

Productive Externalities and Cyclical Volatility

Baxter, Marianne and Robert G. King

Working Paper No. 245
September 1990

University of
Rochester

Productive Externalities and Cyclical Volatility

Marianne Baxter and Robert King

Rochester Center for Economic Research
Working Paper No. 245

PRODUCTIVE EXTERNALITIES AND CYCLICAL VOLATILITY

Marianne Baxter

and

Robert G. King

University of Rochester

Working Paper No. 245

First Draft: August 1987

Revised: August 1988

September 1990

The authors acknowledge financial support from the National Science Foundation.

Abstract

PRODUCTIVE EXTERNALITIES AND CYCLICAL VOLATILITY

Marianne Baxter
and
Robert G. King

This paper begins with the observation that growth in factor inputs is insufficient to explain growth in output, and explores the empirical plausibility of the hypothesis that this fact is due to the presence of productive externalities and increasing returns to scale. We construct a quantitative equilibrium macroeconomic model which incorporates these features, and allows for demand shocks operating at the level of the consumer. We find that this model generates time series which replicate the basic stylized facts of U.S. business cycles. Based on this analysis, we conclude that the increasing returns/productive externalities model is a promising alternative to the standard constant returns framework with technology shocks as a model of the business cycle.

TABLE OF CONTENTS

1. Introduction	?
2. Increasing Returns: Specification and Estimation	?
3. A Business Cycle Model	?
4. Dynamic Properties	?
simulated time series	
impulse responses	
response to government purchases	
5. Comparing Efficient and Inefficient Equilibria	?
6. A Residual Puzzle	?
7. Conclusions	?
References	?

Appendices:

- A. Analysis of Equilibrium Models with Productive Externalities
- B. Data and Model Appendix

Tables and Figures:

Table 1:	Estimates of the Increasing Returns Parameter
Table 2:	Moments of U.S. Data and RBC Model
Table 3:	Notation and Parameter Choices Used In Our Investigation
Table 4:	Moments of Demand Shock Models: IR and CR
Figure 1:	Output Growth and Input Growth in U.S. Private Industry
	A. Constant Returns Version
	B. Increasing Returns Version
Figure 2:	Output, Consumption, Investment, etc.
Figure 3:	Business Cycles with Persistent Technology Shocks
Figure 4:	Business Cycles with Persistent Demand Shifts & Increasing Returns
Figure 5:	Business Cycles with Persistent Demand Shifts & Constant Returns
Figure 6:	Responses to Demand Shifts
Figure 7:	Responses to Government Purchases
Figure 8:	The IR residual and labor productivity

1. Introduction

Since the work of Solow [1957], economists have recognized that measured growth in factor inputs is insufficient to explain output growth. Figure 1-A plots output growth over the postwar period against growth in total factor input defined as growth in labor and capital weighted by factor income shares. Factor input growth is positively correlated with output growth, but fails to explain it in two important ways. First, the growth rate of total input averaged only 2.45 percent per year over the postwar period, while output grew at an average rate of 3.22 percent. Second, total factor input growth is less volatile than output growth, with a standard deviation of 1.75 percent for inputs, compared with 2.96 percent for output.

This paper explores the empirical plausibility of the hypothesis that the aggregate production function exhibits increasing returns to scale, so that growth in total input leads to greater than one-for-one growth in output.¹ The practical implications of increasing returns for business cycle volatility are shown in Figure 1-B, where we scale total input growth by 1.5 which is the scaling factor used in the increasing returns model presented in this paper. Fluctuations in the growth rate of total output are matched much more closely by this scaled input series than by the unscaled series in Figure 1-A.

To retain the use of competitive analysis in the presence of increasing returns, we utilize a device previously employed by Romer [1986] in studying economic growth and by Murphy, Schleifer and Vishny [1989] in business cycle analysis. Under this "Marshallian externalities" modelling strategy, the production technology is constant returns to scale at the individual level, but the existence of productive externalities means that increasing returns operate at the level of society as a whole.

Most of our analysis centers on the effects of "demand shocks" (represented as exogenous shifts in private individuals' marginal utility schedules) within the increasing returns environment. We also briefly consider the effects of a shift in government purchases. However, our previous analysis of fiscal policy (Baxter and King [1990]) has led us to believe that financing issues are much more important quantitatively than the shifts in government demand *per se*. Thus a serious analysis of fiscal policy issues in an increasing returns environment is beyond the scope of this paper.

The paper is organized as follows. Section 2 discusses estimation of the parameter governing the extent of increasing returns, and reviews the dimensions along which the standard technology-shock-driven model captures central features of business cycles. Section 3 lays out our model with increasing returns and productive externalities, and discusses the method used to solve and simulate this economy. Section 4 is the heart of the paper: in this section we examine the response of our model to persistent demand shocks. We ask whether this model is capable of producing "realistic" business cycles, and contrasts the behavior of the increasing returns model with that of the standard constant returns model. This section also considers the effects of an increase in government purchases, and compares the predictions of the increasing returns model and the constant returns model. Section 5 addresses the question of how to compare competitive equilibria to socially optimal equilibria in models in which these two do not coincide. Section 6 discusses a residual puzzle, namely the high correlation between labor productivity and the component of output unexplained by the increasing returns model. Section 7 concludes with a discussion of the paper's main results, and directions for future research.

2. Increasing Returns: Specification and Estimation

As noted above, growth in inputs is insufficient to explain output growth: productive externalities leading to increasing returns is one potential explanation for this phenomenon. Accordingly, we assume that an individual agent (a representative worker-producer) combines capital (K_t) and labor input (N_t) to produce output according to:

$$(2.1) \quad Y_t = A_t F(K_t, N_t) \underline{Y}_t^\epsilon.$$

In this expression, $F(K_t, N_t)$ is a constant returns to scale production function of the Cobb-Douglas form, $F(K, N) = K^{\theta_K} N^{\theta_N}$; A_t is a total factor productivity shock; and \underline{Y}_t^ϵ is per capita output (\underline{Y}_t) raised to the power ϵ . Thus ϵ controls the magnitude of the external effect. Throughout, we use underbars to denote variables which private individuals view as being outside their control. Thus, as in standard in competitive models, the representative worker-producer is assumed to treat \underline{Y}_t as beyond his control. Yet the actions of all the (identical) agents taken together determine the per capita capital stock \underline{K}_t , labor input \underline{N}_t , and output \underline{Y}_t . Thus, equilibrium output is:

$$(2.2) \quad \underline{Y}_t = [A_t F(\underline{K}_t, \underline{N}_t)]^\eta,$$

where $\eta=1/(1-\epsilon)$ indexes the extent of increasing returns.

Estimating η

An estimate of η can readily be obtained: it is just the slope coefficient in a regression of output growth, $\gamma_{Y,t} = \log(Y_t/Y_{t-1})$, on input growth,

$\gamma_{Z,t} = [\theta_K \log(K_t/K_{t-1}) + \theta_N \log(N_t/N_{t-1})]$. If there are no random variations in technology, then we can estimate η consistently with least squares. Table 1 presents statistics on this regression and others to be discussed below. In this regression equation, the OLS point estimate of η is 1.45, corresponding to a value of ϵ of about 1/3.

If there are technology shocks hitting the production function, then an instrumental variables estimator of η must be constructed. We experimented with some measures of public expenditure as instruments. First, we employed three military spending measures as suggested by the work of Hall [1987], [1988]. With these instruments, we obtain an estimate of η equal to 1.81. However, the poor performance of the first stage regression made us concerned about the precision of this estimate. We therefore explored two other sets of instruments: (i) two measures of defense compensation with an associated estimate of η equal to 1.53, and (ii) total nondefense purchases, implying an estimate of η equal to 1.10. In all three instrumental variables regressions there is only minor explanatory power in the first stage regression.

Caballero and Lyons [1989] estimate aggregate and industry-level equations that are similar to ours, and conclude that there are significant economies of scale that are external from the industry point of view, but internal to the U.S. as a whole. Using our notation, the externality parameter preferred by Caballero and Lyons is $\eta=1.5$. Based on our results and those of Caballero and Lyons, the remainder of the paper (and Figure 1-B) uses a value of η equal to 1.5.

Business Cycles with Technology Shocks

As a standard of comparison for the model to be developed below, it is useful to discuss the dimensions along which the standard real business cycle

model with technology shocks—as expositied by Prescott [1986] and Plosser [1989]—captures central elements of economic fluctuations. One standard for evaluation that one would certainly not choose to employ is the fit of the production function, since technology shocks are computed as a residual. Rather, the focus must be on the variability and comovement of time series, given the behavior of the residual.

Economic fluctuations in the United States over the period 1948–1986 are shown in Figure 2.² The top two panels display some well-known business cycle phenomena: (i) consumption and investment covary strongly with output, (ii) consumption has lower amplitude than output, and (iii) investment has higher amplitude. The bottom two panels display information on the cyclic behavior of the labor market. Labor compensation, which has averaged 58% of GNP over the postwar period, moves very closely with output and has similar amplitude. Labor input is positively correlated with output over the postwar period, although only weakly. But major upward and downward movements in output and labor input occur together.

The cyclical behavior of real wages has long been viewed as a macroeconomic puzzle. One measure of an average real wage may be constructed by dividing labor compensation by labor input. Given the close relationship between labor compensation and output, variations in this measure of the average real wage are essentially the same as variations in output per manhour. In the analysis below we report the models' predictions for marginal real wages, which should be proportional to output per manhour in the model. However, we lack direct empirical measures of marginal real wages.

Table 2 presents statistics summarizing aspects of U.S. data (Panel A) and model-generated time series (Panel B). When driven by highly persistent technology shocks, the standard model predicts that consumption is less

variable than output and that investment is more variable than output, as in the U.S. data. On the other hand, labor input is more highly correlated with output in the model than in the data.

Tables of moments are hard to interpret without a metric for evaluating model performance. Recent work by Watson [1990] is attractive because it provides an evaluation method which recognizes that many useful models are nevertheless so simple that they will be rejected by the data using standard statistical procedures (such as likelihood-ratio tests). We shall return, in Section 7, to a discussion of model evaluation in our context. For the present, however, we supplement the presentation of moments with plots of model-generated (simulated) time series to provide an informal method of determining whether these simulated time series "look like" actual time series. Based on simulations of the standard real business cycle model with technology shocks, as plotted in Figure 3, most economists would agree that the artificial data generated by this model resemble U.S. postwar business cycles.³

In assessing the empirical plausibility of our model, which combines aggregate demand shocks with an increasing returns technology, we use a similar approach. Since our model exhibits increasing returns, it necessarily implies that output is more variable than inputs. Thus, our focus will be on selected moment implications of the model and on salient features of simulated business cycles.

3. A Business Cycle Model

Our investigation departs in two ways from the basic neoclassical macroeconomic model.⁴ We have already discussed altering the production technology to introduce external effects and increasing returns. In

addition, we alter preferences to allow for demand shifts. Although our quantitative analysis allows for growth in population and trends in technology, for simplicity we have transformed the model to eliminate growth to in our presentation below. This transformed model possesses a stable stationary state in the absence of transitory shocks to demand (preferences) or supply (technology), so long as the returns to scale parameter η is not too large. Our estimate of η (discussed below) is small enough so that there is a stable steady state in the model, i.e., the externality is not strong enough to generate endogenous growth as in Romer [1986].

Preferences. Each agent has preferences over consumption and leisure as summarized by (3.1a) and (3.1b):

$$(3.1a) \quad U = \sum_{t=0}^{\infty} \beta^t u(C_t, L_t)$$

$$(3.1b) \quad u(C_t, L_t) = \log(C_t - \Delta_t) + \theta_L \log(\nu(L_t)),$$

where the amount of consumption is C_t ; the amount of leisure is L_t , with $\nu(L)$ a positive and increasing function; and Δ_t is a stochastic component of preferences that permits us to analyze demand shifts. A positive innovation to Δ_t represents a positive demand shock, i.e., an increased urgency to consume. In most of our analysis below, we set $\nu(L) = L$ so that the labor supply elasticity is determined by the stationary level of hours as in Prescott [1986] and Plosser [1989]. However, we also conduct some experiments with $\nu(L)$ functions that imply higher and lower values of aggregate labor supply elasticity.

The stochastic preference term Δ_t is a demand shift in the following specific sense. Consider the "Frisch" demand function for consumption which describes date t consumption demand as a function of its price p_t and a

measure of lifetime wealth, the multiplier (Λ) on the intertemporal budget constraint.⁵ Under the preference specification (3.1), that demand function is $C_t = (\Lambda p_t)^{-1} + \Delta_t$. Thus Δ_t can be interpreted as an additive demand shift, holding fixed prices and the wealth measure.

Private and Social Marginal Products: In the presence of productive externalities, it is necessary to distinguish between private and social marginal product schedules. We continue to let underbars denote aggregate quantities beyond the control of the individual. Given the production function facing the individual, $Y_t = A_t K_t^{\theta_K} N_t^{\theta_N} \underline{Y}_t^{\epsilon}$, the private marginal product schedules for labor and capital are:

$$(3.2) \quad \begin{aligned} \text{MPN}_t &= \theta_N (Y_t / N_t) \underline{Y}_t^{\epsilon} = \theta_N Y_t / N_t \\ \text{MPK}_t &= \theta_K (Y_t / K_t) \underline{Y}_t^{\epsilon} = \theta_K Y_t / K_t, \end{aligned}$$

where the latter equality reflects the fact that all agents will be producing the same quantities and selecting the same input choices in equilibrium. The social marginal product schedules for labor and capital are

$$(3.3) \quad \begin{aligned} \text{SMPN}_t &= \eta \theta_N Y_t / N_t \\ \text{SMPK}_t &= \eta \theta_K Y_t / K_t, \end{aligned}$$

which are higher at given values of \underline{K}_t and \underline{N}_t so long as $\eta > 1$. While the levels of these schedules are different, the (constant) elasticities with respect to capital, labor and technology shocks are equal for private and social marginal products. For example, the labor elasticity of the marginal product of labor is simply $\eta \theta_N - 1$ in both cases.

