The Equilibrium Allocation of Investment Capital in the Presence of Adverse Selection and Costly State Verification

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This paper studies the equilibrium allocation of investment capital in an environment with the following features: (a) investments must be financed externally; (b) investment opportunities are heterogeneous, differing in their probability distributions of returns; (c) owners of investment projects are privately informed about the return distributions, and (d) actual returns on any project can be observed by agents other than the owner only at some cost. Thus we consider the market allocation of investment funds in the presence of adverse selection (b and c) and costly state verification (d). Of particular interest in this environment is who obtains funds (or conversely, who might experience credit rationing), and what contractual terms emerge in equilibrium. As will be apparent, the answers to these questions are intimately related.

More specifically, we consider a contracting problem between borrowers and lenders, all of whom are risk neutral. As in Williamson (1987), each borrower is endowed with an investment project, which requires one unit of funds to operate, and each lender is endowed with one unit of funds to invest. Investment projects differ with respect to the probability distribution of returns, with project owners being privately informed about their own return distribution. In addition, realized project returns can be observed by any agent other than the owner only at some cost.[1] Following Williamson (1986, 1987), we assume that borrowers announce contract terms. Funds are obtained only if announced contracts are incentive compatible, and yield at least the expected market return. We state conditions under which (a) truthful revelation of type occurs, (b) equilibrium contracts are so-called "standard debt contracts," and (c) credit-rationing is an observed feature of equilibrium.

We also consider the determination of a full general equilibrium under the assumption that there is an upward sloping supply curve of investment funds. This permits us to derive some comparative static results; for instance concerning the consequences for credit rationing of an improvement in the monitoring technology.

There are several reasons to study this environment. First, there appears to be widespread agreement that credit rationing, as modeled for instance by Stiglitz and Weiss (1981), captures aspects of an important real world phenomenon. However, at a formal level, the Stiglitz-Weiss (1981) results depend on the *assumption* that debt contracts are employed. Since this contractual form is not optimal in their environment, credit rationing may appear to be something of an artifact. More generally, a good deal of the credit rationing literature follows Stiglitz-Weiss in imposing the use of debt contracts; or imposes assumptions (such as two-state return distributions) that essentially force debt contracts to emerge.[2]

We consider an environment which is similar in many respects to that in Stiglitz-Weiss (1981). When there is no costly state verification problem (only an adverse selection problem), the equilibrium *cannot* have all contracts being debt contracts. Moreover, under weak conditions the adverse selection problem will not be binding in equilibrium, so that no credit rationing will be observed. However, when the costly state verification problem is sufficiently severe credit rationing can emerge. In addition, we state conditions under which debt contracts are optimal (assuming that stochastic monitoring is precluded), even when project returns are a continuous random variable. And, when equilibrium contracts are debt contracts, credit rationing must be observed in equilibrium.

Thus, we show that, when both adverse selection and costly state verification problems are present, they interact. This fact can be used to rationalize some commonly used credit rationing formulations, as just argued. Moreover, it bears on observations that are often made about the empirical magnitude of various informational frictions. For instance, Bernanke and Gertler (1990, p. 89) argue that the costly state verification problem considered by Bernanke and Gertler (1989) is not of sufficient empirical importance to rationalize "first-order" macroeconomic effects. Our results indicate that an important consequence of even relatively small verification costs is that they exacerbate other informational frictions (which might otherwise themselves be of minor significance). This constitutes a caution against arguments that any single informational friction is too small, in isolation, to be of empirical significance.

Since problems of costly state verification and adverse selection interact here, we can investigate how changes in the technology of information collection (monitoring) impact on the extent of equilibrium credit rationing. It is often argued by development economists (e.g., McKinnon 1973) that high costs of information acquisition are associated with extensive rationing of credit. However, to date this possibility has not been addressed in the theoretical literature on credit rationing. We show that improvements in the technology for acquiring information (reductions in monitoring costs) will, under weak conditions, result in reductions in credit rationing and increases in the (expected) returns to lenders. We are thus able to formalize this common assertion. This constitutes an ingredient in showing how improvements in technologies for processing information are conducive to increasing investment and economic development.

Another objective of this study is to investigate the robustness of various conclusions that emerged from early work on costly state verification environments. As Townsend (1979) and Gale-Hellwig (1985) demonstrated, simple versions of such environments have a powerful ability to help explain the observed form of several contractual arrangements. A particular success was the derivation of contracts resembling observed debt contracts in economies with a large number of identical (ex ante), risk neutral agents. However, it is of interest to know how well such results survive generalization. For instance, as shown by Mookherjee and P'ng (1989), such contracts do not necessarily survive the introduction of stochastic monitoring. When stochastic monitoring is ruled out, however, such contracts do survive the introduction of risk aversion, and continue to be observed in sufficiently large finite economies (Krasa and Villamil 1990, 1991). We show that debt contracts can also survive the introduction of heterogeneity among borrowers and an additional informational asymmetry, so long as differences among borrower types are not too great (in a sense to be made precise).

Finally, this paper provides ingredients for a sequel (Boyd-Smith 1991) which considers an identical environment, with the additional complication that agents are spatially separate. If inter-location monitoring is more costly than within-location monitoring, inter-location lending will be intermediated. The result is a model in which intermediation, debt contracts, and credit rationing all emerge endogenously. This model is then used to investigate how intermediation improves the allocation of investment capital, a central topic in economic development (Cameron 1967, Goldsmith 1969, McKinnon 1973, and Shaw 1973). It is also used to investigate how the quantity of intermediation and credit rationing respond to changing economic conditions.

The remainder of this paper proceeds as follows. Section I describes the environment, defines equilibrium contract announcements, proves the existence of contracts that result in a separating equilibrium under certain parameter restrictions, and demonstrates that the space of parameters which produce such an equilibrium is not empty. Section II defines a general equilibrium under the assumption that the supply of funds is (at least potentially) adequate to meet demand. It provides conditions necessary for the existence of such an equilibrium, and derives some comparative static results. Section III considers the case where the supply of funds is inadequate to meet demand, and describes how this would affect various results. Section IV concludes.

I. The Model

A. Environment

This section describes an environment in which adverse selection is introduced into Williamson's (1987) model of credit markets. Throughout the notation is kept as close to

Williamson's as possible. Also, we note that our intention in this section is to provide sufficient conditions for debt contracts to emerge in an equilibrium with rationing of credit. No attempt is made to produce the most general conditions that allow this.

Agents in this economy are of three types. A fraction α of the population belongs to the first group, called "lenders," while $1 - \alpha$ belongs to the other two groups ("borrowers"). Borrowers consist of two types; a high average return type (type g), and a low average return type (type b). Borrower type is indexed by $i = g, b$, and a fraction θ of borrowers are of type b.

All borrowers are risk neutral. Each is endowed with an indivisible project that requires one unit of an investment good to operate. Projects, if funded, generate a random return $\tilde{w}$, realized after one period. Realizations of $\tilde{w}$, denoted w, among borrowers of type i are independent and identically distributed. For type g borrowers project returns have the probability distribution G, with associated density function g. g is assumed to be differentiable, with support $[0,\bar{w}]$. Similarly for type b borrowers project returns have the probability distribution F, with the associated (differentiable) density function f. The support of f is also $[0,\bar{w}]$. Throughout we assume that the functions f and g are common knowledge, and that $f(w) > 0$ and $g(w) > 0 \;\forall\; w \in [0,\bar{w}]$.

We impose the following assumptions. First $G > F$ in the sense of first order stochastic dominance; i.e., $F(w) \geq G(w) \;\forall\; w$, with strict inequality for some w. Second, we assume that there exists a value $w^* \in (0,\bar{w})$ such that

(a.1) $$f(w) = g(w) \;\forall\; w \geq w^*.$$

[and hence $F(w) = G(w) \;\forall\; w \geq w^*$]. Assumption (a.1) asserts that return distributions differ only for "low" values of w. This is a purely technical assumption, which plays the following role in the analysis. Departures from debt contracts in a separating equilibrium can occur only for values of w such that $f(w) \neq g(w)$. Then (a.1) implies that such departures can occur only when relatively poor project returns are realized. (a.1) is thus a highly restrictive assumption, made only to simplify the conditions under which debt contracts are optimal. However, we note that (a.1) will be a reasonable assumption in contexts where unobserved differences in project quality (or the ability of the project owner) are important to project returns only in relatively "bad" states of nature.

