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DEBT, ASYMMETRIC INFORMATION, AND BANKRUPTCY

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Abstract

This paper provides a model of credit in an optimal contract setting in which a simple debt contract with a default clause is optimal even when randomization is allowed. An observable liquidation value for projects is the crucial feature that gives rise to debt. The model emphasizes the role of ongoing monitoring of borrowers by rather than *ex post* monitoring. The social costs of bankruptcies are due to inefficient liquidations, as the optimal contract generally involves some liquidation of projects whose value exceeds the liquidation value.

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The essential feature of bankruptcy is a change in ownership and/or control of assets. The extent of the change can of course vary greatly from one case to the next, from relinquishing control over some aspect of a firm's decisions in exchange for rescheduling or forgiving some portion of the debt, to outright liquidation and a complete change of ownership. But in all cases, as a matter of definition, the debtor obtains a reduction of his liabilities in exchange for a giving up some of his assets. It is reasonable to ask what distinguishes this method of exchange from more conventional spot market exchange, and to ask why this peculiar method for changing ownership should be observed so frequently.

An important distinguishing feature of bankruptcy is that it inevitably involves the carrying out of terms of a contract. That is, conditions under which it occurs are spelled out in a contract that is written prior to the receipt of relevant information. Debt contracts also have a number of defining characteristics, i.e. the debtor retains residual rights of control over the assets purchased with the debt (except in the event of default), and payments to the creditor are not directly contingent on the assets' value.

The observed nature of these credit relationships represent data that should be informative about the underlying economic environment. Indeed there is now a large literature on financial contracts that analyzes the prevalence of certain types of contracts such as debt and equity from an optimal contracting standpoint. The most popular explanation for debt contracts is based on a costly auditing story (Townsend, Diamond, Gale and Hellwig, Williamson): Lenders cannot costlessly observe the outcome of borrowers' investments. The standard debt contract minimizes monitoring costs by establishing a fixed payment except in those states of the world in which the borrower is insolvent (i.e. the outcome is sufficiently bad that the borrower cannot make the payment). The social cost of bankruptcy consists of monitoring costs, which are incurred only in the event of "default".

There are several weaknesses in the standard story as an explanation of the widespread use of debt. First, debt turns out to be optimal only in the class of *deterministic* contracts. In fact it is easy to see that a contract that is more like equity with stochastic monitoring dominates debt contracts with deterministic monitoring (see Mookherjee and Png, 1989).[1] For example, under risk–neutrality the contract would have borrowers repay a share of output, with honesty enforced by an arbitrarily small probability of monitoring together with an arbitrarily large reward if the agent is discovered to have been honest. Even if one puts bounds on transfers between principal and agent, stochastic monitoring combined with rewards for honesty will generally be a part of optimal schemes (see Border and Sobel, 1987). Such contracts are perfectly plausible, and are observed in many settings (e.g. auditing of tax returns by the IRS is, from the perspective of the taxpayer, essentially random).

A second problem with this story is that the direct costs of bankruptcy are not believed to be large, and many of those costs mainly have to do with ascertaining priorities in dividing up claims to assets among creditors, not in establishing the value of those assets.[2] Third, there is no distinction between insolvency and illiquidity. Bankruptcy in these models is not really forward–looking, rather it is just a symptom or a consequence of a bad realization between the time of the loan and when it becomes due. Finally, most so–called debt contracts are not pure debt, but have contingencies ("covenants") written on variables that are relatively easy to observe. This suggests that the ongoing monitoring of active loans is also important. Overall these problems suggest that the costly state verification approach is not an ideal basis for models of debt and financial intermediation.

This paper describes an alternative approach that overcomes these weaknesses.

[1]A related point is that the debt contract is not time–consistent: It would pay the lender to threaten to monitor but then not actually carry out the threat.

[2]See Warner (1977) for estimates of the magnitude of the so–called "direct" costs of bankruptcy.

