Indivisible Assets, Equilibrium, and the Value of Intermediary Output

Cooley, Thomas F. and Bruce D. Smith

Working Paper No. 243
September 1990

Indivisible Assets, Equilibrium, and the Value of Intermediary Output

Thomas F. Cooley and Bruce D. Smith

Rochester Center for Economic Research
Working Paper No. 243

September 1990

INDIVISIBLE ASSETS, EQUILIBRIUM, AND THE VALUE OF INTERMEDIARY OUTPUT

Thomas F. Cooley
University of Rochester

and

Bruce D. Smith
Cornell University
and Rochester Center for Economic Research

Working Paper No. 243

September 10, 1990

We have benefitted from conversations with Valerie Bencivenga, Dave Braden, Stan Engerman and Warren Weber, and from the comments of seminar participants at the Federal Reserve Bank of Minneapolis. This research is supported in part by NSF Grant SES-8921346, by the Bradley Policy Research Center and by the John M. Olin Foundation.

ABSTRACT

In this paper we study the behavior of an economy with indivisible primary assets. In particular, we study the consequences of costly intermediation of assets of limited divisilibity for the equilibrium of the economy. When assets are perfectly divisible but there is a fixed cost of participating in asset markets only stationary equilibria exist. With a fixed participation cost and indivisible assets, we show that, if a stationary equilibrium with positive savings exists, there will typically be a continuum of non-stationary equilibria as well.

We state sufficient conditions for intermediaries to form and hold all primary assets directly. These intermediaries allow the (finite) fixed cost of participating in primary asset markets to be borne by a large number of agents, and also allow agents to pool funds and "share" primary assets (in large denominations). If an equilibrium with intermediated assets exists, the set of equilibrium paths qualitatively resembles that for standard homogeneous agent, overlapping generations models. However, for some initial debt levels --levels that would be consistent with equilibrium in the absence of intermediation-- there will be no equilibrium if intermediaries are allowed to form. In particular, if the initial debt level is too high, and if intermediaries raise market returns (as is often argued in the development literature), debt service may "explode" if intermediaries form. Consequently, governments with large initial debt levels may wish to inhibit (or "repress") the formation of intermediaries. That such repression is common in developing countries has been argued by McKinnon (1973) and Shaw (1973) among others.

Finally, the model permits us to define and analyze various measures of the consumer surplus created by the intermediation industry. We show that conventional measures of intermediary output bear no obvious relation to the consumer surplus created by intermediation.

INTRODUCTION

Among the most important outputs of financial intermediaries are liquidity, which is produced by borrowing short and lending long, insurance, which is produced by pooling risks, and divisibility, which is produced by intermediating large denomination assets. While much attention has been devoted to the former two roles, relatively little has been devoted to the latter even though asset indivisibilities are, and have historically been, an important fact of life and even though assets issued in "convenient" denominations are an important product of intermediaries.[1] The dramatic increase over the past two decades in the importance of money market funds and bond funds, which allow agents to purchase shares of assets issued in large minimum denominations, suggests that this form of intermediation is empirically important. There have also been many historical episodes in which currencies, both metallic and paper, were issued primarily in large denominations. Carothers (1930), Hanson (1979, 1980), Rolnick and Weber (1986), and Glassman and Redish (1988) document a number of historical episodes in which coins were of relatively large denomination, could only be divided to a limited extent, and in which shortages of small change caused apparently significant economic disruption.[2] In this paper we study the behavior of an economy with indivisible primary assets. In particular, we study the consequences of costly intermediation of assets of limited divisilibity for the equilibrium of the economy. We also address the issue of how one would measure what the intermediation industry produces when it intermediates indivisible assets.

The indivisibility of commodities - labor for example - has been shown to be important in explaining certain macreconomic phenomena. [Examples include Hansen (1985), Rogerson (1987), Rogerson and Wright (1987), Greenwood and Huffman (1987), and Shell and Wright (1989).] Several authors -- Klein (1973), Farmer (1984), Marimon and Wallace (1987), and Smith (1989) -- have also addressed the problem of the limited divisibility of assets. What is missing from the latter work is an examination of the incentives for intermediaries (mutual funds or banks)[3] to form or the consequences of allowing for their formation. One focus of this paper is on the existence of incentives for the development of costly intermediation, and on the consequences of intermediation for the properties of equilibrium. We also address the issue of why a government might interfere with the development or operation of intermediaries, and how bank notes that are perfectly safe claims to specie can coexist with a specie currency bearing a higher rate of return. Finally, we are able to discuss explicitly how one would measure the value of the services provided by intermediaries, and contrast that to measures of intermediary output that are commonly used.

The model we use to examine these issues is a two-period lived overlapping generations model, in which each generation contains a continuum of identical agents. There is a single non-storable good, and a single primary (outside) asset. This asset can be purchased only in integer multiples of some real quantity $x > 0$. In addition, as in Williamson (1986), there is a fixed cost of participating in the market for this asset. We then consider three situations: (i) all participants in primary asset markets bear a fixed cost, there is no intermediation, and assets are perfectly divisible; (ii) the same circumstances apply except that the asset is indivisible; (iii) intermediaries exist that purchase the primary asset and issue (at a constant marginal cost) perfectly divisible secondary securities. These secondary securities can be thought of as mutual fund shares, or bank notes/bank deposits.

Our findings are as follows. When assets are perfectly divisible but there is a fixed cost of participating in asset markets only stationary equilibria exist. [In contrast, the same model with a zero fixed cost would allow for a continuum of non-stationary equilibria, as in Gale (1973) or Sargent (1987)]. This result is reminiscent of Farmer's (1984) result that indivisibilities can rule out non-stationary equilibria in this class of models. However, with a fixed participation cost <u>and</u> indivisible assets, we show that, if a stationary equilibrium with positive savings exists, there will typically be a continuum of non-stationary equilibria as well. In fact, we show that if a stationary equilibrium with positive savings exists, then one of two situations must obtain: (a) there is a continuum of Pareto ranked stationary equilibria with positive savings; or (b) starting from an initial debt level that coincides with the steady state debt level, there is a continuum of non-stationary equilibrium paths from that point (as well as the steady state equilibrium). Interestingly, all such equilibrium paths (stationary and non-stationary) achieve a constant inflation rate/rate of return in finite time.

We then state sufficient conditions for intermediaries to form and hold all primary assets directly. These intermediaries allow the (finite) fixed cost of participating in primary asset markets to be borne by a large number of agents, and also allow agents to pool funds and "share" primary assets (in large denominations). If an equilibrium with intermediated assets exists, the set of equilibrium paths qualitatively resembles that for standard homogeneous agent, overlapping generations models. However, for some initial debt levels --levels that would be consistent with equilibrium in the absence of intermediation-- there will be no equilibrium if intermediaries are allowed to form. In particular, if the initial debt level is too high, and if intermediaries raise market returns (as is often argued in the development literature), debt service may "explode" if intermediaries form. Consequently, governments with large initial debt levels may wish to inhibit (or "repress") the formation of intermediaries. That such repression is common in developing countries has been argued by McKinnon (1973) and Shaw (1973) among others.

When assets are intermediated at a positive cost, primary assets will bear a higher rate of return than intermediary liabilities. Thus the model explains, for instance, how bank notes that are perfectly safe claims to specie could bear a lower rate of return than specie, a situation that has often been viewed as inconsistent with the class of models at hand. [See, for instance, the discussion in White (1987).]

Finally, the model permits us to define and analyze various measures of the consumer surplus created by the intermediation industry. We think that consumer surplus measures for intermediary services are a natural topic for analysis, since the output of financial intermediaries (as the term is conventionally used) is notoriously difficult to measure. In particular, the exact services provided by intermediaries are often difficult to define, and are typically bundled together (as they are in our model). Moreover, intermediary services are rarely priced directly, making any "price times quantity" calculations impossible. We, on the other hand, define intermediary services to be the creation of consumer surplus, which can be measured in a conventional way. This is consistent with the view that the measurement of economic welfare is the ultimate goal of national income accounting. Simon Kuznets argued that:

> "It is not only permissible but necessary to view national income measures as approximations to economic welfare, since they are, by definition, appraisals of the yield of the country's economy from the standpoint of the wants of its ultimate consumers." (1953, p. 193)

In practice, measures of intermediary output are generally based on some combination of cost data (value of inputs) and the quantity of intermediary assets or liabilities, which are identified as outputs. [See for instance, Berger and Humphrey (1990) or Fixler and Zieschang (1990).] However, we show that the consumer surplus created by intermediaries will, under several definitions and over some range of values, be inversely related both to intermediary costs and to the quantity of intermediary assets or liabilities. This suggests to us that the intermediation of more assets does not imply the creation of more services, and that it is therefore most appropriate to think of measuring the consumer surplus created by the intermediary sector. We henceforth equate the "value of intermediary output" with the value (measured in units of consumption) of consumer surplus generated by intermediaries.[4]

The remainder of the paper proceeds as follows: Section I considers the economy with divisible and indivisible primary assets and a fixed cost of market participation in the absence of intermediation. Section II considers the set of equilibria when intermediaries exist. The incentives for intermediaries to form and persist are discussed in Section III. Section IV develops some measures of intermediary output. Section V contains some concluding remarks, and comments on some possible extensions.

