Consumption, Income, and Cointegration: Further Analysis

Han, Hsiang-Ling and Masao Ogaki

Working Paper No. 305
December 1991

Consumption, Income, and Cointegration
Further Analysis

Hsiang–Ling Han

and

Masao Ogaki

Rochester Center for Economic Research
Working Paper No. 305

Consumption, Income, and Cointegration:
Further Analysis

Hsiang-Ling Han

and

Masao Ogaki

Department of Economics
University of Rochester

The Rochester Center for Economic Research
Working Paper No. 305

December 1991
First Draft: December 1990

We thank Robert King for providing the data used in this paper and for his helpful comments. All remaining errors are ours.

Consumption, Income, and Cointegration:
Further Analysis

Abstract

The paper reexamines the long-run relation between consumption and income in the U.S. that King, Plosser, Stock, and Watson (1991, KPSW) studied. KPSW showed that a version of the permanent income hypothesis (PIH) implies that the log of consumption and the log of income are cointegrated with a known cointegrated vector of (1,-1)'. They also showed that this cointegrating vector also eliminates the deterministic trends arising from drift terms. This restriction, which we call deterministic cointegration restriction, was not tested by KPSW. The purpose of the present paper is to test this restriction. We do not reject this restriction. (*JEL* E21, C32)

1. Introduction

This paper reexamines the long-run relation between consumption and income in the U.S., using the concept of cointegration of Engle and Granger (1987). Two time series, log consumption and log income can be modeled as series with stochastic trends and deterministic trends arising from drift terms. The present paper investigates the relation between the stochastic trends and the deterministic trends arising from drift terms. Previous empirical studies on cointegration of consumption and income focused on the stochastic trends in these series.

In King, Plosser, Stock, and Watson (1991, KPSW), the difference stationarity specification is employed to model nonstationarity. They show that a version of the permanent income hypothesis (PIH) model with a constant intertemporal elasticity of substitution implies that the log of consumption minus the log of income is stationary. This implies that the log of consumption and the log of income are cointegrated with a known cointegrating vector (1, -1)′. KPSW did not find evidence against this cointegration restriction of the PIH.[1] In their economic model, this cointegrating vector eliminates both stochastic trends and deterministic trends arising from the drift terms. This restriction, which Ogaki and Park (1990) called the deterministic cointegration restriction, was not tested by KPSW. The purpose of the present paper is to test this restriction.

Many other authors have tested the cointegration relation of the *levels*

---

[1]Neusser (1991) confirmed this finding for some other industrial countries and for slightly different data for the U.S. Both KPSW and Neusser include other economic variables such as investment to investigate a neoclassical model that includes the PIH. The present paper focuses on the PIH.

of consumption and income rather than the *logs* of consumption and income (see, e.g., Campbell (1987), Engle and Granger (1987), Phillips and Ouliaris (1988), and Park, Ouliaris, and Choi (1988)). As discussed in Cochrane and Sbordone (1988), another version of the PIH model of consumption with a representative consumer maximizing a quadratic utility function implies the level of consumption (C(t)) and the level of income (Y(t)) are cointegrated. Let us assume that the C(t) and Y(t) are difference stationary with drift. Then this version of the PIH also implies the deterministic cointegration restriction. Since C(t)-Y(t) is stationary while Y(t) grows, this version of the PIH implies that the saving rate (C(t)-Y(t))/Y(t) declines as Y(t) grows. This is not consistent with Kuznets's (1946) finding that the saving rate is stable in the long run in the U.S. On the other hand, if the log(C(t))-log(Y(t)) is (strictly) stationary as implied by the model of KPSW, then the ratio C(t)/Y(t) is stationary and the saving rate is stationary. Thus it is more natural to use the version of the PIH studied by KPSW to investigate the long-run relationships between consumption and income especially when both stochastic and deterministic trends are to be investigated.

The deterministic cointegration restriction is implied by many economic models. There are at least two reasons why testing this restriction is important. First, a test for the deterministic cointegration is important as a specification test for models implying cointegration. Since it is difficult to test the existence of stochastic trends [see, e.g., Cochrane (1988), Christiano and Eichenbaum (1989), Campbell and Perron (1991)], it is difficult to test restrictions on only stochastic trends. Because it is arguably easier to test the existence of deterministic trends, tests for the deterministic cointegration restriction can be useful in discriminating competing models.

Second, it is important to impose this restriction on estimators of the cointegrating vector because large efficiency gains can be expected (see, West (1988), Hansen (1989), and Park (1990)). For some models, this restriction cannot be rejected (see, e.g., Cooley and Ogaki's (1990)) and the cointegrating vector is estimated with this restriction imposed. On the other hand, if the deterministic cointegration restriction is not satisfied by the data, the cointegrating vector must be estimated from cointegration of the stochastic trends (stochastic cointegration). The estimators for cointegrating vectors are not consistent when the deterministic restriction is erroneously imposed when it is not satisfied. For example, this restriction has been rejected for some models in Ogaki and Park (1990), Ogaki (1990), which lead these authors to explore (economic) specifications that imply stochastic cointegration but do not imply the deterministic cointegration restriction.

The rest of the paper is organized as follows. Section 2 presents the econometric procedure we use. In Section 3, we report empirical results. In Section 4, we investigates small sample properties of the tests we use. Section 5 discusses directions of future research suggested by our empirical results and contains our conclusions.

