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THE LIFE-CYCLE OF A COMPETITIVE INDUSTRY:
THEORY AND MEASUREMENT†

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**Abstract**

This paper develops on estimable model of industry equilibrium in which firm entry and exit behavior is fueled by invention and innovation. The model's parameters are estimated by maximum likelihood, based on data from the U.S. automobile tire industry, 1906–73.

## I. Introduction

Industries, like people, display a life cycle. In a young industry, firms are few, they produce little, and the price of output is high. Entry then augments the number of firms, output per firm rises, and price falls. Eventually exit reduces the number of firms to some long run value, and output and price both stabilize. See, for example, Gort and Klepper (1982).

Informally, it has been argued that such a time path will emerge if its assumed that demand increases at a decreasing rate, while optimum firm size increases due to learning–by–doing. This, and other arguments are surveyed by Gort and Klepper and by Geroski and Masson (1987).[1] What the existing literature lacks however, is an equilibrium model that naturally gives rise to the life cycle properties mentioned above, much less one that can be estimated.

This paper presents a simple model of entry and exit in a competitive industry. It has few parameters and its equilibrium has the basic life–cycle features described above. Moreover, the equilibrium is simple enough that its parameters can be estimated.

The main ingredients of the model are as follows. The force driving industry life cycles is the growth of technological know–how. Inventions occur at random. The arrival of a new invention creates potential for figuring out exactly how it can be put to use in the production process: innovation. The same applies to subsequent invention of refinements to basic inventions. The present model allows for just two possible inventions: a basic one and a refinement. Once the basic invention has occurred firms may try to implement it. Success is random. Those who succeed may begin to produce, and if they do the industry is born. The basic invention may then be refined, and the timing of this event is random as well. But once the refinement has come on the scene, it too may be implemented, either by new entrants or by established firms. Success here is also random, and, in general, some will succeed earlier than others. At some point firms that have yet to succeed may give up trying since others' success has led to a low price for output. At that point the industry enters an exit phase wherein the number of firms gradually declines, achieving some long run value as the industry reaches "maturity". In brief, new opportunity produces entry, and relative failure to innovate yields exit.

Ignoring cases corresponding to extreme parameter values, the main observables in the model—number of producing firms, price and output—behave as follows in equilibrium. Each new invention yields immediate entry followed by stability in firm numbers. Some time after the refinement arrives, exit begins and is equal to a constant proportion of the stock of operating firms that have so far failed to generate an innovation based on the refinement. During the period after the basic invention and before refinement, output price is stable. But once refinement has occurred, the price of output begins to fall. However, its decline must cease entirely once exit has begun. The time path of output is implied by the just–described price path along with product demand.

The model's parameters describe product demand, the value of alternatives and the discount factor, the structure of production costs utilizing technology employing either the basic invention or its refinement, the stochastic processes of inventions and innovations, and "general productivity growth" and measurement error. Maximum likelihood estimates of the parameters stem from data on the U.S. automobile tire industry, 1906–73. The observed variables are number of operating firms, industry output, and an index of the wholesale price of output.

The next Section sets out and analyzes the model and its equilibrium. Since structural estimation is the goal, the focus is the structure of the model's equilibrium. Section III describes the parameterization, data, estimation methods, and the estimates.

## II. Theory

The goal of this Section is to develop an estimable model of an industry life–cycle. Thus the model's elements are straightforward. New knowledge, or "inventions", emerges constantly in the economy at large, including ideas in both science and industry. Most of this information is of no use whatsoever for any given industry, but some knowledge is applicable. Given some basic invention, firms may try to find a way to put it to use commercially, to "innovate" in the familiar Schumpeterian distinction. Those who succeed in the costly and unpredictable process of innovation start production and the industry begins. Further inventions yield new innovation opportunities for firms currently operating in the industry as well as for others; the latter may find it to their advantage to enter when such an invention occurs, and the growth phase ensues. Innovation being stochastic, however, some will lag behind as progress occurs. Because progress lowers costs for competitors, these laggard firms may find exit maximal; this yields the exit phase. These ingredients—new opportunities generating entry and relative lack of innovative success yielding exit—are key in the analysis to follow.

The model imposes three restrictions that should be discussed in advance. First, there are no *direct* costs associated with attempts to implement a new technology. Such activities are costly because they necessitate foregoing some other activity in the economy and because new techniques cannot be implemented instantaneously once learned. But, if the firm has chosen to participate in the industry in question, learning entails no further outlay. This will imply that if any active firm is using a technology other than the most advanced invented so far, such firms will learn a better technology with positive probability—progress must occur if the industry operates at all.

Second, inventions occur exogenously and *outside* the industry. There are industries in which external invention is common; see Davies (1979).

Finally, there are no spillovers of knowledge within the industry; in particular, one firm's innovative success is not affected by others' luck. In other words, it is easier for the firm to sort out implementation internally (via a R & D department, for example) than it is for it to learn from or imitate others. Under this assumption, the most attractive dates at which to enter the industry are those at which new inventions occur.

These three assumptions greatly ease the analysis, and the essential features of equilibrium do not appear sensitive to moderate relaxations of them. The formal analysis follows.

*Time*, $t$, is discrete and the horizon is infinite: $t \in \{0,1,...\}$.

The *industry* is defined by the commodity firms might produce and sell to consumers. The *consumer side* of the market is summarized by a time–invariant inverse market demand function $D(Q)$, where $Q$ is industry output; define the product price $p$ by $p \equiv D(Q)$. $D$ is continuous, strictly declining and bounded with $\lim_{Q \to \infty} D(Q) = 0$.[2]

The *invention* process is simple. At $t=0$, it is known that there will be at most two inventions of relevance to the industry. The knowledge existing at $t=0$, along with any production techniques based on it, will be referred to as "primitive". The first invention will be called a "basic" invention and the second a "refinement". This structure is intended to be a stylization of an industry's development in terms of fundamental breakthrough and subsequent significant improvement; prop planes and jets are an example, as are mainframe and super computers.

Below, assumptions will be imposed that guarantee no trade will occur prior to implementation of the basic invention. Thus, with respect to observables, there is no loss of generality in letting $t=0$ denote the date at which the basic invention arrives on the scene.

Once the basic invention has emerged, refinement is possible. For any $t \geq 1$, if the refinement has not been discovered earlier, it occurs with fixed probability $\rho \in (0,1)$. Let $T \geq 1$ denote the actual date of refinement.

Altogether, the basic invention arrives at $t=0$ and refinement is possible at any $t \geq 1$, occurring with probability $\rho$.

The *supply side* of the market comprises a fixed continuum of identical firms $[0,N]$. At any date, one option for a firm is participation "elsewhere" in the economy. Doing so yields a per period profit of $\pi^0 > 0$. Assuming a perfect capital market and constant interest rate $i > 0$, the option of producing elsewhere has capital value $\pi^0/(1-\gamma)$ where $\gamma \equiv 1/(1+i) \in (0,1)$.

Participation in the industry requires that $\pi^0$ be foregone and makes possible two activities: innovation and production.

*Innovation* is the implementation of inventions. As discussed earlier the innovation process is assumed to entail no immediate costs apart from foregoing $\pi^0$. Firms that have the know–how to implement only technologies based on primitive inventions will be referred to as "knowing $\theta^0$"; all firms are endowed with this information at $t=0$. Any firm knowing how to put at most the basic invention to work (there may be many ways to do so — for simplicity it will be assumed that all yield the same cost) will be referred to as "knowing $\underline{\theta}$", while a firm that can utilize the refinement (once it has been invented) will be said to "know $\bar{\theta}$".

Prior to $t=0$, all firms necessarily know $\theta^0$. At $t=0$, innovation of production techniques involving the basic technology becomes possible. It will be assumed that when only the basic invention has occurred, any firm knowing $\theta^0$ succeeds in its efforts to innovate (i.e. learns $\underline{\theta}$) with fixed probability $\beta \in (0,1)$ in any period. If a firm learns $\underline{\theta}$ at $t$, it may commence production using the innovated technology at $t+1$ or later.

For $t \geq T$, innovation of techniques based on the refinement (i.e. learning $\bar{\theta}$) is also a possibility. The likelihood of success at doing so may depend on the firm's present state of knowledge. Any firm knowing $\underline{\theta}$ learns $\bar{\theta}$ at $t \geq T$ with probability $r \in (0,1)$. Those knowing only $\theta^0$ learn $\underline{\theta}$ with probability $\underline{r} \in (0,1)$ and $\bar{\theta}$ with probability $\bar{r} \in [0,1]$; learning $\underline{\theta}$ is not a prerequisite to learning $\bar{\theta}$ unless $\bar{r} = 0$.

In brief then, once an invention has occurred implementation is possible and stochastic. When only the basic invention has arrived, participating firms implement it with probability $\beta$. Once refinement occurs, firms knowing how to use the basic technology learn how to use the refinement with probability $\underline{r} \in (0,1)$ ; others may still learn how to use the basic invention, doing so with probability $\underline{r} \in (0,1)$, but they may also skip directly to techniques based on the refinement. For them, this latter possibility has probability $\bar{r} \in (0,1)$.

Turning to *production*, given $\theta$, production activities yield one period profits

$$\pi(p;\theta) \equiv \max_{q \geq 0}\{pq - c(q;\theta)\},$$

where $\theta \in \{\theta^0, \underline{\theta} \text{ or } \bar{\theta}\}$. $c(\cdot\,;\theta)$ gives the factor cost of producing output $q$ using technology based on knowledge $\theta$. Implicit in this specification is that the prices of all factors, including any that might be technology–specific, are constant over time and that there are no direct adjustment costs. One rationalization for this assumption is that any factor specificity is in terms of the underlying inventions rather than in terms of the specific application in this particular industry, and that inventions find applications in numerous industries.

It is assumed that $\pi$ satisfies

i) $\pi[D(0), \theta^0] = 0$ (i.e. $0 = \underset{q \geq 0}{\text{argmax}} \{D(0)q - c(q;\theta^0)\}$); and ii) for all $p > 0$, $\infty > \pi(p, \bar{\theta}) > \pi(p, \underline{\theta}) > 0$. (i) requires that the primitive technology is nonviable even if $\pi^0$ has been foregone; (ii) imposes the condition that if $\pi^0$ has been foregone, production given any $\theta \neq \theta^0$ dominates shutting down, but knowing $\bar{\theta}$ is more profitable than knowing $\underline{\theta}$. Also, define $q(p,\theta) \equiv \underset{q \geq 0}{\text{argmax}}\{pq - c(q;\theta)\}$. $q(p,\theta^0) = 0$, and for $p > 0$ assume $q$ is increasing in $p$ with $q(p,\bar{\theta}) > q(p,\underline{\theta})$. Note that $q(p,\underline{\theta}) > 0$ is implied by $\pi(p,\underline{\theta}) > 0$.

To proceed, consider a fixed refinement date $T$. It will be assumed that while there is individual randomness due to heterogeneity in learning, there is no aggregate risk for given $T$. Given this deterministic aggregate behavior for fixed $T$, some useful expressions describing industry evolution can be set out. This description depends on the timing of events in each period. The within–period timing convention is as follows: inventions occur first, then firms choose whether to participate in the industry, and finally output is produced and any innovations realized.

At any date $t$, firms may operate in whatever industry they chose. Let $n_t^0$ denote the number (strictly, measure) of firms knowing only $\theta^0$ and that participate in the industry at $t$. $n_t^0$ evolves according to

$$n_0^0 \in [0,N],$$

$$n_{t+1}^0 = \begin{cases} (1-\beta)n_t^0 - x_{t+1}^0 & 0 \leq t < T \\ (1-\underline{r} - \bar{r})n_t^0 - x_{t+1}^0 & t \geq T, \end{cases}$$

where $x_t^0$ is net exit (or entry, if $x_t^0 < 0$) by firm's knowing $\theta^0$ at the beginning of period $t$.[3] $n_t^0$ is thus the number of firms knowing $\theta^0$ that are "at risk" with respect to learning $\theta \neq \theta^0$ at $t$. For $0 \leq t < T$, only innovations using basic technology are possible, and these are learned with probability $\beta$; for $t \geq T$ innovations using the refinement may also occur, $\underline{r}$ and $\bar{r}$ being the probabilities of innovation using the basic invention and its refinement respectively.

In an analogous manner, define $\underline{n}_t$ and $\underline{x}_t$ ($\bar{n}_t$ and $\bar{x}_t$) as the number of firms knowing $\underline{\theta}$ ($\bar{\theta}$) that participate or exit at $t$. The implied evolutions are

$$\underline{n}_0 = 0,$$

$$\underline{n}_{t+1} = \begin{cases} \underline{n}_t + \beta n_t^0 - \underline{x}_{t+1} & 0 \leq t \leq T \\ (1-r)\underline{n}_t + \underline{r}\, n_t^0 - \underline{x}_{t+1} & t \geq T, \end{cases}$$

$$\bar{n}_t = 0 \qquad 0 \leq t \leq T,$$

$$\bar{n}_{t+1} = \bar{n}_t + r\underline{n}_t + \bar{r} n_t^0 - \bar{x}_{t+1} \qquad t \geq T.$$

The interpretation of these expressions parallels that given for the evolution of $n_t^0$. To reduce clutter, assume $\bar{x}_t = 0$ and replace $\underline{x}_t$ by $x_t$. The restriction $\bar{x}_t = 0$ will be shown to be nonbinding in equilibrium

Now consider *optimization by an individual firm*. Each firm takes as given the participation decisions and knowledge of others. In this model, as is familiar from standard competitive analysis, the actions and information of others may be summarized by a price sequence. However, in the present case price will generally depend on whether the refinement has been invented; i.e. whether $t \gtrless T$, where $T$ is random. Since the equilibrium price path turns out to be a very simple one, introduction of an elaborate body of notation to describe it is not the most straightforward route; an equivalent and simpler approach will be followed. Price, knowledge and a variable indexing whether the refinement has arrived are modelled as a joint Markov process on $[0,D(0)] \times \{\theta^0, \underline{\theta}, \bar{\theta}\} \times \{0,1\}$. The numerous restrictions implied by the structure set out above—for example that prices are deterministic given $T$—can be left implicit at this point.

Let $E_0$ be the expectations operator at $t=0$ given $p_0 = D(0)$ (recall there is no trade at $t=0$ irrespective of $p$), $\theta = \theta^0$ and that the refinement has not been invented. It is assumed that firms are risk neutral and behave so as to maximize

$$E_0\left\{\sum_{t=0}^{\infty}\gamma^t\tilde{\pi}_t\right\},$$

where $\tilde{\pi}_t$ equals $\pi^0$ if the firm does not participate, and $\pi(p_t;\theta_t)$ if the firm participates at $t$ when price is $p_t$ and knowledge is $\theta_t \in \{\theta^0, \underline{\theta}, \bar{\theta}\}$.

Given the boundedness of $\pi^0$ and $\pi(p;\,\theta)$, this optimization may be represented by a sequence of pairs of functions $\{U_t(\theta), V_t(\theta)\}_0^{\infty}$ where $U_t(\theta)$ represents the expected present value of profits from $t$ onward given i) an optimal participation policy will be followed; ii) knowledge is $\theta$; and iii) the refinement has not been invented either prior to or at $t$. $V_t(\theta)$ has the same interpretation except that it takes as given that the refinement *has* been invented at $t$ or earlier.

