A Reformulation of the Economic Theory of Fertility

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Abstract

When parents are altruistic toward children, the choices of fertility and consumption come from the maximization of a dynastic utility function. The maximization conditions imply first, an arbitrage condition for consumption across generations, and second, the equation of the benefit from an extra child to the net cost of rearing that child. These conditions imply that fertility in open economies depends positively on the world interest rate, on the degree of altruism, and on the growth of child-survival probabilities; and negatively on the rate of technical progress and the growth rate of social security. The growth of consumption across generations depends on changes in the net cost of rearing children, but not on interest rates or time preference. Even when we include life-cycle elements, we conclude that the growth of aggregate consumption per capita depends in the long run on the growth of consumption across generations. Thereby we show that real interest rates and growth rates of consumption per capita would be unrelated in the long run.

1. Introduction

The economic approach to fertility has emphasized the effects of parents' income and the cost of rearing children. The most important determinants of cost have been employment opportunities of children, the value of parents' time spent on child care, monetary and psychological costs of avoiding births through abstinence and birth-control methods, and the interaction between the "quality" and quantity of children.

With the exception of work by Richard Easterlin (1973) and a few others (e.g., Becker [1981, Chapter 7]), studies that use an economic approach have neglected the analytical links between decisions by different generations of the same family. Moreover, despite Malthus's famous precedent, fertility has not been integrated with the determination of wage rates, interest rates, capital accumulation, and other macro variables (exceptions include Razin and Ben Zion [1975] and Willis [1985]).

This paper and a sequel develop an economic analysis of both linkages in fertility across generations and of the interaction between fertility and various macro variables (see Barro and Becker [1985] for an earlier version). In the present paper, wage rates and interest rates are parameters to each family and to open economies; the sequel considers the determination of interest rates, wage rates, population growth, and capital accumulation in closed economies.

Our model is based on the assumption that parents are altruistic toward children--hence, the utility of parents depends on the utility of each child as well as on the consumption of parents. By relating the utility of children to their own consumption and to the utility of their children, we derive a dynastic utility function that depends on the consumption and number of descendants in all generations. We venture to use the word "reformulation" in the title because of our emphasis on dynastic utility functions and the number of descendants in all generations. The reformulated approach provides a new way of looking at the determination of fertility.

Section 2 sets out a model of altruism toward children, and derives the budget constraint and utility function of a dynastic family. The first-order conditions to maximize utility imply that fertility in any generation depends positively on interest rates and the degree of altruism, and negatively on the rate of growth in the consumption of descendants. Consumption of a descendant is positively related to the difference between the cost of rearing a descendant and the value of his lifetime earnings.

Section 3 considers the effects of child mortality, subsidies to (or taxes on) children, and social security and other transfer payments to adults. Among other things, we show that the demand for surviving children rises during the transition to low child mortality. However, this demand returns to its prior level once mortality stabilizes at a low level.

Section 4 considers fertility and population growth in economies linked to an international capital market but not an international labor market. Among other results, we show that fertility is reduced by declines in international interest rates, and by increases in an economy's rate of technological progress. This analysis of fertility in open economies may contribute to the explanation of low fertility in Western countries during the past couple of decades.

Section 5 extends the analysis to include full life-cycle variations in consumption, earnings, and utility. We show that fertility depends on the expenditure on the subsistence and human capital of children, but not on expenditures that simply raise the consumption of children. We show also for demographic steady states, that aggregate consumption does not depend on interest rates, time preference or other determinants of life-cycle variations in consumption.

2. A Model of Fertility and Population Change

We assume that each person lives for two generations, childhood and adulthood. By assuming only one period of adulthood we omit life-cycle considerations. However, we show in section 5 how to combine a life-cycle analysis with the intergenerational forces that we stress in sections 2-4. For simplicity, we pretend that each adult has children without "marriage." We believe that production of children through marriage of men and women would not affect the essence of the analysis. We also bypass issues related to the spacing of children by assuming that parents have all of their children at the beginning of adulthood.

Economic analyses of fertility have assumed that the utility of parents depends on the number and "quality" of children, usually without any specification of how or why children affect utility. Although agnosticism about preferences is common among economists, a more powerful analysis of fertility and population change can be obtained by building on recent discussions of altruism toward children.

The importance of altruism within families began to be recognized systematically by economists during the 1970s (two early studies are Barro [1974] and Becker [1974]). Obviously many parents are altruistic toward their children in the sense that the utility of parents depends positively on the utility of children. This paper relies heavily on the assumption of altruism toward children to generate a dynamic analysis of population change.

If the utility of a parent were an additively-separable function of own consumption, denoted c_0, and the utility of each child, then

$$(1) \qquad U_0 = v(c_0) + \sum_{i=1}^{n} \phi_i(U_{1,i}, n_0),$$

where U_0 is parental utility, v is a standard utility function (with $v' > 0$, $v'' < 0$), $U_{1,i}$ is the utility of the ith child, and n_0 is the number of children. Since reactions by parents to differences among their children are not important for the issues discussed in this paper,[1] we simplify by assuming that siblings are identical; hence the function $\phi_i = \phi$ is the same for all children. If this function is increasing and concave in the utility of each child, $U_{1,i}$, then the parent's utility is maximized when all children attain the same level of utility, $U_{1,i} = U_{1,j}$ for all i and j. Then parent's utility can be written as

$$(2) \qquad U_0 = v(c_0) + n_0\phi(U_1, n_0).$$

With the further assumption that the function ϕ is proportional to U_1, so that $\phi(U_1, n_0) = U_1 a(n_0)$, parent's utility would be given by

[1] See the discussions in Becker (1981, Chapter 6), Sheshinski and Weiss (1982), and Behrman, et al. (1982).

$$(3) \qquad U_0 = v(c_0) + a(n_0)n_0U_1.$$

The term $a(n_0)$ measures the degree of altruism toward each child, and converts the utility of children into that of parents. We assume that, for given utility per child U_1, parental utility is increasing and concave in the number of children n_0. This property, together with equation (3), requires the altruism function to satisfy the conditions,

$$(4) \qquad a(n_0) + n_0a'(n_0) > 0 \text{ and } 2a'(n_0) + n_0a''(n_0) < 0,$$

where we neglect integer restrictions on the number of children.

We assume that the parameters of a parent's utility function are the same for all generations of a dynastic family. Therefore, the utility of each child, U_1, depends as in equation (3) on own consumption, c_1, and on the number, n_1, and utility, U_2, of own children. If the presentation is simplified by neglecting utility during childhood (see section 5), then we have after substituting for U_1 in equation (3)

$$(5) \qquad U_0 = v(c_0) + a(n_0)n_0v(c_1) + a(n_0)a(n_1)n_0n_1U_2.$$

Note that U_2 is the utility of each grandchild, and n_0n_1 is the number of grandchildren.

Utility functions like that in equation (3) have been criticized for neglecting altruism toward grandchildren (and perhaps great-grandchildren, etc.). Equation (5) shows that this criticism is invalid because, indirectly, grandparents are altruistic toward grandchildren. A more subtle claim is that the indirect altruism toward grandchildren must be weaker than the direct altruism toward children. Even this criticism does not necessarily hold, because the utility function in equation (5) does not require the altruism toward grandchildren to be less than that toward children. This property holds only if $a(n_1) < 1$.

