Horizontal and Vertical Equity: A Theoretical Framework and Empirical Results for the Federal Individual Income Tax 1966-1987

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ABSTRACT

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Index numbers summarizing the equity effects of tax systems are often employed by policymakers, economists, and the general public. After surveying the literature, we define and justify through axioms index numbers reflecting the vertical and horizontal equity of a tax system. In contrast with most of the literature, which employs a social welfare function as the primitive, we use assumptions on the index number or ordering directly, thus making the underlying value judgments transparent. Our index numbers, along with many others drawn from the literature, are applied to study the U.S. Federal individual income tax system over 22 years using publicly available individual income tax data from the Internal Revenue Service.

In general, the system is progressive, but horizontal equity is seen to be a problem. After each tax reform, the system becomes more and then less progressive, reflecting taxpayer adjustment to law changes. This adjustment takes about two years. Finally, an empirical, inverse tradeoff is found between the progressivity and the horizontal equity of the tax system over time.

1. Introduction

The systematic characterization of the distribution of income has long interested social scientists, statisticians, and policy makers. The normative content of various statistics which summarize large amounts of information about, for example, the before or after-tax distribution of income, is often quite important in public policy debates about the wisdom of particular tax and spending programs. Indeed, the charge that a particular public policy is "regressive" carries with it significant negative connotations and the implication that such a policy should be withdrawn because it offends our shared values of what a just distribution of income should be.

How one defines and actually measures such emotive terms as "equitable," "inequitable," "progressive," and "regressive" can have a significant impact on public debate on such policies, and are often discussed as election issues.

The Department of Treasury and the Joint Committee on Taxation have, over the years, developed a microsimulation model of the tax code that is used to evaluate policy changes. The model presumes no behavioral reaction on the part of taxpayers other than to itemize or not itemize deductions, and simply takes a random, stratified sample of returns through the changes in law, weights the resulting tax liability to bring the sample to national totals, and reports the results. The model is used primarily to project how revenue changes with policy. However, computations from the model have often been used to characterize the equity aspects of changes in policy. The model generates three items that are given to lawmakers for any tax proposal:

1) The number of taxpayers by income class whose tax liability increases and decreases;

2) The average dollar amount of increase or decrease by income class;

3) The change in tax burden for representative fictitious taxpayers.

These statistics are generally what appear in newspapers when tax legislation is being considered by Congress. To economists, these statistics do not accurately measure the equity aspects of the tax system that should be measured[2]:

Vertical Equity - The degree to which taxpayers with higher ability to pay in fact pay more in taxes.

Horizontal Equity - The degree to which taxpayers in identical circumstances pay the same taxes.

Over the years, a number of statistical measures of these properties of a tax system have been proposed and used. These include the Gini coefficient, median effective tax rates by income class, and the coefficient of variation of effective tax rates. It has become apparent that these simplistic measures do not capture the important horizontal and vertical aspects of taxation, and this has led to the development of more sophisticated and axiomatically justified measures.

Although tax policy is often driven by revenue or perhaps efficiency considerations, the congressional decision process makes it important to provide simple measures of relevant equity aspects of tax legislation to policy makers.

Our purpose in this paper is:

1. to describe two major approaches in the economics literature to developing indices that seek to measure various concepts of "equity";

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[2]See Musgrave and Musgrave [1989, p. 223].

2. to specify a class of equity measures that are theoretically justified by the second of the two approaches to characterizing equity of a tax system, and which are based on the relative position of taxpayers throughout society; and,
3. to apply these measures to annual, random stratified samples of Federal tax returns for the period 1966-87 to identify trends in the equity of our Federal individual income tax.

By way of summary, our major findings are:

1. that this second class of equity measures or index numbers may be derived from a consistent and sensible set of axioms which are somewhat weaker in terms of separability, and therefore more attractive than axioms employed in deriving several of the indices developed under the social welfare function approach;
2. there is substantial evidence that the progressivity and horizontal equity of the Federal individual income tax has declined after each of the major tax reform efforts since the mid-1960's--these declines are conjectured to reflect the behavioral response of taxpayers and their advisors to Congressional "stiffening" of the progressivity of our tax structure. Thus, taxpayers and advisors take advantage of remaining tax preferences that they are able to find, and typically it takes 2 years for such a reaction to take effect;
3. overall, there is more evidence of horizontal inequity [unequal tax treatment of taxpayers with the same ability to pay] than lack of progressivity in our tax system. Overall, better than 80% of all pairs of taxpayers are treated progressively by our tax system over the period 1966-87, while better than 80% of all pairs of comparable taxpayers with the same ability to pay are treated differently by the tax system as it has evolved over the years; and,
4. there is a sizeable, inverse, statistical association between horizontal equity and vertical progressivity across the sample period, and a sizeable, inverse statistical association between horizontal inequity as measured by the coefficient of variation in effective tax rates and the after-tax Gini coefficient of income inequality.

Our results are consistent with those in the recent literature on income inequality and taxes in the 1980's, such as Kern [1990], Gramlich, Kasten, and Sammartino [1991] or Michel [1991]. The main conclusions drawn from that literature are that the before and after tax income inequality increased in the 1980's, and although the federal individual income tax remained progressive over this time period, its progressivity declined. We come to the same conclusions, but augment them. We examine changes in progressivity over a longer time period, and we also examine horizontal equity. The conclusions concerning the adjustment time of taxpayers after tax reform appear new. Our methodology differs from these other studies as well. We do not account for transfers and imputations that might be made, and we use actual post - behavior data rather than data from earlier years that is aged. Finally, we use a variety of measures rather than concentrate on one particular measure.

In a companion paper, Berliant and Strauss [1991], we examine the effects of the Tax Reform Act of 1986 on the state and federal tax systems using the same techniques as in the present paper.

This paper is organized as follows. In section 2, we briefly summarize the literature on the measurement of income and tax inequality. Section 3 defines and discusses the index numbers on which we focus, while section 4 provides characterizations of them in terms of axioms. Section 5 specifies the empirical context and data, while section 6 gives the results from the empirical implementation. Section 7 contains conclusions and suggestions for further work. An appendix contains the formulae of the index numbers from the literature that are applied to the data.

2. Approaches to Characterizing the Distribution of Taxes and Income

From a theoretical standpoint, index numbers describing the distribution of income or tax burdens arise from two directions. First, they can be justified as simple summary statistics to be used by policymakers in evaluating tax systems. In this sense, they are directly connected to a policymaker's preferences. A second way they can arise is by their explicit entry in agents' utility functions (that is, they summarize an externality) or in a social welfare function; see King [1983].

From a pragmatic viewpoint, the first motivation is more important. Recalling the definitions of vertical and horizontal equity given in the previous section, methods for quantifying the *degree* of vertical and horizontal equity embedded in a tax system are needed to evaluate policy changes.

Better than sixty years ago, the English economist Dalton [1925] pointed out that underlying the choice of one statistical inequality index over another (e.g., choosing the GINI coefficient of income inequality rather than the variance of income) is some notion of aggregate or social welfare that would be maximized were the index to reach its limit [say an egalitarian or equal distribution of after-tax income] as a result of deliberate social policy.

Dalton focused attention on the fact that our inference about how desirable specific distributional policies might be is affected by the nature of the index number or summary statistic used to compare present circumstances [say, the current distribution of income] with those resulting from a specific policy.

Over the years, a number of measures of (after tax) income inequality, such as the Gini coefficient (see the appendix for an algebraic statement) have been proposed and used. However, in the specific context of tax policy, these simple measures do not capture the notions of either vertical or horizontal equity. They capture shifts, say, between the before and after tax distributions of income, but do not account for how individuals are treated by the tax system. For example, the relative position of an individual in the before and after tax income distributions might be quite different. The axioms or properties underlying these index numbers have have been examined only relatively recently; see, for example, Thon [1972].

In 1948, Musgrave and Thin proposed some crude progressivity measures. They included the rate of change of the effective or average tax rate as income changes, the rate of change of the marginal tax rate, the elasticity of tax liability with respect to before tax income, and the elasticity of after tax income with respect to before tax income. These could be graphed over the range of before tax incomes, or averaged over this range. These measures are easily calculated for statutory taxes. However, they are hard to calculate for the empirical distribution of taxes, since there is generally considerable variance in the taxes actually paid at any income level; see for example Gouveia and Strauss [1991]. This variety of measure also takes into account statutory law and portions of tax schedules that might apply to nobody; thus, it is important to account for the characteristics of taxpayers who are actually present. Toward the end of their article, Musgrave and Thin propose a measure, the measure of effective progression, that does not suffer from these deficiencies. It is defined by one minus the before tax Gini coefficient divided by one minus the after tax Gini coefficient. However, the most important deficiency of all of these measures is that the value judgments underlying them are not exposed.

Next we turn to the modern development of index numbers of vertical and horizontal equity, which is based on properties that characterize (that is, are necessarily satisfied by and imply the use of) a particular measure.

Two approaches to this problem of how to choose the proper index number for evaluating tax and income distributions suggest themselves: 1] try to derive an index number from an aggregation rule or social welfare function which contains specific value-judgements about how society views individual incomes--we call this approach to index number construction the "welfare approach;" and 2] view an index number as a normative decision tool directly, and choose it on the basis of the plausibility of the value judgements contained in the indices directly. We call this second approach to index number construction the "direct approach." We turn first to the welfare approach.

In a fundamental paper, Atkinson [1970] argued that an index number summarizing the distribution of income should be derived from a well-defined social welfare function [SWF]. Most recent work on index numbers of income inequality as well as poverty lines are generalizations or extensions of this line of analysis and technique of proof. An example may be found in King [1983]. Atkinson [1970] suggests that the social welfare function be of the general additively separable and symmetric form:

$$W = \sum_{i=1}^{n} U(y_i) \qquad (1)$$

where y_i is income of the i'th unit, and U is a monotonic indirect utility function. The concept of equally distributed equivalent income, y_{ede}, per capita income yielding the same social welfare as the true distribution, is defined by:

$$nU(y_{ede}) = \sum_{i=1}^{n} U(y_i). \qquad (2)$$

The inequality index I is defined to be the loss in social welfare, in terms of income, from having income unequally distributed, normalized by mean income. Formally, if m is mean income of the true distribution,

$$I = 1 - y_{ede} / m.$$

If I is assumed to be invariant to proportional shifts in the distribution, i.e.

$$I(y_1,...,y_n) = I(ky_1,...,ky_n)$$

for $k > 0$, then using some mathematics derived in the theory of risk aversion,

$$I = 1 - [\sum_{i=1}^{n} y_i^{1-t}/m]^{1/1-t} \qquad (3)$$

where t is a parameter representing inequality aversion.

Blackorby and Donaldson [1978, 1980] proved that the relationship between homothetic social welfare functions and inequality indices is one-to-one, although under their framework, ordinally equivalent indices do not always lead to ordinally equivalent social welfare functions. A general procedure has recently been proposed by Ebert [1987]. A second ordering, through which the trade-off between the inequality of an income distribution and its mean income is determined, is postulated. When this order is combined with an inequality ordering, the two orderings generate a social welfare function and vice versa.

Lin [1989] has examined the relationship between revenue, tax systems, and index numbers under the welfare approach. If the revenue generated by a certain tax system increases, does progressivity (as measured by a particular index number) increase? If so, then the tax system is *progressive effective* with respect to the inequality measure. In this way, relations between index numbers, tax systems, and social welfare functions were established.

While the derivation of various indices of vertical and horizontal equity from social welfare functions has been a prevalent form of theoretical rationale for particular equity measures, this line of research suffers from certain limitations. What does a social welfare function do? It ranks states of an economy. What does an inequality index do? It ranks states of the economy. What is the difference? What properties do we want each to have? If assumptions are imposed on the social welfare function, why shouldn't the same assumptions be imposed on the index number? For example, the social welfare function of Atkinson is required to be additively separable, but the inequality index I is not additively separable in incomes. The inequality index I is assumed to be homogeneous of degree zero, but the social welfare function does not have this property. The application of different assumptions to the social welfare function and the index number may reflect inconsistencies between fundamental value judgements being entertained about the social welfare function and the index number. Furthermore, since the derivation of this type of index number requires the inversion of a utility function, they are inherently single-variable in nature. This is a limitation if one wishes to characterize social welfare in terms of several variables, such as incomes and effective tax rates or incomes and tax liabilities.

A number of these disadvantages may be overcome if one views index numbers directly as a social welfare function, and simply chooses an index number on the basis of its inherent plausibility.[3] For example, the value judgements underlying an index number are more transparent.

Below, a class of index numbers based on the *relative* position of all pairs of incomes in society is developed. The underlying separability assumptions are weaker than those of King [1983] and Atkinson [1970], and as such are inherently more attractive. In the next section, we shall describe the intuition behind these index numbers. In section 4, we characterize these index numbers with their underlying axioms. Finally, we shall turn to applications of these and other index numbers in the literature. Further applications can be found in Berliant and Strauss [1991]. An appendix contains the formal definitions of many index numbers, all of which are used in the empirical applications below.

