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Working Paper No. 311
January 1992

University of
Rochester

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The Stolper-Samuelson Theorem is still alive and well – and being debated – after fifty years. The original statement of the theorem was couched in a two-factor, two-commodity setting in which technology exhibits constant returns to scale and the absence of joint production. Competitive pressures then ensure that if a policy of tariff protection raises the relative domestic price of importables, the real reward to the productive factor used intensively in the import-competing sector must unambiguously rise. The logic is impeccable, but subsequent attempts to expand the dimensionality of the setting have been less successful in pinning down the effects of protection on real factor returns, a result perhaps neither unwarranted nor undesirable since it reveals that relationships among a small subset of variables can be altered by conditions of technology and relative endowments concerning other variables in the model.

A few years after the appearance of the Stolper-Samuelson Theorem a pair of articles by Lloyd Metzler (1949a, 1949b) suggested the possibility that for a large country a tariff might not be protective. If such a (Metzler) paradox were to prevail, a tariff would be opposed by labor if the import – competing good were labor-intensive. I suspect that the Metzler possibility, suggesting a reversal of the Stolper-Samuelson Theorem as originally stated, was instrumental in causing the theorem to be recast in terms of the effect of changes in domestic commodity prices on the rewards to domestic factors. It was this commodity price-factor price link that was so painstakingly (and frustratingly) analyzed in the late 1960's and early 1970's by Kemp and Wegge (1969), Chipman (1969), Uekawa (1971), Inada (1971) and others. By contrast, a recent piece by Leamer (1991) serves as a reminder of the trade setting of the original argument. Leamer raises the question of the attitude of the scarce

factor towards protection of imports. Suppose labor is a scarce factor and suppose that there is a commodity which is an unambiguous "friend" of labor in the sense that its price rise by itself would increase the wage rate by relatively more.¹ In such a setting is there a necessity that this commodity be imported? If not, would the scarce factor still favor protection? The 3 x 3 model, and the "triangle" techniques employed by Leamer (1987, 1991) and Jones and Marjit (1991), provide the most simple setting in which links among factor intensities in productive techniques, trade patterns, and concepts of factor scarcity and abundance can be analyzed in higher dimensional settings.²

1. Factor Intensities and the Leamer Triangle

In the original two-dimensional setting of the Stolper-Samuelson Theorem the concept of factor intensity ranking was crucial and its meaning clear. The labor-intensive sector was the sector employing a higher ratio of labor to land than the other sector. Some of the difficulties concerned with extending the commodity price, factor price links to higher dimensions turned on the meaning of factor intensity

¹The concept of a commodity being a "friend" or an "enemy" for a factor was introduced in Jones and Scheinkman (1977).

²These two interpretations of the Stolper-Samuelson Theorem – the link between commodity and factor prices on the one hand or the attitude towards protection by the scarce factor on the other – have their parallel in the factor-price-equalization literature. As originally stated by Samuelson (1948, 1949), the question was whether free trade in commodities would, under appropriate circumstances, equate the returns to non-traded factors. By contrast, this literature soon focussed on a different issue, whether the (common) technology supported a unique dependence of factor prices on commodity prices. In the original trade setting, factor endowments of the two countries were required to be fairly "close." By contrast, the univalence literature has no need to mention factor endowments or trade.

comparisons in these dimensions. In the 3x3 context, with which we are concerned, two possibilities were suggested in the literature. Denote the distributive share of factor i in the production of commodity j by θ_{ij} . Chipman (1969) suggested that factor i was used intensively in producing commodity i if

$$\theta_{ii} > \theta_{ij} \quad \text{for } j \neq i.$$

If this held for each factor, Chipman showed that a rise in commodity i 's price would raise the return to factor i by relatively more. Kemp and Wegge (1969) suggested a stronger criterion for factor intensity ranking. Suppose for each commodity i and factor i ,

$$\frac{\theta_{ii}}{\theta_{ki}} > \frac{\theta_{ij}}{\theta_{kj}} \quad \text{for all } i, j \neq i, k \neq i.$$

That is, suppose factor i 's share in industry i , relative to the share of any other factor in industry i , is larger than the corresponding ratio of these factors in any other industry. Then in the 3x3 case, Kemp and Wegge showed that an increase in the price of commodity i not only raises the return to factor i relatively more, it must lower the return to the other two factors. Chipman's criterion is weaker and does not require that other factor returns fall. It turned out, much to their frustration, that neither condition proved sufficient for their results if the number of factors and commodities exceeded three, but that takes us beyond the scope of the present investigation.³

The 3x3 case can be given geometric interpretation by using the "triangle" device pioneered by McKenzie (1955) and intensively exploited by Leamer (1987, 1991) and Jones and Marjit (1991). Figure 1 depicts an equilateral triangle. Following Leamer (1987) I label the vertices labor (L), land (T), and capital (K). Every point in the

³The Kemp–Wegge condition is, as they proved, a necessary condition in higher dimensions. A sufficient condition in higher dimensions was established by Jones, Marjit and Mitra (1991).

triangle has three co-ordinates (barycentric co-ordinates), given by the perpendicular distances to the sides opposite each vertex. Thus point 1, indicative of techniques used to produce the first commodity at a given set of factor prices, is fairly labor-intensive relative to its (rather balanced) use of capital and land. It is a property of equilateral triangles that the sum of co-ordinates for any point is the same, and if this is set at unity the coordinates can be taken to represent each factor's distributive share in that activity. Activity 3 in Figure 1 thus has a land share that is lower than that in activity 2 and higher than that in activity 1. Indeed, there is a matching in each industry of a unique factor whose share exceeds the comparable share in the other two industries (L in X_1 , T in X_2 and K in X_3).⁴ For an economy to produce all three goods using the techniques shown, its endowment point must be contained strictly within the activities triangle.

Two cones, l and k , are explicitly shown in Figure 1. These are examples of what may be termed the *minimal containing cone for factor i* . The rays from the labor vertex through activities 2 and 3 connect all points illustrating the same ratio of land to capital distributive shares as do X_2 and X_3 respectively, and any other activity (X_1) must have an intermediate land/capital distributive share ratio. Note the distinction between cones l and k – for cone l the *facing boundary* of the activities triangle consists of two facets joined at 1, while that for cone k consists of a single facet, the cord 1–3. The number of facets in a facing boundary is crucial; as demonstrated in Leamer (1987) or Jones and Marjit (1991), the relationship between increases in the supply of a factor and corresponding output changes at given techniques depends upon the number of facets in the facing boundary. Thus an increase in the endowment of labor in Figure 1 would raise output of X_1 and reduce

⁴Thus this production triangle satisfies the conditions laid down by Chipman (1969).

the output both of X_2 and X_3 . By contrast, an increase in K alone increases outputs of X_1 and X_3 but reduces the output of commodity 2. To see this, draw a ray from the factor vertex through the activity *not* acting as a support for the containing cone (activity 1 for l and 2 for k). Such a ray intersects the facing boundary at point N for factor K or, for factor L , passes through the activities triangle to intersect the 2-3 side. Consider the ray through K and N (not drawn). Point N is a positive convex combination of activity 2 and an activity that would consist only of inputs of K . Therefore an addition of K alone to the economy could be absorbed by an increase in (fictitious) activity N and a decrease in X_2 . But N is itself a positive convex combination of 1 and 3 (with 3 more heavily weighted) so that an increase in K by itself would be absorbed by an increase in X_1 and X_3 and a reduction in X_2 .

