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Abstract: After the initial breakthrough in the research phase of R&D a new product undergoes a continuous and long process of change, improvement and adaptation to market conditions. We model the strategic behaviour of firms in this development phase of R&D. We emphasise that a key dimension to this competition is the innovations that lead to product differentiation and quality improvement. In a duopoly model with a single adoption choice, we derive endogeneously the level and diversity of product innovations. We demonstrate the existence of equilibria in which firms emerge at different points of the quality spectrum. In such equilibria, no monopoly rent is dissipated and later innovators make more profits. Incumbent firms may well be the early innovators, contrary to the predictions of the "incumbency inertia" hypothesis. The role of policy and underlying factors as consumer diversity, learning and market lock-in, in determining market expectations and hence the innovation outcomes is analysed. Finally, innovative incentives under a cartel and social planner are contrasted with the duopoly outcomes.

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1. Introduction

After the initial breakthrough in the research phase of R&D a new product undergoes a continuous and long process of change, improvement and adaptation to market conditions.[1] In this paper we model the strategic behaviour of firms in this development phase of R&D. We analyze the following questions: How does a single discovery or idea lead to a spectrum of differentiated products? What determines the diffusion and development of a new technology, the process by which a basic research breakthrough is improved and adapted by competing firms before they introduce it into the market? What determines the time of introduction of imperfectly substitutable new products into the market?

Comparatively little attention has been paid in economic theory to the development stage of R&D. The adoption of a newly discovered technology is almost always analyzed under the assumption that at the time of the discovery it is already in a form that can be marketed and undergoes no technical or economic modification afterwards. This assumption that the first blueprint of a new product remains unaltered shifts the balance of the analysis to the research phase: it is a convenient assumption, but it biases the analysis.[2]

Note that if there is no room for differentiation after the first introduction, the only important aspect of R&D activity is the race to be first in the discovery and introduction of a new product. In particular, if the end of the research race is also the end of innovative activity, the attempt to preempt the entry of opponents becomes the dominant feature of the development phase. In such a model therefore, firms dissipate all the rents which go to the first adopter, because any positive profits to the first mover prompts a preemptive move of the others. Earlier contributions (Scherer (1967), Reinganum (1981), Fudenberg and Tirole (1985), Quirmbach (1986)), have all assumed that the key decision that firms have to make is the choice of the adoption time of a homogeneous new technology. Consequently preemption and rent equalization have been the dominant results of this literature. In this

[1] Rosenberg (1982) makes a very forceful case for the historical importance of continuous changes and contains numerous examples. In particular, refer to chapters 1,3 and 5.

[2] By way of evidence that most R&D analyses have focussed exclusively on the research phase, note the extensive literature on patent races (see Reinganum (1989) for a comprehensive survey). In this literature the complete focus is on a single research breakthrough which gives one firm exclusive rights to further development of this breakthrough and indeed this development phase is never explicitly modelled. In the popular press however, there is a growing debate which focusses precisely on the development phase. This debate has been spurred by concerns that American firms, although successful at making basic research breakthroughs, have been unsuccessful at translating these breakthroughs into commercial usage.

paper we show that once we allow for post-breakthrough improvement and adaptation, many of these conclusions change considerably.

The basic intuition we try to formalize is the following: part of the R&D effort is to improve and adapt a research breakthrough before it is introduced into the market. Further, the innovative activity at this stage is primarily one of *vertical differentiation*. Firms devote resources to develop a quality of product or technology that is better than those competitors have developed.[3] The model we study is a synthesis of two literatures: the adoption of a new technology (Reinganum (1981), Fudenberg and Tirole (1985)) and vertical differentiation (Shaked and Sutton (1982)). We argue in the sequel that this synthesis is natural for both lines of research and yields a more complete picture of the development stage of R&D.

The model we analyze is the following. An idea or a new technology arrives exogenously into an industry, it is commonly available, and it can be improved by the firms in the industry. At any point of this development stage, a firm can incorporate the currently available technology into a product and market it: the longer this development phase, the better and more profitable is the product. In this sense innovation at this development stage of R&D is essentially a process of vertical differentiation by firms in the industry. Further the critical resource cost is time: it takes longer to produce a better product. The main conclusions of this model are:

Maturation and Rent Escalation

We have two types of equilibria: the first is the classic preemption equilibrium, with rent equalization and dissipation. A second type of equilibrium appears with different return functions and other primitives. Here the unique subgame perfect equilibrium induces *staggered innovations*: one firm, the leader, enters first with a low quality product, earns monopoly rents while the other firm continues to develop the product and eventually markets a high quality product. We call this a *maturation equilibrium*: a spectrum of differentiated products emerges from the same discovery, and there is *diffusion* of the new technology because different firms adopt it at different times. Finally, there is *rent escalation*: a later entry yields a higher profit, but there is no rent dissipation because there

[3] There is clearly some horizontal differentiation activity as well during the development phase. We believe vertical differentation is a better description of innovative activity in a dynamic context and hence concentrate on that alone.

is no preemption. These results emphasize that for a research breakthrough which can undergo subsequent development the prize need not go to the "swift of foot."

Incumbent Adopts First

A natural conjecture for the staggered equilibrium, and one often made, is that an incumbent will innovate last because any new product cannibalizes the existing one.[4] We argue that this conjecture overlooks a subtle signalling problem which is present with multiple equilibria. In our model the actual innovation pattern is the exact opposite of this "incumbency inertia": any a priori advantage of being an incumbent leads to an early adoption by such a firm and consequent lower profits than the non incumbent. Precisely because of the cannibalization effect on his product, the incumbent would prefer a later entry date than the opponent. This fact is common knowledge and gives the non incumbent the ability to make a credible commitment to be the high quality firm, by simply passing up his own best opportunity to be the first entrant.

Policy Effectiveness in Product Development

Public policies have different effects on the rate at which an initial research breakthrough can be developed into a marketable product. We argue that increasing this rate has ambiguous effects and may not lead to an increase in the level and diversity of innovations. Since subsequent innovations are easier to make, early adopters of a new technology may choose to increase monopoly rents by bringing products to the market more quickly and thereby lower the level of innovatice activity. We do, however, give conditions under which such policy will in fact encourage maturation rather than preemption, and at the same time increase product diversity.

The Market May Overdevelop

A frequent concern in the literature is under-innovation in competitive markets. We show that in our model strategic product development may lead to a level and diversity of innovations which is greater than under either a cartel or a social planner. The reason for this is that a duopolist does not internalize the full costs of introducing a high quality product and ignores the fall in profits of an existing product.

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4 For example, Schumpeter (1934) says, "...it is not essential to the matter--though it may happen--that the new combinations should be carried out by the same people who control the productive or commercial process which is to be displaced by the new. On the contrary, new combinations are, as a rule, embodied, as it were, in new firms which generally do not arise out of the old ones but start producing beside them."

This paper is part of a series in which we explore a view of innovations as a continuous and ongoing process. Much of economic theory has concentrated on the Schumpetarian perspective in which innovations are discrete discontinuous phenomenon, propelled by a major breakthrough whose subsequent adaptation and change is of small magnitude[5]. This contradicts much of the historical[6] and contemporary[7] evidence on the importance of small improvements which individually may be insignificant but have a significant cumulative effect. We investigate the phenomenon of repeat innovations in Dutta-Rustichini (1990b). In Dutta-Rustichini (1990c), we examine the effect of post-breakthrough product development on the organization and level of research leading to the breakthrough and in particular, the viability of cooperative research.

The previous literature on adoption-development includes Reinganum (1981), Fudenberg-Tirole (1985) and Katz-Shapiro (1987) who analyze the timing of adoption of an initial breakthrough, which is publicly available and used to produce a homogeneous good. Reinganum's paper showed that, given precommitment possibilities, there would be a diffusion of adoption times if the fixed costs of adoption decline over time. Fudenberg-Tirole (1985) however demonstrated that in a perfect equilibrium the typical outcome is a race to be the first adopter with consequent rent dissipation.[8] With a small modification, our model contains as a sub-case, the adoption models of Fudenberg-Tirole (1985) and Reinganum (1981).[9] Benoit (1983) and Ramey (1988) consider the consequences of imitation of a new fixed technology, again producing a homogeneous good, when only one firm has access to this technology.

5 "The historic and irreversible change in the way of doing things we call 'innovation' and we define: innovations are changes in production functions which cannot be decomposed into infinitesimal steps. Add as many mail coaches as you please, you will never get a railroad by so doing" (Schumpeter (1935), p7).

6 To take Schumpeter's railroad example, Fishlow (1966) found in his study of the American railroads that at a time of significant cost reductions, the years between 1870 and 1910, the largest cost saving, by far, was due to a succesion of improvements in the design of locomotives and freight trains. The process included no single major break with the past and yet "its cumulative character and lack of a single impressive innovation should not obscure its rapidity" (ibid, p 35).

7 Mansfield et. al. (1981) found from their survey of 48 product innovations, of which 70% were patented, that about 60% were competed against within four years of the patent.

8 Fudenberg-Tirole also showed that for some specification of returns there might be a continuum of joint adoption equilibria which do not involve a race but do result in rent equalization and (in all except one equilibrium) rent dissipation.