I. THE MODEL: NO INTERMEDIATION

The economy consists of an infinite sequence of two-period lived overlapping generations, and an initial old generation. Except for the initial old, each generation is identical in size, and contains a continuum of identical agents of measure one. The initial old generation also has measure one. Time is obviously discrete, and indexed by $t=1,2....$

At each date there is a single non-storable consumption good. Young agents are endowed with a quantity $w_1 > 0$ of the good when young, and with $w_2 > 0$ when old. Letting c_j denote age j consumption ($j=1,2$), these agents have preferences described by the additively separable utility function $u(c_1) + v(c_2)$. The functions u and v are assumed to be strictly increasing, concave, and twice-continuously differentiable. In addition, we assume that $\forall\ c \in R_+$ (the consumption set)

(A.1) $\quad 0 > cv''(c)/v'(c) \geq -1.$

There is a single asset that agents in this economy can hold. We assume that the asset comes in indivisible units with a real value of x, so agents can only hold integer multiples of x.[5] Throughout we let $n=0,1,...$ denote an integer. Also, we assume that $x < w_1$. The indivisibility of this asset has various possible interpretations. One is that the asset is a treasury liability issued in a minimum denomination. Another is that the asset is a specie currency (coins); but where specie is in fixed total supply and has no alternative uses.[6] Finally, we assume that there is a fixed cost $\phi > 0$ associated with acquiring the asset. This cost could be interpreted as a cost of participating in a T-bill market if the asset is a treasury liability, or the cost of a scale (to verify that coins are of full weight and not counterfeited) if the asset is specie. ϕ is assumed to satisfy $\phi < w_1 - x$ and

$$\text{(A.2)} \qquad u'(w_1 - \phi) < v'(w_2)$$

is assumed to hold.

A. Divisible Assets

As a benchmark, we begin by considering the case where assets are held directly and are perfectly divisible, but we retain the assumption of a positive fixed cost. Here the asset can be interpreted as fiat money, but young agents bear a cost of bringing goods to market to be exchanged for currency. (Of course, the other interpretations of the asset mentioned above can be retained as well.)

Let r_t denote the gross real return on the asset at t; let B_t be the real per capita quantity of the asset outstanding; and let

$$f(r_t) = \operatorname{argmax}[u(w_1 - \phi - s) + v(w_2 + r_t s)].$$

Then f is a standard savings function. Assumption (A.1) implies that $f'(r_t) > 0$, and (A.2) implies that $f(1) > 0$. Furthermore, define $H(w_1 - \phi, w_2, r_t)$ by

$$H(w_1 - \phi, w_2, r_t) \equiv u[w_1 - \phi - f(r_t)] + v[w_2 + r_t f(r_t)],$$

so that H is the indirect utility function. We assume that

(A.3) $$H(w_1 - \phi, w_2, 1) > u(w_1) + v(w_2).$$

Also, define $\tilde{r}$ by

$$H(w_1 - \phi, w_2, \tilde{r}) \equiv u(w_1) + v(w_2).$$

To insure consistency, we henceforth assume that $f^{-1}(x)$ exists, and that

(A.4) $$\tilde{r} \leq f^{-1}(x).$$

Then from (A.3) and (A.4), clearly

$$\min[1, f^{-1}(x)] \geq \tilde{r} > f^{-1}(0) = u'(w_1 - \phi)/v'(w_2).$$

An equilibrium for this economy consists of non-negative sequences $\{r_t\}$ and $\{B_t\}$ such that

(1) $$B_t = f(r_t) \qquad \text{(market clearing)}$$

(2) $$B_{t+1} = r_t B_t \quad \forall t,$$

and $r_t \geq \tilde{r}$ if $B_t > 0$. Equation (2) can be interpreted as the government budget constraint,[7] and

$r_t \geq \tilde{r}$ must hold for agents to voluntarily exchange goods for assets.

This economy has two stationary equilibria: (i) $B_t = f(1)$ $\forall t$, and (ii) $B_t = 0$, $\tilde{r} \geq r_t \geq f^{-1}(0)$ $\forall t$. In contrast to standard overlapping generations models, however, it has no non-stationary equilibria. In particular, any non-stationary equilibria have $f(1) > B_1 > B_t > 0$; $t=2,...,T$, and for such t, $r_t \geq \tilde{r}$. However, from (2), for some period T+1, $B_{T+1} > 0$ and $r_{T+1} = f^{-1}(B_{T+1}) < \tilde{r}$. But then $B_{T+1} = 0$, yielding a contradiction. One can see this from Figure 1. In this setting the fixed cost of trade precludes the existence of non-stationary equilibria, just as asset indivisibilities do in Farmer (1984). Here, however, this conclusion depends critically on the asset being divisible.

B. Indivisible Assets

We now require assets to be purchased in integer multiples of x.[8] We let n^* be the smallest integer such that $(n^* + 1)x > (w_1 - \phi)$ and begin by considering the optimal savings behavior of a young agent. We begin with the following lemma.

Lemma 1. Suppose at the interest rate r_t', agents (weakly) prefer saving n_2x to n_1x where n_1 and n_2 are integers. If $n^* > n_2 > n_1$, then n_2x is (strictly) preferred to n_1x $\forall$ $r_t > r_t'$. Similarly, if $n_2 < n_1 < n^*$, n_2x is preferred to n_1x $\forall$ $r_t < r_t'$.

Proof. By assumption, if $n_2, n_1 \geq 1$, then

$$u(w_1 - \phi - n_2x) + v(w_2 + r_t'n_2x) \geq u(w_1 - \phi - n_1x) + v(w_2 + r_t'n_1x).$$

Now define $\Psi(n_1,n_2,r)$ by

$$\Psi(n_1,n_2,r) = u(w_1 - \phi - n_2x) + v(w_2 + rn_2x) - u(w_1 - \phi - n_1x) - v(w_2 + rn_1x).$$

Then

$$\Psi_3 = v'(w_2 + rn_2x)n_2x - v'(w_2 + rn_1x)n_1x$$

and $(n_2 - n_1)\,\Psi_3 > 0$ where the inequality follows from (A.1). (In particular $v'(w_2 + z)z$ is an increasing function of z.) This establishes the result for n_2, $n_1 \geq 1$. If $n_2 > n_1 = 0$, then

$$u(w_1 - \phi - n_2x) + v(w_2 + r_t' n_2x) \geq u(w_1) + v(w_2),$$

and the result is obvious.

For $n < n^*$, we now define r(n) to be the interest rate that makes young agents indifferent between saving nx and (n-1)x. r(n) may or may not exist. If it exists, it is defined by

$$\text{(3)} \qquad u(w_1 - \phi - nx) + v[w_2 + r(n)nx] \equiv u[w_1 - \phi - (n-1)x] + v[w_2 + r(n)(n-1)x]$$

for $n > 1$, and by

$$\text{(4)} \qquad u(w_1 - \phi - x) + v[w_2 + r(1)x] \equiv u(w_1) + v(w_2)$$

for $n = 1$. Clearly r(1) exists [by (A.4)] and is unique. For $n > 1$, r(n) may or may not exist. However, by lemma 1, if a value r(n) exists it is unique. A sufficient condition for r(n) to exist is now stated.

Lemma 2. Suppose that $f^{-1}(nx)$ exists; $n > 1$. Then $r(n)$ exists and satisfies

$f^{-1}[(n-1)x] < r(n) < f^{-1}(nx)$.

Proof. By definition,

$$u(w_1 - \phi - nx) + v[w_2 + f^{-1}(nx)nx] > u[w_1 - \phi - (n-1)x] + v[w_2 + f^{-1}(nx)(n-1)x].$$

Similarly,

$$u(w_1 - \phi - nx) + v[w_2 + f^{-1}[(n-1)x]nx] < u[w_1 - \phi - (n-1)x] + v[w_2 + f^{-1}[(n-1)x](n-1)x].$$

Then by the intermediate value theorem there exists a value $r(n) \in (f^{-1}[(n-1)x], f^{-1}(nx))$ satisfying (3).

We next establish that, for all n such that r(n) exists, r(n) is increasing in n.

Lemma 3. If $r(n)$ exists, then $r(n) > r(n-1)$.

Proof. It is easy to show that if $r(n)$ exists then $f^{-1}[(n-1)x]$ also exists. If $n=2$, then by (A.4) and the definition of $r(1)$, $r(1) \leq f^{-1}(x)$. By lemmas 1 and 2, $r(2) > f^{-1}(x)$. Thus $r(2) > r(1)$. For $n > 2$ we have

(5) $\quad r(n-1) < f^{-1}[(n-1)x] < r(n)$,

by lemmas 1 and 2. This establishes the result.

The last step in characterizing optimal savings behavior is to show that if $r_t = r(n)$, $n > 1$, the savings levels $(n-1)x$ and nx are preferred to any other positive savings levels.

Lemma 4. If $r_t = r(n)$, $(n-1)x$ and nx are preferred to any other positive asset choice.

Proof. (Case 1: $n>1$) Suppose that $\hat{n}x$, with $\hat{n} \neq (n-1), n$, is a (weakly) preferred choice, and suppose $\hat{n} > n$. Then $u(w_1 - \phi - \hat{n}x) + v[w_2 + r(n)\hat{n}x] \geq u[w_1 - \phi - (n-1)x] + v[w_2 + r(n)(n-1)x]$. Moreover, there exists a value $\lambda \in (0,1)$ such that $\lambda\hat{n} + (1-\lambda)(n-1) = n$. Therefore $u[w_1 - \phi - nx] + v[w_2 + r(n)nx] > u[w_1 - \phi - (n-1)x] + v[w_2 + r(n)(n-1)x]$ (since v is strictly concave). But this contradicts the definition of r(n). A similar contradiction is derived if $0 < \hat{n} < n-1$.

(Case 2: $n=1$) Now suppose $\hat{n}x$ is a preferred asset choice. If $\hat{n} > 2$, then 2x is also a preferred asset choice, since $2 = \lambda\hat{n} + (1-\lambda)$ for some $\lambda \in (0,1)$. Thus

(6) $u(w_1 - \phi - 2x) + v[w_2 + r(1)2x] \geq u(w_1 - \phi - x) + v[w_2 + r(1)x]$.

But (6) implies that $r(2) \leq r(1)$, contradicting lemma 3.

Note that the lemmas imply that, if $r_t = r(n)$, then nx and (n-1)x are the only optimal asset choices (including the choice of zero assets).