These changes are reflective of Rybczynski effects, and by the Samuelson (1953) reciprocity conditions also reveal the effect of commodity price changes on factor prices. Thus a rise in p_1 by itself would cause the wage rate to rise, and an increase in either p_2 or p_3 would force wages down. Since a uniform increase in all three commodity prices would raise wages by the same relative amount, an increase in p_1 by itself must have a magnified effect on raising the wage rate. In this sense of the word commodity 1 would be an "unambiguous friend" of labor and, if good 1 were imported, a tariff that caused its domestic price to rise would unambiguously improve the real wage in Stolper-Samuelson fashion. An increase in p_1 would also benefit the nominal return to capital, but perhaps not relatively by as much, so that in real terms the fate of capital's return could be ambiguous. What is clear from the existence of a single facet in the facing boundary for cone k is that a rise in the price of good 2 pushes down the return to capital. Thus for cone l there is a single unambiguous *friend* for labor (industry 1) and for cone k there is a single unambiguous *enemy* for capital (industry 2). In what follows we need to distinguish these two types of cases in

developing a theorem that links *factor scarcity* and demands for protection if the factor has a single unambiguous friend and *factor abundance* and demands for free trade if the factor has a single unambiguous enemy.

Although not drawn, the corresponding t cone from the T-vertex shares with the k cone the property that the facing boundary of the activities triangle consists of a single facet (the 2-3 cord), so that an increase in the price of commodity 1 would unambiguously reduce the real return to land.

Figure 1 can also be used to depict *isoquants* for each of the commodities. Focus on commodity 2. If the endowment proportions happen to correspond exactly to those used to produce the second commodity, the country's entire resource bundle would be used only in X_2 . By contrast, should the endowment bundle lie on the 1-3 cord, full employment of all three factors would entail production of only goods 1 and 3, with a zero output for good 2. Lines α and β , parallel to cord 1-3, illustrate intermediate isoquants for good 2. If the endowment bundle is shown by F , roughly 80% of the economy's resources would be devoted to produce the first commodity and 20% devoted to commodity 2. Similarly, at point G only 20% of resources are devoted to X_2 , the remaining 80% now to produce the third commodity. If the endowment bundle moves along the β -line from F to G , all three factors are reallocated from X_1 to X_3 , leaving the same allocation of resources to produce X_2 . Lines α and β are X_2 -isoquants, with endowments along the α -line showing higher levels of good 2 output.

Although the use of the term "isoquant" is natural in this context, it should be noted that in traditional terms an isoquant shows the bundle of inputs into a particular commodity that leave the output of that commodity unaltered. By contrast, here an isoquant refers to bundles of total factors available to the economy, shown in barycentric coordinates, that would keep the output of one good constant assuming techniques are fixed and full employment prevails.

2. Relative Factor Abundance and Scarcity

So far nothing has been said about relative factor abundance or scarcity. Such a comparison requires two countries or, even more appropriately, one country versus the world. Therefore consider an integrated world economy in which all commodity prices reflect an equilibrium balance between demand and supply and all factors are fully employed. Factor prices in the world economy are determined as well, and with these prices there corresponds a set of techniques for each of the three commodities. These techniques form an activities triangle that contains the world endowment vector. For convenience in drawing the diagram suppose that world endowments of the three factors lead to factor prices resulting in factor endowment shares each equal to one-third. Thus point W in Figure 2 would represent the world endowment point, and if all goods are produced and consumed in the integrated world economy, the activities triangle (not drawn) contains point W .

Two sets of lines have been drawn through point W . The solid lines are parallel to the sides of the triangle. Consider line γ , drawn parallel to the T - K boundary. Any endowment point lying along line γ has the same labor-share ($1/3$) as does world endowment point W . Thus if a country's endowment bundle lies closer to the L -vertex than the γ -line we can say that this country is labor abundant. This simple bilateral comparison divides regions of labor abundance relative to the world from labor scarcity. Letting a country's labor share in the national income be denoted by θ_L , and world labor's share by θ_L^W , labor is a scarce factor in a country if and only if

$$\theta_L < \theta_L^W.$$

The dotted lines through W each aim for a factor vertex. An endowment point anywhere above line ρ would exhibit a higher share of land to capital than does world endowment point W . Now consider a point like B . It lies to the right of the dashed

line from the T-vertex through W (implying $\frac{\theta_L}{\theta_W} < \frac{\theta_K}{\theta_W}$) and above the dashed line

from the K-vertex through W (implying $\frac{\theta_L}{\theta_W} < \frac{\theta_T}{\theta_W}$). Thus if the country's

endowment bundle is at B , it must be the case that labor is the *most scarce* factor in the sense that

$$\frac{\theta_L}{\theta_W} < \frac{\theta_i}{\theta_i} \quad \text{for } i = T, K$$

(Endowment point B lies in the 120° arc indicated by MS, "most scarce", as opposed to the 180° arc for labor labelled S, "scarce"). Finally, consider an endowment point like C . It lies within the 60° arc labelled OS, to the right of the line through W parallel to the L-T edge, thus indicating that capital is an abundant factor, and above the line parallel to the L-K edge, indicating that land is also an abundant factor. Thus labor is the *only scarce* factor if

$$\frac{\theta_L}{\theta_W} < 1 < \frac{\theta_i}{\theta_i} \quad \text{for } i = T, K.$$

As we shall demonstrate, the kind of paradox pointed out by Leamer could take the especially strong form: labor might be the *only scarce* factor and yet the country could export a commodity which is unambiguously labor-intensive and thus labor could be opposed to protection.

Three concepts of factor scarcity have been cited: a factor is said to be *scarce* in a country if its distributive share is smaller than in the integrated world economy. It is the *most scarce* factor if its distributive share relative to the share in the integrated economy is smaller than the comparable ratio for the other two factors, and, if both of the other factors are relatively abundant, it is the *only scarce* factor. The move from labor being *scarce* to *most scarce* to *only scarce* involves increasing *balance* in the

ratios of land and capital. Thus perfect balance would be achieved by a point such as E , at which the country's land/capital endowment ratio would exactly match that in the world. As distinct from this issue of *balance* is the *degree of labor scarcity*. Thus two endowment points with the same degree of labor scarcity have the same values for θ_L (or θ_L/θ_L^W). These concepts of *balance* and *degree of scarcity* (or of factor abundance) have crucial roles to play in formulating generalizations of the Stolper-Samuelson Theorem.

