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Abstract

In this paper we study circumstances under which it is constrained Pareto efficient for a government to issue simultaneously bonds with non-random returns and bonds with extraneously randomized returns to finance a stationary deficit. The model has two types of agents with identical preferences and endowments, but with differential access to an alternative asset. There is private information about type, which creates an adverse selection problem. The government's bond policy amounts to non-linear taxation, involving (self-selected) random taxation of the more advantaged type. The predictions of the model appear to accord well with historical episodes where such government liabilities were actually in use.

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1 Introduction

Governments with large deficits to finance often borrow in a way which amounts to a form of price discrimination or non-linear taxation, issuing indivisible bonds in an array of denominations or maturities, and offering different rates of return on different types of bonds. [Calomiris (1991) provides several historical examples]. It is also common for governments with large deficits to borrow by issuing liabilities with random rates of return. The randomness in these returns is of three main types. First, bonds may have an associated probability of default, which clearly makes the bond's return stochastic. Second, any bond issued with a fixed nominal return in an inflationary environment also has a stochastic real return unless inflation is anticipated perfectly. Finally, history indicates that governments have often issued bonds where the return to individuals was extraneously randomized by an explicit lottery device, while the government faced little or no randomness in its interest obligations. In this paper we study circumstances under which the intentional and explicit introduction of this type of extraneous uncertainty by a government is constrained Pareto efficient.

There are many historical examples of bonds with extraneously randomized returns. In the 18th century, England and France issued bonds where, in exchange for a capital payment, an investor received a title to a bond plus a lottery ticket for a drawing of additional bonds. The payoff from a "winning ticket" often provided an annual income greater than the total capital contributed. The capital contributed generally was not small, possibly being an amount exceeding average per capita income [see Weir and Velde (1989)]. At this same time England and France also borrowed through the use of tontines, where a group of subscribers purchased bonds with fixed payments divided among "survivors." With a large group of subscribers, the government's payments displayed little randomness, but for any individual the returns were random. Interestingly, subscribers could make the payment contingent on the survival of someone other than themselves, and the expected returns on this type of liability were generally favorable [see Weir (1989)]. Finally, during the American Revolution the Continental government attempted to borrow, in Europe, through the use of so-called lottery bonds with randomly determined returns [see Anderson (1982)].

The use of such debt instruments has often been viewed as puzzling because the necessity of a risk premium to compensate (presumably) risk averse borrowers for randomized returns makes this an apparently expensive way to borrow [see the discussion in Weir (1989)]. Further, one might expect the government to be relatively better able to bear risk than individuals, so the *intentional* introduction of extrinsic uncertainty by the government merits explanation. This is especially true in the historical episodes that we describe, because financial and insurance markets that might have allowed individuals to diversify idio-synchratic risk were at best weakly developed. Also, "lottery bonds" and bonds without explicitly randomized returns were often used simultaneously. In addition, in some of the historical examples, bonds with randomized returns were intended to be sold to relatively wealthy investors. This contrasts with the socio-economic characteristics of participants in recent state sponsored lottery "games" [see Clotfelter and Cook (1990)]. Both of these features merit explanation.

We develop a stationary, two-period lived overlapping generations model in which a government with a utilitarian social welfare function must finance a fixed deficit of a given size. It does this by borrowing from two types of agents that are identical in all respects but one: different agents have different access to investment opportunities other than government bonds. This is intended to capture a situation where wealthier investors have access to investment opportunities not open to poorer investors, or where a government seeks to borrow both at home and abroad, and foreign investors have investment opportunities not open to domestic investors. Finally, agent type and outside investment activity are private information. If agents did not have differential access to outside investment opportunities the government would raise revenue from all types equally. However, when the government's deficit is sufficiently large, doing so drives agents with the best investment opportunities (say type 1 agents) out of the bond market. This, in turn, requires all revenue to be raised from type 2 agents, which a utilitarian government regards as undesirable. Thus the government raises as much revenue as it can from type 1 agents without driving them out of the bond market, and this requires that type 1 agents be treated preferentially. However, when type is private information, preferential treatment of type 1 agents creates an adverse selection problem.

The model predicts the use of a kind of price discrimination described by Bryant and Wallace (1984) or Villamil (1988), where government liabilities are issued in minimum denominations, and intermediation is prohibited. As in Villamil, the solution to this problem involves the issue of multiple types of government bonds, all of which involve price discrimination and non-linear taxation. In our model, at least one type of bond bears a non-random return. However, under conditions we describe, it is constrained Pareto efficient for the government to extraneously randomize the return on the other type of bond. Bonds with randomized returns are sold to investors with access to the best outside investment opportunities. In particular, we show that under a condition that amounts to a requirement that absolute risk aversion decrease at a rapid enough rate, extraneously randomized returns are the best way to keep type 1 agents in the bond market, given the adverse selection problem. However, randomization is observed only if the deficit is sufficiently large.

The result that bonds with extraneously randomized returns are constrained Pareto efficient can also be interpreted as asserting the desirability of random taxation. Of course the potential desirability of random taxation in the presence of an adverse selection problem has been previously pointed out [for instance by Stiglitz (1982)]. In our analysis, however, where the focus is on government bond sales, the government cannot compel participation.

This makes our analysis somewhat different from that in standard taxation models. Interpreted in a taxation context, our model could be regarded as one in which only market activities can be taxed, and high enough taxation drives some agents into non-market (or "underground") activities. Thus, taxation must not only raise sufficient revenue, it must be designed to prevent exit from market activities. Note that the adverse selection problem arises in this setting if and only if voluntary participation is a binding constraint.

