for the relative price between food and transportation and the relative price between household operation and transportation.

We conclude this subsection by summarizing empirical results in tables 3 and 4. We did not find evidence against difference stationarity for any of the relative prices we tested at the 10 per cent significance level. We found evidence against trend stationarity for most of the relative prices at the 5 per cent level.

Tests for No Stochastic Cointegration

Suppose that Assumption 2a is satisfied. Then Assumption 2b (together with the stationarity restriction) implies that the logarithm of the relative price is difference stationary as in proposition 1. Thus rather strong evidence in favor of difference stationarity of most of the relative prices supports Assumption 2b. The difference stationarity of the relative price, however, does not imply Assumption 2b because the two consumption series may be stochastically cointegrated with a normalized cointegrating vector other than $[\alpha_1, -\alpha_2]'$. For this reason, we report results of tests for the null hypothesis of no stochastic cointegration in this subsection.

Consider an *OLS* regression

$$(4.3) \qquad y(t) = \hat{\theta} + \sum_{i=1}^{q} \hat{\eta}_i t^i + \hat{\gamma}' X(t) + \hat{\epsilon}(t).$$

where $y(t)$ and $X(t)$ are difference stationary processes, and let $F(p,q)$ denote the standard Wald test statistic in regression (8.3) for the null hypothesis $\eta_{p+1}=\eta_{p+2}=,\ldots,=\eta_q=0$. Define $I(p,q) = (1/T)F(p,q)$. Ouliaris, Park and Choi (1988) showed that $I(1,q)$ converges in distribution to a nondegenerate random variable under the null hypothesis that $y(t)$ and $X(t)$ are not stochastically cointegrated. We reject the null of no cointegration when $I(p,q)$ statistics are smaller than critical values of $I(p,q)$ test statistics tabulated by Park, Ouliaris, and Choi (1988). The $I(p,q)$ tests are consistent against the alternative of stochastic cointegration. The $I(p,q)$ tests basically apply the $J(p,q)$ tests to the residual of regression (4.3). Alternatively, we can apply the Z_α or Z_t tests to the residual as in Phillips and Ouliaris (1988). We did not use these Z_α and Z_t tests because of the serious size distortion problem mentioned above.

Table 5 presents results of the $I(1,5)$ test for the null of no cointegration of $c_1^m(t)$ and $c_2^m(t)$ for various choices of the first and second goods. For each pair of consumption goods, we can choose the first good as the regressand or the second good as the regressand for the $I(1,q)$ tests. No test statistics were significant at the 10 per cent for any of the pairs of consumption goods except for the some of the pairs that involve household operation. The null of no cointegration was rejected at the 1 per cent level for the pair of household operation and transportation. The null of no cointegration was rejected at the 10 per cent level for the pair of food and household operation and the pair of household operation and housing. Thus there is strong evidence against Assumption 2b for the model with household operation and transportation if we maintain Assumption 2a. These results are compatible with Assumption 3 and results in Table 2 if the logarithm of consumption of household operation is trend stationary.

5. Empirical Results of Cointegrating Regressions

This section reports results of cointegrating regressions. The first subsection presents results when Assumption 2 is employed; the second subsection, results when Assumption 3 is employed. One of the main purposes of this section is to compare our estimate of α_1/α_2 with estimates of the ratio of income elasticities from cross-sectional data by Houthakker and Taylor (1970). The relation between the cross-sectional data and the time series data in the NIPA was discussed extensively by Houthakker and Taylor. Table 6 reports estimates of the ratio of income elasticities calculated from estimates of income elasticities in Houthakker and Taylor (1970, Table 6.5). The standard error of these estimates can be calculated by a mean-value approximation (the delta method). Since Houthakker and Taylor did not report the correlation between their estimates, we calculated the standard errors with three alternative assumptions that the correlation was minus one, zero, or one. The calculated standard errors were the largest when the correlation was assumed to be minus one. Even in this case, the standard errors were very small relative to the estimates, indicating that Houthakker and Taylor obtained very sharp estimates of income elasticities.

