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A Specification Test of a Model of Engel's Law and Saving

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Abstract

This paper develops a formal specification test for Atkeson and Ogaki's [1990] model of consumer behavior that explains, at least qualitatively, some stylized facts concerning Engel's law and saving in a unified framework. As an initial step to quantify our predictions about the stylized facts, we estimate and test our model, using Consumption Expendituer Survey (CES) and National Income and Product Accounts (NIPA) in the U.S. We develop a method to estimate preference parameters governing expenditure elasticities and the intertempoarl elasticity of substitution from panel data. We apply the method to the CES, using the CES as synthetic panel data. Then we estimate the same preference parameters from aggregate time series data of NIPA, using Ogaki and Park's [1989] cointegration approach. Then a specification test is formed by comparing our estimates from the CES with our estimates from the NIPA data. We obtain similar estimates from these two set of data, and the specification test does not reject our model.

I. Introduction:

This paper develops a formal specification test for Atkeson and Ogaki's [1990] model of consumer behavior that explains, at least qualitatively, some stylized facts concerning Engel's law and saving in a unified framework. As an initial step to quantify our predictions about the stylized facts, we estimate and test our model, using Consumption Expendituer Survey and National Income and Product Accounts in the U.S.

Some researchers found a rising S shaped pattern of saving rates the process of development and in cross-sectional data. Kuznetes [1960] and Chenery and Syrquin [1975] found that the saving rate is stable or rises only gradually at the initial stage of development; the saving rae rises rapidly in the second stage of development; and the saving rate becomes stable again at a high level in the last stage of development. Using Indian panel data, Bhalla [1980] found an S curve of saving rate as a function of permanent income. Atkeson and Ogaki [1990] argued that this S curve could be related with nonhomothetic preferences over food consumption and nonfood consumption. Using simulations, Atkeson and Ogaki [1990] found that both the linear expenditure system (Klein-Rubin [1947-48], Geary [1950-51] and Stone [1954]) and Houthakker's [1960] addilog utility function that have often been used in the literature of the applied counsumption analysis could generate an S curve of saving rate. Atkeson and Ogaki [1990] argued, however, the linear expenditure system has an counterfactual implication that the expenditure elasiticity of demand for food rises as the budget share for food increases.[1] For this reason, this paper foucuses on the

[1] Ogaki [1990] found that the addilog utility funciton, estimated from

addilog utility function.

The rest of the present paper is organized as follows. Section II presents our model and disucsses that poorer consumers have smaller intertemporal elasticity of substitution. This implies that poorer consumers will save less proportionately. Corresponding to lower saving rates, poorer consumers have lower growth rates for real consumption expenditures. In Section III, we will derive an exact analytical solution for the growth rate of real consumption expenditure, assuming the addilog utility function and utilizing stochastic Euler equations and intraperiod first order conditions. This solution is used to develop a method to estimate the curvature parameters of the utility function from panel data. We use Consumption Expenditure Survey (CES) in the United States as synthetic panel data.

Section IV explains that Ogaki and Park's (1989) cointegration approach provides another method to estimate the same preference parameters from time series data. The cointegration approach focuses on intratemporal first order condtions and uses stochastic and deterministic time trends in the relative price and real consumption expenditures to estimate these preference parameters. We use National Income and Product Accounts (NIPA) in the United States for this purpose. Then a specification test is formed by comparing our estimates from CES with our estimates from NIPA as suggested by Hausman's (1978) test. Thus as Hall (1978) and Hansen and Singleton's (1982, 1983) Euler equation approach, our approach avoids estimation of permanent income.

NIPA time series data, could explain expenditure elasticities estimated from cross-sectional data well.

Section V presents trend properties of the time series data, and Section VI presents cointegration regression results. Our estimates of the curvature parameters from time series data are similar to those from the synthetic panel data in Section III and we easily accept our model by the specification test. Section VII containes our concluding remarks.

II. The Model

A. An Arrow-Debreu Economy

Consider an ecomomy with H consumers and two goods. Let a scalar $s(t)$, $s(t)=1,2,\ldots,S$, denote the state of the world in each period and the vector $e(t)=[s(0),s(1),\ldots,s(t)]$ be the history of the economy. Let the consumer h, $h=1,\ldots,H$, have time and state separable utility with an intratemporal utility function $u(C^h(t,e(t)))$, where $C(t,e(t))=(C_1(t,e(t)),C_2(t,e(t))'$ and let β denote the consumer's discount factor. Let $P(t,e(t))=(P_1(t,e(t)),P_2(t,e(t)))'$ be the intratemporal prices and $E(t,e(t))$ be the total expenditure allocated to consumption at time t. Thus the consumer h maximizes

$$U^h = \sum_{t=0}^{\infty} \sum_{e(t)} \beta^t Prob(e(t)|e(0))u(C^h(t,e(t)) \tag{1}$$

where $Prob(e(t)|e(\tau))$ denotes the conditional probability of $e(t)$ given $e(\tau)$, subject to a life-time budget constraint

$$\sum_{t=0}^{\infty} \sum_{e(t)} (\prod_{\tau=0}^{t} R(\tau-1,e(\tau-1),e(\tau))^{-1} P(t,e(t))' C^h(t,e(t)) \le W^h(0), \tag{2}$$

where $W^h(0)$ is the consumer h's initial wealth. Here we take the first good as the numeraire in each period $(P_1(t)\equiv 1)$ and $R(t-1,e(t-1),e(t))$ is the (gross) asset return of the state contingent security for the event $e(t)$ in terms of the first good in the event $e(t-1)$ at period $t-1$. We assume that consumers' intratemporal preferences are well described by the addilog utility function

$$u(C(t)) = \frac{\theta_1}{1-\alpha_1}(C_1(t)^{(1-\alpha_1)}-1) + \frac{\theta_2}{1-\alpha_2}(C_2(t)^{(1-\alpha_2)}-1) \tag{3}$$

where C_1 is food consumption, C_2 is nonfood consumption and $\alpha_1>\alpha_2>0$.