The functions $V_t(\cdot)$ and $U_t(\cdot)$ must satisfy the optimality conditions.[4]

$$U_t(\theta^0) = \max_{\iota\in\{0,1\}}\Big\{(1-\iota)\{\pi^0 + \gamma[\rho V_{t+1}(\theta^0) + (1-\rho)U_{t+1}(\theta^0)]\}$$
$$+\iota\gamma\{\beta[\rho V_{t+1}(\underline{\theta}) + (1-\rho)U_{t+1}(\underline{\theta})]$$
$$+ (1-\beta)[\rho V_{t+1}(\theta^0) + (1-\rho)U_{t+1}(\theta^0)]\}\Big\},$$

$$\max_{\iota\in\{0,1\}}\Big\{(1-\iota)[\pi^0 + V_{t+1}(\theta^0)] + \iota\gamma[\underline{r}V_{t+1}(\underline{\theta})$$
$$+ \bar{r}V_{t+1}(\bar{\theta}) + (1-\underline{r}-\bar{r})V_{t+1}(\theta^0)]\Big\},$$

$$U_t(\underline{\theta}) = \max_{\iota\in\{0,1\}}\Big\{(1-\iota)\{\pi^0 + \gamma[\rho V_{t+1}(\underline{\theta}) + (1-\rho)U_{t+1}(\underline{\theta})]\}$$
$$\iota\{\pi(p_t,\,\underline{\theta}) + \gamma[\rho V_{t+1}(\underline{\theta}) + (1-\rho)U_{t+1}(\underline{\theta})]\}\Big\}$$
$$= \max_{\iota\in\{0,1\}}[(1-\iota)\pi^0 + \iota\pi(p_t,\,\underline{\theta})] + \gamma[\rho V_{t+1}(\underline{\theta}) + (1-\rho)U_{t+1}(\underline{\theta})],$$

$$V_t(\underline{\theta}) = \max_{\iota\in\{0,1\}}\Big\{(1-\iota)[\pi^0 + \gamma V_{t+1}(\underline{\theta})] + \iota\{\pi(p_t,\,\underline{\theta}) + \gamma[rV_{t+1}(\bar{\theta})$$
$$+ (1-r)V_{t+1}(\underline{\theta})]\}\Big\},$$

$$\text{and} \quad V_t(\bar{\theta}) = \max_{\iota \in \{0,1\}} \left\{ (1-\iota)[\pi^0 + \gamma V_{t+1}(\bar{\theta}) + \iota[\pi(p_t, \bar{\theta}) + \gamma V_{t+1}(\bar{\theta})] \right\}$$
$$= \max_{\iota \in \{0,1\}} [(1-\iota)\pi^0 + \iota\pi(p_t, \bar{\theta})] + \gamma V_{t+1}(\bar{\theta}).$$

Define $U_t(\bar{\theta}) = V_t(\bar{\theta})$. $U_t(\cdot)$ and $V_t(\cdot)$ may be interpreted as follows. Consider $U_t(\theta^0)$. $U_t(\theta^0)$ is the present value of firm profits given knowledge of $\theta^0$ and that the refinement has not been invented by $t$. If a firm knowing $\theta^0$ does not participate ($\iota=0$) it earns current profit $\pi^0$ plus the expected discounted profit associated with certainly knowing no more than $\theta^0$ at $t+1$, where the expectation takes into account that the refinement might be invented at $t+1$. On the other hand, since $q(p,\theta^0) = 0$, the option of participating ($\iota=1$) yields no current period profit, but it does offer the possibility of knowing $\underline{\theta}$ at $t+1$, the value of which will generally depend on whether the refinement is invented at $t+1$. $U_t(\theta^0)$ is the larger of the values following from the participate/not participate decisions.

Now consider *equilibrium*. At each date $t$, firm actions (participation ($\iota$), quantity produced ($q$)) and knowledge ($\theta$) may be summarized by a conditional distribution function $\lambda_t(\iota, q, \theta|\cdot)$ where the conditioning is with respect to whether the refinement occurred at or before $t$. The sequence of such conditional distribution functions, $\{\lambda_t\}_0^\infty$, is defined to be an equilibrium of the economy if given the (market clearing) price sequence $\{p_t\}$ implied by $\{\lambda_t\}$, the optimizing behavior of all firms coupled with the learning technology implies $\{\lambda_t\}$ itself as the sequence of joint conditional distributions over $(\iota, q, \theta)$.

The determination of the *structure of an equilibrium* proceeds as follows. First, equilibrium is constructed for $t = T+1,...$; that is, for the periods following that at which the refinement occurred. This construction takes as given fixed values for $n_T^0 \geq 0$, $\underline{n}_T \geq 0$, and $\bar{n}_T = 0$. These values imply fixed values for the sum $\underline{n}_{T+1} + x_{T+1} = (1-r)\underline{n}_T + \underline{r}n_T^0$ as well as for $\bar{n}_{T+1} = r\underline{n}_T + \bar{r}n_T^0$. It is also assumed that for $t \geq T+1$, $n_t^0 = 0$ and $x_t \geq 0$; both will be shown to hold in equilibrium. Next, the structure of period $T$ is determined. Subsequently, behavior in all periods to $T$ but after t=0 is set out. Finally, period 0 is analyzed.

The material to follow makes use of the following notation. Define $p^*$ as the unique solution to

$$\frac{\pi^0}{1-\gamma} = \pi(p^*, \underline{\theta}) + \frac{\gamma}{1-\gamma}[r\pi(p^*, \bar{\theta}) + (1-r)\pi^0].$$

$p^*$ has the property that if $t \geq T$, and $p_t = p^*$ in the current and all subsequent periods, a firm knowing $\underline{\theta}$ would be just indifferent about current period participation. Let $\bar{n}^*$ be the number of firms knowing $\bar{\theta}$ which as a group would produce output exactly sufficient to yield price $p^*$ if no other firms produced; that is, $\bar{n}^*$ is the unique solution to

$$p^* = D[\bar{n}^* q(p^*, \bar{\theta})].$$

Next, let $\tilde{p}$ ($> p^*$) be the price at which firms knowing $\underline{\theta}$ would earn exactly $\pi^0$ from current production. $\tilde{p}$ solves

$$\pi(\tilde{p}, \underline{\theta}) = \pi^0;$$

$\tilde{p} < D(0)$ is guaranteed by (3) below.

Finally, let $Q_t$ denote industry output at $t$:

$$Q_t \equiv \underline{n}_t q(p_t, \underline{\theta}) + \bar{n}_t q(p_t, \bar{\theta}).$$

In particular, given $p^*$, define $Q^*$ by $p^* = D(Q^*)$.

To avoid a proliferation of less relevant cases, four parameter restrictions will now be imposed. None are binding in the estimation reported on below. First, assume that

$$\pi^0 > \gamma[\bar{r}\pi(p^*, \bar{\theta}) + (1-\bar{r})\,\pi^0]. \tag{1}$$

The restriction implies that if $p_t = p^*$ prevails for all $t$ beginning next period, firms knowing only $\theta^0$ would not enter during the current period. (Recall that $p^*$ is such that those knowing

$\underline{\theta}$ would be indifferent.) Without this restriction it may be that those knowing only $\theta^0$ may learn $\bar{\theta}$ so easily relative to those knowing $\underline{\theta}$, that equilibrium may involve those knowing $\underline{\theta}$ producing at $T$ and exiting at $T+1$, with output sufficient to cause $p_t = p^*$ for all $t \geq T + 1$ being supplied only by firms who entered at $T$ knowing only $\theta^0$, and who learned $\bar{\theta}$ at $T$. Under restriction (1), it will be shown that $\underline{n}_{T+1} > 0$ must hold. It is easy to show that $r \geq \bar{r}$ would be sufficient to guarantee the above inequality.[5] That $r \geq \bar{r}$ is not necessary is, however advantageous, for in a richer model allowing innovation effort to be endogenous (for example, Jovanovic and MacDonald) it is possible that the counterpart to $r < \bar{r}$ can occur easily enough as a result of the greater incentives to innovate faced by firms whose technological knowledge is presently inferior.

The second restriction is

$$q(p^*, \underline{\theta}) > rq(p^*, \bar{\theta}). \tag{2}$$

This inequality is a mild restriction because it is equivalent to the *one period* output of a firm knowing $\bar{\theta}$, when $p_t = p^*$, being smaller than the *total* output a firm presently knowing only $\underline{\theta}$ would expect to produce using technology based on $\underline{\theta}$ if price remained fixed at $p^*$. At the cost of a minor increase in the number of cases examined, (2) can be dropped. Since (2) is not binding in the estimation, this relaxation is ignored.

Next, it will be assumed that $N$ is sufficiently large that the expected present value of profits from participation by firms knowing $\theta^0$ may always be driven to $\pi^0/(1-\gamma)$ by suitably many participating, and, that if all $N$ firms participate, so many would learn $\underline{\theta}$ that the supposed participation is not maximal for any firm knowing only $\theta^0$. As stated, this "free entry" restriction depends on the structure of equilibrium, but it can easily be phrased in terms of primitives.

In a similar vein assume that

$$\pi^0 < \gamma\, min \left\{ \beta\pi[D(0),\underline{\theta}] + (1-\beta)\pi^0,\ \bar{r}\pi[D(0), \bar{\theta}] + \underline{r}\pi[D(0), \underline{\theta}] + (1-\bar{r}-\underline{r})\pi^0 \right\}. \tag{3}$$

Under this restriction, if no firms plan to operate at $t+1$, in which case $p_{t+1} = D(0)$, it would always pay for a firm knowing $\theta^0$ to participate at $t$. This restriction merely serves to guarantees that the industry is "viable" once the basic invention has arrived on the scene.

Periods following T may now be analyzed. Take as given $n_T^0 \geq 0$, $\underline{n}_T > 0$ and $\bar{n}_T = 0$, and consider date $t \geq T + 1$ assuming $x_t \geq 0$ (no entry of firms knowing $\underline{\theta}$) and $n_t^0 = 0$. Recall (1) implies that if $p_t \leq p^*$ for $t \geq T+1$, $n_T^0 = 0$ may also be assumed; also, if $p_t < \tilde{p}$ for all $t \geq T+1$, $\underline{n}_T = 0$ must instead be imposed.

There are three cases to consider. In the first the maximum number of firms that might operate at $t = T+1$, namely $\underline{n}_T + (\underline{r} + \bar{r})n_T^0$, is at most $\bar{n}^*$. In this instance the hypothesized equilibrium evolution is[6]

i) $\quad n_{T+1}^0 = 0$, $\underline{n}_{T+1} = (1-r)\underline{n}_T + \underline{r}n_T^0$ and $\bar{n}_{T+1} = r\underline{n}_T + \bar{r}n_T^0$;

and ii) $\quad \forall t \geq T+1 \quad n_{t1}^0 = 0,$

$$\underline{n}_{t+1} = (1-r)\underline{n}_t,$$

and $\quad \bar{n}_{t+1} = \bar{n}_t + r\underline{n}_t;$

in particular $x_t = 0$ for all $t \geq T+1$. Given $n_T^0 \geq 0$ and $\underline{n}_T > 0$, this evolution is clearly feasible. Moreover, because $q(p,\theta)$ is increasing in $p$ for all $\theta$, and $0 < q(p,\underline{\theta}) < q(p,\bar{\theta})$, it follows that $Q_t \leq Q^*$, and thus that $p_t \geq p^*$. Therefore it is certainly maximal for firms knowing either $\underline{\theta}$ or $\bar{\theta}$ to behave as hypothesized; that $n_t^0 = 0$ is maximal for those knowing $\theta^0$ will be demonstrated below. It follows that the evolution displayed above is equilibrium behavior given $\underline{n}_T + (\bar{r} + \underline{r})\underline{n}_T \leq \bar{n}^*$.

For the second case, suppose instead that $\underline{n}_T + (\bar{r} + \underline{r})n_T^0 > \bar{n}^*$; in particular that $\bar{n}_{T+1}(= r\underline{n}_T + \bar{r}n_T^0) \geq \bar{n}^*$. In this case firms knowing $\bar{\theta}$ at $T$+1 would by themselves produce output sufficient to cause $p_t \leq p^*$ for all $t \geq T + 1$, and should $\underline{n}_t > 0$ for any $t \geq T+1$, $p_t < p^*$ is implied. It follows that the evolution

$$\forall t \geq T+1,\ n_t^0 = 0,\ \underline{n}_t = 0 \text{ and } \bar{n}_t = r\underline{n}_T + \bar{r}n_T^0$$

is equilibrium behavior.

The third and intermediate case—characterized by $\underline{n}_T + (\bar{r} + \underline{r})n^0_T > \bar{n}^*$ and $\bar{n}_{T+1}(=r\underline{n}_T + \bar{r}n^0_T) < \bar{n}^*$—generates a slightly more complicated equilibrium evolution. Recall the evolution in the first case: $x_t \equiv 0$. Given the present parameter values, if this "no exit" evolution obtained, eventually $Q_t < Q^*$ would have to occur, and hence $p^*$. Let $T^* \geq T+1$ denote the first date at which the inequality obtains. The hypothesized evolution is then given by

i) $n^0_{T+1} = 0, \quad \underline{n}_{T+1} = (1-r)\underline{n}_T + \underline{r}n^0_T - x_{T+1}, \bar{n}_{T+1} = r\underline{n}_T + \bar{r}n^0_T;$

ii) $\forall t \geq T+1 \quad n^0_{t+1} = 0,$

$\underline{n}_{t+1} = (1-r)\underline{n}_t - x_{t+1},$

and $\bar{n}_{t+1} = \bar{n}_t + r\underline{n}_t;$

and iii)
$$x_t = \begin{cases} 0 & \text{if } T^* > T+1 \text{ and } T+1 \leq t < T^* \\ (1-r)\underline{n}_{T^*-1} + (\bar{n}_{T^*-1} + r\underline{n}_{T^*-1} - \bar{n}^*)\cdot\dfrac{q(p^*,\bar{\theta})}{q(p^*,\underline{\theta})} & \text{if } t = T^* \geq T+1 \\ \left[\dfrac{q(p^*,\bar{\theta})}{q(p^*,\underline{\theta})} - 1\right]r\underline{n}_{t-1} & \text{if } t > T^*. \end{cases}$$

The path of exit $(x_t)$ by firms knowing $\underline{\theta}$ implies that $p_t = D[\underline{n}_t q(p_t, \underline{\theta}) + \bar{n}_t q(p_t, \bar{\theta})] > p^*$ for $T+1 \leq t < T^*$ (assuming $T+1 < T^*$) and $p_t = p^*$ for $t \geq T^*$. If $T+1 = T^*$, $p_t = p^*$ for all $t \geq T+1$. Note also that $n^0_{T^*-1} = 0$ always holds. If $T^* > T+1$, this is implied by the assumption $n^0_t = 0$; if $T^* = T+1$, the restriction (1) implies $n^0_T\left[=n^0_{T^*-1}\right] = 0$.

The expressions for $x_t \neq 0$ are obtained as follows. First, $x_{T^*}$ is chosen to yield $p_{T^*} = p^*$, requiring $Q_{T^*} = Q^*$, or

$$\underline{n}_{T^*}q(p^*,\underline{\theta}) + \bar{n}_{T^*}q(p^*,\bar{\theta}) = \bar{n}^*q(p^*,\bar{\theta}).$$

Substitution of $\underline{n}_{T^*} = (1-r)\underline{n}_{T^*-1} - x_{T^*}$ (recall $n^0_{T^*-1} = 0$) and $\bar{n}_{T^*} = \bar{n}_{T^*-1} + r\underline{n}_{T^*-1}$ gives the desired expression. Feasibility requires $0 < x_{T^*} < (1-r)\underline{n}_{T^*-1}$. Should $x_{T^*} = 0$, by definition of $T^*$, $p_{T^*} < p^*$ violating the requirement $p_{T^*} = p^*$. Similarly, if $x_{T^*} = (1-r)\underline{n}_{T^*-1}$, it follows that

$$
\begin{aligned}
Q_{T*} &= (\bar{n}_{T*-1} + r\underline{n}_{T*-1})q(p^*, \bar{\theta}) && \text{(by (2))} \\
&< \bar{n}_{T*-1}q(p^*, \bar{\theta}) + \underline{n}_{T*-1}q(p^*, \underline{\theta}) \\
&= Q_{T*-1},
\end{aligned}
$$

implying $p_{T*-1} < p^*$, again violating the definition of $T^*$. Thus $0 < x_{T*} < (1-r)\underline{n}_{T*-1}$.

For $x_t$, given $t > T^*$, $p_t = p^*$ and thus $Q_t = Q^*$ must hold for all $t$, implying

$$\underline{n}_{t-1}q(p^*, \underline{\theta}) + \bar{n}_{t-1}q(p^*, \underline{\theta}) = \underline{n}_t q(p^*, \underline{\theta}) + \bar{n}_t q(p^*, \bar{\theta}).$$

Substitution of $\underline{n}_t = (1-r)\underline{n}_{t-1} - x_t$ and $\bar{n}_t = \bar{n}_{t-1} + r\underline{n}_{t-1}$ gives the desired expression for $x_t$. Once again the feasibility requirement $0 < x_t < (1-r)\underline{n}_{t-1}$ is easily verified under assumption (2).

The role of (2) is now clear. Exit by firms knowing $\underline{\theta}$ must maintain $p_t = p^*$. If (2) fails, learning may be so rapid, or firms knowing $\bar{\theta}$ may be so large, that even simultaneous exit by all firms knowing $\underline{\theta}$ might not prevent $p_t < p^*$.

That $p_t \geq p^*$ for all $t$ implies all those knowing $\bar{\theta}$ will invariably participate. All firms knowing $\underline{\theta}$ at $t \geq T$ will strictly prefer to participate when $t < T^*$, and will be willing to behave as prescribed by the hypothesized evolution for $t \geq T^*$.[7] Again, that $n^0_t = 0$ is maximal remains to be demonstrated.