Canonical Cointegrating Regressions

In this subsection, we assume that all the consumption series are difference stationary, so that Assumption 2 is satisfied for each pair of consumption series. Table 7 presents *CCR* results. As is well known (see, e.g., Engle and Granger [1987]), we can choose any variable as the regressand in the cointegrated systems. For each pair of consumption series, we first chose the logarithm of consumption of the second good, c_2^m, as the regressand. With this choice of the regressand, the parameter of interest, α_1/α_2, is estimated linearly. This choice was made because the mean value approximation is often the source of severe finite sampling errors (see, e.g., Phillips and Park [1988]). Since the $G(1,q)$ test statistics were stabilized with the lag truncation numbers that were greater than 60, we used the lag truncation number of 80 for the results reported in the tables below. The lag truncation numbers of 60 and 100 were also tried. Most of our results were not very sensitive to the choice of these lag truncation numbers. The cases where the results were sensitive in terms of the statistical inference will be reported.

Table 7 reports estimates of α_1, α_2, and α_1/α_2, and the G_R test statistic for the null hypothesis that $1/\alpha_2=\alpha_1/\alpha_2=1$ from regression (3.4). The G_R statistic tests the null hypothesis of that preferences are represented by a Cobb-Douglas utility function. Table 7 also reports the $H(0,1)$ test statistic for the deterministic cointegration restriction from regression (3.5) with q=1 and the $H(1,2)$, $H(1,3)$, and $H(1,4)$ test statistics for stochastic cointegration from regression (3.5) with q= 2, 3, and 4, respectively. When none of these $H(p,q)$ tests rejected the model at the 1 per cent significance level, we applied the *CCR* with p_2^m chosen as the regressand as reported in table 7. For this choice of the regressand, table 7 reports the G_R test statistic for the null hypothesis $\alpha_1=\alpha_2=1$ that is a linear restriction on estimated parameters to test the Cobb-Douglas utility function. When none of the $H(p,q)$ tests with p_2^m as the regressand rejected the model at the 1 per cent significance level, we applied the *CCR* with c_2^m chosen as the regressand as reported in table 7. For this choice of the regressand, the reported G_R statistic tests the null hypothesis $1/\alpha_1=\alpha_2/\alpha_1=1$.

The *G(1,q)* test results reported in the last section provides some ideas about small sample power of the *H(1,q)* tests. For example, the value *G(1,q)* for p_2 will be close to the value of *H(1,q)* in the regression with p_2 as the regressand if estimated α_1 and α_2 are close to zero.

The deterministic cointegration restriction was rejected at the 1 per cent level by the *H(0,1)* test with some choice of the regressand for all the pairs of goods except for the pair of household operation and transportation. Since Assumption 2b was rejected at the 1 per cent level for the pair of household operation and transportation, there were no encouraging results for the model under Assumption 2 in terms of the formal tests.

Point estimates reported in Table 7, however, were encouraging. All the point estimates of α_1 and α_2 had theoretically correct positive sign. Our estimates of α_1/α_2 from the regressions with c_2^m as the regressand were close to Houthakker and Taylor's estimates of the ratio of income elasticities reported in Table 6 for most of the pairs of goods. Houthakker and Taylor's point estimates were within two standard errors from our point estimate covers For the pairs of food and housing, food and household operation, food and transportation, clothing and transportation, housing and household operation. Consider the interval formed by taking three standard errors (calculated with the assumption that the correlation between the two estimates of income elasticities was minus one) from Houthakker and Taylor's estimates and the interval formed by taking three standard errors form our estimates. The only pairs for which these two intervals do not overlap were the pair of food and clothing and the pair of household operation and transportation.

Overwhelming evidence was found against the Cobb-Douglas utility function ($\alpha_1=\alpha_2=1$) for all the paris of goods in terms of the G_R tests. For most of the pairs, our estimate of α_1/α_2 was significantly different from one. Hence there was evidence against the hypothesis of homothetic preferences ($\alpha_1=\alpha_2$).

