Aggregation of Intratemporal Preferences Under Complete Markets

Ogaki, Masao

Working Paper No. 249
September 1990

Aggregation of Intratemporal Preferences Under Complete Markets

Masao Ogaki

Rochester Center for Economic Research
Working Paper No. 249

AGGREGATION OF INTRATEMPORAL PREFERENCES UNDER COMPLETE MARKETS*

Masao Ogaki

University of Rochester

Working Paper No. 249

September 1990

*I thank Lars Peter Hansen for indicating to me that results in Constantinides (1982) and Negishi (1960) can be interpreted as aggregation results. The present paper pursues this interpretation in the context of intratemporal demand. Discussions with Andrew Atkeson, John H. Boyd III, John C. Heaton, and Arthur Lewbel and seminar participants at the University of Rochester were very helpful and conversations with Ariel Pakes motivated this work. All remaining shortcomings are mine.

Aggregation of Intratemporal Demand under Complete Markets

Masao Ogaki

Department of Economics
University of Rochester
Harkness 231
Rochester, New York 14627

Abstract

By extending and applying an aggregation result in financial economics under complete markets to intratemporal demand, the present paper shows that all concave intratemporal utility functions can be aggregated. A fictitious representative consumer is constructed in an economy with heterogeneous consumers whose preferences can be represented by time-additive, von Neumann-Morgenstern utility functions. The distribution of total consumption expenditures does not affect the utility function of the representative consumer. This aggregation result for preferences is valid even when aggregate demand functions do not satisfy usual properties of individual demand functions and depend on the distribution of total consumption expenditures.

I. Introduction

In financial economics, the existence of complete markets has been studied as an important condition for aggregation (see, e.g., Rubinstein (1974)). In particular, Constantinides (1982, Lemma 2) showed thatequilibrium intertemporal asset prices in an economy with heterogeneous consumers can be viewed as equilibrium prices of an economy with one fictitious representative consumer as long as markets are complete and consumers have time-additive, von Neumann-Morgenstern utility functions. On the other hand, in the literature of aggregation for intratemporal demand for multiple goods (see Shafer and Sonnenschein (1982) for a survey), the existence of complete markets has not been studied as a condition for aggregation. The present paper shows that Constantinides's result extends easily to an aggregation result for intratemporal goods and discusses the relation of this aggregation result with some other aggregation results.

This aggregation result applies to all concave intraperiod utility functions and does not need conditions studied in previous work such as restrictions on Engel curves (see, e.g., Gorman (1953), Freixas and Mas-Colell (1987)), restrictions on the form of the cost function (see, e.g., Muellbauer (1976)), or restrictions on the distribution of total consumption expenditures (see, e.g., Hildenbrand (1983) and Lewbel (1988)). This is because the present paper defines aggregation in a different way than those studied in these papers. I will argue that my different definition of aggregation is still useful.

The rest of the present paper is organized as follows. Section II provides a definition of aggregation. Section III develops an aggregation result under complete markets. Section IV discusses an example of this aggregation result developed in Atkeson and Ogaki (1990). Section V discusses the relation of this aggregation result with other aggregation results. Section VI illustrates the use of the aggregation result in Section II. Section VII contains concluding remarks.

II. A Definition of Aggregation

Consider an economy with J consumers, K firms, and n goods. Let $R^n_+=\{x\in R^n: x_i\geq 0,$ for all $i\}$, $R^n_{++}=\{x\in R^n: x_i>0,$ for all $i\}$, $\Gamma=R^n_{++}\times R^1_+$, $\Phi=\{x\in R^J: 0<x_i<1$ for all $i\}$, and $\Lambda=\Gamma\times\Phi$. Let c^j be a consumption vector and $X_j\subset R^n_+$ be a closed and convex consumption set. In each period, consumer j with intraperiod utility function v^j solves an intraperiod problem

$$\text{(P1)}\qquad \underset{c^j}{Max}\ v^j(c^j) \text{ subject to } p'c^j\leq e_j \text{ and } c^j\in X_j,$$

for a price vector p and total consumption expenditure e_j. In order to simplify notations, consider the case where demand functions (rather than demand correspondences) exist. Let $f^j(p,\ e_j)\colon \Gamma\to R^n_+$ be individual demand function that solves (P1) and define an aggregate demand $f^a(p,e_a,\theta^e)=(1/J)\sum_j f^j(p,\ e_j)$, where $e_a\equiv(1/J)\sum_j e_j(t)$ is aggregate (average) total consumption expenditure and $\theta^e=[e_1/(Je_a),\ldots,e_J/(Je_a)]'$ is the distribution of relative total expenditures.