The literature on vertical equity is huge and growing, so it would be futile to try to give a complete survey in the limited space here. We can only say that there are many approaches to this measurement problem, and that many measures are yet to be justified by axiomatic characterizations. References in other strands of the literature include Kakwani [1977] and Suits [1977].

The literature on horizontal equity is more recent but is rapidly developing. Feldstein [1976] ignited interest in this area by discussing its importance in the context of tax reform. He asserted in this paper (p. 83) that the classic definition is related to the principle that the ordering of individuals by utility level should not be changed by a tax system. This led researchers to consider measures of rank reversals in utility or income to be measures of horizontal inequity. Atkinson [1980], Plotnick [1981, 1982], and King [1983] followed up on this line of reserch. For examples demonstrating that such measures are unrelated to the classical concept of horizontal equity defined above, see Berliant and Strauss [1985]. Recent contributions to this literature include Kaplow [1989], Musgrave [1990], and Jenkins [1988]. Currently, there is much debate about the definition of horizontal equity, and how it might be made operational. We prefer the classic definition, and direct axioms or properties that characterize the index numbers.

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[3]Under this second approach, population decomposibility is the axiom employed most frequently; see for example Shorrocks [1980, 1984] Recently, Shorrocks and Foster [1987] have shown that transfer sensitive Pigou-Dalton indices agree on the pairwise inequality ranking of one income distribution obtained from another using favorable composite transfers.

3. Operational Measures of Vertical and Horizontal Equity

We provide below operationalizations of the traditional concepts of horizontal and vertical equity. This is achieved in two steps. First, index numbers based on the equity concepts are developed. Second, they are applied along with other index numbers found in the literature to annual data on Federal individual income tax returns for the years 1966 - 1987.

3.1. Classifications of Progressivity and Horizontal Equity

Two prefatory remarks are in order. First, we shall use economic income as a proxy for individual welfare. This is equivalent to the use of an indirect utility function, and is standard in the literature. Second, we take as given a partition of the income distribution into cells of "equals" for the purpose of separating horizontal and vertical comparisons. We also take as given a partition of the set of effective tax rates into cells, which is used to distinguish "similar" effective tax rates for proportional comparisons. Clearly the index number values depend on the precise nature of these partitions, but the empirical ordering of tax systems generated by index numbers is generally independent of these partitions.

To describe the vertical characteristics of the tax system, we follow Wertz [1975, 1978] and partition comparisons between taxpayers into three groups: the fraction of pairs of taxpayers whose tax liability is progressively distributed, the fraction of pairs of taxpayers whose tax liability is proportionately distributed, and the fraction of pairs of taxpayers whose tax liability is regressively distributed. We shall construct the measures so that they sum to 1. A comparison of taxpayers shows progressivity when both the income and effective tax rate of one taxpayer are greater than the income and effective tax rate of the other taxpayer. Proportionality is said to occur when the incomes of two taxpayers are different, but the effective tax rates are the same. Finally, regressivity is said to occur when one taxpayer has a larger income but a lower effective tax rate than the other taxpayer in the pairwise comparison. Counting the number of paired comparisons that are progressive and dividing by the total number of paired comparisons between taxpayers with different incomes (the vertical comparisons) yields the unweighted progressive index. Similar computations yield the unweighted proportional and regressive index numbers.

We refer to Table 3-1 for a summary of the classifications of these static comparisons as well as certain "dynamic" index numbers which are discussed below.

To ascertain the *extent* to which taxes are distributed progressively, proportionately, and regressively, we take into account not only the *number* of occurrences of each type of comparison, but also the *degree* of income and effective tax rate disparities. Our subjective judgement is that it matters when scoring such comparisons whether taxpayer A with an effective tax rate of 28% and taxpayer B with an effective tax rate of 20% have similar or very different incomes. Thus the actual measurement involves the weighting of each comparison count by the absolute difference in income of each pair of taxpayers.

Similar considerations argue for taking into account the *extent* of differences in effective tax rates. That is, it seems to matter, if taxpayer A has an income twice that of taxpayer B, just how similar (or different) the effective tax rates are for the two taxpayers. For example, should A have an income of $30,000 and B have an income of $15,000, the 'progressiveness' of the tax system would seem to differ if in the first instance the respective effective tax rates were 28% and 20% while in the second instance effective tax rates of 32% and 18%. Clearly, the former would seem to be *less* progressive than the latter.

To account for such differences in effective tax rates, we weight the comparisons by the *ratio* of effective tax rates rather than the *differences* in effective tax rates. We do this for several reasons. First, using the ratio differentiates more effectively between a pair of effective tax rates that are close to each other nominally but not relatively. A pair of effective tax rates of 10% and 14% would seem to be much more disparate than a pair of effective tax rates of 46% and 50%. While the *differences* are both 4%, the former pair of tax rates clearly displays more disparity. Second, using the ratio of rates deals with proportional comparisons when forming the weights for each comparison operation. If one were to form a weight based on the difference in effective tax rates, the weight would be zero, while by using the *ratio* the weight becomes unity.

The weighted vertical index numbers are formed as follows. For each progressive comparison, weight by the difference in incomes and the ratio of effective tax rates, and sum over progressive comparisons. Repeat this procedure for both regressive and proportional comparisons as well. Divide each of these sums by the total weighted sum over all vertical comparisons.

Horizontal equity, unlike vertical equity, does not admit of multiple classifications. Simply put, horizontal equity means either that equals are treated the same, or not. Accordingly, we shall measure the *extent* to which effective tax rates are different or are identical. Again, following Wertz [1975], we classify instances of differential effective tax rates for pairs of taxpayers with identical incomes to be instances of *inequity*, and instances of identical effective tax rates for pairs of taxpayers with identical incomes to be instances of *equity*. Dividing these counts by the total number of horizontal paired comparisons, comparisons between taxpayers deemed to be equals (operationally, in terms of income), the unweighted horizontal equity and inequity index numbers are obtained. By weighting each paired comparison by the ratio of effective tax rates in order to account for the extent of inequitable treatment by a tax system, and then performing the same calculations as for the unweighted horizontal index numbers, the weighted equity and inequity index numbers are obtained. Notice that each weighted count is divided by the sum over all horizontal comparisons of weighted counts.

The weighted horizontal and vertical measures are obtained by making all possible comparisons among pairs of taxpayers, and accumulating the weighted comparisons of each type of classification. Note that in the case of the vertical comparisons, a tax system may be said to have simultaneously progressive, regressive, and proportional components to it. This occurs because comparisons are relative, and the comparisons are numerous. For n individuals in an economy, there are n(n-1) total comparisons.

What we call "dynamic" index numbers are used to compare two tax systems, which we call X and Y. We assume that economic income is independent of which tax system, X or Y, is imposed. In an application in a companion paper, plan X is the federal income tax system, while plan Y is the total income tax system consisting of both federal and state taxes. The question we ask is as follows. Given that both the federal and state tax systems are imposed, what is the marginal effect on equity of the state tax system? We do not seek to address questions concerning the equity effect of repealing a state tax system. Therefore, the assumption that economic income is fixed is needed. For each pair of taxpayers, these dynamic index numbers account for whether the comparison becomes more progressive, regressive or proportional under Y as opposed to X, provided that the comparison is vertical; see Table 3-1. For example, consider a comparison between two taxpayers with unequal incomes. If the ratio of the effective tax rate under plan Y to the effective tax rate under plan X is higher for the taxpayer with higher income, then this comparison is classified as more progressive. If the ratios are the same for the two taxpayers, the comparison is classified as proportional. If the ratio is higher for the taxpayer with lower income, then the comparison is classified as more regressive. The counts in each classification are totalled; no weighting is involved. Dividing each count by the total number of vertical comparisons yields the dynamic vertical index numbers.

Table 3-1: Definition of Static and Dynamic Berliant-Strauss Index Numbers

STATIC Comparison		DYNAMIC More Prog	DYNAMIC No Change	DYNAMIC More Regr
Progressive	$Y_1 > Y_2$ $t_1 > t_2$	$\frac{t'_1}{t_1} > \frac{t'_2}{t_2}$	$\frac{t'_1}{t_1} = \frac{t'_2}{t_2}$	$\frac{t'_1}{t_1} < \frac{t'_2}{t_2}$
Proportional	$Y_1 \neq Y_2$ $t_1 = t_2$	$t'_1 < t'_2$ for $Y_1 < Y_2$	$\frac{t'_1}{t_1} = \frac{t'_2}{t_2}$	$t'_1 < t'_2$ for $Y_1 > Y_2$
Regressive	$Y_1 < Y_2$ $t_1 > t_2$	$\frac{t'_1}{t_1} < \frac{t'_2}{t_2}$	$\frac{t'_1}{t_1} = \frac{t'_2}{t_2}$	$\frac{t'_1}{t_1} > \frac{t'_2}{t_2}$

NOTE: Y is income, person 1,2;
t is effective tax rate in period 1 (initial period); and
t' is effective tax rate in period 2 (after tax changes).

3.2. Algebraic Statement of the Index Numbers

To facilitate the algebraic treatment of the preceding vertical and horizontal equity concepts, let there be j = 1,...,m ordered effective tax rate classes, and i = 1,...,n ordered economic income classes for the first taxpayer, and let there be k = 1,...,m effective tax rates classes and h = 1,...,n ordered economic income classes for the second taxpayer in each comparison.

Let N_i^j be the number of taxpayers in the *ij* the economic income - effective tax rate class which is to be compared to N_h^k, the number of taxpayers in the *hk* economic income - effective tax rate class. Note that higher subscripts and superscripts indicate higher economic income and higher effective tax rates, and that j = k =1 is the lowest negative tax rate class.

The unweighted vertical index numbers can now be specified. The total number of vertical comparisons is as follows.

$$V = \sum_{j=1}^{m} \sum_{i=1}^{n} \sum_{k=1}^{m} \sum_{h=1,\, h \neq i}^{n} [N_i^j N_h^k]$$

The unweighted progressive index is specified as follows.

$$\frac{1}{V} \sum_{j=1}^{m} \sum_{i=1}^{n} \sum_{k<j}^{m} \sum_{h<i}^{n} (N_i^j N_h^k)$$

$$+\frac{1}{V}\sum_{j=1}^{m}\sum_{i=1}^{n}\sum_{k>j}^{m}\sum_{h>i}^{n}(N_i^jN_h^k)$$

The unweighted proportional index number is given as follows.

$$\frac{1}{V}\sum_{j=1}^{m}\sum_{i=1}^{n}\sum_{h=1_{h\neq i}}^{n}(N_i^jN_h^j)$$

The unweighted regressive index is as follows.

$$\frac{1}{V}\sum_{j=1}^{m}\sum_{i=1}^{n}\sum_{k<j}^{m}\sum_{h>i}^{n}(N_i^jN_h^k) +$$

$$\frac{1}{V}\sum_{j=1}^{m}\sum_{i=1}^{n}\sum_{k>j}^{m}\sum_{h<i}^{n}(N_i^jN_h^k)$$

Next the weighted vertical index numbers are specified. Let Y_i^j be the average income in the cell containing taxpayers in the *ij* economic income - effective tax rate cell. We would prefer to use individual economic incomes in the empirical work below, but cannot due to computational limitations, so we use the average income in a cell. The theoretical development in section 4 is based on individual incomes.

To deal with a comparison between a positive and a negative tax rate, we take a ratio of the tax rate *class ranks* (or subscripts) rather that the ratio of the average tax rates in the classes themselves. To be consistent, we also use the ratio of class ranks in comparisons involving two positive tax rates as well as any comparison involving a zero tax rate.

The total of weighted vertical comparisons is specified as follows.

$$\Delta = \sum_{j=1}^{m}\sum_{i=1}^{n}\sum_{k=1}^{m}\sum_{h=1_{h\neq i}}^{n}[N_i^jN_h^k\,max(\frac{j}{k},\frac{k}{j})|Y_i^j - Y_h^k|]$$

The weighted fraction of taxpayers whose tax liability is progressively distributed is obtained by accumulating across comparisons in which the effective tax rate and economic income classes of the second group of taxpayers are *smaller* than those of the first group of taxpayers ($k < j$, $h < i$), and by accumulating across comparisons in which the effective tax rate and economic income of the second group of taxpayers are *greater* than the first group of taxpayers ($k > j$, $h > i$).

Since tax rates vary now in these progressive comparisons, we weight the accumulation by the ratio of the ranks of tax rate classes discussed above. Note that in forming the weight for the tax-rate ratio, we always divide the larger rank by the smaller rank of effective tax rates to insure that comparisons are treated symmetrically. Since the first group of comparisons always entails $k < j$, we form the weight as j/k; similarly, since the second group of progressive comparisons always entails $k > j$, we form the weight as k/j.