Recall that point W is the endowment point for an integrated world economy in which, if all three goods are demanded, equilibrium factor prices and techniques must be such that the activities triangle contains point W . If a country's endowment point lies within this activities triangle, and if (as I assume) its technology is identical with that of the integrated world economy, the country's factor prices and techniques actually adopted will match those in the rest of the world and it will produce all three goods. However, if its endowment point lies outside the world activities triangle, the country must produce fewer than three commodities if it trades freely at world prices. Furthermore, its factor prices will differ from those in the integrated world economy and the techniques adopted will differ as well, and will be such that its activities triangle degenerates into a line if only two goods are produced (or a point if only one good is produced) containing the endowment point. In what follows I assume the country's endowment point lies within the world's activities triangle.

3. Factor Scarcity, Factor Abundance and the Trade Pattern: Some Examples

Figure 3 illustrates the case of a perfectly symmetrical activities triangle. Each sector uses a different factor most intensively (labor in 1, land in 2 and capital in 3), and the shares of the other two factors are identical. As previously argued, the two-faceted nature of the facing boundary for labor ensures that a price rise for the

first commodity unambiguously increases the real wage, while price rises for either of the other two commodities would unambiguously lower the real wage. Furthermore, the slopes of the isoquants for each good, which always match the slopes of the edges of the activities triangle, in this symmetric case match as well the slopes of the large triangle. As a consequence the dashed x_1 -isoquant through world endowment point W also separates the regions for a country in which labor is scarce (areas A, B, and C) from those in which labor is abundant relative to world supplies.

Before turning to trade patterns, some assumptions must be made about tastes. Even in the simple 2x2 case demand differences between countries could result in a labor-abundant country nonetheless importing the labor-intensive commodity. Therefore make the (strong) assumption that taste patterns throughout the world are homothetic and identical.

Any endowment point lying on the dashed X_1 -isoquant through W would involve the country producing exactly enough of the first commodity to satisfy its own demands. If the endowment point lies in an area where labor is scarce (A , B or C), the country must import good 1. Labor would thus favor a tariff on this good. In region A commodity 3 is imported as well (and in B commodity 2 is imported as well as commodity 1), and labor would *not* favor a tariff on this item. However, a uniform tariff on all imports would be of decided benefit to labor in region A (B) since it would be equivalent to a lowering of the price of export-good 2 (3), which would cause real wages to rise. In region C the only import is good 1, and this region corresponds exactly to the case in which labor is the only scarce factor. The statement, "the scarce factor favors protection" holds in the strongest sense in region C.

In Figure 4 I depart from this perfectly symmetric case in one respect - commodity 3 is now even more intensive in its use of capital, with symmetric reductions in the use of land and labor. This is nonetheless still a case in which each

factor unambiguously favors a price rise for a unique (and different) good.⁵ Two regions have been singled out. If the country's endowment lies in region *A*, labor is a scarce factor and yet the first commodity is exported (as is commodity 3). Commodity 2 is the only commodity imported, and labor would be unambiguously opposed to a policy of protection since an increase in p_2 lowers wages. By contrast, in region *B* labor is an abundant factor and yet the labor-intensive first commodity is now imported (along with commodity 3), and protection – either of good 1 or uniformly over both imports – would be favored by labor. Note, however, that should the endowment point lie in region *B*, capital would be the only scarce factor and capitalists would unambiguously favor protection.

To understand this phenomenon consider two economies in which labor is neither scarce nor abundant – economies whose endowments are given by points *C* and *D* in Figure 4. In the *C*-economy commodities X_1 and X_3 only are produced whereas in more land-abundant economy-*D* only X_1 and X_2 have positive outputs. As drawn, X_2 is more labor-intensive than is X_3 , $\theta_{L2} > \theta_{L3}$. As a consequence, the fraction of the national income that can be devoted to labor-intensive X_1 is greater in the *C*-economy than in the *D*-economy. There is a higher X_1 -isoquant passing through *C* than through *D*; the *C*-economy exports good 1, while the *D*-economy imports it.

Figure 5 shares with Figure 4 a situation in which the first commodity remains an unambiguous friend to labor – the facing boundary to the *L*-vertex has two facets and commodity 2 is more labor-intensive than is X_3 . But the asymmetry in labor – requirements for commodities 2 and 3 is now much more striking. The shaded area exhibits potential endowment locations in which (i) labor is a scarce factor, but

⁵This is an example of what Jones and Marjit (1991) call a Produced Mobile Factor Structure, in which rays from the factor vertices of the large triangle passing through the activity which uses that factor intensively (not drawn) all meet in a common point (*W* in Figure 4).

(ii) commodity 1, which is unambiguously a friend to labor, is nonetheless exported (as is commodity 3). This shaded area even includes endowments (northeast of W) revealing labor as the *only scarce* factor. Protection would (barring the Metzler paradox) raise the domestic price of the second commodity, and such a move would unambiguously be opposed by labor.

This latter example illustrates an extreme version of the point Leamer wishes to make – that the scarce factor may oppose protection in any form. Does this serve to invalidate the Stolper–Samuelson result? I think not. With reference to Figure 5 the paradox is clearly revealed by point *a*. But consider points *b* and *c*. Point *c* lies on the ray from the L–vertex through point *a*, and thus has the same degree of balance in land and capital as has point *a*, but represents a higher degree of labor scarcity. If the endowment point is *c*, commodity 1 is imported and labor favors a tariff on X_1 or (to a lesser extent) a uniform tariff on X_1 and X_2 . By contrast, point *b* and point *a* show endowments with the same degree of labor scarcity, but at *b* there is more balance in the relative abundance of non–labor factors, and at *b* the trade pattern changes and the first commodity is (the only commodity) imported. In sum,

if labor is scarce and has a unique commodity friend, a sufficient degree of labor scarcity and/or sufficient balance in the relative abundance of non–labor factors ensures that the real wage rises with protection. The scarce factor would favor protection.

The examples drawn in Figures 3–5 all exhibited a facing boundary for labor that possessed two facets. That is, commodity 1 served as an unambiguous friend to labor. The situation is somewhat different should the facing boundary for labor consist of a single facet (as it does in Figure 1 for capital or land). If so, the single activity not supporting the minimal containing cone for a factor (commodity 2 for capital or commodity 1 for land in Figure 1) is that factor's unique unambiguous enemy. Should

that commodity be imported, the factor would favor free trade over protection. Does this describe the situation in which an abundant factor finds itself? If so, this seems like an appropriate variation of the Stolper–Samuelson Theorem.