Borrower type is assumed to be private information, ex ante. In addition, the realization of $\tilde{w}$ for any agent can be observed at zero cost only by that agent. It can be observed by any other agents at a cost (in effort) of γ. Finally lenders are endowed with second period effort, and borrowers have no funds to be used in operating their projects. We assume that $\forall\ w \in [0,\bar{w}]$

$$\text{(a.2)} \qquad f(w) + \gamma f'(w) > 0; \quad g(w) + \gamma g'(w) > 0.$$

Assumption (a.2) follows Williamson (1987); it implies that the expected return received by lenders is a concave function of the interest rate paid by borrowers (under a debt contract).

If borrowers do not operate their projects (either voluntarily, or because they cannot obtain funding) they engage in an outside alternative. The outside alternative yields a payoff of R_i to type i borrowers. It will be useful to impose the following assumption on the values R_i. Define

$$\hat{w}_g = \int_0^{\bar{w}} wg(w)dw; \quad \hat{w}_b = \int_0^{\bar{w}} wf(w)dw$$

and let I(w) be the indicator function

$$I(w) = \begin{Bmatrix} 1 & \text{if } f(w) \geq g(w) \\ 0 & \text{otherwise} \end{Bmatrix}.$$

Then we assume that

$$\text{(a.3)} \qquad \hat{w}_g - R_g < \hat{w}_b - R_b - \int_0^{w^*} w\,[f(w) - g(w)]\,I(w)dw.$$

As will be apparent when optimal contracts are discussed, (a.3) implies that the expected utility of borrowers of all types declines with increases in expected monitoring costs (ceteris paribus). Note that (a.3) also implies that $R_g > R_b$.

Each lender is endowed with one unit of an investment good. Lenders care only about second period consumption (c) and expenditure of effort on monitoring (e). Lenders are assumed to have the linear utility function $c - e$. Furthermore, lender j is assumed to have some opportunity cost of investment, t_j.[3] The distribution of opportunity costs in the lender population is denoted by H, where $H(t) = \int_{\underline{t}}^{t} h(s)ds$, with $h(s) > 0$; $s \geq \underline{t}$. Finally, we assume that $w^* \leq \underline{t}$. Thus return distributions across different borrower types vary only over returns that are low relative to the expected return required to elicit any funds.

B. Optimal Contracts

Following Williamson (1987), we assume that borrowers offer contract terms to lenders, taking the contracts offered by other borrowers as given. Contract offers must yield a prospective lender at least the market expected return r, given the inferences that a lender draws from contractual terms about a borrower's type. Throughout we focus on separating equilibria, in which type g and b borrowers offer distinct contractual terms. Having derived conditions under which such equilibria exist, we will comment on the possibility of pooling equilibria. And, of course, our focus on separating equilibria implies that in equilibrium lenders correctly infer the types of borrowers.

A loan contract specifies the following set of objects: (i) a probability that credit will be received by a type i borrower, p_i, so that credit is granted stochastically even if a lender is available; (ii) a set of return realizations, $A_i \subset [0,\bar{w}]$, for which monitoring of a type i borrower (costly verification of the state) occurs, and of course a set $B_i = [0,\bar{w}] - A_i$ of realizations for which monitoring does not occur (note that we abstract from stochastic monitoring); (iii) a noncontingent loan repayment (interest rate) x_i if $w \in B_i$; (iv) a repayment schedule $R_i(w)$; $w \in A_i$. We focus only on contracts that induce truthful revelation of whether or not $w \in A_i$.[4] Then feasibility requires that $0 \leq x_i \leq w$, $w \in B_i$; $0 \leq R_i(w) \leq w$, $w \in A_i$, and $R_i(w) \leq x \ \forall \ w \in A_i$.

1. Type b Borrowers

If self-selection of borrower types occurs, then we conjecture (and verify subsequently) that type b borrowers are not constrained by any incentive conditions that require type g borrowers not to want to mimic type b contract announcements. Then type b borrowers choose p_b, A_b, B_b, x_b, and $R_b(w)$ to maximize

$$p_b \left\{ \int_{A_b} [w - R_b(w)]f(w)dw + \int_{B_b} (w-x_b)f(w)dw \right\} + (1-p_b)R_b$$

subject to

$$\int_{A_b} [R_b(w) - \gamma]f(w)dw + x \int_{B_b} f(w)dw \geq r \tag{1}$$

and $0 \leq R_b(w) \leq w$, $w \in A_b$; $R_b(w) \leq x$, $w \in A_b$.

Assuming that r is sufficiently low that type b borrowers are not driven out of the credit market altogether (see below), this problem is identical to that considered by Gale and Hellwig

(1985) or Williamson (1987). Thus $p_b = 1$, and the other contractual terms are those of so-called standard debt contracts; i.e., $A_b = [0,x_b)$, $R_b(w) = w \ \forall \ w \in [0,x_b)$, and x_b satisfies

$$\text{(2)} \qquad \int_0^{x_b} wf(w)dw + x_b[1 - F(x_b)] - \gamma F(x_b) = r.$$

If (2) has more than one solution, then apparently the smallest solution maximizes the expected utility of a type b borrower. Denote the (smallest) solution to (2) by $x_b = \phi(r)$, and note that $\phi'(r) = 1/[1 - F(x_b) - \gamma f(x_b)] > 0$. Finally, for future reference, we define the function $\pi_b(x)$ by

$$\pi_b(x) = \int_x^{\bar{w}} wf(w)dw - x[1 - F(x)]$$

so that $\pi_b(x)$ is the expected profit of a type b borrower, as a function of the interest rate x, under a debt contract. Note that $\pi_b'(x) < 0$, and that the solution to the type b borrowers' problem sets $p_b = 1$ iff $\pi_b[\phi(r)] \geq R_b$, and sets $p_b = 0$ otherwise.

2. Type g Borrowers

An optimal contract for type g borrowers is next derived under the assumption that self-selection of types occurs. Conditions implying that this is an equilibrium outcome are then stated.

Consider the following problem for type g borrowers: to maximize

$$p_g \left\{ \int_{A_g} [w - R_g(w)]g(w)dw + \int_{B_g} (w - x_g)g(w)dw \right\} + (1 - p_g)R_g$$

subject to

$$\text{(3)} \qquad \int_{A_g} [R_g(w) - \gamma]g(w)dw + x_g \int_{B_g} g(w)dw \geq r$$

$$(4) \qquad p_g \left\{ \int_{A_g} [w - R_g(w)]f(w)dw + \int_{B_g} (w-x_g)f(w)dw \right\} + (1-p_g)R_b \leq \pi_b[\phi(r)],$$

$0 \leq R_g(w) \leq w$, and $R_g(w) \leq x \; \forall \; w \in A_g$.

Note that even if a lender is available credit may not be received (if $p_g < 1$); however, if credit is received the lender must obtain an expected return of at least r. Equation (4) asserts that the utility a type b agent can obtain by announcing a type g contract does not exceed that obtained by announcing the type b contract derived previously.

We may immediately observe that (4) must be binding in equilibrium. If this were not the case then the solution to the type g borrowers' problem would set $p_g = 1$ and be a standard debt contract (if r is not too high). But then

$$(5) \qquad \int_0^{x_g} wg(w)dw + x_g[1 - G(x_g)] - \gamma G(x_g) = r$$

would hold, implying $x_g < x_b$. This, of course, would result in a contract violating (4). Thus adverse selection problems must impact on the solution to the problem of type g borrowers. Also, for future reference, denote the (smallest) value of x_g satisfying (5) by $x_g = \psi(r)$. Then $\psi'(r) = 1/[1 - G(x_g) - \gamma g(x_g)] > 0$.

We now state conditions sufficient for the solution to the problem of type g borrowers to be a debt contract with credit rationing.

Proposition 1. Define $\epsilon(r) \equiv \hat{w}_g - R_g - r$ and $\delta(r) \equiv \hat{w}_b - R_b - r - \int_0^{w^*} wI(w)[f(w) - g(w)]dw$. Suppose that the following conditions hold:

$$(6) \qquad \delta(r) - \epsilon(r) \geq \epsilon(r) - \gamma G[\psi(r)]$$

$$(7) \qquad \gamma g(w) \geq 1 - G(w) - \gamma g(w) \qquad \forall \; w \geq \psi(r)$$

(8a) $\quad g(w) \geq f(w) - g(w) \ \forall \ w$

(8b) $\quad f(w) \geq g(w) - f(w) \ \forall \ w.$

Then the optimal contract for type g borrowers sets $A_g = [0,x_g)$, $R_g(w) = w \ \forall \ w \in A_g$, $x_g = \psi(r)$, and $p_g = \{\pi_b[\phi(r)] - R_b\}/\{\pi_b[\psi(r)] - R_b\} < 1$ [since $\phi(r) > \psi(r)$].