Thus, we have:

$$\frac{1}{\Delta}\sum_{j=1}^{m}\sum_{i=1}^{n}\sum_{k<j}^{m}\sum_{h<i}^{n}(N_i^jN_h^k)(\frac{j}{k})|Y_i^j - Y_h^k|$$

$$+\frac{1}{\Delta}\sum_{j=1}^{m}\sum_{i=1}^{n}\sum_{k>j}^{m}\sum_{h>i}^{n}(N_i^jN_h^k)\frac{k}{j}|Y_i^j - Y_h^k|$$

We obtain our measure of the extent to which taxes are proportionately distributed among pairs of taxpayers with different incomes by making all *possible* paired comparisons of taxpayers with different economic income ($i \neq h$), and then add up the number of such proportional comparisons from different effective tax rate classes to the total number of proportional comparisons. Normalization by the sum of weighted comparisons, Δ, provides the fraction or percentage of comparisons which is proportionately distributed:

$$\frac{1}{\Delta}\sum_{j=1}^{m}\sum_{i=1}^{n}\sum_{h=1_{h\neq i}}^{n}(N_i^j N_i^k |Y_i^j - Y_i^k|)$$

The fraction of taxpayers whose tax liability is regressively distributed is obtained by accumulating in the same manner as was used in calculating the fraction of taxpayers whose tax liability is progressively distributed, *except* that in accumulating for this index number, $k < j$ and $h > i$ in the first accumulation, and $k > j$ and $h < i$ in the second accumulation. That is, for the comparison to be *regressive*, the second group of taxpayers either has a lower effective tax rate and greater economic income, *or* higher effective tax rate and lower economic income than the first group of taxpayers. Since in the first accumulation the effective tax rate of the second group of taxpayers is lower than the first group of taxpayers, our tax rate weight for regressivity is formed as j/k. Similarly, since in the second accumulation the effective tax rate of the second group of taxpayers is greater than the first group of taxpayers, our tax rate weight is formed as k/j. We have, then:

$$\frac{1}{\Delta}\sum_{j=1}^{m}\sum_{i=1}^{n}\sum_{k<j}^{m}\sum_{h>i}^{n}(N_i^j N_h^k)\frac{j}{k}|Y_i^j - Y_h^k| +$$

$$\frac{1}{\Delta}\sum_{j=1}^{m}\sum_{i=1}^{n}\sum_{k>j}^{m}\sum_{h<i}^{n}(N_i^j N_h^k)\frac{k}{j}|Y_i^j - Y_h^k|$$

We move next to give formulae for the unweighted horizontal equity index numbers. The total count of horizontal comparisons is given by the following expression.

$$H=\sum_{j=1}^{m}\sum_{i=1}^{n}\sum_{k=1 k\neq j}^{m}[N_i^j N_i^k] + \sum_{j=1}^{m}\sum_{i=1}^{n}[N_i^j(N_i^j - 1)]$$

The unweighted horizontal inequity index is given by the following expression.

$$\frac{1}{H}\sum_{j=1}^{m}\sum_{i=1}^{n}\sum_{k=1_{k\neq j}}^{m}[N_i^j N_i^k]$$

The unweighted horizontal equity index is simply the difference between 1 and this fraction.

With respect to the algebraic statement of the weighted index of horizontal equity, recall first that the economic income of two taxpayers in the same horizontal comparison is close. That is, all analysis is done *within* each economic income class ($i = h$). As a consequence, there can be no differences in economic income to weight by, and only differences in effective tax rates are of interest in accumulating instances of horizontal inequity. We may then compactly define the fraction of taxpayers with the same economic income but different effective tax rates, instances of horizontal inequity as:

$$\frac{1}{\delta}\sum_{j=1}^{m}\sum_{i=1}^{n}\sum_{k=1_{k\neq j}}^{m}[N_i^j N_i^k\, max(\frac{j}{k},\frac{k}{j})]$$

where the sum of all horizontal comparisons, δ, is:

$$\delta=\sum_{j=1}^{m}\sum_{i=1}^{n}\sum_{k=1 k\neq j}^{m}[N_i^j N_i^k\, max(\frac{j}{k},\frac{k}{j})] + \sum_{j=1}^{m}\sum_{i=1}^{n}[N_i^j(N_i^j - 1)]$$

Note that the second term represents the number of comparisons in which the effective tax rates and economic income classes are the same (j = k), (i = h). A total of N_i^{j2} comparisons are possible; however, in order to avoid comparisons of individuals with themselves, there remains $N_i^j(N_i^j-1)$ comparisons. The difference between 1 and the unweighted horizontal inequity index is our measure of horizontal equity, and differs from that suggested by Wertz [1975] in that the *extent* of effective tax rate differences are accounted for by weighting using the ratio of relative ranks.

Finally, we specify formally the (unweighted) dynamic vertical index numbers. Let q be the number of classes of ratios of effective rates. Of course, this can differ in number and classification from the classifications of the effective rates themselves. Let D_i^j be the number of taxpayers in economic income class i and *change* in effective tax rate class j. Recall that the unweighted count of vertical comparisons is V. The dynamic progressive index number is given by the following formula.

$$\frac{1}{V}\sum_{j=1}^{q}\sum_{i=1}^{n}\sum_{k<j}^{q}\sum_{h<i}^{n}(D_i^jD_h^k)$$

$$+\frac{1}{V}\sum_{j=1}^{q}\sum_{i=1}^{n}\sum_{k>j}^{q}\sum_{h>i}^{n}(D_i^jD_h^k)$$

The dynamic proportional index number is given as follows.

$$\frac{1}{V}\sum_{j=1}^{q}\sum_{i=1}^{n}\sum_{h=1_{h\neq i}}^{n}(D_i^jD_h^j)$$

The dynamic regressive index is as follows.

$$\frac{1}{V}\sum_{j=1}^{q}\sum_{i=1}^{n}\sum_{k<j}^{q}\sum_{h>i}^{n}(D_i^jD_h^k)+$$

$$\frac{1}{V}\sum_{j=1}^{q}\sum_{i=1}^{n}\sum_{k>j}^{q}\sum_{h<i}^{n}(D_i^jD_h^k)$$

3.3. Properties of the Index Numbers

What properties should index numbers have? The answer to this question depends on what one is trying to measure, and what types of cardinal assumptions one wants to make. Moreover, it is natural to inquire both whether a property is satisfied by an index as well as whether it is part of some set of (minimal) sufficient conditions for deriving an index. Many index numbers in the literature have been characterized in the sense that necessary and sufficient conditions generating them have been found. Here we concentrate on necessary conditions. Complete characterizations of our index numbers can be found in section 4.

Index numbers pertaining to income inequality tend to be dependent only on after tax income, while index numbers pertaining to vertical and horizontal equity tend to be dependent on before and after tax income, before tax income and effective rates, or before tax income and tax liability. Thus, measures of horizontal and vertical equity have more complex ordinal and cardinal properties.

The first type of property that one might require is that the index depend only on the attributes of taxpayers that actually exist, and not on parts of the tax system that apply to nobody. This condition is satisfied by most index numbers, including ours.

Another important property one might require is that an index not change when various kinds of increasing transformations of variables are taken. For example, if every taxpayer's after tax income is increased by $1, one might require that an index number's value not change, since the relative distribution of taxpayers does not change. For index numbers of income inequality, which depend only on after tax income, such assumptions are evident. For more complex numbers that depend on more than one variable per taxpayer, the formulation of such properties is not as obvious, since there are several variables (before tax income, tax liability, effective tax rate, after tax income) on which the property might hold. (Henceforth, we refer to these variables as taxpayer attributes.) Knowledge of any two of these variables allows one to calculate the other two, so index numbers of equity can be phrased in terms of any two, but cardinality properties obviously differ depending on how the index is formulated.

In addition to these concerns, there are strong and weak cardinality properties. Strongest among these are independence with respect to any increasing (even nonlinear) transformation of any attribute for all taxpayers. Weaker is the assumption that an index is independent of any increasing linear transformation of an attribute, which implies that the index is scale - independent. Finally, the weakest assumptions are of independence with respect to certain types of increasing linear transformations of attributes, such as multiplication by a positive constant or addition of a constant. In all of these cases, it is natural to put the cells of "equal" incomes and "similar" tax rates used to define our index numbers through the same transformations as income and tax rates.

It is easy to check the properties of index numbers given an algebraic statement, so we leave to the reader the derivation of properties of index numbers listed in the appendix. Here we focus on our own index numbers. We choose to focus on before tax income and effective tax rate as the two taxpayer attributes of interest. The reason this choice is made is that it results in comparisons that can be classified, as explained previously. If instead of effective tax rates we chose to use tax liability, the classification of pairwise comparisons would not be as easy or natural. For example, a pairwise comparison between two taxpayers where one taxpayer's income and tax liability were higher than the other's only has the implication that marginal tax rates are positive; it might not be classified as progressive if the effective tax rate of the first is not higher than that of the second.

First consider the unweighted index numbers (of all varieties). These index numbers depend only upon classifications of comparisons, and not on the actual values of the attributes involved. Thus, it is easy to verify that these index numbers are independent of increasing (even nonlinear and discontinuous) transformations of each of the attributes separately. For the static index numbers, this means transformations of the before tax income scale and the effective rate scale. For dynamic index numbers, this means transformations of the before tax income scale and the ratio of plan Y to plan X effective rate scale (which, in fact, can be interpretted as transformations of the plan X and plan Y scales separately).

Now consider the weighted index numbers. For *given* effective rates, they are immune to increasing linear transformations in before tax income, but not to nonlinear transformations. They are also immune to multiplication of the effective rate scale by a positive constant, but not to addition of a constant or nonlinear transformations. In other words, the weighted measures are more cardinal than the unweighted measures. Does this make sense? The answer lies in the intuition given in the previous subsection. If we want to distinguish between comparisons of taxpayers with effective rates of 10% and 14% on the one hand, and 50% and 46% on the other, independence with respect to addition of constants (36, in this case) will not be satisfied. In other words, stronger assumptions of independence with respect to transformations are not always desirable, and are not an end in themselves.

There are many other types of axioms that might be placed on index numbers. For example, population decomposability requires that an index be additive across populations. As can easily be verified, this axiom is satisfied by many after tax income inequality measures, but not by the Gini coefficient nor any of the multivariate index numbers commonly used. Finally, one can check to see the effect on an index if the population is "cloned" so that each taxpayer is represented by two with the same attributes as the original taxpayer. Our vertical index numbers are immune to such an operation, while our horizontal numbers are not.

Many other properties of index numbers have been examined in the literature, and some index numbers, such as the Gini coefficient, possess multiple characterizations in terms of axioms.

Kiefer [1984] tries to provide a taxonomy for sorting index numbers by their properties, narrowing down the class of acceptable index numbers to his own (among those he considers), which he modestly calls K. Most important in this taxonomy is the property that an index *should not* be invariant to multiplication of all effective tax rates by a constant. The arguments for this property are, of course, quite subjective. One could also assert that shifting the pre-tax income distribution by adding a constant to all incomes should yield the same value of the index, since neither the relative pre-tax income distribution nor tax liabilities change with this shift. It is easy to see that Kiefer's index does not satisfy this property. The point is that there is an infinity of ways to classify index numbers, an infinity of properties (desirable or not) that they might satisfy, as well as an infinity of ways to characterize each index.

Differences between axioms underlying index numbers tend to be less relevant from the standpoint of empiricism, since the index numbers tend to be highly correlated and tend to reflect common trends. This was exposed in our earlier work, and will be discussed again in Section 6 below.

4. Characterizations of the Index Numbers

The basic data concerning taxpayers that is needed are their economic income and effective or average tax rates. These can be derived in a variety of ways, taking into account agents' reactions to a tax system, using elasticities drawn from the literature or even computable general equilibrium models. However, in the empirical implementation below, we conduct a purely positive study and use only actual income and taxes after an agent's reaction to the tax system.

In the systems of axioms below, one can employ assumptions on either an underlying social ordering over economic incomes and tax rates of the agents, or on an index itself; the latter object is interpreted as a numerical representation of the social order. In contrast with Atkinson's (and others following) approach, any index is considered to be a social welfare function (or a representation of a social ordering) itself, and thus conditions are imposed directly on this object. Since the index numbers considered are highly cardinal, the assumptions imposed on the ordering or index are cardinal as well. For this reason, as well as simplicity and brevity, assumptions are imposed on the index representations of social orderings rather than on orderings proper.

Let n be the number of taxpayers in the economy (finite and integer), and let A, $B \subseteq \mathbf{R}$ be the sets of admissible economic incomes and effective tax rates, respectively. An index is defined to be a mapping from $(AxB)^n$ to the real line, an evaluation of taxpayer characteristics. Every index induces a social ordering over $(AxB)^n$ in an obvious way.