The previous results imply that optimal savings behavior is as follows. For $r_t < r(1)$, it is optimal not to save. If $r_t = r(1)$, savings of zero or x are optimal. For $n > 1$ such that r(n) exists, if $r_t = r(n)$, (n-1)x and nx are optimal savings choices, while for $r_t \in (r(n-1),r(n))$, (n-1)x is the unique optimal asset choice. Finally, if r(n-1) exists and r(n) does not, then $\forall\, r_t > r(n-1)$, lemmas 1, 3, and 4 imply that (n-1)x is the optimal asset choice for young agents.

Equilibrium

Suppose that the per capita asset supply, B_t, equals $\hat{n}x$ for some integer $\hat{n}$ $(0<\hat{n}<n^*)$ such that $r(\hat{n})$ exists. Then market clearing will occur if $r_t \in [r(\hat{n}),r(\hat{n}+1)]$, since $\hat{n}x$ will be an optimal asset choice for all agents. However, suppose B_t is not an integer multiple of x. Then clearly not all agents can hold the same portfolio in equilibrium. Define $n(B_t)$ to be the smallest

integer satisfying $n(B_t)x \geq B_t > [n(B_t) - 1]x$. Then if $r[n(B_t)]$ exists and $r_t = r[n(B_t)]$, $n(B_t)x$ and $[n(B_t) - 1]x$ will be optimal asset choices. Let μ_t denote the fraction of young agents saving $n(B_t)x$. Market clearing will require that $r_t = r[n(B_t)]$ and

$$\mu_t n(B_t)x + (1-\mu_t)[n(B_t) - 1]x = B_t. \tag{7}$$

Moreover, this is clearly the only way in which the asset market can clear. Then an equilibrium is a set of non-negative sequences $\{r_t\}$, $\{B_t\}$, and $\{\mu_t\}$ such that

$$\begin{aligned} &r_t = r[n(B_t)] \quad \text{if} \quad n(B_t)x > B_t \\ &r_t \in [r[n(B_t)], r[n(B_t) + 1]] \quad \text{if } n(B_t)x = B_t. \end{aligned} \tag{8}$$

$$B_{t+1} = r_t B_t, \tag{9}$$

and such that (7) holds with $\mu_t \in [0,1]$.

There are three possible configurations with respect to equilibrium time paths for this economy.

<u>Case 1</u>. $r(1) > 1$. Then, by lemma 3, $r(n) > 1$ $\forall n$ such that $r(n)$ exists. This situation is depicted in Figure 2. Clearly the only equilibrium has $B_t = 0$ $\forall t$.

<u>Case 2</u>. $r(1) < 1$; $r(n) \neq 1$ $\forall n$ such that $r(n)$ exists. In this case, which is depicted in Figure 3, there is a stationary equilibrium with $B_t = 0$ $\forall t$ and one with $r_t = 1$ $\forall t$. In addition, there is a continuum of non-stationary equilibria with $\lim B_t = 0$. Thus, the indivisible asset economy

behaves substantially differently from the divisible asset economy. Moreover, this case illustrates that Farmer's (1984) result showing that non-stationary equilibria with $B_t \to 0$ cannot exist if assets are imperfectly divisible depends crucially on each generation being of finite size. Here, once $r_t = r(1) < 1$, $B_t \to 0$ because $\mu_t \to 0$.

Notice that the non-stationary equilibria have at least two properties of interest. First, for all $B_t \in ((n-1)x, x)$, the equilibrium rate of return is constant. Thus when assets have limited divisibility (as in a coinage economy) long periods of stable inflation are possible. (For instance, if $r_T = r(1)$, $r_t = r(1) \;\forall\; t > T$.) This represents a form of price stickiness. However, periods of stability can be interrupted by sharp declines in r_t; as for instance when $B_t > x$ but $B_{t+1} < x$. In a coinage economy these will be episodes of a one time discrete jump in inflation. Second, even with B_1 given, limited divisibility can create indeterminacy of equilibrium. For instance, suppose $B_1 = B^* = x$ in Figure 3. Then the economy could remain at the steady state, or follow the non-stationary path shown. Thus limited divisibility <u>creates</u> indeterminacies here in contrast with Farmer (1984).

<u>A Continuum of Pareto-ranked Stationary Equilibria</u>

We discuss the third case separately. Marimon and Wallace (1987) have shown that costly asset divisibility can result in multiple Pareto ranked stationary equilibria with positive savings. [Under mild regularity conditions normal overlapping generations models with divisible assets have at most one stationary equilibrium with positive savings, as in Gale (1973).] Here suppose $r(n) = 1$ for some n. This case is depicted in Figure 4. Clearly there is a continuum of stationary equilibria with $r_t = r(n) = 1$. All such equilibria yield young agents utility equal to $u(w_1 - \phi - nx) + v[w_2 + r(n)nx]$, which is independent of the stationary level of B_t. However, equilibria with higher B_t yield the initial old higher utility, and hence are Pareto superior. Thus a continuum of

Pareto ranked stationary equilibria with positive savings are possible.

Young Welfare

Under assumptions implying increasing savings functions, standard overlapping generations models have the feature that increases in B_t (that are consistent with equilibrium) raise r_t, and hence the welfare of young savers. Here an analogous result obtains. Define

$$G(n) = u(w_1 - \phi - nx) + v[w_2 + r(n)nx].$$

Then for given B_t, $G[n(B_t)]$ is young welfare at t. (Note that this is well defined only if $B_t < n(B_t)x$.) Then clearly if $n(B'_t) = n(B_t)$, young agents have the same utility at B_t and B'_t. However, we now show

Proposition 1. $G(n + 1) > G(n)$.

Proof. By definition,

$$G(n + 1) = u[w_1 - \phi - (n + 1)x] + v[w_2 + r(n + 1)(n + 1)x] \equiv u(w_1 - \phi - nx) + v[w_2 + r(n + 1)nx] > u(w_1 - \phi - nx) + v[w_2 + r(n)nx] \equiv G(n),$$

where the inequality follows from lemma 3.

An Example

Let $u(c_1) + v(c_2) = c_1 + \ln c_2$. Then from (3), $r(n)$, $n > 1$, is defined by

$$w_1 - \phi - nx + \ln[w_2 + r(n)nx] \equiv w_1 - \phi - (n-1)x + \ln[w_2 + r(n)(n-1)x]. \tag{10}$$

Solving (10) for r(n) gives

$$r(n) = \frac{w_2(e^x - 1)}{nx - (n-1)xe^x} \tag{11}$$

r(n) exists (is positive) $\forall$ n satisfying $n/(n-1) > e^x$. Also, r(1) is given by

$$w_1 - \phi - x + \ln[w_2 + r(1)x] \equiv w_1 + \ln w_2.$$

Solving for r(1) yields

$$r(1) = (e^{\phi+x} - 1)w_2/x.$$

In addition, $f(r) = 1-(w_2/r)$, so that $f^{-1}(x) = w_2/(1-x)$. Then $f^{-1}(x) \geq \tilde{r}$ iff $1 \geq (1-x)e^{\phi+x}$. Of course (A.4) could be replaced with the weaker assumption that $r(1) \leq r(2)$, which would hold here iff $1 \geq e^{\phi+x}(2-e^x)$. Finally, it is apparent when $r(1) < 1$ will hold.

II. THE MODEL: INTERMEDIATION

There are two incentives for the formation of intermediaries in this economy. First, if the fixed cost is a cost of one agent participating in a T-bill market, buying a scale, or taking goods to market, one agent could bear this cost while representing others. The fixed cost could then be born by a large number of agents, thereby rendering it negligible. Second, agents could pool their resources and, in effect, share assets of limited divisibility. This allows agents to achieve convex combinations of integer multiples of x. Consequently, if intermediaries could form at zero cost, they clearly would.

We now suppose that intermediaries can form, but at a cost. The intermediary bears the fixed cost ϕ (but divides this cost among an arbitrarily large number of agents), and in addition, can intermediate z units of assets at a cost of γz where $\gamma > 0$. We assume that intermediaries have no assets of their own (capital). Rather the intermediary sells shares in itself in amount q_t per customer. (Alternatively, q_t could be deposits, or the real value of bank notes issued.) The gross return on these shares is R_t with certainty. Then if an intermediary intermediates the entire per capita debt, B_t, it must sell $q_t = (1+\gamma)B_t$ shares to purchase the debt and cover costs (the per capita fixed cost is zero if the intermediary serves a large number of clients).[9] If there is free entry, zero profits requires that $R_t q_t = r_t B_t$. Therefore

$$(12) \qquad R_t = r_t/(1+\gamma).$$

Behavior of Agents

Individuals now save solely (by assumption) in the form of intermediary shares. We state below the conditions under which this is equilibrium behavior. This saving involves no fixed cost. Define

$$g(R_t) = \text{argmax } [u(w_1 - s) + v(w_2 + R_t s)].$$

Thus, $g(R_t)$ is the demand for intermediary shares; g is continuous, and satisfies [by (A.1) and (A.2)] $g(1) > 0$, $g'(R) > 0$.

Equilibrium

An equilibrium is now a set of sequences $\{r_t\}$, $\{R_t\}$, and $\{B_t\}$ satisfying (12),

$$(13) \qquad B_{t+1} = r_t B_t \,,$$

and

$$(14) \qquad (1+\gamma)B_t = g(R_t).$$

Equation (14) requires that agents purchase enough intermediary shares for the intermediary to acquire the debt and cover its costs. Substituting (12) and (14) into (13) gives the equilibrium law of motion for B_t:

$$(15) \qquad B_{t+1} = (1+\gamma)B_t \, g^{-1}[(1+\gamma)B_t].$$

If $(1+\gamma)g^{-1}(0) \geq 1$, there is a unique equilibrium with $B_t = 0 \ \forall$ t. If $(1+\gamma)g^{-1}(0) < 1$, there are two stationary equilibria and a continuum of non-stationary equilibria, as shown in Figure 5.