While the model is related to the costly state verification approach, it differs in that it assumes a freely observable liquidation value for any project. It remains costly to observe the outcome of the project, but because of the observable liquidation value it is never necessary to incur the cost.[3] The model also differs in that the "outcome" of the project at the stage in which debt is repaid is really the value of future cash flows that the project will generate. Thus, in accord with the standard modern treatment of debt and equity, the optimal arrangement between the borrower and lender essentially gives the borrower a call option on his project, the exercise price of which is the face value of the debt.[4]

The underlying assumption is that borrowers have some special skills or attributes that make the value of their capital to them generally different from its value to anyone else. That value, which is essentially the present value of cash flows from the capital, is assumed to be private information. The liquidation value of an investment is the value of the capital in its most productive alternative use, i.e. the market value of the capital after allowing for transactions costs. For example, an individual may wish to borrow money to start a small business. This involves the purchase and renovation of a building, some purchases of equipment, legal costs, etc. The lender has a very good idea of the resale value of the building and the equipment, but much less information about factors that are unique to the entrepreneur.[5]

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[3]Rather, the cost of determining the asset's value in the hands of the borrower is assumed to be sufficiently large relative to the expected liquidation cost (to be defined below) so that it never pays to incur it.

[4]Hart and Moore (1990) have a related model which relies on nonverifiability as opposed to asymmetric information. The distinction is that nonverifiability relates only to what an outside enforcer can act upon. Thus if we assume that contracts can only be enforced by an outsider who cannot verify the outcome of the project, it is not even necessary to assume asymmetric information between borrowers and lenders. The constraint on contracts then would be that it specify actions contingent only on verifiable state variables. The resulting contract might be similar to that found in this paper, but the environment would imply gains from renegotiation when new information arrives, with potentially no inefficient liquidations.

[5]That is, whatever is different about entrepreneurs that leads to ex post heterogeneity

It will turn out that the optimal contract gives rise to unambiguously too much liquidation relative to the symmetric information case. Thus the costs of bankruptcy arise not from monitoring or transactions costs *per se*, but from the misallocation of capital. At the same time bankruptcy can be beneficial (conditional on the new information about the project) if capital is reallocated to a superior use. This occurs (by definition) when the value of a borrower's project falls below the capital's liquidation value.[6]

In addition to providing a basis for debt as a truly optimal contract, the model also has other interesting implications. It suggests that financial intermediation arises from the need for ongoing monitoring of active loans rather than from *ex post* monitoring. There is also a potential for binding liquidity constraints. In this case the distinction between illiquidity and insolvency is explicit, and the optimal contract must take into account the additional source of inefficient liquidations.

1. The Model

We begin with the simplest version of the story that makes the basic point. Consider an economy in which there is one good that can either be used as a capital input to a project, stored, or consumed. There are two types of agents, "firms" and "banks". Each firm is endowed with an investment project and some endowment of the good w that is insufficient capital to undertake the project at its optimal scale. Banks, on the other hand, have plenty of capital but no projects. The projects are "putty–clay" in the sense that once begun there can be no further positive investment, while negative investment can only take place via a liquidation technology described below.

The economy lasts for three periods ($t=0,1,2$). It is assumed that at $t=0$ there

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in outcomes. See Lacker (1990) for a model that relies on heterogeneous preferences to endogenize the borrower's and lender's different valuations of the collateral.

[6] Barro's (1976) model has similar features, though it is not an optimal contract model and does not have private information.

is a large number of identical agents (i.e. a countable infinity), and that outcomes are independent across agents. Hence the standard device of working with proportions and per capita quantities is feasible when discussing aggregate equilibrium conditions. Banks are assumed to be identical, while firms are identical *ex ante* but different *ex post*, so we can start at t=0 with a representative firm and bank. At that point the firm and the bank agree to terms on financing the firm's investment project. These terms include the size of the bank's investment x, and specific terms for repayment. It is assumed that the firm cannot obtain more than one such contract. Banks behave competitively—so that they are price takers in the market for investment capital—and are willing to invest in anything on the margin that gives them a rate of return at least as large as the market risk–free return, which is denoted R. We limit the analysis to partial equilibrium and take R as given.

The three period structure is designed to focus attention on one–period contracts. Repayment can only be enforced by the threat of liquidation. With no further output after t=2 it cannot be incentive–compatible for a contract to have positive repayments at t=2. While it would obviously be more realistic to have longer term contracts, nothing of interest for this paper would be gained by complicating the model in this way. The essential features of the model are that the firm receives information about the value of the project's future output after the contract is agreed upon, and that liquidation can occur before some of that output is realized.

At t=1 the firm learns something about its future prospects—specifically the present value v of its future output flows (contingent on no liquidation)—and some output y_1 from the project is realized. The value of v is private information to the firm, and the firm has complete discretion over what value of v to report to the bank. In effect this means that the firm has discretion over whether and how much to repay. Formally, the firm reports $\hat{v}$, and the firm and the bank carry out whatever actions are specified by the contract, including either partially or completely liquidating the project.