Seemingly Unrelated Canonical Cointegrating Regressions

This subsection reports results under Assumption 3. Tables 8, 9, and 10 report results when the logarithm of one of consumption goods is assumed to be trend stationary and the logarithm of each of the other consumption goods is assumed to be difference stationary. Table 8 reports results when (the logarithm of) food consumption is assumed to be trend stationary; Table 9, when consumption of household operation is assumed to trend stationary; Table 10 when transportation consumption is assumed to stationary.[3] We did not reject the null of trend stationarity for food, household operation, and transportation in the last section.

Table 8, 9, and 10 present *SUCCR* results. For each pair of consumption series, we first chose c_1^m as the regressand. With this choice of the regressand, α_1/α_2 is estimated as μ_z/μ_c. Since the estimators for μ_z and μ_c converge faster than the estimator for γ_x, this is a better choice of the

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[3]Assumption 3 states the case where the first good is difference stationary and the second good is trend stationary. In the following, we sometimes choose the first good to be trend stationary and the second good to be difference stationary. Because of the symmetry, this case is not different from the case stated in Assumption 3.

regressand to estimate α_1/α_2, which is the most important parameter for our purpose. The G_R statistic tests the hypothesis $1/\alpha_1=1$ and $\mu_c=\mu_z$, which is equivalent with the hypothesis $\alpha_1=\alpha_2=1$ in this case. Hence the G_R statistic tests the null hypothesis of the Cobb-Douglas utility function. Tables 8, 9, 10 also report the *H(1,2)*, *H(1,3)*, and *H(1,4)* test statistics for stochastic cointegration from the *SUCCR* system consisting of (3.5) and (3.6) with q= 2, 3, and 4, respectively. When none of these *H(1,q)* tests rejected the model at the 1 per cent significance level, we used p_2^m as the regressand. Tables 8, 9, and 10 report estimates of α_1, α_2, and α_1/α_2, and the G_R test statistic for the null hypothesis $\alpha_1=1$ and $\mu_c=-\mu_z$. Since α_2 = $-\mu_c/\mu_z$, the hypothesis $\mu_c=-\mu_z$ is equivalent with the hypothesis $\alpha_2=1$.

Stochastic cointegration was rejected at the 1 per cent level only for the following three cases: the pair of clothing and household operation and the pair of household operation and transportation when household operation was assumed to be trend stationary and the pair of clothing and transportation when transportation was assumed to be trend stationary.

Point estimates reported in Tables 8, 9, and 10 were encouraging. All the point estimates of α_1 and α_2 had theoretically correct positive sign. Our estimates of α_1/α_2 from the regressions with c_2^m as the regressand were close to Houthakker and Taylor's estimates of the ratio of income elasticities reported in Table 6 for most of the pairs of goods. Houthakker and Taylor's estimate was within two standard errors from our estimate of α_1/α_2 obtained from the regression with c_1^m as the regressand for the pair of food and clothing in Table 8, and the pair of housing and household operation in Table 9, and the pair of food and transportation. Consider the interval formed by taking three standard errors (calculated with the assumption that the correlation between the two estimates of income elasticities was minus one) from Houthakker and Taylor's estimates and the interval formed by taking three standard errors form our estimates. The only pairs for which these two intervals did not overlap were the pair of clothing and household operation and the pair of household operation and transportation in Table 9 and the pair of household operation and transportation in Table 10.

For most of the pairs of goods examined in Tables 8 and 9, our estimate of α_1/α_2 was significantly different from one. Thus there was evidence against homothetic preferences. In Table 10, however, we found little evidence against homothetic preferences except for the pair of food and transportation.

6. Conclusions

The present paper estimated preference parameters governing income elasticities using Ogaki and Park's cointegration approach. We considered two sources of the disturbance terms in our regressions. One source was preference shocks as in the model of Brown and Walker (1989), and the other source was measurement errors. The only requirement we placed on these two sources was that they were stationary stochastic processes. No assumptions about exogeneity and serial correlations were necessary by the nature of the cointegration approach.