Accumulation Technology. The evolution of capital is specified as

$$(3.4) \quad K_{t+1} = [(1 - \delta_K) K_t + I_t],$$

where I_t is gross investment (i.e. the amount of current output to be used in next period's production) and δ_K is the rate of depreciation of capital.

Resource Constraints. In each period, there are resource constraints on goods and time:

$$(3.5) \quad L_t + N_t \leq 1$$

$$(3.6) \quad C_t + I_t \leq Y_t$$

The latter condition need not hold for an individual agent, who may borrow and lend. However, the aggregate resource constraint $\bar{C}_t + \bar{I}_t \leq \bar{Y}_t$ must hold in equilibrium, and will also hold for each individual in our representative agent economy. We therefore impose (3.6) as an equilibrium condition in our analysis.

Analysis of Dynamic Equilibrium

The standard method of solving real business cycle models with constant returns to scale technology and no government-imposed distortions is to solve an associated planner's problem, and to reinterpret as competitive market outcomes the planner's optimal decisions and the associated shadow prices. In our setting, the presence of productive externalities makes that methodology inapplicable. We therefore use an alternative, Euler-equation-based approach. Within this "Euler equation" approach to finding suboptimal dynamic equilibria, there are a variety of methods for approximating the equilibrium laws of motion for macroeconomic prices and quantities.⁶ In this paper we employ the log-linear approximation methods of King, Plosser and Rebelo [1987], which produces certainty-equivalent decision rules describing deviations from steady state values.⁷ The basic logic behind the Euler-equation approach is as follows. In any competitive equilibrium

problem, individuals make privately-efficient decisions which are summarized by first-order necessary conditions. In making these decisions, individuals take as given the paths of per capita quantities. Next, aggregate consistency conditions (resource constraints and rational expectations) are imposed on the first-order conditions. This two-stage procedure generates conditions that restrict the dynamic evolution of the economy, and describes competitive equilibrium even in distorted economies.

Our representative consumer makes consumption, leisure and investment decisions in a manner that is privately efficient: he equates the marginal utility of date t consumption to its opportunity cost; the marginal utility of leisure to the value of foregone earnings; and the opportunity cost of investment to its expected future return. Under certainty equivalence, these conditions are:

$$(3.7) \quad D_1 u(C_t - \Delta_t, L_t) = \lambda_t$$

$$(3.8) \quad D_2 u(C_t - \Delta_t, L_t) = \lambda_t A_t D_2 F(K_t, N_t) Y_t^\epsilon$$

$$(3.9) \quad \beta \lambda_{t+1} [A_{t+1} D_1 F(K_{t+1}, N_{t+1}) Y_{t+1}^\epsilon + 1 - \delta_K] = \lambda_t$$

where λ_t is the Lagrange multiplier attached to the flow budget constraint (3.4), and is interpretable as the shadow value of private consumption at date t . We use the notation $D_1 u(C_t, \dots)$ to represent the marginal utility of consumption (the partial derivative of utility with respect to its first argument), and we use corresponding notation for other marginal utilities and marginal products throughout the paper. By combining these efficiency conditions with the macroeconomic equilibrium conditions, (3.4–3.6) and the production function, we obtain a dynamic system that can be solved to trace out the response of the economy to shifts in A_t or Δ_t .

The log-linear system that we obtain describes the evolution of a vector of state variables, $s_t = [\hat{K}_t, \hat{A}_t, \hat{\Delta}_t]'$, where the circumflex denotes the

proportionate deviation from the stationary level for capital and productivity, $\hat{K}_t = \log(K_t/K)$ and $\hat{A}_t = \log(A_t/A)$. For the demand shock, deviations are computed relative to stationary consumption:

$\hat{\Delta}_t = \log(C-\Delta_t/C)$. The state vector evolves according to $s_t = M s_{t-1} + \xi_t$, where $\xi_t = [0, a_t, d_t]'$ is a vector containing the innovations to technology and demand, and where the matrix M is given by

$$M = \begin{bmatrix} \mu_1 & \pi_{KA} & \pi_{K\Delta} \\ 0 & \rho_A & 0 \\ 0 & 0 & \rho_\Delta \end{bmatrix} .$$

The coefficients in this matrix determine the evolution of the economy's state variables. Specifically, ρ_A and ρ_Δ determine the persistence of exogenous shocks; and the implied reduced form for capital is $\hat{K}_{t+1} = \mu_1 \hat{K}_t + \pi_{KA} \hat{A}_t + \pi_{K\Delta} \hat{\Delta}_t$. Hence μ_1 determines the speed of transition-path dynamics. The impulse responses of capital and other variables to an innovation in ξ are jointly determined by the exogenous propagation mechanisms of the model (parameterized by ρ_A and ρ_K) and the endogenous propagation mechanism (governed by μ_1).

The remainder of the model's variables are simply functions of the state variables. Letting $Z_t = [\hat{C}_t \hat{N}_t \hat{Y}_t \hat{W}_t r_t \dots]$ be the vector of these variables, the model implies that $z_t = \Pi s_t$ with particular numerical values for the elements of the matrix Π .⁸ For example, consumption is governed by the relation $\hat{C}_t = \pi_{CK} \hat{K}_t + \pi_{CA} \hat{A}_t + \pi_{C\Delta} \hat{\Delta}_t$. With this "state space" system in hand, it is direct to compute the stochastic simulations, population moments, and impulse responses discussed in the paper. It also provides a natural basis for model evaluation analysis along the lines of Hansen and Sargent [1980] or Watson [1990].

Parameterizing the Model

In order to undertake a quantitative examination of our model, we need to specify values of the parameters of preferences and technologies. The full set of parameter values used here is given in Table 3, which also provides a convenient review of notation.

4. Dynamic Properties of the Macroeconomic Model

In this section, we investigate whether the time series generated by the model economy with productive externalities and increasing returns broadly resemble U.S. macroeconomic time series. As a benchmark for comparison we use the model with constant returns and no externalities ($\eta=1$). By comparing the responses of these two models we gain insight into the role of increasing returns and productive externalities in determining the response of the economy to shocks. As a useful shorthand, we denote by IR the model with increasing returns and productive externalities, and we denote by CR the standard model with constant returns to scale and no externalities.

Our model is subject to demand shocks which are highly persistent, obeying $\Delta_t = \rho_{\Delta} \Delta_{t-1} + d_t$ with $\rho_{\Delta}=.95$. This value of ρ_{Δ} implies that the half-life of a demand shock is 13.5 quarters. Further, we assume that the standard deviation of d_t is such that the model exactly matches the standard deviation of detrended U.S. output as reported in Panel A of Table 2.

Attributes of Simulated Time Series

Previously, we discussed the dimensions along which the standard real business cycle model (the CR model) captured central elements of business cycles, when driven by technology shocks. Can our model with productive

externalities and increasing returns (the IR model) similarly generate business cycles, when driven by persistent demand shocks? Looking at Figure 4 and Table 4-A, we see that the IR model performs well in terms of its commodity market implications. Both consumption and investment move closely with output, with consumption's standard deviation about six-tenths that of output, and investment's standard deviation about twice that of output. Thus, investment in the model is somewhat more volatile than in the U.S. data in panel A of Table 2.⁹ Turning to labor market implications of the IR model, we see in Figure 4 that the model economy predicts substantial labor input variability with little accompanying real wage movement.

In order to isolate the role played by productive externalities and increasing returns in generating these business cycle phenomena, we take the CR ($\eta=1$) model and feed the same series of demand shocks into this model that were used to construct Figure 4. The resulting series are plotted in Figure 5. Business cycle statistics for the CR model are reported in Table 4-B. The results are striking in three dimensions. First, cyclic volatility of output is five times higher in the IR model than in the CR model. Second, the CR model fails to reproduce basic commodity market attributes of business cycles, in that investment and output are strongly negatively correlated (-.99) and are about equally volatile, and consumption is predicted to be twice as volatile as output. It is interesting to note that Figure 5 shows that the CR model predicts a strong negative correlation between the real wage and labor input, with labor input about twice as volatile as the real wage. Finally, the real interest rate is about 2.5 times as volatile in the IR model compared with the CR model. Evidently, the standard real business cycle model does not necessarily generate realistic cyclic behavior

regardless of the source of shocks to the economy. We take this to be one important finding of the present investigation.

Responses to an Innovation in Demand

Additional insight into the dynamic properties of the IR model is provided by tracing out the impulse response to a demand shock. We consider a shock to demand that would raise consumption by one percent of its steady state level on impact if we held fixed all prices faced by the representative consumer. As noted above, changes in demand are assumed to be highly persistent: with $\rho_{\Delta} = .95$, slightly over one half of the original demand shift will be present after twelve quarters. Figure 6 shows the dynamic response of prices and quantities to the demand shift in the IR model; for comparison, we also plot the responses of the CR model.

Impact Effects: At date $t=1$, when the innovation to demand takes place, the effects of the demand shock on output are much larger in the IR model than in the CR model: output increases by 1.05% of its steady state level in the former, and only .33% in the latter. This increased response can be traced to two sources. First, a given increase in labor input simply yields more output under increasing returns (with $\eta=1.5$, the impact output response via this channel is $1.5 * .33\% = .495\%$). Second, labor input is much more responsive to demand shocks in the presence of increasing returns.

This just pushes the question back one stage however—why does labor input respond more elastically when there are productive externalities and increasing returns? One way to think about the difference between the responses of the CR and IR models is to notice that the external effect operates on the individual's production function, $Y_t = AF(K_t, N_t)Y_t^{\epsilon}$, much like a technology shock. The external effect temporarily raises the position of

the private production function, inducing additional labor input. The magnitude of this "production function shift" is $\epsilon \hat{Y}_t$, which is one-third of the output response displayed in Figure 6 for the IR model.

In a decentralized market system, individuals are induced to alter their behavior by changes in relative prices such as the real interest rate and the real wage rate. Compared with the CR model, the IR model displays larger labor supply responses because (i) the real interest rate displays a larger response to the demand shocks; and (ii) the real wage displays smaller response to demand shocks. The second of these simply reflects the fact that the marginal product of labor declines less sharply with labor input in the increasing returns model. The elasticity of the real wage rate with respect to labor input is $\eta \theta_N - 1$. Under constant returns, this magnitude is $-.42$, compared with a value of $-.13$ under increasing returns with $\eta=1.5$.

The larger response of the real interest rate in the IR model stems from the fact that the demand shock induces increased investment, leading to a higher equilibrium rate of return. This higher return induces intertemporal substitution in consumption and labor supply, so that consumption increases less in response to the demand shock with $\eta=1.5$ than it does with $\eta=1$, and labor supply increases by more.

A notable feature of the impact response of output is that there is a multiplier: output rises by 1.49 times the demand shock.¹⁰ By comparison, the output effect of the demand shift in the CR model shown in Figure 6 is only .49. These relative magnitudes are consistent with the analyses of Aiyagari, Christiano and Eichenbaum [1990] and Baxter and King [1990] who found that that large multiplier effects of demand disturbances required strong supply-side responses of capital and labor. In the present context,

these strong supply-side responses arise in the IR model, but do not arise in the CR model.

Persistence and Comovement: The predictions of the IR model for the persistence and comovement of macroeconomic time series differ markedly from the predictions of the CR model. First, the IR model proceeds more slowly than the CR model along the transition path.¹¹ In the standard CR model, the transitional dynamics coefficient μ_1 is .953, which implies that a 1% drop in the capital stock will be half rebuilt in 14 quarters. In the IR model, $\mu_1 = .987$, which corresponds to a half life of 52 quarters.

Second, the transition path dynamics of the IR model involve positive comovements of labor input and gross investment with the capital stock, while these comovements are negative in the standard CR model. This characteristic means that the IR model contains inherently stronger propagation mechanisms than the CR model. That is, a positive innovation to the capital stock in the IR model leads to increased labor supply, which in turn leads to an increased marginal product of capital and an incentive to invest further. Yet this propagation mechanism is not, by itself, sufficient to generate business cycles, given an arbitrary stochastic process for the demand shocks. In fact, high serial correlation in the demand shocks is necessary if this model is to generate business cycles with realistic amplitude, comovement, and persistence. If the demand shocks were purely temporary, the contemporaneous outcomes generated by the model would bear little resemblance to initial phases of U.S. business cycle expansions or contractions. In particular, with $\rho_\Delta = 0$ (or, indeed, with $\rho_\Delta \leq .9$), investment would respond negatively to shocks to consumption demand, with its role as a buffer dominating the input demand linkages highlighted above. Second, without persistence in demand shocks, there would not be important serial correlation

in output and labor input. To sum up: while the IR model has stronger internal propagation mechanisms than the standard CR model, these propagation mechanisms are still relatively weak. For either model to produce realistic cyclic behavior, it is necessary for the shocks driving the economy to be highly persistent.

Labor Supply and Business Cycles: One recurrent criticism of real business cycle models driven by productivity shocks is that they require too high a labor supply elasticity to be consistent with microeconomic studies (see, for example, Mankiw [1989] and McCallum [1989]). The IR model with demand shocks similarly requires significant labor supply elasticity to generate empirically interesting business cycles. For example, if we reduce the labor supply elasticity to the value suggested by the panel data studies of male labor supply surveyed by Pencavel [1986], then the response to the persistent demand shift differs substantially from that shown in Figure 6. In particular, the impact effect on output is only .15% and the impact effect on investment is $-.83\%$. Thus, our analysis confirms the view expressed by Murphy, Schleifer and Vishny [1989, section 4.2]: business cycle models with increasing returns and productive externalities require substantial labor supply elasticity.