2. Econometric Procedure

Let $X(t)$ be a 2-dimensional difference stationary process: $X(t) - X(t-1) = \mu + v(t)$ for $t \geq 1$, where μ is a 2-dimensional vector of real numbers and $v(t)$ is stationary with mean zero, with each component of $v(t)$ having a positive long run variance. Suppose that $X(t)$ are cointegrated with a cointegrating vector $(1,-\beta)$ and that the deterministic cointegration restriction is satisfied. Then we can apply the Canonical Cointegrating Regressions (*CCR*) procedure developed by Park (1990) to

$$X_1(t) = \psi + \beta X_2(t) + e(t), \qquad (1)$$

This *CCR* procedure, which is asymptotically equivalent to maximum likelihood estimation, only requires us to transform data before running a regression and corrects for endogeneity and serial correlation. The *CCR* estimators have asymptotic distributions that can be essentially considered as normal distributions, so that their standard errors can be interpreted in the usual way.[2]

An important property of the *CCR* procedure is that linear restrictions can be tested by χ^2 tests which are free from nuisance parameters. We can use χ^2 tests in a regression with spurious deterministic trends added to (1) to test for stochastic and deterministic cointegration. For this purpose, the *CCR* procedure is applied to a regression

$$X_1(t) = \psi + \sum_{i=1}^{q} \eta_i t^i + \beta X_2(t) + e(t). \qquad (2)$$

Let $H(p,q)$ denote the standard Wold statistic to test the hypothesis $\eta_p=\eta_{p+1}= \dots =\eta_q=0$ with the estimate of the variance of $e(t)$ replaced by the long run variance of the *CCR* (see Park [1990] for more explanations). Then $H(p,q)$ converges in distribution to a χ^2_{p-q} random variable under the null of

[2]In this special case of a single regressor, the *CCR* procedure is not necessary to obtain normal asymptotic distribution in the regression (1) as long as the regressor has nonzero drift. West (1988) shows that the OLS estimator has asymptotic normal distributions in this case. However, the OLS estimator for the regression (2) is asymptotically biased.

cointegration. In particular, the *H(0,1)* statistic tests the hypothesis $\eta_1=0$ in (17) and thus tests the deterministic cointegrating restriction. On the other hand, the *H(1,q)* tests stochastic cointegration.

Efficiency gains in estimating the cointegrating vector from imposing the deterministic cointegrating restriction was discussed by West (1989) for the one regressor case and by Hansen (1990) and Park (1990) for the general multiple regressors case. There are other estimation procedures which are maximum likelihood or equivalent to maximum likelihood asymptotically such as Johansen (1988, 1991), Phillips (1988, 1991), Stock and Watson (1989), Phillips and Hansen (1990), and Saikkonen (1989)). We use the CCR because Monte Carlo experiments in Park and Ogaki (1991) showed that the CCR estimators are better than Johansen's maximum likelihood estimators in terms of the mean square error (MSE) when the sample size is small and because it is easy to test the null of the deterministic cointegration restriction and the null of stochastic cointegration with the CCR.

The CCR procedure requires an estimate of nuisance parameters such as the the long run covariance of the disturbances in the system. We use Park and Ogaki's (1991) VAR prewhitening method with Andrews's (1991) QS kernel and his automatic band width parameter estimator to estimate the nuisance parameters.[3] The VAR of order one was used for prewhitening. In the first stage, we use a cointegrating regression based on the OLS. The OLS estimates for the cointegrating vector are used to form estimates for the nuisance parameters for the second stage CCR. In our empirical work and

---

[3]Andrews's (1990) automatic bandwidth estimator, S_T, was constructed form fitting AR(1) to each disturbance.

Monte Carlo experiments, we report the third stage CCR estimators based on the estimates of the nuisance parameters from the second stage CCR as recommended by Park and Ogaki (1991). We report the results for H(p,q) test statistics from the fourth stage CCR for which we bound the singular values[4] for the VAR coefficient matrix by 0.99 and the bandwidth parameter by $\sqrt{T}$.[5]

3. Empirical Results

We use the same consumption and income data as those used by KPSW. We use GNP minus government spending, which KPSW called private GNP as income. To obtain per capita values, real consumption and income are divided by total population at the end of month in each quarter. Table 1 presents test results. Either consumption or GNP can be used as the regressand in the CCR.[6] We cannot reject the deterministic cointegration restriction in terms of the *H(0,1)* tests. This result is robust to the choice of the regressand. We do not find evidence against stochastic cointegration from the *H(1,q)* tests. This result is consistent with those of KPSW. They did not reject stochastic cointegration for the logs of consumption and income, using procedures developed by Stock and Watson (1988, 1991) and Johansen (1988).

According to the point estimate of β and their standard errors, we find some evidence against the hypothesis that the cointegrating vector is (1,-1). This is especially true when log(C(t)) is used as the regressand.

---

[4]We follow Andrews and Monahan (1990) in bounding the singular values. We thank Don Andrews for clarifying the relation between the singular values and the eigenvalues.

[5]We do not bound these parameters for the second stage CCR and the third stage CCR because we find that the bounding often deteriorate the MSE and bias in our Monte Carlo simulations.

[6]We use Ogaki's GAUSS CCR package for our empirical work and Monte Carlo simulations. See Ogaki (1991b) for details about this package.

The standard errors reported are based on the asymptotic distributions. It is possible that the asymptotic distributions are not good approximations in small samples. We investigate small sample properties of the CCR procedure.

