Pricing in a Customer Market

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Abstract

In standard pricing models movements in demand are partially offset by price responses. In a customer market, however, I find that optimal pricing is to lower price/marginal cost markup in a period of high demand. Therefore price may magnify, rather than stabilize, demand movements.

I consider a monopolist selling a good of which first-time consumers are uncertain. Repeat customers, however, know the product works, and are therefore willing to pay discretely more than comparable new consumers. The monopolist trades off the pricing objectives of exploiting past customers and attracting new ones. In a period with many new potential customers it is optimal to give more importance to attracting; so the monopolist lowers its markup.

I. Introduction

It is difficult to reconcile the cyclical behavior of real wages with a classical model of pricing in goods markets. Many have pointed this out (e.g., Keynes, 1939, Patinkin, 1956, Barro-Grossman, 1971). The presence of excess capacity in recessions implies marginal cost declines relative to wage rates during downturns. If firms keep prices at a constant ratio to marginal cost, then prices should move procyclically relative to wages; so real wages should be countercyclical. The empirical evidence, however, is that real wages are, if anything, procyclical.[1]

Largely for this reason, a number of papers have extended the Keynesian framework to incorporate price stickiness in output markets (e.g., Patinkin, Solow-Stiglitz, 1968, Barro-Grossman, Malinvaud, 1977). Theoretical underpinnings for price stickiness in output markets represents a particular void in macroeconomics. Price stickiness has been potentially attributed to costs of nominal price adjustment, either due to upsetting customers (Okun, 1981, Rotemberg, 1982), or due to menu costs (Barro, 1972, Mankiw, forth., Blanchard, 1985). Pigou (1927) and Keynes considered such adjustment costs as potentially important, but also considered broader reasons why output prices might fail to move with cyclical movements in marginal cost. In particular, both speculated on reasons why monopoly power might be more important in bad times than in good, thereby causing the desired price markup over marginal cost to be higher in recessions. This could cause real wages to be procyclical despite procyclical movement in marginal cost relative to the wage rate. My effort is very much along this line (as are Stiglitz's, 1984, and Rotemberg-Saloner's, 1984).

I examine pricing in a "customer market". By customer market I mean a market where consumers develop some attachment to a product which they have purchased previously. (Papers developing related themes include Phelps-Winter, 1970, Okun, Schmalensee, 1981, and Glazer, 1984.) In my model this attachment arises because consumers have some uncertainty about whether a firm's product works satisfactorily. This uncertainty is resolved after they have purchased. A consumer who finds the product does work will then be willing to pay discretely more for it than prior to trying it (similarly to Schmalensee's consumers). This gives the firm a stronger pricing position relative to past customers than to new potential customers. In pricing, a firm must trade off the twin objectives of exploiting past customers and attracting new ones. (In this respect the firm's problem is like that of firms in Phelps-Winter.) In periods of high demand, defined as periods with high inflow of potential customers, the firm finds it profitable to give more weight to the market-size objective. Therefore, the firm lowers price relative to marginal cost in a period of high demand.

The next section presents the story for a single monopolist in partial equilibrium. The concluding section considers extending the model to a number of firms and to a general equilibrium.

II. A Customer Market

Consumers' Demand

I consider a monopolist selling its good over an infinite horizon. Consumption of the good is indivisible; in a given time period a consumer purchases one unit or none. Each period a new generation of consumers becomes available to the monopolist. Each generation remains available to the monopolist for only two periods. I assume the monopolist cannot price discriminate. In particular, the monopolist cannot charge different prices for the new and old generations.[2] I assume consumers differ in their valuation of the monopolist's product. Let $Q_t(v)$, for $\underline{v} < v < \overline{v}$, give the number of consumers of generation t willing to pay at least v for a unit of the good that performs to satisfaction (explained momentarily). Each consumer is aware of their own value of v before purchasing.[3] Both the monopolist and I treat $Q_t(v)$ as given; so the model is strictly partial equilibrium.

For some consumers the monopolist's product will be a dud. Being a dud is subjective--what is a dud for some persons is fine for others (e.g., some hate bad food, some hate bad service). Furthermore, consumers only find out the quality of the good by trying it. Such goods are what Nelson (1970) termed "experience goods". Each consumer expects with probability (1-q) that the good will be a dud for them. I assume their expectations are rational; for a share (1-q) of consumers the product is a dud. With uninteresting loss of generality, I assume a dud yields zero utility.

In the first period of life a consumer born in t will choose to purchase the monopolist's good if either condition (1) or condition (2) is satisfied:

$$(1) \quad qv > P_t \quad ,$$

or

$$(2) \quad qv + qdv > P_t + qdP_{t+1} \quad ,$$

where P is the monopolist's price and the parameter d reflects consumers' (common) rate of time discount, $[(1/d)-1]$. (1) says buy in the first period of life even though there is no intention of buying in the second period of life. (2) says buy if the cost of buying this period and buying next period only if it works is less than the expected benefit over both periods of life. For condition (2) consumers require knowledge of next period's price. I assume consumers, as well as the monopolist, have perfect foresight of shocks to the market. I also assume consumers are aware of the profit-maximization problem facing the monopolist. From this knowledge consumers will be able to infer price in t+1 from the monopolist's price in t. By purchasing in the first period of life a consumer gains knowledge of whether the good works. This is beneficial for the second period of life, unless P_{t+1} is significantly higher than P_t. Comparing equations (1) and (2) shows that (2) is the relevant consition as long as P_{t+1} is less than (P_t/q). For equilibria which evolve reasonably smoothly through time this will be true. I restrict attention to such equilibria. What this requires of parameters is discussed at length below.

In the second period of life a consumer who purchased in the first period of life and found the good to be a dud will not purchase. A consumer who purchased and found the good works will purchase again if:

(3) $v > P_{t+1}$.

But given (2) holds, (3) will hold as long as P_{t+1} is less than (P_t/q), which I am imposing. A consumer who did not purchase in the first period of life in t will purchase in t+1 if:

(4) $qv > P_{t+1}$.

But given (2) does not hold, (4) can only be true if P_t is greater than $(1+d-qd)P_{t+1}$. A restriction that price evolves reasonably smoothly, however, rules out this possibility.