Productivity Shocks: While the analysis of this section has concentrated on demand shocks, the response of the IR model to productivity shocks can be easily studied. There are two general implications of the IR specification, which may readily be seen from the equilibrium relation describing the relationship between output and input, $Y_t = [A_t F(K_t, N_t)]^\eta$. First, a given size technical shift will exert a larger influence on output with $\eta > 1$, although in some sense this is simply a rescaling of an unobserved residual. Second, since inputs will respond to variations in A_t^η , there will be internal

amplification mechanisms of the form that we described earlier for demand shocks.

Responses to Basic Government Purchases

In this section, we briefly study how the IR model responds to shifts in government demand for final goods. We concentrate on "basic purchases" that do not shift private marginal product schedules or marginal utility schedules, but simply remove resources that otherwise could be used for private consumption and investment. These purchases are assumed to evolve according to the autoregressive process $G_t = \rho_G G_{t-1} + g_t$ with $\rho_G = .95$, and are assumed to be financed by lump sum taxes or reductions in transfer payments. As above, we use the CR model as a reference point, allowing us to draw on results previously obtained by Ayigari, Christiano and Eichenbaum [1990] and Baxter and King [1990].

Figure 7 shows the response of the IR and CR models to an innovation (g_t) equal to 1% of gross national product. The output effects differ sharply across the two model economies: there is a multiplier of about 1.5 in the IR model, while there is no multiplier in the standard model, since the output response is only about .5. On impact, the response of consumption is about the same in the two models. But investment increases dramatically on impact in the IR model, while it declines in the CR model. In fact, the responses of both investment and labor input in the IR model generally resemble the way in which this economy would respond to a shift in technology, which is consistent with the shift in private marginal product schedules that the external effect brings about. In light of the Rotemberg and Woodford [1989] analysis which stresses that strong negative real wage responses to changes in government purchases are a potentially counterfactual implication of the

CR model, we note that the real wage declines much less in the IR model compared with the CR model. In addition, it is interesting to note that in the IR model the transition path of consumption displays a humped shape, in contrast to the typical monotonic response of the CR model.

One potential route to exploring these empirical implications of the IR model driven by shocks to government purchases model is illustrated by the work of Rotemberg and Woodford [1989] and Ahmed and Yoo [1990]. These analyses examine the dimensions along which the model's dynamic implications are consistent with the behavior of an estimated data generating process. Using the methods of these investigations, it would be direct to (i) estimate a multivariate process for macroeconomic quantities, prices and basic government purchases prices; and (ii) determine whether the preceding impulse responses of alternative models were consistent with the estimated data generating process. However, our prior work on fiscal policy in the basic neoclassical model makes us hesitant to move too quickly along this path: in Baxter and King [1990], we found that (i) different types of government purchases had substantially different macroeconomic effects, so that it is important to appropriately classify purchases by type; and (ii) the financing decision is quantitatively more important than the size of the spending shock. We found, for example, that the output effects of an increase in purchases were sharply negative when government purchases were subject to a balanced budget requirement.

5. Comparing Efficient and Inefficient Equilibria

Given that the competitive equilibrium is inefficient, two natural questions arise. First, as in Cooper and Haltiwanger [1990], how would efficient responses to demand shifts differ from the competitive responses

studied in the previous section? Second, what "stabilization policies" could be used to obtain socially efficient outcomes?

Positive Macroeconomics: Two Versions of the First Question

In addressing the first question, it is necessary to distinguish between two ways in which it may be posed. One may ask "how different are time series generated by models with efficient and inefficient equilibria, given that the parameters of each are chosen to match certain long run properties of the U.S. economy?" Alternatively, one may ask "how different would observed time series be if there were a shift—possibly policy induced—from inefficient to efficient outcomes?" In the former experiment, it is sensible to allow for parameter variation across the two models in a way that gives each model the best possible chance to fit the data. In the latter experiment, it is appropriate to hold fixed the parameters of the model while the nature of market interactions is varied. We view both of these experiments as valid in the appropriate context. It is simply important to be clear about exactly what thought experiment is being undertaken.

Based on some preliminary results with the latter of these thought experiments, we believe that the main implications of increasing returns for variability of macroeconomic fluctuations are already present in the suboptimal equilibrium model presented above. Optimal outcomes with increasing returns necessarily involve higher steady state paths for capital consumption, etc., since a social planner sees the higher social marginal product schedules for capital and labor. But, since the elasticities of marginal schedules are unaffected by whether these are social or private in form, proportionate fluctuations around this higher base may not be too different. In particular, our current results already capture key aspects of

increasing returns for point-in-time responses and propagation of shocks that are also present in the analysis of optimal outcomes. However, the outcomes of optimal and suboptimal models are not exactly the same, since the optimal economy has different steady state shares of investment and levels of labor supply than the suboptimal economy. These differences in shares influence the reduced form system describing proportionate deviations from the steady state.

Normative Macroeconomics: Stabilization Policy Isn't Necessary

In our economy, the introduction of increasing returns does not require stabilization policy. Socially optimal outcomes can be obtained via the simple, non-contingent rule of subsidizing production activity so that social and private marginal product schedules coincide. In our model, the subsidy takes a particularly simple form: for each unit of output produced, the firm should receive a subsidy of $(\eta-1)$ units.

6. A Residual Puzzle

In this paper, we have explored how demand disturbances can lead to business cycles in a model which incorporates productive externalities and increasing returns. This exploration was motivated by the observation—made explicit in the construction of Figure 1—that output growth was more closely matched by input growth if increasing returns were introduced, with an aggregate returns to scale parameter of about 1.5. That parameter value implies that productivity measured by "Solow residuals" corresponds to about one half of total input growth, leading "Solow residuals" to be procyclical in a demand shock environment.

However, that interaction cannot be the full story about the cyclical variation in productivity, as may be seen by referring to Figure 8. In that figure, we plot the growth rate of labor productivity, where labor productivity is conventionally defined as output per manhour, versus the residual from the least squares regression reported in Table 1. The correlation between these two series is .86. If the increasing returns model were true up to small measurement errors in output, this correlation should be very small. Thus we conclude that there is an additional source of fluctuations operating in the U.S. economy beyond the interaction of demand shocks and increasing returns that we have studied in this paper.

7. Summary and Conclusions

In this paper, we have shown that an equilibrium model incorporating productive externalities that lead to increasing returns is capable of producing "realistic" business cycles in the following sense. If the key business cycle phenomena are those stressed by Kydland and Prescott [1982], namely, specific patterns of comovement and persistence of aggregate quantity variables, the IR model with demand shocks has done about as good a job of matching the data as standard real business cycle models embodying constant returns and driven by technology shocks. In addition, we found that the CR model driven by demand shocks does not produce realistic cyclic behavior: one notable deficiency is its prediction of a negative correlation between investment and output. Finally, we found that the IR model contains inherently stronger propagation mechanisms than the CR model, but that very persistent shocks are nevertheless necessary for this model to produce realistic persistence and comovement of output, consumption, and investment.

Like the CR model, the IR model is a fully-articulated equilibrium model, driven by unobservable exogenous shocks. It therefore shares with the CR model a certain vulnerability to the complaint that it is hard to take seriously a model in which the central stochastic element is something one cannot easily link to observable phenomena. We have some sympathy for this point of view, and in the remainder of this section we outline ways in which we think one could make progress on performing data-based evaluation of the IR model. Explorations along the lines sketched below are natural directions for future research.

Measuring Shifts in Productivity and Preferences

As previously mentioned, both the CR and IR models are driven by unobservables. In the CR model, the unobservables are called "technology shocks" or "productivity shocks." Given an aggregate production function, measurements of technical shifts can be produced along the lines of Solow [1957], as carried out in the analyses of Prescott [1986] and Plosser [1989]. These "Solow residuals" can be plotted, and one can then ask whether the resulting series looks reasonably like what one might expect of a series purporting to describe a society's technical capabilities. For example, Watson [1990] notes that technology shocks are sufficiently volatile that the level of technology actually falls about one quarter in three. Additionally, one can undertake statistical tests of hypotheses that should be true, conditional on these "Solow residuals" being exogenous technology shocks. Evans [1990], for example, has found that U.S. postwar Solow residuals are Granger-caused by money, interest rates, and government spending. Taken together, these observations casts doubt on the view that "Solow residuals" measure shifts in technical capabilities facing private firms.

The demand shifts in our model economy are preference shifts, in contrast to the supply shifts (technology shocks) stressed in real business cycle theory. Although technology shocks are not directly observable in the macroeconomic data, "observations" on the technology variable can be constructed, as described above, as residuals from a specified production function. In a similar spirit, Hall [1986] constructs a method of isolating these demand shifts as residuals from Euler equations. In Hall's procedure, the marginal conditions from the utility function relate the unobserved preference shift to observable variables in a manner analogous to that employed by Solow to "measure" shifts to technology.

In our model, the requirement that the marginal rate of substitution between leisure and consumption equals the real wage provides a straightforward method of identifying preference shifts, as follows:

$$\frac{\Delta_t}{C} = \frac{C_t}{C} - \frac{w_t L_t}{wL},$$

where C , w and L denote the steady state levels of consumption, leisure and the real wage.¹² Given a time series on "demand shocks" constructed along these lines, one can then proceed to evaluate the plausibility of this series as an exogenous demand shock series. Mankiw, Rotemberg and Summers [1985] and Eichenbaum, Hansen, and Singleton [1988] have shown that the Euler equations of the representative agent model provide poor descriptions of the U.S. macroeconomy. One interpretation of this finding is that there are large and persistent preference disturbances operating in the economy.

Measuring the Fit of Calibrated Models

A new approach to econometric evaluation of small-scale equilibrium models has been developed by Watson [1990]. In that paper, Watson applies his method to analysis of the standard CR model driven by technology shocks, but his method is equally applicable to the IR model driven by preference shocks. Watson's method could potentially be used, therefore, to determine which model provides the best "fit" to the data.

In particular, Watson notes that the technology shock model does not provide a particularly good description of the data at conventionally-defined business cycle frequencies (6-32 quarters). Since the IR model differs importantly from the CR in terms of its transition path dynamics, which translates to more protracted impulse responses to disturbances, it may have a better ability to mimic the data at these frequencies. On the other hand, the material presented in section 6, which showed that labor productivity is strongly correlated with the component of output unexplained by the IR model, means that the IR model must necessarily be leaving out important influences on the economy.

Concluding thoughts

Based on the foregoing, we conclude that the IR model is a serious alternative to the standard CR model as an equilibrium model of the business cycle. Both models are capable of replicating major features of business cycles. Yet the two models differ substantially in two ways. First, the IR model requires government intervention to achieve a social optimum, while the CR model does not. Second, the two models differ sharply in their predictions concerning the dynamic responses to demand and supply

disturbances. Evidently, the standards of evaluation used to date in the real business cycle literature are not sharp enough to distinguish between these two very different models. But it is clearly important that we devise methods which can do so.

Endnotes

¹Two alternative research paths have been explored which retain the central features of Solow's [1957] analysis, namely (i) use of the aggregate production function as an organizing device for aggregate time series; and (ii) competitive analysis as an organizing device for studying market interactions. First, one branch of research on growth and business cycles has treated the gap between output and input growth a measure of "technical progress" and explored the implications of this hypothesis for the dynamic evolution of the economy, as in the work of Solow [1956] and Prescott [1986]. A second approach path has viewed the input series as imperfectly measured. In the literature on economic growth, this has motivated new measurements designed at improving series on labor and capital input (see e.g. Denison [1962] and Jorgenson, Gollop and Fraumeni [1987]). In the literature on business cycles, this idea has motivated both additional measurement (Kydland and Prescott [1989]) and theory (Eichenbaum and Rebelo [1990] and Rotemberg and Summers [1990]).

²These series were detrended using a common deterministic trend with growth rate equal to the average of the growth rates of consumption, investment, and output.

³King and Plosser [1989] perform a version of the "test of the Adelmans" which indicates that the standard technology shock model generates measures of cyclical amplitude and conformity which resemble those isolated by NBER researchers for the U.S. economy.

⁴Expositions of this model have been provided by Barro [1984] and King, Plosser, and Rebelo [1988a,b].

⁵See, for example, Deaton and Muellbauer [1980].

⁶Baxter [1988] provides a general description of the Euler-equation approach to computing suboptimal equilibria and provides a discrete state space approach that is capable of handling problems that are less well behaved than ours. Taylor and Uhlig [1990] summarizes this and several other strategies for computing equilibria, several of which are useful in the context of distorted economies.

⁷There has been relatively little work on that accuracy of log-linear approximations in the context of distorted economies. However, the preliminary results of Dotsey and Mao [1990] give us confidence in our results, since they show that the King, Plosser and Rebelo [1987] methods are highly accurate in economies with tax distortions that are much larger than the external effects studied here. Their work uses Baxter's [1988] discrete state space Euler equation approach with fine grids to yield "exact solutions" and shows that the KPR log-linear approximations are remarkably accurate.

⁸The system matrices and the MATLAB programs used to construct them are available on request from the authors.

⁹Baxter [1990] shows that these simple measures of cyclic behavior are highly sensitive to detrending methods. She finds that the relative volatility of gross productive investment in postwar U.S. quarterly data ranges from 2.50 to 5.36, depending on the detrending method. Baxter and Crucini [1990] and Canova and Dellas [1990] find, in addition, that the extent to which a model economy "fits" the data (using the informal metric employed here) is also highly sensitive to detrending procedures.

¹⁰To convert the percentage responses of output to commodity units, the responses must be scaled by the steady state shares.

¹¹This can be understood as follows. Suppose that capital is below its steady state level. In both models, this implies that the rate of return is above its steady state level. The magnitude of this increase in the rate of return is governed by the elasticity of the marginal product of capital with respect to capital: this elasticity is $\eta\theta_K - 1$, which is $-.58$ with $\eta=1$ and is $-.13$ with $\eta=1.5$. Thus the rate of return is higher in the CR model than in the IR model. It is the interest rate which signals that consumption should be postponed to undertake the net investment necessary to restore the capital stock to its steady level. Hence consumption will be growing faster in the CR model than in the IR model. The effect is quantitatively important in terms of the transitional dynamics of the capital stock.