Proposition 1 is proved in Appendix A. Note that $\delta(r) - \epsilon(r)$ is independent of r, so (6) holds if r is sufficiently large. Also, $\psi(r) > r$, so (7) holds (for instance) if $2\gamma g(w) \geq 1 - G(w)$ $\forall \ w \geq \underline{t}$. We henceforth proceed under the assumption that a debt contract is optimal for type g agents among the class of separating contracts.[5] We next state conditions implying that a separating (sequential Nash) equilibrium exists with respect to contract announcements.

C. *Existence of a Separating Equilibrium*

Clearly the incentives of borrowers to deviate from the contracts derived above depend critically on the inferences that lenders draw as a result of any deviation. We assume that lenders believe that any borrower announcing a debt contract with $x = \phi(r)$ is a type b borrower with probability one, and any borrower announcing a contract that is maximal for type g borrowers (among the set of contracts satisfying (3) and (4)) is a type g borrower with probability one. We further assume that any other announcement causes lenders to believe that the agent announcing the contract is a type b borrower with probability one. The latter specification guarantees that type b borrowers have no incentive to deviate from the candidate separating equilibrium contracts derived above.[6] Under this specification of lender beliefs we now show that

Proposition 2. Suppose (7) and (8) hold. Then the best deviation from the candidate separating equilibrium for a type g borrower is a debt contract with $x = \phi(r)$. Or, in other words, the best deviation for a type g borrower is to mimic type b borrowers.

Proposition 2 is proved in Appendix B. (7) and (8) are henceforth assumed to hold.

Define the function $\pi_g(x)$ by

$$\pi_g(x) = \int_x^{\bar{w}} wg(w)dw - x[1 - G(x)].$$

Then $\pi_g(x)$ is the expected utility of a type g borrower (conditional on receiving credit) under a debt contract specifying the interest rate x. Proposition 2 implies that type g borrowers have no incentive to deviate from the candidate separating equilibrium iff

$$(9) \qquad p_g\pi_g(x_g) + (1-p_g)R_g \geq \pi_g(x_b).$$

The determination of equilibrium contracts (under the assumption that debt contracts are observed) is depicted in Figure 1. In the figure type i borrowers have indifference curves described by loci of the form $p\pi_i(x) + (1-p)R_i = k$. For $x \geq \psi(r) > r \geq \underline{t} \geq w^*$, $\pi_g(x) = \pi_b(x)$. For all such x, the slope of a type b indifference curve in this space is given by

$$\frac{dp}{dx} = \frac{p[1 - F(x)]}{\pi_b(x) - R_b} = \frac{p[1 - G(x)]}{\pi_b(x) - R_b}$$

while the slope of a type g indifference curve is given by

$$\frac{dp}{dx} = \frac{p[1 - G(x)]}{\pi_g(x) - R_g} = \frac{p[1 - G(x)]}{\pi_b(x) - R_g}.$$

Thus type g indifference curves through any point (x,p) (with $x \geq \psi(r)$) are more steeply sloped than type b indifference curves through the same point (as shown).

In equilibrium, (x_b,p_b) is chosen to maximize $p\pi_b(x) + (1-p)R_b$ subject to $x \geq \phi(r)$ and $p \in [0,1]$. If $\pi_b[\phi(r)] > R_b$, the solution sets $p_b = 1$ and $x_b = \phi(r)$. Similarly, the candidate separating equilibrium values (p_g,x_g) must lie on or below the type b indifference curve through

$(\phi(r),1)$, and on or to the right of $x = \psi(r)$. As shown, the solution sets $x_g = \psi(r)$ and $p_g = [\pi_b(x_b) - R_b]/[\pi_b(x_g) - R_b]$. Apparently this (x_g,p_g) pair is preferred to $(\phi(r),1)$ by type g agents. Hence, by Proposition 2, type g agents have no incentive to deviate in any separating equilibrium, and a separating equilibrium exists.[7]

D. Discussion

Propositions 1 and 2 state conditions under which all borrower types announce debt contracts. These contracts also have the feature that type g borrowers voluntarily experience credit rationing. Since the rationing of credit to relatively high quality borrowers may appear counterintuitive, some discussion is warranted. It first deserves emphasis that in this environment there are no observable (ex ante) differences between borrowers of different types. Our result is therefore more correctly stated as: among borrowers who are *observationally indistinguishable* ex ante, high quality borrowers will experience credit rationing. This result parallels standard results in adverse selection settings (e.g., Rothschild and Stiglitz 1976, Azariadis and Smith 1990, or Smith and Stutzer 1989) that, among agents with the same observable characteristics, "good risks" experience rationing. We might also note that it is commonly asserted in the development literature (for example, McKinnon 1973, p. 8) that agents with high quality investment projects are often rationed while lower quality projects are fully funded. Our analysis offers an explanation for why this is the observed outcome.

It also bears emphasis that Propositions 1 and 2 depend on sufficiently large costs of state verification. In particular, conditions (6) and (7) can be viewed as requiring that γ be sufficiently large. If $\gamma = 0$ (there are no costs of verification) then it is easy to show that type g borrowers never offer debt contracts. Moreover, equation (4) will not bind on the solution to the problem of these borrowers if w* is small enough relative to r. In this case credit rationing will not be observed. Thus the presence of costly state verification is essential in delivering debt contracts as an equilibrium outcome. The use of debt contracts then implies that credit rationing will be observed.

E. Two Examples

We conclude this section by presenting two examples. The first demonstrates that our assumptions are not vacuous. The second satisfies all of our conditions except equation (6). It has the feature that a debt contract is not optimal for type g agents.

Example 1. Let $\bar{w} = 1$, let $g(w) = 1 \ \forall\, w \in [0,1]$, let $w^* = \underline{t} = 0.2$, let $f(w) = 1.5$; $w \in [0,0.1)$, $f(w) = 0.5$; $w \in [0.1,0.2)$, and let $f(w) = 1$ otherwise. In addition, let $\gamma = 0.3$, $R_g = 0.1$, and $R_b = 0$. Then it is straightforward to verify that all of our assumptions (including (a.5) below) are satisfied[8] $\forall\, r \in [0.2,0.225)$.

Example 2. The example is the same as Example 1, except that $R_g = 0.0076$. Then all of our assumptions, including (7) and (8), are satisfied, but (6) is violated. For all $r \in [0.2,0.225]$, it is straightforward but tedious to show that a debt contract is not optimal. Then the optimal contract has the features described in footnote 5.

II. Equilibrium

A. Existence

In this section we consider only the situation in which loan supply is potentially adequate to meet all possible loan demand. In particular, if optimal loan contracts for all borrower types are debt contracts (the case considered), then x_g and x_b are determined by (2) and (5) for any r. But then there is an interest rate, $\tilde{x}$, (which under our assumptions is the same for all borrower types), such that $\psi^{-1}(\tilde{x}) \geq \psi^{-1}(x)$ and $\phi^{-1}(\tilde{x}) \geq \phi^{-1}(x) \ \forall\, x$.[9] We assume that

$$\text{(a.4)} \qquad \alpha H[\phi^{-1}(\tilde{x})] \geq (1-\alpha).$$

Thus the kind of credit rationing discussed by Williamson (1986, 1987) cannot occur. Section III considers credit rationing that results from the failure of condition (a.4).

Under the assumption that (a.4) holds, then, an equilibrium with debt contracts has $x_b = \phi(r)$, $x_g = \psi(r)$, $p_b = 1$, and

$$(10) \qquad p_g = \frac{\pi_b[\phi(r)] - R_b}{\pi_b[\psi(r)] - R_b}.$$

In addition, loan supply must equal loan demand, so that

$$(11) \qquad \alpha H(r) = (1-\alpha)[\theta + (1-\theta)p_g]$$

if borrowers of both types are active in the credit market. This occurs iff $\pi_b[\phi(r)] \geq R_b$ and $\pi_g[\psi(r)] \geq R_g$. We henceforth restrict attention to equilibria with this property (as otherwise adverse selection is uninteresting).

