The Dynamic Behavior of College Enrollment Rates: The Effect of Baby Booms and Busts

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I. Introduction

Higher education has been a growth industry throughout this century. Except for very brief pauses during wars and depressions the enrollments in colleges and universities have climbed steadily. On the other hand the proportion of high school graduates going on to college, i.e. the enrollment rate, has not shown a clear trend. For instance between 1968 and 1973 the proportion of high school graduates entering college fell from .63 to .49. Perhaps not coincidentally, this period also witnessed the arrival of the large post war birth cohorts to the labor market. For instance, the 1957 birth cohort, that entered the market in approximately 1977, is 44% larger than the 1942 birth cohort, that entered in approximately 1962. This paper is concerned, in general, with what is the effect of variation in birth rates on the incentive to attend college and, in particular, what was the effect of the post World War II baby boom of the 1950's and the ensuing bust of the 1960's.

It has been clearly established by Welch (1979), Freeman (1979), Murphy, Plant, and Welch (1983), Berger (1984), and Alsalam (1985) that variation in the frequency distribution of experience in the workforce (due to variation in the number of births) produces corresponding variation in the earnings-experience function. Increases in the number of workers entering the labor market depresses the wages of inexperienced workers relative to their more experienced counterparts. There is even some evidence that the latter effect is stronger among college graduates than high school graduates which would suggest that large birth cohorts are less likely to attend college[1].

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[1]Welch (1979).

There has been little theoretical discussion of why variation in birth rates might have effects on college enrollments, and it is this void that this paper attempts to fill.[2] I make use of the simple fact that school takes time to produce a non-neutral effect of variation in the number of births on the incentive for college attendance. I get the result that increases in the growth rate of births increases enrollment rates. I also find that these increases lead the arrival of the larger cohorts. This suggests that declining college enrollments in the early seventies may have been in anticipation of the smaller birth cohorts that would begin arriving in the late 1970's and early 1980's. This is in constrast to the suggestion that the decline was in response to the worstening labor market for college graduates in the early 1970's, due primarily to the entry of the large baby boom birth cohorts[3].

II. The Model

A. The Basic Structure

The rudiments of the model are described in this section. This model is used to study the impact of variation in the number of births on aggregate school enrollments. The number of births should be intepreted as the number of high school graduates or the number of individuals reaching working age. School enrollments should be interpreted as the number of high school graduates enrolling as college freshman or the number of individuals who delay their entry into the workforce to acquire the "label" of college graduate.

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[2] Wachter and Wascher (1984) is an exception.

[3] Freeman (1975, 1976, 1977).

At each point in time[4], P(t) individuals are born (reach working age). These individuals die (retire) at time t+1, i.e. an individual's potential working life is one unit of time[5]. At birth all individuals are identical, but immediately they must make a choice that will put them in one of two groups -- skilled and unskilled. "Skilled" and "unskilled" are simply labels and do not necessarily connote differences in productivity. If they choose to become skilled they must forego working and the wages of unskilled workers for s units of time -- college[6] However, upon graduation they earn the wages of skilled workers. Otherwise, they earn the wages of unskilled workers throughout their lives.

Aggregate output, y(t), is determined by a linearly homogenous production function which depends on the stocks of skilled, S(t), and unskilled, U(t), workers. The stocks of skilled and unskilled workers depend on the number of births and the number in each cohort that choose to go to school.

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[4]This paper is schizophrenic with respect to its treatment of time. At first, it is treated as continuous because a more general problem can be stated and an aspect of the problem is highlighted that otherwise would not be clear. Then it is treated as discrete to simplify calculating and characterizing a solution.

[5]At this point it is convenient to scale time in this way. Later, it becomes convenient to scale time so that an individual's potential working life is N units of time, and college takes one unit of time.

[6]In this model individuals are simply labelled skilled or unskilled depending on whether on not they went to college. Those who did are not necessarily more productive. Relative productivities depends upon relative scarcities. This view is implicit in Mincer's (1974) work on estimating the returns to schooling and experience, and it is not in Ben-Porath's (1967) formal model of human capital accumulation.

$$S(t) = \int_{t-1}^{t-s} E(v)dv$$

(1)

$$U(t) = \int_{t-1}^{t} P(v)-E(v)dv$$

where E(v) is the number born at time v that decide to go to school.

At a point in time and given the enrollment decision of the most recent generation, supplies of skilled and unskilled workers are fixed. Wages are determined by the marginal products of the aggregate production function. Output is taken as the numeraire, so

$$W_s(t) = \partial f(S(t),U(t))/\partial S$$

(1)

$$W_u(t) = \partial f(S(t),U(t))/\partial U$$

where W_s and W_u are the wages of skilled and unskilled workers, respectively.

Interest rates are exogenous, constant, and individuals may reallocate their consumption over their lifetimes by trading at these prices[7]. Interest rates are taken as given so that we may abstract from the determination of investment, savings, and capital[8].

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[7]To avoid issues of intergenerational contracts, I implicity assume the existence of a "Bank" that survives forever.

[8]To include the determination of the capital stock and the interest rate into the model is likely to be "excess baggage" for the purposes of this paper. However, a tractable alternative suggested to me by Eric Hanushek and Alan

B. Equilibrium and Expectations

The market for college education clears when all[9] individuals are indifferent between attending school and not. This occurs when the present value of expected wage streams are identical[10]. The equilibrium condition is

$$(3) \qquad \int_{t}^{t+1} W_u(v;t)e^{-rv}dv = \int_{t+s}^{t+1} W_s(v;t)e^{-rv}dv \quad t\in(-\infty,\infty)$$

where $W_u(v;t)$ is the wage individuals at time t anticipate unskilled workers will earn at time v. The anticipated wage of skilled workers, $W_s(v;t)$, is similarly defined.

Suppose individuals perfectly foresee the path of future births, i.e.

$$(4) \qquad P(v;t)=P(v;v) \quad \text{for all } v>t$$

Stockman, would be to introduce a cost of adjustment of enrollments. The facilities required to provide college education cannot be instantly acquired or converted to other uses. The supply function of college education is not perfectly elastic even with full anticipation of requirements.

[9] Because all individual's are identical, in equilibrium they are all indifferent between going to college or not and simply toss a coin (with the correct weights) to make their decision. If the population is heterogeneous, only those on the margin are indifferent and inframarginal individuals earn rents.

[10] I assume the existence of perfect capital markets which are equally accessible to all individuals.

Suppose, also, that individuals anticipate enrollments "rationally", i.e. they form expectations of the future path of enrollments in exactly the same way the actual equilibrium path is generated. Their expectations are self-fulfilling. As there is no uncertainty about births, this implies anticipations and realizations of enrollments will be identical.