Overwhelming evidence against the Cobb-Douglas utility function was found for all the pairs of goods in terms of the G_R test statistics. Strong empirical evidence was found against the hypothesis that the relative price is stationary for all the relative prices examined. These two observations imply that the aggregation over goods used in standard neoclassical macroeconomic models (see, e.g., King, Plosser, and Rebelo [1988a, 1988b] references therein) are potentially problematic; neither Hicks's aggregation nor the aggregation based on the Cobb-Douglas utility function works. Thus it may be important to introduce multiple goods and nonhomothetic preferences into neoclassical models. For example, Christino's (1989) neoclassical model could not explain the hump-shaped feature of the Japanese saving rate in the time series data after WWII. Introducing two consumption goods (food and the other consumption goods) into a neoclassical model with nonhomothetic preferences might help explaining this type of rich saving rate dynamics. To quantify such a conjecture, one needs a nonhomothetic utility function that can explain cross-sectional Engel's curves and time series properties of relative prices and consumption expenditures. The present paper showed that the addilog utility function estimated from time series data was consistent with cross-sectional observations on income elasticities. Thus the addilog utility function is a qualified candidate to be used in research that seeks to quantify implications of Engel's law in neoclassical models.

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TABLE 1

TESTS FOR DIFFERENCE STATIONARITY OF CONSUMPTION

	$J(1,2)$	$J(1,3)$	$J(1,4)$	$J(1,5)$	$J(1,6)$	Z_α	Z_t
Food	0.082	0.119*	0.318	0.531	0.626*	-31.152‡	-4.037‡
Clothing	1.423	1.507	1.545	1.731	2.490	-25.774†	-3.679†
Housing	3.907	16.630	19.945	28.602	31.036	0.822	0.553
Household op.	0.251	0.291	0.540	0.540	1.150	-35.828‡	-4.320‡
Transportation	0.535	0.540	1.713	1.957	2.035	-13.942	-2.682

NOTE: The lag truncation number used for the Z_α and Z_t statistics reported in this table was 30. Critical values for the 1per cent, 5 per cent, and 10 per cent significance levels are 0.000086, 0.0023, and 0.0093 for $J(1,2)$; 0.011, 0.055, and 0.12 for $J(1,3)$; 0.055, 0.16, and 0.29 for $J(1,4)$; 0.123, 0.295, and 0.452 for $J(1,5)$; 0.21, 0.43, 0.66 for $J(1,6)$; -29.5, -21.8, and -18.3 for Z_α; -3.96, -3.41, and -3.12 for Z_t. Critical values for $J(p,q)$ are from Park and Choi (1988) when they are reported, and were estimated using 500 observations and 10,000 iterations when they are not reported in Park and Choi (1988). Critical values for Z_α and Z_t are from Fuller (1976, p.371 and p.373).

*Significant at the 10 per cent level.
†Significant at the 5 per cent level.
‡Significant at the 1 per cent level.

TABLE 2

TESTS FOR TREND STATIONARITY OF CONSUMPTION

	$G(1,2)$	$G(1,3)$	$G(1,4)$	$G(1,5)$	$G(1,6)$
Food	1.077 (0.299)	1.515 (0.469)	3.439 (0.329)	4.950 (0.292)	5.493 (0.359)
Clothing	6.427 (0.011)	6.579 (0.037)	6.643 (0.084)	6.936 (0.139)	7.808 (0.167)
Housing	6.005 (0.014)	7.114 (0.029)	7.181 (0.066)	7.287 (0.121)	7.306 (0.199)
Household op.	2.796 (0.095)	3.142 (0.208)	4.880 (0.181)	4.880 (0.300)	7.448 (0.189)
Transportation	3.369 (0.066)	3.391 (0.184)	6.107 (0.107)	6.401 (0.171)	6.484 (0.262)

NOTE: Probability values are in parentheses. The lag truncation number used for the $G(p,q)$ statistics reported in this table was 80.

