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ABSTRACT

This paper describes an endogenous growth economy where the presence of financial markets fosters growth by encouraging specialization. We assume that some specialization is essential to the occurrence of learning-by-doing. Without financial markets, no learning-by-doing occurs and the economy reduces to the Diamond (1965) model. When financial markets exist, specialization is possible. Young agents can invest in education, borrow to put capital in place, and then operate a firm in both middle and old age. We describe conditions under which an equilibrium exists where some agents do this, while others work in youth and middle-age and are then retired. In this situation all agents repeat activities, so that learning-by-doing occurs. In the presence of spillovers, permanent growth is possible. However, it is also possible that financial markets fail to form for endogenous reasons, preventing specialization. In this event an economy becomes "stuck" in a low growth equilibrium, even if equilibria exist with higher real growth rates. Finally, we show that any (positive) constant growth rate equilibrium has the property that the rate of interest equals the rate of growth.

"..it is quite interesting how rarely one sees the discussions of the use of money making the point that was made by Adam Smith and which somehow the Arrow Debreu paradigm stops us from making: that the extent of the market governs the division of labor; ...I think compared with bringing traders together, the function of money in allowing specialized production is of greater importance." Frank Hahn (1980)

Adam Smith's view that money and the financial system are important for economic growth has long been widely held even though, as Hahn laments, it has not been widely explored in modern monetary theory. The foremost chronicler of British financial markets, Walter Bagehot (, p7) argued that England benefitted "enormously" relative to other countries from the development of its credit markets:[1]

> In a new trade English capital is instantly at the disposal of persons capable of understanding the new opportunities and making good use of them. In countries where there is little money to lend...enterprising traders are long kept back, because they cannot at once borrow the capital, without which skill and knowledge are useless.

More recently, McKinnon (1973, p. 8-9) argues that poorly developed financial markets are part of the reason why less-developed countries remain that way.[2] Without a well functioning financial system "fragmentation in the capital market...suppresses entrepreneurial development, and condemns important sectors of the economy to inferior technology." He then asserts that the development of capital markets "is necessary and sufficient" to foster "the adoption of best-practice technologies and learning-by-doing." The common theme in all of these arguments is that financial institutions and markets promote growth by encouraging specialization and learning-by-doing. In this paper we pursue this idea in the context of a model economy where growth occurs endogenously and where the presence of financial markets fosters growth by encouraging specialization in entrepreneurial activity.

The role of financial markets in promoting growth has been explored recently in the context of endogenous growth models. Bencivenga and Smith (1990) and Greenwood and Jovanovic (1990) study models in which financial markets promote growth by providing liquidity. This permits economies to hold a higher proportion of their savings in the form of capital, and less in the form of liquid (but relatively unproductive) assets. They also argue that financial

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1 Similar arguments are to be found in Cameron et. al. (1967) and Goldsmith (1969).

2 This view is echoed in Shaw (1973). For a survey of more recent evidence, see the World Development Report (1989).

markets reduce "fragmentation" of the credit allocation process, permitting a more productive use of available investment funds. Bencivenga and Smith also show how financial markets can promote growth by changing the timing of investments that otherwise might be delayed. However, these papers do not address the view, discussed above, that financial markets foster growth by encouraging specialization and learning-by-doing. That role for the financial system is analysed here.

We pursue this line of reasoning in a model where growth occurs endogenously via learning-by-doing. As in Arrow (1962), Stokey (1988), and Azariadis and Drazen (1990), learning-by-doing generates spill-overs, which permit permanent growth to occur.[3] We further assume that some specialization of labor is conducive to learning, as argued by Arrow (1962). We then show that the presence of financial markets promotes permanent growth.

Our vehicle for exploring these ideas is an overlapping generations model, in which agents are three period lived. When young, agents can either sell labor, or engage in schooling. Only agents who engage in schooling can operate the production process.[4] When middle-aged agents can either sell labor, or if they invested in education when young, they can operate a firm. Old agents cannot sell labor, but can operate firms if they were educated when young.

There is a single consumption good, produced from capital and labor. Capital must be put in place one period in advance of production. Then if there are no financial markets, so that borrowing is precluded, middle-aged agents cannot run firms, since agents who invest in eduction when young have no income, and hence cannot finance investments. Further, in the absence of financial instruments and rental markets in capital, the only way for agents to provide for old period consumption is to operate a firm when old. Then, when there are no financial markets, all agents invest in education when young, work when middle-aged, invest in capital, and operate a firm when old. There is no specialization.

We assume that some specialization is essential to the occurrence of learning-by-doing (and in particular, agents must repeat activities). Then, without financial markets, no learning-by-doing occurs. We structure the model so that, in this event, it reduces to the Diamond (1965) model. Thus permanent growth is not possible.

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[3]Obviously the idea that growth occurs due to endogenous accumulation of knowledge, embodied in capital or otherwise, appears in a variety of places. Examples include Shell (1966, 1973), Romer (1986), Prescott and Boyd (1987), Rebelo (1987), Lucas (1988), and King, Plosser and Rebelo (1988).

