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Sticky Goods Prices, Flexible Asset Prices, Monopolistic Competition, and Monetary Policy (Revised)

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by Lars E.O. Svensson

Abstract

A monetary general equilibrium asset-pricing model with sticky goods prices is developed. Goods prices are set by monopolistically competitive firms that maximize stock market value. Equilibria with underutilization of resources, excess capacity, in some states result, in contrast to previous monetary asset-pricing models. The degree of competition affects capacity utilization. Monetary policy can affect output and resource utilization, in addition to real asset prices, depending upon the amount of information available to the monetary authority.

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by

Lars E.O. Svensson

1. Introduction

Several recent papers have contributed to the old problem of integrating real general equilibrium theory with finance and, in particular, monetary theory, by developing monetary general equilibrium asset-pricing models. Essentially these papers construct monetary versions of Lucas (1978) barter model. Money is introduced either via a cash-in-advance constraint as in Lucas (1982, 1983), Kouri (1983) and Svensson (1985a,b), or via money in the utility function as in LeRoy (1984a, b) and Danthine and Donaldson (1983).

The cash-in-advance constraint can relatively easily be incorporated in a general equilibrium asset-pricing model, as demonstrated by Lucas (1982). That paper, as well as Kouri (1983) and most of the previous cash-in-advance literature, relies on always binding liquidity constraints, which implies the familiar unitary income-velocity quantity equation, which is widely regarded as a very unsatisfactory demand function for money. Lucas (1983) and Svensson (1985b) independently extend the analysis to equilibria with non-unitary income-

[1] This is a revision and extension of IIES Seminar Paper No. 300. I would like to thank Elhanan Helpman, Robert E. Lucas, Torsten Persson, Alan C. Stockman, Jörgen Weibull, participants in seminars at IIES, Norwegian School of Economics, Bergen, Tel-Aviv University, and University of California, Santa Barbara, anonymous referees, and the editors for comments on initial ideas and previous versions. I am particularly indebted to Elhanan Helpman for specific help with the price-setting problem. All errors remain, of course, my own responsibility. I gratefully acknowledge financial support from the Bank of Sweden Tercentenary Foundation. I thank Karin Edenholm and Lotten Bergström for typing and editorial assistance.

velocity of money.[2] Svensson (1985b) relies on previous work by Goldman (1974), Stockman (1980), Helpman and Razin (1982) and Krugman, Persson and Svensson (1985), and presents a detailed analysis of equilibria with variable velocity, including explicit but simple and usable solutions to the endogenous variables. The simplicity of the solution makes the model suitable for a variety of applications.[3] A very rigorous extension of the analysis to a situation where the cash-in-advance constraint applies to a subset of goods is presented in Lucas and Stokey (1985).

One interpretation of these monetary asset-pricing models is the following. Money is treated symmetrically with other, "ordinary", assets. Ordinary assets, which pay some kind of dividends, are priced according to a standard asset-pricing equation. The solution to this equation expresses the value of an asset as the present expected marginal utility of future dividends, divided by the current marginal utility of wealth. Money differs from ordinary assets in that it doesn't pay any direct dividends. Instead it gives liquidity services to its holder. In complete symmetry with ordinary assets, the value of money (the reciprocal of the price level) is given by an asset-pricing equation such that the value of money is equal to the present expected marginal utility of future liquidity services, divided by the current marginal utility of wealth. This nice symmetric treatment of money with other assets requires only that the liquidity services are appropriately defined. With cash in advance, the liquidity services are the shadow prices of the liquidity constraints. With

[2] Lucas (1980) is a cash-in-advance model with variable income velocity of money. Consumers are heterogenous and face idiosyncratic uncertainty that averages out in the aggregate. Money is the only store of value, however, and it seems technically too difficult to include other assets.

[3] Some international finance issues are discussed in Svensson (1985a) and Stockman and Svensson (1985a).

money in the utility function they are the direct marginal utility of money.[4] I believe this view of money is very clarifying, and I think the existing models have contributed considerably to our understanding of the interaction between monetary, financial and real phenomena. In this paper I would, however, like to focus on one particular weakness of the existing models. I do think these models exaggerate the flexibility of the value of money and hence of the price level. The value of money indeed comes out as flexible as any asset price, and it adjusts instantaneously when new information arrives. It is obvious, I think, that in the real world price levels show less variability than, say, stock prices and exchange rates. Therefore, there are good reasons to extend the monetary asset-pricing models to incorporate some stickiness in the price level. This paper is an attempt to such an extension.