Figure 6 reverses the perfect symmetry exhibited in Figure 3 and, indeed, is a case studied by Inada (1971). He examined an economy in which each commodity was associated with a unique factor which, should that commodity price rise, would find its reward depressed. That is, each factor has a unique (commodity) enemy. For labor this is now commodity 1, whereas a rise in either p_2 or p_3 would increase at least the nominal wage rate. Labor is relatively abundant in regions A, B, or C and commodity 1 is imported in any of these (along with X_2 in A or along with X_3 in B). Of course if labor is scarce the country would export X_1 and import either X_2 or X_3 or both, and would favor protection. But what deserves emphasis now is labor's attitude if it is relatively abundant – it supports free trade (or, better yet, import subsidies).

Figure 7 is less symmetric, although again labor's minimal supporting cone has a single facet as the facing boundary of the activities triangle. Commodity 1 is unambiguously an enemy of labor. If the endowment point is shown by a , labor is a scarce factor and commodity 1 is imported. But now no matter how much the degree of labor scarcity is increased, for the same balance in relative endowments of land and capital, e.g. the move from a to c , commodity 1 is still imported and labor still does *not* favor protection. This contrasts with the previous result for scarce factors when there is a unique commodity friend. It is the case, however, that should the balance of land to capital endowments be adjusted for the same degree of labor scarcity, e.g. the move from a to b , commodity 1 would be exported and labor would indeed favor import protection.

On a more positive note consider the endowment point e in Figure 7. Now labor is an abundant factor, although, since commodity 1 is exported, labor would favor

protection. This is similar to the kind of paradox posed by Leamer for a scarce factor. However, it is now possible to get a strong result analogous to the version of the Stolper-Samuelson Theorem in the 3x3 case when labor has an unambiguous friend and is a scarce factor:

If labor is abundant and has a unique commodity enemy, a sufficient degree of labor abundance and/or sufficient balance in the relative endowment of non-labor factors ensures that the real wage rises with a move from protection towards free trade. The abundant factor favors free trade.

The stronger result obtainable for endowment changes that keep the degree of one factor's scarcity or abundance unchanged but alter the proportions of the other two factors more in balance with those of the world economy is easily explained. If the proportions of land and capital are similar for the country and the world, endowment differences assume a bilateral form - one country is labor scarce or abundant relative to the rest of the world, compared with "baskets" of land and capital endowment of roughly similar composition.

4. Concluding Remarks

The Stolper-Samuelson Theorem in its initial garb was stated for a two-factor, two-commodity country engaged in trade with the outside world. How was the theorem stated by the authors? The notion of the scarce factor gaining in real terms by protection was certainly present, but the analytical core of their demonstration focussed on the effect of an increase in the domestic relative price of the import-competing good on real wages (in a small-country setting in which the terms of trade do not change) where it was assumed that labor is used relatively intensively in the import-competing sector. Thus the subsequent development of the literature,

concentrating on the effects on factor rewards of changes in commodity prices, seems justified as a legitimate extension of the original theorem.

To push this argument further, note that the notion of relative factor scarcity or abundance of necessity involves a comparison with other countries' endowments. And yet, to quote from the original Stolper-Samuelson paper,

"Our analysis neglected the other country completely. If factors of production are not comparable between countries, or if production functions differ, nevertheless, so long as the country has only two factors, international trade would necessarily affect the real wage of a factor in the same direction as its relative remuneration. The only loss to our analysis would be the possibility of labelling the factor which is harmed as the 'scarce' (relative to the other country) one." (p. 72)

Indeed, it is often pointed out that the Stolper-Samuelson argument rests on much less strict assumptions than does the Heckscher-Ohlin Theorem (or Factor-Price Equalization Theorem) with its stipulation of commonly-shared technologies and absence of demand dissimilarities. Nothing about the rest of the world need be known.

Having said this there is no doubt that interest in the original paper was certainly aroused since it seemed to justify to a certain extent the "pauper-labor" arguments for protection popular in countries such as the United States thought to be labor-scarce.⁶ Thus Leamer's query as to the attitude of a scarce factor towards protection is well taken. In pursuing this question here I have confirmed Leamer's criticism that in a 3x3 world the scarce factor may not favor a tariff, perhaps on any import. The fault, however, seems to lie less with the Stolper-Samuelson Theorem than with the Heckscher-Ohlin dictum that countries tend to import commodities which make intensive use of factors which are locally scarce. In a multi-factor,

⁶Indeed the paper was originally rejected by the *American Economic Review*. Free trade was the overwhelmingly popular view among economists at the time.

multi-commodity world there are ways in which asymmetries in factor intensities among goods can foul up this logic.

Despite apparent paradoxes, certain tendencies were emphasized. A sufficient degree of factor scarcity or factor abundance or of balance among other factor supplies relative to the rest of the world serve to restore the basic connection between factor endowments and attitudes towards protection and free trade.

Is the 3x3 model sufficiently general to bring out all the issues? The earlier literature warns that this may not be the case - both Chipman (1969) and Kemp and Wegge (1969) were frustrated in extending their versions of the generalized Stolper-Samuelson Theorem from the 3x3 case to the 4x4 case. As dimensions mount it becomes ever more difficult to maintain statements about bilateral relationships among a subset of variables. Nonetheless, let me suggest a pair of $n \times n$ generalizations which appear to be supportable by the kind of "triangle" geometry used here to explore the 3x3 case.

The triangle of this paper has an $(n-1)$ -dimensional "hyper-triangle" version in the n -factor, n -commodity case. Once again a cone can be constructed from each factor vertex that is a *minimal containing cone* for the $(n-1)$ dimensional counterpart of the production triangle. Suppose there exists a factor (say, labor) for which the first $(n-1)$ activities are supports for the minimal containing cone, and consider two possibilities for the n th activity. First, it may form part of the facing boundary, such as industry 1 did for labor in Figures 1-5. In that case the facing boundary consists of $(n-1)$ facets, each of dimension $(n-2)$, joined at a point which represents the input shares for the n th activity. Commodity n would be an unambiguous friend to labor (and all other commodities enemies). A ray from the labor vertex through the n th activity would pass through the hyper-triangle and pierce the single $(n-2)$ dimensional facet representing the convex combination of activities 1, ..., $(n-1)$. Labor scarcity can

be defined precisely as in the 3x3 case, and if labor is sufficiently scarce and/or if non-labor factors in the endowment bundle match closely enough the relative world supplies, the n th commodity would be imported and labor would favor protection for it alone or, indeed, uniform protection for all imports.

Alternatively, the n th activity vector may not form part of the facing boundary for labor (as X_1 in Figures 6, 7). In such a case the facing boundary would consist of a single $(n-2)$ dimensional facet (the convex combination of activities 1, — — —, $(n-1)$), and a ray from the labor vertex through the n th activity would first pass through the facing boundary. Labor would find that activity n is an unambiguous enemy, and if labor were sufficiently abundant, and/or if for a given degree of labor abundance if non-labor factors are sufficiently "balanced," the n th commodity would be imported and labor would work to dismantle protection in favor of free trade.

Of course many other scenarios are possible for the number of facets in the facing boundary once the number of factors and commodities exceeds three. In these few of the strong factor-intensity rankings suggested by Kemp and Wegge or Inada hold and orderly propositions linking factor prices and commodity prices can no longer be obtained.