While projects have diminishing returns to scale as a function of the size of the investment, liquidation exhibits constant returns: If a proportion $1-\gamma$ of a project that has invested x is liquidated at t=1, the liquidation yields $(1-\gamma)\lambda(x)$ units of capital, leaving a project with a value of γv.[7,8] The function $\lambda(x)$ is the liquidation value at t=1 for an investment of size x. For simplicity it is assumed for now that λ is a known concave function of x, with $\lambda(x) \leq x \; \forall x$, and does not depend on any other state variables.[9]

At t=2 the remaining output from the project is realized (provided the project was not fully liquidated in the previous period) and any remaining terms of the contract are fulfilled. The liquidation value at t=2 is assumed to be zero. In any case, liquidation in the final period is ineffective as an enforcement mechanism because the project has no further cash flows.

Following Diamond and others, we will assume that agents are risk–neutral but cannot have negative consumption. In other words, both the lender and the borrower maximize expected utility E(U), where:

$$U = c_1 + \beta c_2, \quad c_t \geq 0. \tag{1}$$

and where c_t is consumption at time t, and $\beta < 1$ is a discount factor. We will

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[7]Note that this builds in an irreversibility to investment: It is more costly to invest x and then liquidate half of the project (even if one receives full value for the liquidated assets) than to invest x/2 to begin with.

[8]Proportional liquidation is mathematically equivalent (given risk–neutrality) to randomized discrete liquidation. Randomized schemes may be somewhat difficult to implement in some contexts (e.g. in the present case when liquidation is costly). Therefore the preferred interpretation is the one in the text.

[9]Nothing of any consequence is lost by the last assumption. Even if the bank could infer θ from λ, the assumption would be that the bank cannot learn θ without liquidating the project. Then $\lambda(x)$ would just be the bank's expectation of the liquidation value over all θ such that (given the specifics of the contract) the firm chooses to liquidate.

assume for now that $\beta = 1/R$. The firm's outputs y_t (t=1,2) are based on the technology

$$y_t = \theta_t f(x), \quad t=1,2, \tag{2}$$

where θ_t stochastic disturbance that differs across firms and that only the individual firm can observe. Thus at t=1 the firm observes θ_1 and gets private information about θ_2. Hence $v = \beta E_1(y_2)$. It is assumed that the support of the distribution of v includes both zero and some number greater than $\lambda(x)$.

The contract specifies an initial transfer of resources from the bank to the firm (denoted by ℓ), the investment x, contingent repayments by the firm h at t=1, and other actions such as liquidation contingent on $\hat{v}$, the borrower's reported value of v. Since v is the private information of the firm, the contract must be incentive–compatible; that is, the contract must be designed so that any contingencies based on v do not induce the firm to misrepresent its value.

As in Hart and Moore (1990) and others, it is assumed that the firm can "conceal" (or consume) any assets (e.g. first period output) other than the firm's capital. This effectively ensures limited liability, since the firm cannot be induced to pay more than the capital's value. Also, required payments must respect the non–negativity constraint on consumption.

Finally, to keep the model as simple as possible it is assumed that w=0, i.e. the firm has zero endowment of capital. It will be clear in what follows that lenders will not lend more than the firm would want to invest, which means that $\ell = x$. Hence the payment by the firm must satisfy the liquidity constraint

$$h \leq y_1. \tag{3}$$

Let $\gamma \in [0,1]$ denote the proportion of the project that is **not** liquidated at t = 1.

We can limit our focus to contracts that specify x, h, and γ, and without loss of generality assume that any actions not specified in the contract are left to the borrower to choose to maximize his own utility.

The Optimal Contract without Liquidity Constraints

We first take x as given, and determine the optimal h and γ functions. It suffices to have h and γ depend on the firm's $\hat{v}$, by the Revelation Principle. Consider the situation at t=1. The firm has y_1. To decide what value of v to announce, it solves

$$(4) \qquad \max_{\hat{v},\xi} \; y_1 - h(\hat{v}) - \zeta + \beta R\zeta + \gamma(\hat{v})v,$$

where ζ is savings at t=1 (which is assumed to be able to earn R). The bank receives $h(\hat{v}) + [1-\gamma(\hat{v})]\lambda$. The incentive-compatibility constraint is that the firm's optimal choice of $\hat{v}$ equal the true value v.