9 In these models the cost of adoption is assumed to decline over time whereas we assume it to be constant. This is however a difference which could easily be added to our formulation.

Vertical differentiation models (as Shaked-Sutton (1982)) or sequential differentiation models (as Prescott-Visscher (1979)), have firms choosing quality first and then engaging in price competition. These models carry exogenously imposed structures on the timing and sequence of quality choice. Our framework is one natural paradigm within which such models can be embedded, since we endogeneously model the quality choice as a choice of timing or technology maturation.

Section 2 introduces the development game. In Section 3, we discuss maturation (and preemption) equilibria, while Section 4 analyzes the incentives of incumbents to adopt new technologies. Section 5 contains the policy analysis and finally, in Section 6, we ask: does the market underdevelop?

2. The Innovation and Product Differentiation Problem

We now formulate precisely the model discussed in the introduction. Suppose the payoff relevant attributes of firm i's product can be represented by a variable $x_i \in R_+$. $x_i(t)$ should be thought of as the level of technology or quality, that is available at instant t, to firm i. Time is continuous and the horizon infinite.[10] Over time the initial research breakthrough, with associated quality x(0), can be improved and matured. One may either imagine that this basic idea grows in a publicly accessible environment like a government or university laboratory or that it grows on account of the private activities of individual firms. The growth rate of quality is in general stochastic and firm-specific but in the current model we abstract from these considerations since much of the analysis of this paper is unchanged if such generalizations are introduced.[11] Here we assume that the quality level can be improved at a (common) constant rate, which we normalize to one.[12] The innovations that firms engage in during the development stage are i) the improvement of the original technology or idea (if it is a private activity) and ii) the selection of the amount of improvement prior to the introduction of the product . Under the simplification that improvements can be made at a constant rate, a firm's innovative activity is completely

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[10] Continuous time is analytically more convenient although we believe that a similar exercise could be carried out in discrete time as well.

[11] See Dutta-Rustichini (1990a). Of course the results are considerably less sharp in the more general formulation.

[12] We shall talk exclusively of product innovation, although much of what we say carries over verbatim to the case of process innovations, i.e. cost-reducing technological innovations.

described by decision ii). Hence, in the sequel we shall sometimes refer to the latter, i.e. the adoption decision by itself, as an innovation.

Consider a duopolistic market, with firms indexed by a generic index, $i = 1,2$. j will index the "other" firm. Either firm can introduce a product at any time, incorporating the currently available technology.[13] As in the adoption of new technology and vertical differentiation literatures referred to above, we shall make the simplifying assumption that firms adopt but once.[14] We shall represent the flow profits of a monopoly selling a product with attribute x, as $\delta R(x)$. If firm i has introduced a product with attribute x_i (at time x_i, and from hereon we will use time and attribute interchangeably), then firm j has a choice on its introduction time. If j introduces a product at x_j, then flow profits to the duopolists from that point on depend on (x_i, x_j) and (x_j, x_i), respectively. These profits could be thought of, for example, as the returns to (one-shot) Cournot or Bertrand competition in the duopoly market. We give examples below.

2.1 Assumptions

The principal simplifying assumption that we make is that duopoly returns depend only on the relative qualities, i.e.

(A1) Firm i's duopoly profits are given by a function, $r_i(x_i - x_j)$, $i = 1,2$, $i \neq j$

This assumption cleans up the exposition considerably. We discuss in Section 7 how the analysis is complicated (although the results are largely unchanged) by the absence of this assumption. From hereon let us place a symmetry restriction on the duopoly profits, i.e. $r_i = r_j = r$, say. This assumption is made so as to not force the identity of who moves first through factors exogeneous to the game. The reader can check that this assumption can be easily relaxed. Also, denote $\theta = x_i - x_j$, the generic difference in qualities. The natural monotonicity assumptions are

(A2) i) Monopoly returns, $R(x)$ are increasing in x.

13 An implicit assumption, then, is that technology is irreversible. Even if technology or quality was reversible, i.e. if at quality x a firm could choose to introduce a product incorporating an attribute $s \in [0,x)$, under natural restrictions on returns they would not.

14 This could be justified through an implicit assumption that fixed costs of adoption are very high. In any case, we ignore repeat innovations since they bring up a set of questions about long term cooperation and appropriate strategies in the dynamic game, that would detract from the level and diffusion of technology questions that we are interested in here (however see Dutta-Rustichini (1990a) for a repeat innovation model).

ii) In a duopoly, $r(\theta)$ is increasing in θ, whenever $\theta \geq 0$.

(A2) ii) merely says that a technological leader has duopoly profits increasing in his lead. Whether a quality laggard makes more or less as θ increases, depends on the market's preferences over diversity. We make no assumptions hence on $r(\theta)$, for $\theta < 0$.

A useful technical quasi-concavity assumption on present discounted profits is

(A3) i) $e^{-\delta x}[aR(x) + b]$ is strictly quasi-concave on R_+, for $a,b \in R_+$ and attains a maximum.
ii) $e^{-\delta\theta}r(\theta)$ is strictly quasi-concave on R_+ and attains a maximum.

Another useful assumption to simplify exposition is:

(A4) r and R are non-negative and differentiable.

To complete the specification of the model, we have to specify what happens if both firms attempt to introduce a product at the same time.

(A5) If both i and j attempt to enter at any period t, then only one of them succeeds in doing so. The probability of i entering is $p \in (0,1)$.

Simultaneous adoption is hence ruled out although adoption at any $\tau > t$ is feasible. An ex-ante justification for this assumption is that typically there is an "adoption technology", like advertising, which may have limited capacity. As with (A1), not making this assumption complicates but does not substantially alter the analysis. We return to this theme in Section 3.

2.2 Two Examples

It should be clear that the only strong assumption on returns is (A1). We present briefly two examples of duopoly profits that satisfy (A1) (and additionally (A2) - (A4)).

Example 2.1 Cournot Competition

This example is adapted from Kreps (1990) and shows that if demand functions are based on relative differences, then this extends to Cournot profits. Consider the following demands for products of vintage x_i, i = 1,2.

$$P_i = a - Q_i - b(x_i - x_j)\,Q_j \qquad i,j = 1,2,\ i \neq j \tag{1}$$

$b(\cdot)$ is a coefficient of substitution which depends on the differences in product attributes. Write $x_1 - x_2 = \theta$ temporarily. It is easy to see that if $b(\theta) \in (0,1)$, for all θ, then the products are imperfect substitutes. Further, if $b(\cdot)$ is decreasing, then the demand for a product is increasing the higher its quality relative to the other product.

Taking costs of production to be zero, straightforward computation yields the Cournot quantities, given vintages x_1 and x_2, to be

$$Q_1(\theta) = \frac{2 - b(\theta)}{4 - b(\theta)b(-\theta)} a$$

$$Q_2(\theta) = \frac{2 - b(-\theta)}{4 - b(\theta)b(-\theta)} a \tag{2}$$

From that it follows that the Cournot profits are

$$r_1(\theta) = [Q_1(\theta)]^2$$

$$r_2(-\theta) = [Q_2(\theta)]^2 \tag{3}$$

(3) shows that returns satisfy (A1). Note that symmetry requires that firm 2's profits be identical to those of firm 1 when $x_2 - x_1 = \theta$, i.e. that $r_1(\theta) = r_2(\theta)$. It is immediate that the returns satisfy symmetry as well. Straightforward calculus shows that under the further assumption, $b'(\theta) = b'(-\theta)$, r_1 is increasing in the relative quality advantage of product 1, and r_2 is decreasing. Finally the returns are non-negative, differentiable and satisfy (A3) under appropriate and straightforward conditions on $b(\theta)$.

Example 2.2 Bertrand Competition

This example is adapted from Shaked-Sutton (1982), (as reported in Eaton-Lipsey (1989)), and gives a construction based on utility functions. Consumers have preferences on quality, with this preference index ranging over $[a,b]$, $b > 2a > 0$. Each consumer has y units of a numeraire good and uses it to buy a single unit from either producer. The m-th consumer's utility from buying a good of vintage x_i is $mx_i + y - p_i$, $i = 1,2$, $m \in [a,b]$.

So, in a duopoly, with qualities x_i, prices p_i yield market shares of $[a,\bar{m}]$ and $[\bar{m},b]$. Letting, without loss of generality, $x_1 > x_2$ (i.e. $\theta > 0$), the high quality customers buy from firm 1 and the market divides at $\bar{m} = \frac{p_1 - p_2}{x_1 - x_2}$ $(p_1 > p_2)$.

Straightforward computation yields prices in Bertrand equilibrium as

$$p_1(\theta) = \frac{2b - a}{3}\theta$$

$$p_2(\theta) = \frac{b - 2a}{3}\theta \tag{4}$$

It follows that

$$r_1(\theta) = \left(\frac{2b - a}{3}\right)^2 \theta, \qquad \theta \geq 0$$

$$r_2(-\theta) = \left(\frac{b - 2a}{3}\right)^2 \theta, \qquad \theta \geq 0 \tag{5}$$

Symmetry gives the returns for $\theta < 0$. In this case it is easy to see that both r_1 and r_2 are increasing in θ, i.e. that the more diverse are the products the greater are profits for both the technological or quality leader as well as the laggard. This is of course directly the phenomenon of differentation lessening the severity of Bertrand price competition. Clearly (A1) - (A4) are satisfied in this example as well.