Let $X=\sum_j X_j$ and choose a set of (p,e_a,θ^e) of interest, say Λ^*, which is a subset of Λ. Preferences are said to be aggregated for Λ^* if there exists a utility function v_a which is increasing in each element and quasi-concave, such that $f^a(p,e_a,\theta^e)$ solves

$$\text{(P2)}\qquad \underset{c}{Max}\ v_a(c) \text{ subject to } p'c\leq e_a \text{ and } c\in X$$

for all (p,e_a,θ^e) in Λ^*. Thus for different Λ^*, a different utility function of a fictitious representative consumer v_a may be chosen. Obviously more general conditions for aggregation in this sense can be obtained by restricting Λ^*, but it is necessary to restrict Λ^* in a way that the aggregation result is still useful.

III. Aggregation under Complete Markets

Let a scalar $s(t)$, $s(t)=1,2,\ldots,S$, denote the state of the world in each period and the vector $h(t)=[s(0),s(1),\ldots,s(t)]$ be the history of the economy. Let consumer j have time and state separable utility with an intratemporal utility function $u_t^j(c^j(t,h(t)))$, where $c^j(t,h(t)) = (c_1^j(t,h(t)),\ldots,c_n^j(t,h(t)))'\in X$ for time t. When $h(t)$ is clear from the context, it will be often suppressed in the following. Let $Prob_j(h(t))$ denote the probability of $h(t)$ assumed by consumer j. Following Constantinides, assume

A1: Consumers agree on the probability, so that $Prob_j(h(t)) = Prob(h(t))$ for all $j=1,\ldots,J$.

and

A2: Consumer j has time-additive, time-separable, von Neumann-Morgenstern utility function

$$(1) \qquad U^j(C^j) = \sum_{t=0}^{T} \sum_{e(t)} Prob_j(h(t))u_t^j(c^j(t,h(t))$$

where $C^j=(c_1^j(0),\ldots,c_n^j(T))$ is his or her contingent consumption plan, and $u_t^j(\cdot)$ is increasing in each element, continuous, and concave for $j=1,\ldots,J$.

An additional assumption is necessary to obtain an aggregation result for intraperiod demand:

A3: The intraperiod utility function is of the form $u_t^j(c^j(t))=\beta(t)v_j(c^j(t))$ for all $j=1,\ldots,J$, where $\beta(t)$ is a common discount factor for time t utility.

Let $w^j(t,h(t)) = (w_1^j(t,h(t)),\ldots,w_n^j(t,h(t))'$ be the vector of goods endowed to consumer j at time t and $W^j=(w^j(0),\ldots,w^j(T))$. Let $Z^k=(z^k(0),\ldots,z^k(T))$ be the contingent production plan of the k-th firm, and Y^k be the possible set of Z^k. Let ξ_{kj} be the proportion of profit of the kth firm distributed to the jth consumer. Let $p(t,h(t)) = (p_1(t,h(t)),\ldots,p_n(t,h(t)))'$ be the intratemporal price vector at time t. Define

$$(2)\qquad V(C) = \sum_{t=0}^{T} \sum_{e(t)} (\prod_{\tau=0}^{t} R(\tau,h(\tau))^{-1} p(t,h(t))' c(t,h(t))$$

for any contingent plan $C=(c_1(0),\ldots,c(T))$. Here we take any good, say the first good, as the numeraire in each period $(p_1(t)\equiv 1)$ and $R(t,h(t))$ is the (gross) asset return of the state contingent security for the event $h(t)$ in terms of the first good at time 0. Let $Q = [p(0),R(1),p(1),R(2),\ldots,p(T),R(T)]$ be the price vector and $A=[C^1,\ldots,C^J,Z^1,\ldots,Z^K]$ be an allocation.