In general the relation between the law of motion for the intermediated and unintermediated economy can be almost anything. For instance, consider the example of section I, with the additional condition $w_1 > 1$. $r(1) > 1$ holds iff $(e^{\phi+x} - 1)w_2 > x$ holds, while $r(1) < 1$ in the opposite case. $g^{-1}(0)(1+\gamma) > (<) \ 1$ iff $(1+\gamma)w_2 < (>) \ 1$. Clearly all configurations are possible. Similarly, the steady state debt level with intermediation, which exists if $(1+\gamma)w_2 < 1$, is $(1+\gamma)^{-1} - w_2$. For the intermediated economy $B_t = x$ can be selected as the steady state equilibrium, with x

$\gtrless (1+\gamma)^{-1} - w_2$ by appropriate choice of parameters. Thus intermediation can raise, lower, or leave unchanged the steady state equilibrium savings level.

The Necessity of Legal Restrictions

As pointed out in the introduction, governments (especially in developing countries) often severely repress the formation of intermediaries.[10] The foregoing discussion suggests a reason why this might be the case. Figure 6 depicts the equilibrium laws of motion for B_t for the intermediated and unintermediated economies of the example under the assumptions that $x = w_2 = 1/2$, $\gamma = .1$, $\phi = \ln(1.9) - x$, and $w_1 = 2$. As is clear, if $B_1 \in (.409, .5]$, the unintermediated economy has an equilibrium. However, for this initial debt level, the intermediated economy would have to have $r_t > 1 \; \forall \; t$, so that interest obligations on the debt would "blow up". In this case, as the development literature often argues, intermediation raises equilibrium returns for a given debt level. Here, returns are raised sufficiently for debt service to blow up. Thus a government with a sufficiently large initial debt will need to prevent intermediaries from forming until $B_t \leq .409$. Note in particular that governments with large debt levels would be the ones that are motivated to repress intermediaries for this reason.[11]

III. THE INCENTIVES FOR THE FORMATION OF INTERMEDIARIES

Thus far the discussion has proceeded under the assumption that intermediaries are either present or absent. We now discuss the incentives for intermediaries to form (and be preserved) in this context.

Consider first the situation when intermediaries are initially absent. Then if the equilibrium asset supply at t is $B_t > 0$, $r_t = r[n(B_t)]$ if $n(B_t)x > B_t$, and $r_t \in [r[n(B_t)], r[n(B_t) + 1]]$ otherwise. In either event, an intermediary could form, divide the fixed cost among a large number of agents, and allow agents to "share" the primary asset. At the prevailing market interest rate r_t (which agents take as given when deciding to form an intermediary), the agents who belong to the intermediary obtain utility equal to $H[w_1,w_2,r_t/(1+\gamma)] \equiv u[w_1 - g(R_t)] + v[w_2 + R_t g(R_t)]$, where $R_t = r_t/(1+\gamma)$.

For the prevailing interest rate r_t, the utility of agents who hold the primary asset directly (and hence bear the fixed cost ϕ) cannot exceed $H(w_1 - \phi, w_2, r_t)$. Thus if $B_t > 0$, there is an incentive to form an intermediary if (but not necessarily only if)

$$\text{(16)} \qquad H[w_1,w_2,r_t/(1+\gamma)] > H(w_1 - \phi,w_2,r_t)$$

for all relevant values of r_t. Since $B_t > 0$, $r_t \in [r(1), 1]$. Then if (16) holds $\forall\ r_t \in [r(1), 1]$, there is always an incentive for the formation of an intermediary. Note that under the same condition, agents will view it as optimal to hold only intermediary shares (i.e., none of the indivisible asset will be held directly). Finally, since $H_3 > 0$, for any $\phi > 0$ there exists a positive value $\tilde{\gamma}$ below which (16) will hold $\forall\ r_t \in [r(1),1]$.

Now suppose that an economy is on an intermediated equilibrium path, as in section II. We ask whether, at the prevailing market rate r_t, agents have any incentives to hold indivisible assets directly. If they do so they bear the fixed cost ϕ, and have utility not exceeding $H(w_1 - \phi,w_2,r_t)$. As holders of (only) intermediary shares, their utility is $H[w_1,w_2,r_t/(1+\gamma)]$. Therefore if (16) holds for all relevant r_t, no agent will have an incentive to hold the indivisible asset directly. It only remains, then, to describe "relevant" values of r_t.

Clearly if $r_t \leq r(1)$, then an agent cannot obtain utility exceeding $u(w_1) + v(w_2)$ by holding the indivisible asset directly. Therefore, if (16) holds $\forall\ r_t \in [r(1), 1]$, no agent will have an incentive to purchase unintermediated primary assets. Of course $r_t \leq r(1)$ can hold only if

$(1+\gamma)g^{-1}(0) \leq r(1)$. Then "relevant" values of r_t lie in the interval $(\max[r(1), (1+\gamma)g^{-1}(0)],1]$.

This discussion raises the question of whether (16) might be satisfied for some relevant values of r_t and be violated for others. In this instance intermediaries might exist during certain time intervals (depending on B_1) and not during others. There might also be existence issues if for a given r_t (16) held, say giving intermediaries an incentive to form, but if their formation resulted in an equilibrium value r_t' that violated (16). At this point we merely raise these as questions and henceforth assume that (16) holds for all relevant r_t.[12]

__An Example__

For the example economy of section I, (16) is

$$\phi > \ln(1+\gamma) - (\gamma w_2/r_t).$$

This condition is satisfied for all relevant r_t by the numerical example of section II.

IV. THE VALUE OF INTERMEDIARY OUTPUT

We define the value of the output of a firm or industry to be the amount agents are willing to pay, at prevailing market prices, for the goods or services it produces. Here the service offered by intermediaries is that they allow agents not to have to hold large assets of limited divisibility directly. Accordingly we ask how much agents are willing to pay to avoid having to hold the primary asset directly [under the assumption, of course, that (16) holds]. We develop three measures of intermediary output; one using prices that prevail absent intermediaries, one using prices that prevail in the presence of intermediaries, and one that allows for the fact that the presence of intermediaries changes market rates of return.

A. The "No-Intermediary" Benchmark

Suppose at date t there is an unintermediated equilibrium with $B_t > 0$. Then the utility of young agents at t in this equilibrium is $u[w_1 - \phi - n(B_t)x] + v[w_2 + r_t n(B_t)x]$, where r_t is the date t equilibrium interest rate. We define date t intermediary output (per customer), y_t, to be the amount agents would be willing to pay not to have to hold primary assets directly. Then y_t is given by

$$(17) \qquad H[w_1 - y_t, w_2, r_t/(1+\gamma)] \equiv u[w_1 - \phi - n(B_t)x] + v[w_2 + r_t n(B_t)x],$$

which gives y_t as a function of B_t whenever r_t is a function of B_t. (This occurs whenever $B_t \neq n(B_t)x$.) We henceforth ignore the (finite) set of values B_t such that B_t is an integer multiple of x (with the integer less than n*). For the remaining values B_t, (17) becomes

$$(17') \qquad H[w_1 - y(B_t), w_2, r[n(B_t)]/(1+\gamma)] \equiv G[n(B_t)].$$

Apparently if $n(B_t) = n(B_t')$, $y(B_t) = y(B_t')$. Thus intermediary services provided are constant $\forall\, B_t \in ([n(B_t) - 1]x, n(B_t)x)$, that is, are independent of the quantity of assets intermediated. Moreover, in practice intermediary output is often measured using costs of operation. Here per capita operating costs are γB_t, which are increasing in B_t. Thus over time intermediary costs can change while intermediary service provision remains constant.

It is also common to measure intermediary output per unit of assets or liabilities. Measured in this way intermediary output is $y(B_t)/B_t$, which is decreasing in B_t $\forall\, B_t \in ([n(B_t) - 1]x, n(B_t)x)$. Costs per unit intermediated are, of course, γ, so again costs will not accurately reflect output movements over time.

B. The Intermediation Benchmark

Now suppose that an intermediated equilibrium exists with $B_t > 0$ and with equilibrium return $r_t = (1+\gamma)g^{-1}[(1+\gamma)B_t]$ at t. We ask how much agents would be willing to pay, at this interest rate, not to have to hold indivisible assets directly. To answer this question, note that if $r_t \in [r(n), r(n+1)]$, agents holding the indivisible asset directly would optimally purchase nx units. Then agents would be willing to pay $y_t = y(B_t)$ not to have to hold the indivisible asset directly, where $y(B_t)$ is defined by

$$H\left(w_1 - y(B_t), w_2, g^{-1}[(1+\gamma)B_t]\right) \equiv u(w_1 - \phi - nx) + v\left(w_2 + (1+\gamma)g^{-1}[(1+\gamma)B_t]nx\right) \tag{18}$$

$\forall$ B_t such that $(1+\gamma)g^{-1}[(1+\gamma)B_t] \in [r(n), r(n+1)]$. Note that $y'(B_t)$ is well defined whenever $(1+\gamma)g^{-1}[(1+\gamma)B_t] \neq r(n)$ for some integer n.

In general the sign of $y'(B_t)$ (when y' exists) is ambiguous. However, it can easily occur that $y'(B_t) < 0$ holds, so that intermediary services are inversely related (over some interval) to both the quantity of assets intermediated, and to intermediary costs (γB_t). A sufficient condition for $y'(B_t) < 0$ is now given.

Proposition 2. Let $(1+\gamma)g^{-1}[(1+\gamma)B_t] \in (r(n), r(n+1))$ for some n. Then $y'(B_t) < 0$ iff $n \geq n(B_t)$ (with strict inequality if $B_t = n(B_t)x$). $n \geq n(B_t)$ if $r_t = (1+\gamma)g^{-1}[(1+\gamma)B_t] \geq r[n(B_t)]$, so $y'(B_t) < 0$ if this holds and $B_t \neq n(B_t)x$.