¹²In deriving this expression from the first-order necessary conditions for the consumer's problems, we have used the fact that $\theta_L C = wL$.

References

- Adelman, I. and F. Adelman, "The Dynamic Properties of the Klein-Goldberger Model," *Econometrica*, vol. 4, (1959), 596-625.
- Ahmed, S. and S. Yoo, "Fiscal Trends in Real Economic Aggregates," working paper, Pennsylvania State University, 1989.
- Aiyagari, R., Christiano, L. and M. Eichenbaum, "The Output, Employment, and Interest Rate Effects of Government Consumption," Federal Reserve Bank of Minneapolis discussion paper no. 25, March 1990.
- Barro, R.J., "Output Effects of Government Purchases," *Journal of Political Economy*, vol. 89, (December 1981), 1086-1121.
- _____, *Macroeconomics*, 1st edition, New York: John Wiley and Sons, 1984.
- _____ and R.G. King, "Time Separable Preferences and Intertemporal Substitution Models of Business Fluctuations," *Quarterly Journal of Economics*, vol. 99, no. 4 (November 1984), 817-839.
- Baxter, M., "Approximating Suboptimal Dynamic Equilibria: An Euler Equation Approach," Rochester Center for Economic Research Working Paper no. 139, June 1988.
- _____, "Business Cycles and the Exchange Rate Regime: Evidence from the U.S.A.," working paper, February 1990.
- _____, and M.J. Crucini "Explaining Saving/Investment Correlations," Rochester Center for Economic Research working paper #224, March 1990.
- _____ and R.G. King, "Fiscal Policy in General Equilibrium," manuscript, June 1990.
- Caballero, R. and R. Lyons (1989) "The Role of External Economies in U.S. Manufacturing," manuscript, September 1989.
- Canova, F. and H. Dellas "Trade Interdependence and International Business Cycles," manuscript, March 1990.
- Cass, D.A., "Optimal Growth in An Aggregative Model of Capital Accumulation," *Review of Economic Studies*, vol. 32, no. 91 (July 1965), 233-240.
- Cooper, R. and J. Haltiwanger, "Macroeconomic Implications of Production Bunching: Factor Demand Linkages," paper presented at the Conference on Instability and Persistence, University of Paris, January 1990.
- Denison, E., *The Sources of Economic Growth in the United States and the Alternatives Before Us*, Washington, D.C.: Committee for Economic Development, 1962.
- Deaton, A. and J. Muellbauer, *Economics and Consumer Behavior*, Cambridge: Cambridge University Press, 1980.

- Dotsey, M., and C. Mao, "On the Accuracy of Alternative Linear Approximation Methods in Distorted Economies," research paper in progress, Federal Reserve Bank of Richmond, 1990.
- Durlauf, S., "Output Persistence, Economic Structure and the Choice of Stabilization Policy," *Brookings Papers on Economic Activity*, no. 2, 1989.
- Eichenbaum, M., L. Hansen, and K. Singleton, "A Time Series Analysis of Representative Agent Models Under Uncertainty," *Quarterly Journal of Economics*, volume 53, no. 1 (February 1988), 51-78.
- Eichenbaum, M., and S. Rebelo, "Labor Hoarding and the Business Cycle," Northwestern University working paper, June 1990.
- Evans, C.L., "Productivity Shocks and the Business Cycle," University of South Carolina working paper, revised May 1990.
- Hall, R.E., "The Role of Consumption in Economic Fluctuations," in R.J. Gordon, ed., *The American Business Cycle: Continuity and Change*, Chicago: University of Chicago Press for National Bureau of Economic Research, 1986, pages 237-266 (including comments by A. Deaton and R.G. King).
- _____, "Productivity and the Business Cycle," in K. Brunner and A. Meltzer, eds., *Carnegie-Rochester Conference Series on Public Policy* 27 (1987), 421-444.
- _____, "The Relationship Between Price and Marginal Cost in U.S. Industry," *Journal of Political Economy*, volume 6, no. 5 (October 1988), 921-947.
- Hansen, G., "Indivisible Labor and the Business Cycle," *Journal of Monetary Economics*, volume 16, no. 3, (November 1985), 309-327.
- Hansen, L.P. and T.J. Sargent, "Formulating and Estimating Dynamic Linear Rational Expectations Models," *Journal of Economic Dynamics and Control*, 1980, reprinted as Chapter 7 in R.E. Lucas, Jr., and T.J. Sargent, eds., *Rational Expectations and Econometric Practice*, Minneapolis: University of Minnesota Press, 1981.
- Jorgenson, D., F. Gollop and B. Fraumeni, *Productivity and U.S. Economic Growth*, Cambridge: Harvard University Press, 1987.
- King, R.G. and C.I. Plosser (1989) "Real Business Cycles and the Test of the Adelmans," Rochester Center for Economic Research working paper.
- King, R.G., Plosser, C.I. and S.T. Rebelo, (1988a), "Production, Growth and Business Cycles: I. The Basic Neoclassical Model," *Journal of Monetary Economics*, vol. 21, no. 2/3 (March/May 1988), 195-232.
- _____, (1988b), "Production, Growth and Business Cycles: II. New Directions," *Journal of Monetary Economics*, vol. 21, no. 2/3 (March/May 1988), 309-342.
- _____, "Production, Growth and Business Cycles: Technical Appendix," unpublished manuscript, University of Rochester, 1987.

- Koopmans, T.J., "Optimal Growth in An Aggregative Model of Capital Accumulation," in *The Econometric Approach to Development Planning*, Chicago: Rand McNally, 1965.
- Kydland, F. and E.C. Prescott, "Time to Build and Aggregate Fluctuations," *Econometrica*, vol. 50, no.6 (November 1982), 1345-1370.
- Long, J.B. and C.I. Plosser, "Real Business Cycles," *Journal of Political Economy*, vol 91, no. 1 (February 1983), 39-69.
- Mankiw, G., "Real Business Cycles: A New Keynesian Perspective," *Economic Perspectives*, vol. 3, no. 3 (Summer 1989), 79-90.
- Mankiw, G., Rotemberg, J., and L. Summers, "Intertemporal Substitution in Macroeconomics," *Quarterly Journal of Economics*, vol. 100, no. (February 1985), 225-251.
- McCallum, B.T., "Real Business Cycles," in R.J. Barro, ed., *Modern Business Cycle Theory*, Cambridge, Ma.: Harvard University Press, 1989.
- Murphy, K., A. Schleifer, and R. Vishny, "Building Blocks of Market Clearing Business Cycle Models," ed. O. Blanchard and S. Fischer, *NBER Macroeconomics Annual 1989*, Cambridge, Ma: MIT Press.
- Pencavel, J., "Labor Supply of Men: A Survey," in O. Ashenfelter and R. Layard, eds., *Handbook of Labor Economics*, Amsterdam: North-Holland, 1986
- Plosser, C.I., "Understanding Real Business Cycles," *Economic Perspectives*, vol. 3, no. 3 (Summer 1989), 51-78.
- Prescott, E.C., "Theory Ahead of Business Cycle Measurement," in K. Brunner and A. Meltzer, eds., *Carnegie-Rochester Conference Series on Public Policy* 25 (1987), 11-66.
- Rogerson, R., "Indivisible Labor, Lotteries and Equilibrium," *Journal of Monetary Economics*, vol. 21, no. 1 (January 1988), 3-16.
- Romer, P., "Increasing Returns and Long Run Growth," *Journal of Political Economy*, vol 94, (1986).
- Rotemberg, J. and M. Woodford, "Oligopolistic Pricing and the Effects of Aggregate Demand on Economic Activity," NBER Economic Fluctuations Research Meeting, February 2, 1990.
- Rotemberg, J., and L. Summers, "Procyclical Productivity as a Consequence of Inflexible Prices,"
- Rouwenhorst, G., "Asset Returns and Business Cycles: A General Equilibrium Investigation," unpublished working paper, W.E. Simon School of Business, University of Rochester, November 1989.
- Shapiro, M., "Are Cyclical Fluctuations in Productivity Due More to Demand Shocks or Supply Shocks?," *American Economic Review Papers and Proceedings*, vol.77, (May 1987), 118-124.

Solow, R.M., "A Contribution to the Theory of Economic Growth," *Quarterly Journal of Economics*, vol. 70, no. 1 (February 1956), 65-94.

_____, "Technical Progress and the Aggregate Production Function," *Review of Economic Studies*, vol 39, (1957), 312-320.

Summers, L., "Some Skeptical Observations on Real Business Cycle Theory," Federal Reserve Bank of Minneapolis *Quarterly Review*, 23-37.

Taylor, J., and H. Uhlig, "Solving Nonlinear Stochastic Growth Models: A Comparison of Alternative Solution Methods," Journal of Business and Economic Statistics 8:1, (January 1990), 1-18.

Watson, M., "Measures of Fit for Calibrated Models," Northwestern University working paper, May 1990.

Table 1

Estimates of Returns to Scale Parameter
Annual U.S. Data, 1953-1985
Total Private Industry*

estimation method:	η	s.e. (η)	D-W	R^2 : stage 1	R^2 : stage 2
OLS	1.45	.155	1.32	—	.73
IV-total defense purchases -civilian defense comp -military defense comp	1.81	.582	1.18	.08	.68
IV-civilian defense comp -military defense comp	1.53	.562	1.27	.08	.73
IV-total nondefense purchases	1.10	2.13	1.60	.01	.69

* Data on output (value added), manhours, capital and labor compensation taken from a larger data base constructed by Shapiro [1987] for his analysis of sectoral Solow residuals. The growth of total input was calculated by the formula $\log(Z_t/Z_{t-1}) = (1-\theta_N) \log(K_t/K_{t-1}) + \theta_N \log(N_t/N_{t-1})$, where $\theta_N = .54$ is the sample average value of labor's share in total private industry.

** Instrumental variables estimates are constructed using variables from the National Income and Product Accounts, table 3.7B. The basic series (CITIBASE mnemonic) are: Federal National Defense Purchases (GGFEN); Compensation of Defense Employees, Military (GGFNCM); Compensation of Defense Employees, Civilian (GGFNCC); and Federal Nondefense Purchases of Goods and Services (GGFEO). Real purchases were created by deflating by the implicit deflator for gross national product. Continuously compounded growth rates of these real quantities were used in the regressions reported above.

TABLE 2

Panel A: Summary Statistics for U.S. Postwar Quarterly Data, 1948:1-1986:4*

	sd	rel sd	autocorrelations			cross-correlations with y										
			r1	r2	r3	12	8	4	2	1	0	-1	-2	-4	-8	-12
y	5.62	1.00	0.96	0.91	0.85	0.39	0.53	0.79	0.91	0.96	1.00	0.96	0.91	0.79	0.53	0.35
c	3.86	0.69	0.98	0.95	0.93	0.59	0.68	0.78	0.83	0.84	0.85	0.82	0.78	0.69	0.43	0.26
i	7.61	1.35	0.93	0.78	0.62	0.18	0.20	0.38	0.51	0.57	0.60	0.59	0.55	0.43	0.22	0.08
N	2.97	0.52	0.94	0.85	0.74	-0.44	-0.31	-0.07	0.03	0.06	0.07	0.07	0.05	-0.01	0.01	0.08

Panel B: The CR model with technology shocks

	sd	rel sd	autocorrelations			cross-correlations with y										
			r1	r2	r3	12	8	4	2	1	0	-1	-2	-4	-8	-12
y	5.62	1.00	0.97	0.93	0.90	0.66	0.76	0.87	0.93	0.97	1.00	0.97	0.93	0.87	0.76	0.66
c	4.27	0.76	1.00	0.99	0.98	0.87	0.91	0.92	0.92	0.91	0.91	0.87	0.84	0.79	0.68	0.59
i	10.71	1.91	0.93	0.86	0.80	0.34	0.48	0.67	0.78	0.85	0.91	0.89	0.86	0.80	0.70	0.61
N	2.02	0.36	0.91	0.82	0.74	-0.00	0.15	0.38	0.52	0.61	0.70	0.67	0.65	0.61	0.54	0.47
w	4.46	0.79	0.99	0.98	0.97	0.83	0.89	0.93	0.94	0.94	0.95	0.91	0.88	0.82	0.71	0.61
q	3.52	0.63	0.91	0.83	0.75	-0.36	-0.21	0.02	0.18	0.27	0.36	0.35	0.34	0.32	0.29	0.25
r	0.51	0.09	0.91	0.83	0.76	-0.42	-0.28	-0.05	0.10	0.19	0.29	0.28	0.28	0.26	0.23	0.21
k	5.44	0.97	1.00	1.00	0.99	0.91	0.92	0.89	0.85	0.83	0.80	0.77	0.74	0.69	0.60	0.51

*Panel A is Table 6 from King, Plosser, and Rebelo [1988].