Individuals know the wage determination process. They know the parameters of the aggregate production function, the existing stocks of skilled and unskilled labor. They understand that the wages they earn in the future either as skilled or unskilled workers depend on the size of future birth cohorts and their schooling decisions. So they forecast the enrollment decisions of all future birth cohorts correctly assuming the younger cohorts will follow the same decision process as they do.

The implication of these assumptions is that all expectations are realized, and the equilibrium/arbitrage condition (3) holds not only in an ex ante expectational sense, but it also holds ex post at each point in time.

$$(5) \qquad \int_{t}^{t+1} W_u(v)e^{-rv}dv = \int_{t+s}^{t+1} W_s(v)e^{-rv}dv \quad t\in(-\infty,\infty)$$

Individuals at time t use the expectations on P(v) v>t and currently available data to forecast future enrollment decisions leading back to their own. The wage an individual will earn tomorrow depends on the number of tomorrow's youth that enroll in school. In turn the number of tomorrow's youth that go to school depends on the number of the day after tomorrow's youth that go to school and so on. So today's youth must make their enrollment decision in the dynamic programming method, working back from the end of time.

Consider the equilibrium condition (3) again. Future wages depend on future stocks. In turn future stocks depend on the size of future birth cohorts and their enrollment rates. For t fixed condition (3), then, is one equation determining E(t) given future enrollments, E(v) v>t, and births, P(v) v>t. Individuals must solve for the future enrollment decisions of future birth cohorts before solving their own. To do this they use the same equilibrium condition that determines their enrollment decision but applied to future generations.

In summary, individuals at time t solve a difference equation in enrollments and births. The solution is a time path for enrollments as a function of a expected future time path of births. Recall, though, that this time path for enrollments is an expected one at time t. The next generation may have different information available to them and will solve for a different time path of enrollments. Only if expectations are realized will the expected time series of the generation born at time t hold for v>t. Implied in this analysis is that individuals' expectations are realized. It is convenient to assume expectations are realized; if for no other reason, this helps avoid constantly distinguishing expectations and realizations of variables. We have a countable set of equations determining the time path of enrollments, E(t), as a function of the time path of births, P(t). The equations are non-linear, because wages are non-linear functions of stocks of skilled and unskilled workers.

III. Steady State Results

The case of constant growth rates of births is examined in this section. On the basis of this, comparative "static" results are presented. For instance, what are the differences in aggregate enrollment rates between economies with different constant growth rates of births? The solution in the constant growth rate case is characterized by a constant enrollment rate, so I describe it as the steady-state case. If the growth rate of births is g, and a fixed proportion, $\bar{e}$, of each age cohort enrolls in school, then the ratio of skilled to unskilled workers at time t, k(t), is:

$$(6) \qquad k(g,\bar{e},s) = \frac{\int_{t-1+s}^{t} \bar{e}P(v)e^{g(t-v)}dv}{\int_{t-1}^{t} (1-\bar{e})P(v)e^{g(t-v)}dv} = \frac{\bar{e}(e^{g(1-s)}-1)}{(1-\bar{e})(e^{g}-1)}$$

Note that k is independent of t. This implies the relative wage of skilled to unskilled workers is constant. The factor ratio, k, depends on the length of schooling, s, the growth rate of births, g, and the enrollment rate, e. Factoring wages from the equilibrium condition (5) and dropping the time subscript yields:

$$(7) \qquad \frac{W_s}{W_u} \equiv h(r,s) = \frac{\int_{0}^{1} e^{-rv}dv}{\int_{s}^{1} e^{-rv}dv} = \begin{cases} \dfrac{1-e^{r}}{1-e^{r(1-s)}} & \text{for } r>0 \\ \dfrac{1}{1-s} & \text{for } r=0 \end{cases}$$

The equilibrium wage premium skilled workers earn depends upon the cost of being skilled, i.e. it depends on the interest rate, r, and the length of schooling, s. Let g(.) be the function that maps factor ratios onto relative wages, and[11] let g^{-1} be the inverse function that maps relative wages onto factor ratios[12]. The factor ratio necessary to maintain the wage premium of skilled workers defined in (5) is $g^{-1}(h(r,s))$. Hence we can express the equilibrium condition (5) as:

$$k(g,c,s) = g^{-1}(h(r,s)) \tag{8}$$

This equation determines c given g (and r and s).

Totally differentiating this identity determines the response of the enrollment rate, c, to changes in the growth rate of births, g, the interest rate, r, or the length of schooling, s.

Results

1. Longer length of schooling decreases the incentive to go to college.

2. Higher interest rates decrease the incentive to go to college.

3. Higher growth rates of births increase the incentives to go to college. Decreases in the length of schooling reduce the postive effect of higher growth rates of births on college attendance.

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[11] For a two factor linearly homogeneous production function, wages (and relative wages) depend only on factor ratios.

[12] g(.) is one to one, and monotonically decreasing.

The first two results are clear. Both inceases in s and and r increase the cost of being skilled and reduce the incentive to being so.

The third result is not immediately obvious. The following example provides the intuition. Consider two economies where acquiring an education requires one half of an individual's potential working life. In both economies the college enrollment rate is 50%, i.e. one half of each age cohort acquires a college education, but the growth rate of births are different. In the first economy it is zero; in the second it is 100%. In the first economy the proportion of the out-of-school workforce that have college degrees is (after some simple arithmetic) one third. In the second economy, however, it is one quarter because the age distribution of the population is skewed towards young people. If relative wages depend upon factor ratios the wage premium earned by college graduates will be greater in the second economy thereby increasing the incentives to attend college.

The empirical importance of this phenomenon depends on both the degree of substitutability across age cohorts within schooling classes, and the length of schooling. We can expect significant effects on the dynamic behavior of enrollments of variation in the number of births even though schooling requires only 1/10th of working life, because workers with different years of experience are not perfect substitues.

Reiterating the intuition behind this result, a positive growth rate of births means the age distribution of the population is skewed towards younger age groups. Higher growth rates increases the skewness towards young age groups. On the other hand, it is young workers in their first four years of (economic) life that go to school[13]. So when a given fraction of these workers withdraw from the labor force to go to school, it leads to a greater fall in the proportion of the workforce who are unskilled in economies with greater growth rates of births.

IV. The Dynamics of Enrollments

The case of non-constant growth of births and the resulting dynamics behavior of enrollments is examined in this section.

The arbitrage condition for the present value of wage streams (5), the stock-flow identities (1), and the wage functions (2) are an infinite set of equations whose solution given the time path of births is the time path of enrollments. In this section, a solution to these equations is found; in the next section an argument for its uniqueness is presented.