Proof. For B_t as stated, $y'(B_t)$ exists and satisfies

$$-H_1(-)y'(B_t)+H_3(-)(1+\gamma)(g^{-1})'-(1+\gamma)^2 v'\left(w_2+(1+\gamma)g^{-1}[(1+\gamma)B_t]nx\right)nx(g^{-1})',$$

where $H_3(-)$ is given by

$$H_3(-)=v'\left[w_2+\left(\frac{r_t}{1+\gamma}\right)g\left(\frac{r_t}{1+\gamma}\right)\right]g\left(\frac{r_t}{1+\gamma}\right).$$

Then, since $H_1 > 0$, $y'(B_t) < 0$ iff

$$(19)\quad v'(w_2+r_t nx)(1+\gamma)nx > v'\left[w_2+\left(\frac{r_t}{1+\gamma}\right)g\left(\frac{r_t}{1+\gamma}\right)\right]g\left(\frac{r_t}{1+\gamma}\right).$$

But, (A.1) implies that $v'(w_2 + z)z$ is increasing in z, so that (19) is equivalent to $(1+\gamma)nx > g[r_t/(1+\gamma)]$, or to

$$(19')\quad (1+\gamma)g^{-1}[(1+\gamma)nx] > r_t = (1+\gamma)g^{-1}[(1+\gamma)B_t].$$

Of course (19′) reduces to $nx > B_t$, or equivalently $n \geq n(B_t)$ (with strict inequality if $B_t = n(B_t)x$).

Furthermore, suppose that $r_t \geq r[n(B_t)]$. Then $n \geq n(B_t)$, which implies that (19′) holds since $B_t \neq n(B_t)x$.

Parenthetically, the economy depicted in Figure 6 satisfies $r_t = (1+\gamma)\, g^{-1}\, [(1+\gamma)\, B_t] > r[n(B_t)]\ \forall\ r_t \leq 1$. Thus economies satisfying this condition for all relevant r_t are easily constructed. All such economies will have intermediary output, as defined by (18), that is inversely related almost everywhere to the quantity of assets intermediated and to intermediary operating costs.

C. A General Equilibrium Measure

In this section a measure of intermediary service provision is constructed that takes account of the fact that intermediation has general equilibrium consequences for rates of return. In particular, per capita intermediary output at t, y_t, is now defined to be the amount that agents are willing to pay not to have to hold divisible assets directly, with any changes in equilibrium rates of return taken into account. Thus if the time t asset supply per capita is B_t, $y_t = y(B_t)$ with $y(B_t)$ defined by

$$(20) \qquad H[w_1 - y(B_t),w_2,g^{-1}[(1+\gamma)B_t]] \equiv G[n(B_t)].$$

It is now demonstrated that $y(B_t)$ cannot be monotone in B_t. In particular, for $B_t \in ((n-1)x, nx)$, $y'(B_t) > 0$ since over this interval $G[n(B_t)]$ is constant. We now show that if $B_t + \epsilon = n(B_t) = B_t' - \epsilon$, then $y(B_t') < y(B_t)$ for $\epsilon > 0$ sufficiently small. To see this, observe that by definition (for ϵ sufficiently small)

$$(21) \qquad H[w_1 - y(B_t'), w_2, g^{-1}[(1 + \gamma)B_t']] - H[w_1 - y(B_t), w_2, g^{-1}[(1+\gamma)B_t]]$$
$$\equiv G[n(B_t) + 1] - G[n(B_t)].$$

As $\epsilon \rightarrow 0$ the right-hand side of (21) is a positive constant (by Proposition 1). If $y(B_t') \geq y(B_t)$ $\forall B_t' > B_t$, then as $\epsilon \rightarrow 0$ the left-hand side of (21) approaches a number bounded above by zero. But this is a contradiction. Thus $y(B_t)$ must be decreasing at certain values of B_t.

Since intermediary costs per capita (γB_t) are monotone in B_t, in certain neighborhoods measured costs will move in the opposite direction from $y(B_t)$. Thus costs do not reliably reflect movements in intermediary services for any of our surplus measures. Furthermore, it has been seen that for at least two of our intermediary output measures, $y(B_t') < y(B_t)$ for some B_t, B_t' with $B_t' > B_t$. Intermediary output can decline while intermediary assets/liabilities increase in real terms. Consequently, changes in intermediary assets or liabilities also need not give any indication of directions (much less magnitudes) of movement in the provision of services by the intermediation industry.

V. CONCLUSIONS

Indivisibilities of assets are an important institutional feature of modern asset markets [Klein (1973)], and perhaps an even more important feature of historical asset markets. Carothers (1930), Hanson (1979, 1980), Rolnick and Weber (1986, 1988), White (1990) and Glassman and Redish (1988) describe a variety of historical episodes in which indivisible assets, indivisible metallic currencies, and small change shortages are central in understanding monetary events, and caused apparent deadweight losses. The preceding sections present a framework for analyzing economies with indivisible assets, and for analyzing the incentives for and equilibrium consequences of the intermediation of such assets.

Not surprisingly, it was seen that asset indivisibilities create considerable scope for indeterminacy.[13] In fact, if $r(1) \leq 1$ there will be either a continuum of stationary equilibria (if $r(n) = 1$ for some n), or indeterminacies even if the initial value B_1 is given. To see the latter point, consult figure 3 and note that even if $B_1 = x$ is given as an initial condition, there is a steady state equilibrium and a continuum of non-stationary equilibria. However, even so, if $B_1 > 0$, the economy will in finite time settle down to a constant inflation rate/rate of return. In particular, either $r_t = 1 \; \forall t$ or $r_t = r(1) \; \forall \; t > T$ will hold. This is of particular interest when we think of our asset as a metallic currency. More specifically, such a currency of limited divisibility (that is in fixed nominal supply) must eventually display great stability of its inflation rate. This is the case even though the indivisibility potentially creates substantial deadweight losses.

It has also been seen that, for appropriate costs associated with asset trading and intermediation, there is always an incentive for intermediaries to form. Intermediated equilibria are qualitatively similar to standard equilibria in homogeneous agent, overlapping generations models [as described, for instance, by Gale (1973) or Sargent (1987)]. However, such equilibria have the feature that the primary asset bears a higher return than intermediary liabilities. It is also possible that over some range of values for the initial debt, intermediation raises rates of return on debt sufficiently that the formation of intermediaries will cause debt service to explode. In this case the existence of equilibria may depend on the government inhibiting the formation of intermediaries.

The fact that the primary asset will dominate intermediary liabilities in rate of return is also of particular interest in economies with a specie currency. For instance, during historical periods with free banking (and other periods as well) intermediaries issued bank notes that were claims to specie.[14] To the extent that there were defaults or noteholder losses, bank notes may appear to have been dominated in rate of return by other assets. White (1987) has made this point, and argued that since free banks were (relatively) unregulated, observations of this type constitute evidence against "legal restrictions theories" of money as articulated by Wallace (1983). However, if we interpret our indivisible asset as a metallic currency, banks will emerge that issue notes that are completely backed by specie, and yet that bear lower rates of return than specie. This situation is entirely consistent with the point of view put forth in Wallace (1983).[15] Finally, we have observed that intermediary output - the consumer surplus created by intermediaries - can be negatively related both to costs and to the quantity of assets intermediated.

It remains to comment on some issues that have not been addressed. First, we have not discussed why assets might be issued in indivisible forms. For historical coinage economies the answer is probably technological.[16] For treasury liabilities or other securities the answer is less obvious, although issuing these in indivisible forms may reduce the costs of issue, as argued by Klein (1973), or permit non-linear pricing, as in Guesnerie and Seade (1982) or, more explicitly, in Bryant and Wallace (1984). However this must remain a topic for future investigation.

Second, we have not considered the possibility of altering the denomination structure, as would occur during recoinages for instance. Relatedly, we have not considered issues of deficit finance with assets of limited divisibility. For a fixed denomination structure (for example, where the asset is a treasury liability) deficit finance raises few new issues. However, if deficit finance must involve recoinages new issues are introduced. The possibility of recoinage, with consequent seigniorage income and changes in denomination structure, would be an interesting topic for further investigation.

REFERENCES

Berger, Allen N., and David B. Humphrey, "Measurement and Efficiency Issues in Banking," manuscript, 1990.

Bryant, John, and Neil Wallace, "A Price Discrimination Analysis of Monetary Policy", *Review of Economic Studies* (51), 1984, p. 279-88.

Carothers, Neil, *Fractional Money*, Wiley and Sons, New York, 1930.

Farmer, Roger E. A., "Bursting Bubbles: On the Rationality of Hyperinflations in Optimizing Models," *Journal of Monetary Economics* (14), 1984, pp. 29-35.

Fixler, Dennis J., and Kimberly D. Zieschang, "User Costs, Shadow Prices, and the Real Output of Banks," manuscript, 1990.

Gale, David, "Pure Exchange Equilibrium of Dynamic Economic Models," *Journal of Economic Theory* (6), 1973, pp. 12-36.

Glassman, Debra, and Angela Redish, "Currency Depreciation in Early Modern England and France," *Explorations in Economic History* (25), 1988, pp. 75-97.

Greenwood, Jeremy, and Gregory Huffman, "A Dynamic Equilibrium Model of Inflation and Unemployment," *Journal of Monetary Economics* (19), 1987, pp. 203-28.

Guesnerie, Roger, and Jesus Seade, "Nonlinear Pricing in a Finite Economy," *Journal of Public Economics* (17), 1982, pp. 157-79.

Hansen, Gary D., "Indivisible Labor and the Business Cycle", *Journal of Monetary Economics* (16), 1985, p. 309-28.

Hanson, John R. II, "Money in the Colonial American Economy: An Extension," *Economic Inquiry* (17), 1979, pp. 281-86.

Hanson, John R. II, "Small Notes in the American Colonies," *Explorations in Economic History* (17), 1980, pp. 411-20.

Hornstein, Andreas, and Edward C. Prescott, "Measures of the Insurance Sector Output," manuscript, 1990.