To summarize what has been shown so far, beginning during the period $(T+1)$ following that in which the refinement was invented, and assuming i) fixed values of $n^0_T \geq 0$ and $\underline{n}_T > 0$, and ii) for all $t > T$, $n^0_t = 0$ and $x_t \geq 0$, the evolution may take on three forms. If $\underline{n}_T + (\bar{r} + \underline{r})n^0_T \leq \bar{n}^*$, $p_t > p^*$ and no exit ever occurs. If $\underline{n}_T + (\bar{r} + \underline{r})n^0_T > \bar{n}^*$ and $r\underline{n}_T + \bar{r}\,n^0_T \geq \bar{n}^*$, all firms knowing $\underline{\theta}$ exit at $T+1$ and $\bar{n}_t = r\underline{n}_T + \bar{r}n^0_T$ for all $t \geq T+1$. Should $\underline{n}_T + (\bar{r} + \underline{r})n^0_T > \bar{n}^*$ and $r\underline{n}_T + \bar{r}n^0_T < \bar{n}^*$, there is no exit prior to some date $T^* \geq T + 1$ at which time exit by firms knowing $\underline{\theta}$ begins; $p_{t+1} < p_t$ for $t < T^*$ and the level of exit maintains $p_t = p^*$ for $t \geq T^*$.

To proceed, consider date $T$, at which time the refinement is invented. In this part it will be supposed that there is a positive number of firms, $\underline{m}_{T-1} > 0$, that have learned $\underline{\theta}$ prior to $T$; thus the number participating is constrained by $0 \leq \underline{n}_T \leq \underline{m}_{T-1}$. The number knowing only $\theta^0$ (prior to $T$) is then $N-\underline{m}_{T-1}$.

The construction for $t > T$ assumed $n_T^0 \geq 0$, $\underline{n}_T > 0$, $\bar{n}_T = 0$, and both $x_t \geq 0$ and $n_t^0 = 0$ for all $t \geq T+1$. These restrictions must be shown to represent optimizing behavior. First consider $n_t^0 = 0$ for all $t \geq T+1$. Under free entry of firms knowing $\theta^0$, the expected present value of participation at date $T$ given $\theta^0$ must not exceed $\pi^0/(1-\gamma)$. In all three evolutions (for $t \geq T+1$) displayed above, the expected present value of participation given $\theta^0$ is as great (or greater) at $t=T$ than at any $t \geq T+1$. Therefore nonparticipation at $t \geq T+1$ is a maximizing choice for these firms: $n_t^0 = 0$ for $t \geq T+1$.

The conditions $n_T^0 \geq 0$ and $\bar{n}_T = 0$ are trivial and need no further consideration. In regard to $x_t \geq 0$ and $\underline{n}_T > 0$, verification requires consideration of periods prior to $t=T$ and is therefore postponed. However, it can be mentioned that those conditions are implied by $\underline{n}_T = \underline{m}_{T-1} > 0$, which is what will be shown below. Thus $\underline{n}_T = \underline{m}_{T-1} > 0$ will be assumed here.

Given $\bar{n}_T = 0$ and $\underline{n}_T = \underline{m}_{T-1}$ (exogenous at $T$), all that needs to be analyzed at $t=T$ is the behavior of those knowing $\theta^0$; in particular, when is $n_T^0 > 0$? Recall that the expected present value of entry at $T$ for such firms cannot, in equilibrium, exceed $\pi^0/(1-\gamma)$. For $\underline{m}_{T-1}$ sufficiently small, given restriction (3), $n_T^0 = 0$ yields any firm knowing only $\theta^0$ expected present value of profits from entry at $T$ in excess of $\pi^0/(1-\gamma)$. Given that $N$ is large and that raising $n_T^0$ augments both $\underline{n}_{T+1}$ and $\bar{n}_{T+1}$, thereby reducing $p_t$ for $t \geq T+1$, there exists some value of $n_T^0$ yielding expected present value of profits exactly equal to $\pi^0/(1-\gamma)$. Moreover, this number is nonincreasing and continuous in $\underline{m}_{T-1}$, since increasing $\underline{m}_{T-1}$ raises $\underline{n}_T$ and may lower $p_t$. Given that $N$ is "large", this value of $n_T^0$ is in fact declining in $\underline{m}_{T-1}$ for $\underline{m}_{T-1}$ large enough, and takes on its minimum value $n_T^0 = 0$ for some value $\underline{m}_{T-1} < N$. Thus, given any $\underline{m}_{T-1}$, the number of firms knowing $\theta^0$ that participate at $t$ is either that $n_T^0 > 0$ which equates the present value of participation at $T$ to $\pi^0/(1-\gamma)$, or, if no positive $n_T^0$ will accomplish this, $n_T^0 = 0$.

Now consider any period $t$ such that $0 < t < T$; i.e. a period after the basic invention and before the refinement. (Since $T=1$ may occur, such $t$ need not exist). It will be assumed that for such $t$, $n_t^0 = 0$. Again, that this behavior is maximal is to be shown. $n_t^0 = 0$ implies the number of firms knowing $\underline{\theta}$ at $t$, $\underline{m}_t$, is equal to the number that learned $\underline{\theta}$ at $t=0$, $\beta n_0^0$, assumed positive.

For firms knowing $\underline{\theta}$, participation at $t < T$ confers no special advantage; in particular, these firms are free to participate as soon as the refinement has been invented. Thus, they will participate at $t < T$ if and only if $p_t \geq \tilde{p}$; i.e. if $\pi(p_t, \underline{\theta}) \geq \pi^0$. The evolution for $1 < t < T$ is given by

$$n_t^0 = 0$$

$$\underline{n}_t = \min(\eta, \underline{m}_t)$$

where $\eta$ solves $\tilde{p} = D[\eta q(\tilde{p}, \underline{\theta})]$.

Notice that $p_t$ is constant for $1 < t < T$, and $p_T$ is not larger than this value, since $\underline{n}_T = \underline{m}_{T-1}$.

Now, consider $t=0$. $\underline{n}_0 = \bar{n}_0 = 0$, by definition. The behavior of firms knowing only $\theta^0$ (i.e. all firms) is to be determined. Given the evolutions for $t \geq 1$ set out above, if $n_0^0$ is sufficiently small, participation by any one yields expected present value of profits in excess of $\pi^0/(1-\gamma)$. Similarly, for $n_0^0$ sufficiently large, the expected payoff falls short of $\pi^0/(1-\gamma)$. It is also easy to verify that the expected payoff is continuous and declining in $n_0^0$, since raising $n_0^0$ augments $\bar{n}_t$, at least. Thus there exists some feasible $n_0^0 > 0$ such that the expected value of participation at $t=0$ is exactly $\pi^0/(1-\gamma)$, and this value is that hypothesized for the evolution at $t=0$.

It remains to check that three restrictions imposed along the way are nonbinding i) $x_t \geq 0$ for $t \geq T+1$; ii) $\underline{n}_T > 0$; and iii) $n_t^0 \neq 0$ only if $t = 0$ or $T$.

Since $\underline{m}_{T-1} = \beta n_0^0 > 0$ under the hypothesized evolutions, both (i) and (ii) will obtain if it can be shown that $\underline{n}_T = \underline{m}_{T-1}$. That is, that $\underline{n}_T = \underline{m}_{T-1}$ implies $\underline{n}_T > 0$ is immediate since $\underline{n}_T = 0$ implies $p_T = D(0)$. In regard to the entry condition $x_t \geq 0$ for $t \geq T+1$, there are two situations. In one, as occurs if, for example $x_{T+1} = (1-r)\underline{n}_T$, that no firm knowing $\underline{\theta}$ would strictly prefer to enter at $t \geq T+1$ is immediate because a payoff of at most $\pi^0/(1-\gamma)$ for firms knowing $\underline{\theta}$ defines this situation. $x_t \geq 0$ is then clearly nonbinding. In the other, the payoff to participation exceeds $\pi^0(1-\gamma)$, and so any firm knowing $\underline{\theta}$ and not participating would seek to do so. But if $\underline{n}_T = \underline{m}_{T-1}$, there are no such firms since all possible entry occurred at $t < T+1$. Thus, if $\underline{n}_T = \underline{m}_{T-1}$, (i) and (ii) will follow.

That $\underline{n}_T = \underline{m}_{T-1}$ is easily shown. Suppose $\underline{n}_T < \underline{m}_{T-1}$. Then the expected present value of participation at $T$ given knowledge of $\underline{\theta}$ cannot exceed $\pi^0/(1-\gamma)$. In particular, $p_t \leq \tilde{p}$ must hold, for simply producing only at $t=T$ is an option. Since $p_t$ is constant for $1 < t < T$ and cannot rise at $T$, the expected present value of profits given knowledge of $\underline{\theta}$ can never exceed $\pi^0/(1-\gamma)$ at any $t$: both $U_t(\underline{\theta})$ and $V_T(\underline{\theta}) \leq \pi^0/(1-\beta)$. Since free entry implies $U_t(\theta^0) = \pi^0/(1-\beta)$, it follows that the value of participation at $t=0$ given $\theta^0$ is at most

$$\gamma\left\{\beta\frac{\pi^0}{1-\gamma} + (1-\beta)\frac{\pi^0}{1-\gamma}\right\} < \frac{\pi^0}{1-\gamma},$$

implying $n_0^0 = 0$. But given the assumption that if no firm planned to enter it would pay for some firm to do so, $n_0^0 = 0$ cannot occur in equilibrium, a contradiction. Thus $\underline{n}_T = \underline{m}_{T-1}$.

Observe that this same argument implies that if $\underline{n}_t < \underline{m}_{T-1}$ for $t \leq T-1$, $x_{T+1} = 0$ must occur; $\bar{n}_{T+1} > \bar{n}^*$ (implying $x_{T+1} > 0$) is thus ruled out for that case. Otherwise, the expected present value of profits given $\underline{\theta}$ is $\pi^0/(1-\gamma)$ for all $t$, in which case no firm knowing only $\theta^0$ would seek to participate at $t=0$. Similarly, if $\underline{n}_t = \underline{m}_{T-1}$, $x_{T+1} > 0$ implies $n_T^0 = 0$.

Finally, checking $n_T^0 \neq 0$ only if $t=0$ or $T$ is straightforward. In all of the evolutions displayed, the value of participating for firms knowing only $\theta^0$ is exactly $\pi^0/(1-\gamma)$ for $0 \leq t < T$, in which case such firms would be content to behave as hypothesized for those dates. For $t \geq T+1$, participation yields *at most* $\pi^0/(1-\gamma)$, so $n_t^0 = 0$ is maximal for those dates as well.

To summarize, the equilibrium path of firm participation is as follows. Define $n_t \equiv \underline{n}_t + \bar{n}_t$; $n_t$ is the total number of producing firms in the industry. $n_t$ is positive and equal to $\min(\beta\, n_0^0, \eta)$ for $t=1,\ldots,T-1$, where $T$ is the refinement date. At $T$, $n_T = \beta\, n_0^0$ must occur. Next, at $T+1$, $n_{T+1}$ exceeds $n_T$ by $(\bar{r} + \underline{r})\, n_T^0 \geq 0$, the number of firms knowing $\theta^0$ at $T$ and who learn either $\underline{\theta}$ or $\bar{\theta}$ during that period. At some $T^* \geq T+1$ (possibly infinite), $n_t$ begins to decline, falling by an amount sufficient to maintain $p_t = p^*$, $t \geq T^*$.

The associated equilibrium price path is constant on $t=1,\ldots,T-1$, may decline at $T$, falls for $t = T+1,\ldots,T^*-1$, and is constant for $t \geq T^*$.

## III. Measurement

### 1. *Parameterization*

This section presents maximum likelihood estimates of a parameterized version of the theory set out in Section II. Parameterization is discussed first and then the data and some additions to the theory are presented. Subsequently, the estimates are discussed, along with the issue of more general specifications of product demand and interpretation of $\bar{\theta}$.

Elements of the theory requiring further structure are product demand and production cost. Demand is assumed to display constant elasticity:

$$D(Q) = d_0\, Q^{-d_1} \tag{4}$$

with $d_0 > 0$ and $d_1 > 0$.[8] Production costs are assumed quadratic in $q$:

$$\begin{aligned} c(q;\, \underline{\theta}) &= cq^2/2, \\ c(q;\bar{\theta}) &= cq^2/[2(1+\bar{\theta})], \end{aligned} \tag{5}$$

where $c > 0$ and $\bar{\theta}$ are to be estimated; implicitly the normalization $\underline{\theta} = 0$ is imposed.

### 2. *Data*

The data used here are (with exceptions noted) a subset of those studied by Gort and Klepper; because a complete and comparatively long series was available, annual data on the U.S. Automobile Tire industry was chosen. The data are available for 1906–73 and consist of the number of producing firms (1906–73), industry output (1910–73) and a wholesale price index (1913–73) for automobile tires (see Table 1 and Figure 1).[9]

As a result of the model's emphasis on information flows, firms, as opposed to plants, are likely the preferred firm entity on which to focus. Note also that from the standpoint of understanding these data, the absence of data on mergers is not as problematic as otherwise might be supposed. If the shakeout stage were accompanied by mergers rather than exit, this would not be inconsistent with this model: If part of what is discovered as the industry progresses is how to operate and coordinate multiple plants effectively, firm data is precisely what is called for, although more information on the structure of costs would be obtained if plants were observed as well. Finally, wholesale prices are presumably what is relevant for firm decision making.

These data display the familiar industry life–cycle features: the number of firms rises sharply, then falls and levels off; output grows steadily, as does output per firm; and price falls sharply and levels off.

Table 1

| Year | Number of Producing Firms[a] | Industry Output[b] (Millions) | Price Index[d] (divided by CPI) |
|---|---|---|---|
| 1906 | 10 | — | — |
| 1907 | 29 | — | — |
| 1908 | 29 | — | — |
| 1909 | 38 | — | — |
| 1910 | 38 | 2 | — |
| 1911 | 46 | 3 | — |
| 1912 | 54 | 5 | — |
| 1913 | 74 | 6 | 7.653 |
| 1914 | 94 | 8 | 6.309 |
| 1915 | 106 | 12 | 5.595 |
| 1916 | 118 | 19 | 5.379 |
| 1917 | 133 | 26 | 5.656 |
| 1918 | 143 | 23 | 5.574 |
| 1919 | 144 | 33 | 4.431 |
| 1920 | 181 | 32 | 4.250 |
| 1921 | 199 | 29 | 3.664 |
| 1922 | 275 | 41 | 2.522 |
| 1923 | 246 | 45 | 2.350 |
| 1924 | 213 | 52 | 1.984 |
| 1925 | 184 | 61 | 2.061 |
| 1926 | 165 | 62 | 2.070 |
| 1927 | 146 | 64 | 1.581 |
| 1928 | 132 | 78 | 1.355 |
| 1929 | 121 | 69 | 1.166 |
| 1930 | 96 | 51 | 1.126 |
| 1931 | 81 | 49 | 1.105 |
| 1932 | 64 | 40 | 1.103 |
| 1933 | 62 | 45 | 1.180 |
| 1934 | 76 | 47 | 1.229 |
| 1935 | 56 | 49 | 1.219 |
| 1936 | 54 | 56 | 1.245 |
| 1937 | 53 | 53 | 1.421 |
| 1938 | 51 | 41 | 1.497 |
| 1939 | 50 | 58 | 1.529 |
| 1940 | 52 | 59 | 1.348 |
| 1941 | 53 | 62 | 1.376 |
| 1942 | 50 | (15) 66[c] | 1.512 |
| 1943 | 49 | (20) 70 | 1.477 |
| 1944 | 46 | (33) 74 | 1.427 |
| 1945 | 45 | (45) 78 | 1.341 |
| 1946 | 46 | 82 | 1.241 |
| 1947 | 51 | 96 | 1.040 |
| 1948 | 51 | 81 | .989 |
| 1949 | 49 | 76 | .968 |
| 1950 | 46 | 93 | 1.103 |
| 1951 | 45 | 83 | 1.203 |
| 1952 | 45 | 90 | 1.142 |
| 1953 | 43 | 96 | 1.111 |
| 1954 | 40 | 89 | 1.135 |
| 1955 | 36 | 112 | 1.266 |

Table 1 (cont'd.)

| Year | Number of Producing Firms[a] | Industry Output[b] (Millions) | Price Index[d] (divided by CPI) |
|---|---|---|---|
| 1956 | 42 | 100 | 1.310 |
| 1957 | 42 | 107 | 1.251 |
| 1958 | 43 | 97 | 1.232 |
| 1959 | 44 | 118 | 1.149 |
| 1960 | 44 | 120 | 1.093 |
| 1961 | 43 | 117 | 1.075 |
| 1962 | 40 | 134 | 1.001 |
| 1963 | 39 | 139 | 1.024 |
| 1964 | 37 | 158 | .998 |
| 1965 | 36 | 168 | .992 |
| 1966 | 34 | 177 | 1.000 |
| 1967 | 32 | 163 | 1.000 |
| 1968 | 32 | 203 | .987 |
| 1969 | 32 | 208 | .932 |
| 1970 | 30 | 190 | .937 |
| 1971 | 32 | 216 | .900 |
| 1972 | 31 | 236 | .872 |
| 1973 | 38 | 223 | .837 |

a. *Thomas Register of American Manufactures*, Thomas Pub. Co., New York, N.Y.

b. U.S. Dept. of Labor, BLS.

c. The data in parentheses are discussed in subsection 3.

d. U.S. Dept. of Labor, BLS Wholesale price index (1967 = 1) divided by CPI (*U.S. Historical Statistics for 1906–70*, and *U.S. Report of the President* for 1971–3; 1967 = 1).