Table 3

Notation and Parameter Values

A. Preferences

momentary utility function: $u(C,L) = \log(C_t - \Delta_t) + \theta_L \log(L_t)$
 θ_L chosen so that $L = .8$ and $N = 1-L = .2$

lifetime utility function: $U = \sum_t \beta^t u(C_t, L_t)$
 β chosen so that steady state real rate is .065

B. Production Function

production function: $Y_t = [A_t K_t^{\theta_K} N_t^{\theta_N}]^{\eta}$

θ_N, θ_K chosen to match U.S. factor share data: $\theta_N = .58, \theta_K = .42$
 η estimated in Table 1

accumulation of private capital: $K_{t+1} - K_t = I_t - \delta_K K_t$

$\delta_K = .10$

TABLE 4

Panel A: The IR Model

	sd	rel sd	autocorrelations			cross-correlations with y										
			r1	r2	r3	12	8	4	2	1	0	-1	-2	-4	-8	-12
y	5.62	1.00	0.98	0.96	0.94	0.79	0.86	0.92	0.96	0.98	1.00	0.98	0.96	0.92	0.86	0.79
c	3.27	0.58	0.98	0.97	0.96	0.83	0.88	0.94	0.97	0.98	1.00	0.98	0.96	0.92	0.85	0.79
i	11.27	2.01	0.98	0.95	0.93	0.77	0.83	0.91	0.95	0.98	1.00	0.98	0.96	0.92	0.85	0.79
N	4.53	0.81	0.96	0.92	0.88	0.59	0.69	0.80	0.87	0.90	0.94	0.92	0.90	0.86	0.80	0.74
w	2.04	0.36	1.00	0.99	0.99	0.87	0.83	0.76	0.72	0.69	0.66	0.65	0.64	0.62	0.58	0.55
q	3.34	0.59	0.94	0.89	0.84	0.32	0.44	0.59	0.68	0.72	0.77	0.76	0.74	0.71	0.65	0.60
r	0.52	0.09	0.94	0.89	0.83	0.32	0.44	0.58	0.67	0.72	0.77	0.75	0.73	0.70	0.64	0.59
k	3.69	0.66	1.00	1.00	1.00	0.92	0.90	0.87	0.85	0.84	0.82	0.81	0.79	0.77	0.72	0.67

Panel B: The CR Model

	sd	rel sd	autocorrelations			cross-correlations with y										
			r1	r2	r3	12	8	4	2	1	0	-1	-2	-4	-8	-12
y	1.05	1.00	0.94	0.89	0.84	0.48	0.62	0.79	0.89	0.94	1.00	0.94	0.89	0.79	0.62	0.48
c	1.89	1.80	0.94	0.88	0.83	0.46	0.60	0.78	0.88	0.94	1.00	0.94	0.89	0.79	0.62	0.48
i	0.97	0.93	0.93	0.86	0.80	-0.38	-0.53	-0.73	-0.85	-0.92	-0.99	-0.93	-0.88	-0.78	-0.61	-0.48
N	2.05	1.95	0.95	0.91	0.87	0.57	0.68	0.82	0.90	0.95	0.99	0.93	0.88	0.78	0.61	0.48
w	1.02	0.97	0.97	0.94	0.91	-0.65	-0.74	-0.85	-0.90	-0.93	-0.96	-0.90	-0.85	-0.75	-0.59	-0.46
q	1.41	1.34	0.97	0.94	0.91	0.65	0.74	0.85	0.90	0.93	0.96	0.90	0.85	0.75	0.59	0.46
r	0.22	0.21	0.97	0.94	0.91	0.64	0.74	0.84	0.90	0.93	0.96	0.91	0.85	0.76	0.59	0.46
k	0.49	0.47	1.00	1.00	0.99	-0.81	-0.80	-0.74	-0.68	-0.65	-0.61	-0.58	-0.54	-0.48	-0.37	-0.29

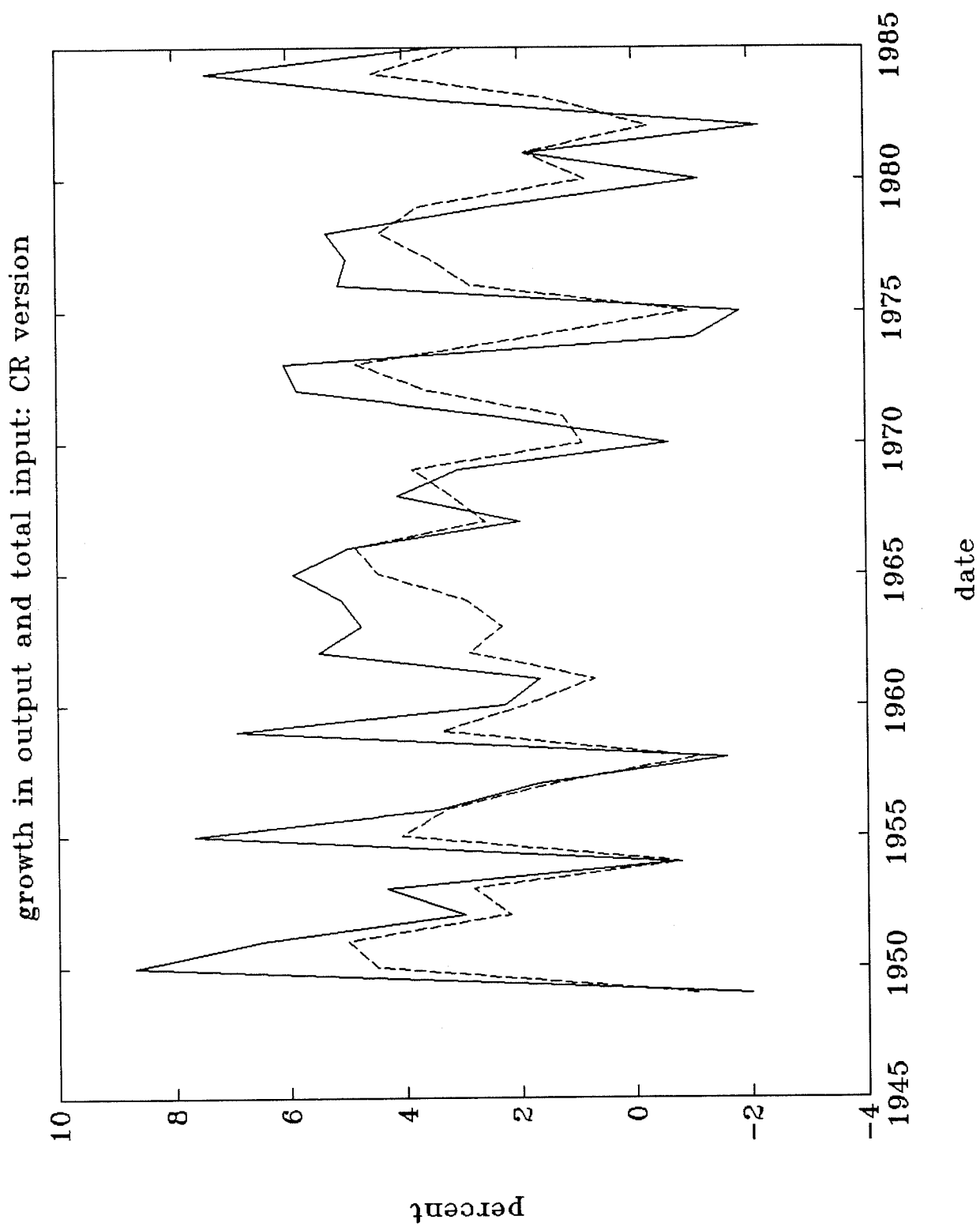


FIGURE 1-A

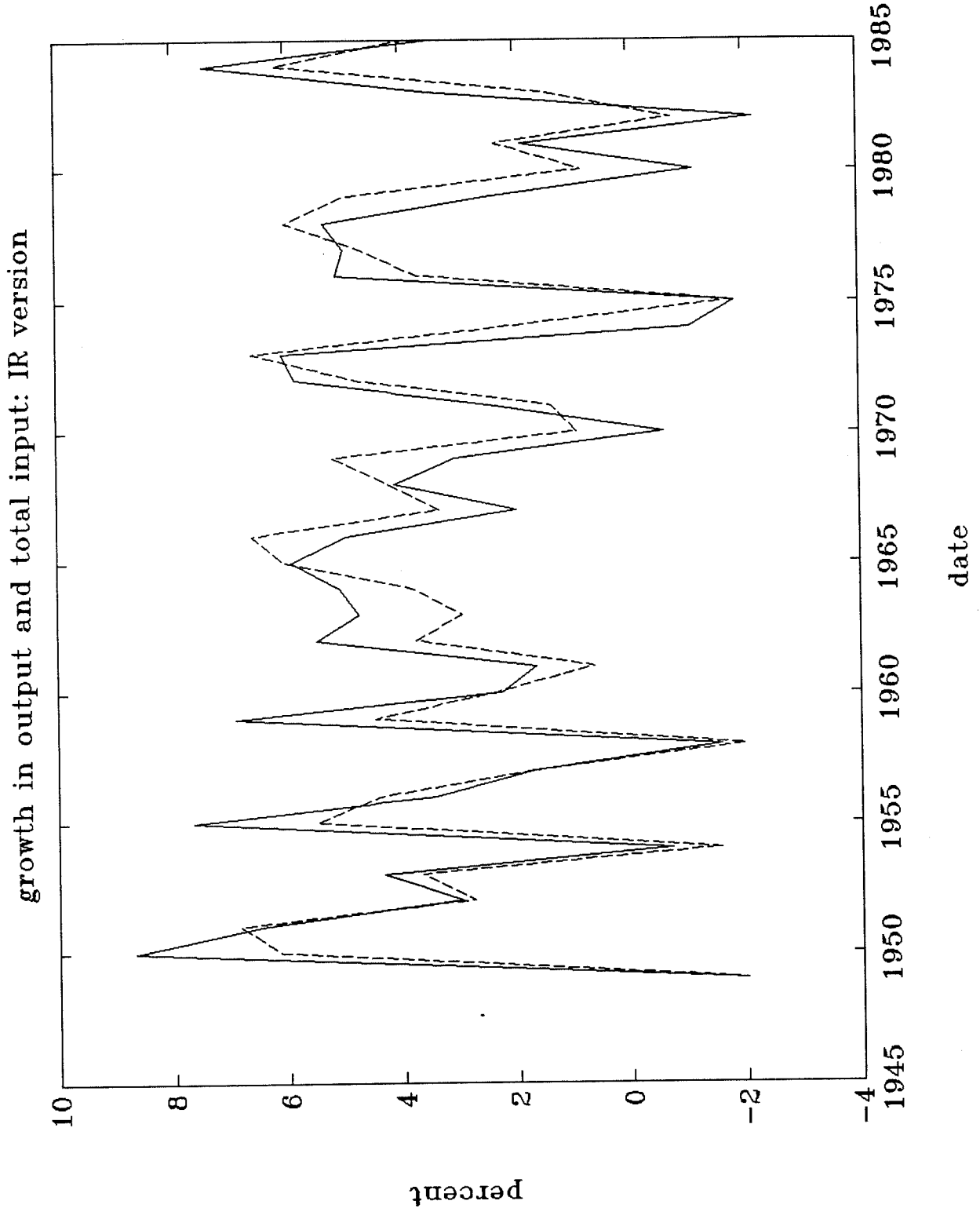


Figure 1-B

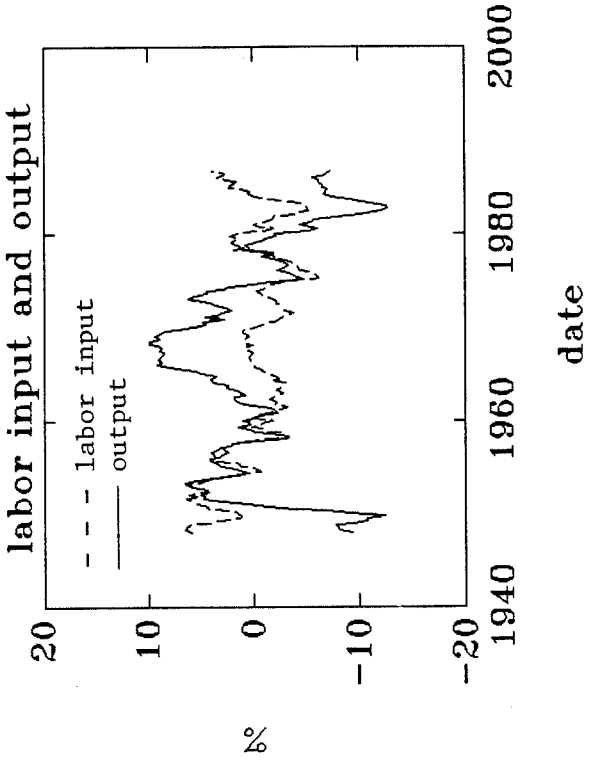
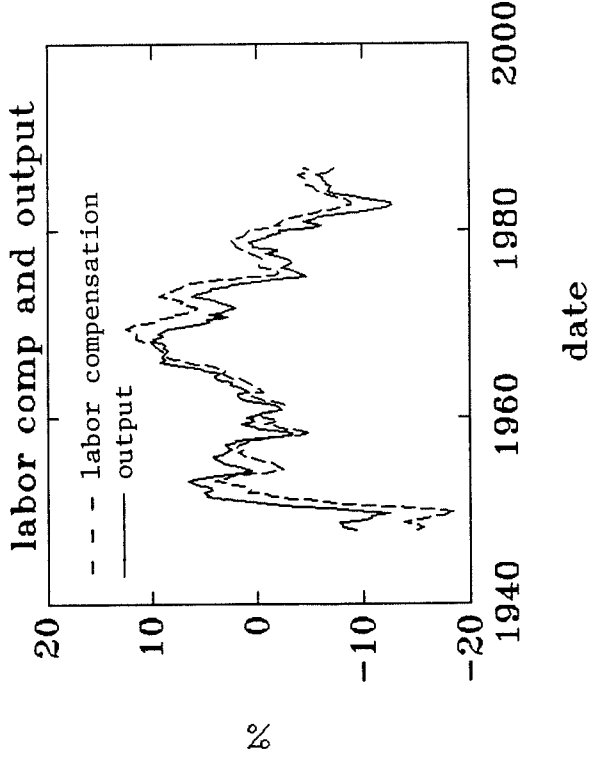
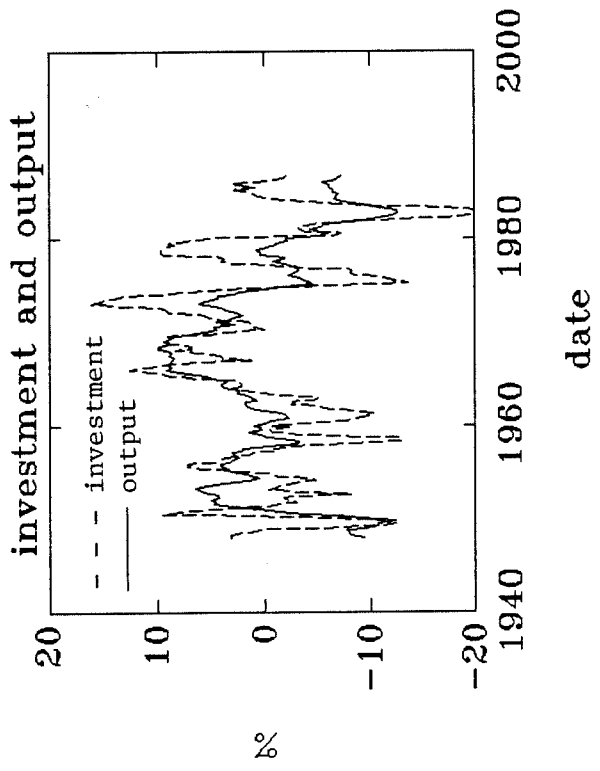
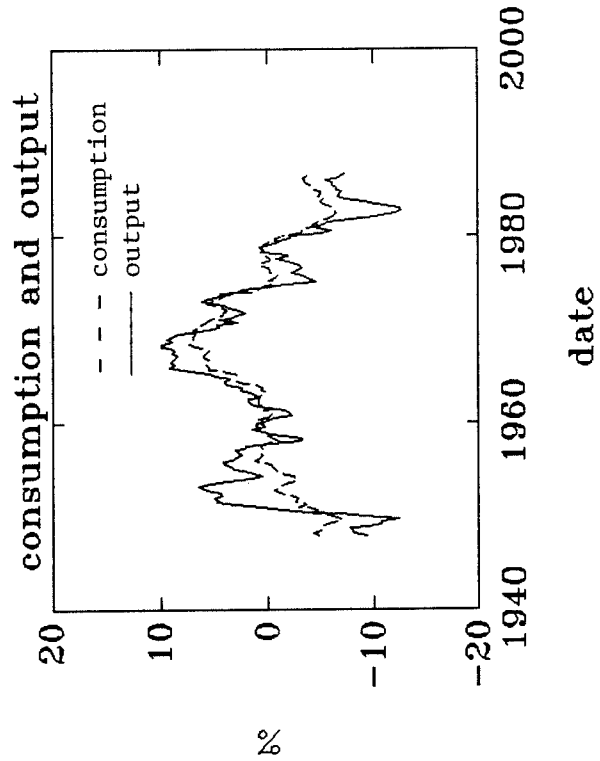