To formulate the types of index numbers discussed in the previous section, the assumptions employ a partition of the n(n-1) paired comparisons of taxpayers into two groups by economic income: "equals" and "unequals". This partition is arbitrary in theory, convenient in practice, and is taken to be exogenous in this section. Similarly, an analogous partition is employed for effective tax rates: tax rates that are "similar" and tax rates that are "dissimilar". These partitions are used to formulate the assumptions. We assume that each element of these partitions is connected.

4.1. Axioms Generating the Unweighted Measures

First we focus on the simplest measures, the unweighted percentages. We use the notation I to represent an arbitrary index.

A0) I is symmetric: For any bijection $\sigma:\{1,...,n\} \rightarrow \{1,...,n\}$,
$I((Y_1,t_1),...,(Y_n,t_n)) = I((Y_{\sigma(1)},t_{\sigma(1)}),...,(Y_{\sigma(n)},t_{\sigma(n)}))$. That is, the order in which taxpayers appear in I is irrelevant.

A1) I is independent of increasing transformations in economic income and effective tax rates.

That is, let $f:A \rightarrow A$ and $g:B \rightarrow B$ be increasing and let $(Y_i,t_i),(f(Y_i),g(t_i)) \in AxB\ \forall i$.

Then $I((Y_1,t_1),...,(Y_n,t_n)) = I((f(Y_1),g(t_1)),...,(f(Y_n),g(t_n)))$.

It is easy to state this assumption in terms of an underlying social order, but such a restatement is unnecessary since subsequent assumptions are cardinal, and hence must be stated in terms of I. Index numbers satisfying A1 are completely independent of scale and units. Implicitly, the partitions defined earlier are subjected to the same transformations f and g. Of course, an interesting example is f and g are linear.

A2) There exists a constant c>0 such that the following holds. Let $[(Y_1,t_1),...,(Y_n,t_n)] \in (AxB)^n$ be ordered in descending order by tax rate, and let V be the number of paired comparisons between unequals.

α) Suppose that $Y_i > Y_{i-1}$, and Y_i and Y_{i-1} are not equals, while t_i and t_{i-1} are dissimilar. Then $I((Y_1,t_1),\ldots,(Y_i,t_{i-1}),(Y_{i-1},t_i),\ldots,(Y_n,t_n)) - I((Y_1,t_1),\ldots,(Y_n,t_n)) = c/V$

β) If either Y_i and Y_{i-1} are equals or t_i and t_{i-1} are similar, then $I((Y_1,t_1),\ldots,(Y_i,t_{i-1}),(Y_{i-1},t_i),\ldots,(Y_n,t_n)) = I((Y_1,t_1),\ldots,(Y_n,t_n))$.

This axiom implies that permuting two incomes so as to result in a new progressive comparison causes the same increase in welfare as any other such permutation, independent of the initial distribution as well. Moreover, permutations changing only comparisons between equals have no effect.

Theorem 1: There exists an index, the unweighted progressivity index, satisfying A0-A2. Moreover, any index satisfying A0-A2 is a linear transformation of the unweighted progressivity index.

Proof: That the unweighted progressivity index satisfies A0, A1 and A2 is an easy verification. Let I be any index satisfying A0, A1 and A2. Pick any $[(Y_1,t_1),\ldots,(Y_n,t_n)] \in (AxB)^n$ and rank the tax rates in descending order (without changing notation) using A0. Let $a \in (AxB)^n$ be $[(Y_1,t_1),\ldots,(Y_n,t_n)]$ with incomes exchanged so as to be in descending order, like tax rates. Using the types of neighbor exchanges described in A2, count the number of exchanges of type α in permuting from $[(Y_1,t_1),\ldots,(Y_n,t_n)]$ to a. Call this number q. Using A2 part α, $I[(Y_1,t_1),\ldots,(Y_n,t_n)] = I(a) - qc/V$ where c is the constant given in A2. Then:

$$\frac{I[(Y_1,t_1),\ldots,(Y_n,t_n)] - I(a)}{c} + 1 = 1 - \frac{q}{V}$$

By A1, I(a) is independent of $[(Y_1,t_1),\ldots,(Y_n,t_n)]$. It is easy to verify that the right hand side is the unweighted progressive index. Q.E.D.

Assumption A1 is, in fact, too strong and could be replaced by A1' which is given below. Similar theorems can be proved for the unweighted proportional and regressive index numbers, but they are repetitious. We move on to the unweighted horizontal index number.

A3) There exists a constant $c>0$ such that the following holds. Let $[(Y_1,t_1),\ldots,(Y_n,t_n)] \in (AxB)^n$ and let H be the number of paired comparisons between equals. Fix i, j, $i \neq j$ and suppose that i and j are equals, but have dissimilar tax rates. Let R_i be the set of taxpayers considered equal to i (including i) with tax rates similar to i, and let R_j be the analogous set for j. Then if $t^* \in B$, let a^* be the same as $[(Y_1,t_1),\ldots,(Y_n,t_n)]$ except replace the tax rates of taxpayers in $R_i \cup R_j$ by t^*. Then:

$$I(a^*) - I[(Y_1,t_1),\ldots,(Y_n,t_n)] = [\#R_i * \#R_j * c]/H$$

where $\#R_i$ is the cardinality of R_i. This axiom implies that changing the tax system so as to give taxpayers with approximately the same incomes approximately the same taxes improves welfare in proportion to the number of pairs affected.

Theorem 2: There exists an index, the unweighted horizontal equity index, satisfying A0, A1, and A3. Moreover, any index satisfying A0, A1, and A3 is a linear transformation of the unweighted horizontal equity index.

Proof: Again, that the unweighted horizontal equity index satisfies A0, A1, and A3 is an easy verification. Now let I be an index satisfying A0, A1, and A3. Fix $t^* \in B$ and fix any element A_i of the partition of A. Let $\bar{a}$ be the distribution $[(Y_1,t^*),\ldots,(Y_n,t^*)]$. Change $[(Y_1,t_1),\ldots,(Y_n,t_n)]$ to $\bar{a}$ using the process described in A3 for all taxpayers in A_i and repeat this process for each element A_i. The net result will be:

$$I(\bar{a}) - I((Y_1,t_1),\ldots,(Y_n,t_n)) = \frac{c}{H}\sum_i \{[\#A_i(\#A_i-1)] - \sum_i \#R_i^j(\#R_i^j-1)\}$$

where i indexes the partition element of A, $\#A_i$ is the cardinality of A_i, R_i^j is the set of taxpayers in A_i as well as tax partition element j, and $\#R_i^j$ is the cardinality of R_i^j. Hence:

$$I((Y_1,t_1),\ldots,(Y_n,t_n)) = I(\bar{a}) - \frac{c}{H}\sum_i \{[\#A_i(\#A_i-1)] - \sum_i \#R_i^j(\#R_i^j-1)\}$$

$$\frac{I((Y_1,t_1),\ldots,(Y_n,t_n))}{c} + 1 = 1 - \frac{1}{H}\{\sum_i [\#A_i(\#A_i-1)] - \sum_i \#R_i^j(\#R_i^j-1)\}$$

$$= \frac{1}{H}\sum_i \#R_i^j(\#R_i^j-1)$$

The last expression is the unweighted horizontal equity index. Q.E.D.

As is evident, a similar argument holds for the unweighted horizontal inequity index. The unweighted dynamic progressive index number has a different domain. For each taxpayer, taking economic income as given, it evaluates two tax systems, and hence is a function of two effective tax rates. Call the plan X effective tax rates t_i and t_j for taxpayers i and j, and the plan Y effective tax rates t'_i and t'_j for taxpayers i and j. A comparison between i and j is said to reflect higher progressivity under Y as opposed to X if i and j are not equals (comparing incomes), $Y_i > Y_j$, and

$$\frac{t'_i}{t'_j} > \frac{t_i}{t_j}.$$

A characterization of the dynamic progressive index can therefore be found using axioms A0-A2 and theorem 1 by replacing all occurrences of t_i by

$$\frac{t'_i}{t_i}.$$

Similar characterizations can be found for the dynamic proportional and regressive index numbers.

4.2. Axioms Generating the Weighted Measures

The characterization of the weighted progressive index is, in fact, quite similar to that of the unweighted progressive index.

A1') If two distributions $[(Y_1,t_1),\ldots,(Y_n,t_n)]$, $[(Y'_1,t'_1),\ldots,(Y'_n,t'_n)] \in (AxB)^n$ have the property that $Y_1 \geq \ldots \geq Y_n$, $t_1 \geq \ldots \geq t_n$, $Y'_1 \geq \ldots \geq Y'_n$, $t'_1 \geq \ldots \geq t'_n$, then $I((Y_1,t_1),\ldots,(Y_n,t_n)) = I((Y'_1,t'_1),\ldots,(Y'_n,t'_n))$.

Clearly, this axiom could be stated using only ordinal constructs, but it will be used in conjunction with a cardinal axiom below. The assumption states that if all vertical comparisons in a distribution are progressive, its value under the index is the same as that of any other such distribution, regardless of scale. It can be viewed as a limited form of A1, and in fact it could be used in place of A1 in Theorem 1.

A2') There exists a constant c>0 such that the following holds. Let $[(Y_1,t_1),\ldots,(Y_n,t_n)] \in (AxB)^n$ be ordered in descending order by tax rate.

α) Suppose that $Y_i > Y_{i-1}$, and Y_i and Y_{i-1} are not equals, while t_i and t_{i-1} are dissimilar. Then

$$I[(Y_1,t_1),\ldots,(Y_i,t_{i-1}),(Y_{i-1},t_i),\ldots,(Y_n,t_n)] - I[(Y_1,t_1),\ldots,(Y_n,t_n)] = c\frac{|Y_i - Y_{i-1}|\frac{t_i}{t_{i-1}}}{\Delta},$$

where Δ is the weighted sum of vertical comparisons as defined in Section 3.

β) If either Y_i and Y_{i-1} are equals or t_i and t_{i-1} are similar, then
$I[(Y_1,t_1),\ldots,(Y_i,t_{i-1}),(Y_{i-1},t_i),\ldots,(Y_n,t_n)] = I[(Y_1,t_1),\ldots,(Y_n,t_n)]$.

Similar to A2, this axiom implies that permuting incomes so as to result in a new progressive comparison causes an increase in progressivity proportional to the product of the income difference and the tax ratio. Moreover, other types of permutations have no effect.

Theorem 3: There exists an index, the weighted progressivity index, satisfying A0, A1', and A2'. Moreover, any index satisfying A0, A1', A2' is a linear transformation of the weighted progressivity index.

Proof: It is easy to verify that the weighted progressivity index satisfies A0 and A1'. Verification of A2' requires a routine calculation, observing that the change in the sums caused by permuting the i and i-1 incomes offset for each term that is multiplied by

$$\frac{t_j}{t_k}.$$

The proof that and index satisfying A0, A1', A2' is a linear transformation of the weighted progressivity index is almost exactly the same as the proof of Theorem 1. Q.E.D.

The weighted proportional and regressive index numbers can be characterized in a similar fashion.

A1'') If two distributions $[(Y_1,t_1),\ldots,(Y_n,t_n)]$ have the property that: for every i, j, with Y_i considered equal to Y_j, t_i is similar to t_j; and for all i, j, with Y'_i considered equal to Y'_j, t'_i is similar to t'_j. Then $I((Y_1,t_1),\ldots,(Y_n,t_n)) = I((Y'_1,t'_1),\ldots,(Y'_n,t'_n))$.

As with A1, this axiom could be stated using only ordinal constructs, but it will be used in conjunction with a cardinal axiom below. The assumption states that if all horizontal comparisons in two distributions are equitable, their value under the index is the same, regardless of scale. It can be viewed as another limited form of A1, and in fact could be used in place of A1 in Theorem 2.

A3') There exists a constant c>0 such that the following holds. Let $[(Y_1,t_1),\ldots,(Y_n,t_n)] \in (AxB)^n$ Fix i, j, $i \neq j$ and suppose that i and j are equals, but have dissimilar tax rates. Let R_i be the set of taxpayers considered equal to i (including i) with tax rates similar to i, and let R_j be the analogous set for j. Then if $t^* \in B$, let a^* be the same as $[(Y_1,t_1),\ldots,(Y_n,t_n)]$ except replace the tax rates of taxpayers in $R_i \cup R_j$ by t^*. Then $I(a^*)*\delta(a^*) - I[(Y_1,t_1),\ldots,(Y_n,t_n)]*\delta[(Y_1,t_1),\ldots,(Y_n,t_n)] = \#R_i\#R_j*c$ where $\#R_i$ is the cardinality of R_i, $\delta(a^*)$ is the weighted sum of all horizontal comparisons in a^* and $\delta[(Y_1,t_1),\ldots,(Y_n,t_n)]$ is the weighted sum of all horizontal comparisons in $[(Y_1,t_1),\ldots,(Y_n,t_n)]$.

Similar to A3, this axiom implies that changing the tax system so as to give taxpayers with approximately the same incomes approximately the same taxes gives a weighted difference proportional to the number of pairs affected.