### 3. *Ad Hoc Additions*

There are four important aspects of the data which go unmentioned in the theory and that require some consideration before proceeding further.

(i) World War II. The theory does not include the second world war. While the number of operating firms and product price were to some degree influenced by the war, output fell dramatically during 1942–45. Evidently, since this large decline in industry output is both the source of much of the total variation in that series, and is an "exceptional" event about which the theory is silent, some type of adjustment must be made. Fortunately, most of the lifecycle behavior emphasized here has concluded by 1940, in which case the results are likely insensitive to the specific adjustment utilized. The method adopted here simply replaces the 1943–5 output figures by a linear interpolation of the 1941–46 data. The numbers given in Table 1 include this adjustment; the original data are in parentheses.[10]

(ii) Quality Change. The theory supposes that whatever it is that firms deliver to consumers or retailers is not changing over time, in particular, that the service flow obtained from a tire is constant. That modern tires are more durable is part of what is captured in $c(\cdot;\theta)$; producing better tires yields a given delivered service flow at lower production cost. Division of price by the CPI also assists in adjusting for quality change since the CPI contains unmeasured elements of increasing quality. In effect, unmeasured quality increase in automotive tires is assumed to be the same as the unmeasured quality increase in the CPI bundle. The latter has been argued to be substantial, although the issue is far from settled; see the papers in Griliches (1971).

(iii) The Model Cannot Fit the Data Exactly. For any parameter values and refinement date the theory delivers a "regular" time path of the number of producing firms ($n_t \equiv \underline{n}_t + \bar{n}_t$), industry output ($Q_t \equiv \underline{n}_t q(p_t;\underline{\theta}) + \bar{n}_t q(p_t;\bar{\theta})$) and price ($p_t$). Obviously the data are not so regular. The discrepancy is reconciled as follows. Letting observed values be distinguished with an asterisk, it is assumed that

$$\ln n_t^* = \ln n_t + \varepsilon_{n_t},$$

$$\ln Q_t^* = \ln Q_t + \varepsilon_{Q_t},$$

and

$$\ln p_t^* = \ln p_t + \varepsilon_{p_t},$$

where $\varepsilon_{n_t}$, $\varepsilon_{Q_t}$ and $\varepsilon_{p_t}$ are i.i.d. $N(0,\sigma_n^2)$, $N(0,\sigma_Q^2)$ and $N(0,\sigma_p^2)$ respectively. The assumption that $\varepsilon_{n_t}$, $\varepsilon_{Q_t}$ and $\varepsilon_{p_t}$ are independent of one another reduces the number of parameters to be estimated and is consistent with the a priori information that these series are collected separately; in particular, the price index is computed from actual price data, as opposed to being inferred from sales and output.

This error specification implies a likelihood function as follows: Let $D_n$, $D_Q$ and $D_p$ be the set of dates for which $n^*_t$, $Q^*_t$ and $p^*_t$ are observed; in the data studied here, $D_p \subset D_Q \subset D_n$. Define $T_n$, $T_Q$ and $T_p$ to be the number of dates in $D_n$, $D_Q$ and $D_p$ respectively. *Given T*, the likelihood of the sample observations is proportional to

$$L(T) \equiv (\sigma_n^2)^{-T_n/2}(\sigma_Q^2)^{-T_Q/2}(\sigma_p^2)^{-T_p/2} \exp -\frac{1}{2}\left\{ \frac{1}{\sigma_n^2} \sum_{t \in D_n} [\ln n^*_t - \ln n_t(T)]^2 \right.$$
$$\left. + \frac{1}{\sigma_Q^2} \sum_{t \in D_Q} [\ln Q^*_t - \ln Q_t(T)]^2 + \frac{1}{\sigma_p^2} \sum_{t \in D_p} [\ln p^*_t - \ln p_t(T)]^2 \right\},$$

where $n_t(T)$, $Q_t(T)$ and $p_t(T)$ are the equilibrium values given $T$. The likelihood is then (using $D_p \subset D_Q \subset D_n$ (and that $D_n$, $D_Q$ and $D_p$ comprise consecutive periods)

$$L \equiv \sum_{T \in D_n} \rho(1-\rho)^{T-1} L(T) + (1-\rho)^{T_n} L(T_n+1).$$

Note that $T$ is not treated as a parameter to be estimated. Rather, it is treated analogously to $\varepsilon_n$, $\varepsilon_Q$ and $\varepsilon_p$; that is, as an unobserved random object. The motivation for doing so is that the number of parameters is reduced by one ($\rho$ must be estimated in any case) and computational difficulties following from $T$ being discrete are avoided.

The issue of identification of the model's parameters requires some discussion. For some regions of the parameter space, identification is not complete. For example, if equilibrium involves $n^0_T = 0$, $\bar{r}$ and $\underline{r}$ do not enter the calculation of the likelihood. For the estimates below, the equilibrium involves $n^0_0 > 0$, $n^0_T > 0$, $r\underline{n}_T + (\bar{r} + \underline{r})n^0_T > n^*$ and $r\underline{n}_T + \bar{r}n^0_T < n^*$. In this case entry occurs at $t = 0$ and $t = T$, and exit is gradual, beginning at $T^*$. It does not appear possible to settle the identification issue entirely. That is, even for the parameters yielding the just–mentioned form of equilibrium it has not been possible to show that the model is identified *for arbitrary data*. However, for the data studied herein, the calculations discussed below show, at least in the sense of doing so numerically, that the likelihood has an unique global maximum in the parameter space, which is sufficient for identification (for given data).

(iv) General Economic Growth. The economy generating the data is growing in various ways, population and "general productivity" in particular; in contrast, the theoretical economy is stationary. In the estimation to follow, it is assumed that these elements can be captured by a single parameter $g \geq 1$ entering demand, cost and the value of alternatives as follows:

demand: $D(Q_t/g^t)$

alternatives: $\pi^0 g^t$

production cost: $c(q;\theta)/g^t$.

This homogeneous growth specification may be derived by assuming the industry to be small relative to the rest of the economy, which is experiencing neutral technological change at rate $g$, and that product demand is unit income elastic. Given growth of this form, $Q_t$ may grow at rate $g$ without a reduction in $p_t$ and, given $p$, $\pi(p;\theta)/\pi^0$ does not depend on $t$. It is straightforward to verify that under this parameterization, provided $\gamma' \equiv \gamma g < 1$, (i) industry equilibrium may be computed as if $g = 1$ with $\gamma'$ replacing $\gamma$ everywhere; (ii) given $\gamma'$, $p_t$ and $n_t$ are independent of $g$; and (iii) the predicted output path is that associated with $\gamma'$, scaled up by $g^t$. In what follows, $\gamma'$ and $g$ are estimated, directly and $\gamma$ inferred as $\gamma'/g$. Whether this simple way of capturing growth is adequate receives some discussion below.

## *4. Maximum Likelihood Estimates*

The model does not have a closed form solution, so that the calculation of an equilibrium and maximization of the likelihood are both numerical exercises. For maximization to proceed, equilibrium — $n_0^0$ and $n_T^0$ in particular — must be calculated to high accuracy. Otherwise, an improvement in the value of the likelihood as parameters vary may occur either because the likelihood, given *exact* values for the equilibrium, has increased, or because the imprecision in the calculation of the equilibrium has produced a predicted equilibrium path that fits the data better even though the exact equilibrium path fits less well.

In all calculations to follow, $n_0^0$ and $n_T^0$ are calculated at "machine accuracy" ($\approx 10^{-12}$). Since time paths for $n_t$, $Q_t$ and $p_t$ exist in closed form given $n_0^0$ and $n_T^0$, this procedure amounts to solving the free entry conditions

$$U_0(\theta^0) = \frac{\pi^0}{1-\gamma}$$

and

$$V_T(\theta^0) = \frac{\pi^0}{1-\gamma}$$

for $n_0^0$ and $n_T^0$.

The natural log of $L$ was maximized using a combination of the method of Davidson, Fletcher and Powell (as implemented in the software GAUSS–386 VM), a restricted grid search, and extensive random search. The standard errors were calculated by a method similar to "jackknifing". That is, treating the estimated parameters (including the variances of $\varepsilon_n$, $\varepsilon_Q$ and $\varepsilon_p$) as fixed throughout, realizations of $T$, $\varepsilon_n$, $\varepsilon_Q$ and $\varepsilon_p$ were produced from a pseudo–random number generator and used to construct artificial data. Given the data the parameters were re–estimated. This procedure was repeated 10 times. The reported standard errors are the empirical standard errors from this sample.

Before discussing the parameters of primary interest, three points should be noted. First, the simple homogeneous method of dealing with general productivity growth appears adequate. Productivity growth is estimated to be 3.55% per annum; for comparison, the real rate of growth of GNP over the sample period was 3.11%. Second, the real rate of interest implied by the estimate of $\gamma$ is 12.4%. This figure is high in comparison to real interest yields on government bonds, but well within the range of recent estimates of real corporate rates of return; see, for example, Bulow & Shoven (1981). Third, since the demand for automobile tires is derived from that for automobiles, and tires make up a small fraction of the total cost of an automobile, it would be expected that demand for tires is inelastic; the estimated price elasticity is –.77 (= –1/1.307).

**Table 2**
**Maximum Likelihood Estimates**

| | Parameter | Estimate | Standard Error |
|---|---|---|---|
| Demand | $d_0$ | 63.167 | 6.26 |
| | $d_1$ | 1.307 | .0272 |
| Cost | $c$ | 38.095 | 3.908 |
| | $\bar{\theta}$ | 13.688 | 1.460 |
| Alternative Opportunity | | | |
| | $\pi^0$ | .0773 | .0027 |
| Discount Factor and Growth Parameter | | | |
| | $\gamma'(\gamma' \equiv \gamma g)$ | .9270 ($\gamma = .8951$) | .0056 |
| | $g$ | 1.0355 | .0010 |
| Invention Process | | | |
| | $\rho$ | .0847 | .0133 |
| Innovation Process | | | |
| | $\beta$ | .0449 | .0051 |
| | $r$ | .0193 | .0040 |
| | $\bar{r}$ | .0011 | .0039 |
| | $\underline{r}$ | .9966 | .2065 |
| Measurement Error Variances | | | |
| | $\sigma_n^2$ | .1383 | .0223 |
| | $\sigma_Q^2$ | .1417 | .0255 |
| | $\sigma_p^2$ | .1612 | .0291 |

Figure 2 displays the data, the predicted (given the estimated parameters) equilibrium values of $n_t$, $Q_t$ and $p_t$ conditional on the expected arrival date of the refinement (E(T) = 1905 + $1/\rho \cong 1917$), and the expected equilibrium values without conditioning an arrival date. These figures are of use in assessing the model's goodness of fit. Also, the simple correlation between actual and predicted values is .51 for number of firms, .92 for output, and .85 for price.[11]

How did technological advance influence costs of production? For the data studied here, better production methods are estimated to have reduced costs of producing a given output by a factor of nearly 15 ($1 + \bar{\theta} = 14.69$)! For comparison, over the whole sample period general productivity growth is estimated to have reduced costs by a factor only about two thirds as large ($1.0355^{68} = 10.72$). Thus, it is estimated that roughly nearly 60% of observed cost reductions over 1906–73 in the tire industry are due to industry–specific technological advance.

Turning to the invention process, ρ is estimated at .085. This implies 1917 as the approximate expected date of arrival of the refinement, just preceding the observed explosive growth in firm numbers.

In regard to the innovation process, β is estimated at .0449. This figure implies that early attempts at entering the industry were risky indeed. However, and necessarily, the possible rewards are estimated to have been large; that is, the expected value of the successful firm in 1906 is estimated to be nearly three times that of others.[12] This is, of course, due to the fact that while technology was primitive, the price of tires was relatively high. In fact, for the estimated parameters, while subsequent technological improvement lowered costs, the rapid decline in price demanded by market clearing, in conjunction with the fact that established firms enjoyed little in the way of advantages in implementing new technology (see below), implies that while new technology created great opportunity for new entrants, it actually *reduced* the capital value of existing firms by 50% (i.e. $V_T(\underline{\theta})/U_T(\underline{\theta}) = .495$).

In regard to the process governing implementation of the refinement, the most striking feature is, as mentioned above, that $\underline{r} = .9966$; ie. at the time of refinement any new entrant could expect to be in the same position as current incumbents — that is, knowing $\underline{\theta}$ — in one period. Evidently incumbency was not of great value during the period of rapid technological advance; in fact, $V_T(\underline{\theta})/[\pi^0/(1-\gamma')] = 1.42$.

The remaining parameter to be discussed is $\pi^0$, estimated at .0773. $\pi^0$ is the only parameter (obviously, aside from $d_0$ and $d_1$) for which the functional form assumed for $D(Q)$ is consequential; for the linear demand specification, $\pi^0 = .113$. Thus the calculations to follow should be viewed as illustrative. Assuming the price of a tire in 1967 to be \$15, and allowing for general productivity growth, $\pi^0 = .0773$ converts to \$10.08 million ($0.773 \times 15 \times 1.0355^{62}$), with 1967 capital value of \$138.08 million.[13] Supposing that by 1967 all producing firms utilized the most modern production techniques (see below) comparable figures for producing firms are about two and a half times larger (i.e. $\pi(1;\bar{\theta}) = (1/2c)(1 + \bar{\theta}) = .193$, $.193/\pi^0 = 2.4$).

## 5. *Nonhomogeneous Demand Growth*

The foregoing analysis emphasized changes in technological know–how in explaining industry life cycles; effectively, the model is a pure supply–side theory. Product demand was permitted to vary over time, but only smoothly and in proportion to general productivity growth in the economy at large. A more traditional route would focus on less homogeneous demand growth, entry and exit being understood as responses to demand variation caused by consumers learning about the product, development of substitute goods, etc.

Presumably, both sets of factors matter, and a complete model would nest them. Development of such an integrated model is beyond the scope of this paper; however, for the data studied herein it is possible to provide some evidence on the quantitative importance of nonhomogeneous demand growth, especially in terms of explaining the observed entry and exit behavior on which this paper focuses.

The approach taken is as follows. Suppose, in addition to the structure imposed earlier, that demand responds positively to a variable $z_t$, where $z_t$ varies with $t$. Then, if $z_t$ is a quantitatively important variable omitted in the measurement undertaken above, in a regression of $n_t^*$ (observed number of firms) on $n_t(T)$ (predicted number, given $T$) and $z_t$, the coefficient of $z_t$ should be positive and significantly different from $0$; the same should occur in regressions of $Q_t^*$ on $Q_t(T)$ and $z_t$, and $p_t^*$ on $p_t(T)$ and $z_t$. Let $a_n(T)$, $a_Q(T)$ and $a_p(T)$ be the coefficients of $z_t$ in these regressions.

The role played by $n_t(T)$, $Q_t(T)$ and $p_t(T)$ in such regressions depends on $T$; for example, if $T$ is taken to be the final period in the data, $n_t(T)$ and $p_t(T)$ do not vary over time and $Q_t(T)$ varies only in proportion to $g^t$. In this case the coefficients of $n_t(T)$, $Q_t(T)$ and $p_t(T)$ are not even identified separately from that of the constant term in the regressions and all variation in $n_t^*$ and $p_t^*$ is left to be explained by $z_t$. For this reason it is essential to weight the estimated $T$–conditional coefficients by the probability that $T$ is the actual date at which refinement occurs. Thus the figures reported below are

$$a_n \equiv \sum_{t \in D_n} \hat{\rho}(1-\hat{\rho})^{T-1}\, a_n(T) + (1-\hat{\rho})^{T_n}\, a_n(T_n),$$

where $\hat{\rho}$ is the value estimated for $\rho$; $a_Q$ and $a_p$ are defined analagously.

Since interest attaches to the hypotheses $a_n = 0$, $a_Q = 0$ and $a_p = 0$, "$t$–statistics" are also reported. These are calculated as weighted averages of the $t$–statistics from the $T$–conditional regressions, with the same weights used to calculate $a_n$. Obviously the sampling theory underlying the use of $t$–statistics makes assumptions that are not likely satisfied here, so these figures should be viewed as suggestive.

How might $z_t$ be represented empirically? Assuming that the price of tires is small in proportion to the overall cost of an automobile, some measure of the number of automobiles is a leading candidate.

Data on total automobile registrations and factory sales of passenger cars — roughly the stock of automobiles and the annual addition to it — are available for the required period (1906–73). The units employed here are thousands.[14] Below, results are reported for $z_t$ equal to the level, first difference and percentage first difference of factory sales. Any of these specifications are reasonable, depending on the underlying model of the tires/automobile relationship. Results on registrations are similar; differences are noted below.