FIGURE 2: U.S. Business Cycles

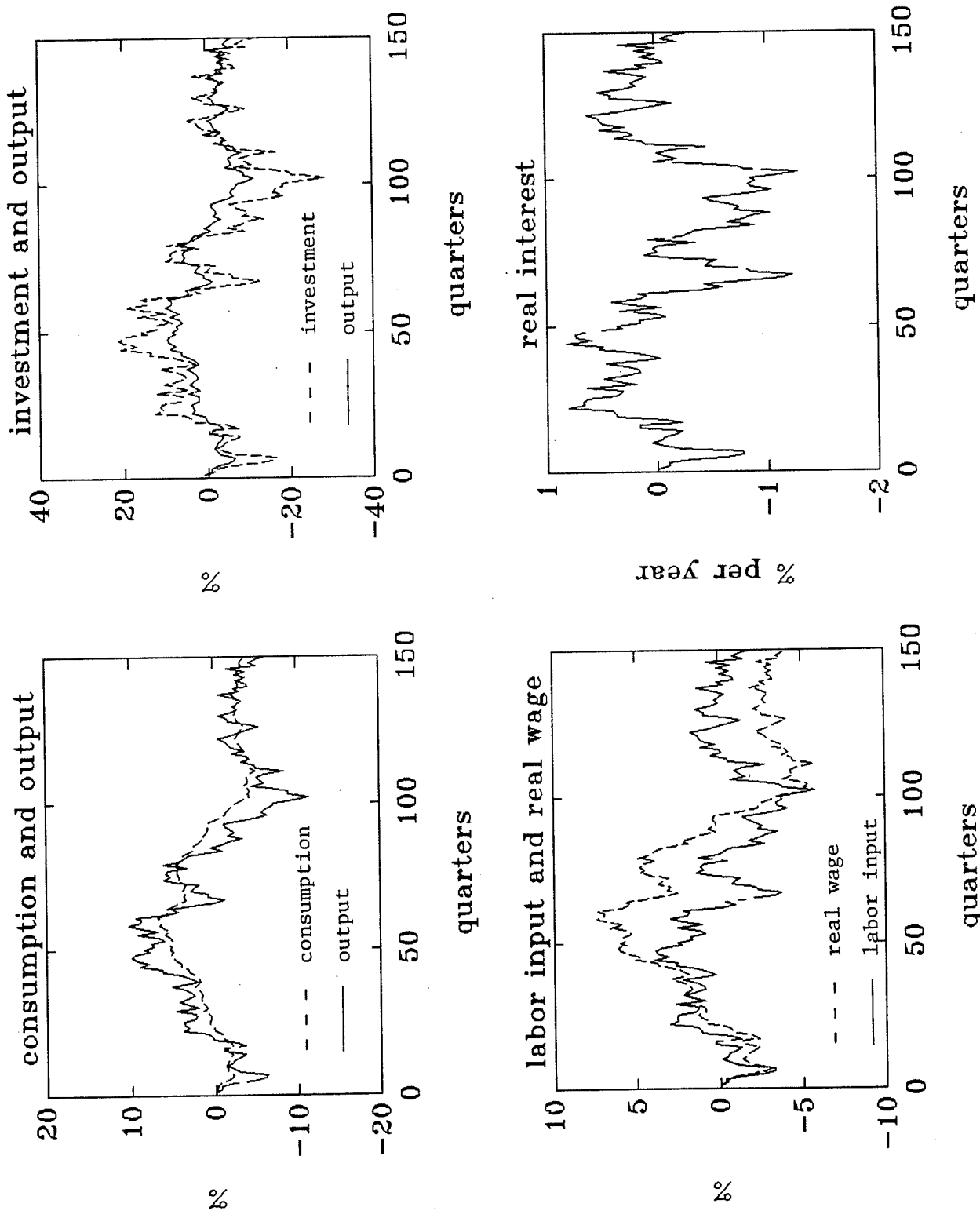


FIGURE 3: Technology shocks in the standard RBC model---Simulated business cycles

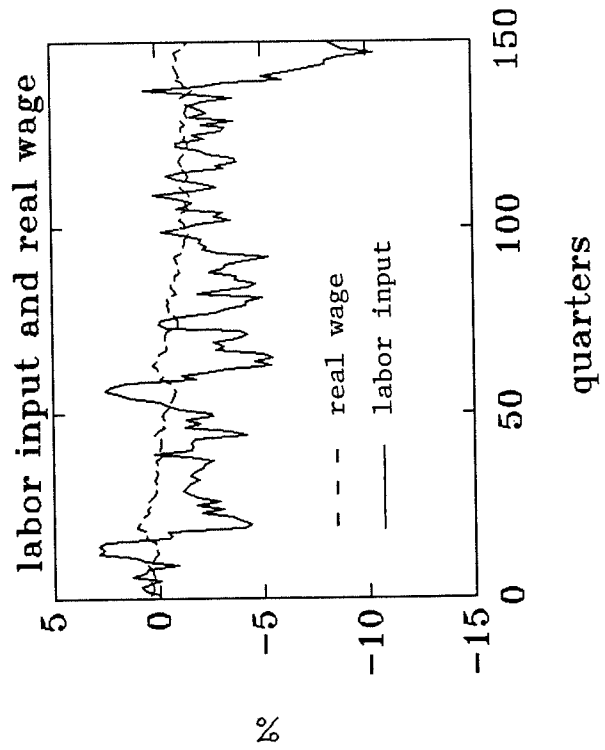
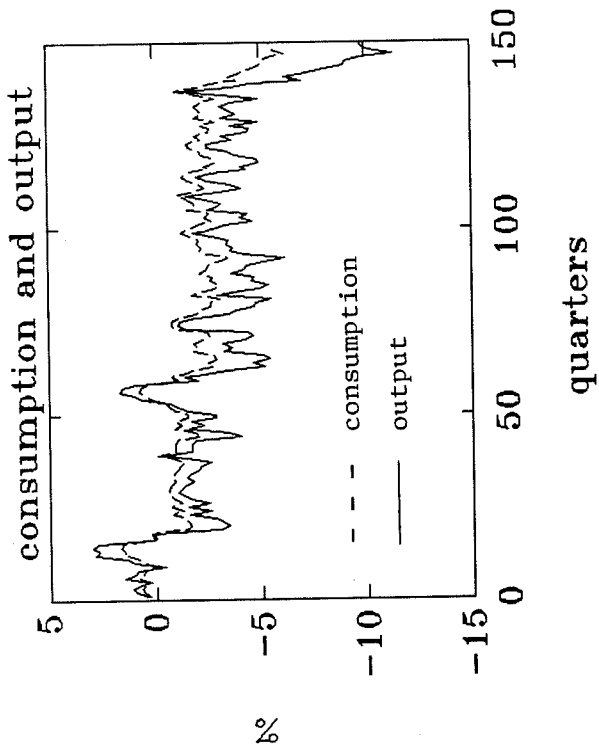
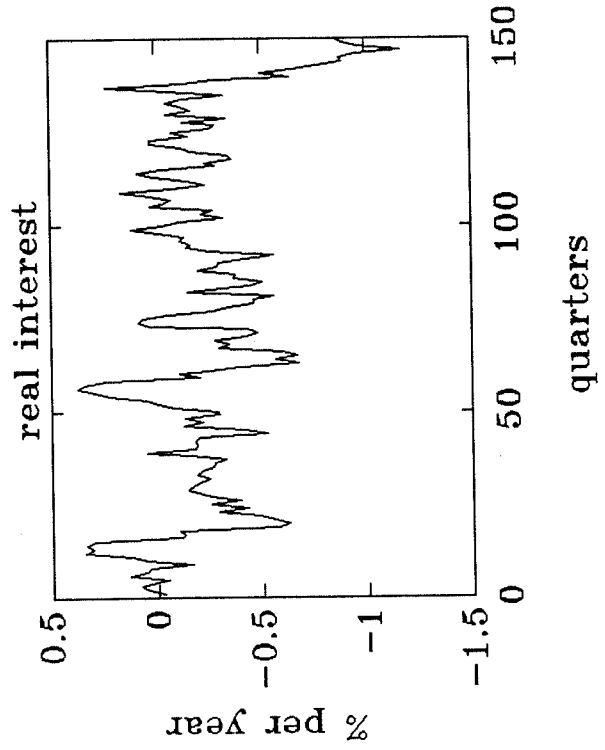
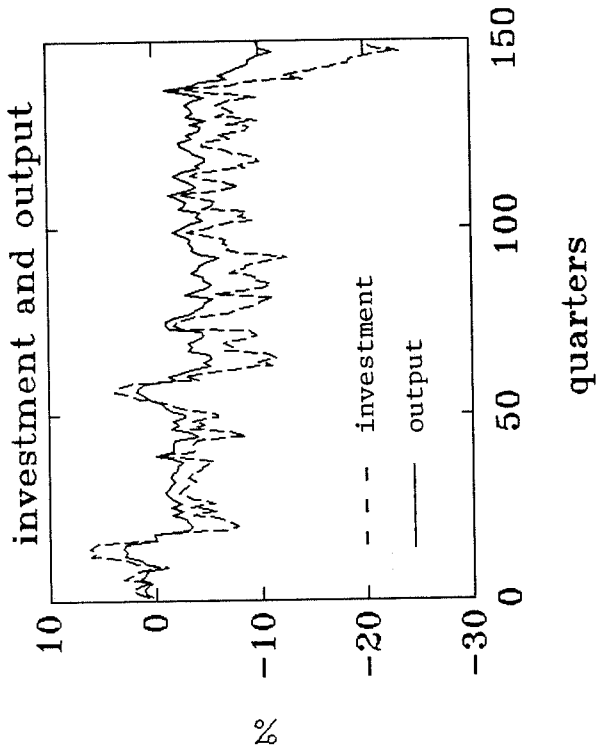