The main idea is the following. Stickiness of nominal goods prices is modelled by making goods prices predetermined. More precisely, nominal goods prices are assumed to be set by firms. Although we of course would like to explain why prices are predetermined in a better way, we use the shortcut of simply assuming that firms face an information constraint in that they must decide on current prices before they observe the current state of the economy, either because it takes time to implement a price change or because the current state can only be observed with a lag. Then prices will depend on past states only, and prices will not respond to current states. The prices are determined as the solution to an optimization problem, where firms in monopolistic competition maximize their stock market values.

The outcome of this setup is stationary stochastic rational-expectations equilibria where nominal goods prices do not respond to the current state of the

[4] In the standard overlapping generations framework there is no separate transactions role for money, and hence no separate liquidity services of money. This leads to well-known difficulties, in that ordinary assets with a direct return exceeding that of money cannot exist. Also, the asset-pricing equation discussed here may have multiple solutions, which corresponds to the non-uniqueness of equilibria observed in monetary overlapping generations models.

economy. This implies that there is excess demand for goods in some states, and excess supply in other states. In the latter case, consumption and actual output is below potential output, i.e. capacity, and there is hence waste and underutilization of resources, what we may call unemployment. We will be able to show that the degree of resource utilization varies with the degree of competition between firms. From this point of view the paper, with its equilibria with endogenous price-setting and underutilization of resources, might be considered as a contribution to the literature on non-Walrasian equilibria, as surveyed for instance by Drazen (1980).

The setup of the paper also allows for a more interesting role for monetary policy, since it may affect the utilization of resources. This is in sharp contrast to previous monetary asset-pricing models where there is full employment in the sense that consumption always equals the exogenous stochastic output.[5] We will consider the effects on utilization and welfare of monetary policy, with special regard to the degree of information available to the monetary authority. Firms' pricing of goods is then affected by the particular monetary policy chosen. We will be able to demonstrate that the effect of monetary policy increases with the amount of available information. To some extent the paper can then be considered as a contribution to the literature on monetary policy with rational expectations and price rigidity, exemplified by Fischer (1977) and Taylor (1980).

The outline of the paper is the following. Section 2 presents the model and characterizes equilibria with a sticky price level. The price-setting problem of the firms is dealt with in Section 3. Section 4 discusses the

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[5] In previous monetary asset-pricing models monetary policy may have real effects in the sense that relative asset prices, including real interest rates, are affected, as in Svensson (1985b), or that non-zero nominal interest rates introduce a distortion and affect welfare, as in Lucas and Stokey (1983, 1985), but there is no excess capacity or unemployment. In Grossman and Weiss (1983) and Rotemberg (1984) a lack of syncronization across individuals causes a distortion, but full employment is maintained.

optimum monetary policy. Section 5 includes some conclusions. An Appendix presents some additional analysis.

2. The Model

We consider a closed monetary economy. The supply $\bar{M}_t$ of money in period t, t = ..., -1, 0, 1, ..., is random and given by

$$(2.1) \qquad \bar{M}_t = \omega_t \bar{M}_{t-1} ,$$

where ω_t, (the gross rate of) monetary expansion, is stochastic. The net rate of monetary expansion is hence $\omega_t - 1$. Generally ω_t can take values either larger or smaller than unity, hence monetary contraction is a possibility. We naturally restrict ω_t to take positive values only. Furthermore, monopolistically competitive firms produce a stochastic output of non-storeable differentiated products. Production is costless up to a stochastic exogenous capacity level. Capacity shocks are perfectly correlated across firms. Throughout this Section it is sufficient to regard the economy's aggregate production, $\tilde{y}_t$, and its capacity, y_t,

$$(2.2) \qquad \tilde{y}_t \leqslant y_t ,$$

as referring to a single non-storeable good. Section 3 will consider disaggregation into differentiated products. We shall call $s_t = (y_t, \omega_t)$ the state in period t. The states are serially independent and their probability distribution is given by the time-independent distribution function $F(s_t)$.

The economy has a representative consumer with preferences in period t given by

$$(2.3) \qquad E_t \, \Sigma_{\tau=t}^{\infty} \, \beta^{\tau-t} u(c_\tau) \ , \ 0 < \beta < 1,$$

where u(c) is a standard concave utility function of consumption c, and E_t is the expectations operator conditional upon information available at t. Again, it is at this stage sufficient to consider aggregate consumption only, whereas disaggregation into differentiated products will be properly dealt with in Section 3.