An equilibrium is a pair $[A^*,Q^*]$ such that (i) consumer j maximizes (i) subject to $c^j(t)\in X$ and the life-time budget constraint

$$(3)\qquad V(C^j) \leq V(W^j) + \sum_k \xi_{kj} V(Z^k) \equiv M^j$$

where M^j is the consumer j's initial wealth for $j=1,\ldots,J$; (ii) firms maximize profit subject to their technology constraints; and (3) markets clear. With additional standard assumptions on Y^k and endowments, competitive equilibrium exists and is optimal. With an additional assumption $C^{j*}\gg 0$, optimality implies that there exist nonnegative numbers λ_j, $j=1,2,\ldots,J$, such that the equilibrium allocation A^* solves a problem to maximize $\sum_j \lambda_j U^j(C^j)$ subject to the feasibility constraints because utility functions are assumed to be concave (see, e.g., Varian (1984, pp.207-208)). Let θ^λ denote the distribution of λ in the economy. Negishi (1960) showed that the the Pareto weight λ_j is inversely related with the equilibrium marginal utility of the initial wealth. Thus λ_j is positively related with the initial wealth and θ^λ represents the distribution of the initial wealth of the economy.

Define intraperiod utility $u_t^a(c^a(t))$ of aggregate average consumption $c^a(t)=(1/J)\sum_j c^j(t)$ by

$$(4) \qquad u_t^a(c^a(t)) = \max_{c^j} \sum_{j=1}^{J} \lambda_j u_t^j(c^j(t))$$

subject to $c^j(t)\in X$ and $c^a(t)=(1/J)\sum_j c^j(t)$. Define life-time utility of aggregate contingent consumption plan by

$$(5) \qquad U^a(C^a) = \sum_{t=0}^{T} \sum_{h(t)} Prob(h(t))u_t^a(c^a(t,h(t))).$$

Proposition 1 (Constantinides's Lemma): Under Assumptions A1 and A2, $u_t^a(\cdot)$ is increasing and concave and $[C^{a*},Z^{1*},\ldots,Z^{K*},Q^*]$ is an equilibrium in an fictitious economy where J consumers are replaced by one fictitious representative consumer with utility $U^a(C^a)$, endowment the sum of the J consumers' endowments, and shares the sum of the J consumers' shares.

Constantinides's Lemma 2 proves this proposition for $n=1$ and it is straightforward to see that his proof does not need the assumption that $n=1$.

Proposition 1 does not provide an intraperiod aggregation result of the form defined in the previous section because u_t^a depends on t. With an additional assumption A3, define an intraperiod utility $v_a(c^a(t))$ of aggregate average consumption, which is indepedent of t, by

(6) $$v_a(c^a(t)) = \max_{c^j} \sum_{j=1}^{J} \lambda_j v_j(c^j(t))$$

subject to $c^j(t) \in X$ and $c^a(t)=(1/J)\sum_j c^j(t)$. Since $\beta(t)$ is common to all consumers, U^a in Proposition 1 now has the form

(5′) $$U^a(C^a) = \sum_{t=0}^{T} \sum_{h(t)} Prob(h(t))\beta(t)v_a(c^a(t,h(t))).$$

As Proposition 1 states, the fictitious representative consumer maximize (5′) subject to the aggregate version of the life time budget constraint. This implies that the representative consumer solves problem 2 in each period in every state. Otherwise, the life time utility can be increased. Hence

Proposition 2: Under Assumptions 1-3 , in any equilibrium (A^*,Q^*), $c^{a*}(t,h(t))$ solves problem 2 for $p=p^*(t,h(t))$ and $e=e_a^* \equiv (1/J)\sum_j e_j^*(t)$.

Hence for the set Λ^* that consists of (p,e_a,θ^e) which is included in (A^*,Q^*), preferences can be aggregated. It should be noted that the utility function of the fictitious consumer $v_a(c)$ will change when θ^λ is changed. Hence $v_a(c)$ is only valid for a fixed θ^λ. However, θ^e may change even when θ^λ is fixed.