FIGURE 4: Demand shocks in the IR model--Simulated business cycles

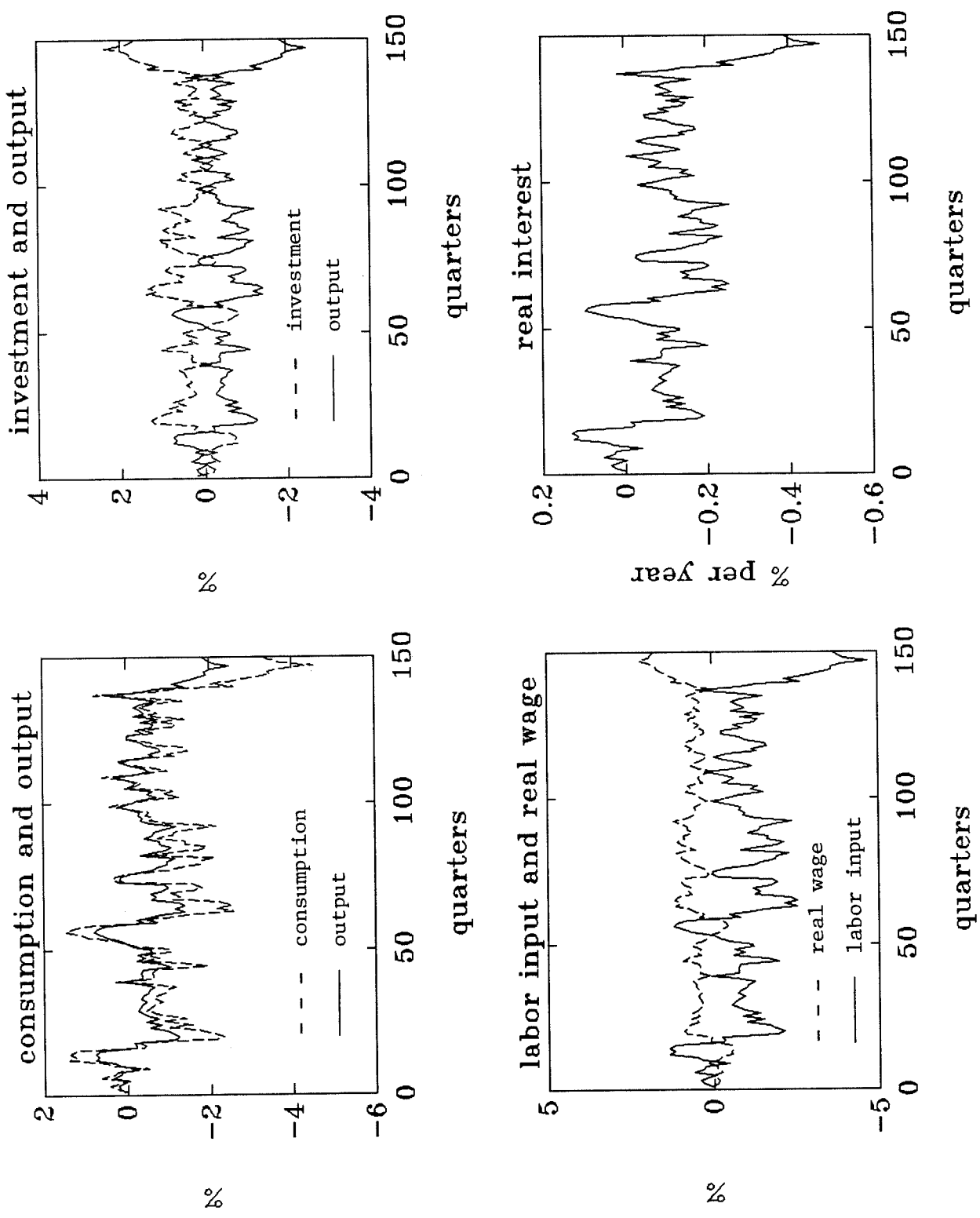


FIGURE 5: Demand shocks in the CR model--Simulated business cycles

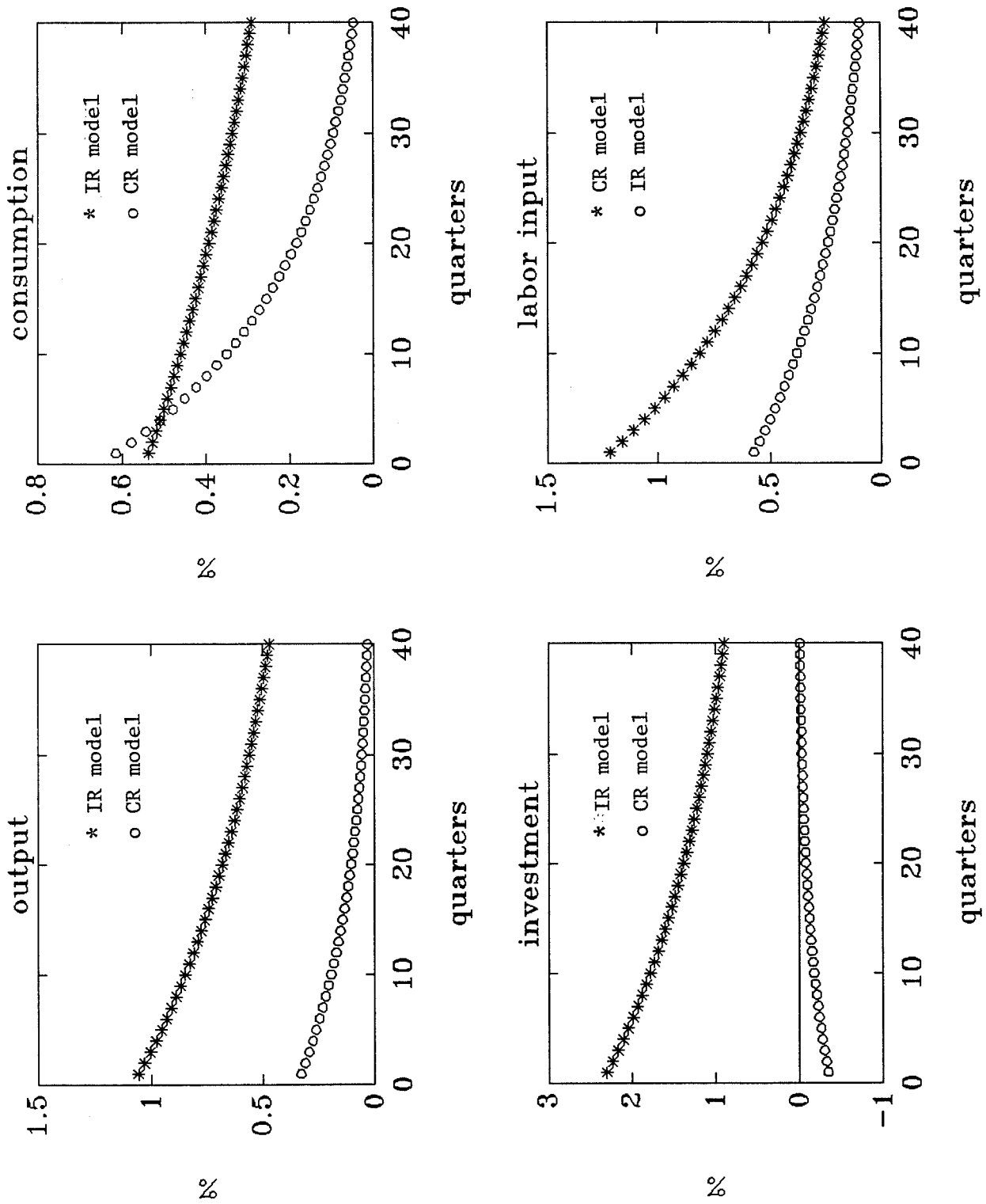


FIGURE 6-A: Impulse responses to demand shocks; quantity variables

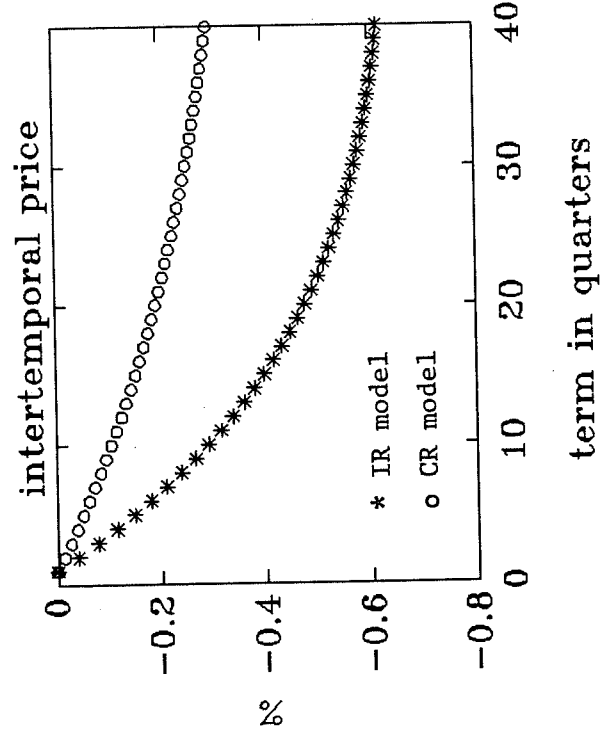
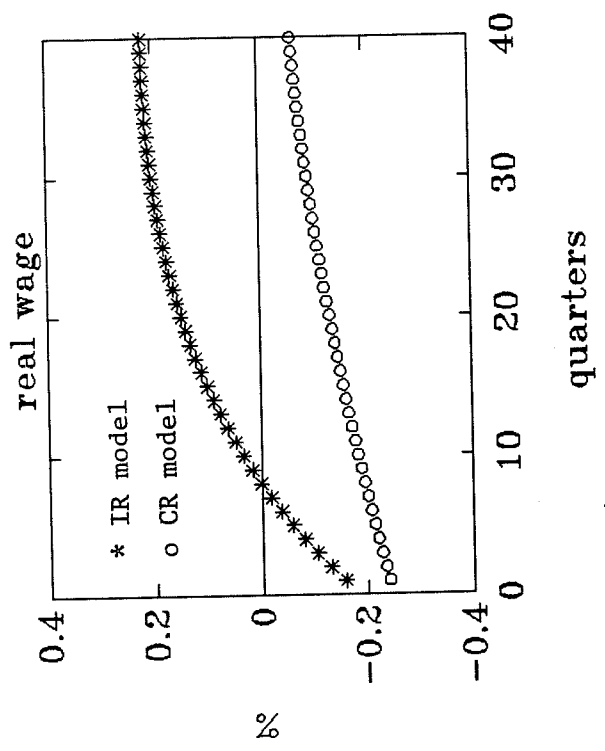
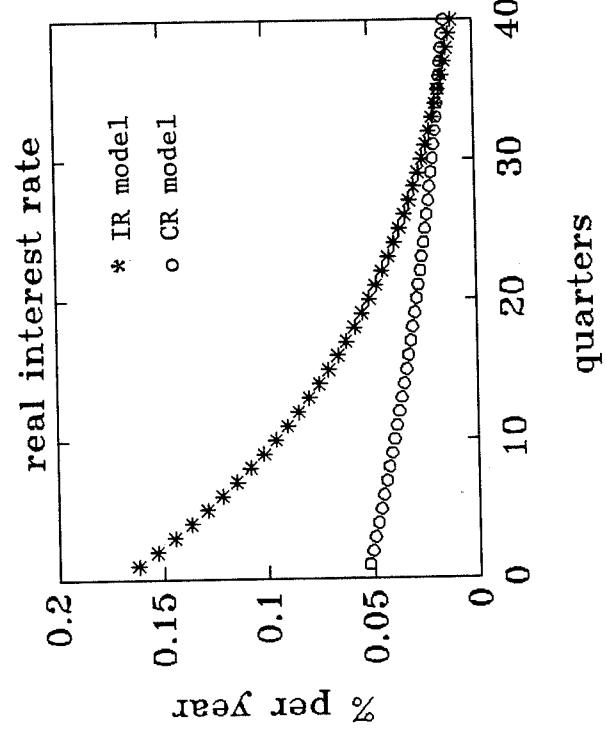
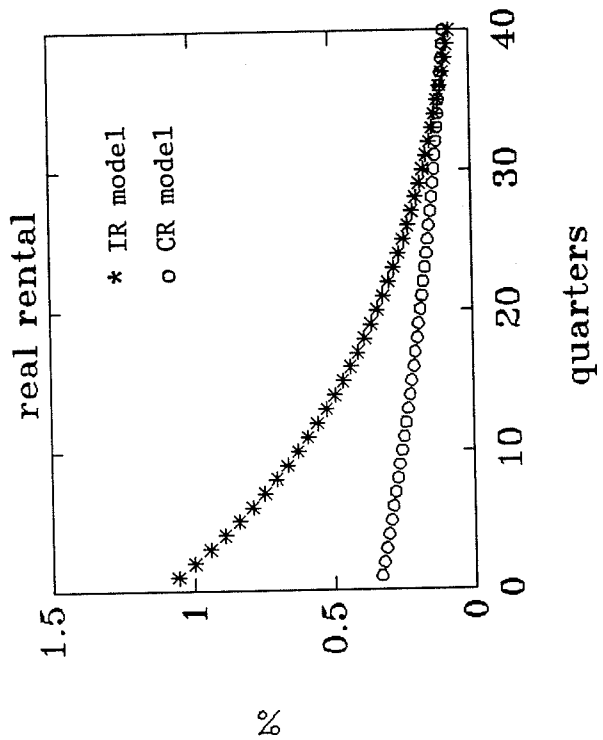