[4]This formulation is similar to that of Freeman and Polasky (1990).

When financial markets exist, specialization is possible. Young agents can invest in education, borrow to put capital in place, and then operate a firm in both middle and old age. We describe conditions under which an equilibrium exists where some agents do this, while others work in youth and middle-age (and are then retired). In this situation all agents repeat activities, so that learning-by-doing occurs. In the presence of spillovers, permanent growth is possible. However, it is also possible that financial markets fail to form for endogenous reasons, preventing specialization. In this event an economy becomes "stuck" in a low growth equilibrium, even if equilibria exist with higher real growth rates. Finally, we show that any (positive) constant growth rate equilibrium has the property that the rate of interest equals the rate of growth.

Section I describes the environment, and section II describes equilibria when financial markets are exogenously precluded. Section III allows for the formation of financial markets. Section IV considers the possibility that financial markets fail to form endogenously; i.e., that they do not form even if they can. Section V concludes.

I. The Model

The economy consists of an infinite sequence of three period lived, overlapping generations. At each date t; $t=0,1,2,\ldots$, a new young generation appears, consisting of a continuum of identical agents of measure one. Also, at each date there is a single produced commodity, which can either be consumed or converted into capital.

Young agents can engage in one of two activities: they can either sell labor to a potential employer, or they can engage in an alternative activity that generates no young period income or output (or utility), but that permits them to run production processes at a later date. In other words a young agent can either work or engage in some kind of "schooling." Only those agents who have this schooling can subsequently manage a production process (i.e., run a firm), and we will call these agents entrepreneurs.[5]

Middle aged agents can either sell labor to a potential employer, or run a firm if they were schooled when young. Old agents have no labor to sell, but if they were schooled when young, they can run a firm. Finally, when labor is supplied it is supplied inelastically (labor generates no disutility).

There is a production technology, available to all who were educated when young, for converting capital and labor, measured in efficiency units, into the single consumption good. Efficiency units are measured as follows. We let $h_t \in \mathbb{R}_+$ denote the stock of "knowledge," or "human capital" at t. This is common to all agents.[6] One unit of actual time supplied at t delivers h_t units of "effective" labor. Then a firm operating at t, which employs L_t agents, each supplying one unit of time, and K_t units of capital produces output equal to $Y_t = \tilde{F}(K_t,h_tL_t,h_t)$, where the third argument of $\tilde{F}$ is managerial input, which is also measured in efficiency units. Each firm has exactly one manager. We restrict attention to the case in which

$$(1) \qquad \tilde{F}(K_t,h_tL_t,h_t) = F(K_t,h_tL_t + h_t).$$

F is assumed to be increasing in each argument, strictly concave, homogeneous of degree one, and

[5] Freeman and Polasky (1990) describe a formulation that is somewhat similar.

[6] The assumption of a freely available common stock of knowledge follows Stokey (1988) or Azariadis and Drazen (1990), who allow a common quantity of human capital to be augmented by individuals who engage in training.

to satisfy standard Inada conditions.

In order to describe the process of human capital accumulation, let $\bar{Y}_t$ denote "average per-firm output" at t, and let μ_t denote the fraction of young agents who choose to become workers at t. Then h_t evolves according to

$$(2) \qquad h_{t+1} = h_t\phi[(\bar{Y}_t/h_t), \mu_t] \ .$$

Thus the rate of growth of knowledge depends on the level of average output relative to current knowledge, and on the fraction of young agents working at t. The former effect represents the consequences of learning-by-doing. Here, as in Stokey (1988), learning-by-doing "spills over" completely, so that no individual has an incentive to consider how his individual behavior affects his knowledge relative to that of others. This dramatically simplifies the analysis. Note also that learning depends on production relative to the current state of knowledge. Thus producing a given amount results in more learning if the current stock of knowledge is relatively low. It seems plausible that more can be learned by producing a given amount for agents who are relatively less knowledgeable to begin with.

The dependence of ϕ on μ_t is meant to capture how learning is increased by specialization. If $\mu_t=0$ (or $\mu_t=1$) all agents engage in the same activity. We assume that heterogeneity of activities is conducive to increasing the aggregate stock of knowledge. In fact, we assume that some specialization is essential to learning, so that $\phi[(\bar{Y}/h), 0] = \phi[(\bar{Y}/h), 1] = 1$ $\forall\ \bar{Y}/h$.[7]

Notice that the formulation in (2) implies constant returns to scale in all dimensions, in that if all inputs are increased in proportion, for a given degree of specialization (μ_t), all outputs (including future knowledge) are increased in the same proportion. We also assume that ϕ is continuously differentiable in its first argument for $\mu \neq 0,1$, with $\phi_1\ [(\bar{Y}/h), \mu] > 0$. In addition, ϕ is assumed to be non-decreasing (non-increasing) in its second argument for $\mu \epsilon\ (0,1/2]$ [$\mu \epsilon$ (1/2,1)], and to satisfy $\phi[(\bar{Y}/h), \mu] > 1$ whenever $\bar{Y}/h > 0$ and $\mu \neq 0,1$. Obviously $\mu = 1/2$ represents the maximal amount of heterogeneity in activity. Lastly, note that we are admitting the possibility that