Equations (10) and (11) constitute two equilibrium conditions jointly determining r and p_g. These conditions are depicted by the solid loci in Figure 2. Apparently (11) defines an upward sloping locus in the figure (since $H' > 0$) which intersects the horizontal axis at $(H^{-1}[\theta(1-\alpha)/\alpha],0)$. Equation (10) intersects the horizontal axis at the value $\bar{r}$, defined by $\pi_b[\phi(\bar{r})] \equiv R_b$. Also, the locus defined by (10) has a slope equal to

$$(12) \qquad \left.\frac{dp_g}{dr}\right|_{10} = \frac{p_g\pi_b'[\phi(r)]\phi'(r)}{\pi_b[\phi(r)] - R_b} - \frac{p_g\pi_b'[\psi(r)]\psi'(r)}{\pi_b[\psi(r)] - R_b},$$

which is of ambiguous sign. However, sensible comparative static results will depend on (10) being downward sloping for values of r and p_g that can simultaneously satisfy (10) and (11). We therefore state conditions such that

$$(13) \qquad \left.\frac{dp_g}{dr}\right|_{10} < 0, \quad \forall\, r \in [\underline{t},\bar{r}].$$

For such values of r, we have $x_b = \phi(r) > \psi(r) = x_g > r \geq \underline{t}$, and hence $F(x_i) = G(x_i)$ and $f(x_i) = g(x_i)$; i = g, b. Then, defining the function $\eta(x)$ by

$$\eta(x) \equiv [1 - G(x)] / [1 - G(x) - \gamma g(x)] [\pi_b(x) - R_b]$$

$$(14) \qquad = [1 - G(x)] / [1 - G(x) - \gamma g(x)] \left[\bar{w} - R_b - x - \int_x^{\bar{w}} G(w)dw \right],$$

(13) holds $\forall\, r \in [\underline{t},\bar{r}]$ iff

$$(15) \qquad \eta[\phi(r)] > \eta[\psi(r)]; \quad r \in [\underline{t},\bar{r}].$$

Since $\phi(r) > \psi(r)\ \forall\, r$, (15) holds if $\eta'(x) > 0\ \forall\, x \in [\underline{t},\bar{r}]$. From (14), $\eta'(x) > 0$ holds everywhere on this interval iff

$$(a.5) \qquad \frac{1 - G(x)}{\pi_b(x) - R_b} + \frac{g(x) + \gamma g'(x)}{1 - G(x) - \gamma g(x)} > \frac{g(x)}{1 - G(x)}$$

$\forall\, x \in [\underline{t},\bar{r}]$. (a.5) is henceforth assumed to hold,[10] in which case the locus defined by (10) is downward sloping as shown.

So long as $H^{-1}[\theta(1-\alpha)/\alpha] < \bar{r}$ and (a.5) hold, then, an equilibrium exists and there is a unique value of r that equilibrates the loan market (when self-selection of types according to announced contracts occurs). We now investigate some properties of this equilibrium.

B. Comparative Statics

Using Figure 2, it is easy to derive comparative static properties of the loan market equilibrium. First, increases in α shift (11) to (11′), as indicated by the dashed line in the figure.

Such parameter changes do not affect (10), and hence reduce the equilibrium expected return. Furthermore such changes raise p_g, and hence reduce the extent of credit rationing. The same qualitative effects occur if θ is reduced.

In order to consider the consequences of varying γ, it is useful to write from (2) that $x_b = \phi(r;\gamma)$ and from (5) that $x_g = \psi(r;\gamma)$. Then (10) is more explicitly written as

$$(10') \qquad p_g = \frac{\pi_b[\phi(r;\gamma)] - R_b}{\pi_b[\psi(r;\gamma)] - R_b}.$$

It is straightforward to verify that reductions in γ shift (10) to the right if (10) is downward sloping (i.e., if (a.5) holds). Then reductions in γ shift (10) to (10′), while leaving (11) unaffected. Thus reductions in monitoring costs raise equilibrium values of r and p_g (and hence reduce credit rationing) if (10) is downward sloping. This result indicates how the adverse selection and costly state verification problems interact in this environment. Loosely speaking, the extent to which p_g is less than one reflects the severity of the adverse selection problem, while the magnitude of γ reflects the severity of the costly state verification problem. As the problem of state verification becomes less severe, so does the adverse selection problem. In the limit, as the costly state verification problem disappears ($\gamma \to 0$), so does the adverse selection problem if w* is sufficiently small.

III. Equilibrium in the Presence of Other Forms of Credit Rationing

In the equilibrium of Sections I and II type g borrowers experience credit rationing. It is feasible for these agents to raise the perceived rate of return for lenders by bidding up the rate of interest, which would in fact increase their probability of receiving credit. However, Propositions 1 and 2 describe conditions under which it is not optimal for them to do so.

Stiglitz and Weiss (1981) and Williamson (1987) consider a somewhat different type of credit rationing where increases in the rate of interest paid by borrowers do *not* raise the perceived (expected) return for lenders. It is therefore possible that the supply of funds is inadequate to meet demand, and that there is no way to elicit more funds by raising the (expected) return faced by lenders. Thus a situation of rationed credit must be observed. Our model can easily accommodate this type of credit rationing as well as the kind described above. In this section we briefly sketch the modifications of the analysis required to accomplish this.

The highest expected return that a type b borrower can offer a lender occurs when these borrowers pay the interest rate $\tilde{x}$; or in other words, $\phi^{-1}(\tilde{x})$ is the greatest expected return that a lender can obtain from a type b borrower. If

$$\text{(16)} \qquad \alpha H[\phi^{-1}(\tilde{x})] < (1-\alpha)\theta + (1-\alpha)(1-\theta)\{\pi_b(\tilde{x}) - R_b\} / \left\{\pi_b[\psi(\phi^{-1}(\tilde{x}))] - R_b\right\},$$

then the supply of funds at this rate of return is inadequate to meet the demand. Moreover, the expected rate of return cannot be increased without making type b borrowers appear inferior to type g borrowers from the perspective of lenders. One possibility, then, is that not all type b borrowers receive credit.

Suppose that a (randomly selected) fraction q of type b borrowers do not receive credit in equilibrium, and that (as in Williamson 1986, 1987) these borrowers view themselves as being unable to influence q (since they cannot bid up the rate of return to lenders). Then the problem of a type b borrower is to choose contractual loan terms to solve the problem

$$\max \; q\left\{\int_{A_b} [w - R_b(w)]f(w)dw + \int_{B_b} (w-x_b)f(w)dw\right\} + (1-q)R_b$$

subject to (1) and the usual nonnegativity constraints. As in Williamson (1986, 1987), the solution to this problem is the same debt contract as previously, and $x_b = \tilde{x}$.

In a separating equilibrium, type g borrowers choose contract terms to maximize

$$p_g\left\{\int_{A_g}[w - R_g(w)]g(w)dw + \int_{B_g}(w-x_g)g(w)dw\right\} + (1-p_g)R_g$$

subject to

$$\int_{A_g}[R_g(w) - \gamma]g(w)dw + x_g\int_{B_g}g(w)dw = r_g \tag{17}$$

$$p_g\left\{\int_{A_g}[w - R_g(w)]f(w)dw + \int_{B_g}(w-x_g)f(w)dw\right\} + (1-p_g)R_b \leq q\pi_b(\tilde{x}) + (1-q)R_b, \tag{18}$$

$r_g \geq r = \phi^{-1}(\tilde{x})$, $0 \leq R_g(w) \leq w$, and $R_g(w) \leq x \;\forall\; w \in A_g$. Assuming that (6) and (7) hold when $r = \phi^{-1}(\tilde{x})$, then Propositions 1 and 2 continue to be true, except that $r_g = r = \phi^{-1}(\tilde{x})$ [so that $x_g = \psi[\phi^{-1}(\tilde{x})]]$, and

$$p_g = q\{\pi_b(\tilde{x}) - R_b\}/\{\pi_b[\psi(\phi^{-1}(\tilde{x}))] - R_b\}. \tag{19}$$

In particular, type g borrowers choose not to bid up the expected return, even though it is feasible for them to do so. Notice also that the two types of credit rationing interact, since q affects p_g.

Factors determining whether a separating equilibrium exists are the same as previously. More specifically, if a type g borrower deviates from the contract just described, he will choose contractual loan terms to maximize

$$q\left\{\int_{A}[w - R(w)]g(w)dw + \int_{B}(w-x)g(w)dw\right\} + (1-q)R_g$$

subject to

(20) $$\int_A [R(w) - \gamma]f(w)dw + x\int_B f(w)dw \geq \phi^{-1}(\tilde{x}),$$

$0 \leq R(w) \leq w$, and $R(w) \leq x \; \forall \; w \in A$. Assuming $\pi_g[\psi(\phi^{-1}(\tilde{x}))] > R_g$, Proposition 2 continues to be true and the solution to this problem is a debt contract (identical to that offered by type b agents). Then a deviating type g borrower has expected utility equal to $q\pi_g(\tilde{x}) + (1-q)R_g$. Therefore, a separating equilibrium exists iff

(21) $$p_g\pi_g[\psi(\phi^{-1}(\tilde{x}))] + (1-p_g)R_g \geq q\pi_g(\tilde{x}) + (1-q)R_g$$

holds, with p_g given by (19). Then the same "single-crossing" argument as in Section I establishes that (21) holds. Thus Propositions 1 and 2 continue to hold, as do conclusions about the existence of a separating equilibrium, when the kind of rationing discussed by Stiglitz-Weiss (1981) and Williamson (1987) is also present.