Atkeson and Ogaki [1990] showed that the intertemproal elasticity of substitution for the consumer h with the addilog utility function is approximately

$$\sigma^h = 1/(\alpha_1\mu^h). \tag{4}$$

where μ^h is the expenditure elasticity of demand for the first good that is given by the expression

$$\mu^h = \frac{\partial \log(C_1^h)}{\partial \log(E^h)} = \left(\frac{C_1^h}{E} + \frac{\alpha_1}{\alpha_2}\frac{P_2C_2^h}{E^h}\right)^{-1} \tag{5}$$

With $\alpha_1>\alpha_2$, the expenditure elasticity of demand for the first good declines from 1 to α_2/α_1 as expenditure E^h rises from zero to infinity. The consumer's intertemporal elasticity of substitution then moves from $1/\alpha_1$ to $1/\alpha_2$ as the budget share spent on food moves from one to zero.

B. Aggregation of Preferences with the Addilog Utility Function

This section describes the aggregation result in Atkeson and Ogaki (1990) for the case of the addilog utility function. Though Atkeson and Ogaki (1990) abstracted from uncertainty, it will be shown that the extention of their aggregation results to the case of uncertainty is straightforsard with the complete set of market. A convenient feature of the addilog utility function is that the preferences of the fictious representative consumer can be represented by another addilog utility function that only depends on the distribution of initial wealth. The distribution of the intial wealteh only affects the reprentative consumer's weighting parameters θ_i's, which will be different from each consumer's weighting parameters. This feature of the addilog utility function implies that one can follow the evolution of the aggregate intertemporal elasticity of substitution by observing the aggregate budget share spent on food. One does not have to observe the evolution of the entire distribution of consumption or wealth to measure the change in aggregate saving behavior over time.

Assume that each of the consumers in the economy has some endowment of the consumption goods and the various factors of production. Furthermore, assume that aggregate production possibilities are described by some set Y of feasible aggregate consumption vectors. Our aggregation result is that, if this economy has a competitive equilibrium, then there exists a parameter D, which only depends on the distribution of the initial wealth, for which the equilibrium prices and the aggregate consumption vector are a competitive equilibrium for an economy with the same production possibility set Y and a single representative consumer who has time separable preferences with an intratemporal utility function given by

$$(6) \qquad u(C(t)) = \frac{\theta_1}{1-\alpha_1}(C_1(t)^{(1-\alpha_1)}-1) + \frac{\theta_2 D}{1-\alpha_2}(C_2(t)^{(1-\alpha_2)}-1)$$

This representative consumer has addilog utility with the same parameters α_1 and α_2 as the individual consumers. Knowledge of these parameters together with the budget share spent on food (either for the individual or for the aggregate) is sufficient to calculate our approximation for both the individuals and the representative consumer's intertemporal elasticity of substitution.

To prove this aggregation result, begin with the assumption that there exists a competitive equilibrium for the original economy with H consumers. Denote individual consumption in period t by $C_i^h(t)$, $i=1,2$, $h=1,2,\ldots,H$. Note that the Euler equations for the intertemporal optimization imply

$$(7)\quad \left(\frac{C_i^h(t,e(t))}{C_i^h(t+1,e(t+1))}\right)^{-\alpha_i} = \beta R(t,e(t),e(t+1)) \frac{P_i(t,e(t))}{P_i(t+1,e(t+1))} Prob(e(t+1)|e(t))$$

for $i=1,2$. The first order conditions for the intratemproal optimizaion impliy

$$(8)\quad \frac{\theta_2}{\theta_1} \frac{C_2^h(t,e(t))^{-\alpha_2}}{C_1^h(t,e(t))^{-\alpha_1}} = \frac{P_2(t,e(t))}{P_1(t,e(t))}.$$

The condition (7) governs the consumer's intertemporal allocation of consumption while the condition (8) governs the consumer's intratemporal allocation of consumption.

The Euler equation for the individual consumers, (7), indicates that consumption growth of each good is the same for all consumers in equilibrium for each peiriod and each state of the world. Hence each consumer's food consumption is a constant fraction over time of aggregate food consumption. We can index the distribution of initial wealth in equilibrium by indexing consumer's by the fraction δ_h defined by $\delta_h C_1^* = C_1^h$. Clearly δ_h must sum to one.

Following Atkeson and Ogaki (1990), the aggregation result can be derived from the fact that the representative consumer's intertemporal and intratemporal first order conditions are satisfied at the equilibrium prices and the equilibrium aggregate consumption vector when $D = \left(\sum_h \delta_h^{\alpha_1/\alpha_2}\right)^{\alpha_2}$. This parameter is clearly one if $\alpha_1=\alpha_2$ (so that intratemporal utility is homothetic). When $\alpha_1>\alpha_2$ as assumed in this model, then the utility function of the representative consumer depends upon the distribution of initial wealth in the economy.

III. Estimation with Panel Data

As noted above, poorer consumers have smaller intertemporal elasticity of substitution in our model. This implies that poorer consumers will save less proportionately. Corresponding to lower saving rates, poorer consumers have lower growth rates for real consumption expenditures. This section derives an analytical solution for the growth rate of real consumption expenditure. This solution is used to develop a method to estimate parameters $1/\alpha_2$ and α_1/α_2 from panel data. We then explain the data and present our empirical results.