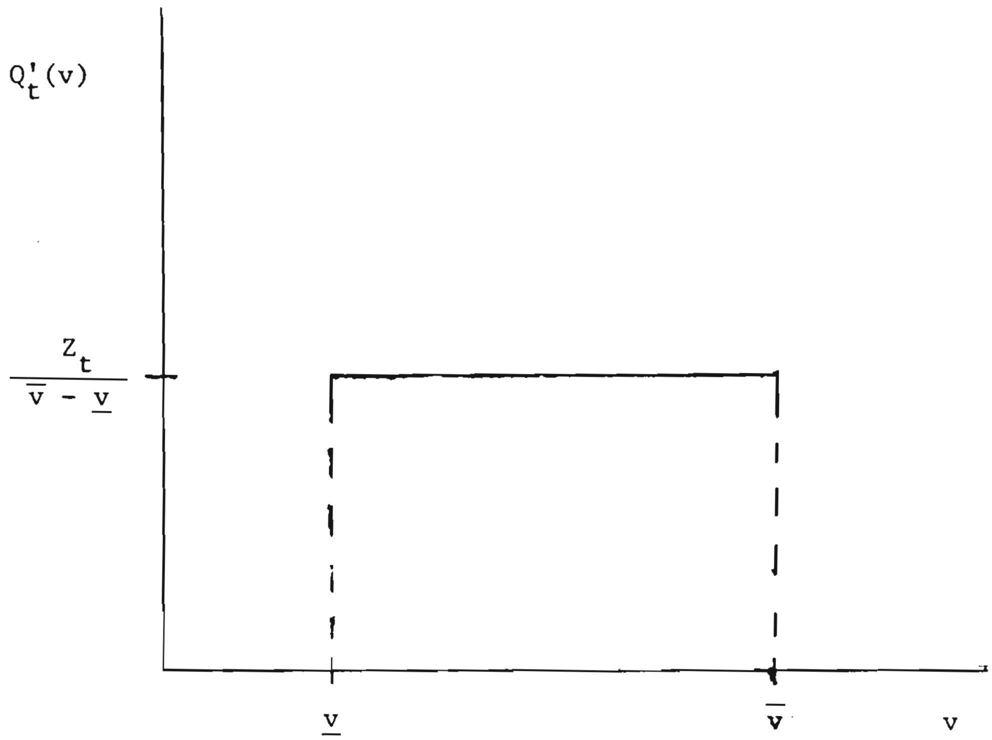
To summarize, for equilibrium where price changes by less than approximately the fraction (1-q), the decision to purchase is given by condition (2). Therefore the number of consumers purchasing in their first period of life in t is given by X_t:

(5) $$X_t = Q_t\frac{(P_t + qdP_{t+1})}{q(1+d)}$$.

Only consumers who purchased in the first period of life purchase in their second period; and a fraction q of these (all those for whom the good works) do purchase. So demand of the older generation in period t+1 is given by qX_t.

To make (5) operational requires an assumption on the form of $Q_t(v)$. I assume individuals are uniformly distributed with regard to v between a minimum value $\underline{v}$ and a maximum value $\overline{v}$, as pictured in Figure 1. The uniform distribution will make X_t a linear function of price. $\underline{v}$ and $\overline{v}$ are assumed constant across generations of consumers. I allow the

Figure 1: Distribution of Consumers



density to be subject to multiplicative shocks through the variable Z_t. The number of consumers at any point v is $Z_t/(\overline{v}-\underline{v})$; and the total number of potential customers in a generation is Z_t. With linear demand, multiplicative shocks generally would not affect elasticity of demand, but will here by shifting the relative importance of old and new market participants.

Given this form for $Q_t(v)$, (5) becomes:

$$(6) \quad X_t = Z_t(a - bP_t - bqdP_{t+1}) \quad , \qquad \text{where:}$$

$$(7) \quad a = \frac{\overline{v}}{(\overline{v} - \underline{v})}$$

$$(8) \quad b = \frac{1}{q(1+d)(\overline{v} - \underline{v})} \quad .$$

Monopolist's Problem

I assume the monopolist maximizes discounted longrun profits. Demand in period t will equal the demand of the new generation plus the demand of the old generation. The discussion above gives this demand as:

$$(9) \quad D_t = X_t + qX_{t-1} \quad .$$

There is a time consistency issue. Demand of the new generation in t depends on price for period t+1; but once t+1 arrives demand of the older generation is unaffected by marginal movements in P_{t+1}. I assume that the monopolist cannot precommit its price for the following period. I examine the time-consistent solution (Strotz's, 1956, terminology) in which the monopolist and consumers infer next period's price by solving the monopolist's problem for the following period. Appendix 1 examines the case when the monopolist can credibly precommit its price for the following period.

The monopolist's future longrun profits as of t are given by:

$$(10) \quad \pi_t = \sum_{i=t}^{\infty} d^{i-t}(P_i - c)(X_i + qX_{i-1}) \quad .$$

I have assumed that the monopolist has the same rate of time discount as consumers. I have taken marginal cost to be constant and equal to c. The monopolist's problem can be written out as:

$$\begin{aligned}(11) \quad \max_{P_t} \ \pi_t = \ & (P_t - c)(X_t + qX_{t-1}) \\ & + \lambda_t[X_t - Z_t(a - bP_t - bqdP_{t+1})] \\ & + d(P_{t+1} - c)(X_{t+1} + qX_t) \\ & + d\lambda_{t+1}[X_{t+1} - Z_{t+1}(a - bP_{t+1} - bqdP_{t+2})] \\ & + \ldots\ldots \quad ,\end{aligned}$$

where λ_i is the Lagrange multiplier for the demand constraint in period i.

First-order conditions are:

$$(12) \quad X_t : \ (P_t - c) + \lambda_t + qd(P_{t+1} - c) = 0 \quad ,$$

$$(13) \quad \lambda_t : \ X_t - Z_t(a - bP_t - bqdP_{t+1}) = 0 \quad ,$$

$$(14) \quad P_t : \ X_t + qX_{t-1} + \lambda_t Z_t b = 0 \quad ,$$

$$(15) \quad P_{t+1} : \ \lambda_t Z_t bq + X_{t+1} + qX_t + \lambda_{t+1} Z_{t+1} b = 0 \quad .$$

Note that the first-order condition for P_{t+1} differs in form from that for P_t updated one period. This reflects the time consistency issue. The solution without precommiting assumes that each period the monopolist chooses price so as to maximize discounted profits from that point forward. Therefore, the relevant first-order condition (14) ignores the effect of P_t on X_{t-1}. By constrast, if in period t-1 the monopolist precommitted for an optimal price for period t, the relevant first-order condition would be (15) back-dated one period. This condition would reflect an effect of P_t on X_{t-1}. (See Appendix 1.)

Solution with No Precommiting

Combining equations (12), (13), and (14) yields a second-order difference equation in P_t.

$$(16)\quad 2qdP_{t+1} + [2 + q^2d(Z_{t-1}/Z_t)]P_t + q(Z_{t-1}/Z_t)P_{t-1} = (1 + qd)c + [1 + q(Z_{t-1}/Z_t)](a/b)$$

This equation has nonconstant coefficients, but for given paths for Z_t has convenient solutions. Momentarily I consider the pricing solution for different hypothetical paths for Z_t.