Incidentally, it is also straightforward in this example to show that the monopoly profits for a product of vintage x is

$$R(x) = \frac{b^2}{4}x \tag{6}$$

It depends only on the upper bound of consumer preference on quality, since a monopolist only services a fraction of the market optimally, and the choice is which of the high quality seeking customers to serve.

2.3 Strategies and Equilibrium

A pure strategy of firm i specifies, at any time t, a decision on "adopt" or "do not adopt", if the firm has not adopted already. This decision is of course conditioned on the available information, which in our model is the knowledge: has j adopted at any $s < t$, and if yes, when. So a history at time t, denoted h_t, is a pair (t_i,t_j) where t_i is firm i's adoption date if i has adopted (in which case $t_i < t$) with the further convention that $t_i = t$ if i has not yet adopted. For well-known reasons[15], we confine attention to pure strategies, (although in asymmetric versions of our game the equilibria we find are unique in the class of <u>all</u> strategies, including mixed strategies). A strategy σ_i associates with every h_t an action (0 for "do not adopt" and 1 for "adopt) with the further convention that $\sigma_i(h_t) = 1$ if $t_i < t$. A strategy pair $\sigma = (\sigma_i, \sigma_j)$ associates, with every history, an outcome[16], a pair of adoption dates (t_i, t_j). Additionally,a strategy pair has associated with it a lifetime discounted return $W_i(h_t)$. For instance if $t \leq t_i \leq t_j$, then writing $\theta = t_i - t_j$ (and hence $\theta < 0$), we have

$$W_i(h_t;\sigma) = e^{-\delta(t_i-t)} \left\{(1 - e^{\delta\theta})R(t_i) + e^{\delta\theta}r(\theta)\right\}$$

$$W_j(h_t;\sigma) = e^{-\delta(t_i-t)} \left\{e^{\delta\theta}r(-\theta)\right\}$$

The lifetime returns associated with other configuration of adoption times are easily computed. The equilibrium concept we employ is that of subgame perfection. Hence a strategy pair $\sigma*$ is an equilibrium if $W_i(h_t;\sigma*) \geq W_i(h_t;\sigma_i,\sigma_j^*)$ for all other strategies σ_i and all histories h_t. We emphasise that we do not restrict ourselves to Nash equilibrium; firms are not committed to any adoption dates and can always adapt to the circumstances that have transpired in the market in making their development decisions.

3. <u>Pre-emption and Maturation Equilibria</u>

In this section we characterize the subgame perfect equilibria of the (symmetric) innovation game of Section 2. We investigate how imperfectly competitive firms trade-off pre-emption and differentiation incentives. In this section we show that a consequence of such conflicting incentives is that either of two equilibria will result. The first or classical <u>pre-emption</u> equilibrium (Fudenberg-Tirole 1985, Tirole 1989), arises from an inability by

[15] See, for example, Rubinstein (1988).

[16] There are well known reasons on account of which continuous time game strategies in which sudden moves are possible, as in the innovation game, may fail to have well-defined outcomes associated with them. All of the anomalies stem from the fact that "instantaneous" reactions are typically admissible in such games but there is no instant after. We assume that all admissible strategies have well-defined outcomes and implicitly therefore assume that instantaneous reactions are not possible. A sufficient condition for this would be to require of all strategies σ that they satisfy $\limsup_{t \to t^*} \sigma(h_t) \leq \sigma(h_{t^*})$. As Section 3 will show, the fact that firm j cannot react instantaneously to firm i's adoption is not a restriction in a best response. That firm j cannot react instantaneously to i's non-adoption (i.e. that strategies of the form "j adopts the first instant after t if i has not adopted till that point" are not allowed) is a restriction but arguably a non-critical one.

firms to sufficiently differentiate their products (in payoff terms). This inability in turn stems from factors as market lock-in by a first entrant, a slow imitation technology or insufficient diversity of consumer preferences. We show that the consequent attractiveness of having a leading position in the market, leads firms to adopt "too early", equalize rents but achieve that by dissipating potential rents. In the second, maturation, equilibrium competition results in an even flow of innovations. A potential technological leader optimally waits to develop a differentiated product. The other firm enters earlier to exploit a temporary monopoly position. None of the rent associated with this monopoly position is dissipated. However, an early entrant makes strictly less in equilibrium than the eventual quality leader. That such rent non-equalization can happen in a symmetric game may appear at first somewhat paradoxical. It results of course from the fact that time (or technology or quality development) is unidirectional. A second entrant can pre-empt the first one if the latter earns higher ex-ante profits. However, a first entrant cannot unilaterally decide to be the follower.

3.1 The Follower's Problem

Consider a continuation subgame after firm i adopts the technology at x. j's problem is to pick an optimal state $x + \theta$, at which to follow. In other words, j solves[17]

$$\max_{\theta \geq 0} \; e^{-\delta\theta} r(\theta) \tag{8}$$

By (A3) ii), $e^{-\delta\theta} r(\theta)$ is single peaked on R_+. Let this peak occur at $\theta^* > 0$. It follows that firm j would follow at $x + \theta^*$. In other words, in equilibrium all strategies prescribe: if the other firm has already moved, then move if and only if θ^* has elapsed since the competitor's adoption.

Let F(x) denote the lifetime returns, to firm j, if it follows optimally a first adoption by firm i, at date x.

$$\begin{aligned} F(x) &= e^{-\delta x}\left\{ e^{-\delta\theta^*} r(\theta^*) \right\} \\ &\equiv e^{-\delta x} \phi \end{aligned} \tag{9}$$

[17]Note that flow returns have been normalized to $\delta r(\theta)$, so the infinite horizon discounted returns are $r(\theta)$.

ϕ, the optimal returns to a follower's differentiation activities, is the direct index of differentiation possibilities in the market. ϕ is determined by underlying factors as diversity in consumer preferences, imitation or learning possibilities and market lock in.

3.2 The Leader's Problem

Following Fudenberg-Tirole (1985, 1989), we develop now the returns to a potential first entrant, or leader. Let L(x) denote the returns to a firm i, evaluated at date 0, if it innovates at quality x, anticipating an optimal follow by j at $x + \theta^*$.

$$L(x) = e^{-\delta x} \left\{(1 - e^{-\delta\theta^*})R(x) + e^{-\delta\theta^*}r(-\theta^*)\right\} \qquad (10)$$

$$\equiv e^{-\delta x} \left\{\lambda_1 R(x) + \lambda_2\right\}$$

Any potential leader evaluates returns from two sources: $\lambda_1 R(x)$, the monopoly phase and λ_2, the phase in which it is a technological laggard in a duopoly. The effect of market differentiation possibilities on λ_k, k = 1,2 are more ambiguous, than on ϕ. We shall develop this in Section 5 when we explore the comparative statics of equilibria.

By (A3) i) L(x) is single-peaked. Further, note that $\frac{L(x)}{F(x)} = \frac{\lambda_1 R(x) + \lambda_2}{\phi}$, which is an increasing function of x. Hence, L and F have at most one intersection and suppose momentarily that there is in fact such an intersection.[18] Denote this x^I and let x^M refer to argmax L. The two types of equilibria correspond to the two possibilities: a) $x^I < x^M$ and b) $x^I \geq x^M$.

(Figs. 1 and 2)

3.3 Equilibria:

Proposition 1 : a) The Race. Suppose that $x^I < x^M$ (figure 1). There is a unique pure strategy equilibrium to the development game. The associated **outcome** is:

both firms try to simultaneously adopt at x^I. Firm i (j) ends up adopting with probability p (1-p) and the remaining firm adopts at $x^I + \theta^*$.

The **strategies** that support this equilibrium are the following:

firms i and j try to innovate at all dates after x^I if neither has

[18]We ignore, for now, the two possibilities: L (resp. F) is forever above F (resp. L).

innovated before that date. If i has innovated already, j innovates iff θ^* has elapsed since i's innovation.

b) Maturation. Suppose that $x^I \geq x^M$ (figure 2). There is a unique pure strategy equilibrium. The associated equilibrium **outcome** is:

firm i unilaterally adopts at x^M and j follows at $x^M + \theta^*$.

The **strategies** that support this equlibrium are the following:

firm i tries to innovate at all dates after x^M while j tries to innovate at all dates after x^I, if neither has innovated till that date. If i has innovated already, j innovates iff θ^* has elapsed since i's innovation. There are two asymmetric equilibria for i=1,2.

Proof: Since the proof of a) is contained in that for b), we only demonstrate the latter. We first show that the strategies outlined form a subgame perfect equilibrium. Given j's strategy, firm i solves the following problem at any x in $[0,x^I)$: (if it has not adopted already)

$$\max_{x^I \geq z \geq x} \; e^{-\delta(z-x)} [\lambda_1 R(z) + \lambda_2]$$

Clearly the solution is x^M if $x \leq x^M$ and x if $x > x^M$. So if j does not adopt till x^I, i's best response is to adopt at any date after x^M, if it has not adopted yet. Moreover for all x in $[x^M,x^I)$, $L(x) < F(x)$ and hence j has no incentive to preempt i's adoption. It is further clear that if the game was ever at $x \geq x^I$, a dominant strategy for either firm is to move immediately (recall (A5): if both try to move, nature selects the actual entrant). So the exhibited strategies in fact form a subgame perfect equilibrium.