A Solution for Expenditure Growth

The first order conditions for the consumer h's ($h=1,\ldots,H$) intratemporal optimization problem include

$$\theta_i C_i^h(t)^{-\alpha_i} = \lambda^h(t) P_i(t) \quad \text{for } i=1,2, \tag{9}$$

$$C_1^h(t) + P_2(t) C_2^h(t) = E^h(t), \tag{10}$$

where $\lambda^h(t)$ is the Lagrange multiplier for the intratemporal budget constraint (10). We employ the normalization $P_1(t)\equiv\theta_1\equiv1$.

From (9), $C_i=\theta_i^{1/\alpha_i}\lambda^{-1/\alpha_i}P_i^{-1/\alpha_i}$. Define $\omega^h(t)=P_2(t)C_2(t)/C_1(t)$. Then

(11) $$\omega^h = \theta_2^{1/\alpha_2} P_2^{1-1/\alpha_2} C_1^{h(\alpha_1/\alpha_2-1)}.$$

Let $\hat{y}(t) = \ln(y(t+1))-\ln(y(t))$ be the growth rate of y for any variable y. As noted in the last section, the Euler equations for the intertemporal optimization imply that $\hat{C}_i^h$ will be equal for all the consumers. Thus $\hat{C}_i^h=\hat{C}_i^*$ where $C_i^*(t) = (1/N)\sum_{h=1}^{H} C_i^h(t)$ is the aggregate equilibrium consumption.[2] Then from (11)

(12) $$\hat{\omega}^h(t) = (1-1/\alpha_2)\hat{P}_2(t) + (\alpha_1/\alpha_2-1)\hat{C}_1^*(t)$$

which implies that all the consumers have the same $\hat{\omega}^h$. Since $E^h \equiv C_1^h(1+\omega^h)$,

(13) $$\hat{E}^h(t) = \ln[1+\omega^h(t)\exp(\hat{\omega}(t))] - \ln[1+\omega^h(t)] + \hat{C}_1^*(t).$$

If $\alpha_1>\alpha_2$, then richer consumers will have higher $\omega(t)$. Suppose that $\hat{\omega}^h>0$. Then relation (13) and the fact that $\hat{\omega}^h$ is the same across the consumers imply that the growth rate of the expenditure is greater for richer consumers.

Synthetic Panel Data

We used the Consumption Expenditure Survey (CES) reported in *Consumer Expenditure Survey* (1989), and *News* (1989a, 1989b) to construct synthetic panel data. In the CES individual households are not followed through time except for a small fraction of the households. Thus the CES is not a panel. However, we can track groups of households as Browning, Deaton, and Irish (1985) did with the British Family Expenditure Survey. The data source

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[2] Note that total expenditure growth will not be the same for all consumers in general since consumption of the two different goods can grow at different rates and consumers can spend different fractions of total expenditure on the two goods.

reports average expenditures for many categories of consumption goods for each quintile of income before taxes. We view each income quintile as an individual household, assuming that most of the households in each income quintile do not move to another different quintile over time.

As the first good, which will be called food, food plus alcoholic beverages were used. As the second good, we used nonfood consumption. We defined nonfood consumption as total consumption minus food consumption where total consumption was average annual expenditures minus the sum of cash contributions and personal insurance and pensions.

Estimation Results

Relation (12) should be exact when all the variables are measured exactly. We used time series data for $\hat{P}_2(t)$ and $\hat{C}^*_1(t)$ and assumed these variables were measured exactly. We used the synthetic panel data for $\hat{\omega}^h(t)$ and assumed that this variable was measured with errors that were independently and identically distributed across consumers and time periods. Under these assumptions, we ran the regression

$$(14) \qquad \hat{\omega}^h(t) = b_1\hat{P}_2(t) + b_2\hat{C}^*_1(t) + \epsilon_p(j)$$

where $j=H(t-1)+h$ for $h=1,\ldots,H$ and $t=1,\ldots,T_1$ and $\epsilon_p(j)$ is i.i.d. for $j=1,\ldots,H\cdot T_1$. Note that there is no constant term in this regression. This regression provides consistent estimates for $1/\alpha_2=1-b_1$ and $\alpha_1/\alpha_2=1+b_2$. Table 1 presents OLS results from $H\cdot T_1=20$ samples given by five income quintiles (H=5) and four time periods ($T_1=4$ from 1984-1987) in the synthetic panel data. Point estimates for $1/\alpha_2$ and α_1/α_2 have theoretically correct positive sign and the point estimate of α_1/α_2 is bigger than one as Engel's law requires.

IV. The Cointegration Approach

This section presents a procedure to testing our model by comparing our estimates in the previous section with estimates from time series data obtained by Ogaki and Park's (1989) cointegration approach. We will show that the first order condition that equates the relative price with the marginal rate of substitution implies cointegration. The cointegrating vector, that can be estimated from cointegrating regressions, involves the curvature parameters.

The Stationarity Restriction

The fixious representative cosumer satisfies the first order condition (8) with θ_2 replaced by $D\theta_2$ for aggregate consumption. Taking the natural logarithm of both sides of this first order condition, which states the relative price is equated with the marginal rate of substitution of the fictitious represetative consumer, yields

$$p_2(t) - \alpha_1 c_1^*(t) + \alpha_i c_2^*(t) = log(D\theta_2/\theta_1)$$

where $p_2(t)=log(P_2(t))$, $c_i^*(t)=log(C_i^*(t))$.