Figure 3 depicts $n_t^*$ and $z_t$ for the three versions of $z_t$. Figures 4 and 5 do the same for $Q_t^*$ and $p_t^*$. The scale for $n_t^*$, $Q_t^*$ and $p_t^*$ appears on the left, and that for $z_t$ on the right.

The figures in Table 3 confirm what might be guessed from examination of Figures 3–5.

Table 3

Estimates of $a_n$, $a_Q$ and $a_p$ †

| | $z_t$ | | |
|---|---|---|---|
| | Level | First Difference | % First Difference |
| $a_n$ | -.002<br>(.79) | .003<br>(.56) | -1.31<br>(.04) |
| $a_Q$ | .008<br>(5.73) | .004<br>(1.98) | 4.67<br>(.87) |
| $a_p$ | -.0001<br>(3.75) | –.00003<br>(.38) | .122<br>(.17) |

† "$t$–statistics" in parentheses

First, irrespective of the way $z_t$ is calculated, $z_t$ does not appear to be an important omitted factor influencing entry and exit. The estimated $a_n$ are small and insignificant by conventional standards. Second, for $z_t$ equal to the level of factory sales, $z_t$ plays a role in the determinations of $Q_t^*$ and $p_t^*$. However, that $a_p$ is *negative* suggests that much of its influence, on $p_t^*$ at least, is due to the fact that sales, $Q_t^*$ and $p_t^*$ are heavily trended. This suspicion is confirmed by the fact that for $z_t$ equal to the first difference of sales, its influence is much reduced, and indeed vanishes as far as $p_t^*$ is concerned; the percentage first difference representation further magnifies this effect.

Results based on automobile registrations are similar. The differences are that for $z_t$ equal to the level of registrations, it is significantly related to $n_t^*$, but *negatively*. Evidently this outcome is due to the interaction of the trend in registrations with the long, downward sloping tail of the $n_t^*$ series. The other difference is that for $z_t$ equal to the percentage first difference, $p_t^*$ is strongly positively associated with $z_t$.

Altogether, while there is some evidence that nonhomogeneous demand growth of some form is an important influence on short–run price and quantity variation — as elementary economic reasoning would indicate — these data would not seem to provide support for the view that nonhomogeneous demand growth is a quantitatively important element whose inclusion would alter the explanation of entry and exit offered by the supply driven model studied herein.

## 6. $\bar{\theta}$?

The theory set out above is based on cost–reducing technological change. In the data studied above, a major technological event seems to have occurred around 1917. While chronologies of the development of tires and the tire industry emphasize various breakthroughs — for example, according to the McGraw–Hill Encyclopedia of Science and Technology, no major developments occurred prior to the late 1930's — Hugh Allen's House of Goodyear (1949, p. 31 ff) discusses production costs explicitly (if not with much precision), along with the industry's attempts at reducing them.

Three events are noteworthy. First, around 1912 it was discovered that "cord" cotton fabric might be a more durable material for providing tires with body and strength in comparison to the "square–woven" fabric currently in use. By 1916 corded tires were the norm throughout the industry. Second, a "complex system of Production Control" (p. 34), allowing greatly improved coordination of production, began to be utilized by Goodyear in 1916; whether this method of production found widespread acceptance quickly is unclear. Finally, and also in 1916, the Banbury mixer was invented. The mixer was a major breakthrough in terms of the slow and space–intensive process of mixing rubber with other compounds; indeed, the time required for mixing was reduced by more than an order of magnitude, and a "large amount of floor space" (p. 45) saved. Again, the time path of the adoption of this new method is not available.

Identification of which, if any, of those advances corresponds to refinement of the basic invention is likely not possible. A case can be made that invention of the corded tire corresponds better to the theory's *basic* invention, in that tires constructed with square–woven cotton were too nondurable for their use ever to be very widespread; Allen certainly discusses corded tires in this way. Thus the improved coordination of production and the Banbury mixer appear to be the more likely candidates.

## IV. Conclusion

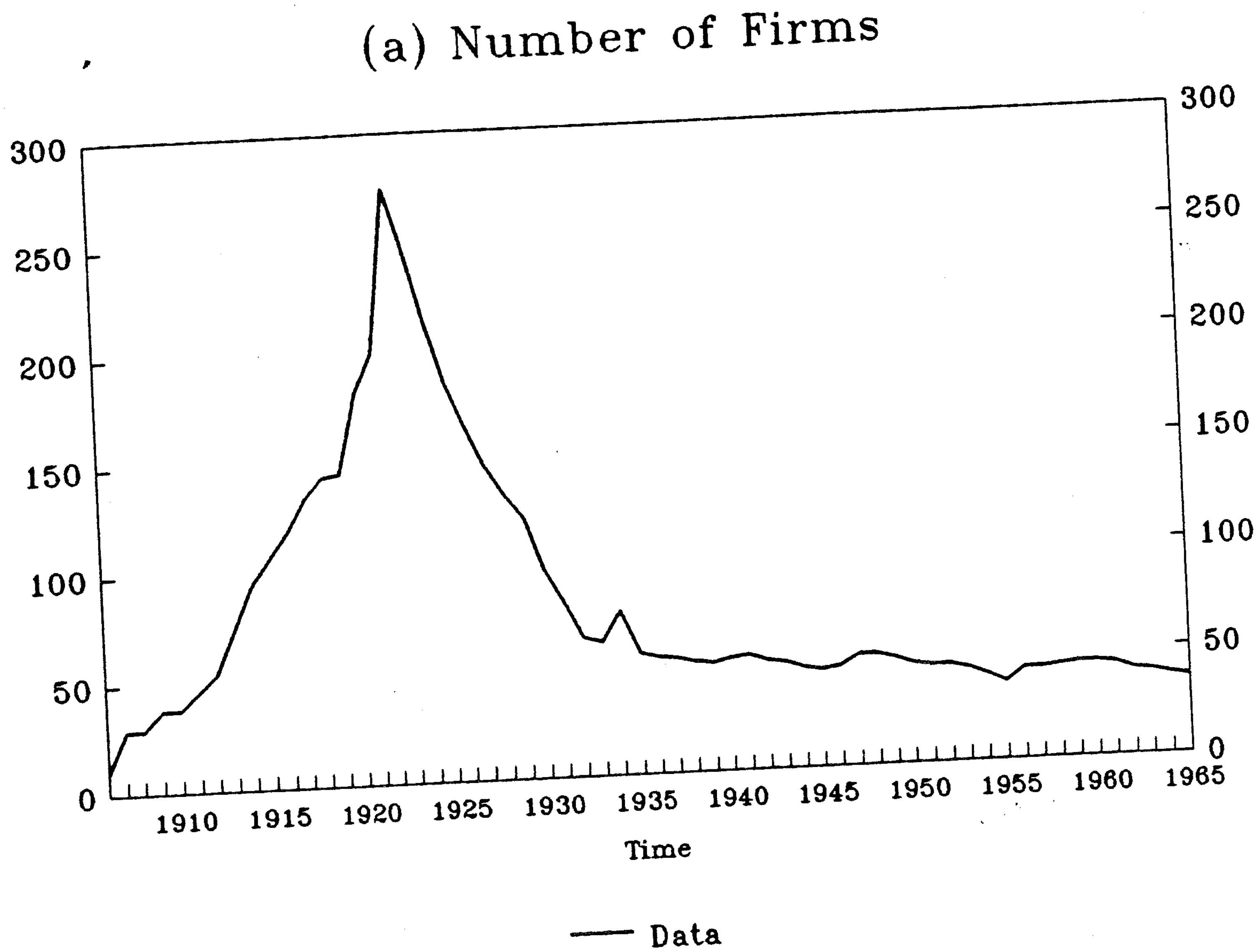
The paper set out first to develop a theory of an industry life cycle meeting two requirements: the model must deliver a specific claim about the form of the industry life cycle; it must also be simple enough that estimation is feasible. The paper's second goal was to estimate the model's parameters.

The model developed above yields an industry equilibrium whose life cycle is roughly in accord with the facts documented for a variety of industries by Gort and Klepper, and herein for the U.S. automobile tire industry. That is, based on the assumptions that a basic breakthrough allows the industry to get underway and a single substantial refinement allows production costs to be reduced, the model predicts growth in the number of producing firms followed by exit and eventual stability in firm numbers.

Opportunity for innovation fuels entry, and relative lack of innovative success drives exit. An initial decline in price followed by stability, along with increasing output, are also implications.

The model's parameters were estimated by maximum likelihood using data on the U.S. automobile tire industry, 1906–1973. The estimates indicate that the explosion in firm numbers occurring during the early 1920's was a consequence of innovations reducing costs, at given output, by more than an order of magnitude. The subsequent collapse in firm numbers was inevitable since firms using the newer production techniques were much larger (given price) and demand is inelastic. Also, the estimated innovation process implies incumbency was of little value during the period of rapid technological change.

Figure 1


(a) Number of Firms
300
250
200
150
100
50
0
1910
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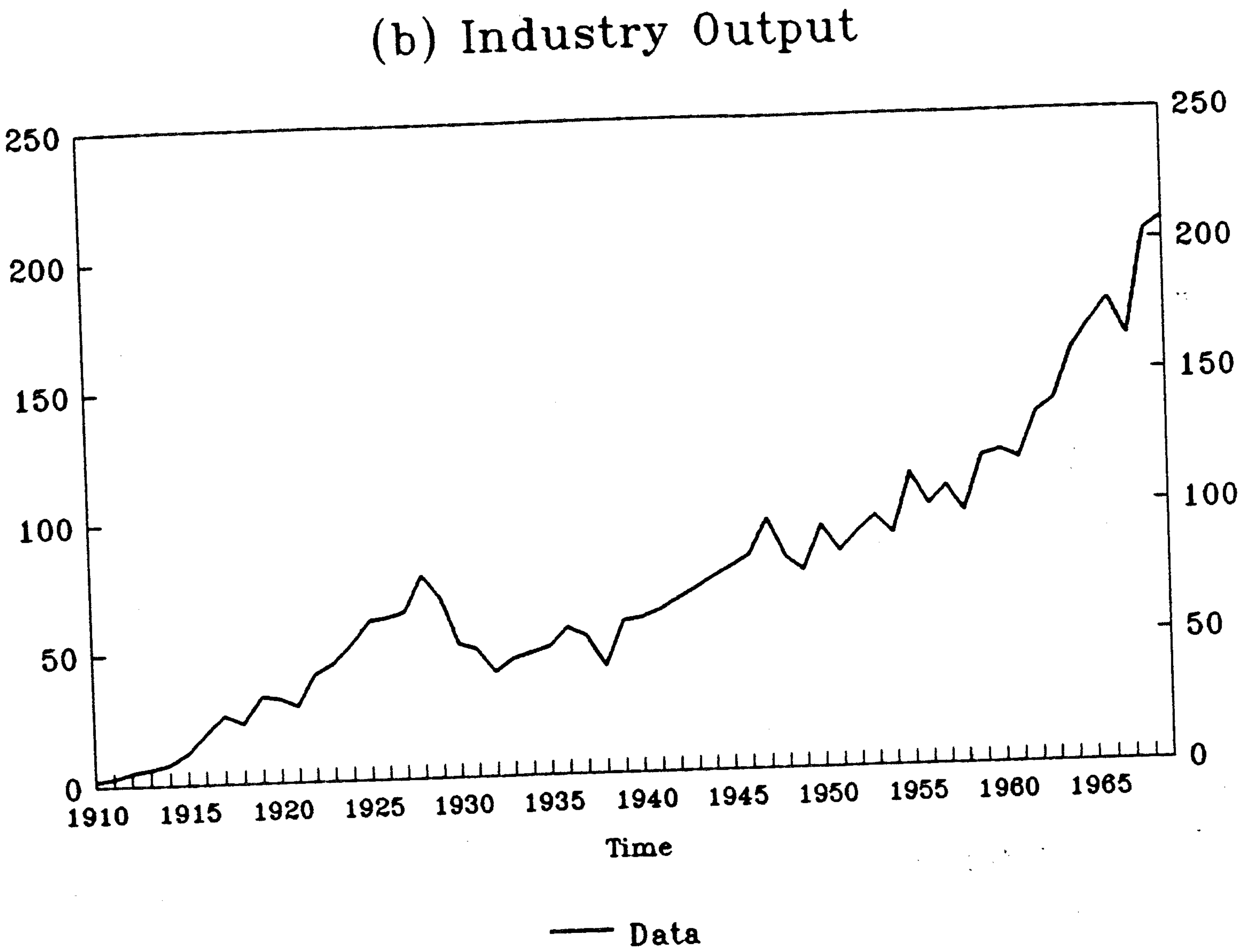
1920
1925
1930
1935
1940
1945
1950
1955
1960
1965
Time
Data

Figure 1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1930
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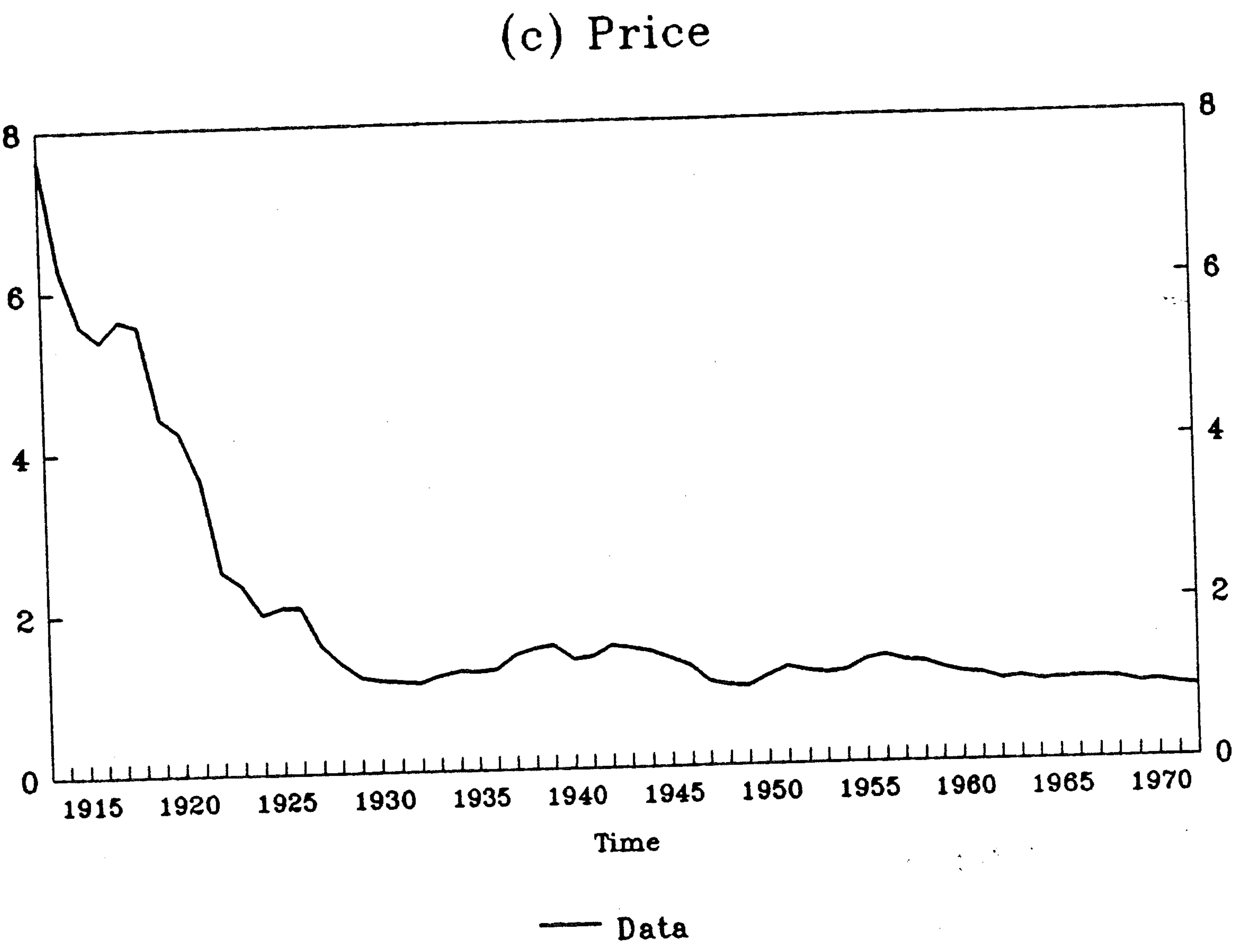
1940
1945
1950
1955
1960
1965
Time
Data