That this is the only pure strategy equilibrium follows by a simple argument. Consider any such equilibrium. Clearly, for any $x < x^M$, it is a dominant strategy to not adopt. We now show that it cannot be the case that there is $x_i < x_j$, $x_i, x_j \in [x^M, x^I)$, with i adopting at x_i and j adopting at x_j (in each case if the other has not adopted till that point). For some $x_1 < x_j$, $F(x_j) > L(x)$, $x \in (x_1, x_j)$. So if i anticipates following at x_j, it is a dominant strategy for him to not adopt for $x \in (x_1,x_j)$. But then, it is a dominant strategy for j to adopt immediately at any such date, rather than wait till x_j. So in fact j moves at x_1 if neither has moved till that point. In turn, there is $x_i < x_2 < x_1$ such that for any $x \in (x_2, x_1)$, i's dominant strategy is to wait and consequently j's is to immediately lead. It is clear that a finite iteration of this logic in fact works back to x_i.[19] But then, i should not be moving at x_i. In other words, there can only be one leader in equilibrium. Finally consider $x \geq x^I$. In this region adopting if neither has adopted before is a dominant strategy. Hence equilibrium strategies are necessarily the given strategies.

For case a) in the region $x \geq x^I$, adoption is no longer a dominant strategy. Note however that no firm can be an uncontested leader in equilibrium. Further, given preemption possibilities, in equilibrium it must be the case that both move imediately at any date after x^I if neither has moved before. Finally, by the argument of (9), the equilibrium differentiation is the smallest possible difference beyond θ^* •

<u>Remark 1</u>: The remaining two possible configurations are: $L \geq F$ always. The equilibrium then is trivial: each firm attempts to move at every instant. The outcome is probabilistic entry by i at 0 and an optimal follow by j at θ^*. Conversely, $F \geq L$ throughout. The equilibrium strategies are: i moves at all $x \geq x^M$, j never moves if i has not moved before. The outcome is the rent preserving one of x^M, $x^M + \theta^*$. These two trivial equilibria are of course special versions of pre-emption and maturation equilibria respectively. From hereon we supress discussion of these trivial cases.

<u>Remark 2</u>: From the arguments above it should be clear that the "no simultaneous move" assumption, (A5), is inessential. Suppose simultaneous moves are possible. Consider Case b). All we require is that there are equilibria in subgames starting at x^I. Clearly, such equilibria result in payoffs which are no more than $L(x^I) = F(x^I)$. The rest of the logic works exactly as stated. Such equilibria beyond x^I may be in mixed strategies or the class of correlated equilibria Fudenberg-Tirole (1985) consider.

<u>Remark 3</u> : Consider the adoption of a (homogeneous) technology. If the fixed cost of adoption is unchanged over time (we have in fact normalized it to zero), then in the model of Reinganum (1981) and Fudenberg-Tirole (1985), $\phi = 0$ and hence $x^I = 0$. Consequently the only possibility is that of a race. In Shaked-Sutton (1982) and other vertical differentiation models there are asymmetric equilibria (like our maturation possibility) but there is no dynamic story for their emergence.

We shall call the equilibrium of Fig. 1, <u>pre-emption</u> equilibrium and that of Fig. 2, <u>maturation</u> equilibrium. Further, (see Tirole, 1989), we shall say that rents are dissipated

[19] Else, there is an accumulation point $x < x^I$, s.t. $F(x) = L(x)$.

if the full returns of a monopoly situation are not realized, i.e. if an equilibrium outcome is $x \neq x^M$, for the first entrant.

Corollary 2 : In a pre-emption equilibrium, rents are equalized and this is achieved through a dissipation of rent. In a maturation equilibrium a follower makes strictly higher profits, although a leader realises the full monopoly rent.

Finally, it should be remarked that although a leader makes less relative to the follower, he makes more of course relative to his alternative option as a follower, for which he has to wait till x^I. One unsatisfactory feature of the asymmetric maturation equilibria is that equilibrium behavior does not of course allow us to say which of the two asymmetric equilibria will actually get played, i.e. whether firm 1 or 2 will actually lead. In the next section, we show that even a little bit of asymmetry in the game will resolve this issue.

3.4 An Illustrative Example

We compute equilibria in the Shaked-Sutton model to illustrate how in this case the extent of consumer diversity determines whether the equilibrium is of the preemption or maturation type. Recall that in this model (Example 2.2) consumer preference for quality (which scales the utility function) is uniformly distributed on [a,b], $0 < a < b$. An increase in diversity could be modelled by a decrease in $\frac{a}{b}$ or a-b. Notice that if b also changes, there is a level effect as well. To concentrate all attention on diversity effects consider (equivalently) decreases in $\frac{a}{b}$ or a-b, with b fixed. It is easy to infer that the optimal product diversity θ^* is unchanged, when consumer diversity changes. Further,

$$L(x) = e^{-\delta x}\left\{\frac{(1-e^{-1})b^2}{4}x + (\delta e)^{-1}\left(\frac{b-2a}{3}\right)^2\right\}$$

$$\equiv e^{-\delta x}\left\{\lambda_1 x + \lambda_2(a)\right\}$$

$$F(x) = e^{-\delta x}(\delta e)^{-1}\left(\frac{2b-a}{3}\right)^2$$

$$\equiv e^{-\delta x}\,\phi(a) \tag{11}$$

It is not difficult to see that we have a maturation equilibrium if and only if $\frac{a}{b} < \xi$, and a pre-emption equilibrium if and only if $\frac{a}{b} > \xi$ where ξ is a positive constant independent of the primitives of the model.[20] It is also clear from (11) that as diversity increases, a potential leader would like to innovate earlier, i.e. $x^M(a)$ is a decreasing function of a. This follows directly from the fact that not adopting has a higher waiting cost, the postponement of greater duopoly returns $\lambda_2(a)$.

4. Entrants Versus Incumbents

In this section we examine, in our model, the cannibalization hypothesis: that incumbent firms are less likely to be innovators in new product development. There are at least two reasons why we investigate this hypothesis in some detail. Incumbency, as we define it here, is one natural way to incorporate asymmetry between firms in an industry and it is a good proxy in many cases to differences in size or experience. Further any argument which says that incumbents are less likely to adopt new technologies also suggests obvious policy prescriptions to correct for such inertia. We argue in this section that although incumbents prefer postponing innovations, in equilibrium they are unable to do so precisely because they are known to have this preference.

It will be very useful to make a distinction between a direct incumbent and an indirect incumbent. A direct incumbent is one who is (at period 0) in the relevant market (and is making some instantaneous profits $\pi \geq 0$). The size of the current profits π is then a measure of incumbent inertia. Our principal finding (Proposition 3) is that there is a critical level of profits, say $\hat{\pi}$, above which the non-incumbent (entrant) does end up adopting first (and making lower lifetime profits). However, for $\pi < \hat{\pi}$ a forward induction argument suggests that in fact, an incumbent is the first to adopt. The intuition for this result is the following: suppose that the firms are unaware which of two equilibria (incumbent high-quality or entrant high-quality) is being played. However it is common

[20] In fact $\xi = 2 - \frac{3}{2}\left[\delta e\left(1 - \frac{1}{e}\right)\right]^{\frac{1}{2}}$.

knowledge that if the entrant were to be the low-quality firm it would only develop the product till date T (whereas the incumbent on account of cannibalization likes to wait till a later date). If date T passes and there is no new product on the market forward induction logic suggests that the only reasonable conclusion that the incumbent can reach is that the potential entrant plans on being the high-quality (second-adopter) firm in the industry. Hence it maximizes its returns by adopting earlier and making lower profits.[21]

An indirect incumbent is one who may not be in the precise market under consideration but has better information about it, perhaps by virtue of selling similar products. A domestic firm facing competition from a foreign enterprise may be said to be in such a situation. As an index of indirect incumbency advantage we shall maintain, as a hypothesis, that the incumbent is better able to exploit a monopoly position. We shall translate this to mean that an incumbent makes mR, $m \geq 1$, in a monopoly situation, whereas an entrant only makes R (as before). m is of course an index of incumbency advantage. We show (Proposition 4) that for <u>any</u> advantage (i.e. for any $m > 1$), the <u>unique</u> equilibrium is one in which the <u>incumbent</u> necessarily adopts first. As long as m is not very large (i.e. the monopoly advantage is not unilaterally big), the incumbent, despite his advantage, makes strictly <u>less</u> in equilibrium. The reasoning is as follows: monopoly rents are higher for the indirect incumbent and consequently the date at which it prefers to adopt rather than be a follower is earlier for such a firm. A backward induction argument then establishes the result.[22]

4.1 <u>Direct Incumbents</u>

From hereon, let firm 1 refer to the incumbent and firm 2 to the entrant. Then, between period 0 and the first adoption x, firm 1 makes a flow profit $\delta\pi$ (and firm 2, the entrant, makes nothing). So,

$$F_1(x) = (1 - e^{-\delta x})\pi + F(x) \qquad (11)$$

$$L_1(x) = (1 - e^{-\delta x})\pi + L(x)$$

Of course, $F_2 = F$ and $L_2 = L$.