The time series data provide measurements $p_2^m(t)$, $c_1^m(t)$, and $c_2^m(t)$ for $p_2(t)$, $c_1^*(t)$, and $c_2^*(t)$. To be consistent with an assumption made in the previous section, we assume that $p_2(t)$ and $c_1^*(t)$ are measured without any error. However, we assume that $C_2^*(t)$ is measured with error. These assumptions can be motivated by the fact that total consumption minus food, which include services, clothing, and durables, is much more difficult than measuring consumption of food. Let $\xi_2(t)=\{C_2^m(t)-C_2^*(t)\}/C_2^*(t)$ be the ratio of the measurement error and consumption. We assume that $\xi_2(t)$ is stationary. Taking the log of both sides of $C_2^m(t)=[1+\xi_2(t)]C_2^*(t)$, we obtain $c_2^m(t)=c_2^*(t)+log[1+\xi_2(t)]$ where $c_2^m(t)=log[C_2^m(t)]$. It should be noted that $log[1+\xi_2(t)]$ is stationary. Since $p_2^m(t) - \alpha_1 c_1^m(t) + \alpha_2 c_2^m(t) = \{p_2(t) - \alpha_1 c_1^*(t) + \alpha_2 c_2^*(t)\} + \alpha_2 log[1+\xi_2(t)\}$, the restriction that $p_2(t)-\alpha_1 c_1^*(t)+\alpha_2 c_2^*(t)$ is a constant implies that $p_2^m(t) - \alpha_1 c_1^m(t) + \alpha_2 c_2^m(t)$ is stationary. We shall call this restriction the stationarity restriction.

Implications of the Stationarity Restriction

In this subsection, implications of the stationarity restriction are discussed. We focus on the relation between the first and second goods. The restriction on trend properties of the variables from the demand side is summarized by the stationarity restriction. For supply side, we need to require that at least one of the endowment process is difference stationary for identification of preference parameters.

Ogaki and Park's (1989) notions of the stochastic cointegration and the deterministic cointegration are useful when economic variables of interest are modeled as difference stationary with drift. The present paper focuses on integrated of order one processes and trend stationary processes around a linear deterministic trend. Suppose that the components of a vector series $Z(t)$ are difference stationary with drift. If a linear combination of $Z(t)$, $\gamma' Z(t)$ is trend stationary, the components of $Z(t)$ are said to be cointegrated with a cointegrating vector γ. It is often convenient to normalize the first element of γ to be one, so that $\gamma=[1,\gamma_x]$, and to call γ_x the normalized cointegrating vector. If $\gamma' Z(t)$ is stationary, then the cointegrating vector γ eliminates the deterministic trends as well as the stochastic trends. This restriction is called the deterministic cointegration restriction. If $Z(t)$ consists of difference stationary and/or trend stationary series, and if $\gamma' Z(t)$ does not have any deterministic trend, then the components of $Z(t)$ are said to be cotrending with a cotrending vector γ. The deterministic cointegration restriction requires the cointegrating vector to be a cotrending vector. We will see that Assumption 2 leads to a stochastic cointegrated difference stationary series that does not satisfy the deterministic cointegration restriction.

First, we consider the case where both the logarithm of the endowment of the first good and that of the second good are difference stationary:

Assumption 1a: The process $\{c_i^*(t):t\geq 0\}$ is difference stationary for $i=1,2$.

Assumption 1b: The processes $\{c_1^*(t):t\geq 0\}$ and $\{c_2^*(t):t\geq 0\}$ are not stochastically cointegrated.

Then $c_2^m(t)$ is the sum of a difference stationary and stationary processes and therefore is difference stationary.

Let $y(t) = p_2^m(t)$ and $X(t) = [c_1^m(t),\ c_2^m(t)]'$, using the notation introduced above. The stationary restriction implies that $y(t) - \gamma_x' X(t)$ is stationary with $\gamma_x=[\alpha_1,\ -\alpha_2]'$. Assumption 1b is equivalent to an assumption that there is no 2-dimensional vector γ_x such that $\gamma_x' X(t)$ is trend stationary. Assumption 1b requires that two equilibrium consumption series possess different stochastic trends. Since $y(t) - \gamma_x' X(t)$ is stationary with $\gamma_x=[\alpha_1,\ -\alpha_2]'$, this implies that $y(t)$, which is the sum of a difference stationary $\gamma_x' X(t)$ and a stationary process, is difference stationary. Thus the stationarity restriction implies that (i) $p_2^m(t)$ is difference stationary, (ii) $p_2^m(t)$ and $[c_1^m(t),\ c_2^m(t)]'$ are stochastically cointegrated with a normalized cointegrating vector $[\alpha_1,\ -\alpha_2]'$, and (iii) the deterministic cointegration restriction is satisfied, under Assumption 1.

Second, we consider the case where the log of the endowment of the first good is difference stationary and that of the second good is trend stationary:

Assumption 2: The process $\{c_1^*(t):t\geq 0\}$ is trend stationary, and the process $\{c_2^*(t):t\geq 0\}$ is difference stationary.

In this case, the stationarity restriction implies that (i) $p_2^m(t)$ is difference stationary, (ii) $p_2^m(t)$ and $c_1^m(t)$ are stochastically cointegrated with a normalized cointegrating vector $\gamma_x=\alpha_1$, and (iii) $p_2^m(t)$ and $[c_1^m(t),c_2^m(t)]$ are cotrended with a normalized cotrending vector $[\gamma_x,\gamma_Z]'$ = $[\alpha_z,\ -\alpha_2]'$.

Econometric Procedures

The first subsection briefly describes Ogaki and Park's (1989) econometic procedure for cointegrated systems. We do not use standard econometric procedures (see, e.g., Engel and Granger [1987]) for reasons described in Ogaki and Park (1989). The second subsection develops our specification test.