The 3x3 version of the model discussed here is useful in showing how the original Stolper-Samuelson results can to some extent be generalized and how factor scarcity and abundance and attitudes towards protection are linked to factor intensity rankings. Leamer is to be credited for pushing the discussion in this direction as well as for exploring the triangle technology. His paradoxes, however, may prove to be "exceptions to the rule," as in much of economic theory.

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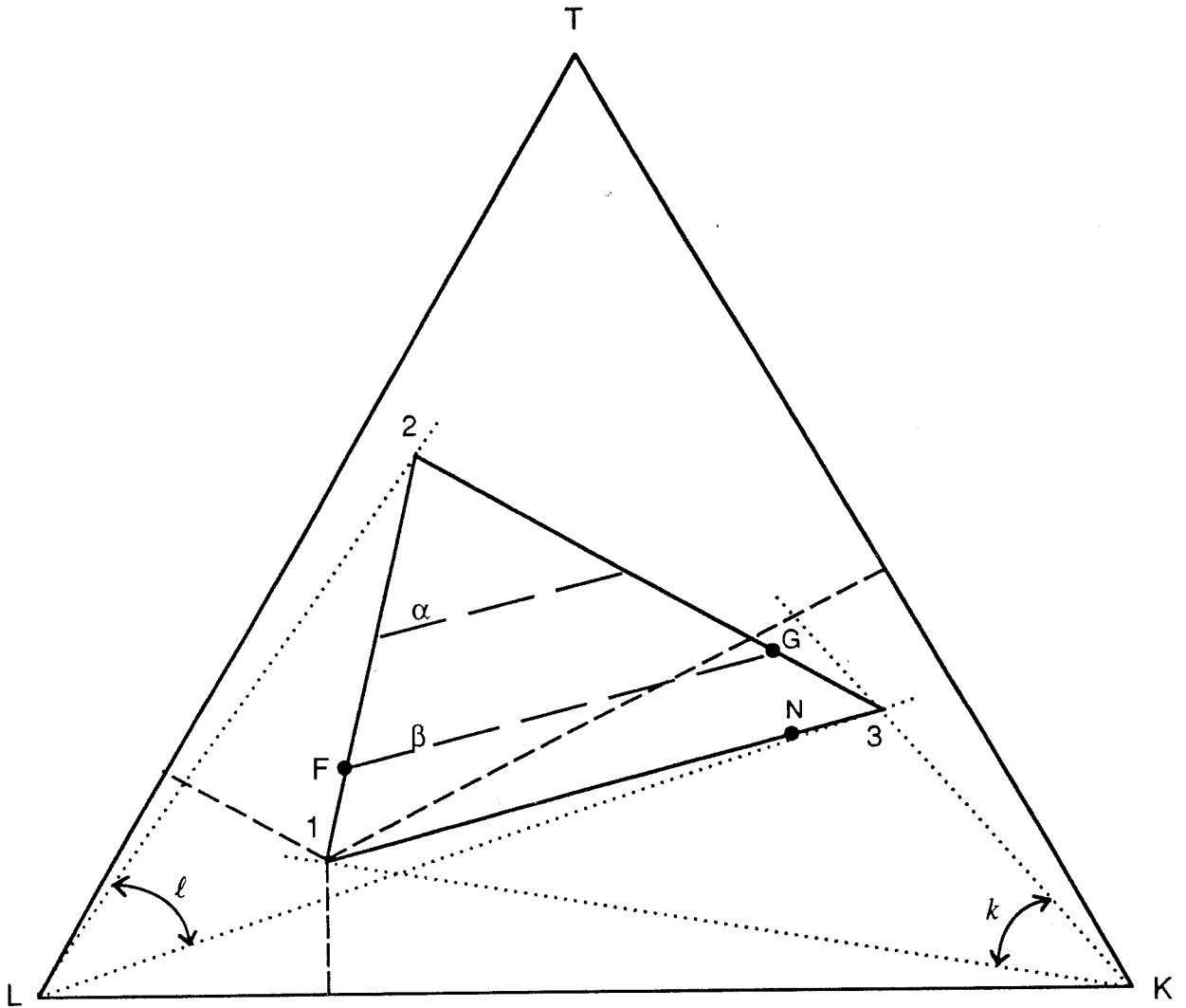