(Fig. 3)

---

21 This is an application of forward induction as introduced by Kohlberg-Mertens (1986).
22 The argument is similar to that in Ghemawat-Nalebuff (1986).

Proposition 3: Suppose the symmetric game had a maturation equilibrium. Then there is a a unique forward induction proof equilibrium and a critical level of incumbent profit $\hat{\pi} > 0$ such that:

i) for $\pi \leq \hat{\pi}$, the outcome is: incumbent adopts at x_1^M and the entrant follows at $x_1^M + \theta^*$.

ii) for $\pi > \hat{\pi}$, the equilibrium outcome is: entrant adopts at x_2^M and the incumbent follows at $x_2^M + \theta^*$.

If the symmetric game had a pre-emption equilibrium, then so does the asymmetric game with an outcome:

probabilistic move by firm i at x^I, j follows at $x^I + \theta^*$.

Proof: It is immediate, from (11), that x_i^I, the intersection of F_i and L_i, are identical for both firms. Let us maintain the notation for this common intersection point and call it x^I. Further, precisely because adopting a new product means foregoing current profits π, if firm 1 had to lead it would lead later than firm 2 in a similar situation, i.e. $x_1^M > x_2^M$. Note that in terms of our earlier notation, $x_2^M = x^M$ and we use the notation interchangeably. It should be easy to see that any increase in π increases x_1^M (i.e. increases incumbency inertia) and leaves x^I and x^M unchanged.

Suppose now that the symmetric game had a maturation equilibrium, i.e. that $x^I > x^M$. We have two cases to consider:

Case 1: $L(x^M) \leq F(x_1^M)$: By arguments identical to those in Proposition 1 one can show that there are only two subgame perfect equilibria. The first has the entrant moving at all $x \geq x_2^M$, if the other has not moved yet while the incumbent (firm 1) only adopts at $x \geq x^I$ if neither has adopted till such point. Of course, as a follower each follows after the optimal gap of θ^*. Beyond x^I both firms will try to adopt if neither has adopted till that point. The second equilibria has the roles reversed with firm 1 (the incumbent) leading at x_1^M, and firm 2 only adopting (together with firm 1) after x^I. Since $L(x_2^M) \leq F(x_1^M)$, the entrant would rather follow at x_1^M, than lead at x_2^M. In fact because of this preference, the forward induction argument of Kohlberg-Mertens (1986) will now be used to show that only the second of the two equilibria survives that refinement. Suppose an incumbent observes at $x \in (x_2^M, x_1^M)$ that firm 2 has <u>not</u> adopted yet. As argued above, there are two possible continuation equilibria at this point which yield player 2, respectively, $L(x)$ and $F(x_1^M)$. Since $L(x) < L(x_2^M)$, and $L(x_2^M)$ is a return that firm 2 could have received by innovating at x_2^M, the only credible inference that the incumbent can draw from this, is that the two firms should coordinate on the second equilibria and it should adopt at x_1^M. Note that firm 1 cannot credibly signal before 2's adoption date, precisely because the cannibalization factor means it is strictly better off not innovating early.

<u>Case 2</u>: $L(x^M) > F(x_1^M)$: It is not difficult to see that there are two subgame perfect equilibria in this case. The first is identical to the first equilibria in case 1 with the entrant leading at x^M. Define x^* through $L(x^*) = F(x_1^M)$. The second equilibrium is: firm 2 adopts for all x in $[x^M, x^*]$ or $x \geq x^I$, if neither has adopted before, whereas firm 1 adopts for all $x \geq x_1^M$. An argument identical to that for case 1 but applied now to the region $[x^*, x_1^M)$ shows that only the second equilibrium is forward induction proof. Of course the outcome in either case is: the entrant adopts at x^M and the incumbent follows at $x^M + \theta^*$.

Since x_1^M is increasing (and hence $F(x_1^M)$ is decreasing) in incumbent profit π, there is a critical profit level $\hat{\pi}$ which divides the two cases above and below which the entrant can credibly signal his unwillingness to lead and force the incumbent, <u>despite</u> the cannibalization effect, into a leadership position. For $\pi \geq \hat{\pi}$, the cannibalization effect dominates. Finally note that if the symmetric game had a preemption equilibrium, i.e. if $x^I < x^M$, then $x^I < x_1^M$ and hence the only equilibrium is one in which both firms try to adopt after x^I. •

4.2 <u>Indirect Incumbent</u>

An indirect incumbent, firm 1, is better able to exploit a monopoly position, and hence makes mR(x) as a monopolist, where $m \geq 1$. Hence,

$$L_1(x) = e^{-\delta x}\left\{(1 - e^{-\delta\theta^*})mR(x) + e^{-\delta\theta^*}r(-\theta^*)\right\} \qquad (12)$$
$$F_1(x) = F(x)$$

For expositional purposes, in this sub-section we assume $r(-\theta^*) = 0$. The reader is invited to check that none of the results are predicated on this; it merely makes the presentation a lot clearer since in this case $L_1(x) = mL(x)$ and consequently $x_1^M = x_2^M$.

We maintain notation and call this common maximum x^M. Of course, $L_2 = L$, $F_2 = F$. Clearly, it follows that (starting from a maturation equilibrium in the symmetric game), we have figure 4.

(Fig. 4)

Proposition 4: Suppose the symmetric game has a maturation equilibrium. Then, there is some critical incumbency advantage $\hat{m}$ s.t.

i) for $m < \hat{m}$, the unique equilibrium has incumbent adopting at x^M, the entrant at $x^M + \theta^*$. No rent is dissipated but the entrant makes strictly more in equilibrium.

ii) for $m \geq \hat{m}$, the unique equilibrium has the same outcome as above, but the incumbency advantage is sufficiently big to overwhelm the first mover disadvantage. The incumbent makes more.

Finally, if the symmetric equilibrium is a pre-emption equilibrium, then so is the asymmetric with the outcome:

incumbent adopts at x^I and entrant follows at $x^I + \theta^*$.

Proof: Suppose the symmetric game has a maturation equilibrium. Note that on account of the indirect incumbency advantage $x_1^I < x_2^I$ and indeed x_1^I is decreasing in m. This of course just says that the opportunity cost of the incumbent for staying out of the market is higher than that of the entrant and this cost is increasing in the extent of monopoly advantage. Clearly, in any equilibrium, a dominant strategy for firm 1 is to adopt beyond x_1^I if neither has adopted before. But then, there is $x^1 < x_1^I$ such that it is a dominant strategy for 2 to not adopt at any $x \in [x^1, x_1^I)$. x^1 is formally defined through

$$x^1 = \max \left\{ z: L(x) \leq F(x_1^I) \quad x \geq z \right\} \tag{13}$$

In the figure, $x^1 = 0$. More generally, there is some left neighborhood of x_1^I, in which firm 2 does better by waiting to follow, than by leading. Given this, firm 1's dominant strategy is to lead on $[x^1, x_1^I)$. An identical argument as in Proposition 1 now leads through an iterated elimination of dominated strategies to: firm 1 adopts at x^M (and any time thereafter). The entrant, firm 2 follows at $x^M + \theta^*$. Note, despite the incumbency advantage, the entrant makes strictly more than the incumbent. As $m\uparrow$, x_1^I decreases and hence at some critical advantage $\hat{m}$, $x_1^I = x^M$. Clearly, for any $m \geq \hat{m}$, the equilibrium outcome is incumbent moves at x^M and makes more than the entrant. •

Remark: Propositions 3 and 4 illustrate one usefulness of a truly dynamic formulation of a vertical differentiation problem. In standard formulations as Prescott-Visscher (1977) or Shaked-Sutton (1982) (see also the survey of such models in Eaton-Lipsey (1989)), quality choices are essentially made in a static model: they are chosen in stage one prior to price competition in stage two. Consequently neither the forward nor backward induction arguments made above can be applied. Hence natural asymmetries between the firms cannot be used to identify which firm is going to be the high quality firm in equilibrium. Put differently even in asymmetric versions of such games typically both of the outcomes contained in the maturation possibility remain equilibrium outcomes. By contrast we have shown that some kinds of asymmetry, no matter how small, can uniquely identify particular outcomes as equilibrium outcomes.

5. Policy Effectiveness and Product Development

For simplicity in the next two sections we return to the symmetric game formulation of Section 3. The question we now turn to is: how does policy (which affects the rate of product development) in turn determine whether pre-emption or maturation equilibrium outcomes obtain? Further what is the effect of policy on the level and diversity of innovation? The role of policy will be to alter the rate at which quality or technology can be improved prior to product adoption. We shall assume from this point on that the rate at which quality improves from the time of discovery to first adoption is ρ^{-1} and its rate of growth between the time of first and second adoption is γ^{1}. The first rate is affected by policies which determine the size of government funding for its own laboratories, if the development was occurring in public facilities, or government coordination of the activities of private laboratories. The second rate is affected by governmental policies on the sharing of market data. Of course γ is also a consequence of market variables as the ease of learning from the experience of rival firms and lock-in of consumer preferences. We examine two conjectures: a) "as maturation becomes more profitable (γ decreases), maturation equilibria are more likely". Further,"the level of innovation or first adoption quality and diversity or differences in adoption qualities increases." b) "as the rate of product development increases (ρ decreases), there are similar effects".