Clearly y_1 has no effect unless the liquidity constraint binds. This section assumes that it does not. Specifically, it is assumed that y_1 is nonstochastic, the same for all firms, and sufficiently large so that (3) does not bind. Weaker assumptions are obviously possible. The case of binding liquidity constraints is discussed in a subsequent section.

Table 1 summarizes the main simplifying assumptions adopted for this section. Noting that ζ falls out of the problem because $\beta R = 1$, (5) boils down to

$$(5') \qquad \max_{\hat{v}} \; \gamma(\hat{v})v - h(\hat{v}).$$

The joint wealth maximizing solution would be for liquidation to take place if and only if $v \leq \lambda$. Suppose the bank sets the terms of the contract to be

$$(6) \qquad \gamma(\hat{v}) = \begin{cases} 1 & \text{if } \hat{v} \geq \lambda \\ 0 & \text{otherwise} \end{cases}$$

and

$$(7) \qquad h(\hat{v}) = \begin{cases} \underline{h} + \lambda & \text{if } \hat{v} \geq \lambda \\ \underline{h} & \text{otherwise ,} \end{cases}$$

where $\underline{h}$ is some constant (determined by the requirement that banks get the market return on loans). This contract is incentive–compatible insofar as the firm has no incentive to misreport v. The problem is that any $\underline{h} > 0$ is not enforceable because the firm can and will consume all of its liquid assets in the first period. Thus in the event that $v < \lambda$, there is no way to make the firm pay anything, since it can claim to have zero liquid assets.

Thus the only enforceable incentive–compatible contract that induces the "correct" liquidation decision is one in which $\underline{h} = 0$. This contract is equivalent to riskless debt, since the bank receives λ regardless of v. While such contracts (i.e "fully secured debt") are commonly observed, they are not the interesting case. Moreover, given the simplifying assumption that the firm has no assets of its own to contribute at t=0 (i.e. that $\ell = x \geq \lambda(x)$), such a contract is not technologically feasible. But even without this assumption the fully secured contract is not generally optimal because it constrains the loan to be such that $\ell \leq \beta\lambda(x)$. It will generally pay to increase loans above this point if the marginal value of additional investment exceeds R at that point. This distorts the liquidation decision (a second order effect at $\ell = \beta\lambda(x)$) but moves investment closer to its efficient level.

To determine the optimal contract it is simplest to think of the scale of investment as being determined by the usual efficiency conditions, but with a technology that is endogenously determined. The "inefficient" liquidations that arise as part of the optimal arrangement reduce the *ex ante* marginal product of capital in the project, and hence the optimal scale of investment below the first–best. For the moment we will take x as given and determine the optimal contract conditional on x. We will also continue to assume that the liquidity constraint (3) does not bind. This means that the project generates enough output at t=1 that liquidity is not an issue. Consequently only an overall constraint on payments that we will call the "solvency constraint" is binding. The solvency constraint is:

$$h(v) \leq (1-\gamma)\lambda + \gamma\max\{\lambda,v\}. \tag{8}$$

The optimal contract should maximize the expected utility of a representative borrower, subject to feasibility, participation by lenders, and incentive constraints. Let the support of v be the interval [0,V], and let G be the c.d.f. of v, assumed to be C^1 with $g(v) = G'(v) < \infty\ \forall v \in [0,V]$. The problem is to choose $\gamma(v)$ and $h(v)$ to solve

$$\text{(P0)} \qquad \max \int_0^V [v\gamma(v) + [1-\gamma(v)]\lambda - h(v)]g(v)dv.$$

The constraints are:

$$\begin{array}{ll}
\text{Feasibility} & \gamma(v) \in [0,1], \\
\text{Solvency} & h(v) \leq (1-\gamma)\lambda + \gamma v, \\
\text{Participation} & \int_0^V h(v)g(v)dv \geq Rx, \\
\text{I–C} & v \in \underset{\hat{v}}{\operatorname{argmax}}\ [1-\gamma(\hat{v})]\lambda + \gamma(\hat{v})v - h(\hat{v}).
\end{array}$$

The Appendix provides a proof of the following important result under some weak regularity conditions on G.

Proposition: The optimal contract conditional on x is a simple debt contract, characterized by a number $F \geq \lambda$, with

$$\gamma(v) = \begin{cases} 1 & \text{if } v \geq F \\ 0 & v < F \end{cases} \tag{9}$$

and

$$h(v) = \begin{cases} F & \text{if } v \geq F \\ \lambda & v \leq F \end{cases}. \tag{10}$$

The optimal value for F is the smallest value that generates the right amount of revenue (satisfies the participation constraint).