[7]This exact specification is inessential to the analysis, but makes the model reduce to that of Diamond (1965) in the absence of financial markets.

$$(2.a) \qquad \phi[(Y/h),\mu] = \begin{cases} 1 & ; \mu = 0,1 \\ \tilde{\phi}(Y/h); & \mu \neq 0,1 \end{cases}$$

Finally, we impose two assumptions on the use of capital. First, capital depreciates entirely after one period. Second, capital must be put in place one period in advance of production. In particular, each producer at t views himself as constrained by past investment decisions: there are no within period rental markets in capital.

It remains to describe the preferences and endowments of agents. Letting $c_j \in \mathbb{R}_+$ denote age j consumption, all young agents have preferences described by the additively separable utility functions $u(c_2) + v(c_3)$. Thus agents do not value period one consumption. This assumption is again inessential; it serves only to make the model reduce to that of Diamond (1965) in the absence of financial markets. u and v are assumed to have standard properties, and in addition we impose a gross substitutes condition:

$$(3) \qquad 0 \geq cv''(c)/v'(c) \geq -1,$$

$\forall\, c \in \mathbb{R}_+$. With respect to endowments, all agents are endowed with one unit of time in youth and middle-age. Time when young is indivisible; agents can work or engage in schooling, but not both. Young agents have no endowment of goods or capital. The initial old have the initial capital stock K_0.

II. Equilibrium: No Financial Markets

We begin by considering an equilibrium for this economy when financial markets are exogenously precluded. This could be a consequence of severe "financial repression," as defined by McKinnon (1973) and Shaw (1973). The possibility that financial markets fail to form endogenously is considered in Section IV.

The absence of financial markets implies that agents can neither borrow nor lend. As a consequence, no specialization will occur. In particular, no agent will choose to work when young, so that $\mu_t \equiv 0$. To see this, suppose some agent did sell labor when young at t, earning wage income $w_t h_t$. Consuming this income generates no utility, and income cannot be saved in the form of any financial instrument. Thus any saving would take the form of capital accumulation. But a middle-aged agent who did not engage in schooling when young cannot operate a production process, and there are no rental markets in capital. This saving would simply be lost. If any value is placed on old age consumption, all agents will engage in schooling when young.

Since $\mu_t \equiv 0$, h_t is constant, say $h_t \equiv 1$. Then, middle-aged agents will work, earning the wage rate w_{t+1} at t+1. In particular, these agents cannot operate firms, since they had no young period income and they were precluded from borrowing. Thus they have no capital in place in middle-age, so that agents must wait until old age to become entrepreneurs.

Middle-aged agents consume some of their wage income, and save the remainder in the form of capital. These agents are old at t+2, and have a capital stock of K_{t+2}. They then operate firms, choosing L_{t+2} to maximize $F(K_{t+2}, L_{t+2} +1) - w_{t+2} L_{t+2}$ (recall $h_t \equiv 1$). Then

(4) $$F_2 (K_{t+2}, L_{t+2} + 1) = w_{t+2}.$$

A middle-aged agent at t+1, then, earns w_{t+1}, and chooses K_{t+2} to maximize

$$u(w_{t+1} - K_{t+2}) + v[F(K_{t+2}, L_{t+2} + 1) - w_{t+2} L_{t+2}],$$

taking account of the dependence of L_{t+2} on K_{t+2}, and taking w_{t+2} as given. Then K_{t+2} satisfies

(5) $$u'(w_{t+1} - K_{t+2}) = F_1 (K_{t+2}, L_{t+2} + 1) v' [F(K_{t+2}, L_{t+2} + 1) - w_{t+2} L_{t+2}].$$

By Euler's Theorem and equation (4),

(6) $$F(K_{t+2}, L_{t+2} + 1) - w_{t+2} L_{t+2} = F_1(\cdot) K_{t+2} + F_2(\cdot) = F_1(\cdot) K_{t+2} + w_{t+2} .$$

Now define $s(w_1,w_2,r)$ to be the savings function of an agent who has income w_1 when middle-aged, w_2 when old, and faces a gross rate of return of r on savings:

(7) $$s(w_1,w_2,r) \equiv \text{argmax } [u(w_1-s) + v(w_2+rs)].$$

Under our assumptions, $s_1 \geq 0 \geq s_2$, and $s_3 \geq 0$. Further, define $k_t \equiv K_t/(L_t+1)$ and $f(k_t) \equiv$

$F(k_t,1)$. Then $f'(k_t) = F_1(k_t,1)$ and from (4), $w_t = F_2(k_t,1) = f(k_t) - k_t f'(k_t) \equiv w(k_t)$.