Time Series Regressions

Let $X(t)$ be a k-dimensional difference stationary process: $X(t)$ - $X(t-1)$ = $\phi_x + \epsilon_x(t)$ for $t\geq 1$, where ϕ_x is a k-dimensional vector of real numbers where $\epsilon(t)$ is stationary with mean zero. Let $y(t)$ be a scalar difference stationary process: $y(t) - y(t-1) = \phi_y + \epsilon_y(t)$. pose that $y(t)$ and $X(t)$ are stochastically cointegrated with a normalized cointegrating vector γ_x, and the components of $X(t)$ are not cointegrated. Then we can apply the Canonical Cointegrating Regressions (*CCR*) procedure developed by

Park (1988) to

$$(15) \qquad y(t) = \theta_c + \phi_c t + \gamma_x' X(t) + \epsilon_c(t),$$

where $\phi_c = \phi_y - \gamma_x' \phi_x$. If $y(t)$ and $X(t)$ satisfies the deterministic cointegration restriction, then ϕ_c is zero and the *CCR* is applied to

$$(16) \qquad y(t) = \theta_c + \gamma_x' X(t) + \epsilon_c(t).$$

This *CCR* procedure only requires to transform data before running a regression and corrects for endogeneity and serial correlation. The *CCR* estimators have asymptotic distributions that can be essentially considered as normal distributions, so that their standard errors can be interpreted in the usual way.

An important property of the *CCR* procedure is that linear restrictions can be tested by χ^2 tests which are free from nuisance parameters. We can use χ^2 tests in a regression with spurious deterministic trends added to (16) to test for stochastic and deterministic cointegration. For this purpose, the *CCR* procedure is applied to a regression

$$(17) \qquad y(t) = \theta_c + \sum_{i=1}^{q} \eta_i t^i + \gamma_x' X(t) + \epsilon_c(t).$$

Let $H(p,q)$ denote the standard Wold statistic to test the hypothesis $\eta_p = \eta_{p+1} = \ldots = \eta_q = 0$ with the estimate of the variance of $\epsilon_c(t)$ replaced by the long run variance of the *CCR* (see Park, Ouliaris, and Choi [1988] or Ogaki and Park [1989] for more explanations). Then $H(p,q)$ converges in distribution to a χ^2_{p-q} random variable under the null of cointegration. In particular, the $H(0,1)$ statistic tests the hypothesis $\phi_c = 0$ in (17) and thus tests the deterministic cointegrating restriction. If $y(t)$ and $X(t)$ are not stochastically cointegrated, then $\epsilon_c(t)$ is difference stationary for any vector of real numbers used as γ_x in (16). In this case, (16) is a spurious regression and $H(1,q)$ statistics diverge in probability. Hence the $H(1,q)$ tests are consistent against the alternative of no stochastic cointegration.

Let us consider a cointegrated system involving a trend stationary process. Let $z(t)$ be a trend stationary process:

$$z(t) = \theta_z + \phi_z t + \epsilon_z(t), \tag{18}$$

where $\epsilon_z(t)$ is stationary with zero mean and $\phi_z \neq 0$. Suppose that an economic model leads to a restriction that $y(t) - \gamma_x' X(t) - \gamma_z z(t)$ is stationary. Since $y(t) - \gamma' X(t) - \gamma_z z(t) = -\gamma_z \theta_z + (\phi_y - \gamma_x' \phi_x - \gamma_z \phi_z)t + \{y^0(t) - \gamma_x' X^0(t)\}$, this restriction implies that

$$\phi_y = \gamma_x' \phi_x + \gamma_z \phi_z \tag{19}$$

and that $y(t)$ and $X(t)$ are stochastically cointegrated with a normalized cointegrating vector γ_x.

For a cointegrated system with a trend stationary process, we can apply the *CCR* to a system of Seemingly Unrelated Regressions *(SUR)* consisting of (17) and (18) to estimate γ_x, ϕ_x, ϕ_c as in Park and Ogaki (1989). We call this system the Seemingly Unrelated Canonical Cointegrating Regressions *(SUCCR)*. We apply the *GLS* to the system of *SUCCR*. Then we can estimate γ_z from (19) and the equation $\phi_c = \phi_y - \gamma_x' \phi_x$.

The Specification Test

A formal specification test for our model is provided by comparing our estimates from the synthetic panel data with our estimates from the time series data. The cointegration approach gives consistent estimates for the curvature parameters even when (i) the relative price and consumption of the first good are measured with error, (ii) some of the consumers are liquidity constrained, (iii) there exist preference shocks, (iv) preferences are time-nonseparable (see Ogaki and Park [1989], and (v) preferences are not additively separable across goods (see Ogaki[1990]). On the other hand, our estimates from the synthetic panel data are not consistent with these factors. Thus comparing two sets of estimates provides a specification test against these alternatives.

First, let us consider the case where Assumption 1 is employed. As is well known (see, e.g., Engle and Granger [1988]), any cointegrated variable can be chosen as the regressand for the cointegrating regression. In order to obtain a joint distribution for the cointegrating regression and the regression with the synthetic panel data, however, it is necessary to choose $c_2^m(t)$ as the regressand. Thus the CCR is applied to

(20) $$c_2^m(t) = \theta_c + \gamma_{x1}p_2^m(t) + \gamma_{x2}c_1^m(t) + \epsilon_c(t)$$

to estimate $1/\alpha_2=-\gamma_{x1}$ and $\alpha_1/\alpha_2=\gamma_{x2}$. Let T_2 be the sample size for the CCR, and assume that $T_2=H\cdot T_1$. Conditioned on the regressors of (20), the asymptotic normal distributions of the CCR estimators of γ_x in (20) and those of the OLS estimators of b in (10) are uncorrelated. We can form a standard Wald statistic for the restriction

(21) $$1-b_1 = -\gamma_{x1} \text{ and } 1+b_2 = \gamma_{x2},$$

following Park (1988) and Park and Ogaki (1990). The statistic, which will be denoted by G_R, has an asymptotic χ^2_2 distribution.