Theorem 4: There exists an index, the weighted horizontal equity index, satisfying A0, A1'', and A3'. Moreover, any index satisfying A0, A1'', and A3' is a linear transformation of the weighted horizontal equity index.

Proof: Almost the same as the proof of Theorem 2.

5. Data Sources and Limitations

The data used to measure over time the vertical and horizontal equity of the U.S. Federal individual income taxes are from publicly available anonymous samples of individual income tax returns created annually by the Statistics of Income Division (SOI) of the Internal Revenue Service, and provided periodically to the National Archives for sale as public use tapes. These data are used by the Internal Revenue Service in their annual publication *Statistics of Income: Individual Income Tax Returns*. A sample of this file is typically provided to the Office of Tax Analysis (OTA), U.S. Treasury Department to be used in conjunction with the Department's microsimulation model of the Federal individual income tax. (This model is used to project the revenue changes from tax reform proposals.) The OTA file is frequently modified further by the addition of imputations for data not contained on the various Federal individual income tax returns, and is reweighted to allow the data to be used to *project* income levels to future time periods.

The SOI files for 1978-1987 are somewhat different from those for 1966-77[4] as they refer to tax returns for the year in question, and thus do not contain returns simply filed in the year in question. Returns filed for other years such as amended returns generally represent less than 1% of the tax paying population.

As is well known, information on the tax position of individuals and families is generally not available from such data sources as the *Current Population Survey* or *CPS*. The *CPS* contains much richer information on transfer income to low income units, and uses a household unit of measurement which differs from that used to administer the Internal Revenue Code. The SOI files do not have information about low income individuals as many are not required to file, and are not in the tax system, and thus have certain limitations.

Since variations in effective rates over time are the primary focus of this paper, we choose to use the richest source on tax information, and sacrifice, as a consequence, somewhat richer information on economic income which is available from such sources as the *CPS*.

Both the *SOI* and *CPS* fail to reflect various types of nonmarket income captured in the national income and product accounts. Personal income, as defined for those purposes, is substantially broader than adjusted gross income, total money income, or the concept of economic income we are able to construct from the available data files. Our income concepts do not capture, for example, interest on state and local bonds, which is tax exempt for federal tax purposes and therefore not reported on the tax forms.

The economic income concept derived from the SOI data averages 85% of personal income, on a national income accounts basis, after the subtraction of government transfer payments, and between 70 to 80% of overall BEA income. Wages in our concept of economic income are 95% of BEA reported wages. Our economic income concept includes wages and salaries, interest and dividend income without regard to the dividend exclusion, the various types of business income from farming, sole proprietorships, rents, and royalties, long and short-term capital gains without regard to any exclusions, gains from installment sales, and all reported pension income. Table 5-1 displays aggregate characteristics of our data for the period 1966-87, and Table 5-2 displays the components of economic income for each year. For each year we have sought to use as broad a definition of economic income as permitted by the data collected by the tax administration system, but have not attempted to make imputations for exempt or excluded items from the tax system or income which might otherwise be attributable to taxpayers.

With between 100,000 to 200,000 observations per year, calculation of the vertical equity measures would require 10^{10} of comparisons of taxpayers (recall that there are n(n-1) comparisons to make) for each of 16 years; this would clearly be too burdensome computationally. Accordingly, the data was grouped into 114 effective tax rate classes,

[4]These files were purchased from the Graduate School of Business, University of Michigan.

and 25 economic intervals.[5]

The data were also grouped into the five statutory filing classes (all filers, head of household, married filing separately, married filing jointly, and single), and grouped on the basis of whether or not the taxpayer chose to itemize his deductions.

The effective tax rate classes utilized were 1% apart, and covered the negative domain as well. The income intervals were chosen each year so that each interval corresponded to 4 percent of the (weighted) number of tax returns each year. It should be emphasized that the intervals in our analysis used are quite different from those used and publicly reported by OTA. Generally, our income classes are much finer in the lower and middle ranges of the income distribution. The Treasury groupings focus attention on higher income taxpayers, e.g. those with income in excess of $100,000. Clearly, for distributional and general statistical analysis, using intervals that reflect the population of taxpayers is the appropriate classification scheme. A prerequisite to obtaining annual classifications by 4 percentage points is that the cumulative distribution of each file had to be calculated and recorded.

6. Empirical Results

During the period 1966-1987, the Federal individual income tax has been significantly revised several times. In 1968, a 10% surcharge was imposed to help finance the Vietnam War. In 1969, a variety of tax shelters (including real estate tax shelters) were either eliminated or substantially restricted. The 1969 act also imposed the alternative minimum tax on individuals for the first time. In 1976, the maximum tax rate on earned income was reduced to 50%, and major changes were made to estate and gift taxes. In 1978, the capital gains exclusion was increased from 50% to 60%, and the earned income tax credit was both liberalized and made fully refundable. The 1981 Economic Recovery Act reduced individual income taxes 25% over a four year period, and reduced the maximum tax rate on unearned income to 50%; this had the effect of imposing a maximum tax rate of 20% on capital gains. In 1982, some of the more generous features of the 1981 capital cost recovery provisions were eliminated.

In 1986, the federal tax system was substantially overhauled by the elimination of any distinction between capital gains and other sources of income, the limitations placed on the amounts of active, positive income which could be offset by negative, passive losses, the phased reduction over time of the top marginal tax rate from 50% to 28/31%, and the doubling of the value of personal exemptions. Because our data ends at 1987, we can not observe the final implications of the Tax Reform Act of 1986; however, we can measure the movement to the transitional tax tables for 1987 and the broadened definitions of income.

Of interest below is how the index numbers for progressivity and horizontal equity have changed with these substantial changes in personal tax law.

6.1. Overall Index Number Results and by Type of Filing Unit

Table 6-1 provides the results of calculating the horizontal and vertical equity indices developed above along with some additional common index numbers using the SOI data. The Gini coefficient is standard, while the coefficient of variation of effective tax rates calculates this coefficient for each income class, and averages these numbers over income classes. Panel A displays the results for all filing units. As is immediately apparent, the U.S. federal

[5]Even this reduction in the dimensionality of the computational problem requires millions of comparisons since the ij matrix has 2850 cells and needs to be compared to 2849 other cells, which implies better than 8 million comparisons. Fortunately, many cells are empty since there are not low income taxpayers with high effective tax rates, etc. The algorithm developed scans and dynamically keeps track of the relative position of non-zero cells in order to achieve computational efficiency.

Table 5-1: Comparison of Economic Income Concept Used to Adjusted Gross Income and BEA Personal Income Concepts ($ billions)

(1) YEAR	(2) ESTIMATE OF NUMBER TAX RETURNS	(3) SAMPLE ECONOMIC INCOME	(4) SAMPLE AGI	(5) BEA PERS. INCOME	(6) SAMPLE WAGES	(7) BEA WAGES	(8) SAMPLE/ BEA WAGES %	(9) ECON.INC./ BEA PERS INC %	(10) AGI/ BEA Inc %
1966	70,159,160	$482.8	$468.5	$600.8	$379.9	$400.3	94.9	80.4	78.0
1967	71,652,056	524.4	504.8	644.5	411.3	428.9	95.9	81.4	78.3
1968	73,744,100	581.8	555.5	707.2	451.6	471.9	95.7	82.3	78.5
1969	75,833,700	623.6	603.2	772.9	497.1	518.3	95.9	80.7	78.0
1970	74,287,800	653.4	631.9	831.8	531.8	551.5	96.4	78.6	76.0
1971	74,576,600	696.0	672.6	894.0	565.1	583.9	96.8	77.9	75.2
1972	77,596,992	775.9	746.8	981.6	621.1	637.8	97.4	79.0	76.1
1973	80,665,600	847.8	818.0	1107.7	687.2	708.7	97.0	76.5	73.8
1974	83,382,384	924.5	909.3	1210.1	759.8	772.6	98.3	76.4	75.1
1975	82,195,576	961.6	933.6	1313.4	794.4	814.6	97.5	73.2	71.1
1976	84,672,576	1099.4	1053.9	1451.4	881.0	899.5	97.9	75.7	72.6
1977	86,634,408	1163.4	1158.5	1607.5	969.4	993.9	97.5	72.4	72.1
1978	89,416,472	1310.3	1296.2	1812.4	1084.5	1119.3	96.9	72.3	71.5
1979	91,435,584	1490.1	1450.4	2034.0	1216.2	1252.1	97.1	73.3	71.3
1980	92,304,024	1643.3	1592.1	2258.5	1331.1	1372.0	97.0	72.8	70.5
1981	93,598,736	1756.0	1746.8	2520.5	1463.6	1510.3	96.9	69.7	69.3
1982	93,477,544	1938.8	1824.5	2670.8	1540.3	1586.1	97.1	72.6	68.3
1983	94,427,064	2075.5	1914.8	2838.6	1619.8	1676.6	96.6	73.1	67.5
1984	97,392,992	2449.0	2104.5	3108.7	1775.3	1838.6	96.6	78.8	67.7
1985	101,661,136	2506.4	2305.8	3327.0	1928.3	1974.9	97.6	75.3	69.3
1986	103,046,456	2707.7	2483.0	3534.3	2031.3	2089.1	97.2	76.6	70.3
1987	106,995,480	2953.4	2773.1	3780.0	2163.2	2248.4	96.2	78.1	73.4

Source: authors' calculations from SOI files.

Table 5-2: Components of Economic Income by Year

Component Of Income	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80	81	82	83	84	85	86	8'
Wages And Salaries	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X
Interest Income	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X
Gross Dividends	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X
Gross Business Or Profession Income	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X
Short-Term Capital Gains	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X
Long-Term Capital Gains	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X
Farm Income	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X
Rental Income	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X
Royalty Income	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X
Partnership Income	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X
Sbc Income	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X
Estate And Trust Income	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X
Capital Gain Distributions							X	X	X		X	X	X	X	X	X	X	X	X	X	X	X
Taxable Portion Of Pensions	X	X				X	X	X	X	1	1	X 1	X	X	X	X	X	X	X	X	X	X
Fully Taxable Pensions							X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X
Gross Pensions			X	X	X					X	X	X	X	X	X	X	X	X	X	X		
Alimony	*2	*2	*2	*2	*2		X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X
State Income Tax Refunds	*2	*2	*2	*2	*2	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X
Premature Distribution From Ira'S And Keogh Plans	*2	*2	*2	*2	*2	*2	*2	*2	*2	X	X		X	X	X	X	X	X	X	X	X	X
Miscellaneous Income				X	X	X	X	X	X	X	X	X	X	X	X	X	X	X				
Supplemental Schedule Gains							X	X	X				X	X	X	X	X	X	X	X	X	X
Ordinary Gains	X	X	X	X	X																	
Other Gains	X	X	X	X	X		X	X												X		
Overseas					X		X		X		X	X	X	X	X				X	X	X	X
Unemployment														X	X	X	X	X	X	X	X	X
Social Security																			X	X	X	X

Notes: (1) Shown separately but also included in gross pension;
(2) included in miscellaneous income.
X denotes available

individual income tax contains, simultaneously, significant progressive, regressive, and proportional elements.[6] Over the time period in question, 1966-1987, as much as 94.1% of the weighted, paired comparisons displayed progressivity.

It is interesting to note that in the year immediately following several of the tax reform measures, progressivity increased, and then declined. For example, progressivity declined in 1970 after the Tax Reform Act of 1969; it increased in 1976 and decreased in 1977 and 1978; it increased in 1979 and decreased in 1980. The 1981 Act was different in that progressivity fell from 1980 to 1981, and rose in 1982. Progressivity again increased in 1986, and fell in 1987. We may conjecture that part of the lagged decline in progressivity reflects behavioral reactions of taxpayers and tax advisors to measures designed to "tighten" the tax system, as well as general economic trends which may affect the distribution of income. An important caution attached to this observation is that these various tax bills had phase-in provisions, while some of the changes were also anticipated by taxpayers, so any statements about timing are necessarily imprecise. Similar patterns can be found in the horizontal equity numbers, although the patterns in these numbers are less evident.[7]

While the progressivity measure displays consistently high levels of progressivity in the U.S. federal individual income tax, there is substantial evidence of horizontal inequity. That is, over the period in question, between 78.9% and 87.8% of the weighted horizontal comparisons displayed horizontal *inequity*.

It is apparent that, overall, the U.S. federal individual income tax has been vertically progressive over a fairly long period of time, and that the level of progressivity has weathered fairly disparate economic circumstances. Also, it is apparent that, overall, the U.S. federal individual income tax has been substantially horizontally inequitable over a fairly long period of time, and that the level of horizontal inequity has remained high through various economic situations.

The high level of progressivity evident in Panel A: All Filers in Table 6-1 is generally apparent in the results for the other filing units. In general, the weighted percentage of paired comparisons displaying progressivity is in the 80-92% range for the various types of filing units. The results for horizontal inequity, however, are quite different.