TABLE 3

TESTS FOR DIFFERENCE STATIONARITY OF RELATIVE PRICES

C_2	$J(1,2)$	$J(1,3)$	$J(1,4)$	$J(1,5)$	$J(1,6)$	Z_α	Z_t
			C_1 = Food				
Clothing	1.862	2.825	6.794	7.082	8.385	-4.391	-1.896
Housing	0.627	3.931	4.031	5.173	5.268	-2.355	-0.778
Household op.	0.241	0.322	0.686	1.256	1.438	-16.218	-2.817
Transportation	0.364	0.381	1.648	3.236	3.294	-8.271	-1.990
			C_1 = Clothing				
Housing	6.176	8.499	44.090	56.675	71.649	0.114	0.093
Household op.	2.507	2.881	20.503	24.923	26.150	-2.953	-1.437
Transportation	1.554	1.682	11.167	16.174	20.927	-3.341	-1.430
			C_1 = Housing				
Household op.	0.403	3.904	4.970	4.972	5.093	3.033	0.990
Transportation	0.025	0.775	3.230	3.988	4.373	-1.601	-0.403
			C_1 = Household op.				
Transportation	0.220	0.222	1.000	1.298	1.631	-13.050	-2.588

NOTE: The lag truncation number used for the Z_α and Z_t statistics reported in this table was 30. See the first footnote of Table 1 for critical values of the test statistics reported in this table.

TABLE 4

TESTS FOR TREND STATIONARITY OF RELATIVE PRICES

C_2	$G(1,2)$	$G(1,3)$	$G(1,4)$	$G(1,5)$	$G(1,6)$
			C_1 = Food		
Clothing	4.846 (0.028)	5.501 (0.064)	6.493 (0.090)	6.527 (0.163)	6.655 (0.248)
Housing	3.330 (0.068)	6.886 (0.032)	6.921 (0.074)	7.239 (0.124)	7.260 (0.202)
Household op.	2.415 (0.120)	3.031 (0.220)	5.057 (0.168)	6.922 (0.140)	7.335 (0.197)
Transportation	2.381 (0.123)	2.461 (0.292)	5.552 (0.136)	6.815 (0.146)	6.843 (0.233)
			C_1 = Clothing		
Housing	5.992 (0.014)	6.229 (0.044)	6.808 (0.078)	6.842 (0.145)	6.866 (0.231)
Household op.	4.977 (0.026)	5.168 (0.075)	6.638 (0.084)	6.693 (0.153)	6.705 (0.243)
Transportation	4.282 (0.039)	4.414 (0.110)	6.459 (0.091)	6.628 (0.157)	6.717 (0.243)
			C_1 = Housing		
Household op.	3.110 (0.078)	8.619 (0.013)	9.013 (0.029)	9.013 (0.061)	9.049 (0.107)
Transportation	0.310 (0.578)	5.597 (0.061)	9.791 (0.020)	10.252 (0.036)	10.436 0.064)
			C_1 = Household operation		
Transportation	1.838 (0.175)	1.852 (0.396)	5.097 (0.165)	5.760 (0.218)	6.321 (0.276)

NOTE: The lag truncation number used for the results reported in this table was 80.

TABLE 5

TESTS FOR NO COINTEGRATION

Regressand	Regressor	$I(1,5)$
Food	Clothing	0.638
Clothing	Food	1.920
Food	Housing	0.530
Housing	Food	28.570
Food	Household operation	0.291*
Household operation	Food	0.298*
Food	Transportation	0.399
Transportation	Food	1.701
Clothing	Housing	0.451
Housing	Clothing	14.725
Clothing	Household operation	4.541
Household operation	Clothing	2.125
Clothing	Transportation	2.199
Transportation	Clothing	2.465
Housing	Household operation	25.080
Household operation	Housing	0.357*
Housing	Transportation	19.961
Transportation	Housing	1.094
Household operation	Transportation	0.086†
Transportation	Household oeration	1.086

NOTE: Critical Values for $I(1,5)$ at the 1 per cent, 5 per cent, and 10 per cent significance levels are 0.103, 0.251 and 0.384, respectively. These critical values are from Park, Ouliaris and Choi (1988).

*Significant at the 10 per cent level.

†Significant at the 1 per cent level.

TABLE 6

ESTIMATES OF THE RATIO OF INCOME ELASTICITIES
FROM CROSS-SECTIONAL DATA

C_1	C_2	α_1/α_2	s.e.*	s.e.†	s.e.‡
Food	Clothing	1.990	0.064	0.045	0.0021
Food	Housing	2.029	0.063	0.044	0.0005
Food	Household op.	1.789	0.047	0.034	0.0084
Food	Transportation	2.865	0.090	0.063	0.0001
Clothing	Housing	1.020	0.033	0.023	0.0013
Clothing	Household op.	0.899	0.025	0.018	0.0052
Clothing	Transportation	1.440	0.046	0.033	0.0014
Housing	Household op.	0.882	0.023	0.017	0.0039
Housing	Transportation	1.412	0.044	0.031	0.0004
Household op.	Transportation	1.601	0.043	0.031	0.0076

NOTE: The results in this table were calculated form those reported in Houthakker and Taylor (1970, Table 6.5).