The steady-state price, $\overline{P}$, is given by:

$$(17)\quad \overline{P} = \frac{(1+q)(a/b)}{(2+q)(1+qd)} + \frac{c}{(2+q)}$$

Note that $\overline{P}$ is independent of the level of demand, Z. For the steady-state markup to be nonnegative requires:

$$(18)\quad (a/b) \geq (1 + qd)c \quad ,$$

which means

$$\bar{v} \geq \frac{(1 + qd)c}{q(1 + d)}$$

Equation (15) can be regrouped into a first-order difference equation in terms of $(P_t + qdP_{t+1})$.

$$(19) \quad (P_t + qdP_{t+1}) + (q/2)(Z_{t-1}/Z_t)(P_{t-1} + qdP_t) = [1 + q(Z_{t-1}/Z_t)](a/b) + (1 + qd)c \quad .$$

This equation and the demand equation (13) together yield a first-order difference equation in new demand, X_t.

$$(20) \quad X_t + (q/2)X_{t-1} = (Z_t/2)[a - (1 + qd)bc] \quad ,$$

which has the solution:

$$(21) \quad X_t = (1/2)[a - (1 + qd)bc] \sum_{j=0}^{\infty} (-q/2)^j Z_{t-j} \quad .$$

Despite the fact that the monopolist and consumers are forward looking, X_t depends only on past realizations of demand, Z. This is because future realizations of Z have precisely offsetting effects on P_t and qdP_{t+1}. Equation (21) shows X_t to be positively related to demand at t and, with dampened magnitude, to demand at lags of even numbers. X_t is negatively related to demand at odd lags. Steady-state X_t, $\bar{X}$, equals:

$$(22) \quad \bar{X} = \frac{\bar{Z}[a - (1 + qd)bc]}{(2 + q)} \quad .$$

Price will be shown to be countercyclical--P_t is low when Z_t is high, and visa versa. Therefore, there is a multiplier effect on X_t. The elasticity of X_t with respect to Z_t evaluated near the steady-state is:

$$(23) \quad (dX_t/dZ_t)(Z/X) = 1 + (q/2) \quad .$$

So less uncertainty (larger q) is associated with a larger multiplier.

Figure 2 displays the behavior of X for the case where Z is constant with the exception of an anticipated, positive blip of one percent in period t. Fiqure 2 gives X relative to its steady-state value $\overline{X}$, assuming a value for q of .5. X is unaffected prior to t. With the multiplier effect, the one-percent increase in Z raises X_t by 1.24 percent. The impact on X beyond t dampens quickly; at t+3 the negative effect on X has magnitude of one-fiftieth of one percent.[4]

Total demand, D_t, is less affected by Z_t than is new customer demand because the previous period's customers are inherited. Total demand is:

$$\begin{aligned} (24) \quad D_t &= X_t + qX_{t-1} \\ &= (1/2)[a - (1+q)bc] \sum_{j=0}^{\infty} (-q/2)^j [Z_{t-j} + qZ_{t-1-j}] \\ &= (1/2)[a - (1+q)bc][Z_t + (q/2) \sum_{j=0}^{\infty} (-q/2)^j Z_{t-1-j} \quad . \end{aligned}$$

The elasticity of D_t with respect to Z_t equals $(1 + q/2)/(1+q)$. By comparison, if prices were constant the elasticity would be only $1/(1+q)$. So there is a magnifying effect on the elasticity of D_t of the same magnitude, $(1 + q/2)$, as the effect on the elasticity of X_t with respect to Z_t.