Let us consider the situation of the consumer and the timing of events in the economy. There are two traded assets; money and shares. Shares are claims to dividends, more specifically cash payments consisting of the firm's revenues from sales of output. (Again, we need only consider aggregate dividends and shares at this stage, since shares in different firms will turn out to be perfect substitutes.) The consumer enters period t with predetermined holdings of money, M_{t-1}, and shares, z_{t-1}. In the beginning of period t, the consumer learns the state, $s_t = (y_t, \omega_t)$ and receives the net money transfer $(\omega_t - 1)\bar{M}_{t-1}$. After that the goods market opens, and the consumer can buy goods for consumption, c_t, at given money prices, P_t, but with cash on hand only. That is, the consumer faces the liquidity constraint

(2.4a) $$P_t c_t \leqslant M_{t-1} + (\omega_t - 1)\bar{M}_{t-1} .$$

As further specified below, goods prices will be predetermined and will not adjust to the current state of the market. Therefore there will be some states with excess supply and some states with excess demand for goods. In the latter states the consumer is rationed. Thus the consumer faces a rationing constraint

(2.4b) $$c_t \leqslant \tilde{y}_t,$$

which may bind in some states.

After the goods trade is completed, the goods market closes, and the asset market opens. The consumer receives dividends on his shares, $P_t\tilde{y}_t z_{t-1}$,[6] and can trade money and shares at given money prices, Q_t, according to the budget constraint

(2.4c) $$M_t + Q_t z_t \leqslant [M_{t-1} + (\omega_t - 1)\bar{M}_{t-1} - P_t c_t] + \\ + (Q_t + P_t\tilde{y}_t)z_{t-1} .$$

[6] Total dividends paid out by the firm in period t is $P_t\tilde{y}_t$, the cash received by selling the firm's output on the goods market. The quantity of shares is normalized such that z_t is the proportion of total outstanding shares. Hence, dividends paid to a consumer holding the proportion z_{t-1} of the shares i period t equal $P_t y_t z_{t-1}$. In equilibrium z_t will obviously equal unity and 100% of the firm is owned by the consumer.

Here M_t and z_t are the new holdings of money and shares to be carried into period t+1. The bracketed expression on the right-hand side is money not spent on the goods market. The other term on the right-hand side is the value of and dividends on initial holdings of shares.

Let $\pi_t = 1/P_t$ and $q_t = Q_t/P_t$ be the "real" prices of money and shares, that is the nominal prices deflated by the nominal goods price. (These real prices are introduced for convenience, in spite of there being no direct exchange between goods and shares). Then the budget, liquidity and constraints can be written

(2.5a) $\qquad c_t + \pi_t M_t + q_t z_t \leqslant w_t$,

(2.5b) $\qquad w_t = \pi_t(M_{t-1} + (\omega_t - 1)\bar{M}_{t-1}) + (q_t + \tilde{y}_t)z_{t-1}$,

(2.5c) $\qquad c_t \leqslant \pi_t(M_{t-1} + (\omega_t - 1)\bar{M}_{t-1})$ and

(2.5d) $\qquad c_t \leqslant \tilde{y}_t$,

where we call w_t real wealth in period t. The decision problem for the consumer is to maximize the objective function (2.3) subject to the budget, liquidity and rationing constraints (2.5).

Before deriving the first-order conditions to this problem, let us consider the pricing of goods. We want to consider a situation when nominal goods prices are in some sense sticky. We choose to represent this by assuming that nominal goods prices are predetermined, in the sense that they do not depend on the current state. The idea is the following. Nominal goods prices for period t+1 are set by representative firms. The firms must set goods prices before they observe the state (capacity and monetary expansion) in period t+1, either because it takes time to implement a price change (retagging of goods) or because it takes time to determine what the state is. Hence, goods prices in period t+1 can only depend on the state variables at t. The state variables at t will be the state $s_t = (y_t, \omega_t)$, the money stock $\bar{M}_t$ and the goods price in period t. The latter is included since it is predetermined by the state variables in period t-1. Then we postulate a stationary pricing function

(2.6a) $$P_{t+1} = P(s_t, \bar{M}_t, P_t) ,$$

or, equivalently, in terms of the real price of money,

(2.6b) $$\pi_{t+1} = \Pi(s_t, \bar{M}_t, \pi_t) .$$

Let us postpone until Section 3 the determination of this pricing function, and note that under this setup the goods price in any period does not respond to the state and the market conditions in the period. It follows that there may arise situations with excess supply or excess demand in the goods market. As is usual we shall assume that the short side rules in the market. When there is excess supply, the firms will be rationed, and the part of capacity that exceeds demand will be wasted. When there is excess demand, the consumer will be rationed. It follows that the goods market equilibrium condition should be written

(2.7a) $$c_t = \tilde{y}_t \leqslant y_t .$$

The waste $y_t - c_t$ can be interpreted as a measure of the underutilization of resources, or unemployment.