4. Small Sample Properties of the H(p,q) Tests

In this section, we study small sample properties of the CCR estimators and the H(p,q) tests, using Monte Carlo experiments.

We use two types of the data generation processes for the Monte Carlo simulations. The data generation process called DGP1 is

$$X_1(t) = \beta X_2(t) + e(t), \tag{3}$$

$$\Delta X_2(t) = \mu_2 + v_2(t), \tag{4}$$

$$\Delta e(t) = \phi\ \Delta e(t-1) + (1-\theta L)(1-\gamma L)\epsilon_1(t), \tag{5}$$

$$v_2(t) = \phi\ v_2(t-1) + (1-\gamma L)\epsilon_2(t), \tag{6}$$

for t=1,...,T, where L is the lag operator, Δ=1-L, $|\phi|<1$, $|\gamma|<1$, and $-1<\theta\leq 1$. Here $\epsilon(t)=(\epsilon_1(t),\epsilon_2(t))'$ is NID with $E(\epsilon_i^2)=1$, $E(\epsilon_1\epsilon_2)=\alpha$, and $|\alpha|<1$. When θ=1, we interpret (5) to mean

$$e(t) = \phi\ e(t-1) + (1-\gamma L)\epsilon_1(t), \tag{5'}$$

so that e(t) is stationary and X_1 and X_2 are cointegrated with the deterministic cointegration restriction. When $|\theta|<1$, e(t) is difference stationary and X_1 and X_2 are not cointegrated. We set β=1. We set μ_2, so that each element of $X(t)$ has a common linear deterministic trend that is comparable to that of the log of GNP. Let $c(t)$ be the log of consumption and $y(t)$ be the log of GNP. We estimate means of $\Delta c(t)$ and $\Delta Y(t)$, restricting the means to be the same. We estimate a common mean to be

0.0052 with the standard error of 0.0006.[7] Since the sample standard deviation of $y(t)$ is 0.014, we set μ_2=0.0052 std(X_2)/0.014, where the std(X_2) is the standard deviation of X_2 that is implied by (6).

The data generation process called DGP2 is based on an error correction model that is estimated from the data we used in the previous section for the purpose of bootstrapping. The estimation is done as follows. We demean $\Delta c(t)$ and $\Delta Y(t)$ by the estimated common mean of 0.0052. Then, assuming that the cointegrating vector is (1, -1)', we use the OLS to estimate the error correction model:

$$\Delta c(t) = \lambda_1 EC(t\text{-}1) + A_1(L)\Delta X(t\text{-}1) + \epsilon_1(t), \tag{7}$$

$$\Delta y(t) = \lambda_2 EC(t\text{-}1) + A_2(L)\Delta X(t\text{-}1) + \epsilon_2(t), \tag{8}$$

for t=1,...,T, where $X(t)=[c(t),\ y(t)]'$, $EC(t\text{-}1)$ is demeaned $c(t\text{-}1)\text{-}y(t\text{-}1)$. We first include four lags of $X(t\text{-}1)$ in the regressors. After excluding terms that are not significant at the 5-percent level, the final model is

$$\Delta c(t) = \underset{(0.024)}{0.058}\ EC(t\text{-}1) + \underset{(0.043)}{0.166}\ \Delta y(t\text{-}1) + \epsilon_1(t), \tag{7'}$$

$$\Delta y(t) = \underset{(0.039)}{0.163}\ EC(t\text{-}1) + \underset{(0.067)}{0.368}\ \Delta c(t\text{-}1) - \underset{(0.083)}{0.196}\ \Delta c(t\text{-}3)$$

$$+ \underset{(0.136)}{0.327}\ \Delta y(t\text{-}4) + \epsilon_2(t), \tag{8'}$$

---

[7] We restrict $\Delta c(t)$ *and* $\Delta y(t)$ to have the same mean, using Hansen/Heaton/Ogaki GMM package that was supported by NSF Grant SES-3512371 (see Ogaki (1991a) for details about this package). The Lagrange Multiplier test statistic for the restriction of a common mean is 0.206 with the p-value of 0.65 when Andrews and Monahan's (1990) prewhitened QS kernel is used with Andrews's (1991) automatic bandwidth parameter estimate. If the system is cointegrated, this Lagrange Multiplier test has a degenerate asymptotic distribution.

where standard errors are in parentheses. When we write (7′) and (8′) as $B(L)X(t)=\epsilon(t)$, $\det B(z)=0$ has one unit root and all other roots outside the unit circle whose smallest absolute value is 1.16. We use sample covariance matrix of the residuals as $E(\epsilon(t)\epsilon(t)')$. After generating $X(t)$ from (7) and (8), a linear deterministic trend $0.0052t$ is added to both elements of $X(t)$, so that the resulting $X(t)$ is cointegrated with the deterministic cointegration restriction.

Table 2 reports results for the DGP1 obtained from 1000 Monte Carlo replications.[8] The sample size T is set to 170 and all regressions include a constant term. We use Park and Ogaki's (1991) VAR prewhitening method based on AR(1) with the Andrews's (1991) automatic bandwidth parameter estimate for the QS kernel. For each parameter values of (ϕ, γ, α), we try both X_1 and X_2 as the regressand and three values θ, 1, 0.9, and 0.5. The system is cointegrated when θ=1. For this case, we report the empirical sizes of H(p,q) tests when the nominal critical values are used. We also report the empirical 5-percent and 95-percent critical values of the t-ratios for the hypothesis β=1, $t_{.05}$ and $t_{.95}$, from the third stage CCR that Park and Ogaki (1991) recommend based on the mean square error and bias in small samples. When θ=0.9 or θ=0.5, the system is not cointegrated. We report size corrected powers of H(p,q) tests and do not report $t_{.05}$ and $t_{.95}$ for this case. We report the results for H(0,1), H(1,2), H(1,4), and H(1,7) test statistics from the fourth stage CCR for which we bound the singular values for the VAR coefficient matrix by 0.99 and the bandwidth parameter by

---

[8] The initial value $X_2(0)$ is set to zero. In the case of γ=0, we draw the initial value e(0) from $N(0,1/(1-\phi^2))$ that is independent from $\epsilon(1),\ldots,\epsilon(T)$.