In our model, the government treats type 1 agents preferentially by designing an asset for them with a randomized return. In contrast, the asset designed for type 2 agents has a lower expected (but non-random) return. Despite the fact that both agent types have identical preferences, endowments, and equal access to the government's assets, the access of type 1 agents to an outside alternative allows them to partially insure against the bad state of nature associated with the randomized return. Type 2 agents, having no access to the outside alternative, prefer the certain return. This taxation policy is substantially different from that in Stiglitz where agents differ in earning ability and it is Pareto efficient to randomize the tax on low ability (i.e., type 2) agents. In his model, government policy is designed to make it less attractive for high ability (i.e., type 1) agents to pretend to be of low ability and hence to work less. Thus, in both models randomization is optimal because it weakens a constraint and hence is welfare improving, but the nature of the randomization policies used to affect welfare improvements are quite different.[1]

[1] Random taxation procedures that minimize monitoring costs (e.g., stochastic auditing, as in Border and Sobel (1987)) are also welfare improving but are unrelated to the questions that we address. However, recent work by Scotchmer and Slemrod (1989) on random enforcement of the tax code is indirectly related to our analysis. On page 23 they discuss an interesting interpretation of random enforcement procedures as risky assets, but the randomness in their model corresponds to the implicit randomness induced by inflation in asset markets (discussed at the outset of this paper). In contrast, we are concerned with the intentional and explicit randomization of a particular asset's returns when a riskless asset is also available.

Finally, we briefly relate our results to two other literatures. First, in the model that we describe, (non-optimal) extraneous uncertainty can be introduced by market factors which allow sunspot equilibria to exist, as in Shell (1977), Azariadis (1981), or Cass and Shell (1983). Our focus is different; we describe conditions under which the government will intentionally, and on welfare grounds, inject extraneous uncertainty into allocations. Second, in adverse selection models where agents have identical underlying preferences, it is generally not the case that randomization of allocations is desirable. [See, for example, Prescott and Townsend (1984) or Arnott and Stiglitz (1988)]. In contrast, we establish the desirability of randomization in an environment where agents have identical underlying preferences and endowments, because different agent "types" have differential access to outside opportunities. We return to the relationship between our work and these literatures in the final section.

The remainder of the paper proceeds as follows. Section 2 describes the model. Section 3 considers non-stochastic planning problems from which Pareto efficient consumption allocations can be derived under three alternative sets of assumptions about the constraints faced by the planner. Section 4 establishes conditions under which randomized allocations are desirable. In both Sections 3 and 4 we describe how the government can implement the optimal allocations. Section 5 concludes.

2 The Model

Consider a discrete time economy populated by an infinite sequence of two-period lived, overlapping generations and an infinitely lived government. Each generation is identical in size and composition, containing a continuum of agents with unit mass. Within each generation there are two types of agents, indexed by $i = 1, 2$. Let θ_i denote the fraction of type i agents in each generation, with $\theta_i > 0$ and $\theta_1 + \theta_2 = 1$. In addition, there is a single consumption good at each date. All agents have endowment w_j of the good in period $j = 1, 2$ of their life, with $w_j \geq 0$.

Agent types are differentiated by their access to a storage technology. In particular, type 1 (and only type 1) agents have access to a constant returns to scale technology for storing the good, where one unit stored at time t returns $x < 1$ units at time $t + 1$. We assume that each agent can store only his or her own good, that the type of each agent is private information (ex-ante), and that the activity of storing the good (or the quantity stored) is unobservable.

All agents have identical preferences, representable by the additively separable utility function $u(c_1^i) + v(c_2^i)$, where $c_j^i \in I\!R_+$ denotes the consumption of a type i agent at age j. We assume that u and v are strictly increasing, strictly concave, and thrice continuously differentiable. For future reference we define $R(c) \equiv -\frac{v''(c)}{v'(c)}$, the coefficient of absolute risk aversion. Finally, let the government have an exogenously given real per capita expenditure level of $g > 0$ each period. We assume that agents derive no utility from this expenditure.[2]

The assumption that type 1 agents can store the good while type 2 agents cannot is meant to represent the situation where a government must sell bonds to finance a deficit, and different potential bondholders have access to different alternative investment opportunities. This differential access limits the ability of the government to extract resources from some agents. The ability of type 1 agents to store the good is intended to capture in a crude way the notion that some potential bondholders have access to better alternative investment opportunities than others. This could occur, for instance, if a domestic government sought to borrow from foreign investors with the option to invest abroad (at the gross rate of return x), while domestic investors (type 2 agents) are prevented from doing so by capital controls. Alternatively, we

[2]Alternatively, let government expenditure affect utility in an additively separable way.

might imagine that wealthier agents have access to investment opportunities not available to poorer agents.[3] Finally, we note that our model can be interpreted as an economy in which direct taxation is employed, but only "market activities" can be taxed. In this case the differential access to the storage technology proxies for different "non-market" opportunities.

For future reference, it will be useful to have a notation for the savings behavior of an agent who pays a lump-sum tax of τ_j at age j, and faces a certain gross rate of return on savings of r. Such an agent chooses a savings level, q, to maximize $u(w_1 - \tau_1 - q) + v(w_2 - \tau_2 + rq)$ subject to non-negativity constraints. The solution to this problem is given by the savings function $q \equiv f(w_1 - \tau_1, w_2 - \tau_2, r)$. Under our assumptions, and assuming interiority, $f_1 > 0 > f_2$. Also, we assume that

$$w_1 > f(w_1, w_2, x) > 0. \qquad (a.1)$$

Finally, we define the indirect utility function V in the standard way:

$$V(w_1 - \tau_1, w_2 - \tau_2, r) \equiv u(w_1 - \tau_1 - f(\cdot)) + v(w_2 - \tau_2 + rf(\cdot)).$$

3 Non-random Pareto Efficient Allocations

In this section we consider planning problems under three alternative sets of assumptions about constraints faced by the planner. In each case we restrict attention to non-random consumption allocations.

3.1 Full Information

As a benchmark, we begin by considering the problem of a social planner under full information; that is, we assume that the planner knows each agent's

[3]Formally, we could let type i agents have an age j endowment of w_j^i, with $w_1^1 > w_1^2$, which resembles the situation in Villamil (1988). However, this formulation complicates our analysis without adding any additional substantive issues, so we do not pursue it here.

type, and can observe and (if desired) prohibit storage of the good. The planner's objective is to find a stationary allocation that maximizes an equally weighted sum of the agents' utilities subject to a resource feasibility constraint. Let k denote the amount of storage by a type 1 agent. Then the full information Pareto problem can be written as follows:

Problem 3.1. *For $i = 1, 2$, choose values c_1^i, c_2^i and k to maximize:*

$$\sum_{i=1}^{2} \theta_i [u(c_1^i) + v(c_2^i)]$$

subject to:

$$\sum_{i=1}^{2} \theta_i (c_1^i + c_2^i) + \theta_1 k \le w_1 + w_2 - g + \theta_1 x k. \tag{1}$$

At an interior optimum, the solution to this problem sets

$$u'(c_1^i) = v'(c_2^i), \tag{2}$$

for $i = 1, 2$, $c_j^1 = c_j^2$, for $j = 1, 2$, and $k = 0$. Notice that, from equations (1) and (2), $c_1^i = w_1 - f(w_1, w_2 - g, 1)$, and $c_2^i = w_2 - g + f(w_1, w_2 - g, 1)$. Thus the utility of agents under this allocation is given by $V(w_1, w_2 - g, 1)$.