The utility of great-grandchildren would appear in the utility function if U_2 in equation (5) were replaced by terms that depend on c_2, n_2 and U_3. By continuing to substitute later consumption and fertility, we arrive at a dynastic utility function that depends on the consumption and number of children of all descendants of the same family line. This dynastic utility function can be expressed as

$$(6) \qquad U_0 = \sum_{i=0}^{\infty} A_i N_i v(c_i),$$

where A_i is the implied degree of altruism of the dynastic head toward each descendant in the ith generation, as given by

$$(7) \qquad A_0 = 1, \ A_i = \prod_{j=0}^{i-1} a(n_j), \qquad i = 1, 2, \ldots,$$

n_j is the number of children per adult in generation j, N_i is the number of descendants in the ith generation, as given by

$$(8) \qquad N_0 = 1, \quad N_i = \prod_{j=0}^{i-1} n_j, \quad i = 1, 2, \ldots,$$

and c_i is the consumption per adult in generation i.

At a point where a parent has one child, $n = 1$, we can say that a parent is "selfish" if the marginal utility of his own consumption exceeds the marginal utility that he derives from his child's consumption. This definition implies $a(1) < 1$ for selfish parents. Since the utility of a dynastic family with stationary consumption per person ($c_i = c$) and a stationary number of descendants ($N_i = 1$) would be bounded only if $a(1) < 1$, we assume that parents are "selfish."

The analysis simplifies greatly if the degree of altruism toward children has a constant elasticity with respect to the number of children--that is,

$$(9) \qquad a(n_i) = \alpha(n_i)^{-\epsilon}$$

In this case the degree of altruism toward descendants, A_i in equation (7), depends only on the number of descendants in generation i, $N_i = \prod_{j=0}^{i-1} n_j$-- specifically, $A_i = \alpha^i(N_i)^{-\epsilon}$. We use this simplification for the subsequent analysis. Then the condition $0 < a(1) < 1$ requires $0 < \alpha < 1$, and the condition that parental utility is increasing and concave in the number of children for given utility per child (as ensured by the inequalities in expression (4)) corresponds to $0 < \epsilon < 1$. By substituting the altruism function from equation (9) into the expression for dynastic utility in equation (6), we get

$$(10) \qquad U_0 = \sum_{i=0}^{\infty} \alpha^i (N_i)^{1-\epsilon} v(c_i)$$

Suppose that we change the number of descendants in generation i, N_i, while holding fixed the <u>total</u> consumption, $C_i = N_i c_i$, for generation i, as well as the number and consumption per person in other generations (N_j and c_j for $j \neq i$). Then the change in U_0 measures the benefit or loss from having more people in generation i to consume a given aggregate quantity of goods. Since the production of children is costly, an increase in N_i in this manner must raise U_0 near a utility-maximizing position (if children are being produced). Otherwise, people would do better with fewer children. The derivative of U_0 in equation (10) with respect to N_i--holding fixed C_i and the values of c_j and N_j for all other generations--is positive only if

$$(11) \qquad \sigma(c_i) < 1 - \epsilon,$$

where $\sigma(c_i) \equiv v'(c_i)c_i/v(c_i)$ is the elasticity of utility with respect to consumption c_i. This condition is important for the subsequent discussion.

Each adult supplies one unit of labor to the market and earns the wage w_i.[2] Adults leave a bequest of (non-depreciable) capital, k_{i+1}, to each child. We assume as a convention that bequests occur at the beginning of period i. Since the capital k_i earns rentals at the rate r_i, an adult in generation i spends his total resources, $w_i + (1 + r_i)k_i$, on own consumption,

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[2] The labor-leisure choice can readily be incorporated by including leisure along with consumption in the v function, and by considering a "full-income" budget equation (see Tamura, 1985).

c_i, on bequests to children, $n_i k_{i+1}$, and on costs of raising children. We assume that each child costs β_i, so that $n_i\beta_i$ is the total cost of raising children to adulthood. Therefore, the overall budget condition for an adult in generation i is

$$(12) \qquad w_i + (1 + r_i)k_i = c_i + n_i(\beta_i + k_{i+1})$$

The parameter β_i represents a cost of raising children that is independent of the "quality" of children (as measured by their consumption, c_{i+1}, wage rate, w_{i+1}, or inheritance, k_{i+1}). To capture the emphasis in the fertility literature on the value of parents' time, we sometimes assume that β_i is proportional to the parent's wage rate, w_i. We assume also that debt can be left to children--that is, bequests k_i can be negative as well as positive.[3]

The optimization problem as seen by the dynastic head is to maximize utility U_0 in equation (10), subject to the budget constraints in equation (12) and to the initial assets k_0. In carrying out this maximization, each individual takes as given the path of wage rates, w_i, interest rates, r_i, and child-rearing costs, β_i. The chosen path of consumption per adult, c_0, c_1,

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[3]Consumption c_i and numbers of descendants N_i must also be non-negative in each generation. However, we neglect integer restrictions on N_i. Ponzi games, in which the debt grows forever as fast as or faster than the interest rate, would be ruled out if the present value (as of period 0) of debt approaches zero asymptotically.

c_2, ...; capital stock per adult; k_1, k_2, ...; and number of descendants; N_2, ..., must be consistent with this maximization problem.[4]

The first-order conditions can be obtained in the usual manner, with allowance for a Lagrange multiplier for each period that corresponds to each of the budget constraints in equation (12).[5] The two sets of first-order conditions are

$$(13)\qquad v'(c_{i+1})/v'(c_i) = (n_i)^{\epsilon}/\alpha(1 + r_{i+1}), \qquad i = 0, 1, \ldots$$

and

$$(14)\qquad v(c_i)[1 - \epsilon - \sigma(c_i)] = v'(c_i)[\beta_{i-1}(1 + r_i) - w_i], \qquad i = 1, 2,$$

where $\sigma(c_i)$ is again the elasticity of $v(c_i)$ with respect to c_i.[6] There is

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[4] We pretend that the dynastic head can pick the entire time path. However, descendants will face a problem of the same form, and would have no incentive to deviate from the choices made initially. In other words, decisions are time-consistent across generations. Note also that, as long as all the capital stocks k_i are positive, bequests from parents to children are also positive.

[5] The second-order condition is $\epsilon + (1 - \epsilon)vv''/(v')^2 < 0$ (see the appendix). If $\sigma(c_i)$ is the constant σ, then this condition reduces to $\sigma + \epsilon < 1$, which is expression (11).

[6] We assume that the parameters of the utility function and budget constraint lead to a finite level of utility. For a steady state with constant values of β, w, r, c and n, this requires $(1 + r) > n$, which is the standard condition that the interest rate exceeds the growth rate (of population). equation (13), a constant c implies $n = [\alpha(1 + r)]^{1/\epsilon}$, so that $(1 + r) > n$ requires $(1 + r)^{(1-\epsilon)} < 1/\alpha$. Hence, utility would be unbounded if the interest rate were too high.

also the dynastic budget constraint, which equates the present value of all resources to the present value of all expenditures,[7]

$$(15) \qquad k_0 + \sum_{i=0}^{\infty} d_i N_i w_i = \sum_{i=0}^{\infty} d_i (N_i c_i + N_{i+1}\beta_i),$$

where $d_i = \prod_{j=0}^{i} (1 + r_j)^{-1}$.

Equation (13) is the usual arbitrage condition for shifting consumption from one generation to the next. Aside from the term that depends on fertility, n_i, this equation expresses the familiar result that the utility rate of substitution between consumption in periods i and i+1, $v'(c_{i+1})/v'(c_i)$, depends inversely on the "time-preference" factor, α, and the interest-rate factor, $1 + r_{i+1}$. The standard conclusion is that a rise in α or r_{i+1} increases c_{i+1} relative to c_i. An increase in fertility, n_i, lowers altruism per child, given by $a(n_i)$, and thereby increases the discount on future consumption. Therefore, higher fertility is associated with a reduction in c_{i+1} relative to c_i, for given values of r_{i+1} and α.