Example: Suppose v is distributed uniformly on the interval [0,V]. Then $G(v) = v/V$, $g(v) = 1/V$. Techniques described in the Appendix lead to the result that

$$\gamma(v) = \begin{cases} 1 & v \geq v^* \\ 0 & \text{otherwise} \end{cases},$$

$$h(v) = \begin{cases} v^* & v \geq v^* \\ \lambda & \text{otherwise} \end{cases}$$

where $v^* = \lambda + \xi(V - \lambda), \xi \equiv \frac{1}{2}\left[1 - \sqrt{1 - 4(Rx-\lambda)/(V-\lambda)}\right]$. Note that ξ is a measure of the extent of inefficient liquidations, and is increasing in R and x. Thus the model would appear to have the standard property that a larger loan or higher interest rate is associated, *ceteris paribus*, with a higher probability of default. (Note also that the condition $(Rx-\lambda)/(V-\lambda) < 1/4$ coincides with the requirement that the participation constraint set be non–empty). We have $\xi \in [0,1/2]$, so v^* (the face value of the debt) is between λ and V. If, for example, $(Rx-\lambda)/(V-\lambda) = 3/16$, then $\xi = 1/4$. The probability of default is v^*/V.

Determination of the optimal loan size x is straightforward, and will not be discussed in detail. F depends on x by the relationship

$$G(F;x)\lambda(x) + (1-G(F;x))F = Rx. \tag{12}$$

Total surplus from investment is

$$\lambda(x)G(F;x) + \int_F^V vg(v;x)dv - Rx. \tag{13}$$

The optimal x can be found by maximizing (13) subject to (12). It has the feature that there will be underinvestment relative to the full information first-best, simply because the expected bankruptcy costs (which arise due to inefficient liquidations) are increasing in x.

Example (cont.): Recall that $v = \beta E_1(y_2)$, and $y_t = \theta_t f(x)$. Suppose $f(x) = bx^\delta$, and $\lambda(x) = ax^\sigma$, where $0 < \sigma < \delta < 1$, $0 < a < b$. Also suppose θ_1 is the same for all firms, while θ_2 has a uniform distribution on $[0,\Theta]$, so that v again has a uniform distribution on $[0,V]$ $(V = \Theta f(x))$. The optimal contract from the first part of this example implies that the present value of a project is

$$E(y_1 + \beta y_2) = y_1 + \frac{v^*}{V}\lambda + (1 - \frac{v^*}{V})\left[\frac{V + v^*}{2}\right] \tag{14}$$

where v* is defined as above. This expression is a complicated function of x that must be maximized subject to the constraint that $(Rx-\lambda)/(V-\lambda) \leq 1/4$. The concavity of (14) is not assured, because ξ is a convex function of x, but a maximum of (14) must exist.

Liquidity Constraints

Another problem that may arise with any of the contracts considered here is that a solvent firm may be illiquid at t=1 and consequently unable to make a required payment. This section shows that if liquidity constraints are present, firms may take actions that shift cash flows from the future to the present, even if such actions are not correct from a net present value standpoint. This is true even if the borrower and lender take account of this possibility in designing their contract. The point is that the lender cannot distinguish between an insolvent and an illiquid borrower.

We now suppose that the constraint (3) may bind, i.e. that at the optimal F with the contract from the previous section some agents may not have enough liquid assets to pay F, even though they are solvent. Clearly if all agents had the same output at t=1, then F would simply be constrained by (3), and would be lower than if (3) were not binding. A more interesting case arises if output at t=1 varies across agents, and is not observable by lenders. We will make the simplest assumption, namely that output is random and uncorrelated with v. This may not be realistic, but allowing for correlation complicates the model without generating much new insight.

The main result is that the optimal contract will be modified to allow for partial default in the event of illiquidity. The contract in terms of v and F will be structured as before, but $y < F < v$ results in $\gamma = y/F < 1$. That is, the optimal γ depends on both v and y as follows:

$$\gamma(v,y) = \begin{cases} \min\{y/F,1\} & v \geq F \\ 0 & v < F \end{cases} \quad (14)$$

It is easy to verify that this contract is incentive-compatible, and that y/F is the largest value that γ can take when $y < F \leq v$ such that borrowers will have no incentive to misrepresent the value of y. Obviously this contract will involve less investment and more inefficient liquidations than in the case without liquidity constraints.