Remark. The allocation given by the solution to Problem 3.1 is identical to the allocation obtained by Bryant and Wallace (1984), and can be supported as they describe. In particular, the government can prohibit goods storage, sell bonds with a minimum real value of F and a rate of return r, and prohibit agents from "sharing" (or intermediating) bonds. If F is chosen to satisfy $F = f(w_1, w_2 - g, 1)$ and r is given by $r = \frac{F-g}{F}$, then each agent type will purchase bonds with a minimum real value of F voluntarily [when $V(w_1, w_2 - g, 1) \ge V(w_1, w_2, 0)$]. This policy permits the government to raise enough revenue to cover its expenditure.

3.2 Voluntary Participation

We now assume that the planner is subject to a voluntary participation constraint, or in other words, that the planner cannot prevent type 1 agents from autarchically storing the good or type 2 agents from consuming their endowments. This represents the situation of a government that must finance a deficit g by selling bonds, with the government being unable to compel bond purchases. Alternatively, we may view this as the situation of a government that cannot tax activities in an "underground" economy. However, we continue to assume that the government observes agents' types directly.

The planner now solves the problem

Problem 3.2. *For $i = 1, 2$, choose c_1^i, c_2^i, and k to maximize:*

$$\sum_{i=1}^{2} \theta_i [u(c_1^i) + v(c_2^i)]$$

subject to: (1) and

$$u(c_1^1) + v(c_2^1) \geq V(w_1, w_2, x); \tag{3}$$

$$u(c_1^2) + v(c_2^2) \geq u(w_1) + v(w_2). \tag{4}$$

There are three possibilities regarding the solution to Problem 3.2.

Case 1: $V(w_1, w_2 - g, 1) \geq V(w_1, w_2, x)$. In this case constraints (3) and (4) are not binding. This occurs, obviously, if g is sufficiently small, in which case the solution to Problem 3.2 is the same as the solution to Problem 3.1.

Case 2: $V(w_1, w_2 - g, 1) < u(w_1) + v(w_2)$. In this case the constraint set is empty. We henceforth abstract from this possibility, which occurs if g is too large.

Case 3: $V(w_1, w_2, x) > V(w_1, w_2 - g, 1) \geq u(w_1) + v(w_2)$. In this case constraint (3) binds. This is the situation of interest to us and we therefore focus exclusively on it. In particular, the solution in this case satisfies constraints (1) and (3) as equalities, (2), and $c_j^1 > c_j^2$, for $j = 1, 2$. In addition, $k = 0$. Thus, due to the government's inability to compel agents to purchase its bonds, type 1 agents must be given incentives not to withdraw from the bond market. Consequently, type 1 agents receive better terms than type 2 agents. Notice, however, that since (2) holds, no inefficiencies result.

Remark. The allocation given by the solution to Problem 3.2 can be supported by the following government policy. Sell bonds to type i agents with a minimum real value of F^i, and pay each type the gross rate of return r^i on these bonds. Then $F^i = w_1 - c_1^i$ and $r^i = \frac{c_2^i - w_2}{F^i}$ hold. Type 2 agents are prohibited from buying type 1 bonds, and intermediation is prohibited ex cathedra. Arguments following those of Bryant and Wallace (1984) establish that type i agents voluntarily purchase F^i units of bonds of type i. It is also easy to verify that this permits the government to raise revenues equal to its expenditures.

3.3 Voluntary Participation and Private Information

We next consider the problem of a planner who wishes to choose non-stochastic Pareto efficient consumption allocations but cannot compel market participation, and in addition, cannot directly observe the type of any agent. Thus, the planner is subject to incentive compatibility constraints, as well as the other constraints specified previously.

The planner now solves the problem

Problem 3.3. *For $i = 1, 2$, choose c_1^i, c_2^i and k to maximize*

$$\sum_{i=1}^{2} \theta_i [u(c_1^i) + v(c_2^i)]$$

subject to: (1), (3), (4), *and the self-selection constraints*

$$u(c_1^1) + v(c_2^1) \geq u(c_1^2) + v(c_2^2); \tag{5}$$

$$u(c_1^2) + v(c_2^2) \geq u(c_1^1 + k) + v(c_2^1 - xk). \tag{6}$$

Equation (5) imposes that type 1 agents (weakly) prefer (c_1^1, c_2^1) to (c_1^2, c_2^2).[4] Equation (6) imposes incentive compatibility for type 2 agents, since a type 2 agent taking a type 1 allocation is not able to mimic the storage of a type 1 agent. Thus such an agent consumes $c_1^1 + k$ when young, and $c_2^1 - xk$ (i.e., c_2^1 less the proceeds of storage) when old.

The solutions to Problem 3.3 are of two general types.

Case 1: $V(w_1, w_2 - g, 1) \geq V(w_1, w_2, x)$. In this case the allocation from Problem 3.1 satisfies conditions (3) through (6), since $c_j^1 = c_j^2$, for $j = 1, 2$.

Case 2: $V(w_1, w_2, x) > V(w_1, w_2 - g, 1)$. In this case the allocation from Problem 3.2 clearly is not incentive compatible, since $c_j^1 > c_j^2$, for $j = 1, 2$, and $k = 0$. In particular, since there is no goods storage and type 1 agents are "better treated" than type 2 agents, all type 2 agents will claim to be of type 1. We now focus on this case. It is apparent that if (3) holds with equality, then (5) will be satisfied. Hence (1), (3), and (6) are the binding constraints in Problem 3.3.

We now characterize the solution to Problem 3.3.

Proposition 1. *The solution to Problem* 3.3 *satisfies* (1), (3), *and* (6) *at equality. In addition, it has* $u'(c_1^1) = xv'(c_2^1)$, $u'(c_1^2) = v'(c_2^2)$, *and* $k > 0$.