Kemmerer, Edwin W., *Seasonal Variations in the Relative Demand for Money and Capital in the United States*, National Monetary Commission, Washington, 1910.

Klein, Michael A., "The Economics of Security Divisibility and Financial Intermediation," *Journal of Finance* (28), 1973, pp. 923-31.

Kuznets, Simon, "National Income and Economic Welfare," in *Economic Change*, W.W. Norton, New York, 1953.

Marimon, Ramon, and Neil Wallace, "Trade Using Assets Divisible at a Cost," *Journal of Economic Theory* (43), 1987, pp. 223-51.

McKinnon, Ronald I., *Money and Capital in Economic Development*, Brookings Institute, Washington, 1973.

Rich, Georg, "Canadian Banks, Gold, and the Crisis of 1907," *Explorations in Economic History* (26), 1989, pp. 135-60.

Rolnick, Arthur J., and Warren E. Weber, "Gresham's Law or Gresham's Fallacy," *Journal of Political Economy* (94), 1986, pp. 185-99.

Rolnick, Arthur J., and Warren E. Weber, "Explaining the Demand for Free Bank Notes," *Journal of Monetary Economics* (21), 1988, pp. 47-71.

Rogerson, Richard, "Indivisible Labor, Lotteries, and Equilibrium", *Journal of Monetary Economics* (21), 1988, p. 3-16.

Rogerson, Richard, and Randall Wright, "Involuntary Unemployment in Economies with Efficient Risk Sharing", *Journal of Monetary Economics* (22), 1988, p. 501-16.

Ross, Victor, *A History of the Canadian Bank of Commerce*, Vol. II, Oxford University Press, Toronto, 1922.

Sargent, Thomas J., *Dynamic Macroeconomics*, Harvard University Press, Cambridge, 1987.

Shaw, Edward S., *Financial Deepening in Economic Development*, Oxford University Press, London, 1973.

Shell, Karl, and Randall Wright, "Contracts, Lotteries, Indivisibilities, and Sunspots", manuscript, 1989.

Smith, Bruce D., "Legal Restrictions, Sunspots, and Cycles," *Journal of Economic Theory* (47), 1989, pp. 369-92.

Wallace, Neil, "A Legal Restrictions Theory of the Demand for Money and the Role of Monetary Policy," Federal Reserve Bank of Minneapolis *Quarterly Review*, 1983, pp. 1-7.

White, Eugene N., "Free Banking, Denominational Restrictions and Liability Insurance," manuscript, Rutgers University, April 1990.

White, Lawrence H., "Accounting for Non-interest-bearing Currency: A Critique of the Legal Restrictions Theory of Money," *Journal of Money, Credit and Banking* (19), 1987, pp. 448-55.

Williamson, Stephen D., "Increasing Returns to Scale in Financial Intermediation and the Non-neutrality of Government Policy," *Review of Economic Studies* (53), 1986, pp. 863-76.

ENDNOTES

1. See Rolnick and Weber (1988) for an argument that many historical banks fulfilled predominantly this latter role.

2. It bears emphasis that "small change shortages" have been observed in economies with paper currencies, and have been observed in relatively modern, developed economies. For instance, in Canada during the early 20th Century, the government enjoyed a monopoly in the issue of notes of less than $5. Non-bank holdings of such notes constituted less than 3% of the aggregate money stock (from 1900-1913). In 1902, the Canadian Bankers Association felt compelled to lobby for an increased issue of small denomination notes. (Parenthetically, small denomination coins were also perceived to be in short supply). These issues are discussed by Ross (1922), p. 220, and by Rich (1989), p. 135-6. For the U.S., Table XIII(a) of Kemmerer (1910) indicates that not all banks were generally able to meet the demand for small bills even at the time of the National Monetary Commission studies.

3. Rolnick and Weber (1988) argue that many historical banks were simply mutual funds that issued circulating liabilities.

4. For another attempt to define (or re-define) intermediary output by redefining intermediary products, see Hornstein and Prescott (1990), who consider output for insurance firms. For an example of attempts to construct "user cost of funds" estimates for banks, and to use these and other costs to estimate intermediary production functions, see Berger and Humphrey (1990).

5. Of course below we allow intermediaries to form. Then the primary asset must be purchased in integer multiples of x, but intermediary liabilities will be assumed to be perfectly divisible.

6. See Marimon and Wallace (1987), who also interpret an asset in fixed supply (paying no dividends and with no alternative uses) and that is divisible only at cost as a specie currency.

7. If the asset is currency issued in a fixed per capita nominal amount M, then $B_t = M/p_t$, where p_t is the time t price level, and $r_t = p_t/p_{t+1}$.

8. Notice that all units purchased earn the return r_t, as would be the case for treasury liabilities or a specie currency.

9. Note that we have abstracted from the possibility that the intermediary sells shares <u>and</u> charges a fee. If agents can buy as many shares as they want at the going rate of return and intermediaries are Nash competitors, it is straightforward to show that in equilibrium the fee charged will be zero.

10. See, e.g., McKinnon (1973) or Shaw (1973).

11. In a coinage economy, the argument just given could be formulated as follows: the government must prevent intermediation until there is a "currency shortage" (i.e., the value of real balances is sufficiently small). Such an argument would appear to account for a number of historical instances.

12. Of course (16) is not a necessary condition for intermediation, so violation of (16) need not indicate that there is no incentive for intermediation to occur.

13. This point is also a theme of Marimon and Wallace (1987) and Smith (1989).

14. See Rolnick and Weber (1988) for an interpretation of free banks as mutual funds - which is, of course, what our intermediaries are.

15. Two points deserve mention. One is that while historically small change often existed, it was also often not in circulation. This point is discussed by Rolnick and Weber (1986) and Glassman and Redish (1988). Second, historically many banks were primarily in the business of intermediating specie. According to Carothers (1930, p. 79-80), after the War of 1812, the per capita supply of outside money in denominations less than 50 cents was less than 25 cents. Carothers argues that many banks existed largely to intermediate specie, issuing notes in denominations of less than 50 cents. Parenthetically, at the time 50 cents was in the neighborhood of a day's per capita income. Finally, while free banks apparently did not intermediate specie, by 1830 many states had prohibited bank note issues in denominations of less than $1. Indeed, Eugene White (1990) documents that a surprisingly large number of states prohibited the issuance of notes in denominations less than $5 and that these restrictions carried over into the free banking era. (Free banking in the U.S. began in 1837). These issues are discussed in some detail by Carothers.

16. See Hanson (1979, 1980), Rolnick-Weber (1986), and Glasman-Redish (1988) on problems dividing coins or inhibiting the creation of small change.