Substituting (6) into (5) and using the previous definitions, it is apparent that

$$(8) \qquad K_{t+2} \equiv k_{t+2}\,(L_{t+2} + 1) = s[w(k_{t+1}),\, w(k_{t+2}),\, f'(k_{t+2})].$$

Furthermore, in equilibrium, there is one worker per firm. Thus $L_{t+2} = 1 \;\forall\, t$, and (8) can be written as (replacing t by t-1),

$$(9) \qquad k_{t+1} = s[w(k_t),\, w(k_{t+1}),\, f'(k_{t+1})]\,/\,2.$$

Equation (9) defines k_{t+1} as an increasing function of k_t, as depicted in Figure 1. Under well-known conditions, [8] (9) has one or more non-trivial stationary solutions, and as in figure 1, no solutions in which $\{k_t\}$ is unbounded. Thus from any initial $k_0 \neq 0$, k_t will approach a non-trivial steady state value, and the economy will approach a situation of zero real growth. Also, convergence to the steady state equilibrium will be monotonic.

III. Financial Markets and Specialization

We now allow for the presence of financial markets, which can be represented as follows. A set of competive intermediaries exists that take deposits and make loans, each paying the competitive gross return r_t at t. Intermediaries behave as if they can make arbitrary loans and raise any desired quantity of deposits at this rate.

In the presence of such intermediaries, it is feasible for agents to specialize. In particular, a fraction μ_t of young agents at t can become workers, earning the wage income $h_t w_t$. This can be saved as a bank deposit, returning $r_t h_t w_t$ at t+1. In middle-age the same agent can work again, now earning $h_{t+1} w_{t+1}$. Similarly, a fraction $1-\mu_t$ of agents can engage in schooling when young. Moreover, they can borrow in order to put capital in place at t+1, so that they can run firms at both t+1 and t+2.[9] We now consider the economy of section I when this specialization occurs.

[8]See, for instance, Azariadis (1988), Chapter 2.

[9]Our formulation also allows these agents to work when middle-aged and run firms when old. It will be apparent that they have no incentive to do so in equilibrium.

A. *Workers*

Agents who work when young at t earn $h_t w_t$, all of which is saved.[10] Then middle-aged workers receive $r_t h_t w_t$ at t+1 as the proceeds of their savings when young. In addition, they earn $h_{t+1}w_{t+1}$ as labor income. These agents then choose a value for middle-age and a value for old age consumption, and a savings level, to maximize $u(c_2) + v(c_3)$ subject to

$$(10) \quad c_2 \leq r_t h_t w_t + h_{t+1} w_{t+1} - s$$

and

$$(11) \quad c_3 \leq r_{t+1}\, s.$$

Notice that $s = s(r_t h_t w_t + h_{t+1} w_{t+1}, 0, r_{t+1}) \equiv \tilde{s}(r_t h_t w_t + h_{t+1} w_{t+1}, r_{t+1})$.

B. *Entrepreneurs*

An agents who runs a firm at t has income equal to $\tilde{F}(K_t, h_t L_t, h_t) - w_t h_t L_t - r_{t-1} K_t$, where we assume without loss of generality that all capital investment is financed by borrowing. In particular, a loan of K_t was taken at t-1 to put K_t units of capital in place at t, so loan repayments are $r_{t-1}K_t$ at t.

Clearly when L_t is chosen to maximize profits,

$$(12) \quad F_2(K_t, h_t L_t + h_t) = w_t.$$

Then (12) and Euler's Theorem imply that firm profits net of payments to labor (but gross of loan repayments) are $[F_1(\cdot) - r_{t-1}]K_t + F_2(\cdot)h_t$. Clearly, then, any equilibrium with $K_t \in (0,\infty)$ has

$$(13) \quad F_1(K_t, h_t L_t + h_t) = r_{t-1},$$

and entrepreneurial income at t is just $F_2(\cdot)h_t = h_t w_t$, by (12).

Entrepreneurs, then, engage in schooling when young, earning nothing and borrowing

[10] Note that w_t is a "per efficiency unit" wage rate.

K_{t+1} units of capital at t. They then earn income $h_{t+1}w_{t+1}$ at t+1 from operating a firm, and earn income $h_{t+2}w_{t+2}$ at t+2 from firm operation.[11] Moreover, these agents have free access to capital markets. Then a middle-aged entrepreneur at t+1 chooses values for middle and old-aged consumption, and a value s for savings, to maximize $u(c_2) + v(c_3)$ subject to

$$(14) \quad c_2 \leq h_{t+1}w_{t+1} - s$$

$$(15) \quad c_3 \leq h_{t+2}w_{t+2} + r_{t+1}\, s.$$

Then optimal savings for middle-aged entrepreneurs is given by $s=s(h_{t+1}w_{t+1}, h_{t+2}w_{t+2}, r_{t+1})$.