On the asset market, the price of shares is flexible and the market equilibrium conditions for money and shares are

(2.7b) $$M_t = \bar{M}_t \quad \text{and}$$

(2.7c) $$z_t = 1 .$$

The pricing function (2.6), the market equilibrium conditions (2.7), and the first-order conditions for the consumer will give the equations determining an equilibrium. We assume that these equations have a unique solution.[7] In a stationary stochastic rational-expectations equilibrium prices of goods and shares, firms output and all other endogenous variables in period t will be functions of the state variables in period t, $(s_t, \bar{M}_t, \pi_t)$. Then the consumers problem to maximize (2.3) subject to (2.5) defines, in the usual way, the value function $v(w_t, M_{t-1}, s_t, \bar{M}_t, \pi_t)$ as the maximum of $u(c_t) + \beta E_t v(w_{t+1}, M_t, s_{t+1}, \bar{M}_t, \pi_t)$ subject to (2.5).[8] The first-order conditions for an optimum, together with the pricing function (2.6b) and the market equilibrium conditions (2.7), will after some manipulation give the equations defining an equilibrium,[9]

(2.8a) $\quad c \leqslant \pi\bar{M} \qquad [\mu \geqslant 0],$

(2.8b) $\quad c \leqslant y \qquad [\nu \geqslant 0],$

(2.8c) $\quad u_c(c) = \lambda + \mu + \nu,$

(2.8d) $\quad \lambda\pi = \beta E[\lambda' + \mu']\Pi(s, \bar{M}, \pi)$ and

(2.8e) $\quad \lambda q = \beta E[\lambda'(q' + c')].$

Here a variable without a prime refers to period t, and with a prime to period t+1. The variables λ, μ and ν are the Lagrange multipliers of the constraints (2.5a), (2.5c) and (2.5d), respectively. By the definition of the value function λ and μ fulfill

(2.9) $\quad v_w = \lambda$ and $v_M = \mu\pi,$

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[7] Existence and uniqueness of equilibrium can be demonstrated along the times of Lucas (1982) and Lucas and Stokey (1985). For instance, one has to make sure that the right hand side of (2.4a) never becomes negative. This can be incorporated by assuming that the probability distribution F(s) is restricted such that there is a small $\varepsilon > 0$ such that ω never take values below ε. Then the constraint $M_t \geqslant (1 - \varepsilon)\bar{M}_t$ is added to the decision problem of the consumer. In equilibrium $M_t = \bar{M}_t$ and the constraint is never binding. Also, one has to assume $\beta E[1/\omega'] \leqslant 1$, that is (roughly) the expected monetary expansion must be sufficiently large. Written $E[1/\omega'] \leqslant 1/\beta = 1+\delta$ where δ is the rate of time preference, we can interpret it as the intuitive condition that the average rate of contraction of the money supply must not exceed the rate of time preference.

[8] In (2.5c) $\bar{M}_{t-1}$ does enter separately, but since it is given by $\bar{M}_t/\omega_t$ we need not enter it as a state variable in the value function.

[9] Equations (2.8c), (2.8d) and (2.8e) are the first-order condition for maximizing the Lagrangean over c, M and z, respectively.

which has been used in deriving (2.8). The variables λ and μ can be interpreted as the marginal utility of real wealth and of real balances, respectively. The notation $[\mu \geqslant 0]$ in (2.8a) refers to the complementary-slackness condition $y \leqslant m$, $\mu \geqslant 0$ and $(m - y)\mu = 0$.

Equations (2.8a) and (2.8b) are the liquidity and rationing constraints, respectively. Equation (2.8c) relates the Lagrange multipliers to the marginal utility of consumption. We note that the marginal utility of wealth, λ, is less than, or equal to, the (total) marginal utility of real balances, $\lambda + \mu$. This expresses the circumstance that shares are less liquid than real balances. The (total) marginal utility of real balances equal the marginal utility of consumption whenever the consumer is not rationed.

Equation (2.8d) corresponds to the asset-pricing equation for money referred to in the Introduction. Since goods prices here are determined instead by the pricing equation (2.6), (2.8d) is here an equilibrium relation between λ and μ. Equation (2.8e) is the asset-pricing equation for shares.