Proof. See Appendix A.

Remark 1. The solution to Problem 3.3 has at least two interesting features. First, as Appendix A shows, goods storage occurs. This is necessary to give type 1 agents a utility level of $V(w_1, w_2, x)$ without having type 2 agents mimic their bond purchases. Second, since $u'(c_1^1) = xv'(c_2^1)$, type 1 agents are "on their savings functions" with respect to storage of the good. Both of these features reflect inefficiencies due to the necessity of treating type 1 agents preferentially and the presence of private information.

[4] Formally, constraint (5) should be written as $u(c_1^1) + v(c_2^1) \geq V(c_1^2, c_2^2, x)$. However, since $u'(c_1^2) = v'(c_2^2) > xv'(c_2^2)$ holds (see below), $V(c_1^2, c_2^2, x) = u(c_1^2) + v(c_2^2)$.

Remark 2. The solution to Problem 3.3 can be implemented as follows. The government issues two types of bonds, and prevents agents from sharing them. Agents who buy type 1 bonds can buy only type 1 bonds, and are permitted to purchase *at most* $\bar{F}$ units (in real terms). These bonds earn the gross return r^1. Agents who purchase type 2 bonds must purchase at least F^2 units (in real terms), and these bonds earn the gross return r^2. The government chooses F^2 and r^2 to satisfy $F^2 = w_1 - c_1^2$ and $r^2 = \frac{c_2^2 - w_2}{F^2}$. The government chooses $\bar{F}$ and r^1 to satisfy

$$c_1^1 = w_1 - f(w_1 - \bar{F}, w_2 + r^1\bar{F}, x); \tag{7}$$

$$c_2^1 = w_2 + r^1\bar{F} + xf(w_1 - \bar{F}, w_2 + r^1\bar{F}, x). \tag{8}$$

Then by (7) and (8), type 1 agents are "on their savings functions." Type 2 agents optimally purchase F^2 units of type 2 bonds, and the government raises revenue with a per capita value of g.

4 Stochastic Pareto Efficient Allocations

We now state conditions under which extrinsic randomization can be Pareto improving. We begin by introducing some notation, and then consider a constrained social planning problem that permits extraneous uncertainty. Our objective is only to show that some extrinsic randomization is desirable, so we proceed as follows. We assume that the planner chooses, for $i = 1, 2$, deterministic values c_1^i for young consumption, and values $c_2^i(s)$ for old consumption that may depend on an extraneous state s. For simplicity we let $s \in \{1, 2\}$, and we let $p \in (0, 1)$ be the probability (which is the same in all periods) that $s = 1$. We assume that realizations of s are independently and identically distributed across agents, and that s is realized at the beginning of old age. This is meant to capture the features of several historical randomization devices employed in government borrowing. Specifically, as we noted in the Introduction, historical evidence indicates that governments have confronted individuals with random returns on some bonds, while the government faced little or no (as here) randomness with respect to interest obligations on these bonds.[5]

We now consider the planning problem from which constrained, Pareto efficient (possibly stochastic) consumption allocations are chosen. To simplify notation, we will sometimes write $E_s h(c_2^i(s)) \equiv ph(c_2^i(1)) + (1-p)h(c_2^i(2))$, where $h(\cdot)$ is an arbitrary function, and E denotes the expectation operator.

The stochastic Pareto problem can be written as follows.

Problem 4.1. *For* $i = 1, 2$ *and* $s = 1, 2$, *choose* c_1^i, $c_2^i(s)$, *and* k *to maximize*

$$\sum_{i=1}^{2} \theta_i [u(c_1^i) + E_s v(c_2^i(s))]$$

subject to:

$$\sum_{i=1}^{2} \theta_i [c_1^i + E_s c_2^i(s)] + \theta_1 k \leq w_1 + w_2 - g + \theta_1 x k; \tag{9}$$

$$u(c_1^1) + E_s v(c_2^1(s)) \geq V(w_1, w_2, x); \tag{10}$$

$$u(c_1^2) + E_s v(c_2^2(s)) \geq u(c_1^1 + k) + E_s v(c_2^1(s) - xk); \tag{11}$$

$$u(c_1^1) + E_s v(c_2^1(s)) \geq u(c_1^2) + E_s v(c_2^2(s)); \tag{12}$$

$$u(c_1^2) + E_s v(c_2^2(s)) \geq u(w_1) + v(w_2). \tag{13}$$

Equation (9) is the resource feasibility constraint, which reflects the fact that there is no aggregate randomness. Equations (11) and (12) are the self-selection constraints, and equations (10) and (13) are the voluntary participation constraints.

[5] While our description treats p as exogenous, clearly randomization is no less desirable on welfare grounds if the government is free to choose p.

Clearly the solution to Problem 4.1 coincides with the solution to Problem 3.1 unless (10) is binding. When (10) binds, so does (11), as in the previous section. Apparently then, (12) cannot bind, and we restrict attention to the case in which (13) is not binding. Thus, for the remainder of the section, constraints (9) through (11) bind. It is easy to verify in this case that the solution to Problem 4.1 has $c_2^2(1) = c_2^2(2)$, so that only (or at most) type 1 agents face extrinsic uncertainty. This captures our observation that historically governments with large deficits have made use of lottery bonds with attractive return distributions that are sold to agents who (presumably) have reasonable alternative investment opportunities. Also, it is easy to check that the solution to Problem 4.1 has $u'(c_1^2) = v'(c_2^2)$. Finally, arguments identical to those in Appendix A can be used to establish that $k > 0$ holds, and that

$$u'(c_1^1) = xE_s v'(c_2^1(s)). \tag{14}$$

4.1 Welfare Improving Randomization

We now state a sufficient condition for $c_2^1(1) \neq c_2^1(2)$ to hold, so that type 1 agents face extraneous uncertainty.

Proposition 2. *Suppose that* $-\frac{(1-x)v''(\tilde{c}_2^1 - x\tilde{k})}{v'(\tilde{c}_2^1 - x\tilde{k}) - u'(\tilde{c}_1^1 + \tilde{k})} > -\frac{v''(\tilde{c}_2^1)}{v'(\tilde{c}_2^1)}$ *holds, where* $\tilde{c}_1^1$, $\tilde{c}_2^1$, *and* $\tilde{k}$ *are solutions to Problem* 3.3. *Then* $c_2^1(1) \neq c_2^1(2)$.