If for any given time path of births, there is an equilibrium time path of enrollments that leaves the relative number of skilled and unskilled workers constant, the enrollment sequence must satisfy:

$$k(E(.),P(.);\ s) = g^{-1}(h(r,s)) \tag{9}$$

where h(r,s) is the relative wage in (7) that support equilibrium,

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[13] The fact that young workers that choose to go to college do so at the beginning of life is not an assumption but a result. This is the strategy that maximizes the length of the period over which a worker can earn returns to his investment. It is possible (in nonsteady state cases) that the wages of unskilled workers be so great in the first period of life that some individuals that otherwise would have gone to college remain out of college. But for this to occur in our homogeneous worker model it would be necessary for all the youngest individuals in the economy to be enrolled for they always have the greater incentive to obtain a college education. If the population were heterogenous, late enrollment would be more likely to occur because the first to enroll would simply be those with the comparative advantage which is not based on age alone.

$g^{-1}(h(r,s))$ is the corresponding factor ratio, and $k(E(.),S(.);s)$ gives the factor ratio as a function of the path of births and enrollments using the stock-flow identity in (1). A linear difference equation determining $E(.)$ as a function of $P(.)$ can be produced from differentiating (9). This is demonstrated below.

Two notes should be made. First, this solution is for the case of perfect foresight and self-fulfilling expectations. Second, the solution does not depend on the substitution properties of the production function -- factor ratios are constant. If expectations are not steb, profit opportunities will exist for those currently making their education decisions, factor ratios will vary temporarily from the long-run equilibrium ratio, and the substitution properties of the production function come into play.

Define ρ as the proportion of the workforce that is skilled which satisfies:

$$g^{-1}(h(r,s)) = \rho/1-\rho \tag{10}$$

Equate the RHS of (10) to $k(E(t),P(t);s)$

$$\frac{\rho}{1-\rho} = k(E(\cdot),\ P(\cdot);\ s) = \frac{\int_{t+s}^{t+1} E(v)dv}{\int_{t}^{t+1} P(v)-E(v)dv} \tag{11}$$

Cross multiplying and differentiating with respect to t, we have:

(12) $$E(t-1)-(1-\rho)E(t-s)-\rho E(t)=\rho(P(t-1)-P(t))$$

or in lead operator notation:

(13) $$(1-(1-\rho)L^{1-s}-\rho L)E(t-1) = \rho(1-L)P(t-1)$$

This is the fundamental difference equation that describes a constant factor ratio equilibrium path of enrollments.

At this point it is useful to make particular choices for the value of s. Suppose $s=1/q$ for q an integer greater than one. Rescale time so that $Z=L^s$. We can then rewrite the above as:

(14) $$(1-(1-p)Z^{q-1}-\rho Z^q)E(t)=\rho(1-Z^q)P(t)$$

Each of the polynomials in the lead operator have the common factor (1-Z). Operating on both sides by the inverse of the polynomial in $E(\cdot)$, and removing the common factor from both sides, gives us:

(15) $$E(t) = \frac{\rho(1+Z+Z^2+ \cdots +Z^{q-2}+ Z^{q-1})}{(1+Z+Z^2+ \cdots +Z^{q-2}+\rho Z^{q-1})} P(t)$$

This difference equation governs the time path of enrollments for any fully anticipated path of births. It is possible to calculate the time path of enrollments, given the time path of births.

Result:

Current enrollment depend on the future growth rate of births, not past growth rates. The important question to the current birth cohort is the size of the birth cohort entering the market at the time they graduate from college? If it is unusually large, there will be an unusually large number of unskilled workers entering the workforce (even if the proportion enrolled in college increases) which will increase the wages of skilled workers to whom they are complementary. This increases the incentives for the current birth cohort to attend.

A useful analogy is of teachers and students. A large increase in the number of births today means the demand for elementary school teachers will increase in 6 years. This increases the incentives for today's 18 year olds contemplating their career plans to choose elementary education.

V. Uniqueness

The previous section demonstrated the existence of an equilibrium time path of enrollments given a time path of births, by producing such a path. This section presents an argument for its uniqueness. The strategy is to show the equivalence of the equations solved above, and the necessary and sufficient conditions of a well-behaved maximization problem, for which uniqueness is known.

Consider the following maximization problem:

$$\max_{\{E(v):\ v=0,1,..\}} \sum_{v=0}^{\infty} f(U(v),\ S(v))\frac{1}{(1+r)^{v}}$$

(16) where

$$U(v) = \sum_{i=0}^{L-1} P(v-1) - E(v-1)$$

$$S(v) = \sum_{i=1}^{L-1} E(v-1)$$

Given are the enrollment decisions of those born in the past:

$$E(-L),\ E(-L+1),\ E(-L+2),\ \ldots,\ E(-1)$$

and the size of all living and future birth cohorts:

$$P(-L),\ P(-L+1),\ P(-L+2),\ldots,\ P(-1),\ P(0),\ P(1),\ldots$$

The solution to this problem maximizes the present value of social output.

School takes 1 period of time. Working life is L periods of time. Here the length of schooling is restricted to be a fraction 1/L of working life. Time has been rescaled and reinterpreted as discrete, so that s units of time is now one period[14]. The aggregate production function is f(.,.), and is globally strictly concave. The choice variables are E(0), E(1), E(2),... The choice set is convex.

$$0 \leq E(0) \leq P(0),\ 0 \leq E(1) \leq P(1),\ \ldots$$

The concavity of the production function and the convexity of the choice set, together imply that the first order conditions are both necessary and sufficient for the existence of a unique maximum. I assume in addition that the time path of births is such that there is an interior solution for enrollments, i.e., it is never the case that no one or everyone goes to college.

The criterion function after substitution of the equations for the stocks of unskilled and skilled workers is:

$$\mathcal{L} = \sum_{t=0}^{\infty} e^{-rt} f\left[\sum_{v=0}^{L-1} P(t-v)-E(t-v),\ \sum_{v=1}^{L-1} E(t-v)\right]$$

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[14]These are not important changes. The main reason for changing the scaling is to remove fractional powers from the polynomials. Although when the model is constructed, time is treated as continuous, the problem still generates <u>difference</u> equations.