Second, let us consider the case where Assumption 2 is employed. In this case, c^m_2 is chosen as the regressand $y(t)$ in (15) and c^m_1 is used as $z(t)$ in (18). Then the restriction

$$(22) \qquad 1\text{-}b_1 = -\gamma_x \text{ and } 1+b_2 = \phi_c/\phi_z$$

can be tested by a Wald statistic, denoted by G_R, that has an asymptotic χ^2_2 distribution.

V. Trend Properties of the Time Series Data

In this section, we test empirical validity of Assumptions 1 and 2. The first subsection describes the time series data. The second subsection reports results of tests for difference stationarity and trend stationarity of time series of real consumption expenditures and relative prices. The third subsection reports results of tests for Assumption 1b.

Time Series Data

Annual data in the National Income and Product Accounts (NIPA) were used for the time series data. We used food consumption as the first good, and nonfood consumption (denoted NFC) as the second good. Our data incorporates the revision of the data reported in July 1989 *Survey of Current Business*. The data for real per capita consumption expenditures were constructed by dividing personal consumption expenditures in constant 1982 dollars by the total population including armed forces overseas obtained from the CITIBASE.[3] The implicit deflator was used as the price for each consumption series. The implicit deflators for each series was constructed by dividing personal consumption expenditure in current dollars by that in constant 1982 dollars. The sample period was from 1947 to 1988.

Gordon (1990) argued that the measurement of durable goods prices were not precise in the NIPA and constructed new data for 1947 to 1983. We also used his data to construct the data for the second good. This series for the second good is denoted NFCG.

Tests for Difference and Trend Stationarity

Tests for Time Series of Consumption Expenditures

Our next results are concerned with discrimination between trend stationarity and difference stationarity of consumption series. Let $\{X(t)\}$ be the process of interest. We are interested in whether $X(t)$ is difference stationary or trend stationary. Consider an *OLS* regression

$$X(t) = \sum_{i=0}^{q} \hat{\eta}_i t^i + \hat{\epsilon}(t), \tag{23}$$

and define $\hat{\sigma}^2 = (1/T)\sum_{t=1}^{T} \hat{\epsilon}(t)^2$. Let $F(p,q)$ denote the standard Wald test statistic in regression (23) for the null hypothesis $\eta_{p+1}=\eta_{p+2}=,\ldots,=\eta_q=0$. Let $J(p,q) = (1/T)F(p,q)$ and $G(p,q) = (\hat{\sigma}^2/\hat{\Omega})F(p,q)$, where $\hat{\Omega}$ is an consistent estimate of the long run variance of $\hat{\epsilon}(t)$ in (23). Then $J(1,q)$ converges in distribution to a nondegenerate random variable under the null hypothesis

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[3] We incorporated the revision of population estimates reported in *Current Population Reports* (Series p-25, No.1036) issued in March 1989 by Bureau of the Census after the release of the version of the CITIBASE we used.

that $X(t)$ is difference stationary; $G(1,q)$, to a χ^2_{q-1} random variable under the null hypothesis that $X(t)$ is trend stationary (see Park and Choi [1988]). Hence $J(1,q)$ can be used to test the null of difference stationarity against the alternative of trend stationarity. We reject the null of difference stationarity when the $J(1,q)$ statistic is smaller than critical values tabulated by Park and Choi (1988). The $G(1,q)$ statistic can be used to test the null of trend stationarity against the alternative of difference stationarity. We reject the null of trend stationarity when the $G(1,q)$ statistic is larger than critical values. These tests are consistent.

For the null of difference stationarity, Dicky and Fuller's (1978) tests and their corrections in Said and Dickey (1984), Phillips (1987), Phillips and Perron (1988) have often been used. These corrections require estimation of the long run variance when the disturbance is serially correlated, which is known to cause severe size distortions in some cases (see Schwert [1987] and Phillips and Perron [1988] for the size distortion problem, Cochrane [1987] and Christiano and Eichenbaum [1989] for discussions about difficulties in estimating the long run variance, and Kahn and Ogaki [1990] and references therein for power of unit root tests.) Park and Choi's J tests do not require estimation of the long run variance and have much less size distortions (see Park and Choi [1988]).

Table 2 presents results of the $J(1,5)$ test with the null of difference stationarity of the logarithm of real per capita consumption expenditures. According to $J(1,5)$ tests, there was no evidence against difference stationarity of consumption series for all the goods examined at the 1 per cent significance level. The $J(1,5)$ test, however, rejected the null of difference stationary food consumption at the 5 per cent level.

In table 3, we report results of the *G(1,2),..., G(1,6)* tests for the null hypothesis of trend stationarity. The sample period used was 1948-1988 for food and NFC and 1948-1983 for NFCG. For estimation of the long run variance, Parzen's lag window was used and the lag truncation numbers of 1, 4, 7, 10, and 13 were tried. The lag truncation number used for the results in table 2 was 7. The *G(1,2)* test results for all the series were stable for the lag truncation numbers of 7, 10, and 13 in terms of statistical inference based on the 1, 5, and 10 per cent significance levels. Most of the test statistics were not stable for the lag truncation numbers that were less than 7, indicating that the lag truncation numbers were not large enough. At least, one of the *G(1,q)* tests rejected the null of trend stationarity in favor of the alternative of difference stationarity at the 5 per cent level for NFC and NFCG. On the other hand, no test statistics were significant at the 5 per cent level for food.

In the light of these results, we will employ the following specifications for the rest of the empirical results reported in the present paper. We will specify that the log real consumption series is difference stationary for NFC and NFCG. We will try both the specifications that the log consumption is difference stationary and that the log consumption is trend stationary for food, though empirical evidence is in favor for the latter specification .