Proof. See Appendix B.

In the remainder of this section, we provide necessary and sufficient conditions for the inequality in Proposition 2 to hold. We then interpret our result. In particular, we find that when the elasticity of absolute risk aversion with respect to old age consumption is sufficiently large, then extrinsic randomization is welfare improving.

The inequality in Proposition 2 can be rewritten

$$\frac{R(\tilde{c}_2^1 - x\tilde{k})(1-x)v'(\tilde{c}_2^1 - x\tilde{k})}{v'(\tilde{c}_2^1 - x\tilde{k}) - u'(\tilde{c}_1^1 + \tilde{k})} > R(\tilde{c}_2^1). \tag{15}$$

Note that $\frac{v'(\tilde{c}_2^1 - x\tilde{k})}{v'(\tilde{c}_2^1 - x\tilde{k}) - u'(\tilde{c}_1^1 + \tilde{k})} < \frac{1}{1-x}$, so it is apparent that a necessary condition for (15) is decreasing absolute risk aversion (with respect to old age consumption).

We now derive a sufficent condition for (15) to hold.

Define the function G by

$$G(c_1^1, c_2^1, k) \equiv \frac{R(c_2^1 - xk)(1-x)v'(c_2^1 - xk)}{v'(c_2^1 - xk) - u'(c_1^1 + k)} - R(c_2^1). \tag{16}$$

Then for all values (c_1^1, c_2^1) satisfying $u'(c_1^1) = xv'(c_2^1)$, it is apparent that $G(c_1^1, c_2^1, 0) = 0$. Furthermore, if $G_3 > 0$, then for all such (c_1^1, c_2^1) pairs, $G(c_1^1, c_2^1, k) > 0$ whenever $k > 0$. Thus, $G(\tilde{c}_1^1, \tilde{c}_2^1, \tilde{k}) > 0$ will hold, which is exactly (15).

Straightforward differentiation of (16) establishes that $G_3 > 0$ iff

$$-\frac{xR'(c_2^1 - xk)}{R(c_2^1 - xk)} > -\{\frac{u'(c_1^1 + k)}{v'(c_2^1 - xk) - u'(c_1^1 + k)}\}\times$$

$$\{\frac{u''(c_1^1 + k)}{u'(c_1^1 + k)} - xR(c_2^1 - xk)\}. \tag{17}$$

Since $xv'(\tilde{c}_2^1 - xk) \geq u'(\tilde{c}_1^1 + k)$ for all $k \geq 0$, a sufficient condition for $G_3(\tilde{c}_1^1, \tilde{c}_2^1, k) > 0$, for all $k \geq 0$, is

$$-\{\frac{(1-x)R'(\tilde{c}_2^1 - xk)}{R(\tilde{c}_2^1 - xk)}\} \geq xR(\tilde{c}_2^1 - xk) - \frac{u''(\tilde{c}_1^1 + k)}{u'(\tilde{c}_1^1 + k)}. \tag{18}$$

An alternative statement of (18) is obtained by multiplying both sides by $(\tilde{c}_2^1 - xk)$ to get

$$-\{\frac{(1-x)R'(\tilde{c}_2^1 - xk)(\tilde{c}_2^1 - xk)}{R(\tilde{c}_2^1 - xk)}\} \geq xR(\tilde{c}_2^1 - xk)(\tilde{c}_2^1 - xk) -$$

$$\{(\frac{\tilde{c}_2^1 - xk}{\tilde{c}_1^1 + k})\frac{u''(\tilde{c}_1^1 + k)(\tilde{c}_1^1 + k)}{u'(\tilde{c}_1^1 + k)}\}. \tag{18$'$}$$

Equation (18$'$) provides the result. In particular, it asserts that $G_3(\tilde{c}_1^1, \tilde{c}_2^1, k) > 0$ for all $k \geq 0$ if the elasticity of absolute risk aversion (with respect to old age consumption, $\frac{R'(\cdot)(\cdot)}{R(\cdot)}$), is sufficiently large. Note that $R'(\cdot) < 0$ if the utility function exhibits everywhere strictly decreasing absolute risk aversion. Decreasing absolute risk aversion implies that in a choice between a safe and a risky asset, the risky asset is a normal good. This is a common assumption about preferences.

4.2 A Special Case

We now consider the special case in which $u(c_1) = \phi\frac{c_1^{1-\rho}}{1-\rho}$ and $v(c_2) = \frac{c_2^{1-\rho}}{1-\rho}$, with $\phi \geq x$ and $\rho > 0$. Then $-\frac{cR'(c)}{R(c)} \equiv 1$ for all c. In addition, $u'(\tilde{c}_1^1) = xv'(\tilde{c}_2^1)$ implies that $\tilde{c}_2^1 = \tilde{c}_1^1(\frac{x}{\phi})^{1/\rho} \leq \tilde{c}_1^1$, and consequently, that $\tilde{c}_2^1 - xk \leq (\tilde{c}_1^1 + k)(\frac{x}{\phi})^{1/\rho}$ for all $k \geq 0$. In this case (18$'$) reduces to

$$1 - x \geq x\rho + \rho\frac{(\tilde{c}_2^1 - xk)}{(\tilde{c}_1^1 + k)} \tag{19}$$

for all $k \geq 0$, which of course holds if

$$1 - x \geq x\rho + \rho(\frac{x}{\phi})^{1/\rho}. \tag{20}$$

Thus $G_3(\tilde{c}_1^1, \tilde{c}_2^1, k) > 0$ for all $k \geq 0$ holds if ρ is sufficiently small. This, in turn, implies that the inequality in Proposition 2 holds, and that randomization is desirable on welfare grounds.

4.3 Supporting the Optimal Stochastic Allocation

We must slightly augment our notation from Section 2 in order to describe how to implement the optimal stochastic allocation. Consider the savings problem of a young agent who faces a random lump-sum tax of $\tau_2(s)$ when old, $s = 1, 2$, where the probability that $s = 1$ is p, and who faces a deterministic gross rate of return r. This agent's problem is to choose a savings level q to maximize $u(w_1 - \tau_1 - q) + pv(w_2 - \tau_2(1) + rq) + (1-p)v(w_2 - \tau_2(2) + rq)$. The solution to the problem is a savings function $q \equiv \tilde{f}(w_1 - \tau_1, w_2 - \tau_2(1), w_2 - \tau_2(2), r; p)$.