FIGURE 6-B: Impulse responses to demand shocks: prices and interest rates

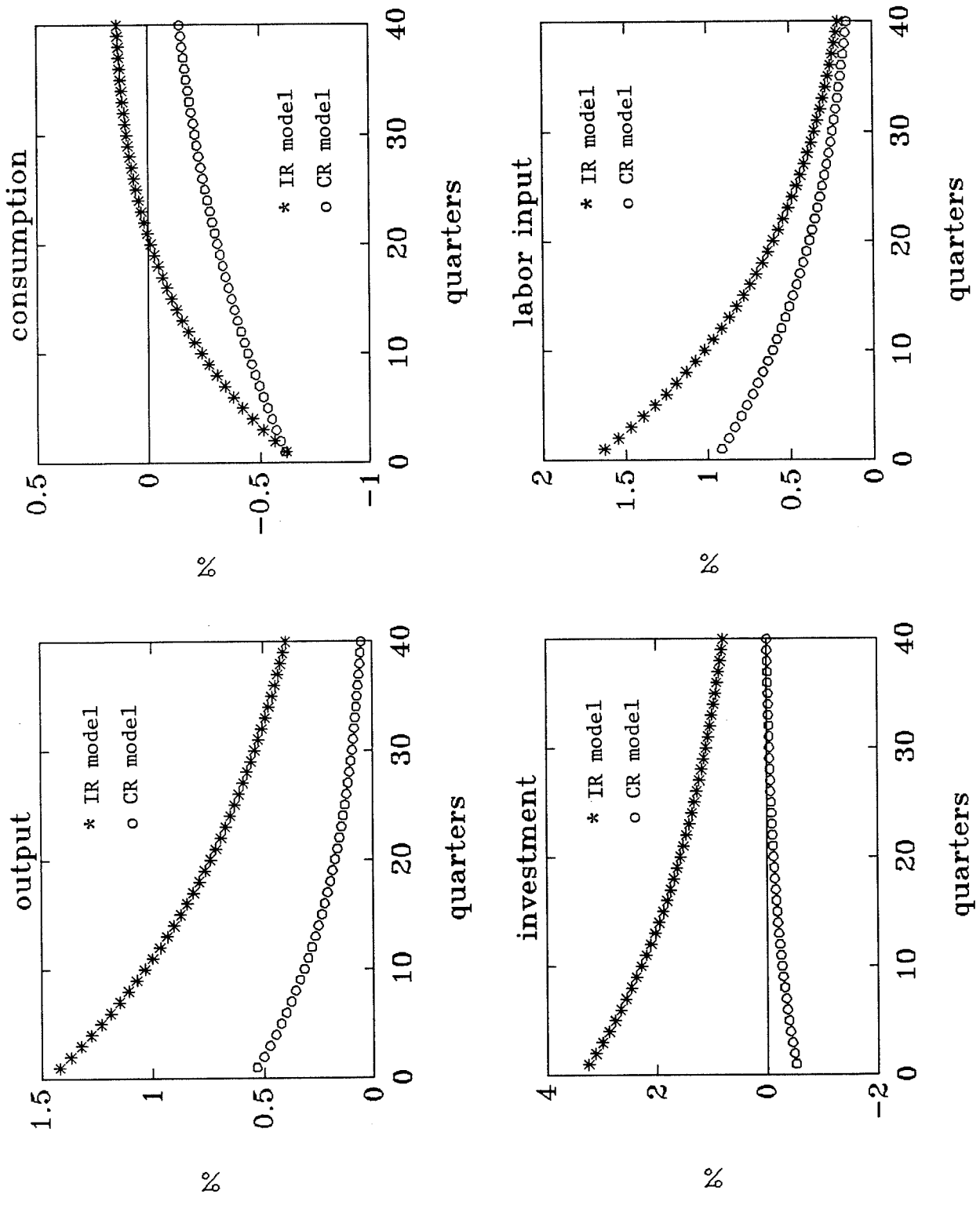


FIGURE 7-A: Impulse responses to government purchases: quantity variables

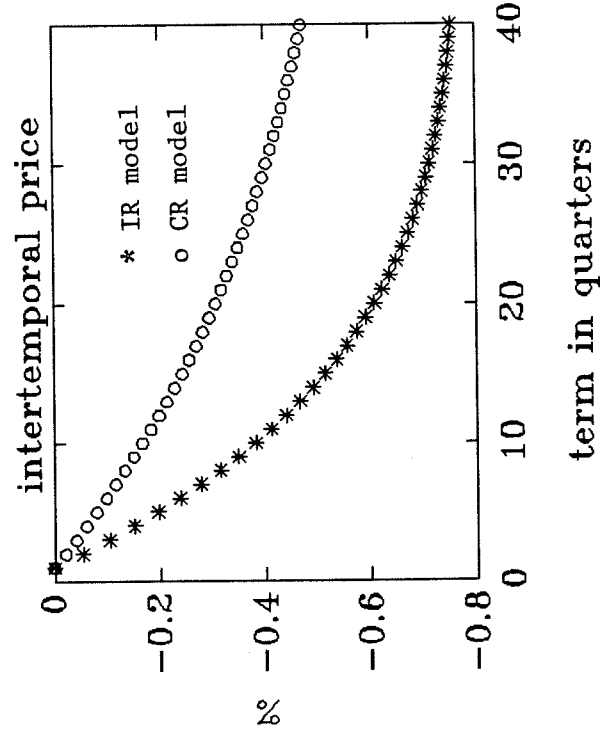
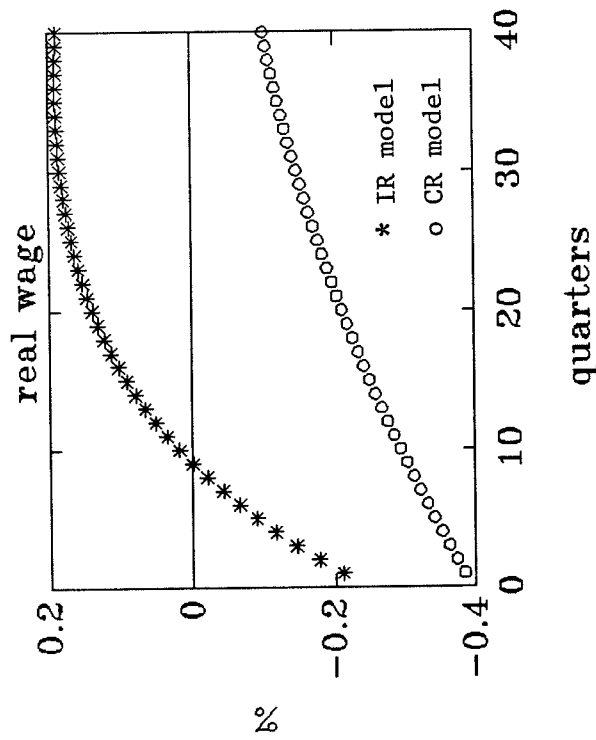
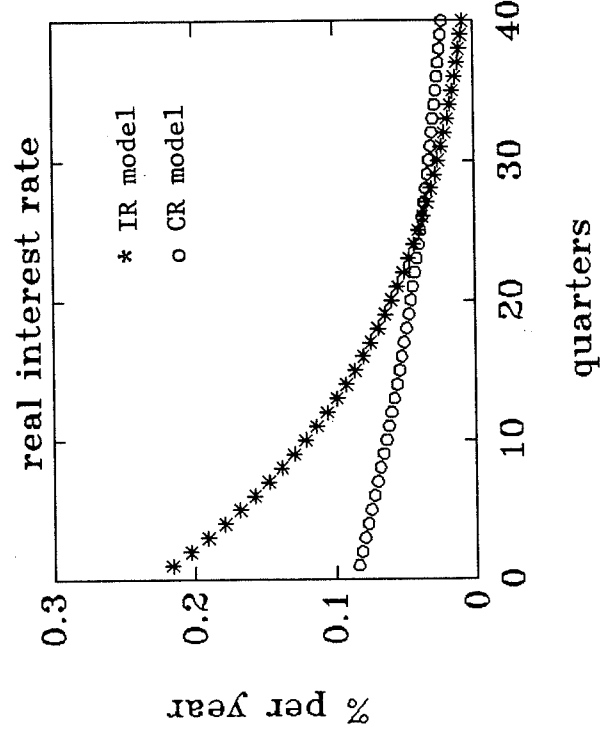
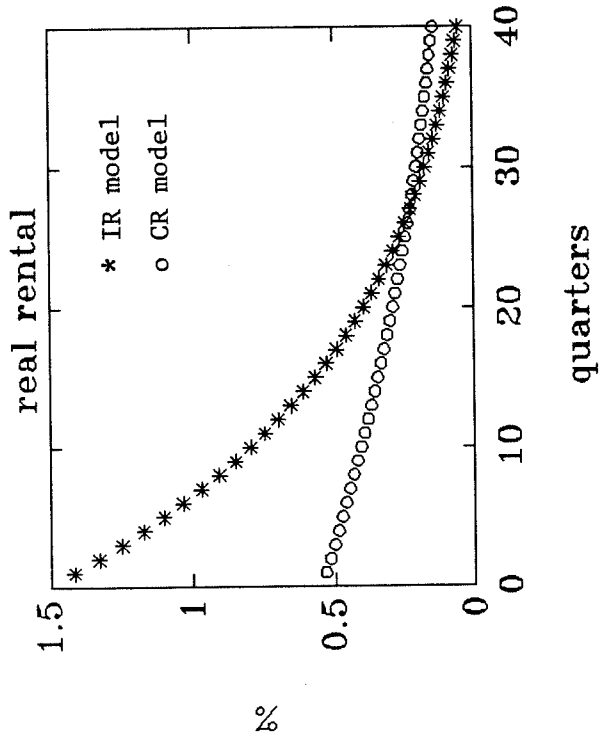


Figure 7-B: Impulse responses to government purchases: prices and interest rates

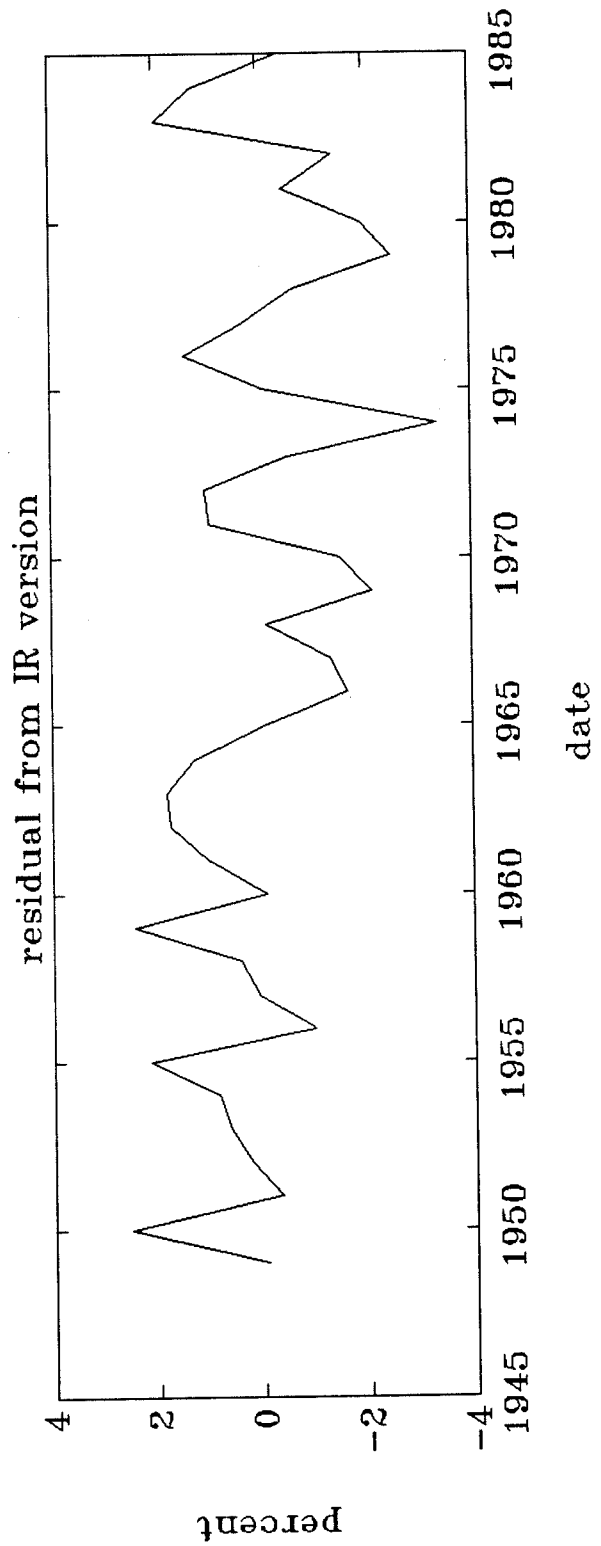
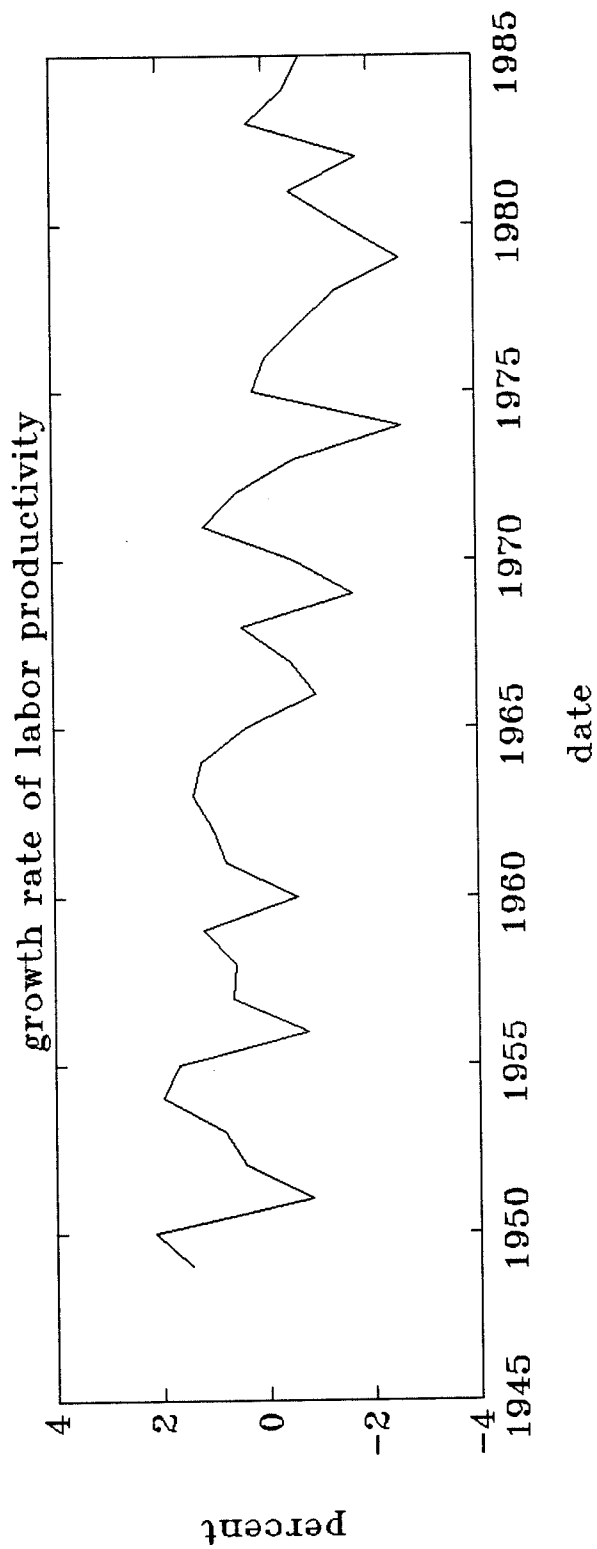


Figure 8: A residual puzzle