The equilibrium value of q is determined by the resource balance condition

(22) $$\alpha H[\phi^{-1}(\tilde{x})] = (1-\alpha)\theta q + (1-\alpha)(1-\theta)p_g,$$

with p_g given by (19). It is the case that the presence of Stiglitz-Weiss/Williamson rationing does affect the results of comparative statics exercises. For instance, local changes in α or θ affect only q. However reductions in γ raise the equilibrium value of r and reduce credit rationing, as before.

IV. Conclusions

In the environment studied, both adverse selection and costly state verification complicate the allocation of investment capital. Conditions under which optimal contracts are debt contracts (for all classes of borrowers) and credit rationing occurs have been derived. Neither of these statements would generally hold in the absence of either adverse selection or costly state verification. Thus these problems interact, and they do so in a way which can rationalize standard credit rationing formulations. This observation constitutes a caution against arguments that any single informational friction is too small to be of empirical significance, since even individually small frictions can exacerbate each other. Finally, as demonstrated in Section III, various kinds of credit rationing can co-exist, and different forms of credit rationing will interact as well.

The analysis of this paper also provides ingredients for a sequel (Boyd-Smith 1991) which adds the features that there are multiple locations, and that within-location monitoring is less costly than inter-location monitoring. If locations differ with respect to population mix (values of α and/or θ), this will result in inter-regional rate of return and interest rate differentials, and/or differential credit rationing. These differentials will create incentives for funds to flow between locations. Efficiency will demand that these flows be intermediated, so that the analysis will provide a role for endogenous intermediation. How intermediation affects return differentials and credit rationing can be studied. Also, how improvements in monitoring technology (reductions in γ) affect the quantity of intermediation and the amount of credit rationing can be examined. That these topics are of great importance in development contexts is widely accepted (Cameron 1967, McKinnon 1973).

Finally, we comment on one other possible extension of the analysis. It would be natural to undertake a welfare investigation of equilibrium, particularly in Sections I and II. In the absence of adverse selection this equilibrium would be Pareto optimal (as in Williamson 1987). However, the presence of adverse selection makes the economy behave similarly to that analyzed by Rothschild and Stiglitz (1976), and in that economy separating equilibria need not be Pareto optimal. An analysis of Pareto optima, and of policies that support Pareto optima in our decentralized context, would be a natural topic for further investigation. This would be particularly true in light of the fact that informational frictions and credit rationing are a reason commonly given for government interventions in financial markets.

Appendix

A. Proof of Proposition 1

We conjecture, and then verify, that the solution to the problem of type g borrowers satisfies (3) with equality. Under the stipulation that (3) holds with equality, we can make the following observations.

Observation 1. $[0,x_g) \subset A_g$ and $w^* < x_g$.

Proof. $[0,x_g) \subset A_g$ is obvious. Also, $w^* \leq \underline{t} \leq r < \psi(r) \leq x_g$, since $\psi(r)$ represents the smallest feasible value of x_g.

Observation 2. Type g expected utility is decreasing in expected monitoring costs, $(\gamma \int_{A_g} g(w)dw)$.

Proof. Solving (4) (at equality) for p_g yields

$$p_g = \frac{\pi_b[\phi(r)] - R_b}{\int_{A_g} [w - R_g(w)]f(w)dw + \int_{B_g} (w-x_g)f(w)dw - R_b}. \tag{A.1}$$

Furthermore

$$\begin{aligned}\int_{A_g} [w - R_g(w)]f(w)dw + \int_{B_g} (w-x_g)f(w)dw &= \hat{w}_b - \int_{A_g} R_g(w)f(w)dw \\ - x_g \int_{B_g} f(w)dw &= \hat{w}_b - \int_{A_g} R_g(w)g(w)dw - x_g \int_{B_g} g(w)dw \\ &- \int_{A_g} R_g(w)[f(w) - g(w)]dw - x_g \int_{B_g} [f(w) - g(w)]dw.\end{aligned} \tag{A.2}$$

Substituting (3) at equality into (A.2) gives

$$(A.3)\qquad \int_{A_g} [w - R_g(w)]f(w)dw + \int_{B_g} (w-x_g)f(w)dw = \hat{w}_b - r - \gamma \int_{A_g} g(w)dw$$
$$- \int_{A_g} R_g(w)[f(w) - g(w)]dw - x_g \int_{B_g} [f(w) - g(w)]dw.$$

Moreover, since $f(w) = g(w)\ \forall\ w > w^*$ (and hence $\forall\ w \in B_g$, by observation 1), (A.3) reduces to

$$(A.3')\qquad \int_{A_g} [w - R_g(w)]f(w)dw + \int_{B_g} (w-x_g)f(w)dw$$
$$= \hat{w}_b - r - \gamma \int_{A_g} g(w)dw - \int_0^{w^*} R_g(w)[f(w) - g(w)]dw.$$

Substitution of (A.3′) into (A.1) gives

$$(A.4)\qquad p_g = \{\pi_b[\phi(r)] - R_b\}/\left\{ \hat{w}_b - R_b - r - \gamma \int_{A_g} g(w)dw - \int_0^{w^*} R_g(w)[f(w) - g(w)]dw \right\}.$$

Apparently, $p_g \leq 1$ iff

$$(A.4')\qquad \gamma G[\phi(r)] \geq \gamma \int_{A_g} g(w)dw + \int_0^{w^*} R_g(w)[f(w) - g(w)]dw$$

$p_g \geq 0$ is implied by $\pi_b[\phi(r)] \geq R_b$ and (A.4′).

We now write type g expected utility as

$$(*)\qquad R_g + p_g \left\{ \int_{A_g} [w - R_g(w)]g(w)dw + \int_{B_g} (w-x_g)g(w)dw - R_g \right\}.$$

Substituting (3) at equality and (A.4) into (*) gives the following expression for type g expected utility:

$$(**) \qquad R_g + \{\pi_b[\phi(r)] - R_b\}\left[\hat{w}_g - R_g - r - \gamma \int_{A_g} g(w)dw\right] \Big/ \left[\hat{w}_b - R_b - r - \gamma \int_{A_g} g(w)dw - \int_0^{w^*} R_g(w)[f(w) - g(w)]dw\right].$$

The expression in (**) is decreasing in $\gamma \int_{A_g} g(w)dw$ iff

$$(A.5) \qquad \hat{w}_b - R_b - \int_0^{w^*} R_g(w)[f(w) - g(w)]dw > \hat{w}_g - R_g.$$

But since $0 \leq R_g(w) \leq w$, (a.3) implies that (A.5) holds. □

Observation 3. There exists an optimal contract with $A_g = [0,x_g)$.

Proof. $[0,x_g) \subset A_g$, by Observation 1. We now show that there exists an optimal contract with $A_g \subset [0,x_g)$.

Suppose then, that there exists an optimal contract specifying p_g', A_g', x_g', and $R_g'(w)$, $w \in A_g'$, such that $A_g' = [0,x_g) \cup \tilde{A}_g$ with $\tilde{A}_g \cap [0,x_g) = \varnothing$ and $\int_{\tilde{A}_g} g(w)dw > 0$. We show that a feasible contract can be constructed that yields type g borrowers strictly greater expected utility (contradicting the supposed optimality of the original contract). There are two possibilities.

Case 1. $\int_{w^*}^{x_g'} R_g'(w)g(w)dw > 0$. Then we construct a new contract as follows. Choose $x_g'' = x_g'$ and $R_g''(w) = R_g'(w)\ \forall\ w \in [0,w^*]$. Further, choose $S \subset \tilde{A}_g$ so that $\int_S g(w)dw > 0$ and choose $A_g'' = A_g' - S$. Then choose $R_g''(w)$, $w \in A_g'' - [0,w^*]$ so that $R_g''(w) = R_g'(w)$; $w \in \tilde{A}_g - S$, and

$$\text{(A.6)} \qquad \int_S [x_g'' + \gamma - R_g'(w)]g(w)dw + \int_{w^*}^{x_g''} [R_g''(w) - R_g'(w)]g(w)dw = 0.$$

(Note that the repayment is x_g outside A_g''.) p_g'' is chosen to satisfy (A.4). By hypothesis such a construction is feasible, and by (A.6) it satisfies (3) with equality. In addition, expected monitoring costs are reduced (by $\gamma \int_S g(w)dw$). Thus by observation 2 the expected utility of type g borrowers is increased by the new contract, and by (A.4'), $0 < p_g'' < p_g \leq 1$.