The optimal random consumption allocation can be supported by the following policy. The government sells two types of bonds, and imposes restrictions which prohibit agents from sharing bonds. The bonds sold to type 2 agents are sold in a minimum denomination of F and bear a deterministic return r. The govenment chooses F and r to satisfy $c_1^2 = w_1 - F$ and $c_2^2 = w_2 + rF$. The bonds sold to type 1 agents are sold only in the indivisible amount $\hat{F}$, and bear a gross return $\hat{r}(1)$ with probability p, and $\hat{r}(2)$ with probability $1-p$. The government chooses $\hat{F}$, $\hat{r}(1)$, and $\hat{r}(2)$ to satisfy

$$c_1^1 = w_1 - \tilde{f}[w_1 - \hat{F}, w_2 + \hat{r}(1)\hat{F}, w_2 + \hat{r}(2)\hat{F}, x; p]; \tag{21}$$

$$c_2^1(1) = w_2 + \hat{r}(1)\hat{F} + x\tilde{f}(\cdot); \tag{22}$$

$$c_2^1(2) = w_2 + \hat{r}(2)\hat{F} + x\tilde{f}(\cdot). \tag{23}$$

This construction works since, by (14), type 1 agents are "on their savings functions."

5 Conclusions

We have described an environment in which a government must finance a fixed deficit of a given size. When some agents have access to reasonable investment opportunities other than government bonds, government borrowing is constrained by the desirability of keeping these agents in the bond market. However, treating some agents preferentially creates an adverse selection problem. The optimal solution to these two problems involves price discrimination by the government, and may involve the simultaneous use of bonds with random and non-random returns. Interestingly, agents with the best outside investment opportunities purchase bonds with random returns, and extraneous randomization of bond returns is observed only when the government's revenue needs are sufficiently large. These two features seem to accord well with the historical observations cited in the Introduction. Moreover, this borrowing mechanism could also be interpreted as one in which inflation is random and both indexed and non-indexed government bonds co-exist, or as one where there is a hierarchy of claims against the government, and bonds bearing high expected returns are subject to some risk of partial default. Thus the model has the potential to confront a number of ways in which a government can borrow using bonds bearing a randomly determined return.

The following features of our model, and their relationship with the recent literature on randomization, are of some interest. First, in our model randomization is desirable even though agents have the same underlying utility functions and endowments. This contrasts with the results in Prescott-Townsend (1984) and Arnott-Stiglitz (1988), and is due to the fact that agents have access to (and make differential use of) different non-market activities. Thus agents' indirect utility functions differ in such a way that randomized allocations may be desirable. This insight is essentially the same as that in Benhabib, Rogerson, and Wright (1990). Second, a type of randomization consistent with the predictions of our model has been observed historically. This is of interest because Arnott and Stiglitz (1988) (among others) note that it is puzzling that randomization of contracts does not occur as frequently as theory suggests. The analysis of government debt contracts in our model may provide some insight into this puzzle.

Arnott and Stiglitz discuss six reasons why randomization might not be observed: (1) agents do not understand that randomization is optimal (i.e., they are only boundedly rational); (2) contracts involving randomization are costly to enforce; (3) secondary markets/randomization insurance neutralize the effects of randomization; (4) lotteries are viewed as unfair; (5) von Neumann-Morgenstern expected utility theory is deficient; and (6) individuals do not trust randomization mechanisms. Our theory and historical observations suggest that reasons (1), (2), (5), and (6) are not persuasive.[6] We also regard reason (4) as unpersuasive because randomization of the type that we describe (i.e., weakening the voluntary participation constraint by treating type 1's preferentially rather than imposing randomization on the relatively disadvantaged type 2's, as in Stiglitz (1982)) need not be percieved as unfair by either group. For example, if bonds with randomized returns were sold to foreign investors, it is unlikely that domestic investors would view the implicit taxation of foreign investors as unfair. In contrast, reason (3) may be an important reason that randomization (even of the type we describe) has been only periodic throughout history.[7]

Because the bond policy that we describe is a form of price discrimination, it is clear that the existence of secondary markets would render the government unable to implement its program. That is, implementaion of the constrained Pareto efficient bond policy that we study requires the government to impose legal restictions that prohibit the intermediation of bonds with randomized returns. It is interesting to note that poorly developed financial markets are common in many high inflation countries that choose to monetize their deficits. For simplicity, we assume an ex cathedra prohibi-

[6]Recall our discussion of Weir and Velde (1989) in the Introduction. They describe one famous randomization mechanism, the "thirty French girls," that was very transparent. Lists of young girls from Genevan families with reputations for longevity and who had survived smallpox were compiled for use as "nominees" (recall that subscribers could make payments contingent on the survival of someone other than themselves, but payment required proof of the nominees' survival). The most common group size was thirty because administrative costs rose with the number of nominees and the marginal reduction in variance became small after thirty.

[7]Of course the size of g and x (see the restictions associated with case 2 in Section 4 that make this policy optimal) also vary over the course of an economy's business cycle.

tion against the intermediation of assets. However, Bencivenga and Smith (1991) study the optimal degree of financial repression in a developing economy faced with a sustained deficit that must be monetized. They find that a government with a deficit (that is either unwilling or unable to decrease spending or increase explicit taxes) may be required by simple feasibility to engage in financial repression to support its monetization program. Such repression is much more difficult in more developed countries and may be one reason why "lottery bonds" have not been observed in well developed financial markets.

Finally, another reason that the type of randomization policy we study has been observed only periodically throughout history may be related to the question of whether the policy supports a unique stationary equilibrium. In particular, an open question is whether the method of decentralization that we describe, based on Bryant and Wallace (1984), also supports other equilibria. While this must remain a topic for future research, Cooley and Smith (1990) have shown that the decentralization scheme in the Bryant and Wallace model can result in severe indeterminacies. In addition, the kinds of government borrowing schemes we describe may easily permit stationary sunspot equilibria, such as those described by Shell (1977), Azariadis (1981), and Cass and Shell (1983), to be observed. [A suggestive example along these lines appears in Smith (1989)]. Thus the market might add extraneous uncertainty to that already created by the government. These kinds of possibilities again raise the following interesting question for future research—how well can the government do, in a welfare sense, if it is constrained to schemes that have a unique (or a unique stationary) equilibrium?