Figure 1


(c) Price
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1915
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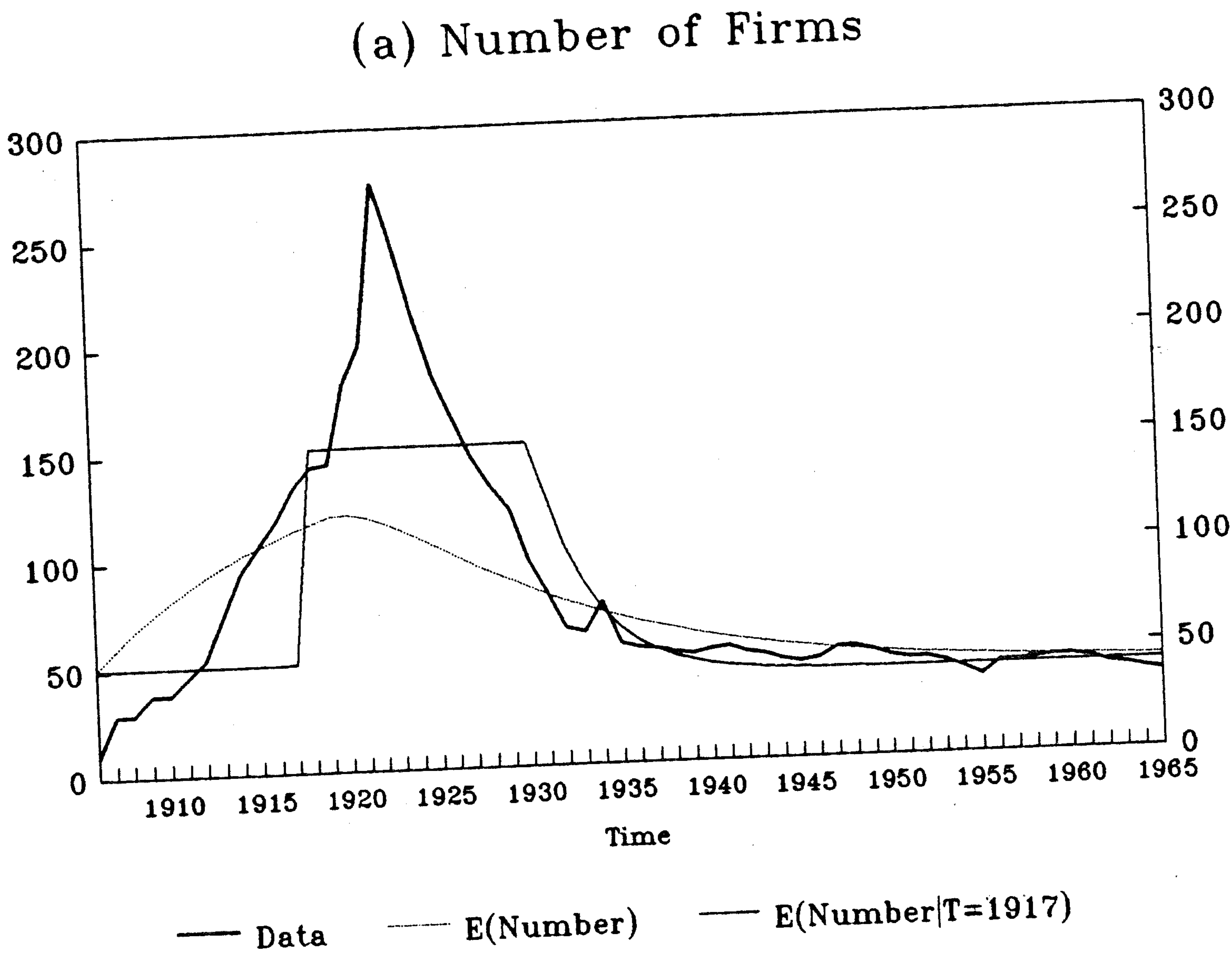
1955
1960
1965
1970
Time
Data

Figure 2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250
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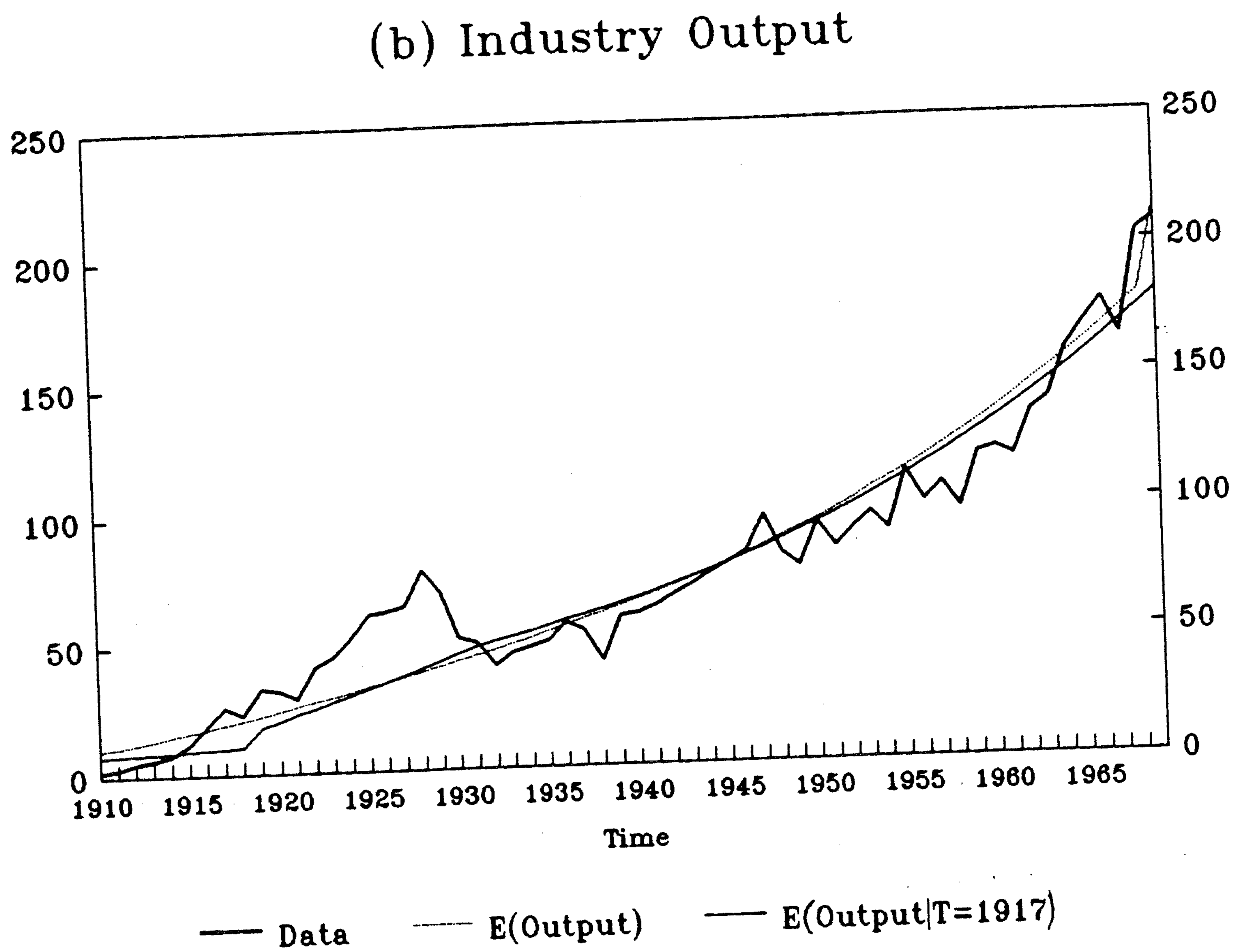
1910 1915 1920 1925 1930 1935 1940 1945 1950 1955 1960 1965
Time
Data
E(Number)
E(Number|T=1917)

Figure 2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200
150
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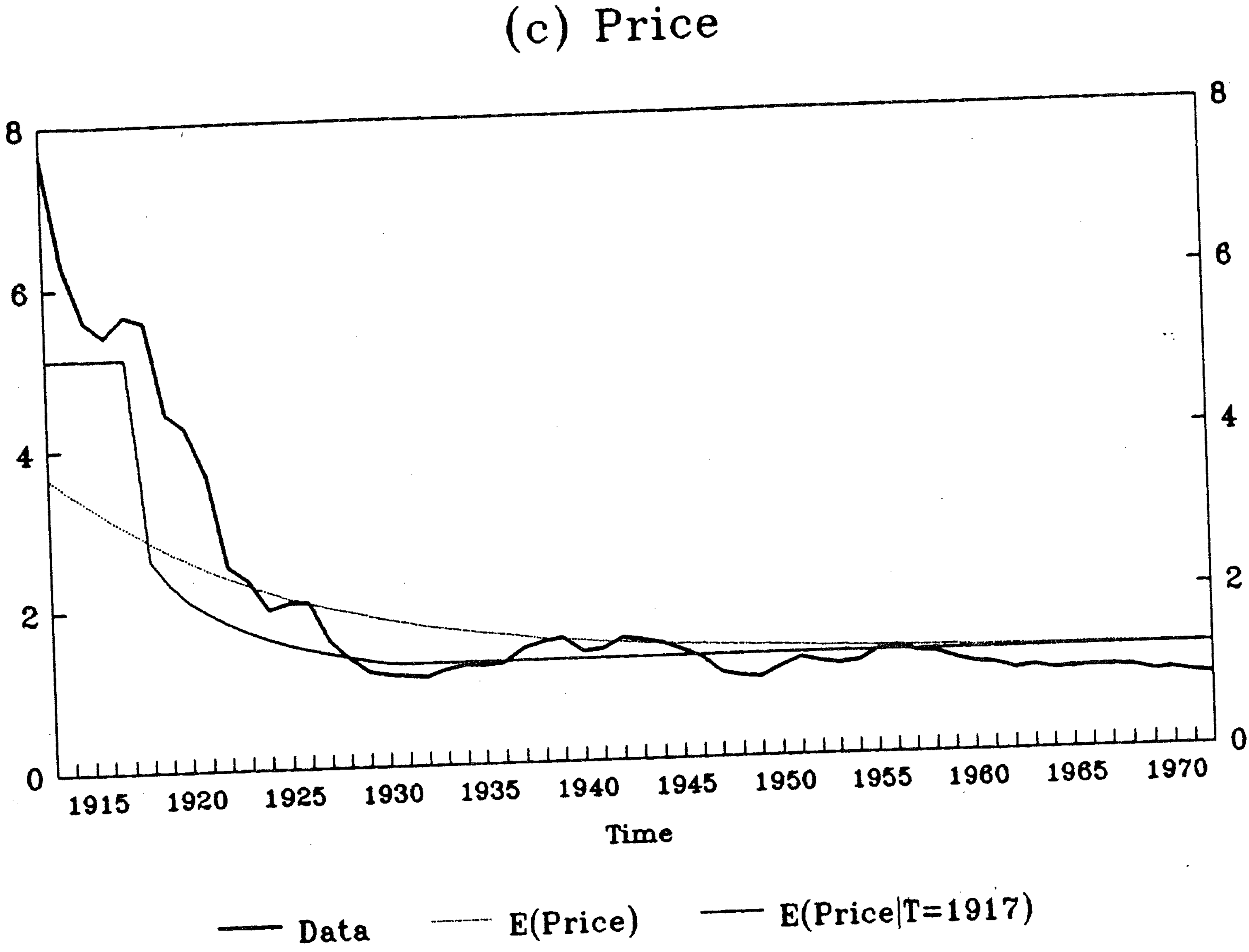
1930
1935
1940
1945
1950
1955
1960
1965
Time
Data
E(Output)
E(Output|T=1917)

Figure 2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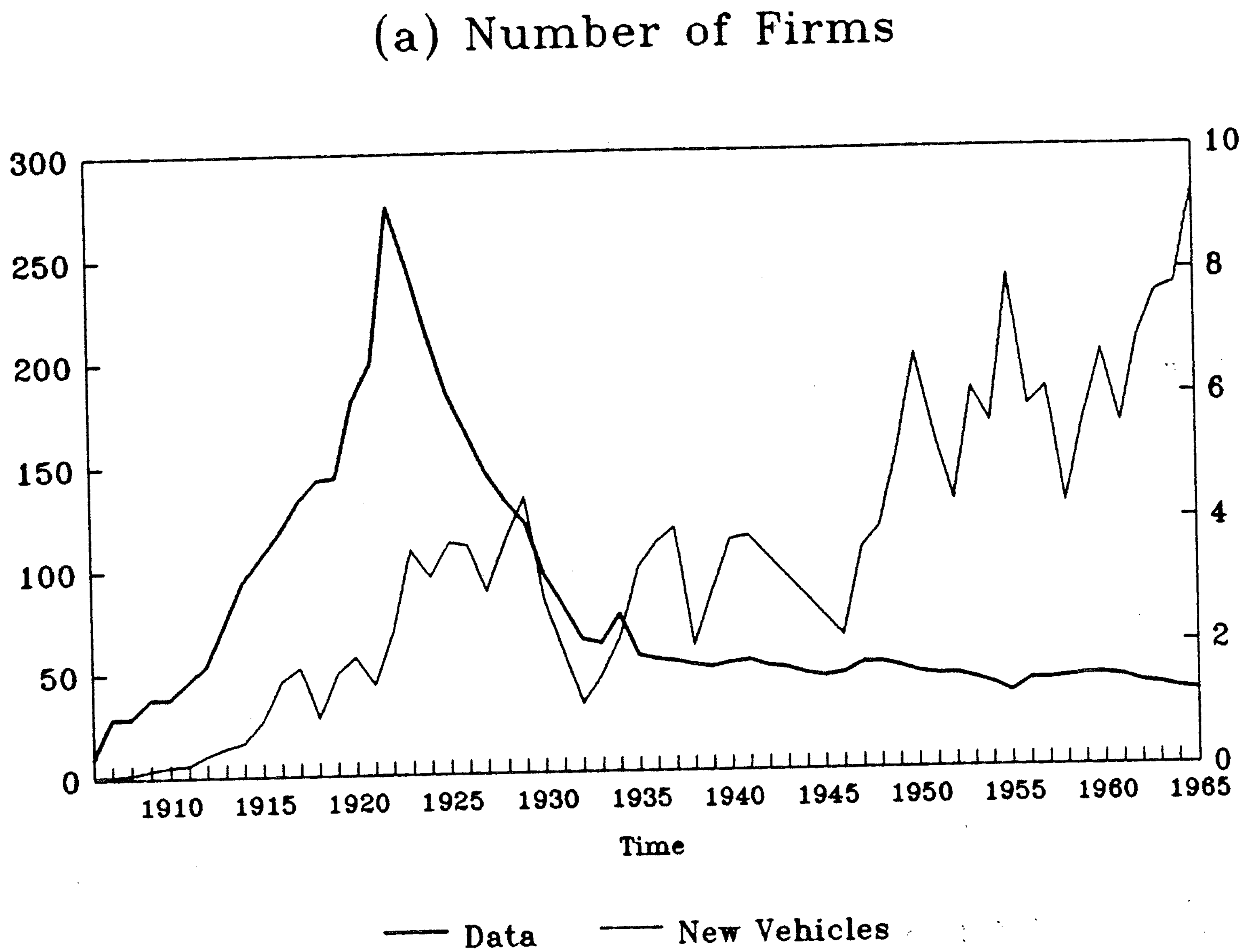
1960
1965
1970
Time
Data
E(Price)
E(Price|T=1917)