<u>Case 2</u>. $\int_{w^*}^{x_g'} R_g'(w)g(w)dw = 0$. If this holds then $x_g' > R_g'(w)\ \forall\ w \in [0,x_g')$, since $R_g'(w) \leq w$. Therefore it is incentive compatible to implement a contract chosen as follows. Select $S \subset \tilde{A}_g$ so that $\int_S g(w)dw > 0$, and set $A_g'' = A_g' - S$. Further, set the borrower repayment equal to $x_g'' < x_g'$ on $[0,\bar{w}] - A_g''$. Let $R_g''(w) = R_g'(w)\ \forall\ w \in A_g''$. Moreover, x_g'' and S should be chosen so that $x_g'' > w^*$ and

$$\text{(A.7)} \qquad \int_{B_g'} (x_g' - x_g'')g(w)dw = \int_S [x_g'' + \gamma - R_g'(w)]g(w)dw.$$

Finally, p_g'' is chosen to satisfy (A.4). This construction is feasible, and by (A.7) satisfies (3) with equality. Again expected monitoring costs are reduced, so that type g expected utility increases. □

<u>Observation 4</u>. There exists an optimal contract with $R_g(w) = w\ \forall\ w \in (w^*,x_g)$.

<u>Proof</u>. By assumption $w^* \leq \underline{t}$, and $\underline{t} \leq r \leq \psi(r) \leq x_g$, so (w^*,x_g) is nonempty. Then suppose $R_g(w) < w$ for some $w \in (w^*,x_g)$. If the original contract is $[p_g,A_g,x_g,R_g(w)]$ [note that by Observation 3 we may take $A_g = [0,x_g)$], then we show that a utility improving contract can be constructed as follows. Set $R_g'(w) = R_g(w)$; $w \in [0,w^*]$, and set $R_g'(w) = w$; $w \in (w^*,x_g')$. In addition let $A_g' = [0,x_g')$, choose $x_g' < x_g$ so that

$$\text{(A.8)} \qquad \int_{w^*}^{x_g'} wg(w)dw + x_g'[1 - G(x_g')] - \gamma G(x_g') = \int_{w^*}^{x_g} R_g(w)g(w)dw + x_g[1 - G(x_g)] - \gamma G(x_g),$$

and choose p_g' to satisfy (A.4). By construction the new contract is feasible, and by (A.8) it satisfies (3) with equality. Moreover, from (**) all affects on type g expected utility derive from the reduction in monitoring costs $[\gamma G(x_g) - \gamma G(x_g')]$. Thus the new contract raises type g expected utility. □

<u>Observation 5</u>. There exists an optimal contract with $R_g(w) = w \ \forall\ w \in [0,w^*]$ such that $f(w) \geq g(w)$.

<u>Proof</u>. Again suppose that there exists an optimal contract $[p_g,A_g,x_g,R_g(w)]$ with $A_g = [0,x_g)$ and $R_g(w) < w$ for some $w \in [0,w^*]$ with $f(w) \geq g(w)$. We then construct a new contract as follows. Set $R_g'(w) = R_g(w) \ \forall\ w \geq w^*$ and $\forall\ w \leq w^*$ such that $g(w) > f(w)$, and set $R_g'(w) = w$ otherwise. Choose $A_g' = [0,x_g')$, and choose $x_g' < x_g$ so that

$$\text{(A.9)} \qquad \int_0^{w^*} [R_g'(w) - R_g(w)]g(w)dw + x_g'[1 - G(x_g')] - \gamma G(x_g') = x_g[1 - G(x_g)] - \gamma G(x_g).$$

Finally, p_g' is chosen to satisfy (A.4). This contract satisfies (3) at equality, and achieves a reduction in expected monitoring costs. Moreover, $\int_0^{w^*} R_g'(w)[f(w) - g(w)]dw \geq \int_0^{w^*} R_g(w)[f(w) - g(w)]dw$. From (**), this also has the effect of increasing expected utility for type g borrowers (if the inequality is strict). Thus we have contradicted the assumed optimality of the original contract if we show that the new contract satisfies (A.4′). Obviously a sufficient condition for this is

$$\gamma G(x_g) - \gamma G(x_g') \geq \int_0^{w^*} [R_g'(w) - R_g(w)]\,[f(w) - g(w)]dw.$$

From (A.9) we have that $Q(x_g) - Q(x_g') = \int_0^{w^*} [R_g'(w) - R_g(w)]g(w)dw$, where $Q(x) \equiv x[1 - G(x)] - \gamma G(x)$. Moreover, (8) and the construction of $R_g'(w)$ imply that $Q(x_g) - Q(x_g') \geq \int_0^{w^*} [R_g'(w) - R_g(w)][f(w) - g(w)]dw$. Then we are done if $\gamma[G(x_g) - G(x_g')] \geq Q(x_g) - Q(x_g')$. But

$$\begin{aligned}\gamma[G(x_g) - G(x_g')] &= \int_{x_g'}^{x_g} \gamma g(w)dw > \int_{x_g'}^{x_g} [1 - G(w) - \gamma g(w) - wg(w)]dw \\ &= Q(x_g) - Q(x_g'),\end{aligned}$$

where the inequality follows from (7). This establishes the desired result. □

Define $Z = \{w \in [0,w^*]: g(w) > f(w)\}$. It now remains to consider the optimal specification of $R_g(w)$; $w \in Z$. By Observation 2, an optimal contract for type g borrowers maximizes (**) subject to (3) (at equality). Since R_g, R_b, and r are taken as given in this maximization, finding an optimal contract reduces to the problem of maximizing

$$\left[\hat{w}_g - R_g - r - \gamma \int_{A_g} g(w)dw\right] \Big/$$
$$\left\{\hat{w}_b - R_b - r - \gamma \int_{A_g} g(w)dw - \int_0^{w^*} R_g(w)[f(w) - g(w)]dw\right\}$$

subject to (3) (at equality). Moreover, by Observation 3, attention can be restricted to contracts with $A_g = [0,x_g)$, and by Observations 4 and 5 to contracts with $R_g(w) = w$; $w \in [0,x_g) - Z$. Then $\int_{A_g} g(w)dw = G(x_g)$, and

$$\int_0^{w^*} R_g(w)[f(w) - g(w)]dw = \int_0^{w^*} wI(w)[f(w) - g(w)]dw$$

$$+ \int_Z R_g(w)[f(w) - g(w)]dw.$$

Recalling the definitions $\epsilon(r) = \hat{w}_g - R_g - r$ and $\delta(r) = \hat{w}_b - R_b - r - \int_0^{w^*} wI(w)[f(w) - g(w)]dw$ it follows that an optimal contract for type g borrowers maximizes $[\epsilon(r) - \gamma G(x_g)]/\{\delta(r) - \gamma G(x_g) - \int_Z R_g(w)[f(w) - g(w)]dw\}$ subject to (3) at equality. Then we have

<u>Observation 6</u>. There exists an optimal contract with $R_g(w)$ chosen to solve the problem

(P) $$\max \int_Z R(w)[f(w) - g(w)]dw; \quad \text{subject to}$$

(A.10) $$\int_0^{x_g} wI(w)g(w)dw + x_g[1 - G(x_g)] - \gamma G(x_g) + \int_Z R(w)g(w)dw = r.$$

Now define

$$k(x_g,r) \equiv r + \gamma G(x_g) - x_g[1 - G(x_g)] - \int_0^{x_g} wI(w)g(w)dw.$$

Then the problem (P) can be written as

(P′) $$\min \int_Z R(w)[g(w) - f(w)]dw; \quad \text{subject to}$$

$$\int_Z R(w)g(w)dw = k(x_g,r).$$

A solution exists if $0 \leq k(x_g,r) \leq \int_Z wg(w)dw$, and involves choosing a value $\lambda \in [0,1]$ and a function $R(w)$ such that

$$\text{(A.11)} \qquad R(w) = \begin{Bmatrix} w; & (1-\lambda)g(w) \leq f(w) \\ 0; & \text{otherwise} \end{Bmatrix}.$$

In addition, defining $Z_\lambda = \{w \in Z: (1-\lambda)g(w) \leq f(w)\}$, λ should be chosen so that

$$\int_{Z_\lambda} R(w)g(w)dw = k(x_g,r).$$

For fixed $k(x_g,r) = k$, denote the minimized value of the objective in (P′) by $H(k)$, and the associated repayment schedule by $R(w;k)$. Clearly if $k' > k$, $R(w;k') \geq R(w;k)$ $\forall$ $w \in Z$ (with strict inequality for some w).

Observation 7. Let $k' > k \geq 0$ hold (with $k' \leq \int_Z wg(w)dw$). Then $k' - k \geq H(k') - H(k) \geq 0$.