Taking derivatives of $\mathcal{L}$ with respect to E(0), E(1), ... E(k),... we get:

$$\partial\mathcal{L}/\partial E(0) = \sum_{t=0}^{L-1} e^{-rt} \left[f_u(U(t),S(t))(-E(0))\right] + \sum_{t=1}^{L-1} e^{-rt} \left[f_s(U(t),S(t))(E(0))\right]$$

$$\partial\mathcal{L}/\partial E(1) = \sum_{t=1}^{L} e^{-rt} \left[f_u(U(t),S(t))(-E(1))\right] + \sum_{t=2}^{L} e^{-rt} \left[f_s(U(t),S(t))(E(1))\right]$$

and so forth or

$$\partial\mathcal{L}/\partial E(k) = \sum_{t=k}^{L-1} e^{-rt} \left[f_u(U(t),S(t))(-E(k))\right] + \sum_{t=k+1}^{L-1+k} e^{-rt} \left[f_s(U(t),S(t))(E(k))\right]$$

for k = 0, 1, 2,...

Simplifying and setting these to zero we get

$$\sum_{t=0}^{L-1} e^{-rt} \left[f_u(U(t),S(t))\right] = \sum_{t=1}^{L-1} e^{-rt} \left[f_s(U(t),S(t))\right]$$

$$\sum_{t=1}^{L} e^{-rt} \left[f_u(U(t),S(t))\right] = \sum_{t=2}^{L} e^{-rt} \left[f_s(U(t),S(t))\right]$$

and so forth, or,

$$\sum_{t=k}^{L-1+k} e^{-rt} \left[f_u(U(t),S(t))\right] = \sum_{t=1+k}^{L-1+k} e^{-rt} \left[f_s(U(t),S(t))\right]$$

for k=0,1,2,...

This sequence of first order conditions discrete time analog of the arbitrage condition (3) from which (12) was generated. The system of equations has a <u>unique</u> solution.

VI. Illustrative Examples

Equation (15) provides (subject to restrictions that keep enrollment rates strictly between zero and one) the number of enrollments in a cohort, as a function of a perfectly foreseen path of future births. Direct interpretation of the solution as provided by equation (15) is difficult. As an alternative several example paths are used to study the dynamics of enrollments (15) implies.

The first example is a change in a constant growth rate of births. This is a useful one because (1) the timing of behavioral responses to the change in the dynamic behavior of birth is very clear, and (2) it represents a non-stationary time series pattern of births[15].

The second example is a cyclical pattern in the number of births. This is useful because (1) the actual time series of births is dominated by cyclical components[16], (2) any stationary time series can be represented by the (possibly infinite) sum of cyclical components, and (3) the smooth change in the growth rate of births inherit in a cyclical pattern of births is in stark contrast to the discontinuous change in the first example.

The third example is a smoothed actual and projected time series of high school graduates. The model presented in this paper does not lend itself to direct estimation and testing, because (1) it ignores technological growth or the increasing demand for college graduates relative to high school graduates and the increasing demand for high school graduates relative to individuals with less education, and (2) it ignores the changing pattern of the direct cost of education. However, the dynamics of enrollments implied by the model for the actual path of high school graduates is interesting to calculate and informally compare to the actual path of enrollment rates.

This paper has focused soley only a perfect foresight dynamic model. Elsewhere, Alsalam(1981), a very similar model where new information may

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[15] Formulating the problem studied here as a dynamic programming problem would make arguing for existence and uniqueness of a solution more straight forward. It would also help in characterising the solution in ways that are not natural for the approach taken here. However, the dynamic programming formulation has the disadvantage of not being particularly useful when the recursive relation must be subscripted by t. Examining the behavior of enrollments when the birth series is non-stationary would make this necessary.

[16] A very large fraction of the variation in the actual and Census projected time series of births from 1926 to 2075 can be explained with five frequencies and associated amplitudes.

arrive is studied. The cost of generalizing in this direction is (1) restriction of the aggregate production function to a quadratic (so wages are linear in stocks), and (2) the ability to solve the s=.50 and s=.33 cases only. However, it is interesting to contrast the results in these cases when the constant growth rate of births changes unexpectedly to the cased when the change is perfectly foreseen. This is the fourth example examined.

A. <u>A One Time Change in a Constant Birth Rate</u>

Suppose the birth rate changes from .00 to .08 per year, and the change is perfectly foreseen. Figure 1 illustrates the resulting behavior of enrollments. Time is scaled in schooling periods -- four years is one schooling period. The growth rate of births is zero up to period 0. In period 0, the growth rate increases to .08 per year. Growth rates are calculated looking forward. The next year the first unusually large cohort arrives.

First, note that all of the adjustments lead the arrival of the first unusually large cohort. Enrollments depend on current and future births only. The past behavior of births is irrelevant when changes are perfectly foreseen. Past cohort have fully adjusted for any effects of the past behavior of births. Second, note that the adjustments have an oscillatory pattern. Suppose the first unusually large cohort arrives at the market place at time 1, and that s=.50. The cohort entering at time 0 views the future growth of births as constant at the new higher rate, and, reflecting this, their enrollment rate is higher. The number of unskilled workers entering the market in time 0 will be unusually small. The cohort entering at time -1 does not overlap with the first unusually large cohort, but does with the time 0 cohort. In fact, the time -1 cohort college graduates earn their lifetime wages in period 0 and these wages are positively related to the number of unskilled workers from cohort 0 entering the market. Hence the incentive for the time -1 cohort to attend college declines from the zero growth steady state rate. The number of unskilled workers entering the market in time -1 will be unusually large. This in turn increases the incentive for the time -2 cohort to attend college. The incentive and enrollment rate oscillates back through time.

Now suppose s=.33. The argument is the same for the rise in the enrollments for members of cohort 0 to the steady state rate consistent with the new higher growth rate of births as it was in the previous case. The cohort arriving at time -2 is the latest cohort that does not overlap with the first unusually large cohort. The time -1 and 0 cohorts following it will overlap and their enrollments will be up to benefit from the unusually large number of unskilled workers entering in time 1 -- cohort 0 moreso than cohort -1. The latter sees fewer unskilled workers entering in time 0 as cohort 0 enrolls in larger numbers. Cohort -1 enrollment incentives are less than those of cohort 0, and those of cohort -2 are below those of cohort -1. However, cohort -3 has increased incentives, because average enrollments among the future cohorts it overlaps with are down. The incentive and enrollment rate oscillates back through time at a half-cycle period equal to the length of schooling.

B. Cyclical Variation in the Number of Births

Suppose the time series of births is cyclical (sinusoidal), with a forty year cycle and a 50% variation above and below its mean value, and the path is perfectly foreseen. Figure 2 illustrates the resulting behavior of enrollments.

First, as one would expect, the enrollment series is itself cyclical. The level of enrollments has the same cyclical behavior as births. Enrollment peaks correspond to birth peaks and vice versa, independent of the length of schooling relative to working life[17].