Figure 1
Non-existence of Non-stationary Equilibria

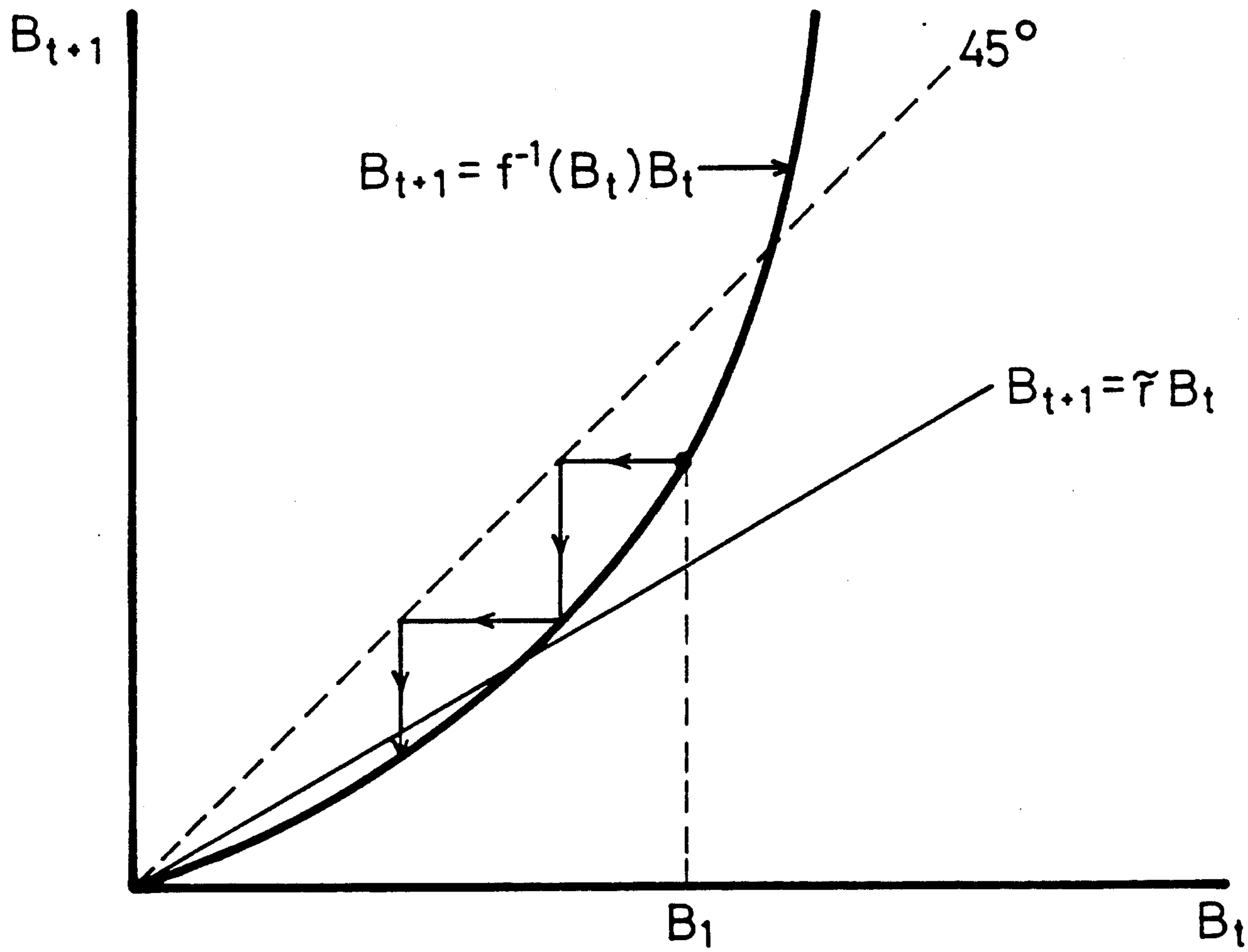

Figure 2

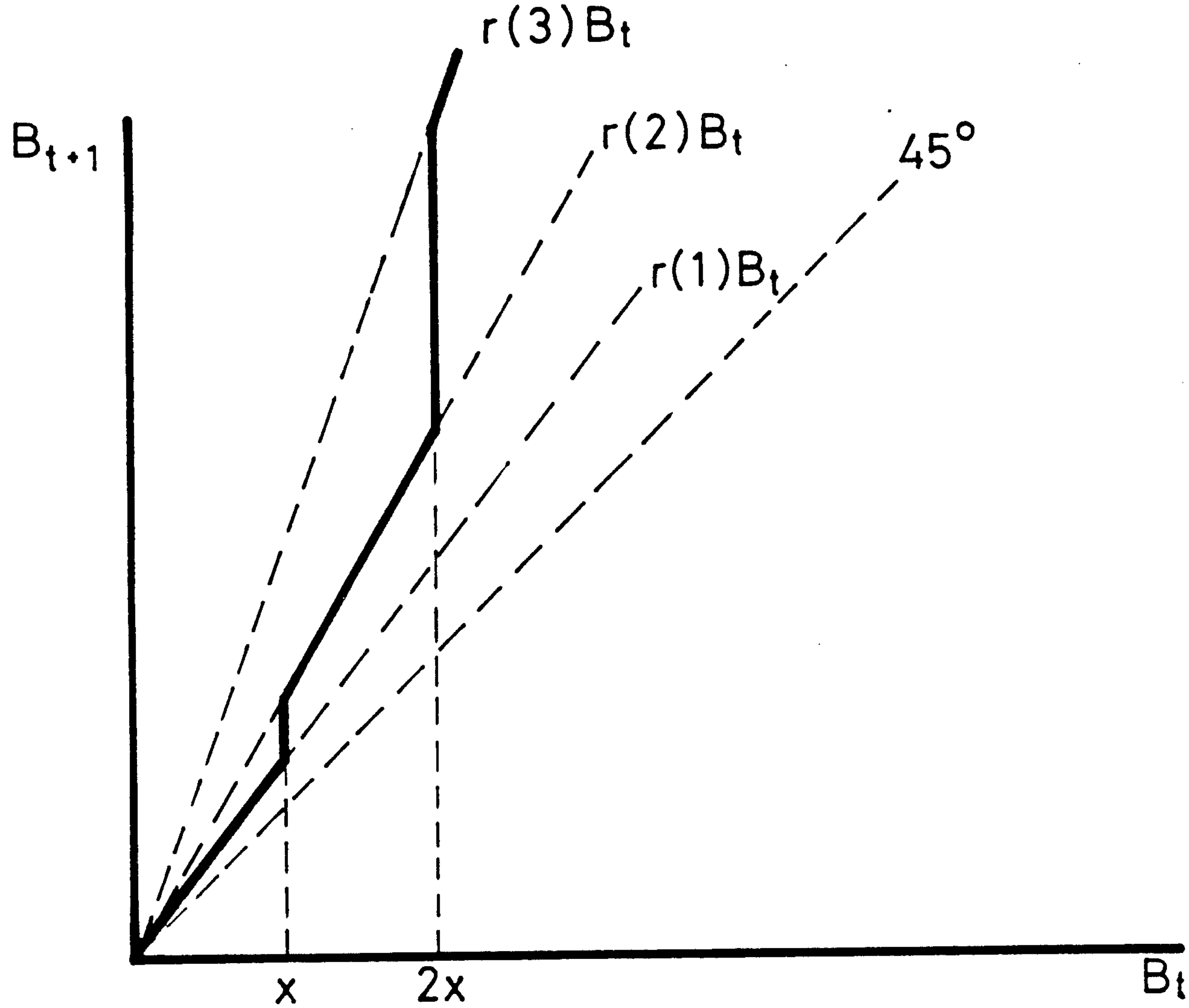

Figure 3

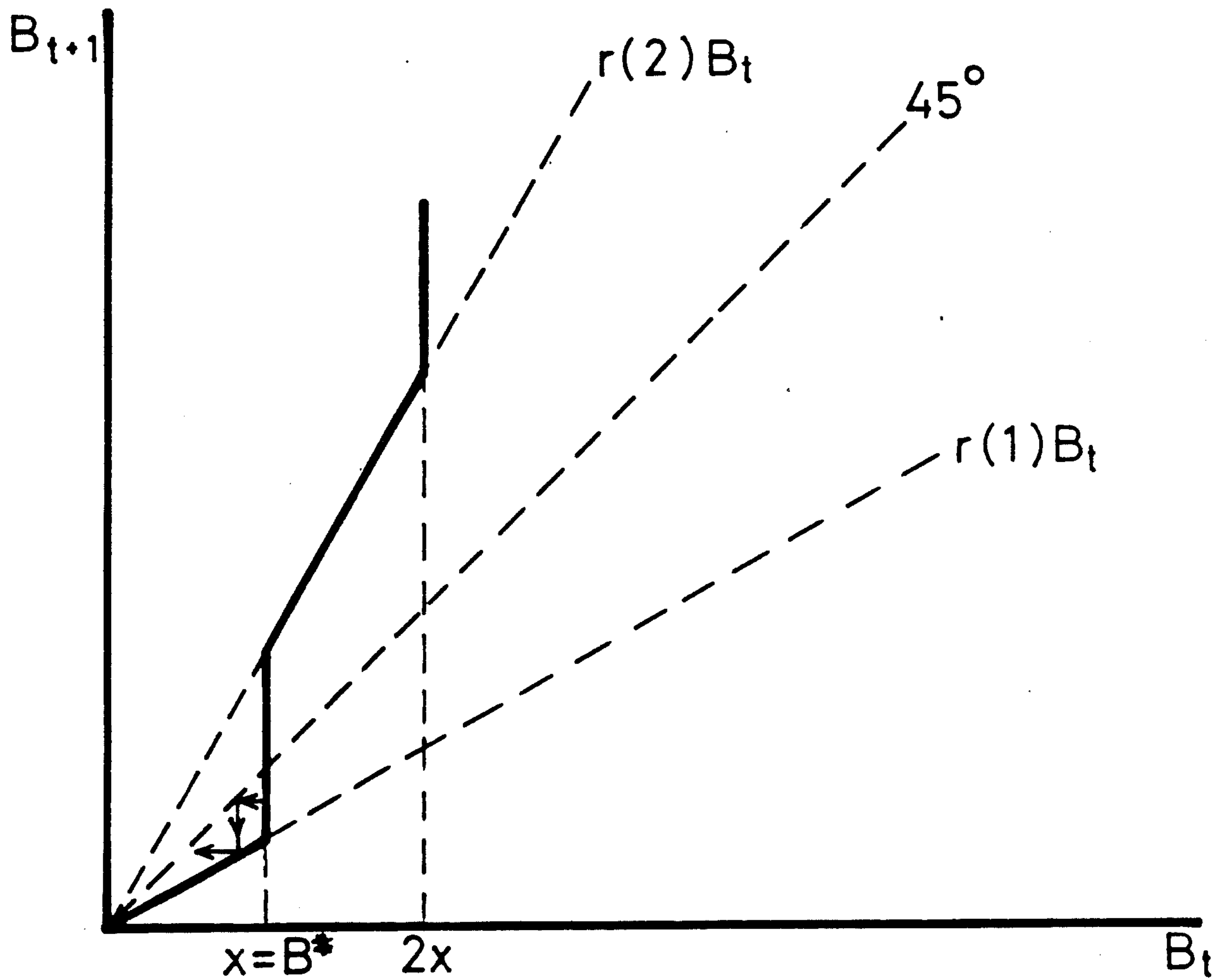