Financial Intermediation

Existing theories of financial intermediation also rely heavily on the costly state verification approach. The presence of costly monitoring in such models (e.g. Diamond, 1985, Williamson, 1986) means that decentralized lending involves a free rider problem and potential duplication of effort. This is because large projects require multiple lenders. Financial intermediaries arise, according to this story, as a means of coordinating monitoring (and thereby minimizing monitoring costs) among large groups of lenders.

The same mechanism does not arise in this setting, since there is no *ex post* monitoring cost. Other types of monitoring would be needed, however, that would also give rise to financial intermediation. These include *ex ante* monitoring (i.e. screening), and ongoing monitoring. The latter may arise because of the divergence in interests between borrowers and lenders with debt. Borrowers, for example, will take zero– or even negative–value risks with their projects because of the convexity of their payoff functions with respect to output.

A natural assumption is that the monitoring of a single borrower need only be done by a single agent, i.e. that simultaneous monitoring by more than one agent is redundant. Then arguments analogous to those in Diamond (1985) and Williamson (1986) can show that intermediation drive out decentralized lending. This section describes a simple extension of the model that allows for this effect. The idea is that borrowers have the ability to take an action at t=1 that induces a mean–preserving spread on the distribution of y. For a fixed monitoring cost any lender can observe whether this action has been taken, and can make it verifiable. The contract can therefore specify a penalty (up to the liquidation value of the project) for taking the action.

Conclusions

This paper has described a model in which debt with a default option is an optimal contract. This is in contrast to commonly used models in which debt is optimal only in a sharply restricted space of contracts. The model is also realistic in a number of dimensions, including its emphasis on ongoinn monitoring of borrowers by intermediaries (as opposed to *ex post* monitoring of defaultors), the option aspect of the contract between borrowers and lenders, and the possibility of liquidity constraints. It also provides a simple framework that may be useful for other issues related to credit markets and financial intermediation such as analyzing the welfare effects of policies that affect the debt–equity funding margin.

One inherent limitation of the asymmetric information approach is that if there are observable variables that are correlated with the underlying unobservable variable (in this case v), the contract should be made contingent on them. This applies to to the assumption of nonverifiability as well (see fn. 4). There may be good reasons why such contingencies are not explicitly found in contracts, but such reasons cannot be found in this model. In practice, however, such contingencies may be allowed for by the discretion lenders have in the event of technical default. In practice, lenders need not liquidate if they choose not to, and it is possible that they do implicitly take account of observable state variables in their actions. One way to investigate this would be to extend the model to allow for such a variable, and see what its implications are for, say, the behavior of Chapter 7 versus Chapter 11 bankruptcies over the business cycle. This is a subject for further research.

Although not explicit in the model, the view of bankruptcy in the paper is essentially forward–looking, i.e. it is a decision about how to allocate a durable productive resource. The liquidation value is determined by the capital's best alternative use, which in general is endogenous and depends on expectations about future disturbances of one sort or another. This should not be controversial, and is not inconsistent with allowing for other factors such as direct limitations on contract complexity (e.g. institutional requirements that contracts be written in nominal dollars). It is an empirical question as to which approach is more successful in understanding the world.

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Appendix: Proof of Proposition 1

The following problem is easily seen to be equivalent to (9)–(13): Choose functions $\bar{h}(\tau)$ and $\bar{\gamma}(\tau)$ to solve

$$\text{(P1)} \qquad \max \int_0^T [\tau\bar{\gamma}(\tau)-\bar{h}(\tau)]\bar{g}(\tau)d\tau$$

subject to:

$$\text{(A1)} \qquad \bar{\gamma}(\tau) \in [0,1],$$

$$\text{(A2)} \qquad \bar{h}(\tau) \le \tau\bar{\gamma}(\tau),$$

$$\text{(A3)} \qquad \int_0^T \bar{h}(\tau)\bar{g}(\tau)d\tau \ge K,$$

$$\text{(A4)} \qquad \tau\bar{\gamma}(\tau) - \bar{h}(\tau) \ge \bar{\gamma}(s)\tau - \bar{h}(s) \ \forall s,t \in T,$$