The first conjecture may or may not be true. One problem is that a first adopter position may simultaneously also become more profitable. On the other hand if the returns to being a first innovator decline substantially a firm may be willing to lead only if the monopoly rents can be earned for a sufficiently long period. However the outcome might still be a preemption one since following after such a long lag is less attractive than earlier pre-emption. Similarly, the effect on the level and diversity of innovations is ambiguous since such effects again depend on the marginal increases in follower profits rather than absolute increases. Changing γ only changes the absolute returns but has no immediate implications for the changes in marginal valuations. In what follows we isolate robust sufficient conditions under which these anomalous behaviors do not arise and in fact the conjecture is exactly true. In particular, we establish critical levels of the growth rate of quality between the first and second adoption such that there are maturation equilibria above these levels and pre-emption equilibria below them. Under these same conditions the level of innovation and diversity increases as γ decreases. Note that for any γ, there is an optimal entry gap θ_γ determined from the follower's problem (see Section 3.1). In particular, firm j enters after a time period $\gamma\theta_\gamma$, during which the state has grown by the amount θ_γ. So,

$$
\begin{aligned}
F_\gamma(x) &= e^{-\delta x}\left\{ e^{-\delta\gamma\theta_\gamma} r(\theta_\gamma) \right\} \equiv e^{-\delta x}\phi(\gamma) \\
L_\gamma(x) &= e^{-\delta x}\left\{ (1 - e^{-\delta\gamma\theta_\gamma})\, R(x) + e^{-\delta\gamma\theta_\gamma} r(-\theta\gamma) \right\} \\
&\equiv e^{-\delta x}\left\{ \lambda_1(\gamma)\, R(x) + \lambda_2(\gamma) \right\}
\end{aligned}
$$

When the differentiation technology improves, i.e. γ decreases, it clearly increases the follower's returns, $\phi(\gamma)$. The effect for a potential leader could be either through monopoly returns ($\lambda_1(\gamma)$) or duopoly returns ($\lambda_2(\gamma)$). The two effects could be in conflict; for example if $\gamma\theta_\gamma$ increases, then so does $\lambda_1(\gamma)$ but clearly $\lambda_2(\gamma)$ simultaneously decreases. To simplify the analysis, we consider only the first effect, and assume for this sub-section that $r(-\theta) \equiv 0$ (and hence, $\lambda_2(\gamma) = 0$). So, $L_\gamma(x) = e^{-\delta x}R(x)\lambda_1(\gamma)$, and we have the situation of Figure 5 (where L' and F' refer to $e^{-\delta x}R(x)\lambda_1(\gamma')$ and $e^{-\delta x}\phi(\gamma')$ respectively and $\gamma' < \gamma$).

(Fig. 5)

Fig. 5 shows a case where an increase in learning possibilities (γ' rather than γ), leads to a shorter time-lag of adoption, thereby lowering the leader's returns. Of course, the follower must necessarily make more as a faster learner. Since the monopoly returns, $\lambda_1(\gamma)$, depend precisely on the adoption time-lag, as long as the follower takes less time imitating, when the imitation or differentiation technology improves, decreasing γ makes maturation more likely.

Proposition 5: Suppose $\gamma\theta_\gamma$ is in fact increasing in γ. There is a critical γ^* s.t.

i) for all $\gamma < \gamma^*$, the equilibrium is a maturation equilibrium with outcomes x^M (independent of γ) and $x^M + \theta_\gamma$.

ii) for all, $\gamma > \gamma^*$ the equilibrium is a pre-emption equilibrium, with outcomes $x^I(\gamma)$ and $x^I(\gamma) + \theta_\gamma$. $x^I(\gamma)$ is decreasing in γ, i.e. the less productive is the differentiation technology, the lower is the level of innovation.

Finally, if r is concave on R_+ then the diversity of innovations θ_γ is decreasing in γ in both cases, i.e. the more productive the differentiation technology, the greater the actual differentiation.

Proof: Under the hypotheses, x^M is independent of γ whereas x^I is decreasing in γ. Since we have maturation equilibria if and only if $x^M < x^I$, the first part of the proposition follows. The first order conditions for the follower's optimal choice of diversity is

$$r'(\theta_\gamma) = \delta\gamma\, r(\theta_\gamma) \qquad (17)$$

If r is concave the second part of the proposition immediately follows. •

Let us now ask what happens if the underlying technology grows at a rate ρ^{-1} till the first adoption (and then, at 1 thereafter). In particular we examine the second conjecture. This case is not equivalent to changes in imitation technology as we will see.

$$F_\rho(x) = e^{-\rho\delta x}\,\phi \qquad (18)$$

$$L_{\rho}(x) = e^{-\rho\delta x}\left\{\lambda_1 R(x) + \lambda_2\right\}$$

Arguments as above establish

Proposition 6: Suppose R is concave. Then, there is a critical level of technological growth, say $\bar{\rho}$, such that

i) If $\rho > \bar{\rho}$ then we have a maturation equilibrium. Entry times are decreasing and profits are decreasing in ρ.

ii) If $\rho < \bar{\rho}$ then we have a pre-emption equilibrium, all of whose outcomes (independently of ρ) are x^I and $x^I + \theta^*$. Firm's profits are decreasing in ρ.

Remark: An alternative interpretation of the analysis in this section is that it we do comparative statics with respect to market as well as policy variables. On account of market variables like learning from the experience of competitors, γ may be different from the growth rate of the underlying technology (i.e. 1), . Firms may learn about market conditions, consumer preferences, ease of marketing new products, unknown and intangible factors like consumer habits, available substitutes etc. In such cases, $\gamma < 1$. On the other hand, casual empiricism strongly suggests that there are "lock-in" effects, with early innovators keeping more than half the market share, when "me too" brands appear. In this case, of course $\gamma > 1$.

6. Does the Market Under Develop?

In much of the R&D literature one encounters the following conjecture: in imperfectly competitive markets there is a tendency to generate too little of innovation, as a consequence of threats of pre-emption. The bench-mark for such a comparison could be taken to be either a cartel's or a social planner's choices. Since neither is faced with the possibility of a rival pre-empting, the argument goes that the level of innovation, i.e. the quality or time of first entry should be higher in such contexts. We examine the validity of this conjecture and show that product differentiation possibilities fundamentally affect the market's innovation incentives. In general, there are two conflicting determinants of the comparison between the two classes of outcomes. Consider a cartel. On the one hand, the cartel internalises the returns to differentiated products and hence may innovate earlier since the waiting costs are consequently greater. On the other hand, the cartel is better able to control the flow of innovations and may be able to generate monopoly returns for a longer period. This would lead to a later first innovation, since it is important to have a better quality in the first innovation. The fact that there are these conflicting incentives suggests of course that the comparative statics are ambiguous. As an illustration we give sufficient conditions under which in any maturation equilibrium, the low technology leader in fact innovates too late relative to both the cartel and social optimum. For a social planner the trade-offs are similar although the returns are different. We restrict ourselves to Example 2.2 and derive the "over innovation of duopoly" result.

Product diversity, i.e. θ, will differ in the three contexts. Cartels and social planners have to trade-off "cannibalisation effects" against "efficiency effects" and the outcome is hence, ambiguous.

6.1 The Cartel Solution

A cartel picks two adoption dates x and x + θ. Let $P(\theta)$ denote the cartel's optimum profits when two products, of diversity θ, are in the market and suppose that $P(\theta) > r(\theta)$ for all θ. To maintain consistency with the duopoly problem, we assume that this optimum profit only depends on relative quality. Also to keep the two structures comparable, we assume that a cartel will in fact innovate twice. The second adoption problem is

$$\max_{\theta\geq 0} (1 - e^{-\delta\theta})R(x) + e^{-\delta\theta}P(\theta) \tag{19}$$

Let us suppose that a solution to this problem exists, denote a solution by θ_x and consider the first adoption problem.

$$\max_{x\geq 0} \; e^{-\delta x} \left\{ (1 - e^{-\delta\theta_x})R(x) + e^{-\delta\theta_x} P(\theta_x) \right\} \tag{20}$$

A first order condition for this problem is:

$$[R'(x_c) - \delta R(x_c)]\,(1 - e^{-\delta\theta_x}) = \delta e^{-\delta\theta_x} P(\theta_x) \tag{21}$$

In contrast a leader in a potential duopoly maximizes rents by selecting a level of innovation x^M such that

$$[R'(x^M) - \delta R(x^M)]\,(1 - e^{-\delta\theta^*}) = \delta e^{-\delta\theta^*} r(-\theta^*) \tag{22}$$

Comparing the trade-offs in (21) - (22) yields

Proposition 7: Suppose that monopoly returns are concave in quality. Suppose further that either i) $r(-\theta^*) = 0$ or ii) $\theta_x \leq \theta^*$. Then, $x_c \leq x_M$, i.e. a cartel innovates earlier than a duopoly in a maturation equilibrium.