$\sqrt{T}$.[9]

When there is no MA component (γ=0), the empirical critical values of the t-ratios are close to those of the standard normal distribution and the H(0,1) and H(1,2) tests have reasonable size properties under the null unless the AR coefficient ϕ is close to one. This is true even when X_2 is used as the regressand and/or α is nonzero. The H(1,4) and H(1,7) tests have somewhat less stable sizes than the H(0,1) and H(1,2) tests in this case. When ϕ is as large as 0.8, and the t-ratios and the H(p,q) tests become less reliable under the null. The H(1,4) and H(1,7) especially have very low empirical sizes and very high empirical sizes depending on which variable is used as the regressand. When γ is nonzero the results under the null are similar to the case where ϕ=0.8 except that the H(0,1) and H(1,2) tests show more size distortion problems when γ=0.5. When α=0 and thus X_2 is exogenous, the H(0,1) test often has lower power than the H(1,q) tests. On the other hand, when $\alpha \neq 0$ and X_2 is not exogenous, the H(0,1) test is more powerful than the H(1,q) tests. Since one of the most appealing feature of cointegration is that structural parameters can be estimated without exogeneity assumptions, this is evidence in favor of the H(0,1) test.

Based on these size and power properties, we recommend the H(0,1) test and the H(1,q) test with small values of q when the sample size is small. These tests performs reasonably well in general, but we find two cases where the test overrejects the null hypothesis. First, when the autoregressive coefficient ϕ is close to one, the system is close to no cointegration.

---

[9] We do not bound these parameters for the second stage CCR based on the OLS and the third stage CCR based on the OLS because we find that the bounding often deteriorate the MSE and bias in our preliminary simulations.

Second, when the moving average coefficient γ is positive and close to one, the random walk component in $X_2(t)$ becomes small compared with the stationary component in the sense of Cochrane (1988). Both of these are cases where the near-observational equivalence of stationary and integrated processes (see, e.g, Campbell and Perron (1991)) is severe. Thus it is natural that these tests have small sample problems in these cases.

Table 3 reports results for the DGP2 from 1000 Monte Carlo replications. The sample size is set to 167 as in the actual data we used in the previous section. The initial values of $X(0)$ are set to zero and we create $X(t)$ for t=1,...,268 and sample from t=102 to t=268 for each Monte Carlo replication.[10] The H(0,1) test is conservative when $y(t)$ is used as the regressand and slightly overrejects the null of the deterministic cointegration restriction when $c(t)$ is used as the regressand. From t_{05} and t_{95}, the computed standard errors tend to be too small when c is used as the regressand. Thus the evidence that we found in the previous section against the hypothesis that (1, -1)′ is a cointegrating vector is likely to be due this small sample problem.

5. Conclusions and Future Research

The present paper showed that the U.S. post war quarterly data supports the joint hypothesis of the difference stationary consumption and income and a version of the permanent income hypothesis. We tested the deterministic cointegration restriction, which has often been neglected in the literature, and found that we cannot reject this joint hypothesis. Our finding is also consistent with Kuznets's (1946) finding that the saving rate is stable in

---

[10]We also use X(101) for estimating the nuisance parameters for the CCR.

the long-run in the U.S.

REFERENCES

Andrews, D. W. K., 1989, Heteroskedasticity and autocorrelation consistent covariance matrix estimation, Econometrica, forthcoming.

Andrews, D. W. K. and Monahan., 1990, An Improved heteroskedasticity and autocorrelation consistent covariance matrix estimator, manuscript.

Campbell, J. Y., 1987, Does saving anticipate declining labor income? An alternative test of the permanent income hypothesis, *Econometrica* 55, 1249-1273.

Campbell, J.Y. and Perron, P. "Pitfalls and Opportunities: What Macroeconomists Should Know About Unit Roots," manuscript.

Christiano, L. J., and M. Eichenbaum, 1989, "Unit Roots in Real GNP - Do We Know and Do We Care?" Paper presented at the Carnegie-Rochester Public Policy Conference, April 21-22, 1989.

Cochrane, J. H.,1988, How Big is the Random Walk in GNP? *Journal of Political Economy*, 96, 893-920.

Cochrane, J. H. and A. M. Sbordone, 1988, Multivariate estimates of the permanent components of GNP and stock prices, *Journal of Economic Dynamics and Control* 12, 255-96.

Cooley, T. F. and M. Ogaki, 1991, A time series analysis of real wages, consumption, and asset returns under optimal contracting: a cointegration-Euler equation approach, RCER Working Paper No. 285, University of Rochester.

Engle, R. F. and C. W. J. Granger (1987): "Co-Integration and Error Correction: Representation, Estimation, and Testing," *Econometrica*, 55, 251-276.