6 Appendix A: Proof of Proposition 1

For convenience we restate Problem 3.3 with only the binding constraints displayed

Problem 3.3.' *For $i = 1, 2$, choose c_1^i, c_2^i, and k to maximize*

$$\sum_{i=1}^{2} \theta_i [u(c_1^i) + v(c_2^i)]$$

subject to

$$\sum_{i=1}^{2} \theta_i (c_1^i + c_2^i) + \theta_1 k \leq w_1 + w_2 - g + \theta_1 x k \tag{A.1}$$

$$u(c_1^1) + v(c_2^1) \geq V(w_1, w_2, x) \tag{A.2}$$

$$u(c_1^2) + v(c_2^2) \geq u(c_1^1 + k) + v(c_2^1 - xk). \tag{A.3}$$

Proof of Proposition 1. Let $\lambda_n \geq 0$ be the Lagrange multiplier associated with constraint $(A.n)$, and observe the following.

At interior solutions for c_1^1 and c_2^1, the relevant first order conditions are

$$u'(c_1^1)(\theta_1 + \lambda_2) - \lambda_3 u'(c_1^1 + k) = \theta_1 \lambda_1. \tag{A.4}$$

$$v'(c_2^1)(\theta_1 + \lambda_2) - \lambda_3 v'(c_2^1 - xk) = \theta_1 \lambda_1. \tag{A.5}$$

The first order condition for k is

$$\lambda_3 [v'(c_2^1 - xk)x - u'(c_1^1 + k)] - \lambda_1 \theta_1 (1 - x) \leq 0, \tag{A.6}$$

with equality if $k > 0$.

Finally, the first order conditions for c_1^2 and c_2^2 at an interior optimum are

$$u'(c_1^2)(\theta_2 + \lambda_3) = \theta_2 \lambda_1. \tag{A.7}$$

$$v'(c_2^2)(\theta_2 + \lambda_3) = \theta_2 \lambda_1. \tag{A.8}$$

Of course $(A.7)$ and $(A.8)$ imply that $u'(c_1^2) = v'(c_2^2)$.

Now multiply both sides of $(A.5)$ by x, subtract the result from $(A.4)$, and use $(A.6)$ to eliminate $\theta_1 \lambda_1 (1 - x)$ to obtain

$$(\theta_1 + \lambda_2)[x v'(c_2^1) - u'(c_1^1)] \leq 0, \tag{A.9}$$

with equality if $k > 0$.

Clearly, if we establish that $k > 0$ the proof is complete. Consequently, suppose by way of contradiction that $k = 0$. Then $(A.4)$ and $(A.5)$ imply that $u'(c_1^1) = v'(c_2^1)$. Moreover, since $(A.2)$ is binding, it follows that $c_j^1 > c_j^2$, for $j = 1, 2$. But then $(A.3)$ is violated, giving the desired result. This proves Proposition 1.

7 Appendix B: Proof of Proposition 2

We begin by considering the following augmented version of Problem 4.1.

Problem 4.1′: *For $i = 1, 2$ and $s = 1, 2$, choose c_1^i, $c_2^i(s)$, and k to maximize*

$$\sum_{i=1}^{2} \theta_i \{u(c_1^i) + pv(c_2^i(1)) + (1-p)v(c_2^i(2))\}$$

subject to: (9) *through* (11) *and*

$$u'(c_1^1) = xpv'(c_2^1(1)) + x(1-p)v'(c_2^1(2)). \qquad (B.1)$$

Since the solution to Problem 4.1 satisfies $(B.1)$, imposition of this constraint does not alter the optimal choices for the social planner.

Equations (9) through (11), which hold as equalities, and $(B.1)$ constitute four equations involving c_1^1, $c_2^1(1)$, $c_2^1(2)$, k, c_1^2, and c_2^2 [since $c_2^2(1) = c_2^2(2) = c_2^2$]. We now use (9), (11), and $(B.1)$ to eliminate c_1^1, $c_2^1(1)$, and k from Problem 4.1′.

First, let $(B.1)$ define c_1^1 as a function of $c_2^1(1)$ and $c_2^1(2)$. In particular, define $c_1^1 \equiv \alpha(c_2^1(1), c_2^1(2))$. Apparently, $\tilde{c}_1^1 = \alpha(\tilde{c}_2^1, \tilde{c}_2^1)$ holds. In addition, differentiation of $(B.1)$ yields

$$\alpha_1(c_2^1(1), c_2^1(2)) = px\frac{v''(c_2^1(1))}{u''(c_1^1)} > 0. \qquad (B.2)$$

$$\alpha_2(c_2^1(1), c_2^1(2)) = (1-p)x\frac{v''(c_2^1(2))}{u''(c_1^1)} > 0. \qquad (B.3)$$

Second, substitute $c_1^1 = \alpha(c_2^1(1), c_2^1(2))$ into (9) at equality. This gives k as a function of $c_2^1(1)$, $c_2^1(2)$, c_1^2, and c_2^2. Thus define $k \equiv \beta(c_2^1(1), c_2^1(2); c_1^2, c_2^2)$. Observe that $\tilde{k} = \beta(\tilde{c}_2^1, \tilde{c}_2^1; \tilde{c}_1^2, \tilde{c}_2^2)$ holds, and that differentiation of $\beta(\cdot)$ yields:

$$\beta_1 = -(\frac{\alpha_1 + p}{1-x}) < 0. \qquad (B.4)$$

$$\beta_2 = -(\frac{\alpha_2 + 1 - p}{1-x}) < 0. \qquad (B.5)$$

Third, substitute $c_1^1 = \alpha(c_2^1(1), c_2^1(2))$ and $k = \beta(\cdot)$ into (11) at equality. This defines $c_2^1(2)$ as a function of $c_2^1(1)$, c_1^2, and c_2^2; say $c_2^1(2) \equiv \gamma(c_2^1(1); c_1^2, c_2^2)$. As before $\tilde{c}_2^1 = \gamma(\tilde{c}_2^1; \tilde{c}_1^2, \tilde{c}_2^2)$. Moreover, differentiation of $\gamma(\cdot)$ yields

$$\gamma_1(\tilde{c}_2^1; \tilde{c}_1^2, \tilde{c}_2^2) = -\frac{p}{1-p}. \qquad (B.6)$$