The single filing units display considerably greater amounts of horizontal equity. From 1975-78, the fraction of horizontal comparisons displaying horizontal equity for heads of household always *exceeded* 20% and climbed to over 30% in 1978, and then fell off dramatically until 1981 when almost 30%. of the horizontal comparisons displayed horizontal equity. Since 1982, however, horizontal equity has averaged about 10% for heads of households.

The lowest horizontal equity scores occur for taxpayers who are married and filing jointly. In 1976, 10.6% of these units showed horizontal equity, while in 1987, only 8.6% showed horizontal equity.

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[6]This result is consistent with the earlier results of Wertz(1975,8), Berliant and Strauss (1983,5), and Kiefer and Nelson(1986).

[7]See, however, the following section which utilizes time series regressions to examine associations between vertical and horizontal equity.

Table 6-1: Progressivity and Horizontal Equity Index Values: 1966-87

Panel A: All Filers

YR	PROG %	REGR %	PROP %	EQUITY %	INEQ %	GINI	CV RATE
1966	0.885	0.089	0.026	0.175	0.825	0.452	0.273
1967	0.886	0.089	0.025	0.174	0.826	0.457	0.273
1968	0.892	0.088	0.020	0.158	0.842	0.462	0.301
1969	0.891	0.090	0.019	0.148	0.852	0.458	0.319
1970	0.888	0.088	0.023	0.179	0.821	0.441	0.277
1971	0.905	0.072	0.022	0.195	0.805	0.448	0.279
1972	0.910	0.067	0.023	0.211	0.789	0.451	0.272
1973	0.899	0.080	0.021	0.199	0.800	0.456	0.352
1974	0.885	0.095	0.020	0.187	0.813	0.463	0.416
1975	0.881	0.096	0.023	0.200	0.800	0.466	0.477
1976	0.901	0.079	0.020	0.204	0.796	0.456	0.307
1977	0.868	0.114	0.019	0.211	0.789	0.476	0.511
1978	0.886	0.095	0.019	0.198	0.802	0.465	0.493
1979	0.941	0.048	0.011	0.134	0.866	0.444	0.340
1980	0.939	0.050	0.011	0.129	0.871	0.449	0.348
1981	0.883	0.101	0.016	0.149	0.851	0.466	0.543
1982	0.920	0.067	0.013	0.119	0.881	0.475	0.358
1983	0.892	0.093	0.015	0.108	0.892	0.500	0.405
1984	0.881	0.101	0.019	0.111	0.889	0.493	0.357
1985	0.916	0.070	0.013	0.097	0.903	0.486	0.376
1986	0.918	0.069	0.013	0.097	0.903	0.509	0.390
1987	0.903	0.086	0.011	0.079	0.921	0.511	0.444

Panel B: Single

YR	PROG %	REGR %	PROP %	EQUITY %	INEQ %	GINI	CV RATE
1966	0.947	0.032	0.021	0.459	0.541	0.486	0.206
1967	0.943	0.037	0.020	0.464	0.536	0.497	0.181
1968	0.936	0.046	0.018	0.448	0.552	0.495	0.225
1969	0.951	0.035	0.014	0.417	0.583	0.485	0.193
1970	0.939	0.035	0.026	0.526	0.474	0.475	0.183
1971	0.949	0.027	0.024	0.572	0.428	0.489	0.220
1972	0.943	0.031	0.026	0.625	0.374	0.495	0.151
1973	0.939	0.041	0.020	0.564	0.435	0.482	0.348
1974	0.926	0.055	0.018	0.523	0.477	0.487	0.434
1975	0.919	0.058	0.023	0.522	0.478	0.481	0.495
1976	0.925	0.051	0.024	0.563	0.437	0.470	0.244
1977	0.888	0.091	0.021	0.526	0.474	0.481	0.522
1978	0.917	0.063	0.019	0.499	0.501	0.461	0.488
1979	0.963	0.021	0.016	0.506	0.494	0.442	0.147
1980	0.958	0.026	0.015	0.497	0.503	0.450	0.194
1981	0.922	0.065	0.012	0.364	0.636	0.461	0.543
1982	0.944	0.040	0.015	0.437	0.563	0.474	0.238
1983	0.906	0.076	0.018	0.389	0.611	0.499	0.320
1984	0.896	0.082	0.022	0.379	0.621	0.493	0.257
1985	0.919	0.061	0.019	0.368	0.632	0.486	0.263
1986	0.905	0.076	0.019	0.362	0.638	0.503	0.273
1987	0.886	0.095	0.018	0.294	0.706	0.508	0.367

Panel C: Married Filing Jointly

YR	PROG %	REGR %	PROP %	EQUITY %	INEQ %	GINI	CV RATE
1966	0.910	0.068	0.021	0.107	0.893	0.342	0.231
1967	0.915	0.064	0.020	0.106	0.894	0.347	0.225
1968	0.918	0.065	0.017	0.091	0.909	0.352	0.247
1969	0.913	0.071	0.016	0.087	0.913	0.343	0.268
1970	0.905	0.075	0.020	0.097	0.903	0.337	0.241
1971	0.911	0.069	0.020	0.106	0.893	0.346	0.229
1972	0.920	0.061	0.019	0.106	0.894	0.344	0.235
1973	0.900	0.081	0.018	0.101	0.899	0.345	0.275
1974	0.882	0.100	0.017	0.094	0.906	0.350	0.310
1975	0.881	0.101	0.018	0.105	0.894	0.363	0.376
1976	0.901	0.083	0.015	0.104	0.896	0.351	0.254
1977	0.860	0.126	0.014	0.108	0.892	0.365	0.396
1978	0.873	0.113	0.014	0.098	0.902	0.355	0.401
1979	0.949	0.043	0.008	0.085	0.915	0.329	0.276
1980	0.945	0.046	0.008	0.078	0.922	0.335	0.284
1981	0.869	0.118	0.012	0.079	0.921	0.365	0.450
1982	0.924	0.065	0.011	0.079	0.921	0.376	0.306
1983	0.886	0.101	0.013	0.080	0.920	0.406	0.347
1984	0.872	0.111	0.017	0.075	0.925	0.401	0.302
1985	0.914	0.073	0.012	0.075	0.925	0.387	0.311
1986	0.920	0.068	0.012	0.075	0.925	0.416	0.319
1987	0.912	0.078	0.010	0.086	0.914	0.404	0.339

Panel D: Married Filing Separately

YR	PROG %	REGR %	PROP %	EQUITY %	INEQ %	GINI	CV RATE
1966	0.858	0.102	0.040	0.218	0.782	0.421	0.304
1967	0.846	0.117	0.036	0.204	0.796	0.416	0.269
1968	0.896	0.081	0.023	0.169	0.831	0.415	0.268
1969	0.817	0.164	0.019	0.181	0.819	0.461	0.289
1970	0.879	0.094	0.027	0.173	0.827	0.405	0.282
1971	0.897	0.076	0.027	0.274	0.726	0.416	0.213
1972	0.875	0.095	0.030	0.213	0.787	0.394	0.273
1973	0.869	0.108	0.023	0.236	0.764	0.420	0.307
1974	0.798	0.171	0.031	0.225	0.775	0.429	0.343
1975	0.863	0.104	0.032	0.252	0.748	0.454	0.377
1976	0.866	0.114	0.020	0.210	0.790	0.407	0.290
1977	0.790	0.188	0.022	0.247	0.752	0.492	0.419
1978	0.889	0.086	0.025	0.209	0.791	0.445	0.349
1979	0.925	0.054	0.021	0.229	0.771	0.418	0.199
1980	0.918	0.065	0.017	0.228	0.772	0.419	0.221
1981	0.878	0.108	0.014	0.185	0.815	0.443	0.473
1982	0.889	0.093	0.018	0.279	0.720	0.505	0.238
1983	0.867	0.110	0.023	0.194	0.806	0.564	0.275
1984	0.814	0.154	0.032	0.296	0.704	0.555	0.232
1985	0.911	0.070	0.019	0.212	0.788	0.572	0.303
1986	0.836	0.145	0.019	0.193	0.807	0.624	0.295
1987	0.752	0.228	0.019	0.189	0.810	0.587	0.265

Panel E: Head of Household

YR	PROG %	REGR %	PROP %	EQUITY %	INEQ %	GINI	CV RATE
1966	0.896	0.082	0.023	0.158	0.842	0.353	0.222
1967	0.862	0.114	0.024	0.136	0.864	0.332	0.225
1968	0.919	0.061	0.019	0.115	0.885	0.329	0.229
1969	0.920	0.061	0.019	0.171	0.829	0.370	0.285
1970	0.885	0.092	0.024	0.176	0.824	0.347	0.238
1971	0.908	0.068	0.024	0.157	0.843	0.341	0.294
1972	0.905	0.071	0.024	0.172	0.828	0.339	0.292
1973	0.897	0.077	0.026	0.180	0.819	0.355	0.206
1974	0.871	0.105	0.024	0.151	0.849	0.340	0.332
1975	0.882	0.083	0.035	0.233	0.767	0.351	0.344
1976	0.891	0.072	0.038	0.313	0.687	0.366	0.332
1977	0.864	0.110	0.026	0.252	0.748	0.383	0.424
1978	0.907	0.059	0.034	0.324	0.676	0.369	0.325
1979	0.983	0.014	0.003	0.106	0.894	0.337	0.525
1980	0.984	0.013	0.003	0.115	0.885	0.347	0.518
1981	0.913	0.058	0.029	0.295	0.705	0.362	0.358
1982	0.967	0.029	0.004	0.099	0.901	0.359	0.440
1983	0.963	0.033	0.004	0.081	0.919	0.382	0.571
1984	0.968	0.027	0.006	0.097	0.903	0.372	0.475
1985	0.981	0.016	0.003	0.098	0.902	0.379	0.724
1986	0.981	0.016	0.003	0.096	0.904	0.402	0.730
1987	0.966	0.032	0.002	0.116	0.884	0.393	0.988

It is well-known that taxpayers who choose to itemize do so in order to reduce their net tax liability. The question naturally arises whether this discretion accorded to taxpayers, and the attending incentives contained in the various itemized deductions provided over the years, materially impacts on the vertical and horizontal equity of the tax structure. Some evidence on this question is contained in Table 6-2, which displays the results of calculating the vertical and horizontal index values by choice of itemization.

It is immediately apparent that non-itemizers display substantially greater horizontal equity than itemizers. For itemizers, the percentage of weighted, paired comparisons never displays more than 9% of horizontal equity. On the other hand, non-itemizers display horizontal equity of at *least* 15% in all but four years of those under consideration, and *exceeded* 30% in eleven of the years under consideration. Evidently, the opportunity to itemize deductions is an important source of horizontal inequity in our tax structure, and evidently a *price* of the incentives which such itemized deductions are generally thought to provide.

Table 6-2: Vertical and Horizontal Equity Scores: Itemizers and Standard Filers: 1966-87

	Prog %		Regress %		Proport %		Equity %		Inequity %	
Year	Itemiz	Stand	Itemiz	Stand	Itemiz	Stand	Itemiz	Stand	Itemiz	Stand
1966	0.888	0.890	0.091	0.073	0.021	0.037	0.086	0.314	0.913	0.686
1967	0.889	0.888	0.090	0.075	0.020	0.037	0.085	0.313	0.915	0.687
1968	0.892	0.896	0.091	0.074	0.017	0.030	0.073	0.300	0.927	0.700
1969	0.881	0.909	0.102	0.064	0.016	0.027	0.070	0.303	0.930	0.697
1970	0.872	0.901	0.106	0.061	0.022	0.038	0.074	0.393	0.925	0.607
1971	0.883	0.916	0.097	0.049	0.020	0.035	0.082	0.363	0.918	0.637
1972	0.897	0.914	0.084	0.053	0.019	0.033	0.080	0.357	0.920	0.643
1973	0.876	0.907	0.106	0.063	0.018	0.030	0.079	0.334	0.921	0.666
1974	0.864	0.889	0.119	0.082	0.016	0.028	0.077	0.311	0.923	0.689
1975	0.846	0.877	0.138	0.084	0.016	0.038	0.069	0.317	0.931	0.683
1976	0.893	0.877	0.092	0.090	0.015	0.033	0.072	0.320	0.928	0.680
1977	0.809	0.854	0.178	0.112	0.013	0.033	0.067	0.301	0.933	0.699
1978	0.847	0.864	0.139	0.104	0.013	0.033	0.063	0.295	0.937	0.705
1979	0.924	0.919	0.063	0.065	0.013	0.016	0.064	0.172	0.936	0.828
1980	0.918	0.919	0.070	0.065	0.012	0.016	0.060	0.174	0.940	0.826
1981	0.846	0.866	0.141	0.108	0.013	0.026	0.054	0.237	0.946	0.763
1982	0.881	0.909	0.102	0.074	0.016	0.017	0.061	0.172	0.939	0.828
1983	0.833	0.876	0.150	0.104	0.018	0.019	0.062	0.148	0.938	0.852
1984	0.805	0.872	0.171	0.102	0.023	0.025	0.059	0.161	0.941	0.839
1985	0.862	0.898	0.119	0.083	0.019	0.019	0.060	0.128	0.940	0.872
1986	0.875	0.878	0.106	0.103	0.018	0.019	0.060	0.126	0.940	0.873
1987	0.845	0.865	0.140	0.117	0.015	0.018	0.067	0.088	0.933	0.912

6.2. Graphical Analysis of Progressivity and Horizontal Equity Index Number Scores

Another way to display the empirical results is to graph the various index number scores across time. Figure 6-1 displays the progressivity scores for all filers for the period 1966-1987, and Figure 5-2 displays the horizontal inequity scores for the same period. The graphical display of the progressivity and horizontal inequity scores show rather dramatically that horizontal inequity has been growing since 1977, and that vertical progressivity has increased and decreased over time.