Figure 3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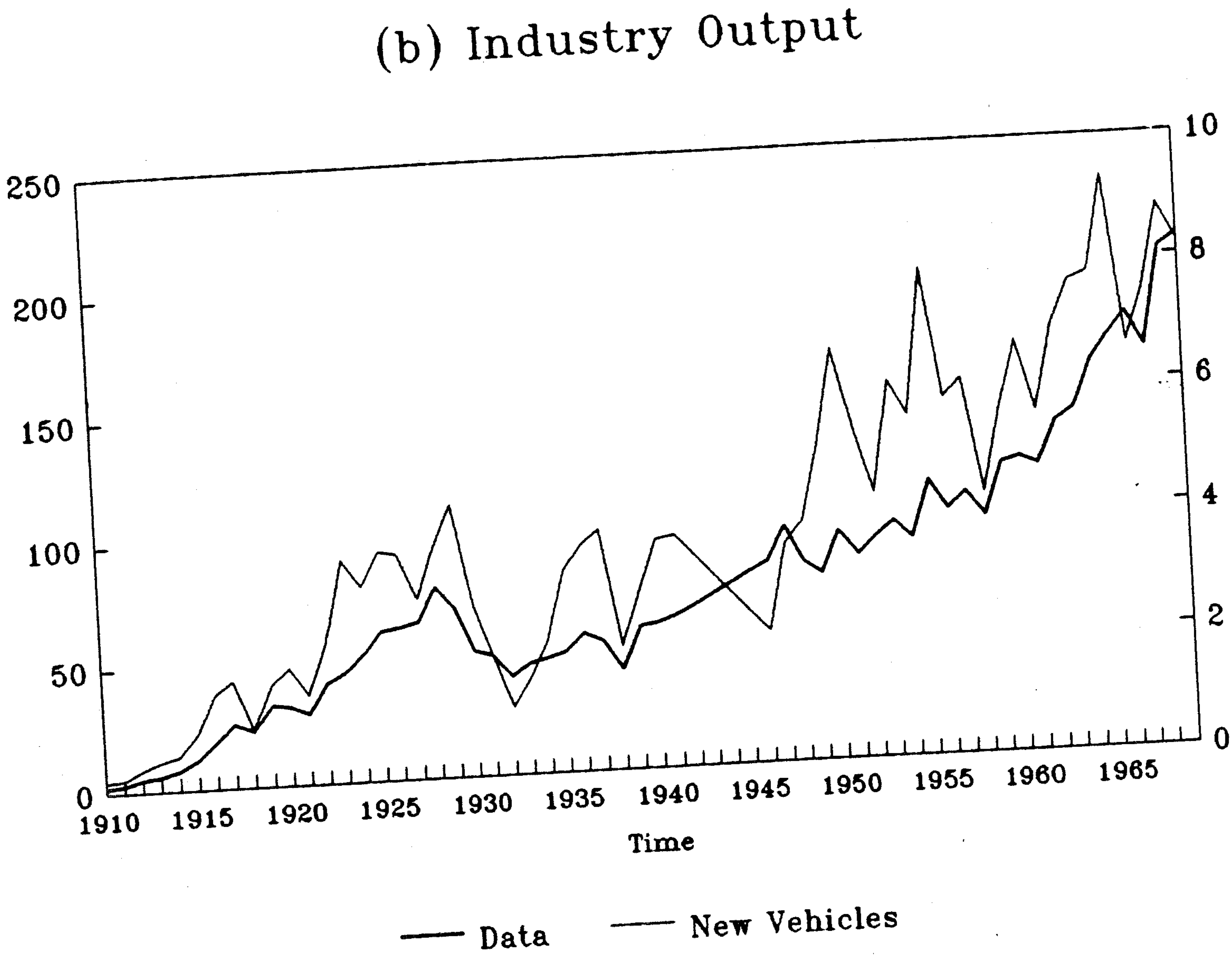
1965
Time
Data
New Vehicles

Figure 3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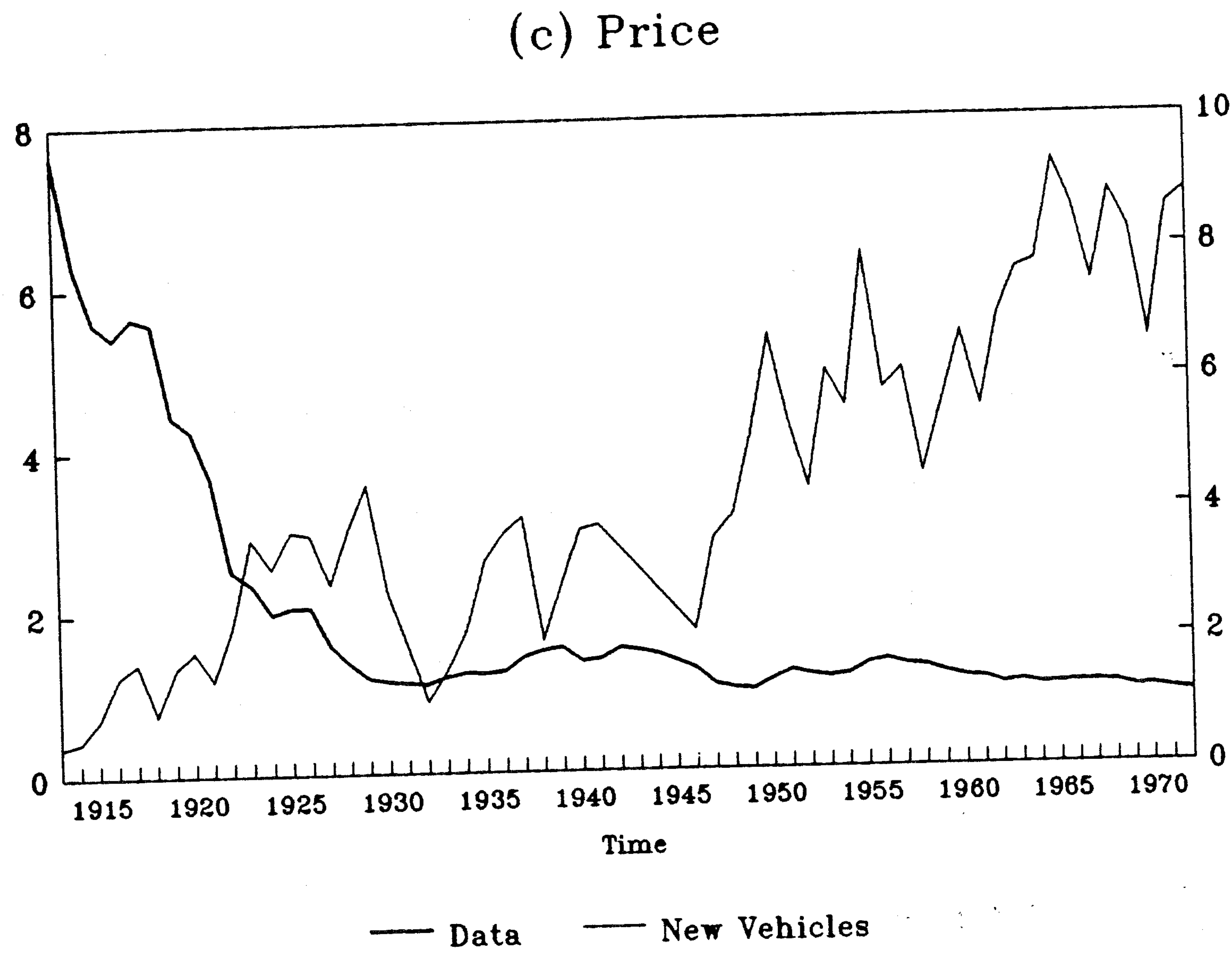
Time
Data
New Vehicles

Figure 3


(c) Price
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10
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4
2
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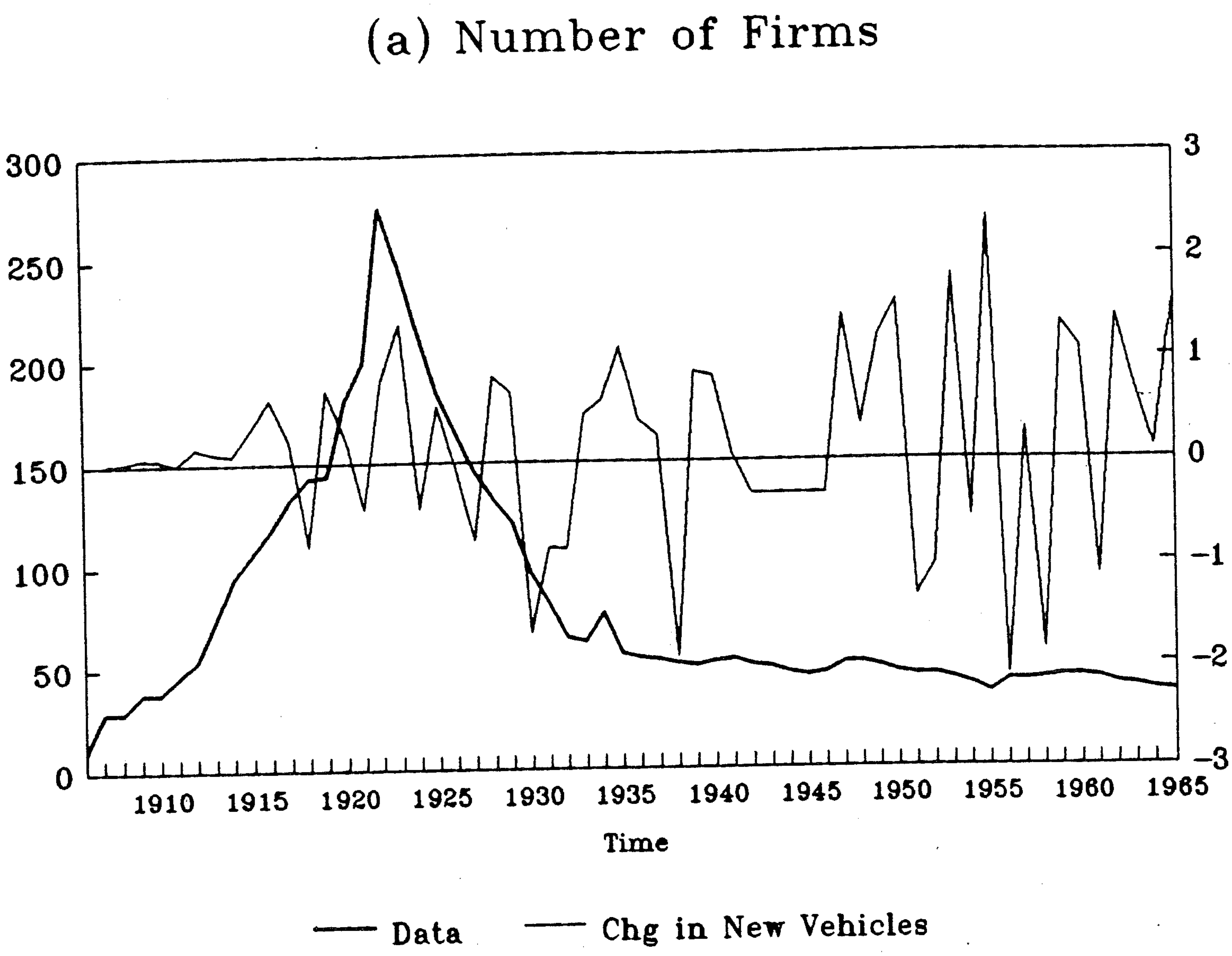
1915 1920 1925 1930 1935 1940 1945 1950 1955 1960 1965 1970
Time
Data
New Vehicles

Figure 4


(a) Number of Firms
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250
200
150
100
50
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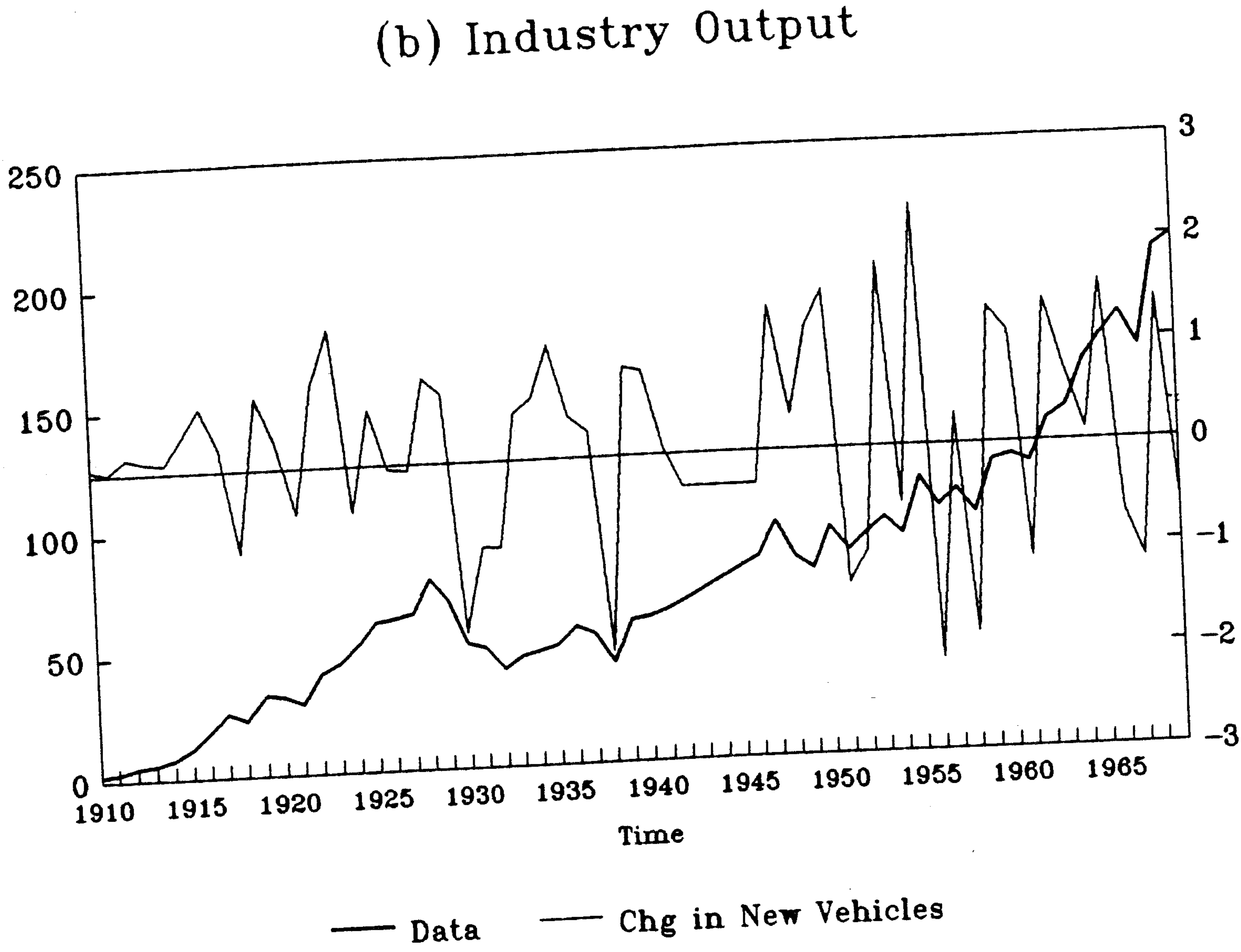
2
1
0
-1
-2
-3
1910 1915 1920 1925 1930 1935 1940 1945 1950 1955 1960 1965
Time
Data
Chg in New Vehicles

Figure 4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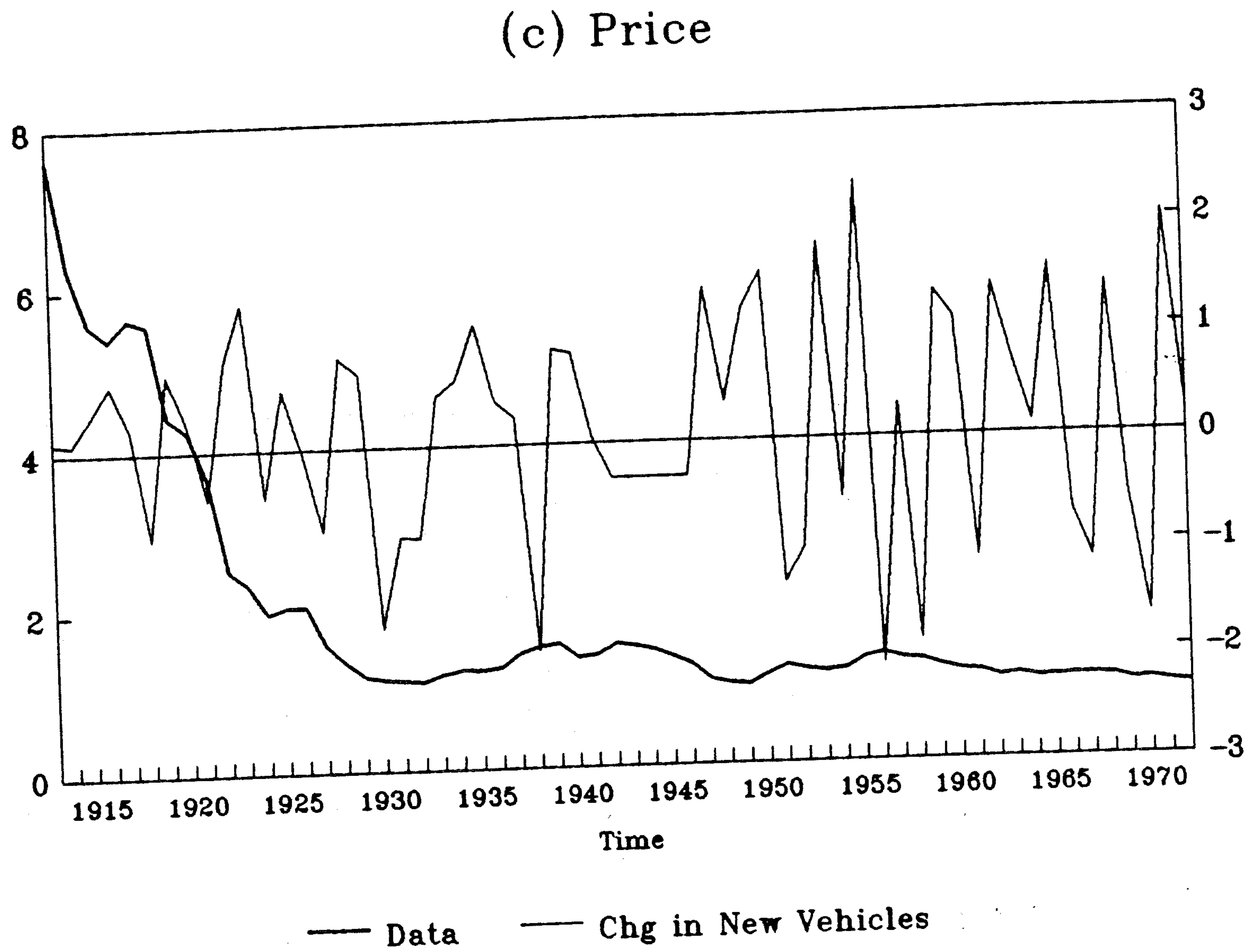
Time
Data
Chg in New Vehicles