Proof. $H(k') \geq H(k)$ is obvious. For the remainder note that

$$\begin{aligned} H(k') &\equiv \int_Z R(w;k')\,[g(w) - f(w)]dw = \int_Z R(w;k)[g(w) - f(w)]dw \\ &\quad + \int_Z [R(w;k') - R(w;k)]\,[g(w) - f(w)]dw \equiv H(k) \\ &\quad + \int_Z [R(w;k') - R(w;k)]\,[g(w) - f(w)]dw \\ &\leq H(k) + \int_Z [R(w;k') - R(w;k)]g(w)dw = H(k) + k' - k, \end{aligned}$$

where the inequality follows from $R(w;k') \geq R(w;k)$ $\forall$ $w \in Z$. □

From the above it follows that the choice of an optimal contract for type g borrowers now reduces to finding a value x_g to maximize the expression

$$\text{(A.12)} \qquad U(x_g) \equiv \frac{\epsilon(r) - \gamma G(x_g)}{\delta(r) - \gamma G(x_g) + H[k(x_g,r)]}$$

(with r taken as given).

Observation 8. (6) and (7) imply that $U(x_g)$ is nonincreasing in x_g.

Proof. The proof is by contradiction. Suppose there exist feasible values x', x with $x' > x$ and $U(x') > U(x)$. (Feasible means $k(x',r) \leq \int_z wg(w)dw$ and $k(x,r) \geq 0$.) Noting that the denominator of (A.12) must be positive for x' and x, (A.12) implies that $U(x') > U(x)$ iff

$$\text{(A.13)} \qquad \begin{aligned} &\gamma[\delta(r) - \epsilon(r)]\,[G(x') - G(x)] + \gamma\{G(x')H[k(x,r)] - G(x)H[k(x',r)]\} \\ &\quad < \epsilon(r)\{H[k(x,r)] - H[k(x',r)]\}. \end{aligned}$$

Furthermore, since $G(x') \geq G(x)$, (A.13) implies that

$$\text{(A.14)} \qquad \gamma[\delta(r) - \epsilon(r)][G(x') - G(x)] < [\epsilon(r) - \gamma G(x)]\{H[k(x,r)] - H[k(x',r)]\},$$

since $H[k(x,r)] \geq 0$.

Now (6) implies that $\delta(r) - \epsilon(r) \geq \epsilon(r) - \gamma G[\psi(r)] \geq \epsilon(r) - \gamma G(x)$ (since $x \geq \psi(r)$ must hold). Then (A.14) requires that $H[k(x,r)] - H[k(x',r)] > \gamma[G(x') - G(x)]$. But by Observation 7, this implies that

$$\text{(A.15)} \qquad k(x,r) - k(x',r) > \gamma[G(x') - G(x)].$$

Moreover, the definition of $k(x,r)$ implies that (A.15) is equivalent to

$$\text{(A.15')} \qquad \int_x^{x'} [1 - G(w) - \gamma g(w)]dw > \gamma \int_x^{x'} g(w)dw.$$

However (A.15) and $x' > x \geq \psi(r)$ contradict (7). □

Since $U(x_g)$ is nonincreasing in x_g, the smallest feasible value of x_g is optimal, i.e., x_g should be minimized subject to $k(x_g,r) \leq \int_Z wg(w)dw$. The solution obviously sets $k(x_g,r) = \int_Z wg(w)dw$. Thus $R_g(w) = w \; \forall \; w \in Z$ (and hence $\forall \; w \in A_g$), so that type g expected utility is maximized by selecting a debt contract. □

Observation 9. (3) must hold with equality for any optimal contract announcement.

Proof. Suppose an optimal contract exists that satisfies

$$\int_{A_g} [R_g(w) - \gamma]g(w)dw + w_g \int_{B_g} g(w)dw = r + \epsilon$$

for $\epsilon > 0$. Then all previous arguments can be repeated with r replaced by $r + \epsilon$. The result will be that type g expected utility will be equal to

$$R_g + \{\pi_b[\phi(r)] - R_b\}\{\hat{w}_g - R_g - r - \epsilon - \gamma G[\psi(r+\epsilon)]\} /$$
$$\left\{\hat{w}_b - R_b - r - \epsilon - \gamma G[\psi(r+\epsilon)] - \int_0^{w^*} w[f(w) - g(w)]dw\right\}.$$

This can be increased by reducing ϵ. □

B. Proof of Proposition 2

Under the specification of lender beliefs in the text, the best a deviating type g borrower can do is announce a contract (consisting of values p and x, a set A of monitoring states, and a repayment schedule R(w); $w \in A$) to maximize

$$p\left\{\int_A [w - R(w)]g(w)dw + \int_B (w-x)g(w)dw\right\} + (1-p)R_g$$

subject to

$$\text{(A.16)} \qquad \int_A [R(w) - \gamma] f(w)dw + x \int_B f(w)dw \geq r$$

and $0 \leq R(w) \leq w$; $R(w) \leq x \; \forall \; w \in A$. (A.16) imposes that lenders receive the expected return r, given that they believe the contract is offered by a type b agent. Since there is no incentive for type b agents to deviate, given this belief, there is no need to impose a self-selection constraint.

We now simplify the problem of a deviating type g borrower. First, clearly, $x \geq \phi(r) > r$ must hold, and $r \geq \underline{t} \geq w^*$. Moreover, $[0,x) \subset A$, so $g(w) = f(w) \; \forall \; w \in B$. Then (A.16) (at equality) can be written as

$$\int_A [R(w) - \gamma] f(w)dw + x \int_B f(w)dw = \int_A [R(w) - \gamma] f(w)dw + x \int_B g(w)dw$$

$$= \int_A [R(w) - \gamma] g(w)dw + x \int_B g(w)dw + \int_A [R(w) - \gamma][f(w) - g(w)]dw = r.$$

Further, since $f(w) = g(w) \; \forall \; w > w^*$, this expression reduces to

$$\text{(A.17)} \qquad \begin{aligned} &\int_A [R(w) - \gamma] g(w)dw + x \int_B g(w)dw \\ &= r - \int_0^{w^*} [R(w) - \gamma][f(w) - g(w)]dw. \end{aligned}$$

Substituting (A.17) into the maximand of a deviating type g agent and integrating yields the expression

$$p \left\{ \hat{w}_g - r - \gamma \int_A f(w)dw - \int_0^{w^*} R(w)[g(w) - f(w)]dw \right\} + (1-p)R_g$$

which is to be maximized subject to the expected return constraint

(A.18) $$\int_A R(w)f(w)dw = r + \gamma \int_A f(w)dw - x \int_B g(w)dw.$$

Now if $R_g \geq \hat{w}_g - r - \gamma \int_A f(w)dw - \int_0^{w^*} R(w)[g(w) - f(w)]dw$ for all choices of A and R(w), clearly type g agents have no incentive to deviate. Thus we need only concern ourselves with the case where this inequality fails. Then $p = 1$. Moreover, arguments used in the proof of Proposition 1 establish that there exists a solution with $A = [0,x)$, and with $R(w) = w \; \forall \; w \in [0,x) - Z$, where it will be recalled that $Z = \{w \in [0,w^*]: g(w) > f(w)\}$. Thus the only issue concerns the optimal choice of $R(w)$; $w \in Z$.

Apparently, for any x, R(w), $w \in Z$, should be chosen to solve the problem

(Q) $$\min \int_Z R(w)[g(w) - f(w)]dw$$

subject to

(A.19) $$\begin{aligned}\int_Z R(w)f(w)dw &= r + \gamma F(x) - x[1 - G(x)] - \int_0^x wI(w)f(w)dw \\ &= r + \gamma G(x) - x[1 - G(x)] - \int_0^x wI(w)f(w)dw,\end{aligned}$$

where the latter equality follows from $F(x) = G(x) \; \forall \; x > w^*$. Then define

$$\ell(x,r) \equiv r + \gamma G(x) - x[1 - G(x)] - \int_0^x wI(w)f(w)dw.$$

The problem (Q) reduces to

(Q') $$\min \int_Z R(w)[g(w) - f(w)]dw$$

subject to $\int_Z R(w)f(w)dw = \ell(x,r)$. As above, the solution to this problem chooses a value $\lambda \geq 0$, sets

$$\text{(A.20)} \qquad R(w) = \begin{Bmatrix} w; & (1+\lambda)f(w) \geq g(w) \\ 0; & \text{otherwise} \end{Bmatrix},$$

and sets λ so that $\int_{Z_\lambda} R(w)f(w)dw = \ell(x,r)$, where $Z_\lambda = \{w \in Z: (1+\lambda)f(w) \geq g(w)\}$.