Tests for Time Series of Relative Prices

As shown in Section V, the stationary restriction implies that the log relative price is deference stationary under Assumption 1 or Assumption 2. We now test this implication of the model.

Table 4 presents results of the *J(1,5)* test for the log of the relative price. The relative price of NFC and Food and that of NFCG and Food were used. Our test statistic was not significant at the 10 per cent level for either of these series.

Table 5 presents results of *G(1,2),...,G(1,6)* tests for the null of trend stationarity. The main purpose of Table 5 is to give some ideas about small sample power of *H(1,q)* tests as we will discuss in the next section. The lag truncation number of 7 was used. At least one of the test statistics was significant at the 5 per cent level for each series of the relative price we used.

Tests for No Stochastic Cointegration

When Assumption 1a is employed, Assumption 1b is an important identifying assumption. For this reason, we report results of tests for the null hypothesis of no stochastic cointegration in this subsection.

Consider an *OLS* regression

$$y(t) = \hat{\theta} + \sum_{i=1}^{q} \hat{\eta}_i t^i + \hat{\gamma}' X(t) + \hat{\epsilon}(t). \qquad (24)$$

where $y(t)$ and $X(t)$ are difference stationary processes, and let $F(p,q)$ denote the standard Wald test statistic in regression (24) for the null hypothesis $\eta_{p+1}=\eta_{p+2}=,\ldots,=\eta_q=0$. Define $I(p,q) = (1/T)F(p,q)$. Ouliaris, Park and Choi (1988) showed that $I(1,q)$ converges in distribution to a nondegenerate random variable under the null hypothesis that $y(t)$ and $X(t)$ are not stochastically cointegrated. We reject the null of no cointegration when $I(p,q)$ statistics are smaller than critical values of $I(p,q)$ test statistics tabulated by Park, Ouliaris, and Choi (1988). The $I(p,q)$ tests statistics tabulated by Park, Ouliaris, and Choi (1988). The $I(p,q)$ tests are consistent against the alternative of stochastic cointegration. The $I(p,q)$ tests basically apply the $J(p,q)$ tests to the residual of regression (24).

Table 6 presents results of the $I(1,5)$ test for the null of no cointegration of $c_1^m(t)$ and $c_2^m(t)$ for various choices of the first and second goods. For each pair of consumption goods, we can choose the first good as the regressand or the second good as the regressand for the $I(1,q)$ tests. No test statistics were significantly small at the 10 per cent level for any of the pairs of consumption goods. Thus we accept Assumption 1b when we maintain Assumption 1a.

VI. Empirical Results of Cointegrating Regressions

This section reports results of cointegrating regressions. The first subsection presents results when Assumption 1 is employed; the second subsection, results when Assumption 2 is employed.

Canonical Cointegrating Regressions

In this subsection, we assume that all the consumption series are difference stationary, so that Assumption 1 is satisfied for each pair of consumption series. Table 7 presents *CCR* results. It reports estimates of $1/\alpha_2$, and α_1/α_2, and the G_R test. The G_R statistic tests the restriction (21) and provides the specification test discussed in Section V. Table 6 also reports the $H(0,1)$ test statistic for the deterministic cointegration restriction from regression (17) with q=1 and the $H(1,2)$, $H(1,3)$, and $H(1,4)$ test statistics for stochastic cointegration from regression (17) with q= 2, 3, and 4, respectively. Since the $G(1,q)$ test statistics for real consumption expenditures were stabilized with the lag truncation numbers that were greater than or equal to 7, we used the lag truncation number of 7 for the results reported in the tables below. The lag truncation numbers of 10 and 13 were also tried. Most of our results were not very sensitive to the choice of these lag truncation numbers.

The G_R test results were encouraging. We easily accepted our model in terms of this test for both NFC and NFCG. All the point estimates of $1/\alpha_2$ and α_1/α_2 had theoretically correct positive sign. Since α_1/α_2 is significantly greater than one, we found evidence against homothetic preferences ($\alpha_1=\alpha_2$).

However, the deterministic cointegration restriction was rejected at the 1 per cent level by the *H(0,1)* test. Thus there was substantial specification error when Assumption 1 was employed. This is not surprising given that we rejected the difference stationarity for food consumption at the 5 per cent level.

Seemingly Unrelated Canonical Cointegrating Regressions

This subsection reports *SUCCR* results under Assumption 2. Tables 8 reports results when the log food consumption is assumed to be trend stationary. We did not reject the null of trend stationarity for food, in the last section.

For each pair of consumption series, the G_R statistic provides the specification test as we discussed in Section V. Tables 8 also reports the *H(1,2)*, *H(1,3)*, and *H(1,4)* test statistics for stochastic cointegration from the *SUCCR* system consisting of (17) and (18) with q= 2, 3, and 4, respectively.

The results in Table 8 are encouraging. The G_R tests easily accepted our model. All the point estimates of $1/\alpha_2$ and α_1/α_2 had theoretically correct positive sign. Stochastic cointegration was never rejected at the 10 per cent level. Our results for two measures of nonfood consumption, NFC and NFCG were very similar. Since our estimate of α_1/α_2 was significantly different from one, there was evidence against homothetic preferences.

VII. Conclusions

If poorer consumers have smaller intertemporal elasticity of substitution, then they will save less proportionately. Corresponding to lower saving rates, poorer consumers have lower growth rates for real consumption expenditures. We derived an analytical solution for the growth rate of real consumption expenditure, assuming the addilog utility function. This solution was used to develop a method to estimate the curvature parameters of the utility function from the CES data. We also estimated the same parameters from the NIPA time series data, using Ogaki and Park's (1989) cointegration approach. Then a specification test was formed by comparing our estimates from the CES with our estimates from the NIPA data. The specification test did not reject our model.