Figure 6-3 displays the calculated Gini coefficient of income for after-tax income for all filers for the same period, while Figure 6-4 displays the coefficient of variation in effective tax rates averaged across income groups. Of immediate interest is that income inequality has generally increased since 1979, and that the coefficient of variation in effective tax rates has increased and decreased dramatically over time.

Figure 6-1: Progressivity of Federal Individual Income Tax

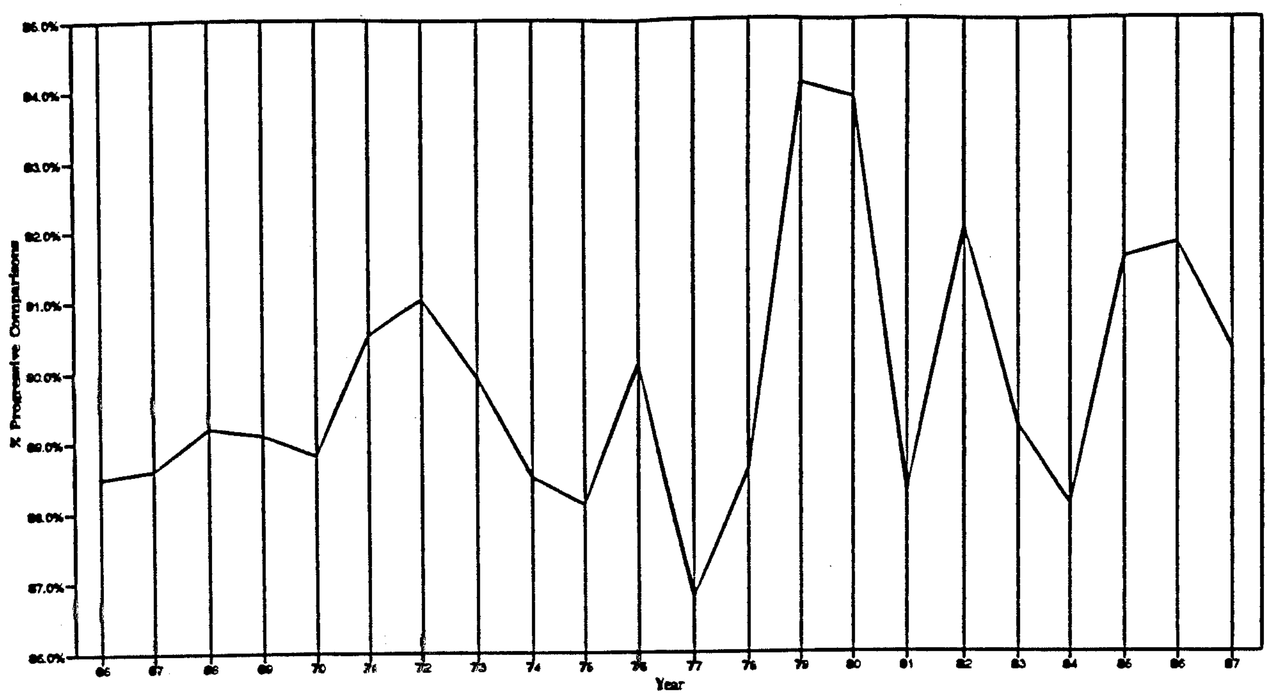


Figure 6-2: Horizontal Inequity of Federal Individual Income Tax

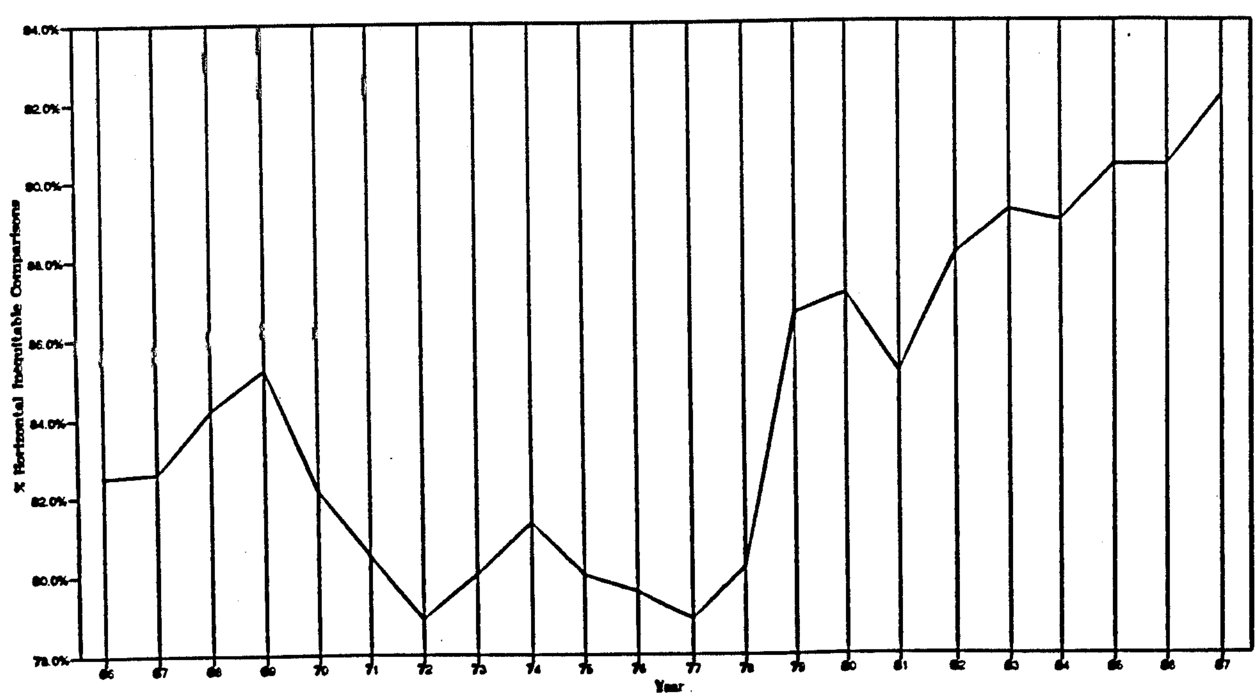


Figure 6-3: Gini Coefficient of Income Inequality

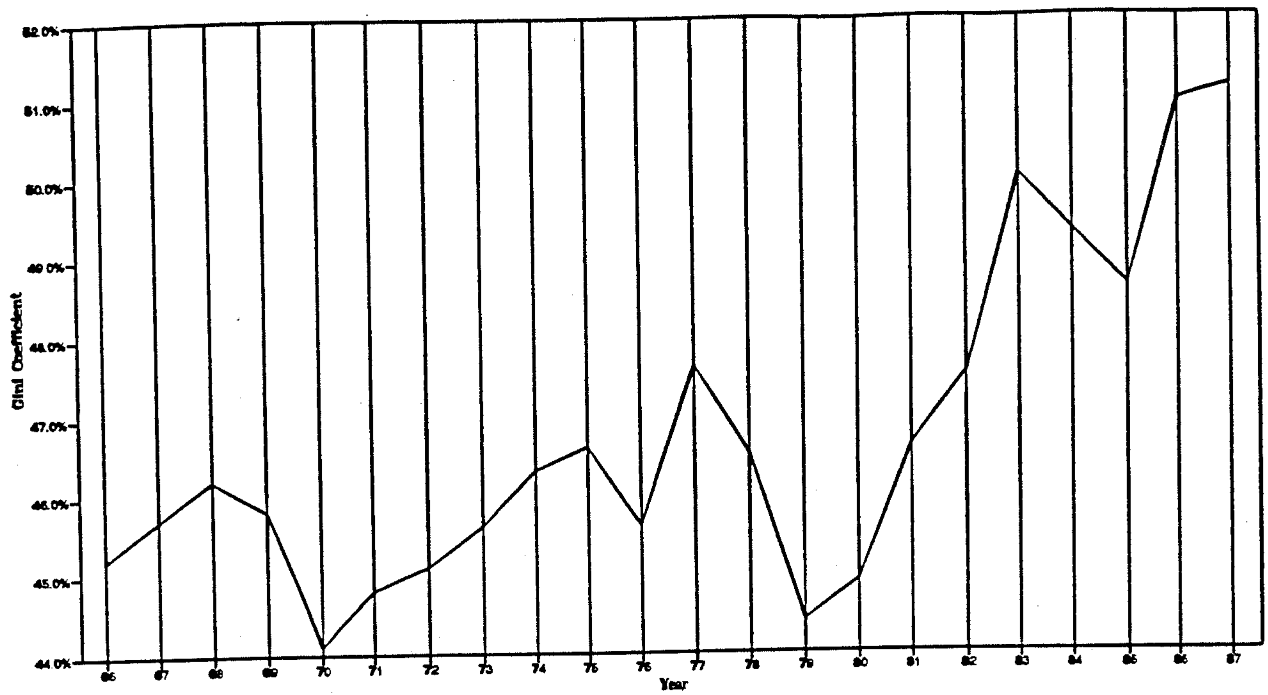
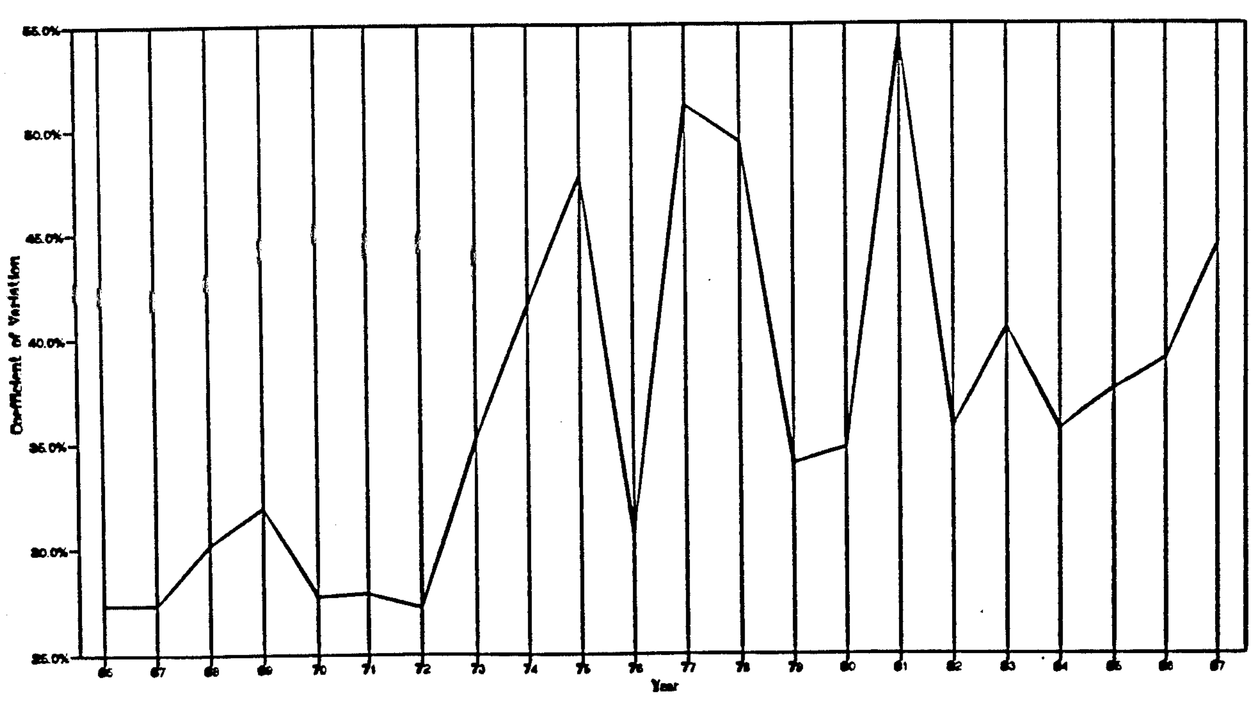


Figure 6-4: Coefficient of Variation in Effective Rates



6.3. Regression Analysis of Index Number Values from Tables 6-1 and 6-2

We may summarize the great deal of information generated by our empirical analysis of the period 1966-87 by estimating several summary double-log regressions. Following Formby and Sykes [1984], we include in these descriptive regressions measures of inflation (the GNP deflator), and the level of real percapita personal income. Of immediate interest is whether or not vertical progressivity and horizontal equity are related to each other after taking into these broad economic trends. Also, we examine the relationship between the coefficient of variation in effective tax rates and the Gini coefficient of income inequality to ascertain if there are associations between these two, traditional measures of horizontal inequity and income inequality.