where $T = V-\lambda$, $\tau \equiv v - \lambda$ for $v \ge \lambda$, $\bar{h}(\tau) = h(\tau+\lambda) - \lambda$, $\bar{\gamma}(\tau) \equiv \gamma(\tau+\lambda)$, $\bar{g}(\tau)$ is the density corresponding to $\bar{G}(\tau) = [G(\tau+\lambda)-G(\lambda)]/[1-G(\lambda)]$, and $K = Rx - \lambda$. This formulation of the problem differs from the one in the text only in that it shifts the origin by λ. This reflects the fact that given x, $\lambda > 0$ simply induces a number of constant terms to the problem that do not alter the nature of the solution. It is clear that there is no conflict of interest in having $\gamma(v) = 1$ and $h(v) = \lambda \ \forall v \le \lambda$. Thus the only interesting part of the problem is where $v > \lambda$, i.e. $\tau > 0$.

Incentive–compatibility can be incorporated into the problem relatively easily by use of the following Lemma:

Lemma: (A2) and (A4) are satisfied if and only if:

(A5) $\qquad \bar{\gamma}(\tau)$ is non–decreasing in τ

(A6) $\qquad \tau\bar{\gamma}(\tau) - \bar{h}(\tau) = \int_0^{\tau} \bar{\gamma}(s)ds.$

(**Proof of Lemma:** Let $U(\tau,s) = \tau\bar{\gamma}(s) - \bar{h}(s)$. We then have

(A7) $$U(\tau,s) = U(s,s) + (\tau-s)\bar{\gamma}(s).$$

Incentive–compatibility implies $U(\tau,\tau) \geq U(s,s) + (\tau-s)\bar{\gamma}(s)$. This yields

(A8) $$(\tau-s)\bar{\gamma}(s) \leq U(\tau,\tau) - U(s,s) \leq (\tau-s)\bar{\gamma}(\tau),$$

which proves (A5). Noting that (A8) holds for any $\delta = \tau-s > 0$, and (with a slight abuse of notation) letting $U(s) \equiv U(s,s)$, we have

(A9) $$\delta\bar{\gamma}(s) \leq U(s+\delta) - U(s) \leq \delta\bar{\gamma}(s+\delta).$$

This implies $U(\tau) = U(0) + \int_0^{\tau} \bar{\gamma}(s)ds$ by the theory of Riemann integration, and (A2) requires that $U(0) = 0$, which yields (A6). Finally, it is easy to check the converse, that (A5) and (A6) imply (A2) and (A4).

We can use this result to get a more useful expression for the maximand:

$$\begin{aligned}\int_0^{T} \bar{h}(\tau)\bar{g}(\tau)d\tau &= \int_0^{T} [\tau\bar{\gamma}(\tau) - \int_0^{\tau} \bar{\gamma}(s)ds]\bar{g}(\tau)d\tau. \\ &= \int_0^{T} \tau\bar{\gamma}(\tau)\bar{g}(\tau)d\tau - \int_0^{T} \bar{\gamma}(s)\left[\int_s^{\bar{\tau}} \bar{g}(\tau)d\tau\right]ds\end{aligned}$$

which yields

$$\text{(A10)} \qquad \int_0^T \bar{h}(\tau)\bar{g}(\tau)d\tau = \int_0^T \bar{\gamma}(\tau)\Big[\tau\bar{g}(\tau) - [1-\bar{G}(\tau)]\Big]d\tau.$$

(The above derivation makes use of Fubini's Theorem to change the order of integration.) By the same line of reasoning we have

$$\text{(A11)} \qquad \int_0^T [\tau\bar{\gamma}(\tau)-\bar{h}(\tau)]\bar{g}(\tau)d\tau = \int_0^T \bar{\gamma}(\tau)[1-\bar{G}(\tau)]d\tau$$

The problem now becomes:

$$\text{(P2)} \qquad \max_{\bar{\gamma}(\tau)} \int_0^T \bar{\gamma}(\tau)[1-\bar{G}(\tau)]d\tau$$

subject to the conditions (A1) ($\bar{\gamma}(\tau) \in [0,1]\ \forall\tau$), $\bar{\gamma}(\tau)$ non-decreasing, and

$$\text{(A12)} \qquad \int_0^T \bar{\gamma}(\tau)\Big[\tau\bar{g}(\tau) - [1-\bar{G}(\tau)]\Big]d\tau \geq K.$$

We will restrict our attention to functions $\bar{G}$ such that the following "regularity" condition holds:

Regularity Assumption: The function $\varphi(\tau) \equiv [1-\bar{G}(\tau)] - \tau\bar{g}(\tau)$ satisfies (1) $\exists\ \tau < \infty$ such that $\varphi(\tau) < 0$; and (2) $\varphi(\tau) > 0 \Rightarrow \varphi(\tau)/[1-G(\tau)]$ is decreasing in τ.