Figure 2: Behavior of X

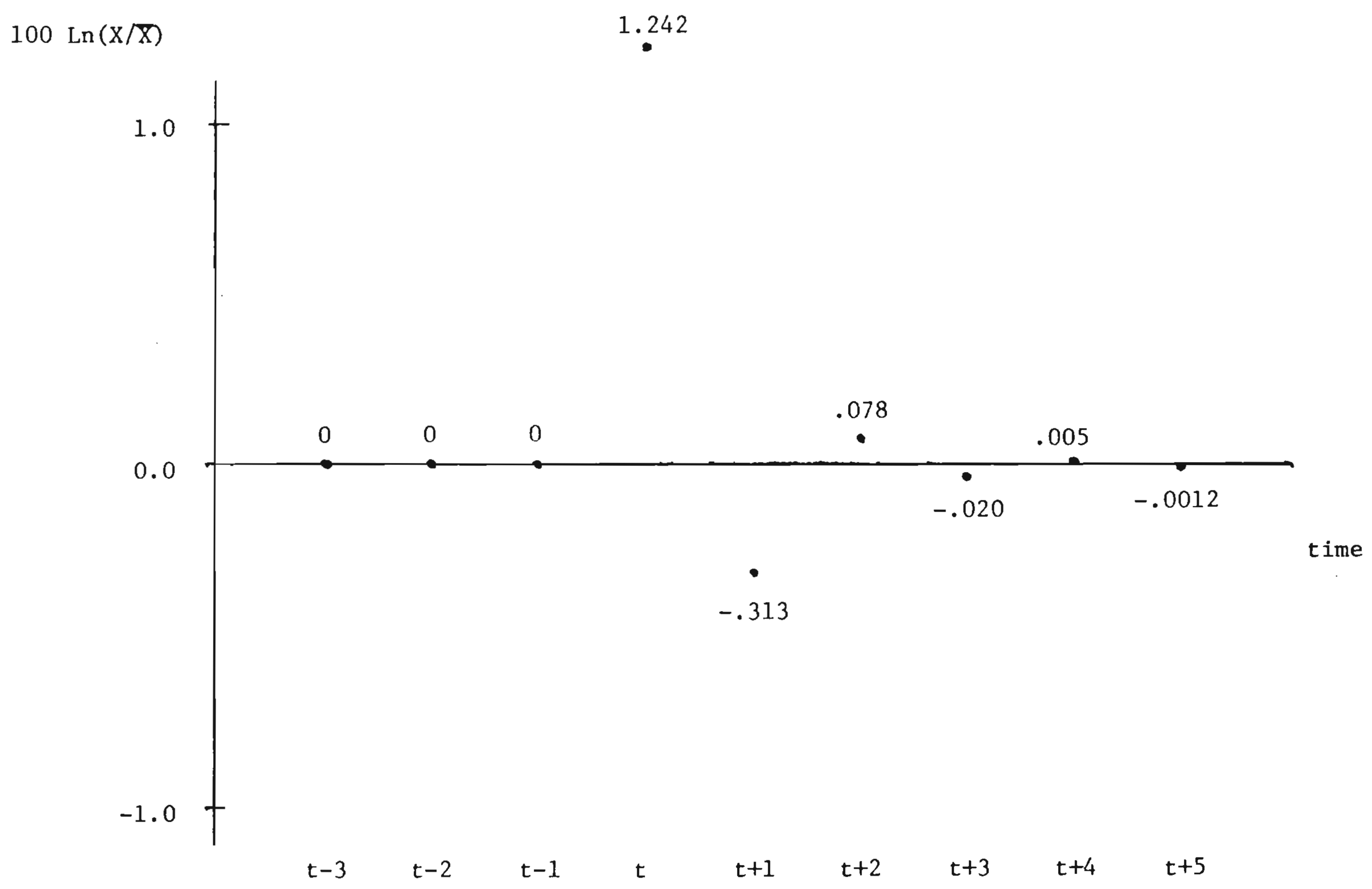


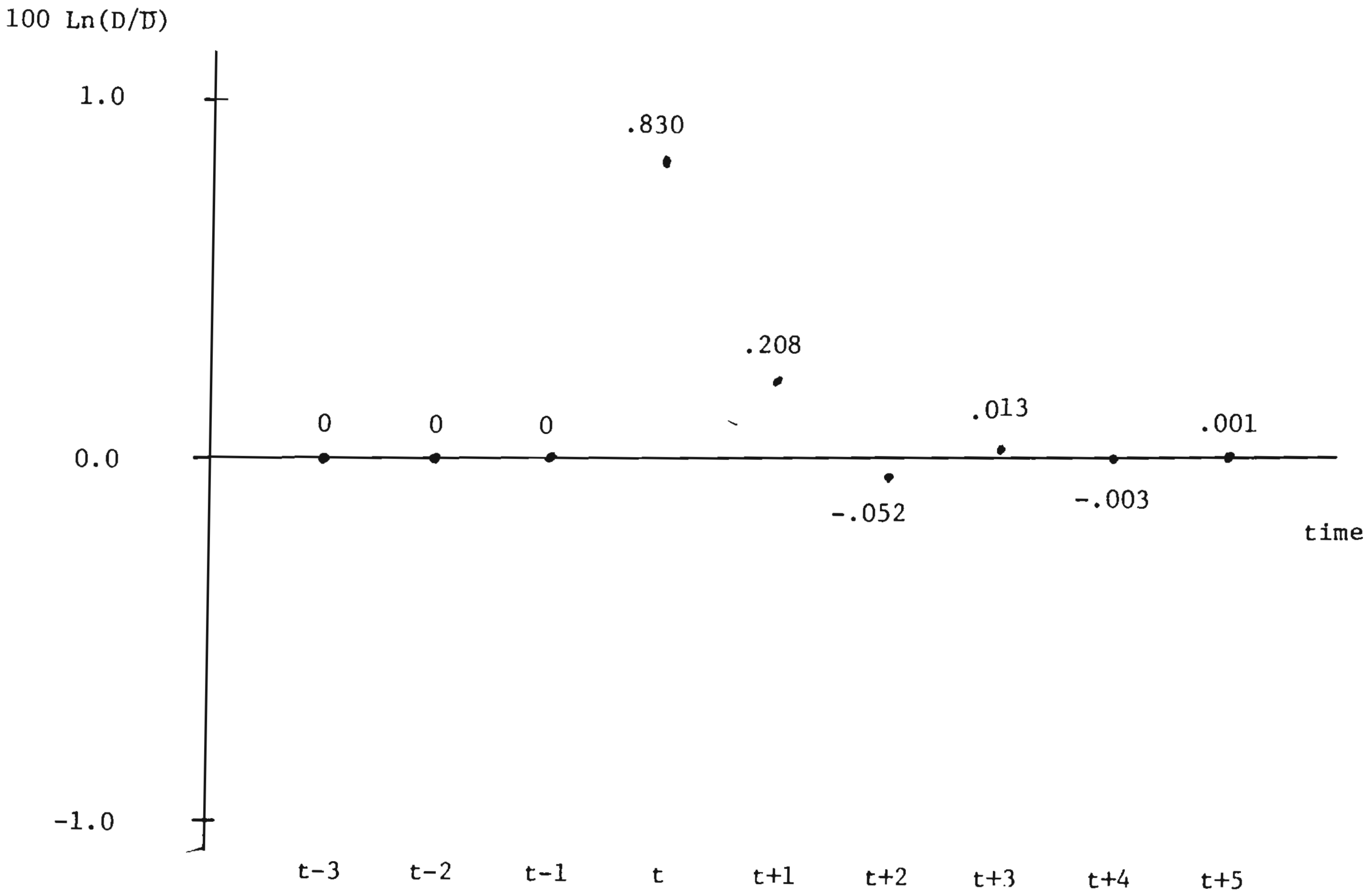
Figure 3 graphs the response of D to the one-time, one-percent blip in Z at time t, again assuming q equals .5. D is constant prior to t. At time t, D is .830 percent above its steady-state value. The positive effect remains through period t+1, with D_{t+1} being raised by .208 percent. The influence of the blip dampens quickly.

Deviations

The time-consistent equilibrium was derived under the presumption that the monopolist takes the period t demand of second-generation consumers as given at X_{t-1}. This is only correct if the monopolist prices within the range where demand of second-generation consumers is unaffected by marginal price movements. There are two ways the monopolist might conceivably deviate from this "smooth-pricing" equilibrium. The monopolist could disregard attracting any new customers in period t, and instead fully exploit those customers inherited from period t-1. Alternatively, the monopolist could lower P_t sufficiently below P_{t-1} to induce some consumers who failed to purchase in their first period in t-1 to purchase in their second period in t.

Appendix 2 derives restrictions on the parameters q and marginal cost c that are sufficient to rule out these deviations as unprofitable (see equations B9 and B11). The appendix treats the particular case of constant demand Z, and a zero discount rate (d equal to 1). For sufficiently small variations in Z and a sufficiently small discount rate, however, the sufficient conditions would be arbitrarily close to these. The restrictions required on q and c are highly nonlinear, and so are difficult to characterize in general terms. For c equal to zero, q less than 0.7 is sufficient for deviations to be unprofitable. For

Figure 3: Behavior of D



higher values of c, the acceptable range for q (from top to bottom) is generally smaller than this.

Demand Fluctuations

To this point I have solved for the optimal price only for the steady state (though $[P_t + qdP_{t+1}]$ is implicitly given by equation (21)). I now consider how price responds to demand fluctuations. I find that price is low relative to marginal cost when demand, Z, is high, and high relative to marginal cost when demand is low. Because marginal cost is taken as constant, movements in price relative to marginal cost are given simply by movements in price. I consider two varieties of demand variations. The first is Z fluctuating forever between periods of high and low demand. The second is a one-time high value for Z.