Figure 1

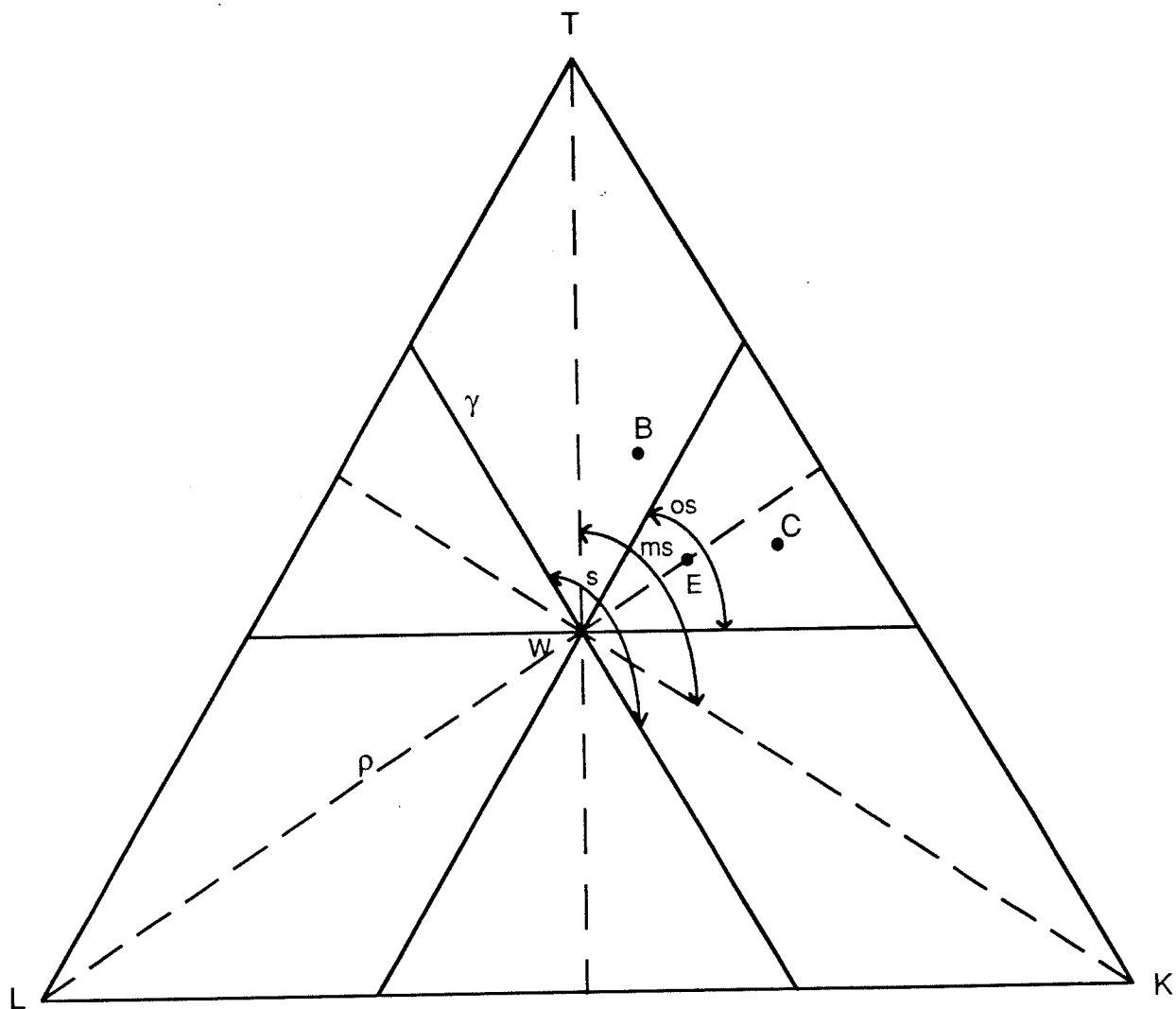


Figure 2

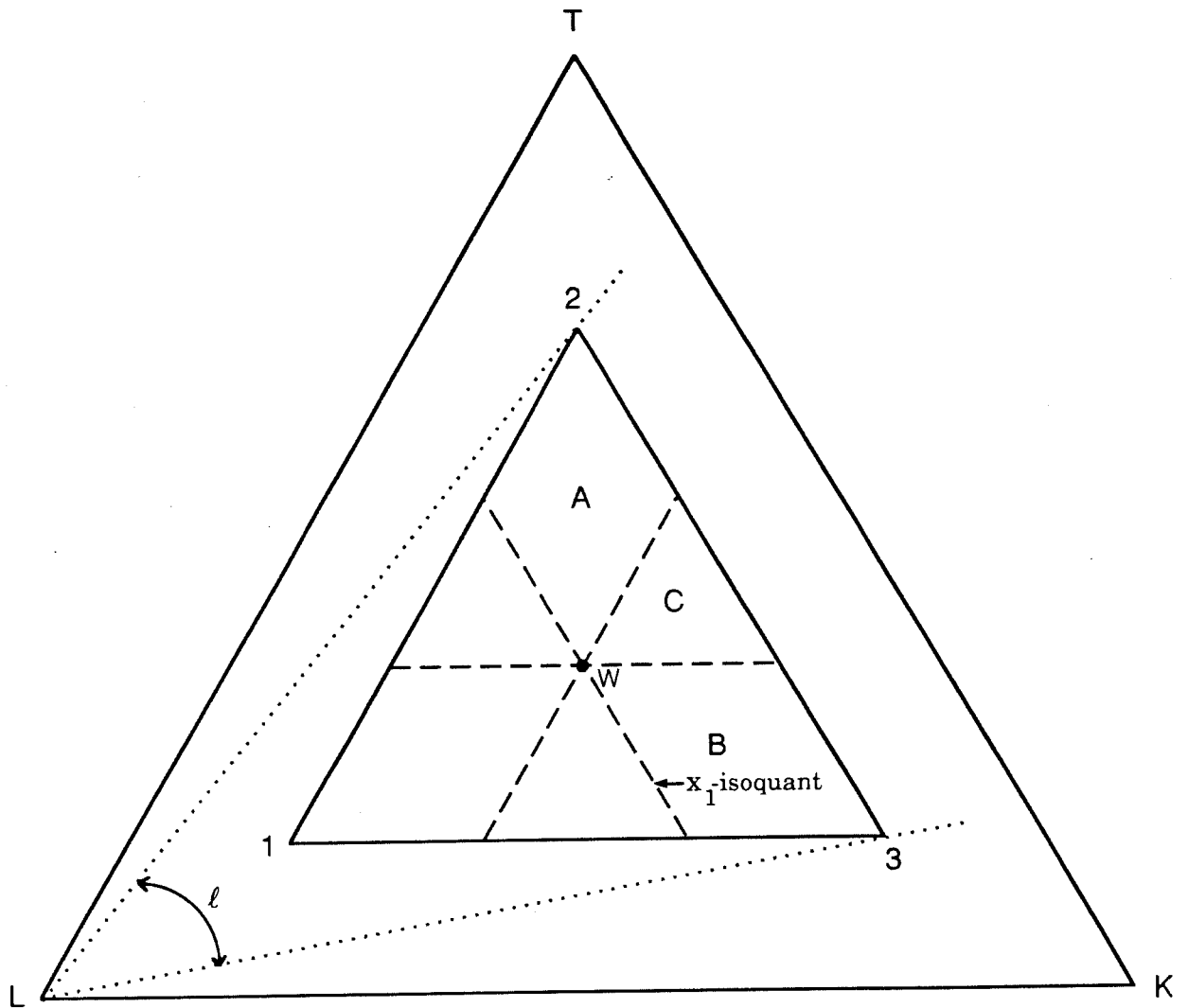