Second, the enrollment rate series is also cyclical. The enrollment rate cycle is not symmetric. It rises quickly (14 years) and falls slowly (26 years), but leads (is out of phase with) the birth series by approximately one quarter cycle or ten years[18]. Stated roughly, when births are accelerating most quickly, enrollment rates are at their peak, and vice versa.

Changes in the amplitude or periodicity of the birth cycle lead to corresponding changes in the amplitude and periodicity of the enrollment rate series. The length of working life does not significantly affect the dynamic behavior of enrollment rates as it did in the first example. The length of the lead (10 years or 1/4 cycle) is (essentially) constant.

If exponential growth were added to a cyclical birth path, the effect would be to increase the level of the enrollment rate, but not to change the

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[17] The enrollment peaks precede the birth peaks by 1 unit of time. A unit of time can be arbitrarily small relative to the length of schooling.

[18] The length of the lead is one unit of time longer for each increment in working life in schooling periods units.

character of the dynamic behavior. An upward trend in enrollment rates can only be produced by an ever increasing birth rate.

C. Actual and Projected Number of High School Graduates

Figure 3 shows part of the smoothed actual and projected (to year 2080) annual time series of high school graduates -- the real world analog of "births" in the abstract model. The number of high school graduates rose sharply throughout the 50's, 60's, and early 70's, due both to an increasing propensity to finish high school and the rising post depression and post war birth rate. In the middle 70's the number began to fall due to the shrinking size of age cohorts, and is projected to continue falling through the 80's. In the 90's the numbers will increase as the baby boom's babys reach maturity. Figure 3 also illustrates the dynamics of college enrollment rates from 1950 to the year 2000 implied by this series.

First, college enrollment rates are projected to have risen quickly in the 50's due to the number of high school graduates foreseen to be fast growing between 1950 and 1970. Second, college enrollment rates are projected to have declined throughout the 60's and 70's due the the slowing growth and decline in the 70's and 80's. In fact, college enrollment rates grew steadily throughout the 1950's and 60's, dropped sharply between 1969 and 1975, and then leveled out.

D. Unforeseen One Time Change in a Constant Birth Rate

Suppose the number of births grows at a constant rate and individuals believe that it will always grow at this rate. At some point in time t_1, however, it becomes known that the rate will change at time t_2 to a new (constant) rate. These two points in time may coincide.

For the case of quadratic production, t_2 equal to t_1, the following equations describe enrollment behavior for the two and three period models. For the case of s=.50 it is:

$$E(t) = \frac{\rho(1+L)}{(1+\rho L)} P(t) + d_{-1}(-\rho e^{r/2})^t \qquad t=1,\ 2,\ \ldots$$

where d_{-1} is the difference between the actual and fully anticipated values of the enrollment rate, one period before the change, time 0.

For the case of s=.33 it is:

$$E(t) = \frac{\rho(1+L+L^2)}{(1+L+\rho L^2)} P(t) + \delta_1\lambda_1^t + \delta_2\lambda_2^t \qquad t=1,2,\ldots.$$

where

$$\begin{bmatrix} 1 & 1 \\ \lambda_1^{-1} & \lambda_2^{-1} \end{bmatrix} \begin{bmatrix} \delta_1 \\ \delta_2 \end{bmatrix} = \begin{bmatrix} d_{-1} \\ d_{-2} \end{bmatrix}$$

and

$$\lambda_1\ ,\ \lambda_2 = \begin{cases} (-\frac{1}{2}\, e^{r/3})(1 \pm \sqrt{1-4\rho}\,) & \rho \leq .25 \\ (-\frac{1}{2}\, e^{r/3})(1 \pm i\sqrt{1-4\rho}\,) & \rho > .25 \end{cases}$$

and d_{-1} and d_{-2} are the differences between the actual and fully anticipated enrollment values one and two periods before the change, respectively.

Figure 4 illustrates the behavior of enrollment rates implied by these equations when the growth rate of births increases unexpectedly from zero to .32 per schooling period. Enrollments oscillate after the change, and the oscillations gradually die out. The size of the initial oscillations are larger (1) the greater is the magnitude of the change in enrollment rates and (2) the smaller is the lead time in discovering the change. In both the two and three period examples, the enrollment rate "overshoots" at the announcement date and then oscillates at a half-cycle period equal to the schooling period. In constrast to the fully anticipated case where all adjustments precede the change, here all adjustment fall the change.

VII. Summary and Conclusions

On a priori grounds there is no reason to expect that increases in the number of births will reduce the rate of return to a college education (or that a fall in the number of births will increase it.) This paper examins a model in which changes in the birth rate has non-neutral effects on the wages of educated and uneducated workers. The key feature of this model is that school takes time. The dynamic behavior of enrollment rates is derived for various time series of births (high school graduates.)

It is found that anticipated increases in the number of births increases the incentive to get a college education. It is also found that the response to an increase in the birth rate leads the time of the increase. Current enrollment rates increase if it is anticipated that the number of uneducated workers, who enhance the productivity of educated workers, will increase. Just as the decision to go to college is based on anticipations of wages in the future, so are they based on anticipations of sizes of new cohorts entering the market in the future. Unlike the preponderance of previous work, the model emphasizes, not current conditions for college graduates, but likely future conditions.

A complete model of enrollment rates suitable for estimation and testing would be flexible enough to allow for technological change or an increase in the demand for college educated labor relative to less educated workers. It would allow for changes over time in the direct cost of college as public support of education changes. It would allow for temporary conditions that change the incentive to attend school such as the state of the economy and the implicit value of a deferment from military service. The model in this paper is not a complete model, but it does suggest a formulation for a dynamic specification that is an alternative to the simple cobb-web dynamics.

Abstracting from the limitations mentioned above, the model does provide a new way of interpretting the decline in enrollment rates that began in 1968 or 69 and continued through 1974. The high school graduates of the late sixties and early seventies did not have the prospect of a flow of large, younger cohorts entering the labor force and college, and increasing the demand for their services. It was clear by this time that the birth rate was falling off quickly. Their enrollment rates fell reflecting the anticipated fall in the demand for their services. The Viet Nam War provided a short run return to college attendance in the form of avoidance of military service. The post Sputnik increase in the public support of higher education reduced the cost of attending college. In the absence of these factors it is likely that college enrollment rates would have fallen sooner than they did.

Individual are homogenous in the model. This with the perfect foresight assumption implies that rates of return to schooling are constant and relative wages are constant. The supply of college graduates adjusts to keep relative wages constant. If individuals are heterogenous with respect to their comparative advantage in attending college, then supply adjustments will not completely eliminate changes in relative wages. To the extent that the homogeneity assumption is a useful approximation and the model is an accurate appraisal of behavior, we should see relatively little movement in wages, net of cyclical influences, relative to movement in enrollments. This interpretation of the evidence suggests that the decline of enrollment rates was not a harbinger of the end of the golden era of a college education, but a natural supply response to anticipated changes in demand conditions.