Proceeding, suppose that Z is high in odd time periods and low in even time periods; let (Z_{i-1}/Z_i) equal (1-y) for odd period i, and (Z_{j-1}/Z_j) equal (1+y) for even period j. Equation (16) can be written as:

$$(25) \quad \{2qdF + [2 + q^2d(Z_{t-1}/Z_t)] + q(Z_{t-1}/Z_t)L\} P_t = (1+q)(a/b) + (1+qd)c + q(a/b)[(Z_{t-1}/Z_t)-1] \quad ,$$

where L is the backward shift or lag operator, and F is the forward shift operator (equal to L^{-1}). For odd periods (25) becomes:

$$(26) \quad \{2qdF + [2 + q^2d(1-y)] + q(1-y)L\} P_t = (1+q)(a/b) + (1+qd)c + q(a/b)[(Z_{t-1}/Z_t)-1] \quad ,$$

Factoring gives:

$$(27)\quad P_i = -(q/2)(1-y)P_{i-1} + (1+q)(a/b)/(2+2qd) + (c/2) - qy(a/b)/(2+2qd) \quad ,$$

Solving similarly for even periods gives:

$$(28)\quad P_j = -(q/2)(1+y)P_{j-1} + (1+q)(a/b)/(2+2qd) + (c/2) + qy(a/b)/(2+2qd) \ .$$

To solve for the absolute value of the odd period price I use the fact that P_{i-1} equals the even period price P_j, and P_{j-1} equals the odd period price P_i. Therefore, combining (27) and (28) gives the odd period price as:

$$(29)\quad P_i = \frac{[1-(q/2)(1-y)][(1+q)(a/b)/(1+qd) + c] - [1+(q/2)(1-y)]qy(a/b)/(1-qd)}{2 - (q^2/2)(1-y^2)}$$

Doing likewise for even periods:

$$(30)\quad P_j = \frac{[1-(q/2)(1+y)][(1+q)(a/b)/(1+qd) + c] + [1+(q/2)(1+y)]qy(a/b)/(1-qd)}{2 - (q^2/2)(1-y^2)}$$

Of primary interest is P_i relative to P_j. Comparison of equations (29) and (30) shows price to be countercyclical. For example, for d equal to one, q equal to .5, y equal to .01, and c equal to 0, (P_i/P_j) equals .970. Thus a one percent movement in demand is associated with a three percent countercyclical movement in price. For c equal to (a/b)(1+q) (the largest value giving positive steady-state profits) and keeping the other parameters as before, (P_i/P_j) equals .984; that is a 1.6 percent countercyclical movement in price.[5]

Now consider the case where Z remains constant with the exception of a one-time upward blip in period t; let (Z_{i-1}/Z_i) equal (1-y) for i equal to t, (1+y) for i equal to t+1, and one for all other periods. For period t the appropriate equation in P_t is given by equation (27), which can alternatively be written as:

$$(31)\quad P_t = \frac{(1+q)(a/b)/(1+qd) + c}{2 + q(1-y)} + \frac{(q/2)(a/b)[(Z_{t-1}/Z_t)-1]}{[1 + (q/2)(1-y)L]\quad [1 + qdf]}$$

which, in turn, can be written as:

$$(32)\quad P_t = [2+q(1-y)]^{-1}[(1+q)(a/b)/(1+qd) + c] + \frac{(q/2)(a/b)}{[1 - (q^2d/2)(1-y)]}\left[\frac{1}{(1 + qdF)} - \frac{(q/2)(1-y)L}{[1 + (q/2)(1-y)L]}\right][(Z_{t-1}/Z_t)-1] .$$

The solution for P_t is:

$$(33)\quad P_t = \frac{(1+q)(a/b)/(1+qd) + c}{2 + q(1-y)} - \frac{q(a/b)(1+qd)y}{2 - q^2d(1-y)} .$$

Solving similarly for P_{t+1} gives:

$$(34)\quad P_t = \frac{(1+q)(a/b)/(1+qd) + c}{2 + q(1+y)} + \frac{q(a/b)(1+qd)y}{2 - q^2d(1+y)} .$$

(P_t/P_{t+1}) is clearly less than one. For prior to t price is given by:

$$(35)\quad P_{t-j} = \frac{(a/b)(1+q)/(1+qd) + c}{2+q} + \frac{(a/b)q(-1)^{j-1}(qd)^{j}(1+qd)y}{2 - q^2 d} \qquad \text{for } j>0 \quad .$$

This oscillates around the steady-state price, being above at odd lags from t and below at even lags. The oscillations dampen moving away from t. For after t+1 price is given by:

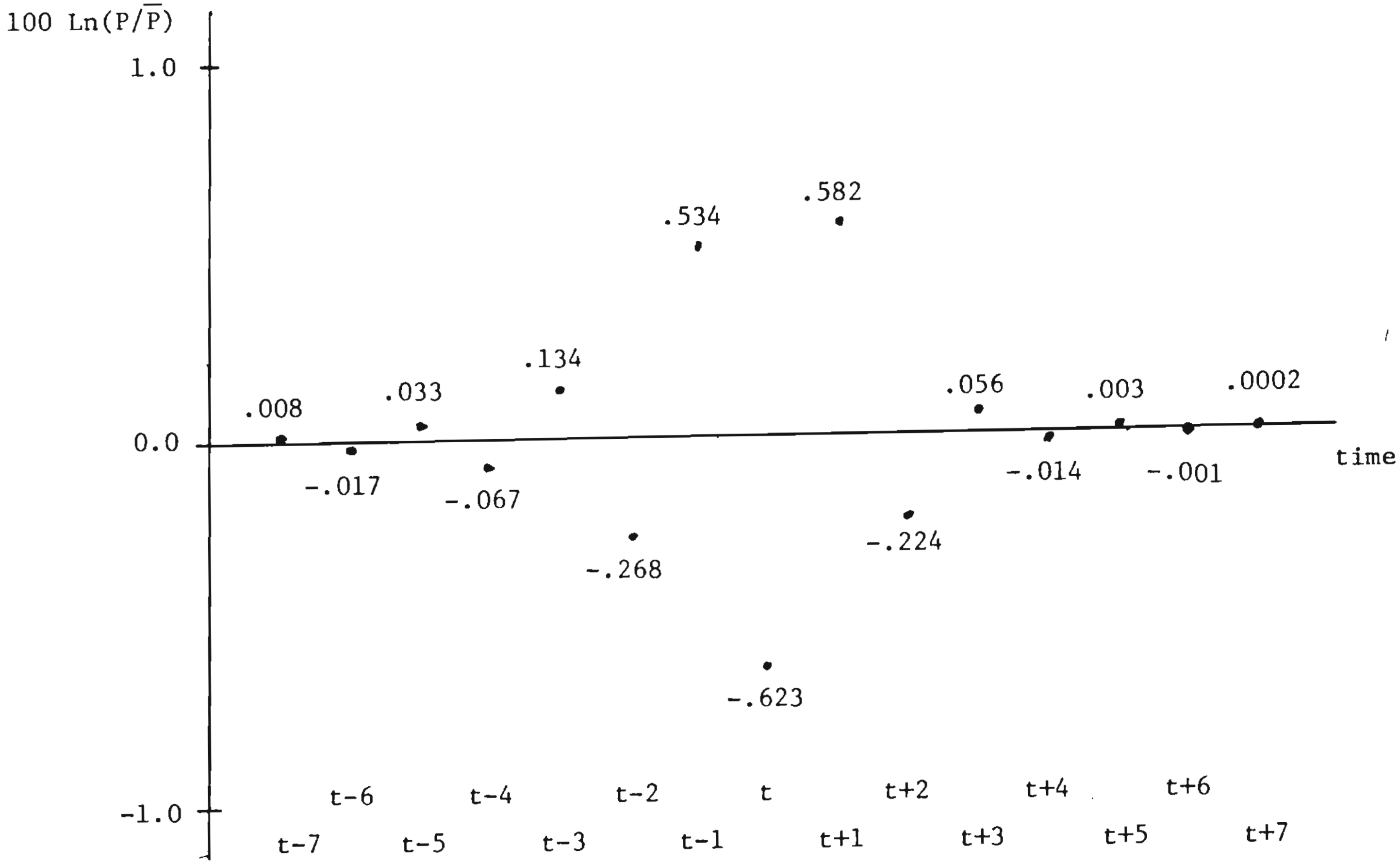
$$(36)\quad P_{t+j} = \frac{(a/b)(1+q)/(1+qd) + c}{2+q} + \frac{(a/b)q(-q/2)^{j-1}(1+ q/2)y}{2 - q^2 d} \qquad \text{for } j>1 \quad .$$