C. *Equilibrium*

In equilibrium, four conditions must be satisfied at each date. First, if $\mu_t \in (0,1)$, young agents must be indifferent between becoming workers or entrepreneurs. From (10), (11), (14), and (15), it is apparent that the required indifference obtains iff the two activities generate the same discounted present value of future income; that is, iff

$$(16) \quad r_t h_t w_t + h_{t+1}w_{t+1} = h_{t+1}w_{t+1} + (h_{t+2}w_{t+2}/r_{t+1}).$$

Second, the labor market must clear. At t there are μ_t young workers, and μ_{t-1} middle-aged workers. In addition there are $1-\mu_{t-1}$ middle-aged entrepreneurs, and $1-\mu_{t-2}$ old entrepreneurs. Then per-firm employment at t, L_t, must satisfy

$$(17) \quad L_t = (\mu_{t-1} + \mu_t) \,/\, (2 - \mu_{t-1} - \mu_{t-2}).$$

Third, savings must equal investment. Given our complete depreciation assumption, this requires that the time t+1 capital stock equals time t savings. Since K_{t+1} is the <u>per firm</u> capital stock at t+1, and since $2 - \mu_t - \mu_{t-1}$ is the mass of entrepreneurs at t+1, the capital stock at t+1 is (2 -

[11]Since entrepreneurs could earn $h_{t+1}w_{t+1}$ at t+1 from selling labor and still operate a firm at t+2, it is apparent that they are indifferent between doing so and running a firm in each period. It is easy to verify that, under constant returns to scale, it is irrelevant which any middle-aged entrepreneur chooses to do. To economize on notation we let all middle-aged entrepreneurs run firms.

$\mu_t - \mu_{t-1}) K_{t+1}$. Savings by young workers at t is $\mu_t h_t w_t$, while saving by middle-aged workers is $\mu_{t-1} \tilde{s}(r_{t-1}h_{t-1}w_{t-1} + h_t w_t, r_t)$. Saving by middle-aged entrepreneurs is $(1-\mu_{t-1})\, s(h_t w_t, h_{t+1} w_{t+1}, r_t)$. Then savings equals investment iff

$$(18)\ (2 - \mu_t - \mu_{t-1}) K_{t+1} = \mu_t h_t w_t + \mu_{t-1} \tilde{s}(r_{t-1}h_{t-1}w_{t-1} + h_t w_t, r_t) + (1 - \mu_{t-1})\, s(h_t w_t, h_{t+1}w_{t+1}, r_t).$$

Fourth, since all firms are identical, $\bar{Y}_t = Y_t = F(K_t, h_t L_t + h_t)$ in equilibrium. Therefore,

$$(19) \quad h_{t+1}/h_t = \phi[F(K_t,h_tL_t+h_t)/h_t,\mu_t].$$

Finally, of course, (12) and (13) describe the determination of wage and interest rates.

We now transform these equilibrium conditions as follows. Define $\hat{k}_t \equiv K_t/h_t$ to be the stock of physical relative to human capital. Then, using $1+L_t = (2 + \mu_t - \mu_{t-2}) /(2 - \mu_{t-1} - \mu_{t-2})$, from (17), we have that

$$(20) \quad w_t = F_2\, [\hat{k}_t, (2 + \mu_t - \mu_{t-2}) / (2 - \mu_{t-1} - \mu_{t-2})],$$

and

$$(21) \quad r_{t-1} = F_1[\hat{k}_t, (2 + \mu_t - \mu_{t-2}) / (2 - \mu_{t-1} - \mu_{t-2})].$$

In addition, (16) reduces to

$$(22) \quad r_t r_{t+1} = h_{t+2}w_{t+2} / h_t w_t,$$

while (19) becomes

$$(23) \quad h_{t+1}/h_t = \phi\{F[\hat{k}_t, (2 + \mu_t - \mu_{t-2}) / (2 - \mu_{t-1} - \mu_{t-2})], \mu_t\}.$$

Finally, it is possible to simplify (18) as follows. Since workers and entrepreneurs who are middle-aged at t have the same discounted present value of income and face the same interest rate, they have the same middle-aged consumption levels. Therefore,

$$(24)\quad r_{t-1}h_{t-1}w_{t-1} + h_t w_t - \tilde{s}(r_{t-1}h_{t-1}w_{t-1} + h_t w_t, r_t) = h_t w_t - s(h_t w_t, h_{t+1}w_{t+1}, r_t).$$

Then, substituting (24) into (18) gives

$$(25)\quad (2 - \mu_t - \mu_{t-1})\, K_{t+1} = \mu_t h_t w_t - (1 - \mu_{t-1})\, r_{t-1}h_{t-1}w_{t-1} + \tilde{s}(r_{t-1}h_{t-1}w_{t-1} + h_t w_t, r_t)$$