*The standard error was calculated with the assumption that the correlation of the two estimates of income elasticities was minus one.

†The standard error was calculated with the assumption that the correlation of the two estimates of income elasticities was zero.

‡The standard error was calculated with the assumption that the correlation of the two estimates of income elasticities was one.

TABLE 7

CANONICAL COINTEGRATING REGRESSION RESULTS

Regress-and	α_1^*	α_2^*	α_1/α_2^*	$G_R^\dagger$	$H(0,1)^\dagger$	$H(1,2)^\dagger$	$H(1,3)^\dagger$	$H(1,4)^\dagger$
			C_1 = Food, C_2 = Clothing					
c_2^m	1.893 (0.261)	1.347 (0.089)	1.405 (0.110)	28.968 (0.000)	5.278 (0.022)	5.230 (0.022)	6.899 (0.032)	7.021 (0.071)
p_2^m	1.445 (0.295)	1.636 (0.099)	1.242 (0.158)	2.724 (0.256)	9.162 (0.002)	4.152 (0.042)	4.521 (0.104)	5.617 (0.230)
			C_1 = Food, C_2 = Housing					
c_2^m	2.590 (0.547)	1.223 (0.230)	2.118 (0.125)	81.959 (0.000)	14.645 (0.000)	3.512 (0.061)	4.505 (0.105)	5.308 (0.151)
			C_1 = Food, C_2 = Household operation					
c_2^m	1.443 (0.248)	0.748 (0.121)	1.928 (0.082)	136.978 (0.000)	7.977 (0.005)	1.866 (0.172)	2.800 (0.247)	5.494 (0.139)
			C_1 = Food, C_2 = TRANSPORTAION					
c_2^m	2.713 (0.395)	1.143 (0.158)	2.375 (0.086)	255.563 (0.000)	6.613 (0.010)	3.645 (0.056)	4.058 (0.131)	8.549 (0.036)
p_2^m	1.195 (0.300)	0.516 (0.116)	2.317 (0.198)	201.519 (0.000)	0.703 (0.386)	2.545 (0.111)	2.693 (0.260)	10.067 (0.018)
c_1^m	3.035 (0.532)	1.142 (0.190)	2.657 (0.112)	1537.692 (0.000)	23.385 (0.000)	0.662 (0.416)	0.680 (0.712)	3.641 (0.303)
			C_1 = Clothing, C_2 = Housing					
c_2^m	1.812 (0.130)	1.469 (0.157)	1.233 (0.053)	347.109 (0.000)	25.531 (0.000)	2.349 (0.123)	3.305 (0.192)	5.006 (0.171)

TABLE 7 - *Continued*

Regress-and	α_1^*	α_2^*	α_1/α_2^*	$G_R^{\dagger}$	$H(0,1)^{\ddagger}$	$H(1,2)^{\ddagger}$	$H(1,3)^{\ddagger}$	$H(1,4)^{\ddagger}$
				C_1 = Clothing, C_2 = Household operation				
c_2^m	1.801	1.634	1.102	477.107	9.339	0.776	2.654	4.436
	(0.121)	(0.178)	(0.051)	(0.000)	(0.002)	(0.378)	(0.265)	(0.218)
				C_1 = Clothing, C_2 = Transportation				
c_2^m	1.536	1.084	1.417	584.261	33.267	0.004	5.238	18.206
	(0.048)	(0.059)	(0.043)	(0.000)	(0.000)	(0.952)	(0.073)	(0.000)
				C_1 = Housing, C_2 = Household operation				
c_2^m	1.507	1.711	0.881	16.396	0.502	0.115	0.336	0.445
	(0.285)	(0.355)	(0.032)	(0.000)	(0.479)	(0.734)	(0.846)	(0.931)
p_2^m	0.614	0.603	1.017	8.450	2.914	0.001	2.174	2.721
	(0.153)	(0.191)	(0.101)	(0.015)	(0.088)	(0.979)	(0.337)	(0.437)
c_1^m	1.216	1.304	0.932	3.889	13.470	4.463	7.684	7.718
	(0.208)	(0.242)	(0.034)	(0.143)	(0.000)	(0.035)	(0.021)	(0.052)
				C_1 = Housing, C_2 = Transportation				
c_2^m	1.060	0.946	1.121	7.376	0.003	1.256	4.091	7.584
	(0.130)	(0.128)	(0.047)	(0.025)	(0.959)	(0.262)	(0.129)	(0.055)
p_2^m	0.418	0.336	1.245	106.236	0.312	1.922	28.641	30.283
	(0.064)	(0.064)	(0.105)	(0.000)	(0.576)	(0.166)	(0.000)	(0.000)