This also oscillates around the steady-state price, being above at odd steps ahead and below at even. The oscillations dampen moving away from t.

As an example, I consider the case where d equals one, q equals .5, y equals .01, and c equals zero. Figure 4 graphs the behavior of price. The one percent increase in new demand in t lowers P_t by .623 percent below the steady-state price, $\overline{P}$. P_{t-1} and P_{t+1} are .534 and .582 percent above $\overline{P}$ respectively. Before t-1 and after t+1 the effect on price dampens much more quickly.

Appendix 1 shows that price markup remains coutercyclical when the monopolist is able to precommit its future price.

Figure 4: Behavior of P



III. Extensions

To briefly summarize, I find that a multiplicative increase in the number of potential new market participants leads the monopolist to lower price relative to marginal cost. For the case of constant marginal cost (and no precommiting of price) the decline in price induces a multiplier effect on the quantity demanded by new customers.

A natural extension of the model is to abandon the monopolist for monopolistically competitive firms. The essential results, however, would remain. Having sold to a consumer, a firm would possess market power with respect to that consumer because the consumer knows its product works. If there were no inflow of new consumers, firms would simply exploit their existing customers at a price above marginal cost (this is Diamond's, 1971 result). A high inflow of new market participants, however, makes attracting new customers more attractive relative to exploiting existing customers; therefore it will cause firms to lower price markups.

A more interesting extension is from partial to general equilibrium. Of particular interest is whether plausible shocks to consumers would lead to a relative increase in the number of consumers considering entering a customer market. The simplist general equilibrium setting is to suppose that there exists only two goods: the customer market good and a divisible good, utility from which is certain and equal to $f(S)$, where S is its consumption. I assume $f'(S)$ is greater than zero and $f''(S)$ is less than zero. Let all consumers receive a nonstorable endowment of the certain good equal to E in each period of life; and let the price of the certain good be internationally determined at the numeraire value of one.[6] Consumers continue to differ in their valuation of the monopolist's good, its utility being indexed by v. All consumers for whom equation (37) holds will purchase the monopolist's good.

$$(37) \quad v > f(E) - [f(E - P_t) + qdf(E - P_{t+1})]/(1+qd) \quad .$$

Suppose a generation arrives with a higher than usual endowment. The concavity of f implies demand is increasing in E. The increase in demand, however, takes the form of an additive increase to $\underline{v}$ and $\overline{v}$ (a shifting right of the distribution in figure 1). Such a shift will not lead the monopolist to lower price. The monopolist just enjoys the fact that more consumers purchase at any given price.

To obtain the result of a reduction in markup requires a richer version of a customer market than I have presented. If the monopolist is dropped in favor of monopolistically competitive firms then the result should obtain. Such firms price along two margins. A reduction in price attracts more consumers into the market and attracts customers away from competitors. The uncertainty about untried firms causes this second margin to effectively evaporate for existing customers in the market. Therefore, an increase in demand of the type described, though not altering the first margin, will significantly increase the importance of the second margin, causing firms to lower price markups.

The result may also obtain if the assumptions of perfect foresight and consumers living only two periods are abandoned. Let consumers be available to the monopolist for T periods. Suppose all consumers experience an unexpected increase in their permanent income in period t. The effect on first-generation consumers will be as described directly above. There will now, however, also be an important effect on the demand by second-generation and older consumers who did not purchase the good prior to t. There will be consumers from generations aged 2 through T who have never purchased that would now be attracted by a marginal decline in price because their incomes have risen. At the same time, for some of these older-generation consumers the increase in permanent income will be sufficiently large that they will now purchase regardless of marginal price movements. But for a reasonably small change in permanent income the proportionate increase in inframarginal consumers will be quite small relative to the proportionate increase in consumers at the pricing margin. This should lead the monopolist to lower its price markup.

Appendix 1--Precommiting Solution

I now suppose that each period the monopolist can credibly precommit its price for the following period. This could occur through contracting, or conceivably through reputational effects.

The relevant first-order condition for P_t takes into account the effect of P_t on X_{t-1}. This condition is equation (15) lagged one period.

$$\text{(A1)} \quad \lambda_{t-1}Z_{t-1}bq + X_t + qX_{t-1} + \lambda_t Z_t b \;=\; 0 \quad .$$

Combining equations (12), (13), and (A1) yields a difference equation in P_t.

$$\text{(A2)} \quad qdP_{t+1} + [1 + q^2 d(Z_{t-1}/Z_t)]P_t + q(Z_{t-1}/Z_t)P_{t-1} \;=\; (1/2)[1 + q(Z_{t-1}/Z_t)][(a/b) + (1+qd)c] \quad .$$

The steady-state price is:

$$\text{(A3)} \quad \overline{P} \;=\; (1/2)(a/b)/(1+qd) \;+\; (1/2)c \quad .$$

This is lower than the time-consistent price given in equation (17).

The conditions yield a first-order difference equation for new demand, X_t.

$$\text{(A4)} \quad X_t + qX_{t-1} \;=\; (1/2)[Z_t + qZ_{t-1}][a - (1+qd)bc] \quad ,$$

which has the solution:

$$(A5)\quad X_t = (Z_t/2)[a - (1+qd)bc] \quad .$$

The elasticity of X_t with respect to Z_t equals one; so there is no multiplier effect.

The difference in steady-state profits between precommiting and not precommiting equals:

$$(A6)\quad DIF = \frac{Z(1+q)}{4(2+q)^2}\left[\frac{(4q+q^2-4d)}{(1+qd)}(a^2/b) - (4q+2q^2-4d)ac + q^2(1+qd)bc^2\right] \quad .$$

This can be positive or negative. Thus, even if it is possible to credibly precommit, the monopolist may not wish to do so.