Equation (14) says that the marginal benefit of an additional child (or equivalently of an additional adult descendant for the next period) balances the marginal cost. The right side of the equation is the net lifetime cost of an additional adult in generation i. The earnings of each adult, w_i,

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[7]The dynastic budget equation follows from the constraints for each period, as shown in equation (12), as long as the transversality condition is satisfied: the present value of the future capital stock must approach zero asymptotically. We also use the constraint on borrowing, which is discussed in n. 3 above.

subtract from the cost of rearing a child in generation i-1, valued in goods of generation i ($\beta_{i-1}(1 + r_i)$). The left side of equation (14) is essentially the effect on utility from adding an additional adult descendant in generation i, N_i, while holding fixed the total consumption C_i of that generation.[8] As discussed earlier, this marginal utility must be positive near an optimal position, which implies $1 - \epsilon - \sigma(c_i) > 0$ (see expression (11)).

Equation (14) indicates that children would be a financial burden to altruistic parents: the cost of rearing an additional child would exceed his <u>lifetime</u> earnings. Otherwise, altruistic parents would maximize utility by having as many children as were biologically feasible. Caldwell (1982), among others, argues that parents in less-developed countries want as many children as possible because children are profitable in the sense that their earnings as child laborers exceed the cost of rearing them. Altruistic parents would want as many children as possible even when child labor was not profitable, if the <u>lifetime</u> earnings of children exceeded the cost of rearing the marginal child.

With the definition, $\sigma(c_i) \equiv v'(c_i)c_i/v(c_i)$, equation (14) can be rewritten as

$$(16)\quad c_i[1 - \epsilon - \sigma(c_i)]/\sigma(c_i) = \beta_{i-1}(1 + r_i) - w_i, \quad i = 1, 2, \ldots$$

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[8] Differentiate the appropriate term in equation (10) with respect to N_i while holding fixed C_i. Aside from the factor, $\alpha^i(N_i)^{-\epsilon}$, the result is the left side of equation (14).

The left side would be proportional to c_i if $\sigma(c_i)$ were constant. Otherwise, we assume that $\sigma(c_i)$ either falls or increases slowly enough with c_i so that the left side is increasing in c_i. Then equation (16) implies that c_i is a positive function of the net cost of producing another descendant in generation i. It follows that consumption per person, c_i, grows across generations only if the net cost of creating descendants also grows. Hence, descendants have the same consumption if they are equally costly to produce. In contrast, the usual models of optimal consumption over time imply that consumption grows (or falls) over time if the interest rate exceeds (or is less than) the rate of time preference. In our analysis, the rate of growth between generations of consumption per person is essentially independent of the level of interest rates, and also does not depend on the rate of altruism or time preference, as summarized by the parameter α.

The main effects from changes in the level of interest rates or in the degree of altruism show up on fertility, n_i. We can rewrite equation (13) to solve out for the fertility rate:

$$(17) \qquad n_i = [\alpha(1 + r_{i+1})v'(c_{i+1})/v'(c_i)]^{1/\epsilon}, \qquad i = 0, 1, \ldots$$

Equation (16) pegs the intertemporal-substitution term, $v'(c_{i+1})/v'(c_i)$, for $i = 1, 2, \ldots$, because c_i in each future generation depends only on the net cost of producing descendants. For example, if the cost were the same for all generations, the intertemporal-substitution term would be unity. Since substitution in consumption is pegged, the fertility rate, n_i for $i = 1, 2, \ldots$, rises with increases in the interest rate, r_{i+1}, or the rate of altruism, α. Note that these responses satisfy the arbitrage condition for shifting consumption across generations, even though the time path of consumption per person does not change. For this given behavior of consumption per person, higher values of interest rates or of the rate of altruism motivate a family to have descendants later rather than sooner--that is, fertility rates rise.

Richard Easterlin, a pioneer in analyzing the effects of intergenerational relations on fertility, has argued (1973) that fertility depends negatively on the wealth of parents compared to own wealth because growing up in a wealthy family shifts preferences toward own consumption at the expense of children. Put differently, fertility is said to be positively related to the growth in wealth from the previous to the present generation. Fertility in our model also depends on the rate of growth between generations, but it depends <u>negatively</u> on the growth between own consumption and consumption per capita of the <u>next</u> generation. Moreover, in our model, preferences are invariant with wealth and have the same form for each generation.

Another important property of the model concerns the effect of changes in wealth, which we can represent by shifts in the initial assets k_0. Equation (16) implies that consumption per person in each future generation, c_i for $i = 1, 2, \ldots$, depends only on the net cost of descendants, $\beta_{i-1}(1 + r_i) - w_i$. If a shift in wealth leaves unchanged the cost of raising children, β_{i-1}, and the wage rate of descendants, w_i, then there is no effect on future consumption per person, c_i. In this case it also follows from equation (17) that future fertility, n_i for $i = 1, 2, \ldots$, does not change with a shift in wealth. With future consumption per capita and future fertility unchanged, the dynastic budget equation (15) implies that either initial consumption, c_0, or fertility, n_0, must change. Using equation (17) for $i = 0$, we can see that an increase (or decrease) in c_0 must be accompanied by an increase (or decrease) in n_0. That is, wealthier persons consume more and also have larger families.

The results imply that an increase in say inherited wealth would increase only the scale of a dynastic family. The number of descendants, N_i, and aggregate consumption, C_i, in each future generation would increase by the same proportion as the increase in initial fertility, n_0. We can see this result directly by recalling that $N_i = n_0 n_1 \ldots n_{i-1}$ for $i = 1, 2, \ldots$ Substitution for each fertility rate from equation (17) leads to

$$(18) \qquad N_i = \left\{\alpha^i [v'(c_i)/v'(c_0)] \cdot \prod_{j=1}^{i} (1 + r_j)\right\}^{1/\epsilon}, \qquad i = 1, 2, \ldots$$

An increase in wealth raises c_0 and thereby lowers the marginal utility of wealth, $v'(c_0)$. Since all future values of c_i are unchanged, equation (18) shows that all future values of N_i rise by the same proportion.

Future capital per person, k_i for $i = 1, 2, \ldots$, would not change with a shift in wealth because future consumption per person, c_i, and fertility, n_i, are unchanged. This result follows from the budget conditions in equation (12) and from the dynastic budget constraint in equation (15). Consequently, bequests to each descendant of the dynastic head are unaffected by a change in dynastic wealth.

Stated differently, wealth completely regresses to the mean between parents and each child because wealthier parents would spend all of their additional resources on their own consumption and on raising larger families. A positive relation between wealth and fertility may help to explain the significant regression toward the mean in the wealth of parents and each child in the United States and other countries (see Becker and Tomes, 1986, Table 2). Although our analysis goes too far by implying complete regression to the mean over one generation, we show below that this extreme result no longer holds if the cost of having children, β_i, depends on the number of children.

Dynastic utility in equation (6) is a time-separable function of the number of descendants and consumption in each generation, and does not depend explicitly on the fertility of any generation. Demand functions derived from time-separable utility functions depend only the marginal utility of wealth and the prices of variables with the same date. Consequently, if we hold constant the marginal utility of wealth, the number of descendants and consumption in any generation would not be affected by price changes in other generations. For example, an increase in the net cost of producing descendants for generation j alone would not change the number of descendants in later generations, i > j. Fertility would rise sufficiently between the jth and (j + 1)st generations to maintain the number of descendants in all later generations.