Finally, define the function $\delta(\cdot)$ as follows

$$\delta(c_2^1(1); c_1^2, c_2^2) \equiv u(\alpha(c_2^1(1), \gamma(\cdot))) + pv(c_2^1(1)) + (1-p)v(\gamma(\cdot)). \qquad (B.7)$$

Observe that $\delta(\cdot)$ is the left-hand-side of constraint (10), the (binding) voluntary participation constraint for type 1 agents. The function $\delta(\cdot)$ expresses the left-hand-side of this constraint solely as a function of $c_2^1(1)$ and c_j^2, for $j = 1, 2$. The strategy of the remainder of the proof is to show that $\delta(\cdot)$ is locally convex in $c_2^1(1)$, so local randomization relaxes the voluntary participation constraint on type 1 agents and consequently is welfare improving.

Now observe that Problem 4.1′ reduces to the following:[8]

Problem 4.1″. *Choose c_1^2, c_2^2, and $c_2^1(1)$ to maximize*

$$u(c_1^2) + v(c_2^2)$$

[8]This follows since $u(c_1^1) + pv(c_2^1(1)) + (1-p)v(c_2^1(2)) = V(w_1, w_2, x)$ holds.

subject to:

$$\delta(c_2^1(1); c_1^2, c_2^2) \geq V(w_1, w_2, x). \tag{B.8}$$

If $c_2^1(1) = \tilde{c}_2^1$ at an optimum, then the solution to Problem 4.1″ coincides with the (non-stochastic) solution to Problem 3.3.

We now establish the following properties of $\delta(\cdot)$:

$$\delta_1(\tilde{c}_2^1; \tilde{c}_1^2, \tilde{c}_2^2) = 0, \tag{B.9}$$

and if the inequality in Proposition 2 holds,

$$\delta_{11}(\tilde{c}_2^1; \tilde{c}_1^2, \tilde{c}_2^2) > 0. \tag{B.10}$$

Then setting $c_2^1(1) \neq \tilde{c}_2^1$ (in some neighborhood of $\tilde{c}_2^1$) relaxes constraint (B.7) in Problem 4.1″. It follows that at an optimum, $c_2^1(1) \neq \tilde{c}_2^1$, and consequently $c_2^1(1) \neq c_2^1(2)$. Thus there will be extraneous randomization of the allocation received by type 1 agents.

It remains, then, to establish that (B.9) and (B.10) hold. For (B.9), straightforward differentiation of (B.7) gives

$$\delta_1(\tilde{c}_2^1; \tilde{c}_1^2, \tilde{c}_2^2) = u'(\alpha(\cdot))[\alpha_1 + \alpha_2\gamma_1] + pv'(\tilde{c}_2^1) + (1-p)v'(\tilde{c}_2^1)\gamma_1. \tag{B.11}$$

Substitution of (B.2), (B.3), and (B.6) into (B.11) gives (B.9)

For (B.10), further differentiation yields

$$\delta_{11}(\tilde{c}_2^1; \tilde{c}_1^2, \tilde{c}_2^2) = u''(\tilde{c}_1^1)[\alpha_1+\alpha_2\gamma_1]^2 + u'(\tilde{c}_1^1)[\alpha_{11}+\alpha_{12}\gamma_1+\alpha_{21}\gamma_1+\alpha_{22}(\gamma_1)^2+\alpha_2\gamma_{11}]$$

$$+pv''(\tilde{c}_2^1) + (1-p)v''(\tilde{c}_2^1)(\gamma_1)^2 + (1-p)v'(\tilde{c}_2^1)\gamma_{11}. \tag{B.12}$$

It is straightforward but tedious to establish that, when evaluated at ($\tilde{c}_2^1$, $\tilde{c}_1^2$, $\tilde{c}_2^2$),

$$\alpha_{11} + \alpha_{12}\gamma_1 + \alpha_{21}\gamma_1 + \alpha_{22}(\gamma_1)^2 + \alpha_2\gamma_{11} =$$

$$\frac{1-p}{p}\alpha_1\{\gamma_{11} + [\frac{p}{(1-p)^2}]\frac{v'''(\tilde{c}_2^1)}{v''(\tilde{c}_2^1)}\}; \tag{B.13}$$

$$\alpha_1 + \alpha_2\gamma_1 = 0; \tag{B.14}$$

and

$$\gamma_{11}(\tilde{c}_2^1; \tilde{c}_1^2, \tilde{c}_2^2)(p + x\alpha_1) = -\frac{px\alpha_1 v'''(\tilde{c}_2^1)}{(1-p)^2 v''(\tilde{c}_2^1)} -$$

$$\frac{p^2 v''(\tilde{c}_2^1 - x\tilde{k})(1-x)}{(1-p)^2[v'(\tilde{c}_2^1 - x\tilde{k}) - u'(\tilde{c}_1^1 + \tilde{k})]}. \tag{B.15}$$

Substituting (B.13) through (B.15) into (B.12) gives

$$\delta_{11}(\tilde{c}_2^1; \tilde{c}_1^2, \tilde{c}_2^2) = \frac{pv''(\tilde{c}_2^1)}{1-p} + \frac{x\alpha_1 v'(\tilde{c}_2^1)v'''(\tilde{c}_2^1)}{(1-p)v''(\tilde{c}_2^1)} + (1-p)v'(\tilde{c}_2^1)\gamma_{11}[1 + \frac{x\alpha_1}{p}] =$$

$$\frac{pv''(\tilde{c}_2^1)}{1-p} - \frac{v'(\tilde{c}_2^1)p(1-x)v''(\tilde{c}_2^1 - x\tilde{k})}{(1-p)[v'(\tilde{c}_2^1 - x\tilde{k}) - u'(\tilde{c}_1^1 + \tilde{k})]}. \tag{B.16}$$

Apparently, $\delta_{11}(\tilde{c}_2^1; \tilde{c}_1^2, \tilde{c}_2^2) > 0$ if the inequality in Proposition 2 holds. This completes the proof.

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