Price (and therefore the markup) remains countercyclical. Consider the text example of (Z_{t-1}/Z_t) fluctuating between (1-y) in odd periods and (1+y) in even periods. The relative price between odd and even periods is:

$$(A7)\quad \frac{P_{odd}}{P_{even}} = \frac{[1-q(1-y)](1+q)/(1+qd) - [1+q(1-y)](qy)/(1-qd)}{[1-q(1+y)](1+q)/(1+qd) + [1+q(1+y)](qy)/(1-qd)}$$

As an example, for d equal to one, q equal to .5, and y equal to .01, (P_{odd}/P_{even}) equals 0.961; so a one percent movement in new demand is associated with a 4 percent countercyclical movement in price.

Appendix 2--Sufficiency Conditions for the Smooth Pricing Equilibrium

In any period the monopolist has three potential sources of demand: New-generation consumers, old-generation customers who tried and liked the product, and old-generation consumers who did not try the product. The text treats the case where pricing is reasonably smooth across periods so that a marginal price change would repel no old-generation customers who like the product nor attract any old-generation consumers who did not try the product. Here I derive sufficient conditions for the smooth pricing path to be the most profitable. I consider the particular simplified case of d equal to one (zero discount rate) and constant demand Z.

The noncontinuity of demand makes a standard dynamic programming approach to calculating profits under diverging intractable. Instead I exploit the fact that the monopolist's price in t only affects its demand curve for the immediately following period, t+1. Given the monopolist diverges from the smooth pricing equilibrium in t, it will follow some pricing path from t+1 forward. Alternatively, suppose the monopolist followed this same price path from t+1 forward after <u>not</u> diverging in t. Profits would be the same with respect to all generations of consumers arriving after t. Profits from selling to the old generation in t+1, however, will differ. I choose the price, P^*_{t+1}, that makes this difference most favorable for diverging in t. Therefore, I need not consider the divergent path for prices beyond t+1. Given P^*_{t+1}, by dynamic programming I choose the divergent price P^*_t, and a path of prices leading up to P^*_t, that makes diverging look most favorable. In maximizing profits the monopolist would in general charge a different price than P^*_{t+1}. This implies the true comparison of profits between diverging and not diverging would be less favorable to diverging than I calculate; so the conditions I obtain for diverging to be unprofitable are sufficient but not necessary.

I first consider the monopolist diverging by increasing price in t to exploit the demand curve of old-generation customers who learned in t-1 that they like the product. I then consider diverging by lowering price in t to attract the demand of old-generation consumers who did not purchase in t-1. The monopolist's initial divergence must be of one type or the other. If neither is profitable, then no price path containing a divergence can be optimal.

Suppose the monopolist raises price in t to exploit its inherited customers from t-1. I need only consider the case where this price is sufficiently high that no first generation consumers purchase--the monopolist did not price at the upper region of the new generation's demand curve when this meant losing no inherited customers, so it certainly will not if it will lose some of these customers. Therefore, having diverged in t, the monopolist carries over no customers to t+1. This is a drawback to revenue in t+1 in that no second generation consumer will pay more than qv in t+1. It is a benefit, however, in that no consumers know the good to be a dud. If the monopolist had charged the smooth price in t everyone with v greater than v^* would have purchased, where:

$$\text{(B1)} \quad v^* = (1+q)[(a/b) + c]/[2q(2+q)] \quad .$$

The benefit in t+1 of having diverged in t is maximized when P_{t+1} is in the range between qv^* and v^*--so that having not diverged the monopolist would attract no first-time, second-generation consumers. The difference in profits in t+1 for having diverged versus not having diverged equals:

$$\text{(B2)}\quad Dif_{t+1} = (P_{t+1}-c)\left[\int_{P_{t+1}/q}^{\bar{v}} [Z/(v-v)]dv - q\int_{v^*}^{\bar{v}} [Z/(v-v)]dv\right] \quad .$$

This is maximized by:

$$\text{(B3)}\quad P^*_{t+1} = [(a/b) + (4+3q+q^2)(c/2)]/[2(2+q)] \quad ,$$

at which:

$$\text{(B4)}\quad Dif^*_{t+1} = Zb[(a/b) - (4+q-q^2)(c/2)]^2/[2(2+q)^2] \quad .$$

This is unambiguously positive and independent of P^*_t.

Turning to period t, it is straightforward to show that the diverging monopolist should charge the highest price at which no inherited customers will leave. This price is:

$$\text{(B5)}\quad P^*_t = (1/4q)(2-q^2)P^*_{t-1} + [(a/b) + c]/4 \quad .$$

Profits diverging in t equal:

$$\begin{aligned}\text{(B6)}\quad (P^*_t-c)qX_{t-1} = &- (Z/8)b(2-q^2)^2(P^*_{t-1})^2 \\ &+ (Z/8)[2(1-q)a + 2qbc]P^*_{t-1} \\ &+ (Z/8)[(2q-q^2)(a^2/b) - (6q-2q^2)ac + 3q^2bc^2] \quad .\end{aligned}$$

By comparison, profits not diverging in t are given by:

$$\text{(B7)} \quad (\overline{P}-c)(1+q)\overline{X} = Zb[(1+q)/(2+q)^2][(a/b) - (1+q)c]^2 \quad .$$

I allow the diverging monopolist to have chosen the nondiverging prices in t-1, t-2, and on back, which make diverging appear most profitable. Optimal prices before t-1 can be written as a function of P^*_{t-1}. The nondiverging monopolist would simply charge the steady-state price P at all preceeding periods. The difference in profits between the two paths equals:

$$\begin{aligned}
\text{(B8)} \quad \mathrm{Dif}^*_{<t-1} = & - Z(2-q^2)(b/2)(P^*_{t-1})^2 \\
& + Z[q(2-q^2)/(2+q)](a + bc)P^*_{t-1} \\
& + Zb(2-q^2)(1-2q)[(a/b) + c]^2/[2(2+q)^2] \\
& + Z[(4+2q-3q^2+q^4)a + (4-2q-10q^2-2q^3+2q^4+q^5)bc] \\
& \quad [P^*_{t-1} - (a/b + c)/(2+q)]/[2(1+q)(2+q)] \quad .
\end{aligned}$$

The total difference in profits diverging versus not diverging is given by combining equations (B4), (B6), (B7), and (B8).