To illustrate these results, consider a tax on raising children in generation j, which raises β_j but does not change β_i for $i \neq j$. Furthermore, to abstract from wealth effects (which we have already discussed), assume a compensating increase in initial assets, k_0, that leaves the marginal utility of wealth, $v'(c_0)$, unchanged. Equation (16) indicates that c_{j+1} rises, but all other c_i do not change. Equation (17) indicates that fertility in generation j falls (children are now more costly to produce), while fertilty in j + 1 rises. Moreover, equation (18) implies that the increase in fertility in generation j + 1 exactly offsets the fall in generation j. Hence, the number of descendants after the (j + 1)st generation is not affected by the tax in generation j.

Similarly, a decrease in wages for one generation, compensated to hold constant the marginal utility of wealth, reduces fertility in the previous generation and raises fertility in the same generation by equal percentages. Again, the number of descendants in later generations does not change. As it were, the fertility rates of adjacent generations are perfect substitutes in the production of descendants: any change in the net cost of producing descendants in one generation causes enough substitution from the fertility in the succeeding generation to leave unchanged the number of descendants in subsequent generations.

Consider now a permanent increase in the cost of children that raises the net cost of children, $\beta_i(1 + r_{i+1}) - w_{i+1}$, by the same proportion for each generation $i \geq j$. If we again hold constant the marginal utility of wealth, $v'(c_0)$, equation (16) implies that consumption per person rises in generation j + 1 and in each subsequent generation. Further, if we now assume as an approximation that the elasticity of utility is the constant σ, then the increases in c_{j+1}, c_{j+2}, ... are equiproportional. The arbitrage condition for consumption over time in equation (13) simplifies in this case to

$$(19) \qquad (c_i/c_{i+1})^{1-\sigma} = (n_i)^{\epsilon}/\alpha(1 + r_{i+1}), \qquad i = 0, 1, \ldots$$

Therefore, the equiproportional increases in c_i for $i = j + 1, \ldots,$ imply that fertility in generation j falls (because c_j/c_{j+1} declines), while fertility in all other generations is unchanged. Consequently, given interest rates, even a permanent (compensated) tax on children reduces fertility only in the generation where the tax is first enacted. However, the effect on incentives is permanent because the utility function of an altruistic, dynastic family depends on the number of descendants, rather than on fertility itself, and the decline in fertility in one generation alone reduces the number of descendants in all later generations.

The permanent increase in β_i for $i \geq j$ means also that consumption per person, c_i for $i > j$, is permanently higher. Since β_i is higher while n_i is unchanged (for $i > j$), the right side of equation (12) indicates a larger level of expenditure per person in each period. This greater expenditure is financed by higher levels of capital and bequests per person, k_i for $i > j$.[9]

The assumption that the cost of rearing each child is independent of the number of children is crucial to our conclusion first, that wealth completely regresses to the mean over one generation, and second, that a permanent tax on children reduces fertility only in the initial generation. The qualitative properties survive natural modifications of this specification, but adjustments to disturbances can now take several generations.

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[9] This result assumes that the family's total capital stock is growing at a rate below the interest rate--that is, if the capital stocks are positive, $n_i k_{i+1}/k_i < 1 + r_i$. This condition must hold in a steady state, where $k_{i+1} = k_i$ and $1 + r > n$--see n. 6 above.

To illustrate, let the cost of rearing n children be given by

$$(20) \qquad b(n), \text{ where } b' > 0 \text{ and } b'' \gtrless 0.$$

Previously, the marginal cost, $b'(n)$, equaled the constant β. Now marginal costs increase or decrease with n, depending on the sign of $b''(n)$. Forces that favor $b'' > 0$ are a rise in the shadow price of parents' time as the amount of child-care increases, and an increase in the physical burden of bearing children as the number of children increases. On the other hand, b can be negative over some range because of economies of scale in raising children, and because the opportunity cost of preventing births would fall when the number of children rose. We assume that the first set of forces eventually dominates, so that $b'' > 0$ applies in the range of our analysis.

This modification to the cost of rearing children does not affect the arbitrage condition for shifting consumption across generations, as given in equation (13). However, the other set of first-order conditions changes fr that shown in equation (14) to

$$(21) \quad c_i[1 - \epsilon - \sigma(c_i)]/\sigma(c_i) = b'(n_{i-1})(1 + r_i) - w_i - n_i[b'(n_i) - b(n_i)/n_i],$$
$$i = 1, 2, \ldots$$

The expression on the right side is the net cost of raising an additional descendant for generation i when the number of descendants in other generations is held constant. The term, $b'(n_{i-1})(1 + r_i) - w_i$, corresponds to that in our previous analysis (where $b'(n_{i-1}) = \beta_{i-1}$). The new term on the far right arises because an increase in n_{i-1}, and hence in N_i, means that n_i would fall for a given value of N_{i+1}. This new term is negative if the marginal cost of raising children in generation i, $b'(n_i)$, exceeds the average cost, $b(n_i)/n_i$, for then a decrease in n_i lowers the average cost of raising children in generation i.

Even with a rising or falling marginal cost of rearing children, equation (21) implies that the steady-state value of c depends on the steady-state value of n, and on the values of $b'(1 + r)$ and w. Equation (13), in turn, implies that the steady-state fertility rate still depends only on the interest rate and the rate of altruism, α.

When the marginal cost of children is constant, we have shown that transitions between steady states take only one generation. Steady states remain stable even when marginal costs are increasing (see Section 2 of the appendix), but transitions now take several generations.

As an example of the dynamic effects that arise, consider an increase in initial assets, k_0. When the marginal cost of children was assumed to be constant, we found that an increase in k_0 increased c_0 and n_0, but did not change future values of c_i and n_i. If marginal costs are rising ($b'' > 0$), an increase in n_0 raises $b'(n_0)$, which increases c_1 (by equation (21)). An increase in c_1 raises n_1 (by equation (13)), which increases c_2, and so on. In this way, the increase in wealth would be spread over increases in consumption per person and fertility in several generations. However, the effects on consumption per person and fertility become smaller over time as the steady state is approached. Although consumption per person and assets per person still regress to the mean from parents to children, the process is now a gradual one across generations, rather than being completed in a single generation.

III. Economic Growth, Child Mortality and Social Security

Return now to the simpler case where the cost per child β_i is independent of the number n_i. Then a temporary change in the cost of children in one generation induces an oscillation in fertility over that generation and the subsequent one, whereas a permanent change in cost starting in a particular generation alters fertility only in that generation. A permanent fall in fertility would require a continuous rise in the cost of children. Assume, for example, that the interest rate is the constant r, while wages, the cost of rearing children, and hence the net cost of descendants are each rising at a constant rate--due say to steady economic growth. Then, if the elasticity of utility is the constant σ, equation (16) implies that the consumption of descendants, c_i, would rise at the same rate as the net cost of descendants. A higher rate of growth in consumption per person would reduce fertility permanently, given the values of α and r (see equation (19)).

The decline in fertility observed since the mid-19th century in most Western countries has sometimes been explained by rapid economic growth that continues to raise the cost of children through raising the value of parents' time (see, for example, Becker, 1981, Chapter 11). This explanation has not been based on a formal model that links fertility to economic growth, and our model does not have this implication. As we have seen, a steady rate of economic growth that induced a steady growth in the net cost of descendants would permanently lower fertility, but would not generate a persistent fall in fertility. A persistent fall requires either that interest rates fall steadily, or that economic growth continues to accelerate, or that the net cost of descendants accelerates for other reasons.

The secular decline in fertility has also been explained by the secular decline in child mortality that continued to reduce the number of births required to produce a target number of surviving children. Our analysis also has novel implications about the effects of declines in child mortality on birth rates and the demand for surviving children.