Hansen, B. E. (1989): "Efficient Estimation of Cointegrating Vectors in the Presence of Deterministic Trends," manuscript, University of Rochester.

Johansen, S., 1988, Statistical analysis of cointegration vectors, Journal of Economic Dynamics and Control 12, 231-254.

———, 1991, Estimation and hypothesis testing of cointegrating vectors in Gaussian vector autoregressive models, *Econometrica* 59, 1551-1580.

King, R. G., C. I. Plosser, and S. T. Rebelo, 1988, Production, growth, and business cycles: I. The basic neoclassical model, Journal of Monetary Economics 21, 195-232.

King, R. G., C. I. Plosser, J. H. Stock, and M. W. Watson, 1991, Stochastic Trends and Economic Fluctuations, American Economic Review 81, 819-40.

Kuznets, Simon, 1946, *National Income, A Summary of Findings*, New York: National Bureau of Economic Research.

Neusser, K., 1991, Testing the long-run implications of the neoclassical model, Journal of Monetary Economics 27, 3-37.

Ogaki, M., 1990, "Engel's Law and Cointegration." RCER Working Paper No. 228, University of Rochester.

Ogaki, M., 1991a, The generalized method of moments: a user's guide, manuscript, University of Rochester.

Ogaki, M., 1991b, Canonical cointegrating regressions: a user's guide, manuscript, University of Rochester.

Ogaki, M., and J. Y. Park, 1990, "A Cointegration Approach to Estimating Preference Parameters," manuscript, University of Rochester.

Park, J. Y., 1990a, "Canonical Cointegrating Regressions," CAE Working Paper No.88-29R, forthcoming in *Econometrica*.

Park, J. Y., 1990b, "Testing for Unit Roots and Cointegration by Variable Addition," *Advances in Econometrics* 8: 107-133.

Park, J. Y. and B. Choi (1988): "A New Approach to Testing for a Unit Root," CAE Working Paper No. 88-23, Cornell University.

Park, J. Y., and M. Ogaki, 1991, Inference in cointegrated models using VAR prewhitening to estimate shortrun dynamics, RCER Working Paper No. 281, University of Rochester.

Park, J. Y., S. Ouliaris, and B. Choi (1988): "Spurious Regressions and Tests for Cointegration," CAE Working Paper No. 88-07, Cornell University.

Phillips, P.C.B., 1988, Spectral regression for cointegrated time series, Cowles Foundation Discussion Paper no. 872, (Yale University, New Haven, CT).

Phillips, P. C. B., 1991, Optimal Inferences in cointegrated systems, Econometrica 59, 283-306.

Phillips, Peter C. B., and Hansen, Bruce E. "Statistical Inference in Instrumental Variables Regression with I(1) Processes," *Review of Economic Studies* 57 (January 1990): 99-125.

Phillips, P. C. B. and S. Ouliaris, 1988, Testing for cointegration using principal components methods, *Journal of Economic Dynamics and Control* 12, 205-30.

Phillips, P. C. B. and P. Perron (1988): "Testing for a Unit Root in Time Series Regression," *Biometrica*, 75, 335-46.

Saikkonen, P., 1989, Asymptotic efficient estimation of cointegration regressions, Mimeo. (University of Helsinki, Helsinki, Finland).

Stock, James H., and Watson, Mark W. "Testing for Common Trends," *Journal of the American Statistical Association*, December 1988, 83, 1097-1107.

Stock, James H., and Watson, Mark W. "A Simple Estimator of Cointegrating Vectors in Higher Order Integrated Systems," Manuscript, 1991.

West, K. D. (1988): "Asymptotic Normality, When Regressors Have a Unit Root," *Econometrica*, 56, 1397-1417.

TABLE 1

CANONICAL COINTEGRATING REGRESSION RESULTS

Regressand	β [a]	$H(0,1)$ [b]	$H(1,2)$ [c]	$H(1,4)$ [c]	$H(1,7)$ [c]
Consumption	1.048 (0.016)	0.311 (0.577)	0.120 (0.729)	5.663 (0.129)	8.047 (0.235)
GNP	0.953 (0.024)	1.810 (0.179)	0.022 (0.881)	1.250 (0.741)	3.062 (0.801)

NOTE: Park and Ogaki's (1991) VAR prewhitening method based on the Andrews's (1991) QS kernel with automatic bandwidth estimator was used.

[a]Standard errors are in parentheses.

[b]P-values are in parentheses. This statistic tests the deterministic cointegration restriction.

[c]P-values are in parentheses. These tests the null of stochastic cointegration.

TABLE 2

MONTE CARLO RESULTS BASED ON DGP1[a]