Table 6-3 reports the results of a simple log relationship between the two Berliant-Strauss index measures, real percapita personal income (before tax), and the GNP deflator. It is immediately evident that there is a very sizeable inverse relationshipo between vertical progressivity and horizontal equity. A 1% increase in vertical progressivity is associated with a 2.6% decline in horizontal equity. Also, note that neither measure of economic well being, real, before tax, percapita personal income, or the inflation rate, is associated with horizontal equity.

Table 6-3: Multiple Regression Analysis of All Filers Horizontal Equity Values

(t ratio in parentheses)

	Intercept	Log Progress.	Log Real PCY	Log GNP Defl	Adjusted R2	Final 1/ Durbin-Watson after correction
Log Equity =	4.078 [.38]	-2.6532 [-2.72]	-.7017 [-.60]	-.3508 [-.83]	.5132	1.627

1/ Results displayed reflect application of the Prais-Winsten correction for autocorrelation. Rho = .89.

The statistical relationship between income inequality and horizontal inequity, displayed in Table 6-4 below, provides similar information. A 1% increase in the Gini coefficient of income inequality is associated with a 3.6% increase in the extent of relative variation in effective tax rates. Again, the other measures, suggested by Formby and Sykes [1984], show no relationship with the measure of horizontal inequity.[8]

It should be noted, however, that in both instances, barely half of the variation in measured horizontal equity or inequity has been explained over the time period in question.

[8] It should be noted, however, that Formby and Sykes [1984] explore the relationship between a distributional measure at the *state* level, and do not explain the variation in the distributional measure with another equity measure on the left-hand side of their statistical relationship.

Table 6-4: Multiple Regression Analysis of All Filers Coefficient of Variation in Effective Tax Rate

(t ratio in parentheses)

	Intercept	Log Gini	Log Real PCY	Log GNP Defl	Adjusted R2	Final 1/ Durbin-Watson after correction
Log CV = Tax Rate	-3.7094 [-.22]	3.7095 [2.08]	.5633 [.31]	-.0904 [-.17]	.2816	2.175

1/ Results displayed reflect application of the Prais-Winsten correction for autocorrelation. Rho = .48.

7. Summary and Conclusions

We have sought in this paper to develop a class of index numbers which may be used to measure the progressivity and equity of a tax system. The index numbers, which are viewed as a social ordering over taxpayers' incomes and taxes, have desirable mathematical properties.

Empirically, these indices were systematically applied to annual, anonymous samples of Federal individual income tax returns for the period 1966-1987. The Federal individual income tax displays generally high levels of progressivity in the sense that higher income taxpayers generally pay taxes at higher rates, and also generally high levels of horizontal inequity. That is, taxpayers with the same income levels often [as much as 92% of the time] face different effective tax rates.

It is evident that the changes to the Federal individual income tax made by the 1981 Economic Recovery Act materially reduced the horizontal equity of the Federal tax system, and decreased the progressivity of the Federal individual income tax. We observe an inverse statistical association between horizontal equity and vertical progressivity, and an inverse association between the coefficient of variation in effective tax rates and the Gini coefficient of income inequality.

I. Algebraic Statement of Other Index Numbers

Key to Symbols:

n = # of economic income classes
a = # of after-tax income classes
m = # of effective rate classes
N_i^j = population in economic income class i, rate class j
Y_i^j = average income in economic income class i, rate class j
Z_i = average income in after-tax income class i
P_i = population in after-tax income class i
POP = total population
INC = total after-tax income

(6) = INC/POP

$$(7) = \frac{1}{POP}\sum_{i=1}^{a}(Z_i - AVINC)^2 * P_i$$

$$(8) = \sqrt{VAR}/AVINC$$

$$(9) = \frac{1}{POP}^2 \sum_{i=1}^{a}\sum_{j=1}^{i-1} P_i * P_j * |Z_i - Z_j|$$

$$(10) = MD/AVINC$$

$$(11) = GINI/2$$

$$(12) = \frac{1}{AVINC * POP * (POP-1)} \sum_{i=1}^{a}\sum_{j=1}^{i-1} |Z_i - Z_j| * P_i * P_j$$

$$(13) = 1-[\sum_{i=1}^{a}(\frac{1}{AVINC})^{1-\epsilon}\frac{1}{POP}]^{\frac{1}{1-\epsilon}}$$

$$(14) = 1000 * \log (\sum_{i=1}^{a} \exp [(AVINC - Z_i) * \frac{1}{1000}] \frac{P_i}{POP})$$

$$(15) = \frac{1}{POP}\sum_{i=1}^{m} |\frac{Z_i}{AVINC} - 1| * P_i$$

$$(16) = RMD1/2$$

$$X_i = Z_i/INC$$

$$(17) = \sum_{i=1}^{a} P_i * X_i * \log(X_i)$$

$$(18) = \sum_{i=1}^{a} P_i * Z_i * \log(POP * Z_i)$$

$$(19) = \frac{1}{POP}\sum_{i=1}^{a} \mathrm{Sign}(Z_i) * P_i * \log(|Z_i|)$$

$$(20) = \frac{1}{POP}\sum_{i=1}^{a} P_i * (\log(|Z_i/AVINC|))^2$$

$$(21) = \frac{1}{POP}\sum_{i=1}^{a} (\mathrm{Sign}(Z_i) * \log(|Z_i|) - THEIL3)^2 * P_i$$

$$(22) = \sum_{i=1}^{a} P_i * |\frac{Z_i}{INC} - \frac{1}{POP}| = RMD1$$

$$(23) = \frac{1}{POP}\sum_{i=1}^{n} \{[\frac{\sum_{j=1}^{m} N_i^{j2}}{\sum_{j=1}^{m} N_i^j * j}] * [\frac{\sum_{j=1}^{m}\sum_{k=j+1}^{m}(j-k)^2 * N_i^k * N_i^j}{\sum_{j=1}^{m}\sum_{k=j+1}^{m}(N_i^k * N_i^j) + \frac{1}{2}\sum_{j=1}^{m} N_i^j * (N_i^j - 1)}]^{\frac{1}{2}}\}$$

Key to Equations:

# of Eq.	Index Number	Reference
(6) =	Average after-tax income	
(7) =	Variance	Kondor 1975
(8) =	Coefficient of variation	Atkinson 1970; Fields and Fei 1978
(9) =	Mean difference	Kendall 1947
(10) =	Gini coefficient	Pyatt 1976
(11) =	Atkinson Gini	Atkinson 1970
(12) =	Coefficient of concentration	Kondor 1975
(13) =	Atkinson	Atkinson 1970 AT1: $\varepsilon = .3$ AT2: $\varepsilon = .7$
(14) =	Kolm	Kolm 1976
(15) =	Relative mean deviation #1	Atkinson 1970
(16) =	Relative mean deviaiton #2	Kondor 1975
(17) =	Theil #1	Bourguignon 1979
(18) =	Theil #2	Fields and Fei 1979; Theil 1967
(19) =	Theil #3	Theil 1967
(20) =	Standard deviation of logarithms	Atkinson 1970
(21) =	Logarithmic variance	Kondor 1975
(22) =	Kuznets ratio = RMD1	Fields and Fei 1979
(23) =	Average coefficient of variation of effective rates	

8. Other Index Number Values: All Filers

Table 8-1: Other Index Number Values: All Filers

YR	(6) Average After Tax Income	(7) Variance of Income	(8) Co. Of Var.	(9) Mean Diff.	(10) After-Tax Gini	(11) Atkinson Gini	(12) Coef of Concen	(13) Atkinson Index: .3	Atkinson Index: .7
1966	6081.01	3.244E7	0.937	2749.24	0.452	0.226	0.452	0.108	0.262
1967	6441.80	3.825E7	0.960	2941.24	0.457	0.228	0.457	0.110	0.266
1968	6773.43	4.709E7	1.013	3130.52	0.462	0.231	0.462	0.114	0.271
1969	6978.37	4.407E7	0.951	3195.18	0.458	0.229	0.458	0.109	0.266
1970	7634.27	4.697E7	0.898	3369.30	0.441	0.221	0.441	0.101	0.249
1971	8187.23	5.663E7	0.919	3664.97	0.448	0.224	0.448	0.104	0.255
1972	8787.94	6.639E7	0.927	3962.19	0.451	0.225	0.451	0.106	0.256
1973	9199.47	7.236E7	0.925	4199.17	0.456	0.228	0.456	0.108	0.267
1974	9584.22	7.937E7	0.930	4436.26	0.463	0.231	0.463	0.111	0.278
1975	10176.1	9.08E7	0.936	4744.84	0.466	0.233	0.466	0.113	0.285
1976	11263.2	1.073E8	0.920	5133.63	0.456	0.228	0.456	0.106	0.259
1977	11580.6	1.227E8	0.957	5510.79	0.476	0.238	0.476	0.115	0.291
1978	12553.9	1.362E8	0.930	5838.71	0.465	0.232	0.465	0.111	0.278
1979	13985.5	1.504E8	0.877	6211.28	0.444	0.222	0.444	0.100	0.244
1980	15140.4	1.813E8	0.889	6797.22	0.449	0.224	0.449	0.101	0.249
1981	15759.5	2.123E8	0.925	7349.30	0.466	0.233	0.466	0.111	0.281
1982	18007.8	3.223E8	0.997	8551.79	0.475	0.237	0.475	0.113	0.273
1983	19383.1	4.599E8	1.106	9684.86	0.500	0.250	0.500	0.125	0.300
1984	22642.3	6.766E8	1.149	11170.1	0.493	0.247	0.493	0.129	0.296
1985	22467.9	5.698E8	1.062	10927.8	0.486	0.243	0.486	0.122	0.288
1986	24511.9	8.516E8	1.191	12488.2	0.509	0.255	0.509	0.136	0.311
1987	24174.1	7.819E8	1.157	12343.2	0.511	0.255	0.511	0.134	0.315

YR	(14) Kolm	(15) Rel Mean Dev #1	(16) Rel Mean Dev #2	(17) Theil #1	(18) Theil #2	(19) Theil #3	(20) Std. Dev. Log	(21) Log Variance	(23) Avg Rate Indx
1966	5959.00	0.638	0.319	-17.883	1.16E13	8.201	1.233	2.717	0.273
1967	7224.00	0.642	0.321	-17.910	1.26E13	8.253	1.266	2.714	0.273
1968	7523.00	0.650	0.325	-17.909	1.36E13	8.293	1.282	2.693	0.301
1969	10140.0	0.647	0.324	-18.028	1.45E13	8.328	1.276	2.842	0.319
1970	10990.0	0.623	0.311	-18.022	1.55E13	8.451	1.135	2.772	0.277
1971	11950.0	0.632	0.316	-18.021	1.68E13	8.506	1.166	2.911	0.279
1972	13690.0	0.638	0.319	-18.018	1.88E13	8.585	1.245	2.599	0.272
1973	12390.0	0.651	0.325	-18.067	2.06E13	8.590	1.263	3.167	0.352
1974	14200.0	0.659	0.330	-18.193	2.22E13	8.587	1.292	3.941	0.416
1975	13940.0	0.665	0.333	-18.185	2.33E13	8.617	1.266	4.553	0.477
1976	19820.0	0.651	0.325	-18.188	2.66E13	8.812	1.184	3.219	0.307
1977	20720.0	0.676	0.338	-18.450	2.81E13	8.714	1.247	5.450	0.511
1978	18000.0	0.668	0.334	-18.235	3.16E13	8.850	1.230	4.292	0.493
1979	26720.0	0.638	0.319	-18.239	3.61E13	9.070	1.122	2.793	0.340
1980	30840.0	0.644	0.322	-18.302	3.95E13	9.130	1.130	3.160	0.348
1981	26540.0	0.669	0.334	-18.422	4.19E13	9.045	1.178	5.293	0.543
1982	40140.0	0.675	0.337	-18.436	4.8E13	9.232	1.158	4.266	0.358
1983	43120.0	0.705	0.352	-18.528	5.25E13	9.227	1.220	5.367	0.405
1984	30190.0	0.701	0.351	-18.080	6.37E13	9.441	1.305	3.360	0.357
1985	42820.0	0.691	0.345	-18.316	6.44E13	9.431	1.258	4.019	0.376
1986	45100.0	0.723	0.361	-18.274	7.33E13	9.477	1.338	4.099	0.390
1987	50380.0	0.723	0.362	-18.456	7.51E13	9.430	1.427	4.807	0.444

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