To simplify the presentation, we assume that wage rates and interest rates are constant over time. Also, parents ignore the uncertainty about child deaths, and respond only to changes in the fraction, p, of offsprings that survive childhood. If n_b is the number of births, then the expected number of survivors is

$$(22) \qquad n = pn_b$$

Let β_s be the cost of rearing a child to adulthood, and β_m the cost of a child that dies prior to becoming an adult. Since the latter cost includes any psychic losses from child mortality, β_m could exceed β_s. The expected cost of n_b births is $[p\beta_s + (1 - p)\beta_m]n_b$. Equation (22) implies that the ratio of this expected cost to the expected number of survivors n--which corresponds to our previous cost per (surviving) child--is

$$(23) \qquad \beta = \beta_s + \beta_m(1 - p)/p.$$

As before, parents choose own consumption, the expected number of surviving children, and bequests to surviving children, subject to a budget constraint that depends on β.

A permanent decline in the level of child mortality--that is, a rise in p--that starts in the jth generation would lower the cost of raising surviving children, β_i for $i \geq j$. Our prior analysis implies that the demand for surviving children per adult (n_i) rises in the jth generation, but would not be affected in later generations.[10] Although the demand for surviving children increases in generation j, birth rates may fall because the higher probability of survival, p, reduces the number of births, n_b, needed to produce a given number of survivors (see equation (22)). Birth rates definitely fall in later generations because the demand for surviving children in these generations would not be affected by the increase in p.

The demand for surviving children per adult would increase for more than one generation if the probability of surviving childhood continued to rise, because then the cost of rearing surviving children would continue to fall over time. However, the rate of increase in the survival probability must slow down once this probability approaches unity, as it has in the West during the past forty years. As the rate of increase slows, the rate of decline in the cost of producing survivors also slows and eventually more or less ceases. Thereafter, the cumulative increase in child survival probability would not affect the demand for surviving children, but would reduce birth rates by the same percentage as the increase in survival probability.

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[10] If the marginal cost of a child increases with the number of children ($b'' > 0$ in expression (20)), future values of n_i for $i > j$ would rise in response to a permanent decline in mortality rates starting in generation j. But eventually, the demand for surviving children would return to its previous value.

Our analysis of altruistic dynastic families explains why the transition to regimes of low child mortality may have only temporary effects on population growth, and why changes in birth rates often lag behind changes in child mortality. The analysis implies a rise in population growth during the early phases of the transition, but eventually population growth would return to that prevailing prior to the transition. Correspondingly, the declines in birth rates accelerate, until the percentage decline from prior levels equals the percentage increase in the probability of surviving to adulthood.

The secular decline in fertility has also been explained partly by the growth in social security and other transfer payments to the elderly. Public transfers to old persons would reduce the demand for children if support from children had been helping to protect parents against ill health, low earnings, and other conditions of old age. Our model of altruistic families implies that <u>growing</u> transfer payments to the elderly would reduce the demand for children, even when children do not support elderly parents.

The model is not set up to incorporate social security precisely because we have only one period of adulthood, and cannot introduce payments to old (retired) adults that are financed on a pay-as-you-go basis by taxes on young (working) adults. However, similar results obtain if we imagine (unrealistically) that transfers to adults are financed by levies on children.

Let s_i be the transfer received by the representative adult in generation i, and τ_{i+1} be the tax paid during generation i by each child (or by parents on behalf of their children). Then the government's budget constraint is $s_i N_i = \tau_{t+1} N_{i+1}$, which implies

$$(24) \qquad \tau_{i+1} = s_i/n_i$$

For given values of fertility, the benefits from social security and the taxes to finance them have exactly offsetting effects on the dynastic wealth of the representative family. Therefore, a change in the scale of the social security program would not affect intergenerational patterns of consumption if fertility were unchanged. Parents would use their social security benefits to pay their children's taxes; in a more general context, parents would raise their bequests sufficiently to enable their children to pay these taxes without cutting back on their consumption (see Barro, 1974).

However, this so-called "Ricardian Equivalence Theorem" must be modified when parents choose the number of children. An extra child in generation i would pay the tax, $\tau_{i+1} = s_i/n_i$, and would receive the transfer s_{i+1} when he becomes an adult. Thus, the social security program imposes the net tax per child of

$$(25) \qquad s_i/n_i - s_{i+1}/(1 + r_{i+1}),$$

where $s_{i+1}/(1 + r_{i+1})$ is the present value of the future transfer. With constant benefits per person ($s_{i+1} = s_i = s$), the net tax is positive if $1 + r_{i+1} > n_i$.[11] Recall that this condition must hold in the model in a steady state.

[11] More generally, we need that total social security payments grow slower than the interest rate--that is, $n_i s_{i+1}/s_i < 1 + r_{i+1}$.

With $1 + r_{i+1} > n_i$, an increase in the scale of the social security program (an increase in s) would raise the net cost of a child and would have the same substitution effects as an increase in the cost of raising a child, β_i. Therefore, our previous analysis of changes in the costs of children applies to changes in social security. For example, a permanent increase in the level of social security benefits starting in generation j is analogous to a permanent increase in β_i at this date. Holding fixed the marginal utility of wealth, $v'(c_0)$, we found that fertility n_j declines, fertility in later generations, n_i for $i > j$, does not change, and the number of descendants, N_i for $i > j$, falls by the same percentage in each future generation. Therefore, a permanent increase in social security benefits tends to reduce fertility (temporarily) even when children do not support their parents.[12]

We also found before that a permanent increase in child-rearing costs in generation j would raise consumption and wealth per person in future generations, c_i and k_i for $i > j$. In the same way, the negative effect of higher social security benefits on the number of descendants would be associated with an increase in "capital intensity." This finding contradicts the usual argument, as in Feldstein (1974), that social security lowers capital intensity. That argument treats fertility as exogenous and neglects the interplay between consumption and intergenerational transfers.

Our model implies that the dramatic growth in transfer payments to the elderly during the past 50 years reduced fertility rates. However, it also implies that fertility would return to its prior value once the growth in

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[12] For discussions of the initial impact of social security, see Becker and Tomes 1976, n. 15, Wildasin 1985, and Willis 1986.

these transfers slowed appreciably. Therefore, in the long run, a larger social security program would not affect fertility rates, but would lower population levels and raise consumption and wealth per person.

4. Open Economies and Western Fertility

Our formulation can be used to analyze the determinants of fertility in an open economy, defined as an economy connected to an international capital market with a single real interest rate. Wages are determined in each economy separately because labor is assumed to be immobile across national boundaries. Wage rates (per unit of human capital) would differ between economies with the same interest rate if production functions differed, if returns to scale were increasing or decreasing, or if wages were taxed at different rates.

If the elasticity of the utility function is the constant σ, then equation (19) implies

$$(26)\quad n_i^j = \left[\alpha^j(1 + r_{i+1})\right]^{1/\epsilon^j}\left[c_i^j/c_{i+1}^j\right]^{(1-\sigma^j)/\epsilon^j},$$

where the superscript j denotes the country and r_{i+1} is the world interest rate. Defining $n_i^j = 1 + \rho_i^j$, where ρ_i^j is the (natural) growth rate of the adult population in country j between generations i and i+1, we have

$$(27)\quad \rho_i^j \approx \frac{\log(\alpha^j)}{\epsilon^j} + \frac{r_{i+1}}{\epsilon^j} - \left[\frac{1-\sigma^j}{\epsilon^j}\right]g_i^j,$$

where g_i^j is the growth rate of consumption per person in country j between generations i and i+1. Equation (16) implies that this growth rate is

$$(28) \qquad g_i^j = \frac{c_{i+1}^j - c_i^j}{c_i^j} = \frac{\beta_i^j(1 + r_{i+1}) - w_{i+1}^j}{\beta_{i-1}^j(1 + r_i) - w_i^j} - 1.$$

The first term on the right-hand side of equation (27) indicates that population grows more rapidly in economies where parents are more altruistic (α^j), and where the degree of altruism is less responsive to the number of children (ϵ^j). The second term shows that population growth is more rapid when the world real interest rate is higher. Moreover, the responses in population growth exceed variations in interest rates because $\epsilon^j < 1$. For small values of ϵ^j, even small fluctuations in world interest rates would induce sizable fluctuations in population growth rates. The term on the far right indicates that population grows more rapidly in open economies where consumption per person grows more slowly. Open economies would differ more in population growth than in consumption growth because $(1 - \sigma^j)/\epsilon^j > 1$ in each economy (see expression (11)).