For fixed $\ell(x,r) = \ell$, denote the minimized value of the objective in (Q′) by $M(\ell)$, and the associated repayment function by $R(w;\ell)$. Clearly, if $\ell' > \ell$ holds (and $\int_Z wf(w)dw \geq \ell' > \ell \geq 0$), $R(w;\ell') \geq R(w;\ell)\ \forall\ w$, with strict inequality for some w. Then it also follows that

$$\begin{aligned} M(\ell') &\equiv \int_Z R(w;\ell')[g(w) - f(w)]dw = \int_Z R(w;\ell)[g(w) - f(w)] \\ &\quad + \int_Z [R(w;\ell') - R(w;\ell)]\,[g(w) - f(w)]dw \\ &\equiv M(\ell) + \int_Z [R(w;\ell') - R(w;\ell)]\,[g(w) - f(w)]dw \\ &\leq M(\ell) + \int_Z [R(w;\ell') - R(w;\ell)]f(w)dw = M(\ell) + \ell' - \ell, \end{aligned}$$

where the inequality follows from (8) and $R(w;\ell') \geq R(w,\ell)$.

The expected utility of a deviating type g agent can now be written as

$$\hat{w}_g - r - \gamma F(x) - M[\ell(x,r)] - \int_0^{w^*} wI(w)[g(w) - f(w)]dw.$$

Moreover, this expression is nonincreasing in x. To see this, suppose $x' > x$ and

$$\text{(A.21)} \qquad \gamma F(x') + M[\ell(x',r)] < \gamma F(x) + M[\ell(x,r)].$$

(A.21) implies that

(A.21′) $\gamma[F(x') - F(x)] < M[\ell(x,r)] - M[\ell(x',r)] \leq \ell(x,r) - \ell(x',r).$

But by the definition of $\ell(x,r)$, (A.21′) implies that

$$\text{(A.22)} \qquad \int_x^{x'} [1 - G(w) - \gamma g(w)]dw > \gamma \int_x^{x'} f(w)dw = \gamma \int_x^{x'} g(w)dw,$$

where the latter equality follows from $f(w) = g(w)\ \forall\ w \geq w^*$ (and $x \geq \phi(r) > r \geq w^*$). (A.22) contradicts (7), however, establishing the desired result.

It now follows that a deviating type g borrower should minimize x subject to $\ell(x,r) \in [0, \int_z wf(w)dw]$. The solution sets $\ell(x,r) = \int_z wf(w)dw$. Thus the optimal contract is a debt contract, and by (A.16), $x = \phi(r)$. □

Footnotes

[1]The allocation of investment capital in environments with risk neutral agents and costly monitoring is considered by Diamond (1984), Gale and Hellwig (1985), Boyd and Prescott (1986), and Williamson (1986, 1987). Of these only Boyd and Prescott (1986) also consider the presence of adverse selection, as we do. Also, our monitoring costs derive from costly state verification, as in Townsend (1979), Gale-Hellwig (1985), and Williamson (1986, 1987).

[2]See, for instance, Besanko and Thakor (1987), Smith and Stutzer (1989), or Azariadis and Smith (1991).

[3]We can assume that lenders vary according to their opportunity costs of investment, or alternatively, we could assume that lender j has the utility function $c_1 + (c_2 - e)/t_j$, where c_k is consumption in period k.

[4]If $w \in B$ but the borrower announces $w \in A$, we may assume that $R(w) = w$.

[5]The arguments in Appendix A establish that, if a debt contract fails to be optimal, the optimal contract will have $A_g = [0, x_g)$, $R_g(w) = w \; \forall \; w \in A_g - Z$, $R_g(w) = w \; \forall \; w \in Z_\lambda$, and $R_g(w) < w \; \forall \; w \in Z - Z_\lambda$, with Z and Z_λ defined in Appendix A. p_g will then be given by (A.4), and $x_g > \psi(r)$ will hold. Thus the contract will resemble a debt contract except that $R_g(w) < w$ will hold for some w.

[6]This leaves open the question of whether other specifications of lender beliefs would support other (pooling or semi-separating) equilibria. While this is clearly possible, such specifications seem unlikely to survive the refinements suggested by Cho and Kreps (1987). Also, we note that our specifications of lender beliefs reduce the question of equilibrium determination essentially to that considered by Rothschild and Stiglitz (1976).

[7]Note that this argument does not rely on the separating equilibrium contract for type g agents being a debt contract.

[8]Except for the assumption that f is differentiable everywhere. It is straightforward to modify the proofs of Propositions 1 and 2 to accommodate this exception for the example.

[9]$\tilde{x}$ is defined by $1 - F(\tilde{x}) - \gamma f(\tilde{x}) = 0$. Since $\tilde{x} > \underline{t} \geq w^*$ must hold for any funds to be offered by lenders, $G(\tilde{x}) = F(\tilde{x})$ and $g(\tilde{x}) = f(\tilde{x})$, so $\tilde{x}$ also satisfies $1 - G(\tilde{x}) - \gamma g(\tilde{x}) = 0$.

[10]A sufficient condition for (a.5) to hold $\forall\ x \leq \bar{r}$ is that $g'(x)[1 - G(x)] \geq -g(x)^2\ \forall\ x \in [\underline{t}, \bar{r}]$.

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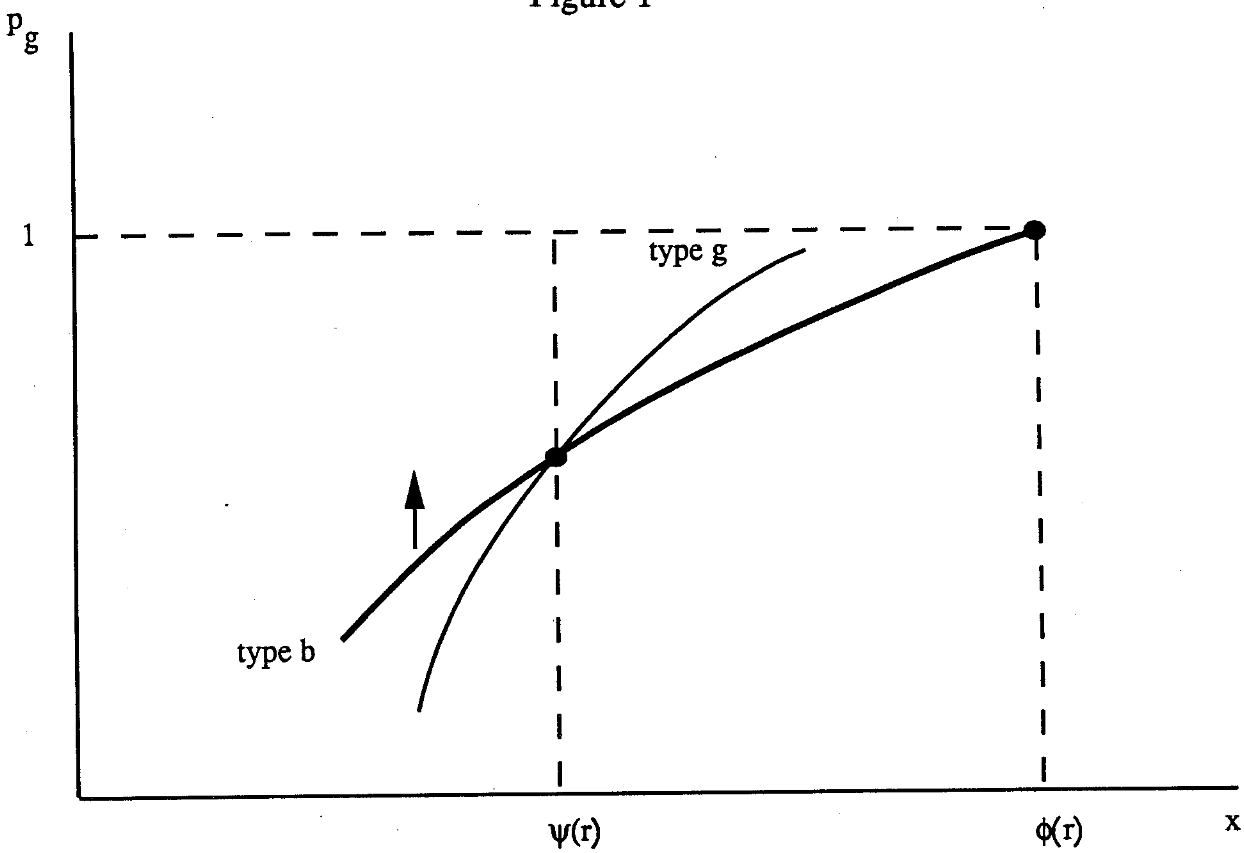
Figure 1



Utility increases in the direction shown.

Figure 2