Figure 4

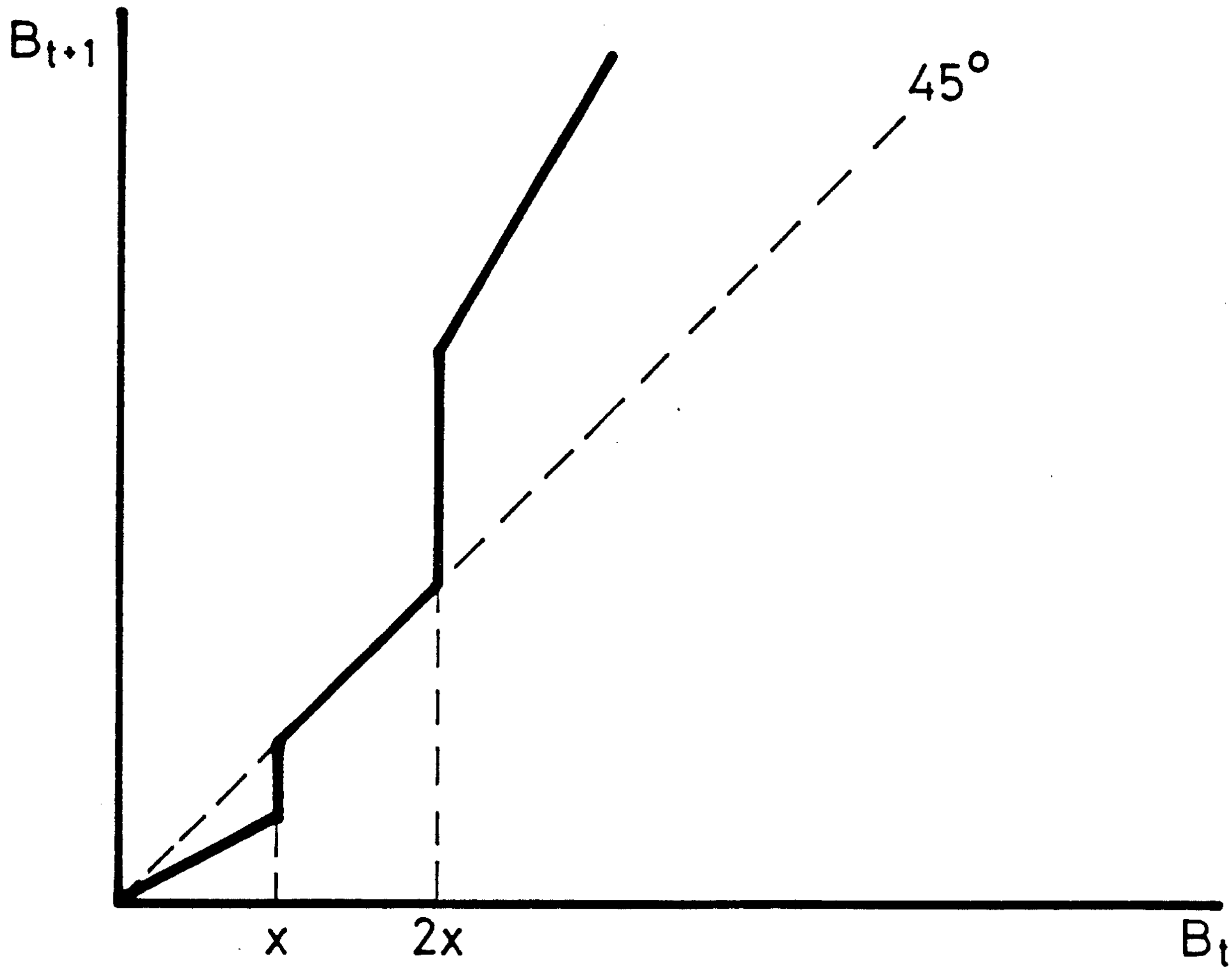

Figure 5

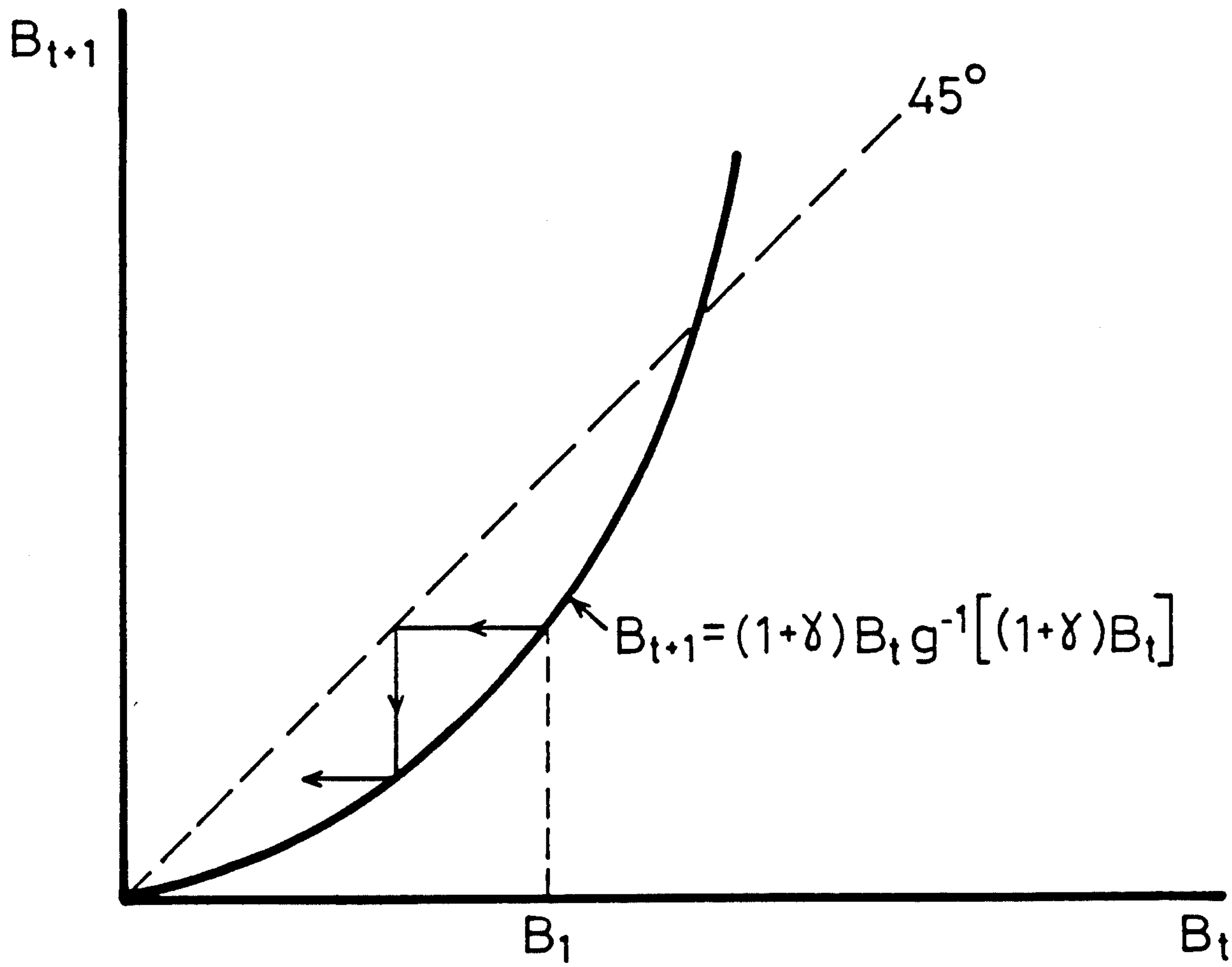

Figure 6

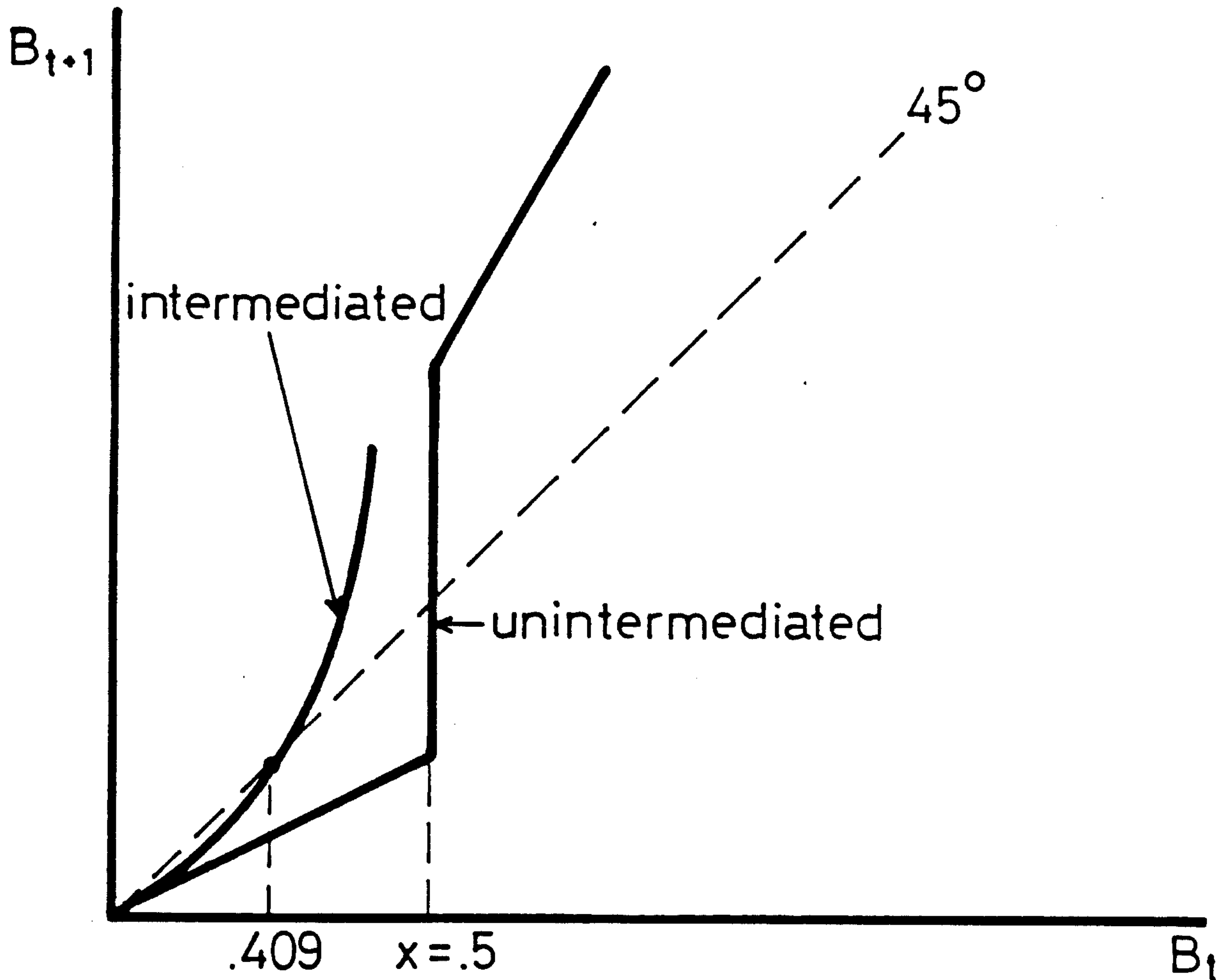

Figure 7