The growth in consumption per person between generations equals the growth in the net cost of descendants [see equation (28)]. The latter is negatively related to growth in the probability of child survival, and is positively related to growth in social security benefits or in other taxes on children. Faster technical progress also raises the growth of consumption per person, at least if the cost of raising children, β_i^j, tends to grow along with wage rates. Therefore, population growth should be lower in open economies that have more rapid technological progress, more rapid increases in social security benefits, and slower declines in child mortality.

These implications of our analysis seem relevant to the low fertility in Western countries since the late 1960s. World interest rates were low until the 1980s--for example, interest rates on short-term U.S. government securities averaged 1.8% per year from 1948 to 1980 after adjusting for anticipated inflation (see Barro, 1986 Ch. 7). Economic growth was rapid--specifically, the annual rate of growth in per capita real GDP averaged 3.7% per year from 1950 to 1980 in 9 industrialized countries that include the United States (Barro, 1986 Ch. 11). Child mortality in the West was already quite low by 1950 and did not improve much further. Social security payments and other transfers to adults expanded dramatically during the past thirty years. For example, per capita real social security payments in the United States and Great Britain grew by 7 1/2 and 5 percent per year, respectively, from 1950 to 1982 (see Hemming, 1984, and U.S. Department of Commerce, 1965, 1984). All of these forces tended to hold down population growth.

These considerations also suggest that Western fertility will rise during the next decade if the high real interest rates of the 1980s continue into the 1990s, if the growth in social security and other transfer payments slows appreciably--as eventually it must--and if the slowdown in economic growth that began in the early 1970s continues. Moreover, fertility could respond sharply even to small changes in interest rates, in the growth rate of transfer payments, and in rates of economic growth, because changes in these variables are magnified into larger changes in fertility.

5. Life-Cycle and Aggregate Consumption

To simplify the presentation, we have assumed that childhood and adulthood are the only periods of life, and that childhood provides no utility. However, we can readily incorporate a full life cycle into the model. We use this extended model to compare the determinants of consumption over the life cycle with the determinants between generations, and also to show how aggregate consumption relates to life-cycle and generational consumption.

We continue to neglect uncertainty about age of death, but now assume that everyone lives for ℓ years. A parent is assumed to have all his children when he is h years old, where the value of h determines the length of a generation. We assume additive preferences over the life cycle, where $v_j(c_{ij})$ is the utility at age j of each descendant in generation i from the consumption, c_{ij}. These current period utilities over the life cycle are discounted by the constant time-preference factor, δ. Therefore, the utility generated by the lifetime consumption of someone in generation i is

$$(29) \qquad v_i = \sum_{j=1}^{\ell} \delta^{j-i} v_j(c_{ij})$$

As before, the overall utility of the dynastic head is

$$(30) \qquad U_0 = \sum_{i=0}^{\infty} A_i N_i v_i ,$$

where A_i is the weight attached to the utils of generation i. We now note explicitly that A_i incorporates both time preference and the degree of altruism toward children. As before, the degree of altruism toward each child varies inversely with the number of children. Specifically, we again assume a function of the form $\tilde{\alpha}(n_i)^{-\epsilon}$. Then the weight A_i in equation (30) is

$$(31) \qquad A_i = (\tilde{\alpha}\delta^h)^i (N_i)^{-\epsilon} = \alpha^i (N_i)^{-\epsilon}$$

Note that the parameter α (which corresponds to the parameter α in our previous model) equals $\tilde{\alpha}\delta^h$--that is, it includes both altruism ($\tilde{\alpha}$) and time preference (δ^h).

Substitution from equation (31) into equation (30) leads to the expression for the head's utility,

$$(32) \qquad U_0 = \sum_{i=0}^{\infty} \alpha^i (N_i)^{1-\epsilon} v_i ,$$

where v_i is given in equation (29). Note that life-cycle utilities, $v_j(c_{ij})$ in equation (29), are discounted only by time preference (δ), while generational utilities, v_i in equation (32), are discounted also by the degree of altruism toward descendants (that is, by $\alpha^i (N_i)^{1-\epsilon} = (\tilde{\alpha}\delta^h)^i (N_i)^{1-\epsilon}$). Some have claimed that rational individuals would not discount future utility. However, even if people did not discount the future ($\delta = 1$), the coefficient A_i in equation (31) need not equal unity because rational individuals might prefer their children's consumption to their own, or vice versa. The latter condition holds in biological models of gene maximization when a parent only has some genes in common with each offspring.

If parents are "selfish" ($\tilde{\alpha} < 1$), dynastic utility would be bounded in a stationary state ($N_i = 1$ and $c_{ij} = c_{kj}$ for all i, k) even without preference for the present (even if $\delta \geq 1$) as long as $\delta < (\tilde{\alpha})^{-1/h}$. Models of infinitely lived individuals--which do not distinguish between time preference and altruism toward descendants--typically assume preference for the present ($\delta < 1$) in order to bound the utility function.

The dynastic budget equation with a full life cycle is

$$k_0 + \sum_{i=0}^{\infty} (1 + r)^{-hi} N_i \bar{w}_i = \sum_{i=0}^{\infty} (1 + r)^{-hi} [N_i \bar{c}_i + N_{i+1} \beta_i] \tag{33}$$

where

$$\begin{aligned} \bar{w}_i &= \sum_{j=1}^{\ell} (1 + r)^{-(j-1)} w_{ij} \\ \bar{c}_i &= \sum_{j=1}^{\ell} (1 + r)^{-(j-1)} c_{ij} \end{aligned} \tag{34}$$

are the present values at the beginning of generation i of lifetime earnings and consumption for generation i. For convenience, we now assume a constant interest rate, r.

When dynastic utility in equation (32) is maximized subject to dynastic resources in equation (33), the first-order conditions are

$$(35) \qquad \frac{v'_{j+1}(c_{i,j+1})}{v'_j(c_{ij})} = \frac{1}{\delta(1+r)}, \qquad \text{for } i = 0, 1, \ldots, \text{ and } j = 1, 2, \ldots, \ell,$$

and

$$(36) \qquad \frac{v'_j(c_{i+1,j})}{v'_j(c_{ij})} = \frac{n_i^{\epsilon}}{\alpha(1+r)^h}, \quad \text{for } i = 0, 1, \ldots, \text{ and } j = 1, 2, \ldots, \ell.$$

Equation (35) is the usual arbitrage relation for life cycles--specifically, it involves the interest rate, r, and the time-preference factor, δ. Since the right-hand side of equation (35) is independent of the generation, i, the ratio of marginal utilities of consumption over the life cycle is the same for all generations.

Equation (36), which is the arbitrage relation across generations, is essentially the same as equation (13). Since the right-hand side of equation (36) is independent of age, j, we find that the ratio of marginal utilities of consumption between generations is the same for all ages.