Costello (1990) constructed data on Solow residuals for several industries in the United States. She rejected the unit root hypothesis for the food industry at the 1 per cent level and accepted the unit root hypothesis for the other industries at the 5 per cent level, using $J(p,q)$ tests. Her finding is consistent with our results in Sections V and VI.

Since the crucial identification assumption in the cointegration approach is that the equilibrium consuption series has autoregressive unit root, it is important to check sensitiviy or our results with respect to the unit root hypothesis. Ogaki (1988, 1989) developed a method to estimate the curvature parameters of the addilog utility function when all the relevant consumption series are trend stationary, utilizing information in deterministic trends summarized in the stationarity restriction. Point estimates of the curvature parameters for consumption goods studied by Ogaki were not very sensitive to the specification about trends. It is of interest to pursue this sensitivity analysis for the series we used.

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TABLE 1

REGRESSION RESULTS FROM THE SYNTHETIC PANEL DATA

$1/\alpha_2$	standard error	α_1/α_2	standard error	R^2
2.732	1.592	2.406	1.068	0.106

TABLE 2

TESTS FOR DIFFERENCE STATIONARITY OF CONSUMPTION

	Sample Period	*J*(1,5)
Food	1947-1988	0.135[a]
NFC	1947-1988	2.431
NFCG	1947-1983	3.166

NOTE: Critical values for the 1 per cent, 5 per cent, and 10 per cent significance levels are 0.123, 0.295, and 0.452. These critical values for *J(1,5)* are from Park and Choi (1988).

[a]Significant at the 5 per cent level.

TABLE 3

TESTS FOR TREND STATIONARITY OF CONSUMPTION

	Sample Period	$G(1,2)$	$G(1,3)$	$G(1,4)$	$G(1,5)$	$G(1,6)$
Food	1947-1988	0.750 (0.102)	0.220 (0.896)	0.473 (0.925)	2.005 (0.735)	2.206 (0.820)
NFC	1947-1988	5.102 (0.024)	7.289 (0.026)	7.743 (0.052)	7.814 (0.099)	8.071 (0.152)
NFCG	1947-1988	6.012 (0.014)	7.621 (0.022)	7.622 (0.055)	7.814 (0.099)	8.071 (0.152)

NOTE: Probability values are in parentheses. The lag truncation number used for the $G(p,q)$ statistics reported in this table was 7.

TABLE 4

TESTS FOR DIFFERENCE STATIONARITY OF RELATIVE PRICES

Second Good	Sample Period	$J(1,5)$
NFC	1947-1988	3.514
NFCG	1947-1983	4.376

NOTE:Critical values for the 1 per cent, 5 per cent, and 10 per cent significance levels are 0.123, 0.295, and 0.452. These critical values for $J(1,5)$ are from Park and Choi (1988).

TABLE 5

TESTS FOR TREND STATIONARITY OF RELATIVE PRICES

Second Good	Sample Period	*G*(1,2)	*G*(1,3)	*G*(1,4)	*G*(1,5)	*G*(1,6)
NFC	1947-1988	1.981 (0.159)	7.483 (0.024)	7.601 (0.055)	8.831 (0.116)	8.839 (0.183)
NFCG	1947-1988	4.484 (0.034)	7.019 (0.030)	7.890 (0.048)	7.938 (0.094)	7.954 (0.159)

NOTE: Probability values are in parentheses. The lag truncation number used for the *G*(p,q) statistics reported in this table was 7.

TABLE 6

TESTS FOR NO COINTEGRATION

Sample Period	Regressand	Regressor	*I*(1,5)
1947-1988	Food	NFC	1.830
1947-1988	NFC	Food	2.443
1947-1983	Food	NFC G	1.842
1947-1983	NFCG	Food	2.943

NOTE: Critical Values for *I(1,5)* at the 1 per cent, 5 per cent, and 10 per cent significance levels are 0.103, 0.251 and 0.384, respectively. These critical values are from Park, Ouliaris and Choi (1988).

TABLE 7

CANONICAL COINTEGRATING REGRESSION RESULTS

	$1/\alpha_2$[a]	α_1/α_2[a]	G_R[b]	$H(0,1)$[b]	$H(1,2)$[b]	$H(1,3)$[b]	$H(1,4)$[b]
NFC	0.416 (0.129)	2.579 (0.071)	2.382 (0.304)	14.153 (0.000)	0.045 (0.832)	0.046 (0.977)	0.862 (0.835)
NFCG	0.461 (0.154)	2.670 (0.096)	2.380 (0.302)	11.264 (0.001)	0.140 (0.708)	0.199 (0.905)	1.061 (0.787)

NOTE: The lag truncation number used for the results in this table was 7. The sample period used were 1947-1988 for results with NFC and 1947-1983 for those with NFCG.

[a]Standard errors are in parentheses.

[b]Probability values are in parentheses.

TABLE 8

SEEMINGLY UNRELATED CANONICAL COINTEGRATING REGRESSION RESULTS WITH THE ASSUMPTION OF TREND STATIONARY FOOD CONSUMPTION

	$1/\alpha_2$[a]	α_1/α_2[a]	G_R[b]	$H(1,2)$[b]	$H(1,3)$[b]	$H(1,4)$[b]
NFC	0.323 (0.066)	2.656 (0.072)	2.668 (0.263)	0.948 (0.330)	2.026 (0.363)	4.121 (0.249)
NFCG	0.287 (0.074)	2.822 (0.100)	2.968 (0.227)	0.909 (0.340)	4.174 (0.124)	4.734 (0.192)

NOTE: The lag truncation number used for the results in this table was 7. The sample periods used were 1947-1988 for results with NFC and 1947-1983 for those with NFCG.

[a]Standard errors are in parentheses.

[b]Probability values are in parentheses.