IV. Aggregation of the Extended Addilog Utility Function

Atkeson and Ogaki's (1990) aggregation result is a special case of the aggregation result in the previous section.[1] Their result provides an example of how v_a depends on v_j's and the distribution of initial wealth. In an economy with two goods, they assume that all consumers have identical intraperiod utility function

---

Though Atkeson and Ogaki (1990) treated the case with no uncertainty, their analysis extends the case with uncertainty without any difficulty as discussed in Ogaki and Atkeson (1990).

$$(7) \qquad v(c_1,c_2) = \frac{\rho_1}{1-\alpha_1}\,[(c_1-\gamma_1)^{(1-\alpha_1)}-1] + \frac{\rho_2}{1-\alpha_2}\,[(c_2-\gamma_2)^{(1-\alpha_2)}-1],$$

which may be called the extended addilog utility function. Here γ_1 and γ_2 are subsistence levels of consumption of the two goods. This utility function contains as special cases two utility functions commonly used in demand studies; the linear expenditure system and Houthakker's (1960) addilog utility function. Atkeson and Ogaki showed that the utility function of the fictitious representative consumer is

$$(8) \qquad v_a(c_1,c_2) = \frac{\rho_1}{1-\alpha_1}\,[(c_1-\gamma_1)^{(1-\alpha_1)}-1] + \frac{\rho_2 D}{1-\alpha_2}\,[(c_2-\gamma_2)^{(1-\alpha_2)}-1],$$

where D is a parameter that depends on the distribution of the initial wealth in general. This representative consumer has utility with the same parameters α_1, α_2, γ_1, and γ_2 as the individual consumers.

V. Other Aggregation Results

Gorman (1986)'s definition of aggregation requires the aggregate demand $f^a(p,e_a,\theta^e)$ to be independent of θ^e for all (p,e_a,θ^e) in Λ. For this exact linear aggregation, Engel curves must be linear. Eichenbaum, Hansen, and Richard (1987) linked Gorman's results with results in financial economics. Hildenbrand (1983), Freixas and Mas-Colell (1987), among others, studied conditions under which $f^a(p,e_a,\theta^e)$ given θ^e satisfies various properties of usual individual demand functions such as the strong or weak axiom of revealed preferences. Lewbel (1988) studied conditions under which $f^a(p,e_a,\theta^e)$ is homogeneous in p and e_a as θ^e changes.

Let $g^a(p,e_a)$ be demand of the representative consumer that solves problem (P2). Clearly, $g^a(p,e_a)$ is independent of θ^e and satisfies all properties of the usual individual demand functions for all (p, e_a) in Γ.

However, $g^a(p,e_a)$ is guaranteed to coincide with $f^a(p,e_a,\theta^e)$ only for those (p,e_a,θ^e) in the set Λ^* specified by the equilibrium used to construct v_a. In general, g^a will be different from f^a for other (p,e_a,θ^e). In this respect, my requirements for aggregation are weaker than those of Gorman, Hildenbrand, Freixas and Mas-Colell, and Lewbel. It should be noted, however, that I require a stronger condition that g^a to be independent of θ^e for all θ^e included in Λ^* rather than fixing θ^e. This is important for applications discussed in the next section because θ^e does change over time in practically all time series data.

Muellbauer's (1976) definition of aggregation does not require that the aggregate total expenditure, e_a, to be the total expenditure given to the representative consumer. On the other hand, Muellbauer required his condition of aggregation to be satisfied by all admissible (p, e) in Γ and hence the cost function had to be restricted.

As a result of my different definition of aggregation, all concave utility functions can be aggregated with weaker conditions on the distribution of total consumption expenditures. It is easy to see that concave utility functions allow nonlinear Engel curves and cost functions which are not be of the form that Muellbauer's (1976) Theorem 2B or Theorem 3B require. As a concave utility function that violates Freixas and Mas-Colell's uniform curvature condition, we can take the utility function (9) with $\gamma_1>\gamma_2>0$ and $\alpha_2>\alpha_1>0$. Since the initial distribution of wealth can be changed arbitrarily, my aggregation result places no restriction on the shape of the distribution of total consumption expenditures at a single point of time unlike the work of Hildenbrand. My aggregation result does restrict how θ^e changes over time as Lewbel. This restriction depends on the individual utility functions. As an example of the economy in which

Lewbel's mean scaling condition is not satisfied, we can take the economy studied by Ogaki and Atkeson (1990) in which all consumers have identical utility function (9) with $\gamma_1=\gamma_2=0$. Using the exact solution for the growth rates of individual total consumption expenditures derived by Ogaki and Atkeson, it is admissible to see that θ^e depends on e_a.[2]

VI. Examples

This section illustrates how the aggregation result in Section II can be used.