TABLE 7 - *Continued*

Regress-and	α_1*	α_2*	α_1/α_2*	G_R†	$H(0,1)$†	$H(1,2)$†	$H(1,3)$†	$H(1,4)$†
			C_1 = Household operation, C_2 = Transportation					
c_2^m	1.328 (0.278)	1.125 (0.232)	1.181 (0.040)	20.569 (0.000)	2.676 (0.102)	2.849 (0.091)	2.849 (0.241)	10.139 (0.017)
p_2^m	0.424 (0.103)	0.365 (0.084)	1.160 (0.078)	95.089 (0.000)	2.263 (0.133)	1.834 (0.176)	2.133 (0.344)	5.494 (0.139)
c_1^m	1.442 (0.358)	1.140 (0.279)	1.265 (0.048)	17.831 (0.000)	0.009 (0.925)	0.017 (0.991)	0.598 (0.897)	0.648 (0.958)

NOTE: The lag truncation number used for the results in this table was 80.

*Standard errors are in parentheses.

†Probability values are in parentheses.

TABLE 8

SEEMINGLY UNRELATED CANONICAL COINTEGRATING REGRESSION RESULTS WITH THE ASSUMPTION OF TREND STATIONARY FOOD CONSUMPTION

Dependent Variable	α_1*	α_2*	α_1/α_2*	G_R†	$H(1,2)$†	$H(1,3)$†	$H(1,4)$†
			C_1 = Food, C_2 = Clothing				
c_1^m	4.413 (0.060)	2.138 (0.292)	2.064 (0.192)	70.293 (0.000)	4.174 (0.041)	5.546 (0.062)	5.619 (0.132)
p_2^m	0.535 (0.488)	0.876 (0.158)	0.611 (0.110)	1.204 (0.548)	2.646 (0.104)	5.427 (0.066)	5.443 (0.142)
			C_1 = Food, C_2 = Housing				
c_1^m	5.085 (0.362)	2.060 (0.147)	2.468 (0.135)	1158.747 (0.000)	3.446 (0.063)	3.617 (0.164)	8.773 (0.032)
p_2^m	4.008 (0.470)	1.692 (0.168)	2.369 (0.235)	333.758 (0.000)	3.058 (0.080)	4.826 (0.090)	6.758 (0.080)
			C_1 = Food, C_2 = Household operation				
c_1^m	2.774 (0.464)	1.322 (0.221)	2.098 (0.096)	365.625 (0.000)	0.017 (0.896)	1.373 (0.503)	3.141 (0.370)
p_2^m	1.041 (0.272)	0.531 (0.119)	1.962 (0.441)	141.840 (0.000)	1.248 (0.264)	1.464 (0.481)	3.736 (0.291)
			C_1 = Food, C_2 = Transportation				
c_1^m	2.928 (0.358)	1.198 (0.147)	2.444 (0.088)	206.293 (0.000)	4.843 (0.028)	4.889 (0.087)	5.394 (0.145)
p_2^m	1.077 (0.272)	0.454 (0.096)	2.372 (0.503)	154.513 (0.000)	1.219 (0.270)	1.361 (0.506)	9.977 (0.019)

NOTE: The lag truncation number used for the results in this table was 80.