These conditions have the following interpretation: Consider a debt contract in which the borrower pays $F > 0$ if the project value τ exceeds F, and zero otherwise. Then $\varphi(F)$ is marginal expected revenue, and the above conditions assure that expected revenue as a function of F (which equals $F[1-\bar{G}(F)]$) is concave, has positive slope at the origin, and attains its maximum at some finite value of F. These conditions seem reasonable, and are satisfied for many commonly used distributions (e.g. the uniform and the exponential).[10]

To ensure that the constraint set is non–empty, we also need the following:

Feasibility Assumption: Let $\overline{\tau} \equiv \inf\{\tau | \varphi(\tau) = 0\}$. Then $K \leq \int_{\overline{\tau}}^{T} \Big[\tau\overline{g}(\tau) - [1-\overline{G}(\tau)]\Big] d\tau$.

Let $\overline{\gamma}^*(\tau)$ denote the solution to P2. It is clear that $\forall\tau > \overline{\tau}$, $\overline{\gamma}^*(\tau) = 1$, since both the maximand and the integrand in (A12) are positive and increasing in $\overline{\gamma}$ over this range. Hence we can confine ourselves to a further subset of the problem. Let $K' = \int_{\overline{\tau}}^{T} \Big[\tau\overline{g}(\tau) - [1-\overline{G}(\tau)]\Big] d\tau - K$. The Feasibility Assumption implies that $K' \geq 0$. If $K' = 0$ then the solution must be $\overline{\gamma}^*(\tau) = 0 \ \forall\tau < \overline{\tau}$. The interesting case occurs when $K' > 0$. We can reformulate the maximization problem yet again as follows:

$$\text{(P3)} \qquad \max_{\overline{\gamma}(\tau)} \int_0^{\overline{\tau}} \overline{\gamma}(\tau)[1-\overline{G}(\tau)]d\tau$$

subject again to the conditions (A1) ($\overline{\gamma}(\tau) \in [0,1] \ \forall\tau$), $\overline{\gamma}(\tau)$ non–decreasing, and

$$\text{(A13)} \qquad \int_0^{\overline{\tau}} \overline{\gamma}(\tau)\Big[1 - \overline{G}(\tau) - \tau\overline{g}(\tau)\Big] d\tau \leq K'.$$

The Regularity Assumption ensures that the constraint that $\overline{\gamma}(\tau)$ be non–decreasing is

[10]The second condition is slightly stronger than needed to get the hump–shaped expected revenue curve described in the text, since $\varphi(\tau)$ decreasing would be sufficient (though it is only imposed on the range of τ where φ is positive). The stronger condition is needed only to simplify the solution to the maximization problem.

not binding. It is straightforward to solve (P3) without this constraint: Standard Lagrangian methods ensure that there is a number $\mu > 0$ such that

$$\bar{\gamma}^*(\tau) = \begin{cases} 0 & \varphi(\tau)/[1-G(\tau)] > \mu \\ 1 & \text{otherwise} \end{cases}.$$

The critical value τ^* defined by $\varphi(\tau^*)/[1-G(\tau^*)] = \mu$ must be such that

$$\int_{\tau^*}^{\bar{\tau}} \left[1 - \bar{G}(\tau) - \tau\bar{g}(\tau)\right] d\tau = K'.$$

Finally, this solution for $\bar{\gamma}$ implies that the solution for $\bar{h}$, denoted $\bar{h}^*$, satisfies

$$\bar{h}^*(\tau) = \begin{cases} 0 & \tau < \tau^* \\ \tau^* & \text{otherwise} \end{cases}.$$

Thus we have shown that the optimal contract is a standard debt contract as described in the Proposition. The remaining step of shifting the origin to take account of the positive liquidation value is left to the reader.

Table 1: Notation and Parametric Assumptions

w	firm's endowment of capital
ℓ	bank's "loan" to firm
x	firm's investment in project
y_t	output from project in period t
$v, \hat{v}$	actual and reported discounted value of period 2 output at t=1
R	required rate of return
β	discount factor
λ	liquidation value

Assumptions for Section 2:

w=0 (=> ℓ=x)

y_1 deterministic and the same for all firms

$R = 1/\beta$

x, λ, and R are fixed and exogenous