Estimation of Demand Functions or Utility Functions

In many applications, only available data for econometricians are aggregated over consumers at certain levels. Econometricians often impose restrictions from economic theories in estimating demand functions or utility functions. This type of exercise is often criticized in the literature of aggregation by pointing out that the aggregate demand $f^a(p,e_a,\theta^e)$ need not satisfy all of the usual properties of individual demand functions. Econometricians can avoid this type of criticism if they assume the existence of complete markets.

Suppose that an econometrician observes equilibrium prices and quantities $[p^*(t),c^{a*}(t),e_a^*(t)]$ for $t=1,2,\dots,T$ as one realization of the possible states of the world. From the time series data, the econometrician

---

[2]Let $\omega_j=p_2c_2^j/c_1^j$, $\omega_a=p_2c_2^a/c_1^a$, $\delta_j=c_1^j/c_1^a$, and $\Delta_j=\omega_j/\omega_a$. It is easy to see that consumption for each good and ω_j grows at the same rate for all consumers in this economy. It follows that δ_j and Δ_a are constant over time and across the states of the world. Then $\theta_j^e(t)$ = $\{c_1^j(t)(1+\omega^j(t))\}/\{c_1^a(t)(1+\omega^a(t))\}$ = $\delta_j(1+\Delta_j\omega_a(t))/(1+\omega_a(t))$. This relation implies that θ^e depends on $\omega_a(t)$ and hence on $e_a(t)$.

can estimate demand $g(p,e_a)$ of a fictitious representative consumer, which satisfies all the properties of the usual demand functions even when $f^a(p,e_a,\theta^e)$ does not. The econometrician may estimate the utility function, $v_a(c)$. As long as time series data on $[p^*(t),c^{a*}(t),e_a^*(t)]$ are generated by one Arrow-Debreu equilibrium, the econometrician will never observe those $[p(t),c^a(t),e_a(t)]$ for which f^a violates any of the properties of the usual demand functions.

Thus the aggregation result provides a basis for applications of econometric methods that interpret aggregate time series data of prices and quantities as those generated from one Arrow-Debreu equilibrium. There are many examples of such applications in macroeconomics and labor economics, in which aggregation results of Eichenbaum, Hansen, and Richard (1986) do not apply because of nonhomothetic preference specifications. For example, Mankiw, Rotemberg, and Summers (1985), Miron (1986) and Ogaki (1988, 1989) applied Hansen and Singleton's (1982) Generalized Method of Moments (GMM) approach. Ogaki (1990) and Ogaki and Atkeson (1990) applied Ogaki and Park's (1990) cointegration approach. When an Arrow-Debreu equilibrium is fixed, the marginal utility of initial wealth is fixed. Thus demand (or supply) functions in an Arrow-Debreu equilibrium are called λ-constant, marginal utility of wealth constant, or Frisch demand (or supply) functions (see Heckman (1976), McCurdy (1981), and Browning, Deaton, and Irish (1985)). Though some applications of λ-constant functions used individual data, other applications involve a certain degree of aggregation (see, e.g., Browning, Deaton, and Irish).

It is clear that the assumption of complete markets cannot be taken as literally true in any applications. However, it is not clear whether or not the complete markets assumption is a good enough approximation for particular problems. The complete markets assumption is often taken as the null hypothesis and tests for this null hypothesis are conducted. In constructing test statistics under the this null hypothesis, the aggregation problem does not cause difficulties.