Figure 3

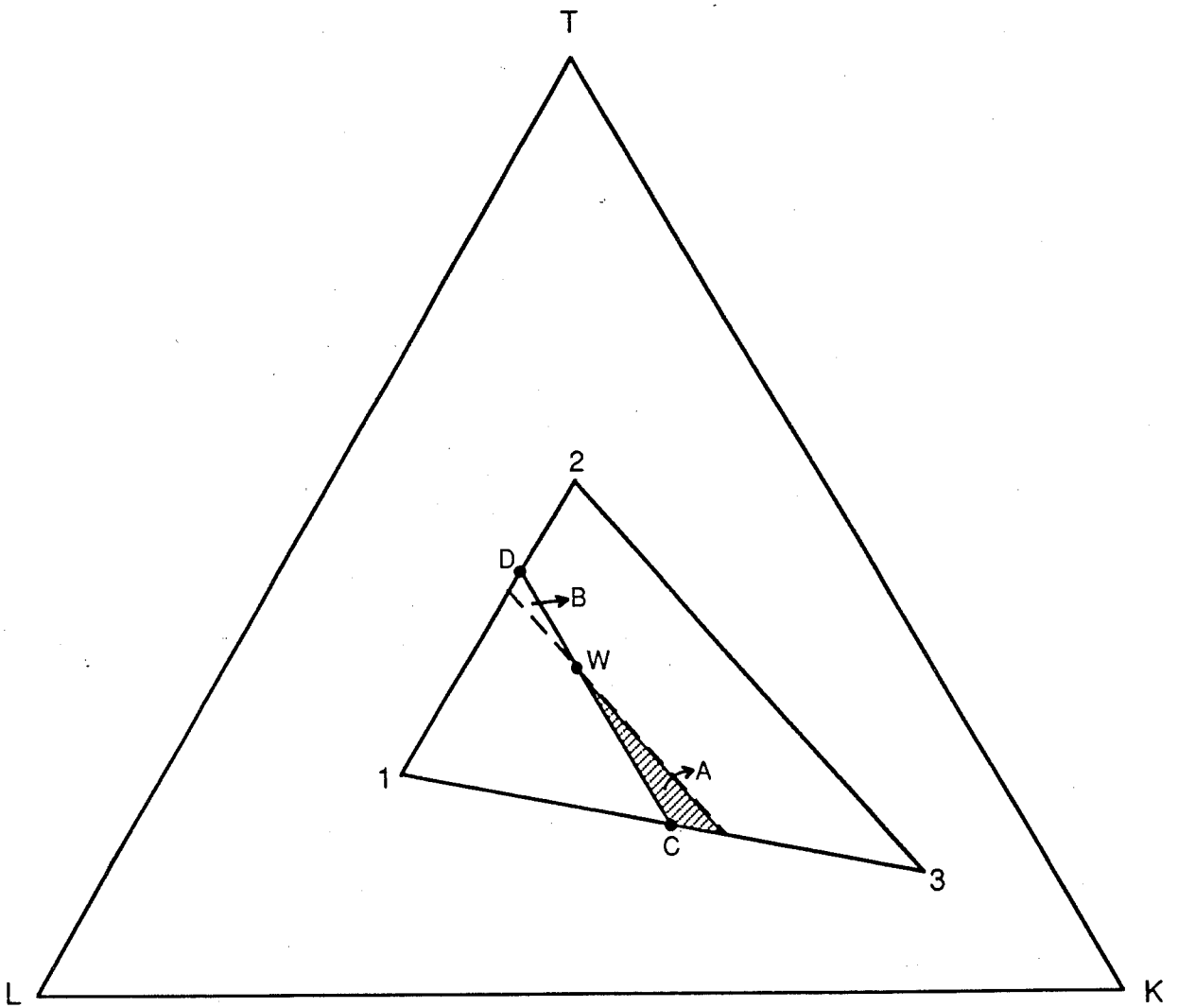


Figure 4

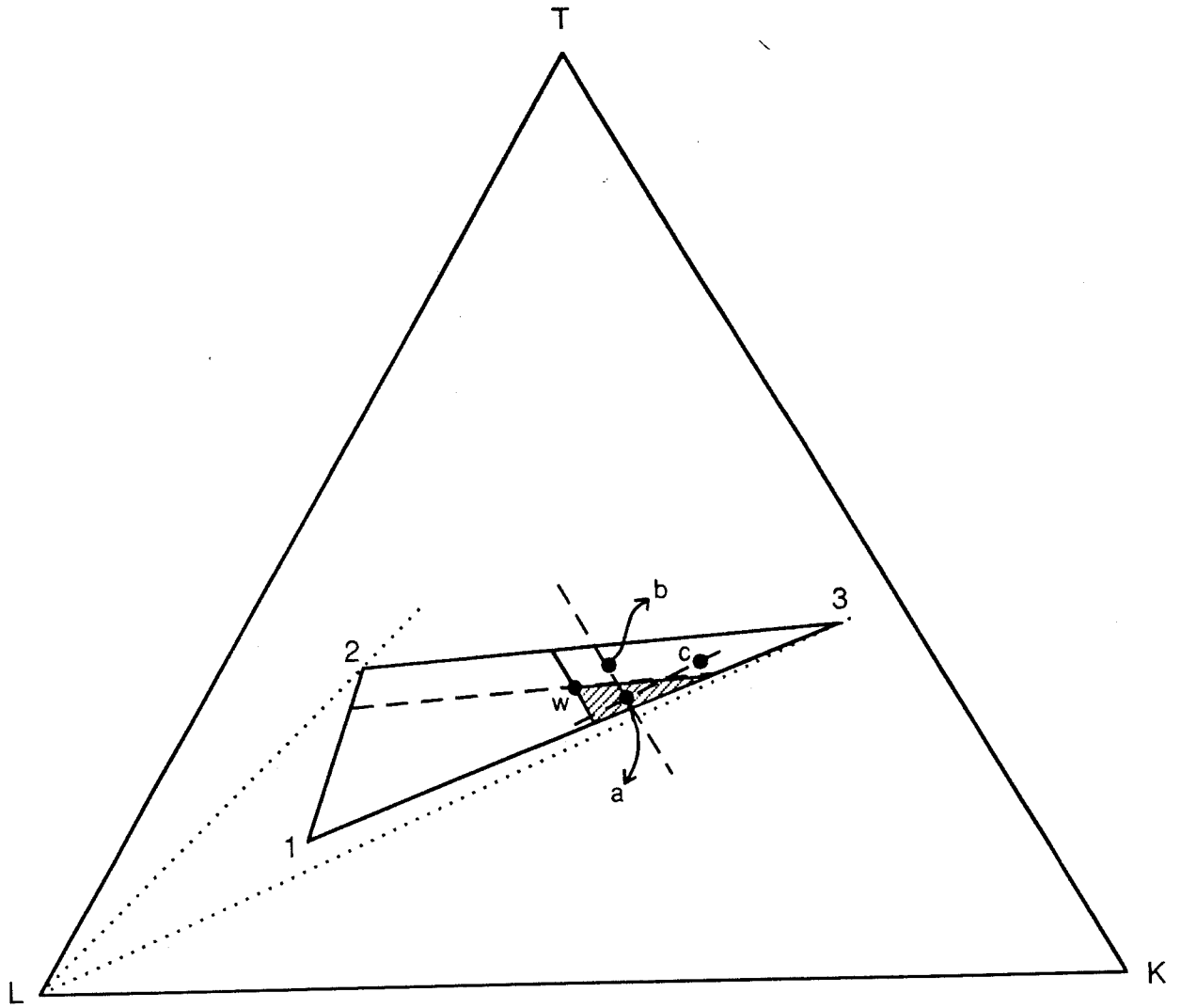


Figure 5