We henceforth assume that agents have homothetic preferences, so that

$$(26)\quad \tilde{s}(r_{t-1}h_{t-1}w_{t-1} + h_t w_t, r_t) \equiv \psi(r_t)(r_{t-1}h_{t-1}w_{t-1} + h_t w_t),$$

where $\psi(r) \in [0,1]\ \forall\, r$ and $\psi'(r) \geq 0$. Then (25) becomes

$$(27)\quad (2 - \mu_t - \mu_{t-1})\hat{k}_{t+1} = \mu_t h_t w_t /h_{t+1} - (1 - \mu_{t-1})r_{t-1}h_{t-1}w_{t-1} /h_{t+1} + \psi(r_t)(r_{t-1}h_{t-1}w_{t-1} + h_t w_t)/h_{t+1}.$$

Equations (20)-(23) and (27) constitute the equilibrium conditions for this economy. It is straightforward to see that this system of equations can be collapsed into a pair of third order, non-linear difference equations in μ_t and $\hat{k}_t$. Thus, relative to the situation absent financial markets (in which dynamics are first order), the presence of financial markets allows for substantially richer dynamic behavior. However, in light of the apparent difficulty of describing general dynamic behavior in this economy, we now turn our attention to characterizing "steady state" equilibria (equilibria with constant values of μ and $\hat{k}$).

D. *Steady State Equilibria*

When μ and $\hat{k}$ are constant, equations (20) and (21) imply that

$$(28)\quad w_t = F_2[\hat{k}, 1/(1-\mu)]\ \forall\, t,$$

$$(29)\quad r_{t-1} = F_1[\hat{k}, 1/(1-\mu)]\ \forall\, t.$$

Then (23) becomes

(30) $h_{t+1}/h_t = \phi\{F[\hat{k}, 1/(1-\mu)], \mu\}$

and (22) reduces to

(31) $r = h_{t+1} / h_t.$

Finally, using (31) and (27), we obtain

(32) $2(1 - \mu)\, \hat{k} = [2\psi(r) - (1 - 2\mu)]\, w/r$

as the final steady state equilibrium condition.

From (31) it is immediate that

Proposition 1. Any constant growth rate equilibrium has the real interest rate equal to the real growth rate.

We now proceed to state conditions under which a (positive) constant growth rate equilibrium exists.

Substituting (29) and (30) into (31) yields

(33) $F_1[\hat{k}, 1/(1-\mu)] = \phi\{F[\hat{k}, 1/(1-\mu)], \mu\}.$

Similarly, substituting (28) and (29) into (32) and rearranging yields

(34) $(1 - \mu)\, \hat{k}F_1\, [\hat{k}, 1/(1-\mu)] \,/\, F_2\, [\hat{k}, 1/(1-\mu)] = \psi\{F_1\, [\hat{k}, 1/(1-\mu)]\} + \mu - 1/2\ .$

Equations (33) and (34) constitute two conditions determining μ and $\hat{k}$.

Now define $z \equiv (1 - \mu)\hat{k}$ to be the capital-labor ratio, with labor measured in efficiency units, and define $f(z) \equiv F(z,1)$. Then $f'(z) = F_1(z,1)$ and $F_2(z,1) = f(z) - zf'(z)$. Using these relations in (33) and (34) respectively gives

(35) $f'(z) = \phi[f(z)/(1-\mu), \mu]$

and

(36) $\mu = 1/2 + \{zf'(z)/[f(z) - zf'(z)]\} - \psi[f'(z)] \equiv g(z).$

We now state sufficient conditions for (35) and (36) to have a solution with $\mu \epsilon (0,1)$ and $z > 0$.

Proposition 2. Suppose that $\phi(0,\mu) < \infty \ \forall \ \mu$, that

(i) $\lim_{z \to 0} f'(z) = \infty$

(ii) $\lim_{z \to \infty} f'(z) = 0$

(iii) $g(z) \ \epsilon \ (0,1) \ \forall \ z$

hold, and that ϕ is continuous in each argument for $\mu \neq 0,1$. Then (35) and (36) have a solution with $\mu \epsilon (0,1)$ and $z > 0$.

Proof. Substituting (36) into (35) gives

(37) $q(z) \equiv f'(z) - \phi\{f(z)/[1-g(z)], g(z)\} = 0.$

(i)-(iii) imply that

$$\lim_{z \to 0} q(z) > 0 > \lim_{z \to \infty} q(z) \ .$$

Then, since q is continuous, (37) has at least one solution, say z^*, with $z^*>0$. Further, $\mu = g(z^*) \ \epsilon (0,1)$, by (iii).

Clearly (i) and (ii) are standard conditions, while (iii) is not. We demonstrate by an example in the sequel that (iii) can easily be violated, and hence must be imposed by assumption.

We now state conditions under which (35) and (36) have a unique solution. To do so, we define the elasticity of substitution, σ, in a conventional way:

$$\sigma \equiv [dz/d(w/r)]\ (w/r)/z.$$

Then

<u>Proposition 3</u>. Suppose that (i)-(iii) in proposition 2 are satisfied, that $\sigma \geq 1$ and $\psi'(r) \geq 0$ hold, and that ϕ has the form given in equation (2.a). Then (35) and (36) have a unique solution.