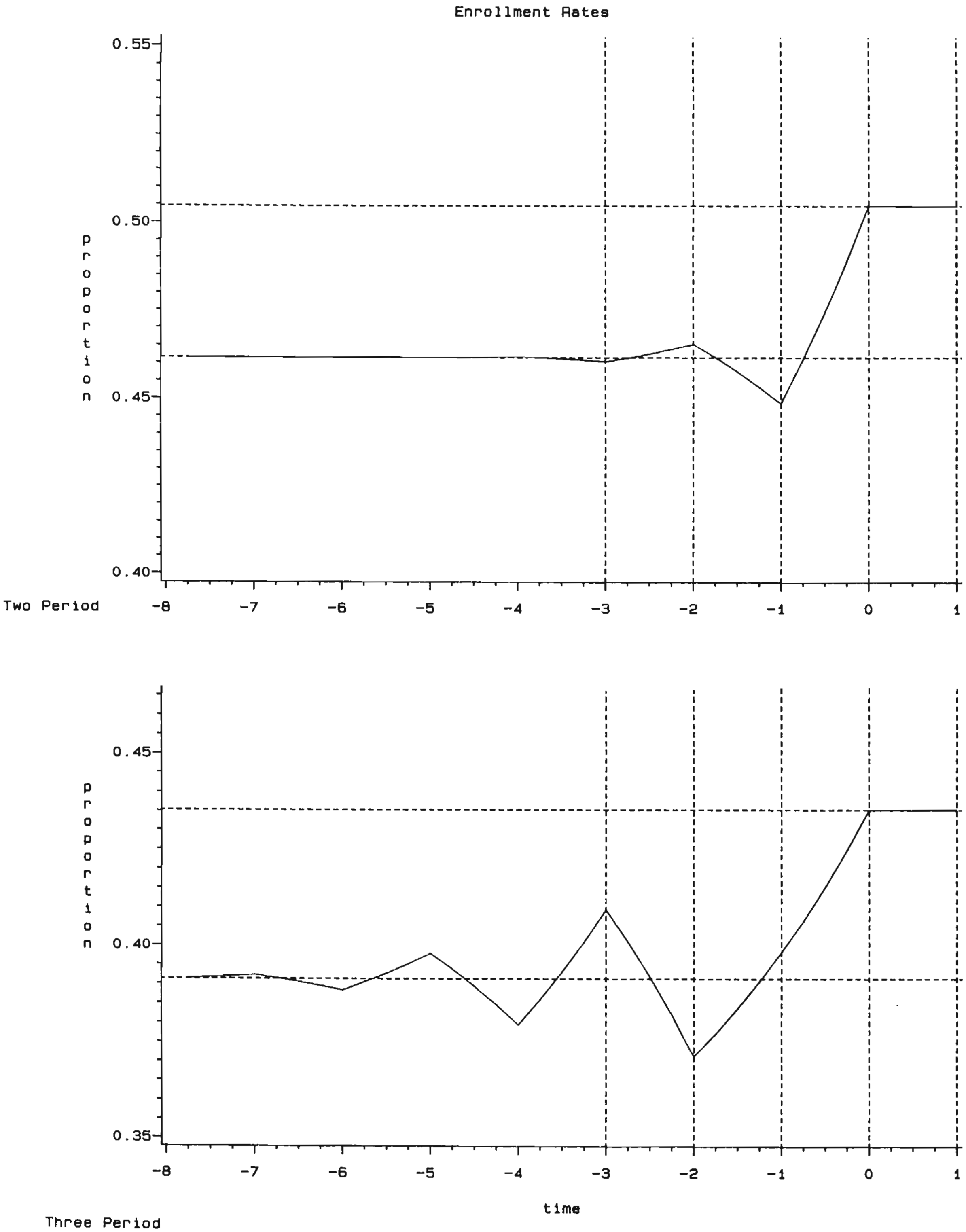

Figure 1.: Enrollment rate responses to a foreseen increase in the growth rate of births at time 0