$$\begin{aligned}
\text{(B9)} \quad \mathrm{DIF}^* = & Zb[(a/b) - (4+q-q^2)(c/2)]^2/[2(2+q)^2] \\
& - (Z/8)b(2-q^2)^2(P^*_{t-1})^2 \\
& + (Z/8)[2(1-q)a + 2qbc]P^*_{t-1} \\
& + (Z/8)[(2q-q^2)(a^2/b) - (6q-2q^2)ac + 3q^2bc^2] \quad . \\
& - Zb[(1+q)/(2+q)^2][(a/b) - (1+q)c]^2 \\
& - Z(2-q^2)(b/2)(P^*_{t-1})^2 + Z[q(2-q^2)/(2+q)](a + bc)P^*_{t-1} \\
& + Zb(2-q^2)(1-2q)[(a/b) + c]^2/[2(2+q)^2]
\end{aligned}$$

$$+ Z[(4+2q-3q^2+q^4)a + (4-2q-10q^2-2q^3+2q^4+q^5)bc]$$
$$\cdot [P^*_{t-1} - (a/b + c)/(2+q)]/[2(1+q)(2+q)] \quad .$$

I still need to define P^*_{t-1}. Maximizing (B9) over P_{t-1} gives:

$$\text{(B10)} \quad P^*_{t-1} = \frac{(12+14q-4q^2-7q^3+q^5)(a/b) + (8+8q-6q^2-8q^3-3q^4+q^5)c}{(6-q^2)(2+q)(2-q^2)(1+q)}$$

Sufficiency will require that (B9), given (B10), be less than zero.

I carry out a similar procedure with respect to diverging in t to attract second-generation consumers who did not purchase in t-1. The equation corresponding to equation (B9) is:

$$\text{(B11)} \quad DIF^* =$$
$$Zb(q^2/8)(2-q^2)[(2-q^2)P^*_t + (a/b) - 3c][(a/b + c)/(2+q) - P^*_t]$$
$$+ (Z/2)(P^*_t-c)[(1-2q+q^3)P^*_{t-1} - (6-q^2)P^*_t + (2+2q-q^2)(a/b) - (q^2/2)c]$$
$$- Zb[(1+q)/(2+q)^2][(a/b) - (1+q)c]^2$$
$$- Z(2-q^2)(b/2)(P^*_{t-1})^2 + Z[q(2-q^2)/(2+q)](a + bc)P^*_{t-1}$$
$$+ Zb(2-q^2)(1-2q)[(a/b) + c]^2/[2(2+q)^2]$$
$$+ Z[(4+2q-3q^2+q^4)a + (4-2q-10q^2-2q^3+2q^4+q^5)bc]$$
$$\cdot [P^*_{t-1} - (a/b + c)/(2+q)]/[2(1+q)(2+q)] \quad ,$$

with:

$$\text{(B12)} \quad P^*_t = \frac{2(1-2q+q^3)\, P^*_{t-1}}{(24-4q^4+q^6)} + \frac{(16+24q-6q^3+q^5+q^6)(a/b) + (48+24q+4q^2-10q^4-3q^5+q^6)c}{2(2+q)(24-4q^4+q^6)}$$

and:

$$\text{(B13)} \quad P^*_{t-1} = \frac{(4+2q-q^2-q^3)(a/b) + [(4+2q-6q^2-4q^3+q^5)/(1+q)]c}{2(2+q)(2-q^2)} \quad .$$

A sufficient condition for divergences to be unprofitable is that equations (B9) and (B11) both be less than zero. The text discusses for what parameter values this is true.

Notes

1. For a summary of the evidence see Geary-Kennan (1982).

2. If the monopolist is allowed to charge different prices for old and new customers the paper's basic result would still follow. The monopolist would charge a discreetly higher price to old customers (because they are willing to pay it). In a period with a proportionately large number of new customers the relative number of customers receiving the lower price would rise; so the <u>average</u> markup being charged would decline.

3. Alternatively, one could imagine that consumers become aware of their value for v only after purchasing. (This is probably true for virtually all goods.) This is actually in the same spirit as my assumption that with probability (1-q) consumers learn the product is a dud for them. I restrict the form the uncertainty can take for the sake of tractability. The more general problem should presumably lead to similar conclusions.

4. The result that X oscillates above and below X after t is clearly dependent on the assumption that customers are available for only two periods. Suppose, more generally, that customers remain for a number of periods. The monopolist would still respond to a high level of Z_t by lowering P_t, making X_t greater than X. After t, however, the monopolist would have a number of periods to profit from the large number of customers attracted in t. P would remain above P, and X below X, for a number of periods (gradually dampening toward $\overline{P}$ and $\overline{X}$).

5. The response of price to demand is decreasing in marginal cost, c, because the markup is decreasing in c. A low markup means new

customers appear relatively less attractive.

6. I further assume that the monopolist conjures up its good at zero marginal cost so that demand does not feed back to consumers' budget constraint. Also, the monopolist spends its profits only on the certain good.

References

Barro, R.J., "A Theory of Monopolistic Price Adjustment." Review of Economic Studies 39 (1972): 17-26.

Barro, R.J., and Grossman, H.I., "A General Disequilibrium Model of Income and Employment." American Economic Review 61 (March 1971): 82-93.

Blanchard, O.J., "Monopolistic Competition, Small Menu Costs, and Real Effects of Nominal Money." Manuscript, M.I.T., March 1985.

Diamond, P.A., "A Model of Price Adjustment." Journal of Economic Theory 3 (June 1971): 156-68.

Geary, P.T. and Kennan, J., "The Employmnent-Real Wage Relationship: An International Study." Journal of Political Economy 90 (1982): 854-71.

Glazer, A., "The Client Relationship and a "Just" Price." American Economic Review 74 (Dec. 1984): 1089-95.

Keynes, J.M., "Relative Movements of Real Wages and Output." Economic Journal 49 (March 1939): 34-51.

Malinvaud, E., The Theory of Unemployment Reconsidered. Basil Blackwell, 1977.

Mankiw, N.G., "Small Menu costs and Large Business Cycles: A Macroeconomic Model of Monopoly." Quarterly Journal of Economics (Forthcoming).

Nelson, P., "Information and Consumer Behavior." Journal of Political Economy 78 (March/April 1970): 311-29.

Okun, A.M., Prices and Quantities: A Macroeconomic Analysis. Brookings, 1981.

Phelps, E.S., and Winter, S.G., Jr., "Optimal Price Policy under Atomistic Competition." In E.S. Phelps, et. al., Microeconomic Foundations of Inflation and Employment Theory. New York: Norton, 1970.

Pigou, A.C., Industrial Fluctuations. London: MacMillan and Co., 1927.

Rotemberg, J.J., "Monopolistic Price Adjustment and Aggregate Output." Review of Economic Studies 49 (1982): 517-31.

Rotemberg, J.J., and Saloner, G., "A Supergame-Theoretic Model of Business Cycles and Price Wars During Booms." M.I.T. Working Paper No. 349, July 1984.

Schmalensee, R., "Product Differentiation Advantages of Pioneering Brands." American Economic Review 72 (June 1982): 349-65.

Stiglitz, J.E., "Price Rigidities and Market Structure." American Economic Review 74 (May 1984): 350-55.

Strotz, R.H., "Myopia and Inconsistency in Dynamic Utility Maximization." Review of Economic Studies 23 (1956): 165-80.

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