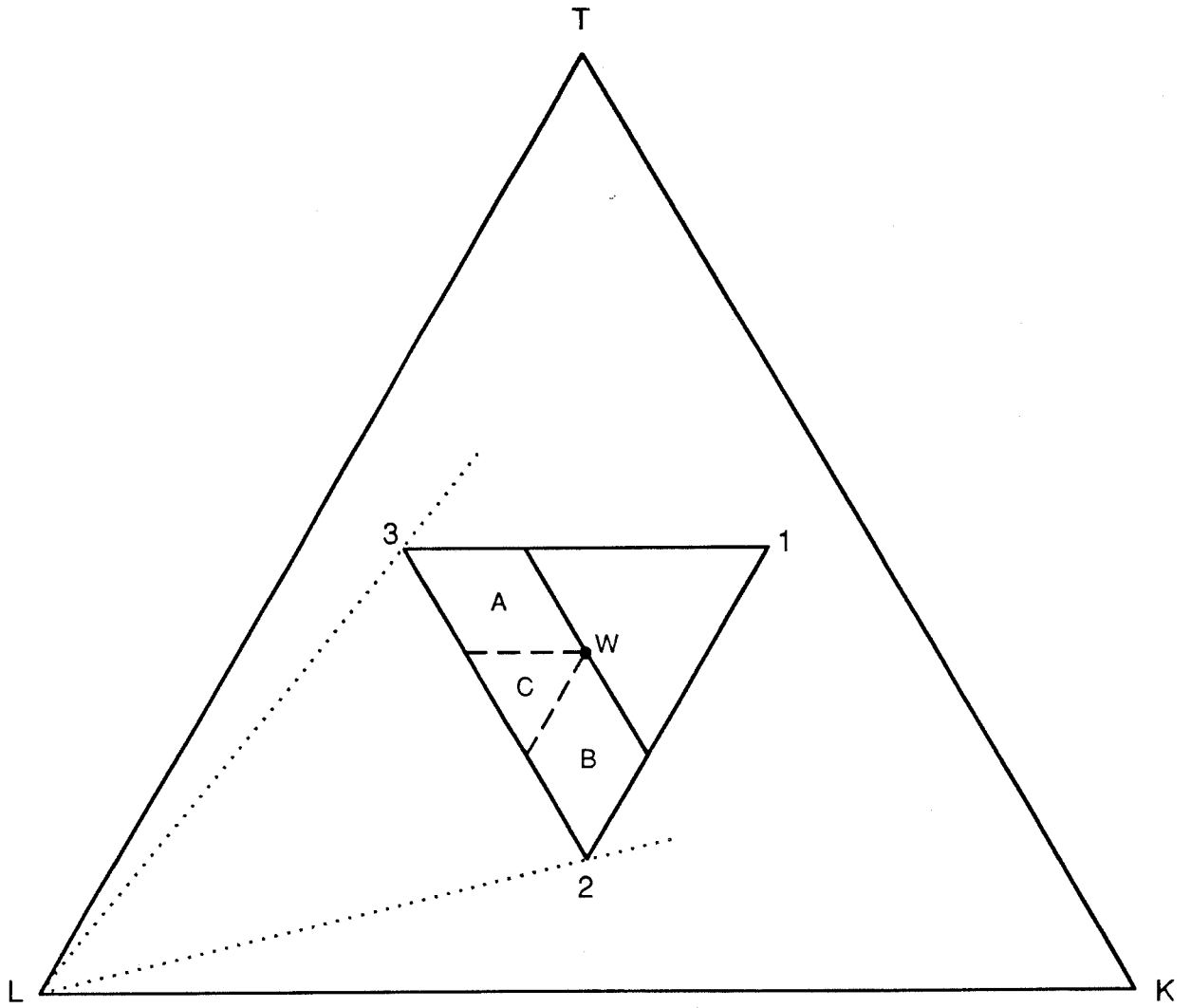


Figure 6

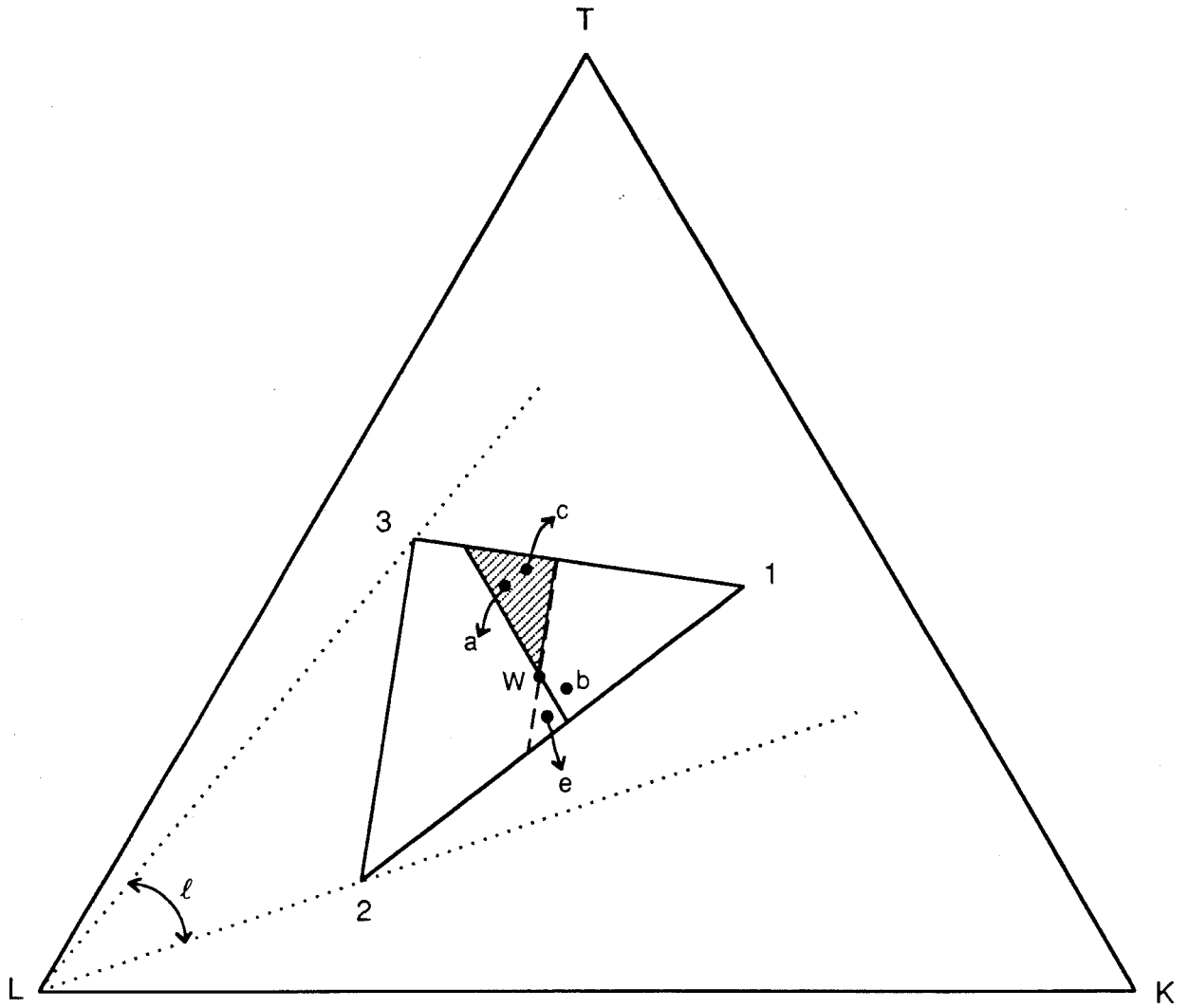


Figure 7