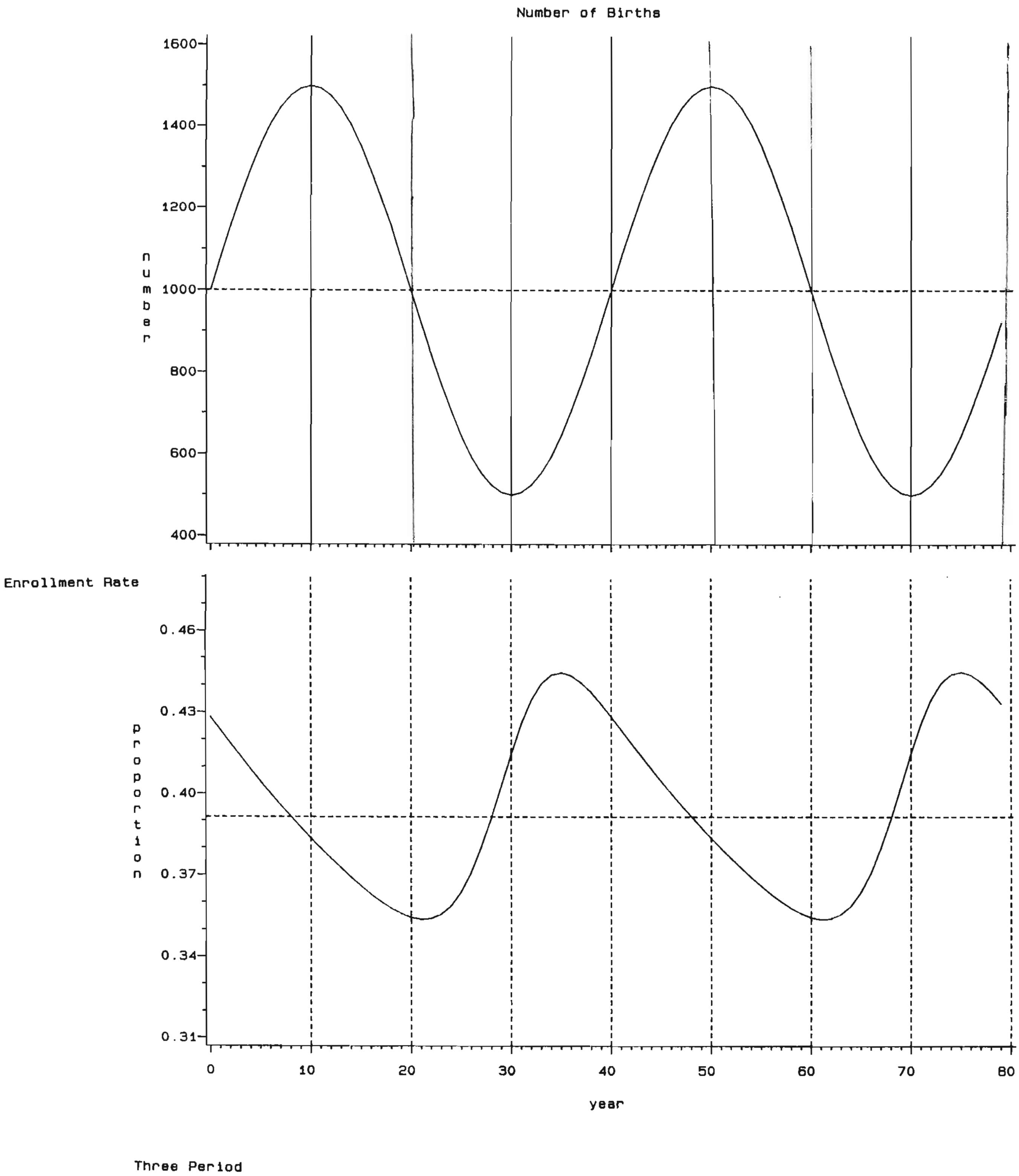

Figure 2.: Enrollment rate responses to a foreseen cyclical path of births

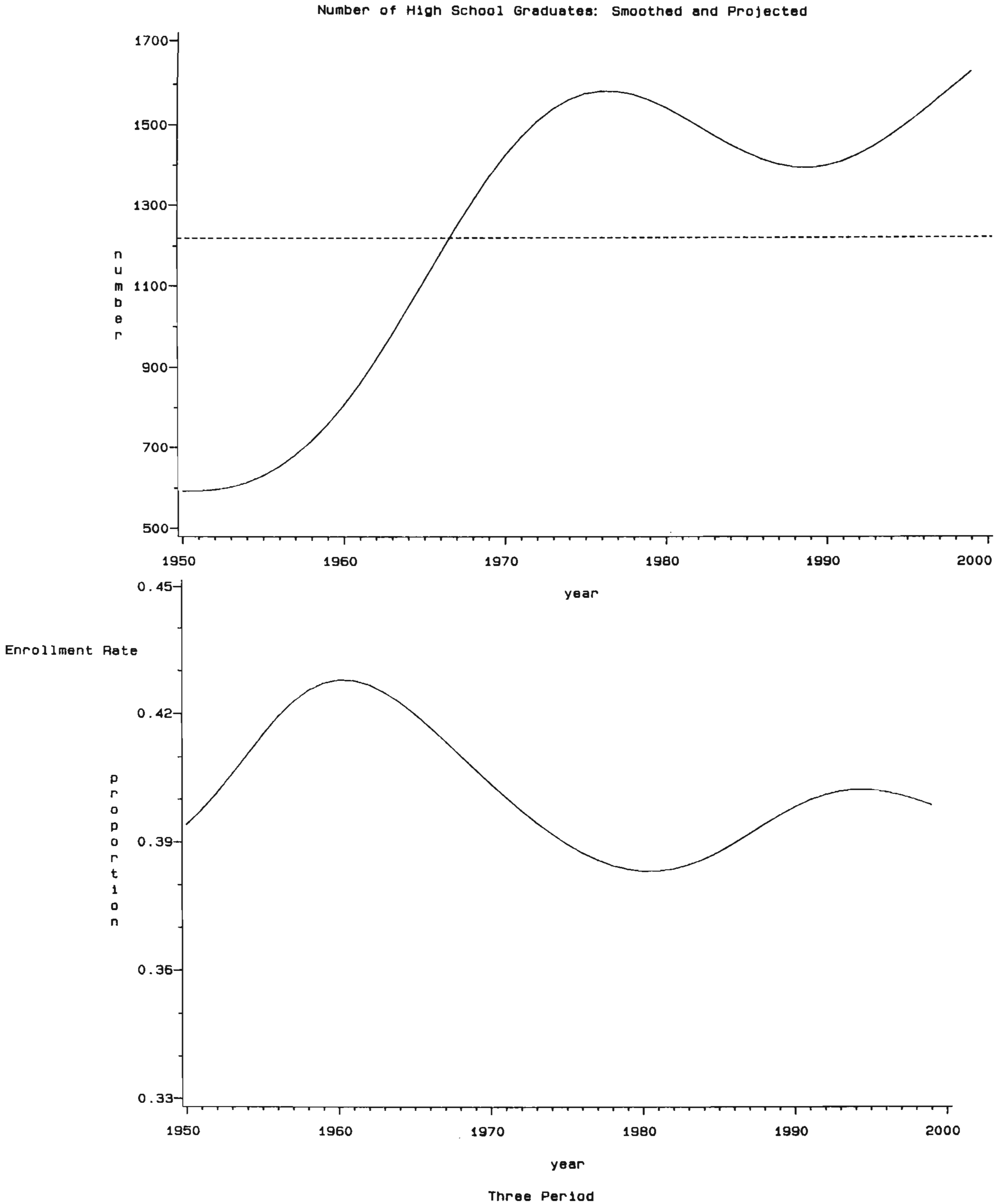

Figure 3.: Enrollment rate responses to the actual and projected number of high school graduates