Remark: The optimal product diversity may, in general, be more or less than the amount of product differentiation in a duopoly equilibrium, θ^*. Further information on joint profits $P(\theta)$ would be required to answer that question.

6.2 The Social Optimum

Consider the social planner's problem of maximizing some index of social welfare by the choice of an innovation and differentiation decision. The obvious indices would be consumer surplus (if the primitives are demand functions) or aggregate utility (if the underlying consumer preferences are specified). We illustrate with Example 2.2, using aggregate utility.

It is easy to see that the social welfare problem is

$$\text{Max } e^{-\delta x} \left\{ x(1 - e^{-\delta\theta}) + (x + \theta)e^{-\delta\theta} \right\} \tag{23}$$

It is straightforward to see that the social diversity $\theta_S = \theta^*$. Comparing the social planner's first adoption problem with that of a potential leader in a duopoly, some straightforward algebra yields

Proposition 8: In Example 2.2, a social planner adopts at a date sooner than that in a duopoly maturation equilibrium. The amount of product diversity is identical in the two cases.

7. Extensions

Let us briefly discuss some extensions of the current model. The principal assumption which facilitated the analysis is (A1), that duopoly returns depend only on relative qualities. Dropping this assumption changes some details but not the main substance of the results. It is clear that in a general setting the optimal amount of product differentiation engaged in by a follower will depend on the level of the first innovation. Denote this dependence $\theta(x)$. The principal complication arises from not knowing, in a general formulation, any qualitative features about this optimal reaction function. For example, if $\theta(x)$ is increasing in x, the above analysis and results remain completely unchanged. Else, it would still be the case that there are only two kinds of equilibria. There may however be several possible maturation or pre-emption equilibria in a single game.

The assumption of deterministic technological change chosen in this paper is virtually inessential to the general analysis. In fact in many interesting applications, like the timing of asset sales or learning about market characteristics, an uncertainty formulation is natural. A similar comment pertains to the requirement that the maturation process be a non-decreasing process as in this paper. For quality of information that is a natural restriction, but not for content of information. Dutta-Rustichini (1990b) develop a general theory for games of entry which can handle such issues.

The controversial element of our formulation is our simplification that there is a once for all improvement of the basic idea which firms engage in befor they introduce the product in the market. There are two possible generalizations: the first allows "learning by doing", i.e. that a product in the market can continue to be improved. This generalization can be straightforwardly incorporated into our formulation provided the rate of improvement prior to an introduction (in the laboratory) is greater than the rate of improvement after the introduction (in the market). A second generalization is more difficult and that relates to repeat innovations. Repeat innovations are an important stylised fact of the innovative process. In a sense the repeated innovation problem is the classic repeated oligopoly problem except that quality or product attributes are costly and less reversible choices than prices or quantities. Hence the issue here is much closer to that of a dynamic game with historical product choices affecting current decisions. One such dynamic analysis is contained in Dutta-Rustichini (1990a). Finally, it should be pointed out that our analysis remains unchanged if we drop the irreversibility of quality assumption. Similar issues as in the case of level-dependent returns arise.