We can solve the arbitrage relations for the rates of growth of consumption between ages and generations if we again assume that the elasticity of utility with respect to consumption is the constant σ:

$$(37) \qquad v_j(c_{ij}) = \frac{\phi_j c_{ij}^{\sigma}}{\sigma}, \quad \text{for all } i, j.$$

The term ϕ_j captures any effects of age on the marginal utility of consumption. For example, the relatively low consumption of young children that has stimulated the literature on child-equivalent scales implies relatively low values of ϕ at young ages. By substituting equation (37) into equations (35) and (36), we get

$$(38) \qquad \frac{\phi_{j+1}}{\phi_j}\left[\frac{c_{ij}}{c_{i,j+1}}\right]^{1-\sigma} = \frac{1}{\delta(1+r)}, \qquad \text{for all } i,\ j$$

and

$$(39) \qquad \left[\frac{c_{ij}}{c_{i+1,j}}\right]^{1-\sigma} = \frac{n_i^{\epsilon}}{\alpha(1+r)^h}, \qquad \text{for all } i,\ j.$$

If σ is constant, the first-order condition that relates consumption to the net lifetime cost of descendants is

$$(40) \qquad \frac{(1-\epsilon-\sigma)}{\sigma}\,\bar{c}_i = (1+r)^h\beta_{i-1} - \bar{w}_i, \qquad \text{for all } 1,\ 2,\ \ldots$$

This equation is the same as equation (16), except that the present values of lifetime consumption and earnings in generation i ($\bar{c}_i$ and $\bar{w}_i$) replace the consumption and earnings during adulthood. By substituting equations (34) and (38) into equation (40), we can solve explicitly for the growth in consumption at age j between any two generations:

$$(41)\qquad \frac{c_{i+1,j}}{c_{ij}} = \frac{(1+r)^h\beta_i - \bar{w}_{i+1}}{(1+r)^h\beta_{i-1} - \bar{w}_i}, \qquad \text{for } i = 1,\ 2,\ \ldots, \text{ and } j = 1,\ 2,\ \ldots\ \ell.$$

The rate of growth in consumption per descendant across generations is the same at all ages, and equals the rate of growth between these generations in the net cost of children. Given the rate of growth of these net costs, the growth of consumption per descendant does not depend on time preference (δ), the degree of selfishness ($\tilde{\alpha}$), or the interest rate (r). In contrast, equation (38) shows that the rate of growth of consumption over the life cycle does not depend on the cost of children, but does depend in the usual way on the interest rate and time preference. Therefore, even when parents are not "selfish" (say $\tilde{\alpha} = 1$), the rates of growth in consumption over the life cycle and between generations are equal only by accident. Once again we find that models with infinitely lived individuals who do not reproduce have very different implications from models with reproducing generations.

Equations (39) and (41) can be solved for the fertility rate in the ith generation

$$(42)\qquad n_i = \alpha^{1/\epsilon}(1 + r)^{h/\epsilon}\left[\frac{(1+r)^h\beta_{i-1} - \bar{w}_i}{(1+r)^h\beta_i - \bar{w}_{i+1}}\right]^{\frac{1-\sigma}{\epsilon}},$$

where recall that $\alpha = \tilde{\alpha}\delta^h$. Fertility is positively related to the extent of altruism ($\tilde{\alpha}$), to the time-preference factor (δ), and to the interest rate (r). Consumption per descendant does not depend on interest rates and time preference essentially because fertility does. When interest rates increase, a dynastic family accumulates capital in the form of additional descendants rather than additional capital per descendant.

Fertility also depends negatively on the growth between generations in the net cost of producing children. Net costs equal the difference between the fixed costs of children ($(1+r)^h\beta$) and the present value of their lifetime earnings ($\bar{w}$), both measured in units of goods in the same generation. Notice that the relevant measure of cost does not depend separately on the part of earnings that children receive prior to leaving home or prior to reaching their majority. Given parental altruism, the children's earnings as adults also matter. The "fixed" costs of raising children include the expenditures necessary to produce the lifetime earnings of children: not only the cost of giving birth, but also expenditures on subsistence and on investments in the human capital of children. Expenditures on children's consumption that simply raise the utility of children are not part of the net costs of having children and hence do not affect the demand for children. In particular, expenditures on the consumption of children who live at home would not affect the cost of children relevant to analyses of the demand for children. Yet empirical studies of the cost of children typically include all (net) expenditures on children up to a particular age, such as age eighteen, without any discussion of whether these estimates are relevant to the study of fertility.

Our analysis has important implications for the behavior of aggregate consumption. Per capita consumption is a weighted average of the consumption of persons at different ages, where the weights are the fraction of persons at each age. The rate of growth in per capita consumption between two time periods is

$$(43) \qquad \frac{\Delta c_t}{c_t} = \sum_{j=0}^{\ell} \left[\left(\frac{c_{jt}}{c_t}\right) \Delta\theta_{jt} + v_{jt}\left(\frac{\Delta c_{jt}}{c_{jt}}\right) \right],$$

where c_t is consumption per capita at time t, c_{jt} is the consumption of a person aged j at time t, $\theta_{jt} \equiv N_{jt}/N_t$, with N_{jt} the number of persons aged j at time t and N_t the total population, and $v_{jt} \equiv \theta_{jt}c_{jt}/c_t$ is the proportion of total consumption accounted for by persons of age j. The symbol Δ denotes the change in a variable between one time period and the next (for a given value of age, j).

The first term on the right-hand side of equation (43) depends on the change over time in the age distribution of the population. This term would be zero in a demographic steady state, where $\Delta\theta_{jt} = 0$ for all j. Moreover, basic theorem of demography states that a closed population with constant birth and death rates would eventually approach a demographic steady state (see, for example, Coale, et al., 1983).

The second term on the right-hand side of equation (43) depends on the rate of growth of consumption between generations. Equation (41) indicates that the growth in consumption equals the growth in the cost of children. the rate of growth of net costs is the constant g, then in a demographic steady state equation (43) becomes

$$(44) \qquad \frac{\Delta c_t}{c_t} = g.$$

The rate of growth of per capita consumption is then independent of time preference, the degree of altruism, and the interest rate, and depends only on the rate of growth of the cost of children.

Many have recognized that changes over time in per capita consumption are independent of life-cycle changes in consumption when the age distribution is constant (see Deaton, 1985, for a recent discussion). Some studies justify the use of life-cycle models to interpret the data on aggregate consumption by assuming that the representative person can be modeled as if he lives forever. This procedure is sometimes rationalized by the assumption that parents are altruistic toward children (see, e.g., Summers, 1981, p. 537).

Altruism does justify the assumption that heads of dynastic families effectively have infinite lives. However, when fertility is endogenous, models with effectively infinite lives that result from parental altruism have implications for consumption that differ substantially from those of models with infinite lives of representative persons. In our model, steady-state consumption per descendant is independent of time preference and interest rates because fertility fully absorbs the effects of these variables. As a result, changes over time in per capita consumption in demographic steady states do not depend on interest rates and time preference, even though each dynastic family effectively lives forever.

Consider once again an open economy that initially has a steady level of fertility and a constant rate of growth of consumption between generations. Assume also that the age distribution is stable initially. A rise in the world interest rate would increase fertility (by equation (26)), which would induce a transition to a new stable age distribution with a younger population. The rate of growth of per capita consumption might change during the transition to the younger population, but would be the same in the new steady state as in the initial state (as long as the rate of growth of the net cost of producing descendants were unaffected by the rise in the world interest rate). Perhaps this conclusion can explain what has been a puzzling finding; namely, that long term growth rates of per capita consumption in the United States apparently have not been affected by changes in long term real interest rates (see).