Comparative Statics and Simulations

Demand functions or utility functions are estimated for comparative statics and policy evaluations in many applications. My aggregation result is relevant for λ-constant comparative statics that uses λ-constant demand or supply functions. Examples of applications of λ-constant comparative statics include Heckman (1976), McCurdy (1981), Browning, Deaton, and Irish (1985) who studied intertemporal effects such as labor supply response to growth in real wages. McLaughlin (1989) showed many empirical studies on intertemporal elasticity of substitution can be interpreted as studies to quantify λ-constant comparative statics. Rosen (1985) used λ-constant comparative statics across the states of the world rather than in the intertemporal context. King (1990) used λ-constant comparative statics to analyze macroeconomic effects of taxation.

If consumers insure against all possible government policies, then the government does not have ability to change θ^{λ} even when it can change θ^{e}. In this case, estimated λ-constant demand functions $g^{a}(p,e_{a})$ or the utility function $v_{a}(c)$ can be used to evaluate government policies. On the other hand, in some applications, it may be more realistic to assume that the government may be able to change θ^{λ} by introducing a policy that surprises agents in the economy. In this case, the shape of $g^{a}(p,e_{a})$ and that of $v_{a}(c)$ will change after a policy changes θ^{λ}. Therefore, estimated $g^{a}(p,e_{a})$ or $v_{a}(c)$ are not useful for policy evaluations in general. However, some preference parameters may be known to be independent of θ^{λ}, in which case, estimating $v_a(c)$ can be useful for policy evaluations. For example, if preferences of all consumers in the economy can be represented as the extended addilog utility function explained in Section III of the present paper, then parameters α_1, α_2, γ_1, and γ_2 are not affected by any change in θ^{λ}.

Simulation analyses have been the main vehicle for many areas of economics. Models with one consumer who face some technology constraints are often simulated. It is convenient for researchers to be able to interpret a simulation result as that for a whole economy. My aggregation result provides a basis for such interpretations in models with complete markets (see, e.g., Atkeson and Ogaki (1990) for such an application).

VII. Concluding Remarks

The present paper showed that all concave intratemporal utility functions can be aggregated in the framework of an Arrow-Debreu economy in which consumers have time-additive, von Neumann-Morgenstern utility functions. The distribution of total consumption expenditures changes but does not affect the utility function of the representative consumer as long as the distribution of initial wealth is fixed. This aggregation result is valid even when $f^a(p,e_a,\theta^e)$ does not satisfy usual properties of indivisual demand functions and depends on θ^e. Hence empirical findings that aggregate demand functions depend on θ^e do not necessary mean that aggregation fails. The previous section discussed that this aggregation result is relevant for empirical applications of λ-constant comparative statics and the econometric approaches that interpret aggregate time series data as those generated from one Arrow-Debreu equilibrium.

The present paper assumed that preferences are time-separable. This assumption can be relaxed to some extent in a similar way as Eichenbaum, Hansen, and Richard (1987). I assumed away private information and moral hazard. However, the existence of complete markets insures Pareto optimality even with these factors as Prescott and Townsend (1984a, 1984b) showed. Therefore, it may be possible to obtain an aggregation result in a similar way as Section III in economies with these factors. When incomplete intertemporal markets are introduced, aggregation errors will lead to random shocks to the utility function of a representative consumer. It depends on specific functional forms and estimation methods whether or not these aggregation errors make estimators of demand functions or utility functions inconsistent. For example, Ogaki and Park (1990) discussed conditions under which aggregation errors do not disturb consistency of their estimators of preference parameters in the case of the addilog utility function.

REFERENCES

Atkeson, Andrew and Ogaki, Masao, "Engel's law and Saving," Rochester Center for Economic Research Working Paper No.240, University of Rochester (September 1990).

Browning, Martin, Angus Deaton, and Margaret Irish, "A Profitable Approach to Labor Supply and Commodity Demands over the Life-Cycle," *Econometrica*, 53 (May 1985), 503-43.

Constantinides, George M., "Intertemporal Asset Pricing with Heterogeneous Consumers and without Demand Aggregation," *Journal of Business*, 55 (1982), 253-67.

Eichenbaum, Martin S., Lars Peter Hansen, and Scott F. Richard, "Aggregation, Durable Goods and Nonseparable Preferences in an Equilibrium Asset Pricing Model," NORC Discussion Paper No. 87-9, (1987).