ϕ	γ	α	θ	Regress-and	$t_{.05}$[b]	$t_{.95}$[c]	$H(0,1)$[d]	$H(1,2)$[d]	$H(1,4)$[d]	$H(1,7)$[d]
0.0	0.0	0.0	1.0	X_1	-1.86	1.75	0.051	0.046	0.038	0.039
							(3.87)	(3.66)	(7.51)	(12.0)
0.0	0.0	0.0	0.9	X_1	. . . .	. . .	0.287	0.378	0.466	0.484
0.0	0.0	0.0	0.5	X_1	. . . .	. . .	0.359	0.462	0.612	0.621
0.0	0.0	0.0	1.0	X_2	-1.77	1.80	0.047	0.048	0.046	0.042
							(3.77)	(3.81)	(7.42)	(12.2)
0.0	0.0	0.0	0.9	X_2	. . . .	. . .	0.272	0.338	0.434	0.423
0.0	0.0	0.0	0.5	X_2	. . . .	. . .	0.307	0.444	0.622	0.686
0.0	0.0	0.5	1.0	X_1	-1.86	1.75	0.052	0.046	0.039	0.039
							(3.86)	(3.64)	(7.53)	(12.1)
0.0	0.0	0.5	0.9	X_1	. . . .	. . .	0.557	0.367	0.448	0.467
0.0	0.0	0.5	0.5	X_1	. . . .	. . .	0.815	0.454	0.623	0.660
0.0	0.0	0.5	1.0	X_2	-1.80	1.84	0.051	0.053	0.045	0.055
							(3.85)	(3.99)	(7.59)	(12.7)
0.0	0.0	0.5	0.9	X_2	. . . .	. . .	0.527	0.301	0.363	0.317
0.0	0.0	0.5	0.5	X_2	. . . .	. . .	0.375	0.206	0.248	0.183
0.0	0.0	-0.5	1.0	X_1	-1.86	1.75	0.050	0.046	0.038	0.039
							(3.80)	(3.66)	(7.50)	(11.9)
0.0	0.0	-0.5	0.9	X_1	. . . .	. . .	0.576	0.356	0.441	0.475
0.0	0.0	-0.5	0.5	X_1	. . . .	. . .	0.820	0.482	0.475	0.664
0.0	0.0	-0.5	1.0	X_2	-1.85	1.82	0.047	0.052	0.050	0.061
							(3.71)	(4.02)	(7.80)	(13.2)
0.0	0.0	-0.5	0.9	X_2	. . . .	. . .	0.570	0.300	0.401	0.397
0.0	0.0	-0.5	0.5	X_2	. . . .	. . .	0.852	0.576	0.835	0.921

TABLE 2 - *Continued*

ϕ	γ	α	θ	Regress-and	$t^{b}_{.05}$	$t^{c}_{.95}$	$H(0,1)^{d}$	$H(1,2)^{d}$	$H(1,4)^{d}$	$H(1,7)^{d}$
0.4	0.0	0.0	1.0	X_1	-1.90	1.79	0.040 (3.53)	0.037 (3.41)	0.032 (6.98)	0.031 (11.6)
0.4	0.0	0.0	0.9	X_1	. . .	. . .	0.302	0.323	0.372	0.329
0.4	0.0	0.0	0.5	X_1	. . .	. . .	0.065	0.114	0.057	0.032
0.4	0.0	0.0	1.0	X_2	-1.77	2.00	0.076 (4.39)	0.066 (4.33)	0.075 (9.08)	0.082 (14.6)
0.4	0.0	0.0	0.9	X_2	. . .	. . .	0.259	0.267	0.272	0.235
0.4	0.0	0.0	0.5	X_2	. . .	. . .	0.213	0.296	0.398	0.489
0.4	0.0	0.5	1.0	X_1	-1.90	1.79	0.041 (3.52)	0.036 (3.41)	0.032 (6.93)	0.029 (11.5)
0.4	0.0	0.5	0.9	X_1	. . .	. . .	0.654	0.376	0.475	0.510
0.4	0.0	0.5	0.5	X_1	. . .	. . .	0.592	0.341	0.448	0.463
0.4	0.0	0.5	1.0	X_2	-1.81	2.02	0.051 (3.89)	0.046 (3.67)	0.035 (7.25)	0.028 (11.4)
0.4	0.0	0.5	0.9	X_2	. . .	. . .	0.368	0.225	0.248	0.188
0.4	0.0	0.5	0.5	X_2	. . .	. . .	0.128	0.153	0.209	0.264
0.4	0.0	-0.5	1.0	X_1	-1.90	1.79	0.038 (3.53)	0.037 (3.48)	0.032 (6.98)	0.030 (11.4)
0.4	0.0	-0.5	0.9	X_1	. . .	. . .	0.652	0.363	0.475	0.510
0.4	0.0	-0.5	0.5	X_1	. . .	. . .	0.601	0.353	0.460	0.491
0.4	0.0	-0.5	1.0	X_2	-1.93	2.19	0.092 (5.17)	0.093 (5.32)	0.140 (11.0)	0.205 (17.2)
0.4	0.0	-0.5	0.9	X_2	. . .	. . .	0.520	0.252	0.317	0.320
0.4	0.0	-0.5	0.5	X_2	. . .	. . .	0.660	0.450	0.700	0.809