*Standard errors are in parentheses.

†Probability values are in parentheses.

TABLE 9

SEEMINGLY UNRELATED CANONICAL COINTEGRATING REGRESSION RESULTS WITH THE ASSUMPTION OF TREND STATIONARY HOUSEHOLD OPERATION CONSUMPTION

Regress-and	α_1*	α_2*	α_1/α_2*	G_R†	$H(1,2)$†	$H(1,3)$†	$H(1,4)$†
			C_1 = Food, C_2 = Household operation				
c_1^m	2.856	1.357	2.104	234.572	1.521	2.555	5.242
	(0.670)	(0.319)	(0.099)	(0.000)	(0.217)	(0.279)	(0.155)
p_2^m	0.903	0.463	1.953	170.158	4.601	4.606	8.046
	(0.197)	(0.098)	(0.426)	(0.000)	(0.032)	(0.100)	(0.045)
			C_1 = Clothing, C_2 = Household operation				
c_1^m	2.624	2.772	0.947	353.022	10.681	10.920	12.797
	(0.269)	(0.284)	(0.062)	(0.000)	(0.001)	(0.004)	(0.005)
			C_1 = Housing, C_2 = Household operation				
c_1^m	2.036	2.394	0.851	77.538	0.794	1.277	1.288
	(0.267)	(0.314)	(0.028)	(0.000)	(0.373)	(0.528)	(0.732)
p_2^m	1.171	1.316	0.890	6.905	1.237	7.063	7.777
	(0.167)	(0.205)	(0.127)	(0.032)	(0.266)	(0.029)	(0.509)
			C_1 = Household operation, C_2 = Transportation				
c_1^m	1.497	1.286	1.164	24.854	8.889	8.903	24.608
	(0.246)	(0.211)	(0.033)	(0.000)	(0.003)	(0.012)	(0.000)

NOTE: The lag truncation number used for the results in this table was 80.

*Standard errors are in parentheses.
†Probability values are in parentheses.

TABLE 10

SEEMINGLY UNRELATED CANONICAL COINTEGRATING REGRESSION RESULTS WITH THE ASSUMPTION OF TREND STATIONARY TRANSPORTATION CONSUMPTION

Regress-and	α_1*	α_2*	α_1/α_2*	G_R†	$H(1,2)$†	$H(1,3)$†	$H(1,4)$†
			C_1 = Food, C_2 = Transportation				
c_1^m	11.044	4.462	2.475	256.925	1.302	1.354	4.784
	(7.237)	(2.924)	(0.283)	(0.000)	(0.254)	(0.508)	(0.188)
p_2^m	1.010	0.429	2.352	73.835	1.759	1.772	6.284
	(0.263)	(0.118)	(0.613)	(0.000)	(0.185)	(0.412)	(0.099)
			C_1 = Clothing, C_2 = Transportation				
c_1^m	3.775	3.858	0.978	549.837	11.464	11.474	11.965
	(0.455)	(0.465)	(0.136)	(0.000)	(0.001)	(0.003)	(0.008)
			C_1 = Housing, C_2 = Transportation				
c_1^m	3.048	3.032	1.005	173.726	5.915	6.931	10.391
	(0.475)	(0.472)	(0.073)	(0.000)	(0.015)	(0.031)	(0.016)
p_2^m	1.095	1.011	1.082	1.023	1.067	2.222	4.568
	(0.253)	(0.251)	(0.250)	(0.600)	(0.302)	(0.329)	(0.206)
			C_1 = Household operation, C_2 = Transportation				
c_1^m	1.0×10^3	8.7×10^2	1.153	76.647	2.203	2.973	6.566
	(1.2×10^5)	(1.0×10^5)	(0.125)	(0.000)	(0.138)	(0.226)	(0.087)
p_2^m	0.119	0.087	1.377	67.974	1.405	1.422	5.866
	(0.109)	(0.104)	(1.253)	(0.000)	(0.236)	(0.491)	(0.118)

NOTE: The lag truncation number used for the results in this table was 80.

*Standard errors are in parentheses.
†Probability values are in parentheses.