Proof. Under these conditions, (35) becomes,

$$(35') \quad f'(z) = \tilde{\phi}[f(z)/(1-\mu)]$$

which defines a downward sloping relationship between z and μ, as shown in Figure 2. Equation (36) defines a locus in Figure 2 which has a positive slope if $g'(z) \geq 0\ \forall\ z$. From (36),

$$(38) \quad g'(z) = \{[ff' + zff'' - z(f')^2] / [f(z) - zf'(z)]^2\} - \psi'f'' \ .$$

Since

$$\sigma = -f'(z)\ [f(z) - zf'(z)] / zf(z)\ f''(z) \geq 1,$$

each term on the right-hand side of (38) is non-negative, giving the desired result.

Clearly (35) and (36) will not generally have a unique solution if ϕ is of the more general form given in (2), and if $\sigma > 1$ and/or $\psi' > 0$. This point is illustrated in Figure 3.

Under reasonable conditions, then, the model supports a steady state equilibrium with a positive constant growth rate when financial markets are present. These markets permit specialization to occur, which in turn makes learning-by-doing possible. This provides an obvious sense in which the presence of financial markets fosters growth.

Moreover, while our assumptions are designed to illustrate this point, they are stronger than necessary to do so. The main point is that financial markets prevent investment from being delayed (relative to what would occur in the absence of financial markets). To the extent that there is some learning-by-doing associated with this investment, it occurs earlier when financial markets exist. Agents are more productive over more of their lifetimes, increasing output and savings. In an endogenous growth context, the latter effect will promote growth.

E. *An Example*

Let $u(c_2) + v(c_3) = \ln c_2 + \beta \ln c_3$, and let $F(K, hL + h) = AK^\alpha (hL + h)^{1-\alpha}$; $\alpha \in (0,1)$. Then $\psi(r) = \beta/(1+\beta) \equiv \psi \; \forall \, r$, and (36) reduces to

$$(39) \qquad \mu = 1/2 - \psi + \alpha/(1-\alpha) \ .$$

Then if $1/2 - \psi + \alpha/(1-\alpha) \in (0,1)$, (39) gives $\mu \in (0,1)$. (35) then gives a unique solution for $z > 0$. Notice that if $1/2 - \psi + \alpha/(1-\alpha) \notin [0,1]$ then no equilibrium exists. Since this is exactly $g(z) \notin [0,1]$, it is clear that (iii) must be imposed in Proposition 2.

IV. Endogenous Absence of Financial Markets

We now pose the following question: can the equilibrium of Section II continue to be observed even if financial intermediaries are free to form? To answer this question, we assume that a steady state equilibrium obtains in the Section II economy, and now allow intermediaries to form. In order to attract any funds intermediaries must offer a rate of return $r \geq f'(k^*)$, where k^* is the (a) stationary solution of (9), while to make any loans, $r \leq F_1$ must hold. Thus let $r = f'(k^*)$.

At this interest rate and the wage rate $w(k^*)$, young agents are content to engage in schooling when young, work when middle-aged, and run a firm when old iff this career path delivers no less income, in a discounted present value sense, than any other career path. If $\mu_t = 0$, $h_{t+1}/h_t = 1$, so an agent who does not specialize has income with a discounted present value of $w(k^*)/r + w(k^*)/r^2 = w(k^*)(1+r)/r^2$. An agent who works in youth and middle-age has income with a discounted present value of $w(k^*) + w(k^*)/r = w(k^*)\,(1+r)/r$. Finally, an agent who runs a firm in middle and old age receives income with a discounted present value of $w(k^*)/r + w(k^*)/r^2 = w(k^*)(1+r)/r^2$. Consequently, non-specialization is always weakly preferred to specializing as an entrepreneur, while non-specialization is (weakly) preferred to working in each period iff $w(k^*)\,(1+r)/r^2 \geq w(k^*)\,(1+r)/r$. Then we have

Proposition 4. When intermediaries are free to form, there is an equilibrium with no specialization iff $r = f'(k^*) \leq 1$, where k^* is a stationary solution to (9).

If the steady state capital stock of the section II economy is no less than the "golden rule" capital stock (with no specialization), then there is an equilibrium in which no agents specialize. In the absence of specialization there is no need for financial markets, so there is also an equilibrium with no financial markets, even though intermediaries are free to form. In this event an economy can get stock in a low (zero) growth equilibrium, even though other equilibria with positive growth rates exist. This is reminiscent of Bagehot's discussion of English financial markets relative to those elsewhere.