Summary and Conclusions

This paper develops the implications of altruism toward children, where utility of parents depends on their own consumption and the utility of each of their children. Altruism toward children implies that the welfare of all generations of a family are linked through a dynastic utility function that depends on the consumption and number of descendants in all generations. The head of a dynastic family acts as if he maximizes dynastic utility subject to a dynastic budget constraint, which involves the wealth inherited by the head, interest rates, the cost of rearing children in all generations, and the earnings of all descendants.

Utility maximization implies first, an arbitrage condition for consumption over time, and second, the equation of the marginal benefit of an additional child to the net cost of producing that child. This net cost equals the expenses for child rearing less the present value of the child's wages. We show that the optimization conditions relate the level of consumption per person to the net cost of creating that person. Then the arbitrage condition for consumption over time ends up implying a response of fertility--but not of the growth of consumption per person--to variations in interest rates and the degree of altruism.

We show that the number of descendants in each generation depends on the net cost of producing those descendants. Thus fertility--which determines the change in the number of descendants from one generation to the next--depends on changes in these net costs. For example, a permanent tax on children lowers fertility in the generation that first faces the tax, and permanently lowers the number of descendants in all subsequent generations. But fertility in later generations is unchanged. We use this result to show that a permanent reduction in the mortality rate initially raises population growth, but has no long-run effect on this growth. Similarly, we find that an expansion of social security has a temporary negative effect on population growth.

We also consider representative dynastic families in open economies that are linked to an international capital market with a single interest rate. Fertility in open economies depends positively on the world interest rate, on the degree of altruism, and on the growth of child-survival probabilities. Fertility depends negatively on technological progress, and on the growth rate of transfer payments. We conjecture that this analysis is relevant for explaining fertility in Western countries during recent decades.

We incorporate life-cycle elements by allowing for consumption at various ages. Life-cycle consumption is discounted by time preference, whereas a child's consumption is discounted by time perference and the degree of altruism. We now get the standard result that the pattern of consumption over the life cycle depends on the interest rate and on time preference.

Nevertheless, we still find that the growth rate of consumption between generations depends on the growth rate of the net cost of creating descendants, and not on the interest rate, time preference, or the degree of altruism. At least in the long run, the growth of consumption between generations will dictate the changes in consumption per person for the entire economy. Therefore, we can explain the puzzling finding from long-term aggregate data that real interest rates and the growth rate of per capita consumption are unrelated.

Thus far, our analysis neglects uncertainty, marriage, the spacing of births, and capital-market constraints over life cycles or across generations. Nevertheless, even the simplified model of altruism toward children and the behavior of dynastic families appears to us to capture important aspects of the dynamic behavior of fertility and consumption. If so, a new approach would be warranted to the analysis of trends and long-term fluctuations in consumption, fertility, and population growth.

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Appendix

1. Second-Order Conditions

We can write the maximization problem in the form of the Lagrangean expression,

$$\text{(A.1)} \qquad H = \sum_{i=0}^{\infty} \alpha^i N_i^{1-\epsilon} v(c_i) + \lambda[k_0 + \sum_{i=0}^{\infty} d_i(w_i N_i - c_i N_i - \beta_i N_{i+1})],$$

where d_i is the discount factor as given by $d_i = \prod_{j=0}^{i} (1 + r_j)^{-1}$, $N_0 = 1$, and λ is a Lagrange multiplier on the dynastic budget constraint. The first-order conditions come from

$$\text{(A.2)} \qquad \partial H/\partial c_i = v' \cdot \alpha^i N_i^{(1-\epsilon)} - \lambda d_i N_i = 0, \qquad i = 0, 1, \ldots$$

and

$$\text{(A.3)} \qquad \partial H/\partial N_i = \alpha^i (1 - \epsilon) N_i^{-\epsilon} v + \lambda d_i[w_i - c_i - (1 + r_i)\beta_{i-1}] = 0, \qquad i = 1, 2, \ldots$$

where we use the condition, $d_{i-1} = d_i(1 + r_i)$.

The second derivatives are

$$\text{(A.4)} \qquad \partial^2 H/\partial c_i^2 = \alpha^i N_i^{(1-\epsilon)} v'' < 0,$$

$$\text{(A.5)} \qquad \partial^2 H/\partial c_i \partial N_i = (1 - \epsilon) v' \alpha^i N_i^{-\epsilon} - \lambda d_i,$$

$$\text{(A.6)} \qquad \partial^2 H/\partial N_i^2 = -\epsilon(1 - \epsilon)\alpha^i N_i^{-\epsilon-1} v < 0.$$

Since all second partials from different periods (e.g., $\partial^2 H/\partial c_i \partial c_j$) are zero (and $\partial^2 H/\partial c_i^2 < 0$, $\partial^2 H/\partial N_i^2 < 0$), the second-order condition for a maximum is

$$\text{(A.7)}\quad \det.\begin{bmatrix} \dfrac{\partial^2 H}{\partial c_i^2} & \dfrac{\partial^2 H}{\partial c_i \partial N_i} & -d_i N_i \\ \dfrac{\partial^2 H}{\partial c_i \partial N_i} & \dfrac{\partial^2 H}{\partial N_i^2} & d_i[w_i - c_i - \beta_{i-1}(1 + r_i)] \\ -d_i N_i & d_i[w_i - c_i - \beta_{i-1}(1 + r_i)] & 0 \end{bmatrix} > 0$$

where the terms in the right column and bottom row are derivatives of the budget constraint with respect to c_i and N_i, respectively. By substituting the first-order conditions (A.2) and (A.3) and simplifying, the condition for a positive determinant is equivalent to $\epsilon + (1 - \epsilon)vv''/(v')^2 < 0$. If $v(c) = \frac{1}{\sigma} c^{\sigma}$, this condition reduces to $\epsilon + \sigma < 1$.

2. Stability Conditions

Costs of rearing children are now $b(n)$ with $b' > 0$, $b'' > 0$. The elasticity of utility is assumed to be the constant σ, and w_i and r_i are constant. Then equations (19) and (21) imply

$$\text{(A.8)}\quad \frac{(n_i)^{\epsilon}}{\alpha(1+r)} = \left[\frac{c_i}{c_{i+1}}\right]^{1-\sigma} = \left[\frac{b'(n_{i-1})(1+r) - w - n_i b'(n_i) + b(n_i)}{b'(n_i)(1+r) - w - n_{i+1} b'(n_{i+1}) + b(n_{i+1})}\right]^{(1-\sigma)}$$

Linearizing around the steady-state value, $n = [\alpha(1 + r)]^{1/\epsilon}$, leads to the second-order difference equation in n_i:

$$(A.9) \qquad (n_{i+1} - n) + a_1(n_i - n) + a_2(n_{i-1} - n) = 0,$$

where

$$(A.10) \qquad a_1 = -\left[1 + \left[\frac{1+r}{n}\right] + \epsilon\theta/b''(1 - \sigma)n^2\right],$$

$$(A.11) \qquad a_2 = (1 + r)/n,$$

(A.12) $\theta = b'(1 + r - n) - w + b > 0$ (since the steady-state c is positive in equation (21)), and

(A.13) $(1 + r) > n$ for utility to be bounded.

Given $b'' > 0$ the roots of $m^2 + a_1 m + a_2 = 0$ are real. One root exceeds one and can be excluded by the transversality condition since eventually n_t would exceed $1 + r$ for all subsequent t. The other root satisfies

$$(A.14) \qquad 2m = -a_1 - \sqrt{(a_1)^2 - 4a_2}$$

and is positive and less than one (for $\epsilon > 0$). It follows that the path of n_i is (locally) stable and exhibits direct convergence to the steady-state value n.

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