Figure 4


(c) Price
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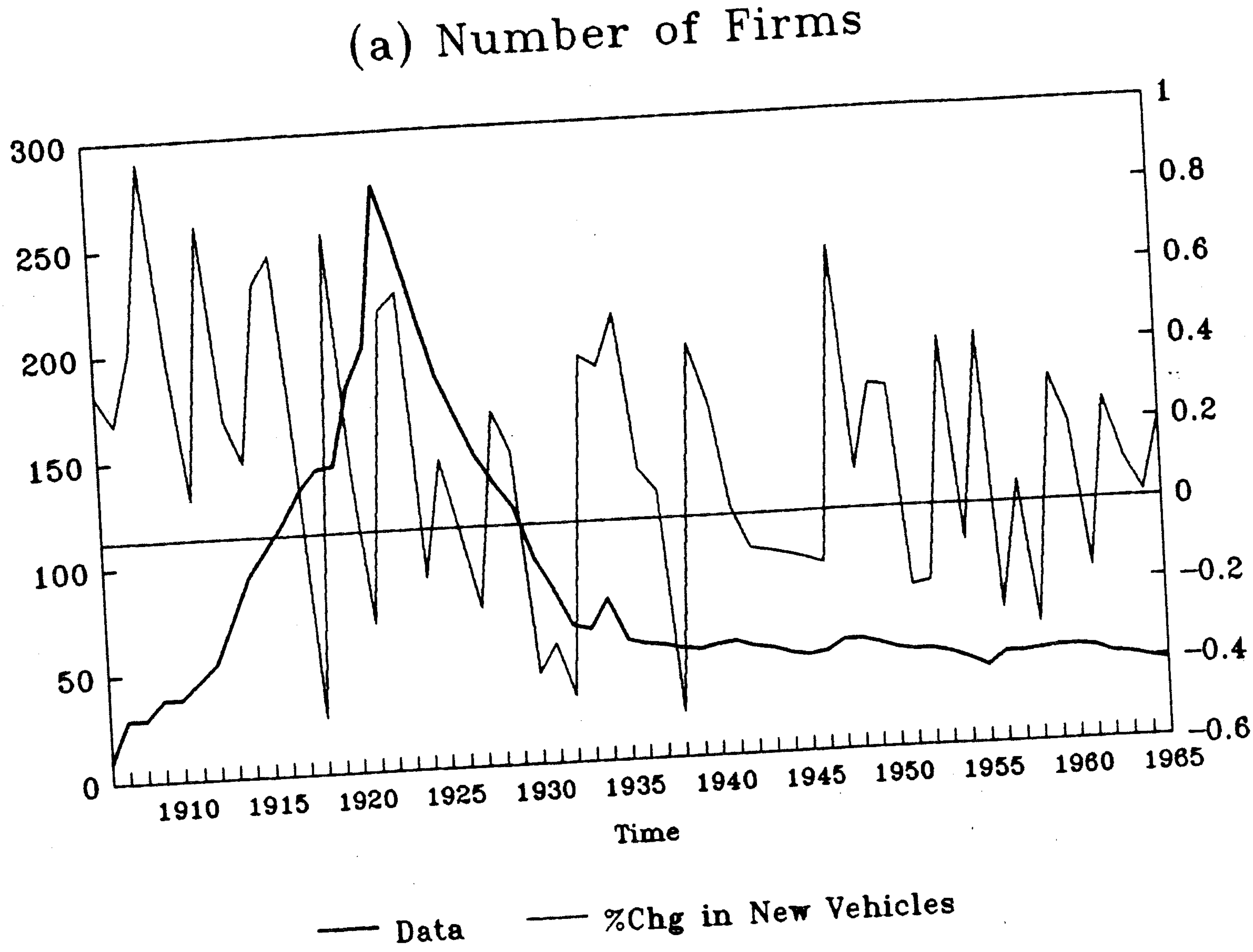
Time
Data
Chg in New Vehicles

Figure 5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250
200
150
100
50
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1
0.8
0.6
0.4
0.2
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-0.2
-0.4
-0.6
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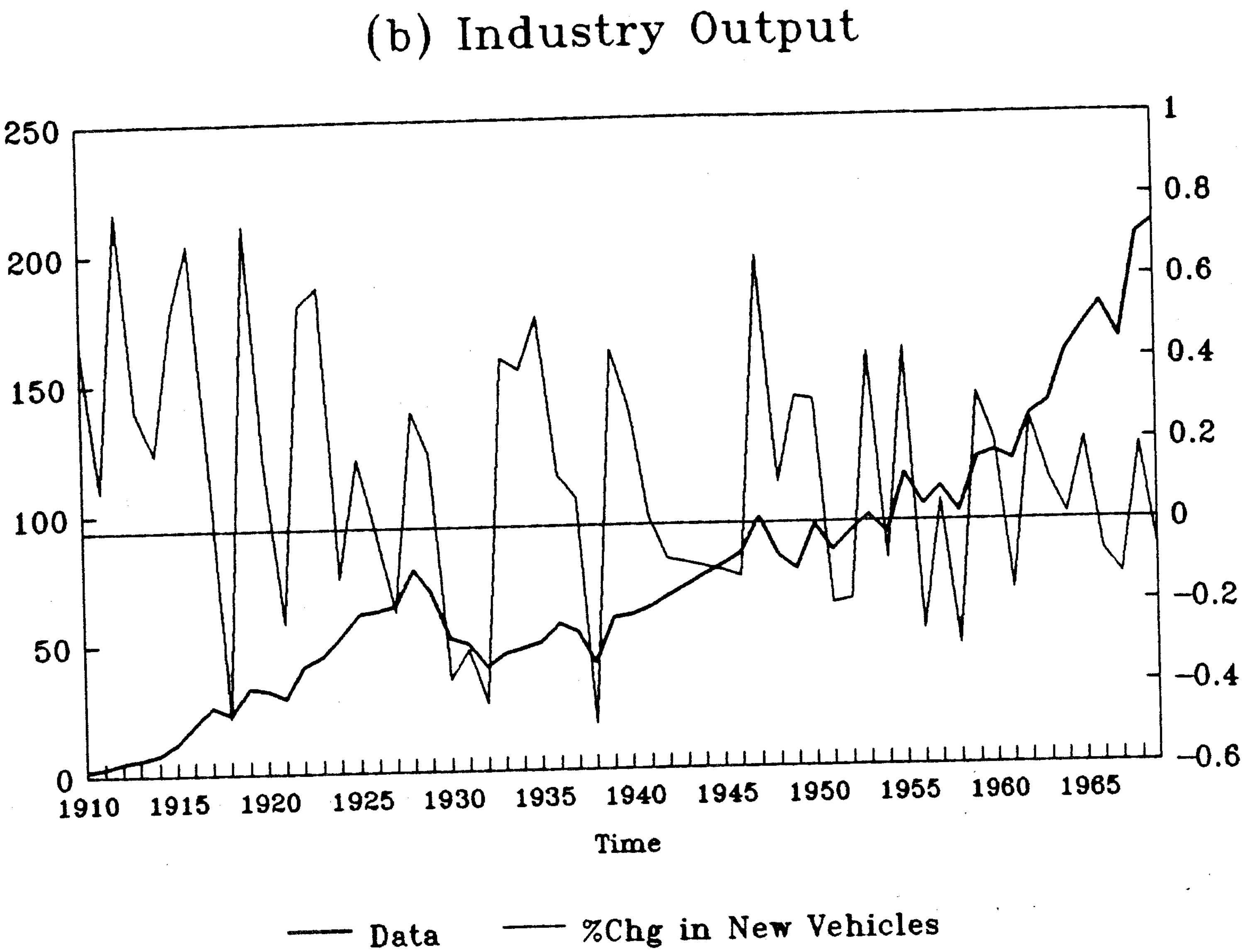
1965
Time
Data
%Chg in New Vehicles

Figure 5


(b) Industry Output
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−0.4
−0.6
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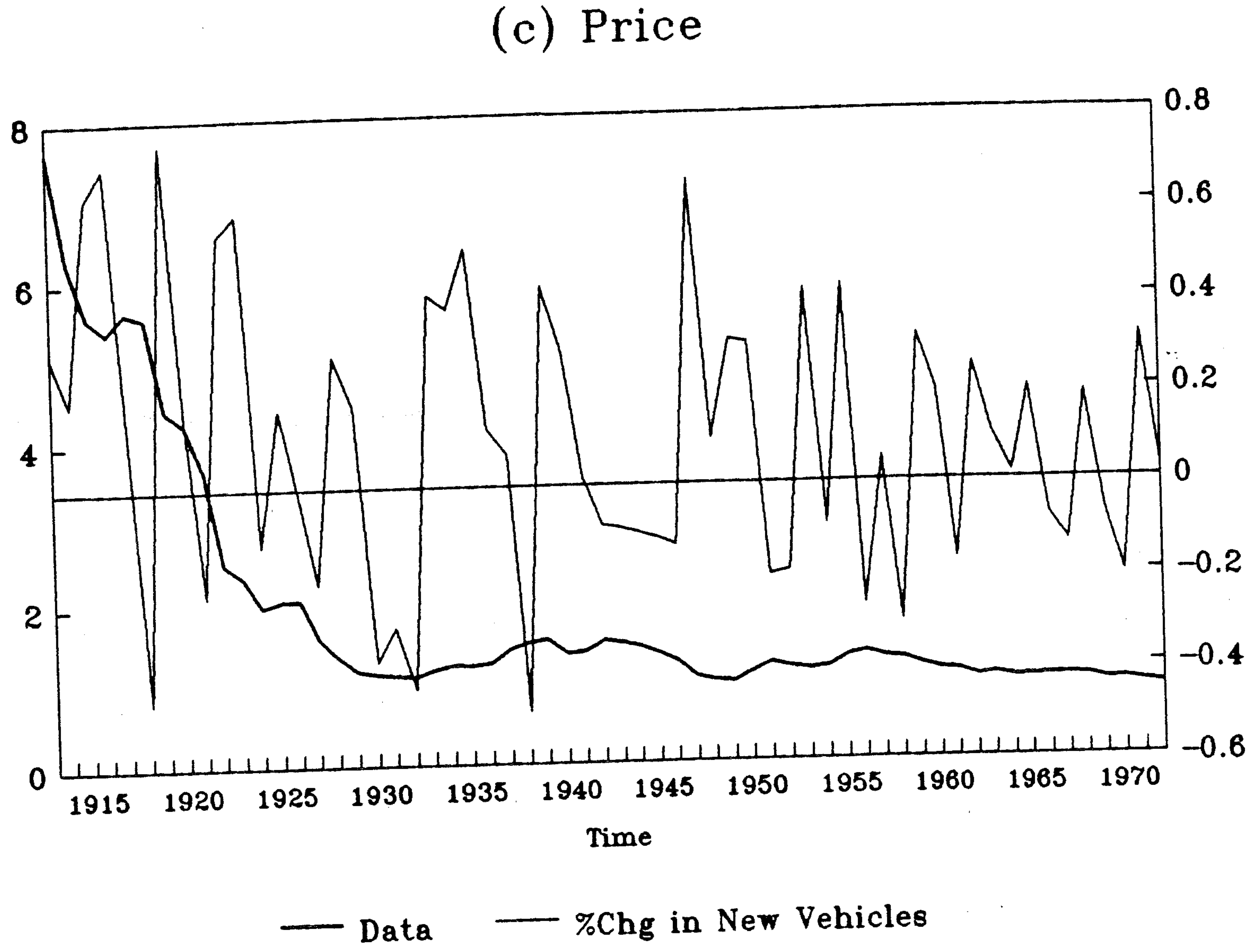
Time
Data
%Chg in New Vehicles

Figure 5


(c) Price
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-0.4
-0.6
1915
1920
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1935
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1945
1950
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1960
1965
1970
Time
Data
%Chg in New Vehicles

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## Footnotes

[1]Ericson and Pakes (1989), Hopenhayn (1989) and Chari and Hopenhayn (1989) are also relevant in that they generate entry/exit implications based on technological know–how. However, their main interest is in the long run features of industry behavior as opposed to the life cycle behavior focused on herein; also, see Brock (1972) and Smith (1974). In a general model, Jovanovic and MacDonald (1990) study technological diffusion but do not focus on entry and exit. Jovanovic (1982) allows for them in a learning model, but provides no restrictions on their behavior.

[2]The time invariance assumption causes all lifecycle behavior to be attributed to supply–side factors. Whether doing so in appropriate for the data studied below is the subject of Sec. III.5.

[3]Obviously $x_t^0$ is constrained both above and below. These constraints are left implicit during the development, but certainly satisfied in the equilibrium. Constraints on $\underline{x}_t$ and $\bar{x}_t$ (below) are treated in the same manner as those on $x_t^0$.

[4]In the expressions to follow $p_t$ is price at $t$ *and taking into account whether the refinement has occurred.*

[5]i.e.

$$\frac{\pi^0}{1-\gamma} \equiv \pi(p^*, \underline{\theta}) + \left[r\frac{\pi(p^*,\bar{\theta})}{1-\gamma} + (1-r)\frac{\pi^0}{1-\gamma}\right]$$

$$> \gamma\left[r\frac{\pi(p^*,\bar{\theta})}{1-\gamma} + (1-r)\frac{\pi^0}{1-\gamma}\right]$$

$$\geq \gamma\left[\bar{r}\frac{\pi(p,\bar{\theta})}{1-\gamma} + (1-\bar{r})\frac{\pi^0}{1-\gamma}\right]$$

if $r > \bar{r}$.

[6]For brevity, a complete description of $\lambda_t(\iota,q,\theta,|\cdot)$ is omitted. All information can, however, be obtained from the evolution displayed, together with $q(p_t, \theta) = \frac{\partial\pi(p_t,\theta)}{\partial p}$, $p_t = D(Q_t)$ and $Q_t = \underline{n}_t q(p_t, \underline{\theta}) + \bar{n}_t q(p_t, \bar{\theta})$.

[7]If $\underline{n}_T$ is less than the number of firms knowing $\underline{\theta}$ at $T$, it will be necessary to verify that nonparticipants knowing $\underline{\theta}$ would be willing to eschew participation at $t \geq T+1$. It will turn out that consideration of this possibility is not required because $\underline{n}_T$ will take on its maximum possible value; see below.

[8]This specification for $D$ is not bounded above, as was assumed in Section II. A minor modification to $D$ regains consistency but plays no role in the measurement and so is not pursued here. Measurement was also undertaken with $D(Q) = d_0 - d_1 Q$ assumed instead. With the exception of measured values for $\pi^0$, and obviously $d_0$ and $d_1$, this modification yields only a trivial changes in the estimated values. Consequently only the constant elasticity specification is reported.

[9]A few firms may have existed prior to 1906. In the *Thomas Register*, 1906 is the earliest date at which producing firms were recorded. 1905 therefore corresponds to $t = 0$ in

the theory.

[10]That most lifecycle behavior had ended by 1940 is also the reason why additional data (beyond 1973) are likely not needed.

[11]These correlations are a weighted average of correlations given the refinement date, where the weights are from the estimated probability distribution of *T*. The procedure is analogous to that discussed in Section III.5 below.

[12]Letting the expected value given success be *E*, free entry in 1905 ensures

$$\frac{\pi^0}{1-\gamma'} = \gamma' \left[\beta E + (1-\beta)\frac{\pi^0}{1-\gamma'}\right].$$

For the estimated $\gamma'$ and $\beta$, $E/[\pi^0/(1-\gamma)] \cong 2.75$.

[13]In the Gort/Klepper data, figures on actual tire prices are available for a few years.

[14]For 1906–70, the data are from the *U.S. Historical Statistics*; 1971–3 came from the *U.S. Report of the President*. Recall that the impact of second world war was removed from $Q_t^*$. Since the data studied here are also influenced by the war, its affect is removed in the same manner. Doing so avoids mistakenly interpreting any lack of significance of $z_t$ as lack of importance of nonhomogeneous demand growth when it is merely due to eratic fluctuations of the automobile data during the war years.