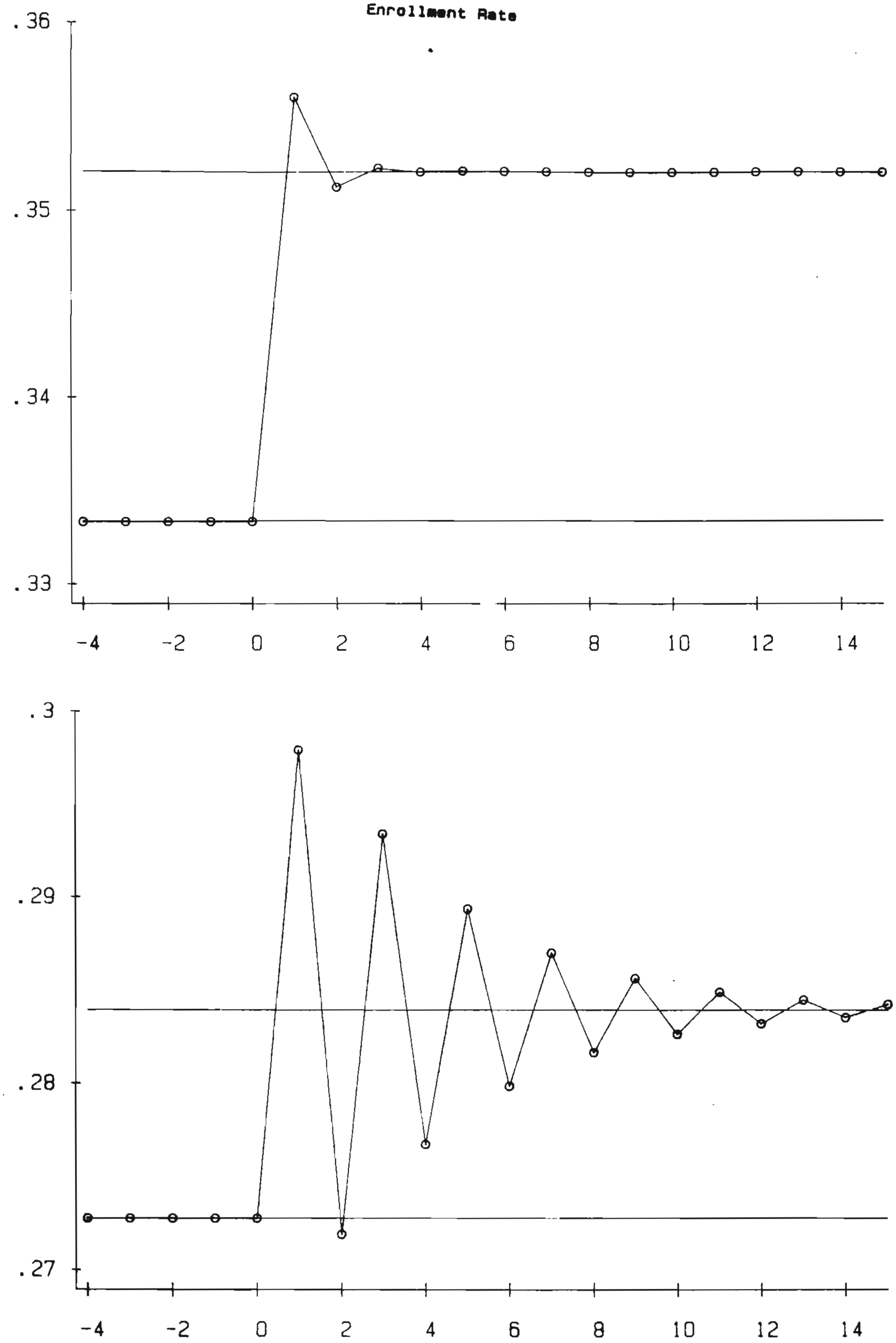

Figure 4: Enrollment rate responses to an unanticipated increase in the birth rate at time 0

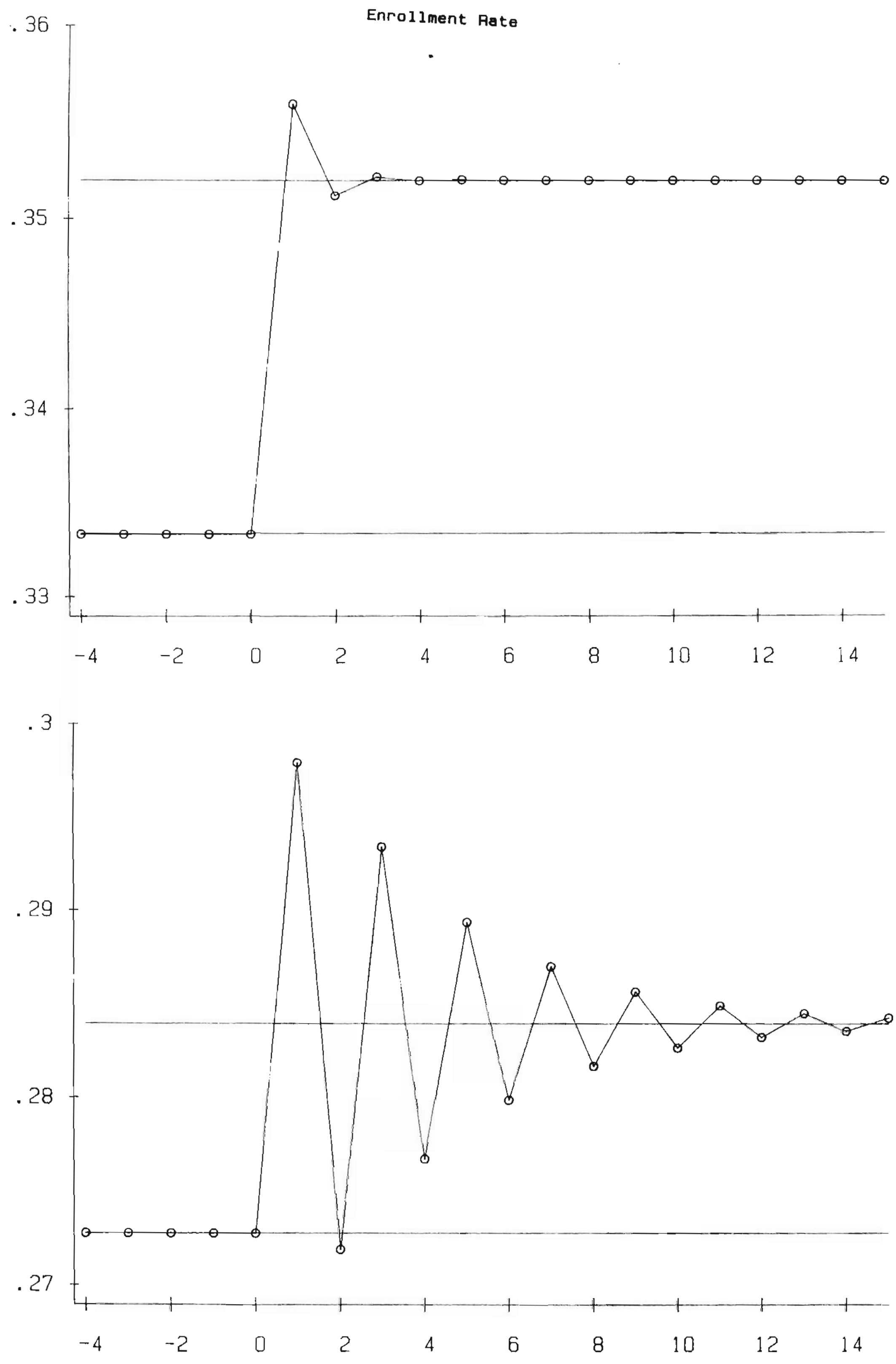

Figure 4: Enrollment rate responses to an unanticipated increase in the birth rate at time 0

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