TABLE 2 - *Continued*

ϕ	γ	α	θ	Regress-and	$t^b_{.05}$	$t^c_{.95}$	$H(0,1)^d$	$H(1,2)^d$	$H(1,4)^d$	$H(1,7)^d$
-0.4	0.0	0.0	1.0	X_1	-1.73	1.73	0.056 (4.06)	0.045 (3.88)	0.059 (7.54)	0.049 (12.4)
-0.4	0.0	0.0	0.9	X_1	. . .	. . .	0.276	0.384	0.510	0.535
-0.4	0.0	0.0	0.5	X_1	. . .	. . .	0.506	0.642	0.886	0.936
-0.4	0.0	0.0	1.0	X_2	-1.81	1.87	0.093 (5.47)	0.072 (4.54)	0.079 (9.35)	0.098 (14.9)
-0.4	0.0	0.0	0.9	X_2	. . .	. . .	0.211	0.328	0.392	0.385
-0.4	0.0	0.0	0.5	X_2	. . .	. . .	0.333	0.516	0.746	0.815
-0.4	0.0	0.5	1.0	X_1	-1.74	1.72	0.055 (4.06)	0.052 (3.93)	0.047 (7.66)	0.050 (12.5)
-0.4	0.0	0.5	0.9	X_1	. . .	. . .	0.448	0.347	0.446	0.437
-0.4	0.0	0.5	0.5	X_1	. . .	. . .	0.823	0.548	0.743	0.771
-0.4	0.0	0.5	1.0	X_2	-1.88	1.83	0.118 (5.65)	0.079 (4.83)	0.087 (9.05)	0.108 (15.3)
-0.4	0.0	0.5	0.9	X_2	. . .	. . .	0.562	0.314	0.399	0.351
-0.4	0.0	0.5	0.5	X_2	. . .	. . .	0.791	0.360	0.480	0.423
-0.4	0.0	-0.5	1.0	X_1	-1.90	1.79	0.056 (4.00)	0.053 (3.87)	0.045 (7.54)	0.048 (12.4)
-0.4	0.0	-0.5	0.9	X_1	. . .	. . .	0.492	0.332	0.450	0.441
-0.4	0.0	-0.5	0.5	X_1	. . .	. . .	0.821	0.553	0.748	0.772
-0.4	0.0	-0.5	1.0	X_2	-1.73	1.76	0.094 (5.50)	0.067 (4.53)	0.093 (9.65)	0.140 (17.4)
-0.4	0.0	-0.5	0.9	X_2	. . .	. . .	0.825	0.402	0.575	0.637
-0.4	0.0	-0.5	0.5	X_2	. . .	. . .	0.897	0.620	0.838	0.912

TABLE 2 - *Continued*

ϕ	γ	α	θ	Regress-and	$t^{b}_{.05}$	$t^{c}_{.95}$	$H(0,1)^{d}$	$H(1,2)^{d}$	$H(1,4)^{d}$	$H(1,7)^{d}$
0.8	0.0	0.0	1.0	X_1	-2.22	2.17	0.028 (3.25)	0.027 (3.04)	0.012 (6.08)	0.007 (9.61)
0.8	0.0	0.0	0.9	X_1	. . .	. . .	0.145	0.171	0.180	0.119
0.8	0.0	0.0	0.5	X_1	. . .	. . .	0.214	0.183	0.202	0.188
0.8	0.0	0.0	1.0	X_2	-2.40	3.31	0.202 (7.96)	0.195 (8.43)	0.340 (16.5)	0.475 (25.0)
0.8	0.0	0.0	0.9	X_2	. . .	. . .	0.074	0.093	0.092	0.101
0.8	0.0	0.0	0.5	X_2	. . .	. . .	0.072	0.041	0.026	0.020
0.8	0.0	0.5	1.0	X_1	-2.22	2.16	0.026 (3.19)	0.026 (3.05)	0.013 (6.09)	0.007 (9.58)
0.8	0.0	0.5	0.9	X_1	. . .	. . .	0.574	0.356	0.507	0.561
0.8	0.0	0.5	0.5	X_1	. . .	. . .	0.301	0.215	0.264	0.279
0.8	0.0	0.5	1.0	X_2	-1.92	2.64	0.084 (4.70)	0.078 (4.67)	0.088 (8.90)	0.095 (14.7)
0.8	0.0	0.5	0.9	X_2	. . .	. . .	0.099	0.101	0.133	0.114
0.8	0.0	0.5	0.5	X_2	. . .	. . .	0.193	0.096	0.125	0.099
0.8	0.0	-0.5	1.0	X_1	-2.24	1.20	0.027 (3.19)	0.026 (3.00)	0.012 (6.12)	0.006 (9.53)
0.8	0.0	-0.5	0.9	X_1	. . .	. . .	0.584	0.357	0.513	0.573
0.8	0.0	-0.5	0.5	X_1	. . .	. . .	0.324	0.208	0.238	0.256
0.8	0.0	-0.5	1.0	X_2	-2.69	3.97	0.302 (3.84)	0.276 (3.84)	0.508 (7.81)	0.743 (12.6)
0.8	0.0	-0.5	0.9	X_2	. . .	. . .	0.313	0.178	0.206	0.244
0.8	0.0	-0.5	0.5	X_2	. . .	. . .	0.051	0.091	0.100	0.081