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Figure 1

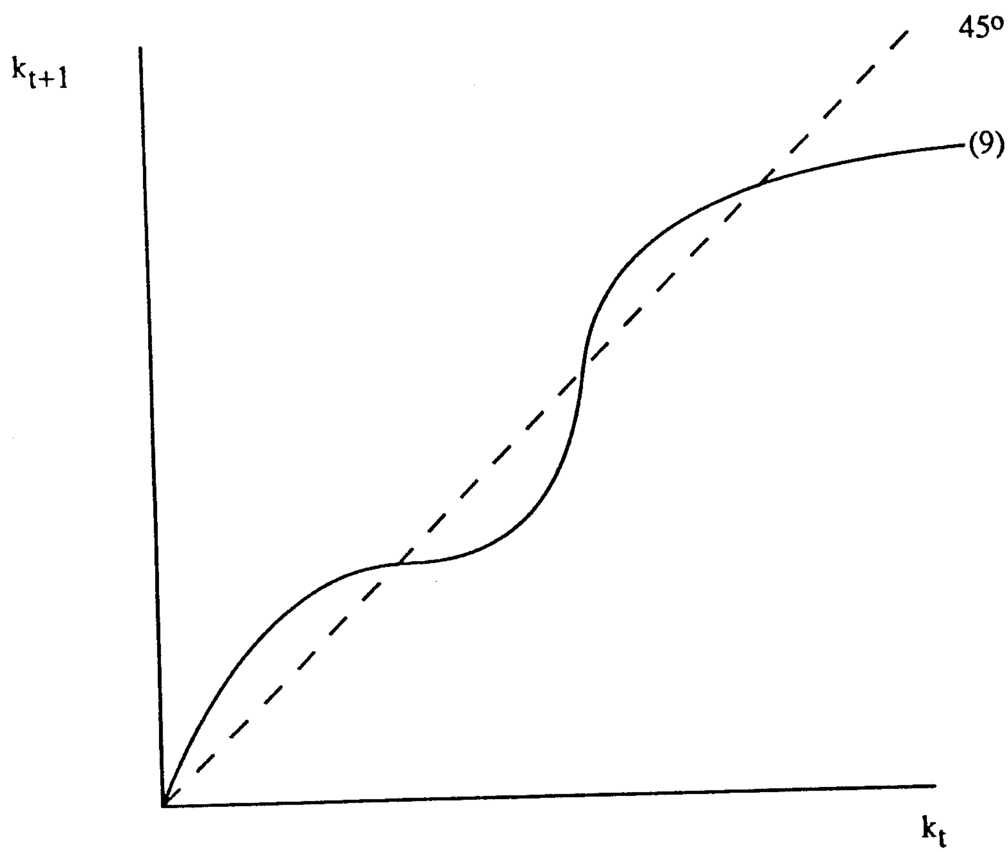

Figure 2

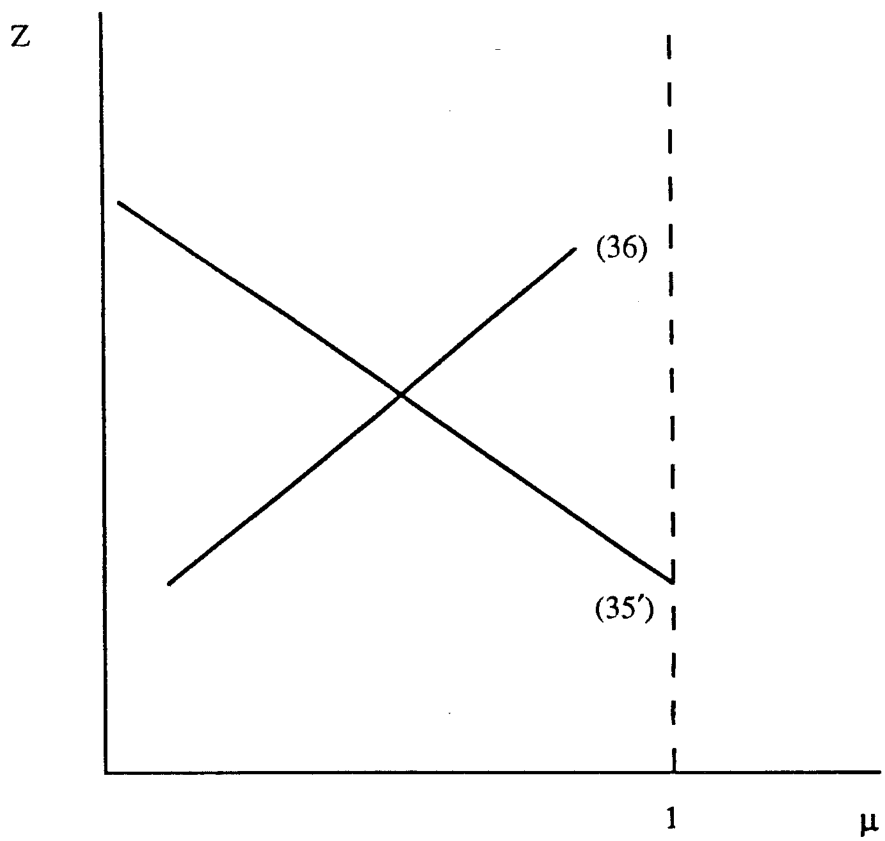

Figure 3