Freixas, Xavier and Andreu Mas-Colell, "Engel Curves Leading to the Weak Axiom in the Aggregate," *Econometrica*, 55 (May 1987) 515-31.

Hansen, Lars Peter and Kenneth J. Singleton, "Generalized Instrumental Variable Estimation of Nonlinear Rational Expectations Models," *Econometrica*, 50, (1982), 1269-86.

Heckman, James, "Life Cycle Consumption and Labor Supply: An Explanation of the Relationship Between Income and Consumption over the Life Cycle," *American Economic Review*, 64 (March 1976) 188-94.

Hildenbrand, W., "On the 'Law of Demand'," *Econometrica*, 51 (1983), 997-1020.

Houthakker, H. S. "Additive Preferences." *Econometrica* 28 (April 1960): 244-56.

King, Robert G., "Observable Implications of Dynamically Optimal Taxation," manuscript, University of Rochester (August 1990).

Lewbel, Arthur. "Exact Aggregation, Distribution Parameterizations, and a Nonlinear Representative Consumer," in G.G. Rhodes and T. B. Fomby, eds., *Advances in Econometrics*, vol. 7, 1988, pp.253-90.

Mankiw, N. Gregory, Julio Rotemberg, and Lawrence H. Summers, "Intertemporal Substitution in Macroeconomics," *Quarterly Journal of Economics*, 100, (February 1985), 225-51.

McLaughlin, Kenneth J., "Intertemporal Substitution and λ-Constant Comparative Statics," Rochester Center for Economic Research Working Paper No.184, 1989.

McCurdy, Thomas E.,"An Empirical Model of Labor Supply in a Life-Cycle Setting," *Journal of Political Economy*, 89 (1981), 1059-85.

Muellbauer, John, "Community Preferences and the Representative Consumer," *Econometrica*, 44 (September 1976), 979-99.

Miron, Jeffrey A., "Seasonal Fluctuations and the Life Cycle-Permanent Income Model of Consumption," *Journal of Political Economy*, 94, (1986), 1258-79.

Negishi, Takashi, "Welfare Economics and Existence of an Equilibrium for a Competitive Equilibrium for a Competitive Economy. *Metroeconomica* 12, (Agosto-Dicembre 1960).

Ogaki, Masao. *Learning about Preferences from Time Trends*, Ph.D. dissertation, University of Chicago (1988).

———. "Information in Deterministic Trends about Preferences." Manuscript, University of Rochester (February 1989).

———. "Engel's Law and Cointegration," Rochester Center for Economic Research Working Paper No. 228, University of Rochester (1990)

Ogaki, Masao and Andrew Atkeson, "A Specification Test for a Model of Engel's Law and Saving," Rochester Center for Economic Research Working Paper No. 236, University of Rochester (1990).

Ogaki, Masao and Joon Y. Park, "A Cointegration Approach to Estimating Preference Parameters," presented at the Conference on Empirical Applications of Structural Models organized by *Econometrica* and Social Systems Research Institute, University of Wisconsin, Madison, Wisconsin, May 1990.

Prescott, Edward C. and Robert M. Townsend, "General Competitive Analysis in an Economy with Private Information," *International Economic Review* 25, 1984, 1-20. (a)

Prescott, Edward C. and Robert M. Townsend, "Pareto Optima and Competitive Equilibria with Adverse Selection and Moral Hazard," *Econometrica* 52, 1984, 21-46. (b)

Rosen, Sherwin, "Implicit Contracts: A Survey," *Journal of Economic Literature* 23 (September 1985), 1144-75.

Rubinstein, Mark, "An Aggregation Theorem for Securities Markets," *Journal of Financial Economics* 1 (1974), 225-44.

Shafer, Wayne and Hugo Sonnenschein, "Market Demand and Excess Demand Functions," in K.J. Arrow and M.D. Intriligator eds., *Handbook of Mathematical Economics*, vol. II, Amsterdam: North-Holland, 1982, 671-93.

Varian, Hal R., *Microeconomic Analysis*, second ed. New York: W.W.Norton, 1984.