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Figure 1

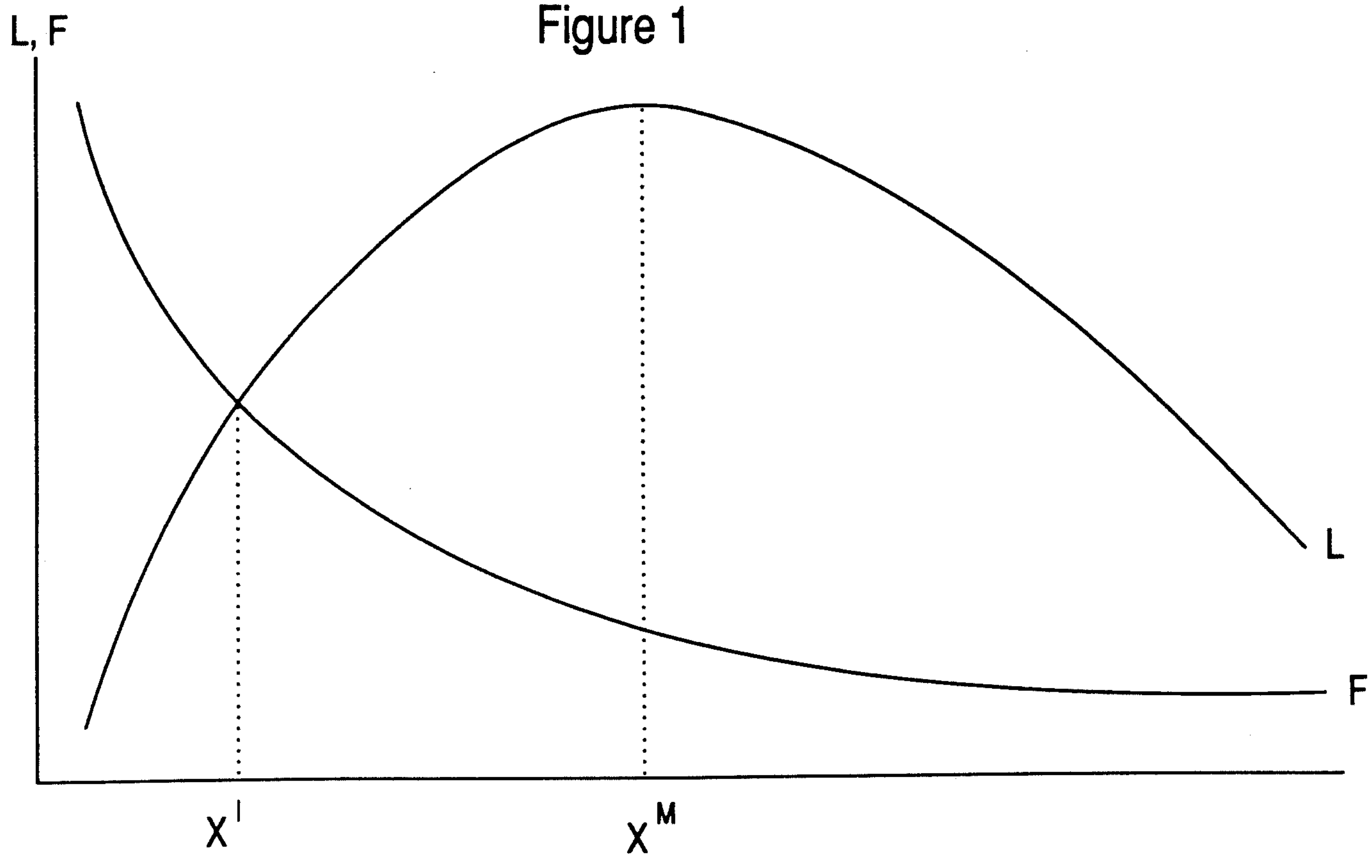

Figure 2

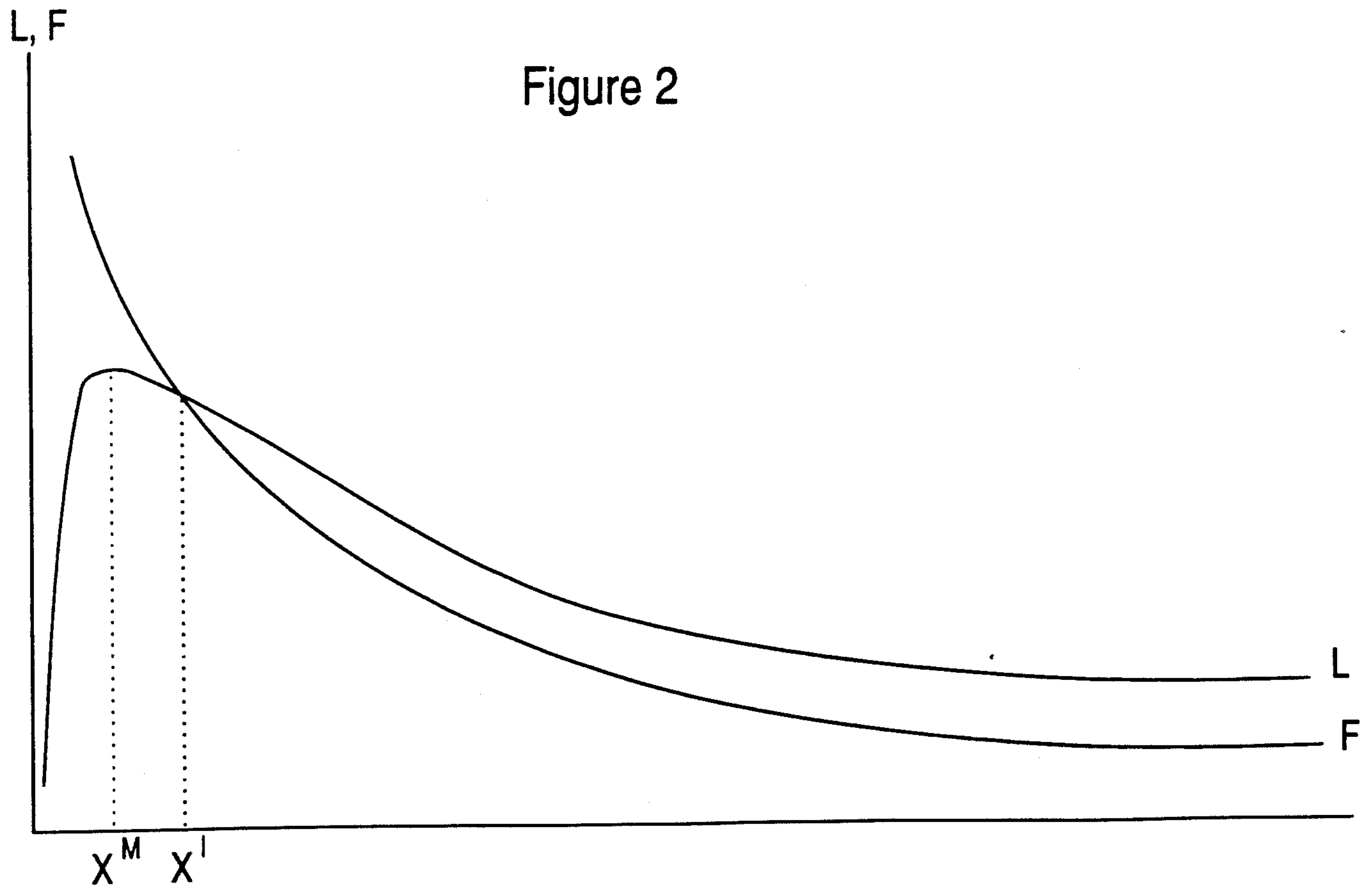

Figure 3

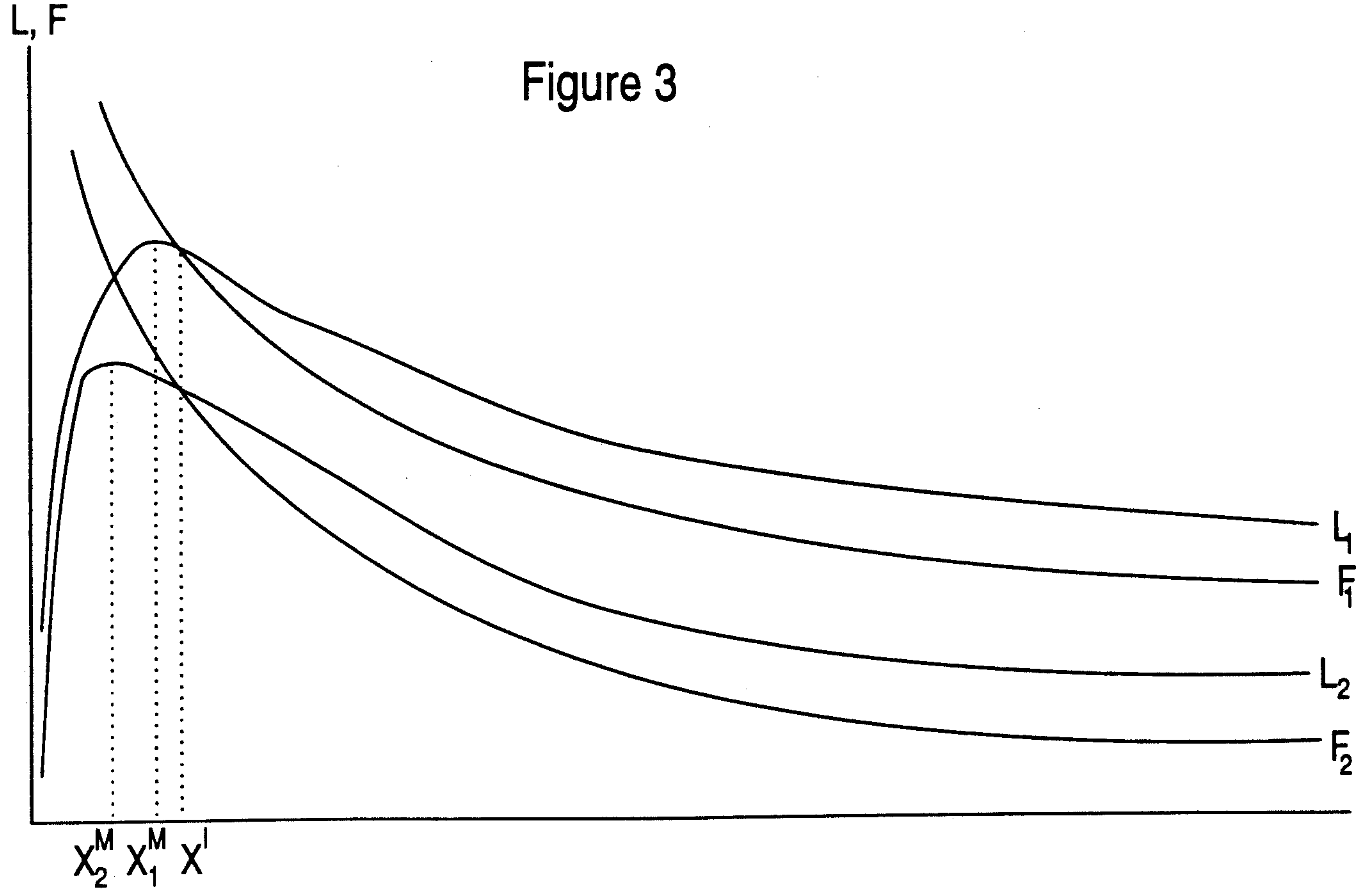

Figure 4

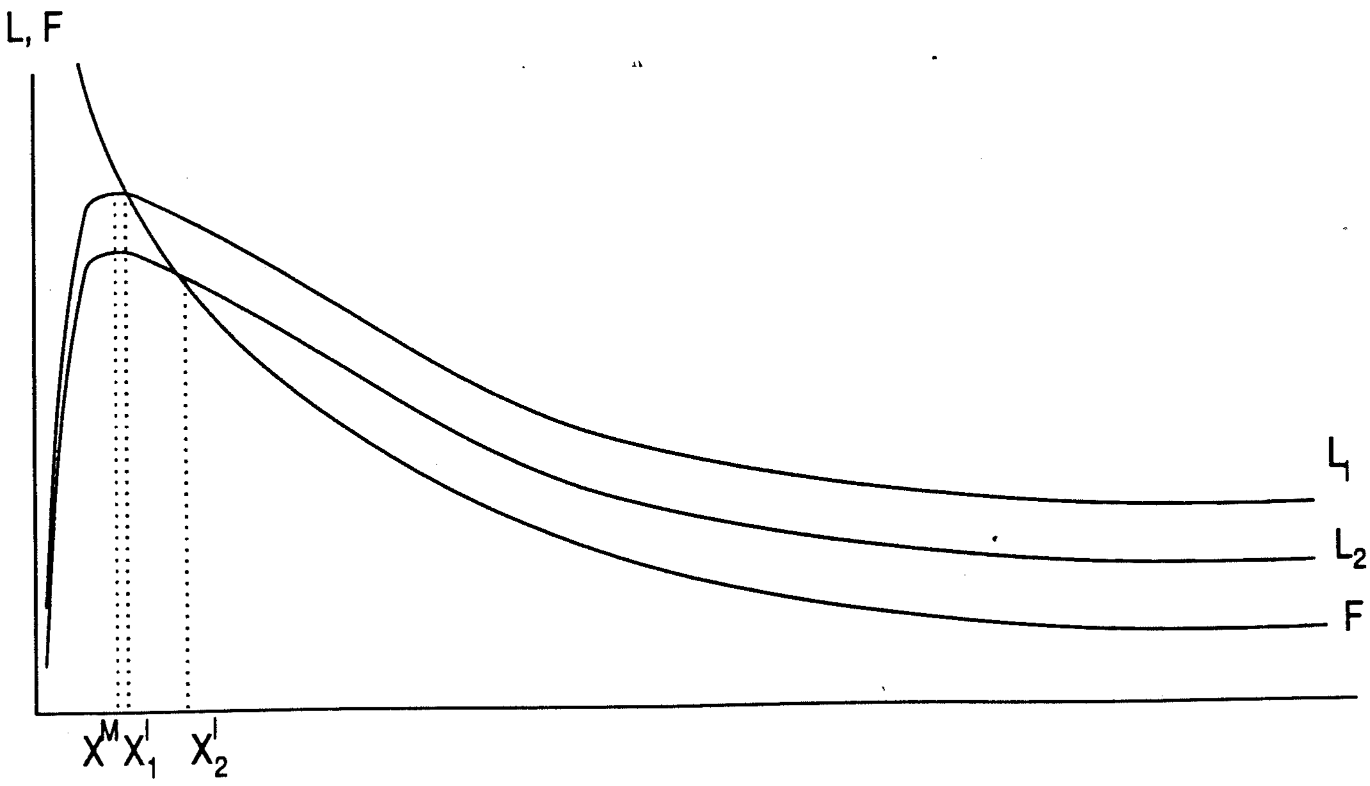

Figure 5