TABLE 2 - *Continued*

ϕ	γ	α	θ	Regress-and	$t^b_{.05}$	$t^c_{.95}$	$H(0,1)^d$	$H(1,2)^d$	$H(1,4)^d$	$H(1,7)^d$
0.0	0.5	0.0	1.0	X_1	-1.33	1.24	0.009 (2.10)	0.010 (2.26)	0.005 (4.50)	0.002 (8.08)
0.0	0.5	0.0	0.9	X_1	. . .	. . .	0.283	0.384	0.518	0.529
0.0	0.5	0.0	0.5	X_1	. . .	. . .	0.585	0.730	0.950	0.986
0.0	0.5	0.0	1.0	X_2	-1.45	1.51	0.257 (11.5)	0.103 (6.37)	0.139 (13.1)	0.187 (20.7)
0.0	0.5	0.0	0.9	X_2	. . .	. . .	0.045	0.186	0.170	0.116
0.0	0.5	0.0	0.5	X_2	. . .	. . .	0.188	0.505	0.722	0.800
0.0	0.5	0.5	1.0	X_1	-1.33	1.25	0.009 (2.13)	0.009 (2.26)	0.005 (4.46)	0.003 (7.97)
0.0	0.5	0.5	0.9	X_1	. . .	. . .	0.469	0.368	0.494	0.515
0.0	0.5	0.5	0.5	X_1	. . .	. . .	0.885	0.665	0.874	0.902
0.0	0.5	0.5	1.0	X_2	-1.47	1.47	0.116 (6.39)	0.053 (3.98)	0.047 (7.73)	0.057 (12.8)
0.0	0.5	0.5	0.9	X_2	. . .	. . .	0.347	0.253	0.300	0.250
0.0	0.5	0.5	0.5	X_2	. . .	. . .	0.775	0.460	0.626	0.615
0.0	0.5	-0.5	1.0	X_1	-1.33	1.25	0.010 (2.17)	0.009 (2.25)	0.004 (4.45)	0.002 (8.02)
0.0	0.5	-0.5	0.9	X_1	. . .	. . .	0.497	0.353	0.497	0.501
0.0	0.5	-0.5	0.5	X_1	. . .	. . .	0.879	0.649	0.862	0.898
0.0	0.5	-0.5	1.0	X_2	-1.33	1.40	0.267 (12.8)	0.119 (7.16)	0.181 (15.4)	0.265 (27.4)
0.0	0.5	-0.5	0.9	X_2	. . .	. . .	0.923	0.459	0.661	0.730
0.0	0.5	-0.5	0.5	X_2	. . .	. . .	0.902	0.555	0.750	0.801

TABLE 2 - *Continued*

ϕ	γ	α	θ	Regress-and	$t^{b}_{.05}$	$t^{c}_{.95}$	$H(0,1)^{d}$	$H(1,2)^{d}$	$H(1,4)^{d}$	$H(1,7)^{d}$
0.0	-0.5	0.0	1.0	X_1	-1.63	1.58	0.014	0.022	0.008	0.001
							(2.71)	(2.61)	(5.42)	(8.97)
0.0	-0.5	0.0	0.9	X_1	...	...	0.290	0.350	0.417	0.375
0.0	-0.5	0.0	0.5	X_1	...	...	0.116	0.200	0.147	0.058
0.0	-0.5	0.0	1.0	X_2	-1.63	1.96	0.079	0.060	0.066	0.063
0.0	-0.5	0.0	0.9	X_2	...	...	0.237	0.288	0.333	0.306
0.0	-0.5	0.0	0.5	X_2	...	...	0.275	0.314	0.449	0.494
0.0	-0.5	0.5	1.0	X_1	-1.63	1.55	0.014	0.022	0.008	0.002
							(2.71)	(2.63)	(5.47)	(9.11)
0.0	-0.5	0.5	0.9	X_1	...	...	0.676	0.392	0.511	0.544
0.0	-0.5	0.5	0.5	X_1	...	...	0.784	0.406	0.553	0.549
0.0	-0.5	0.5	1.0	X_2	-1.49	1.90	0.058	0.041	0.032	0.019
							(3.95)	(3.58)	(7.25)	(11.3)
0.0	-0.5	0.5	0.9	X_2	...	...	0.289	0.230	0.225	0.178
0.0	-0.5	0.5	0.5	X_2	...	...	0.110	0.148	0.181	0.204
0.0	-0.5	-0.5	1.0	X_1	-1.63	1.56	0.014	0.024	0.009	0.001
							(2.69)	(2.59)	(5.46)	(9.04)
0.0	-0.5	-0.5	0.9	X_1	...	...	0.695	0.399	0.513	0.542
0.0	-0.5	-0.5	0.5	X_1	...	...	0.777	0.431	0.553	0.568
0.0	-0.5	-0.5	1.0	X_2	-1.78	2.06	0.080	0.080	0.103	0.146
							(4.81)	(4.61)	(9.56)	(15.5)
0.0	-0.5	-0.5	0.9	X_2	...	...	0.551	0.273	0.336	0.321
0.0	-0.5	-0.5	0.5	X_2	...	...	0.761	0.490	0.707	0.772

NOTE: The results are based on 1000 Monte Carlo replications.

[a]Equations (3)-(6) define the data generation process DGP1.

[b]The empirical 5-percent critical value of the t-ratio for the hypothesis $\beta=1$.

[c]The empirical 95-percent critical value of the t-ratio for the hypothesis β=1.
[d]The rows with θ=1 presents size based on asymptotic critical values with the empirical 5-percent critical values in parentheses; the remaining rows presents the size adjusted power.

TABLE 3

MONTE CARLO RESULTS BASED ON DGP2[a]

Regress-and	$t^{b}_{.05}$	$t^{c}_{.95}$	$H(0,1)^{d}$	$H(1,2)^{d}$	$H(1,4)^{d}$	$H(1,7)^{d}$
c	-2.62	2.39	0.057 (4.10)	0.114 (5.41)	0.146 (11.5)	0.263 (23.2)
y	-1.54	1.75	0.012 (2.55)	0.007 (2.25)	0.002 (4.20)	0.003 (6.63)

NOTE: The results are based on 1000 Monte Carlo replications.
[a]Equations (7)-(8) define the data generation process DGP2.
[b]The empirical 5-percent critical value of the t-ratio for the hypothesis β=1.
[c]The empirical 95-percent critical value of the t-ratio for the hypothesis β=1.
[d]The rows with θ=1 presents size based on asymptotic critical values with the empirical 5-percent critical values in parentheses; the remaining rows presents the size adjusted power.