The system of equations (2.8) can be solved to give consumption and output c, and the other endogenous variables λ, μ, ν and q as functions of the state variables $(s, \bar{M}, \pi)$, for any given pricing function (2.6). As mentioned, we shall give a rigorous derivation of the pricing function in Section 3, when firms in monopolistic competition choose prices so as to maximize their stock market values. That derivation will result in the following simple pricing function,

(2.10a) $\quad P_{t+1} = \bar{M}_t/n$, or

(2.10b) $\quad \pi_{t+1} = n/\bar{M}_t$,

where n is a constant that depends upon the parameters of the model, including the probability distribution of the states.

The price level firms' set is hence simply proportional to the current money stock. The reason the pricing function is this simple is, first, that because stocks are serially independent, current shocks does not influence expectations about future states. Hence, the optimal prices will in some sense be independent of current shocks. Second, there will be no money illusion in the model. Hence prices will be proportional to the current money stock. (If we like, we can think of prices being proportional to next period's expected money stock, $E_t\bar{M}_{t+1} = (E\omega_{t+1})\bar{M}_t$, which is proportional to the current money stock). To simplify the presentation, we here postulate that the pricing function looks like (2.10) for some constant n, whereas we, as mentioned, return to the problem of finding the optimal prices in Section 3.

Under this pricing function, the equilibrium equations (2.8a-e) can be written

(2.11a) $\quad c \leqslant \omega n \qquad [\mu \geqslant 0]$,

(2.11b) $\quad c \leqslant y \qquad [\nu \geqslant 0]$,

(2.11c) $\quad u_c(c) = \lambda + \mu + \nu$ and

(2.11d) $\quad \lambda = A/\omega$, where $A = \beta E[\lambda' + \mu']$, and

(2.11e) $\quad \lambda q = A^*$ where $A^* = \beta E[\lambda'(q'+c')]$.

For a given n, equations (2.11a) - (2.11c) can be solved for c, λ, μ and ν as functions of s, independent of the money stock $\bar{M}$ and the predetermined price of money π. For constant n, A and A^* are constant.[10] We note in (2.11d) that the marginal utility of wealth, λ, is decreasing in monetary expansion and independent of capacity. The share price q can be solved for in (2.11e). More precisely, the solution can be expressed as follows. Let us consider the case with a utility function (2.3) which has an intertemporal elasticity of substitution larger than unity. A high degree of intertemporal institution implies that intertemporal substitution effects are relatively strong, which as usual leads to more "normal" results. (The case with elasticity of substitution

[10] The constant A in (2.11d) can be written $A = \beta E[\mu']/(1-\beta E[1/\omega'])$. It is the discounted expected (total) utility of real balances. It is bounded under the intuitive condition $\beta E[1/\omega'] \leqslant 1$ (cf. footnote 7).

less than unity is presented in the Appendix). Let us for simplicity consider the case with a constant intertemporal elasticity of substitution. It is practical to denote this constant elasticity of substitution by $1/r$. Then this amounts to assume the instantaneous utility function

$$u(c) = c^{1-r}/(1 - r), \quad 0 < r < 1. \tag{2.12}$$

This is the familiar utility function with constant relative risk aversion, the degree of relative risk aversion being r. The constant r is hence assumed to be less than unity. (As is well known, for an additively separable intertemporal utility function the intertemporal elasticity of substitution is the reciprocal of the instantaneous degree of risk aversion).

The solution is illustrated in Figure 1. There are three different regions in (y,ω)-space that are relevant. We first define the critical state $s^* = (y^*, \omega^*)$ as the one for which the liquidity and rationing constraints (2.11a) and (2.11b) are fulfilled with equality and the corresponding Lagrange multipliers are zero. This gives

$$\omega^* = Ay^{*r} \text{ and } \omega^* = y^*/n, \tag{2.13}$$

which equations determine y^* and ω^*.[11] Then region I is defined by $\omega \leq \min(Ay^r, \omega^*)$ and there

$$c = u_c^{-1}(\lambda) = A^{-1/r}\omega^{1/r} \leq \min(\omega n, y), \tag{2.14a}$$

$$\lambda = A/\omega \text{ and } \mu = \nu = 0.$$

Region II is given by $y \geq y^*$ and $\omega^* \leq \omega \leq y/n$, and there

$$c = \omega n \leq y, \tag{2.14b}$$

$$\lambda = A/\omega,$$

$$\mu = u_c(c) - \lambda = n^{-r}\omega^{-r} - A/\omega \geq 0 \text{ and } \nu = 0.$$

Region III, finally, is given by $\omega \geq \max(Ay^r, y/n)$. There

$$c = y \leq \omega n, \tag{2.14c}$$

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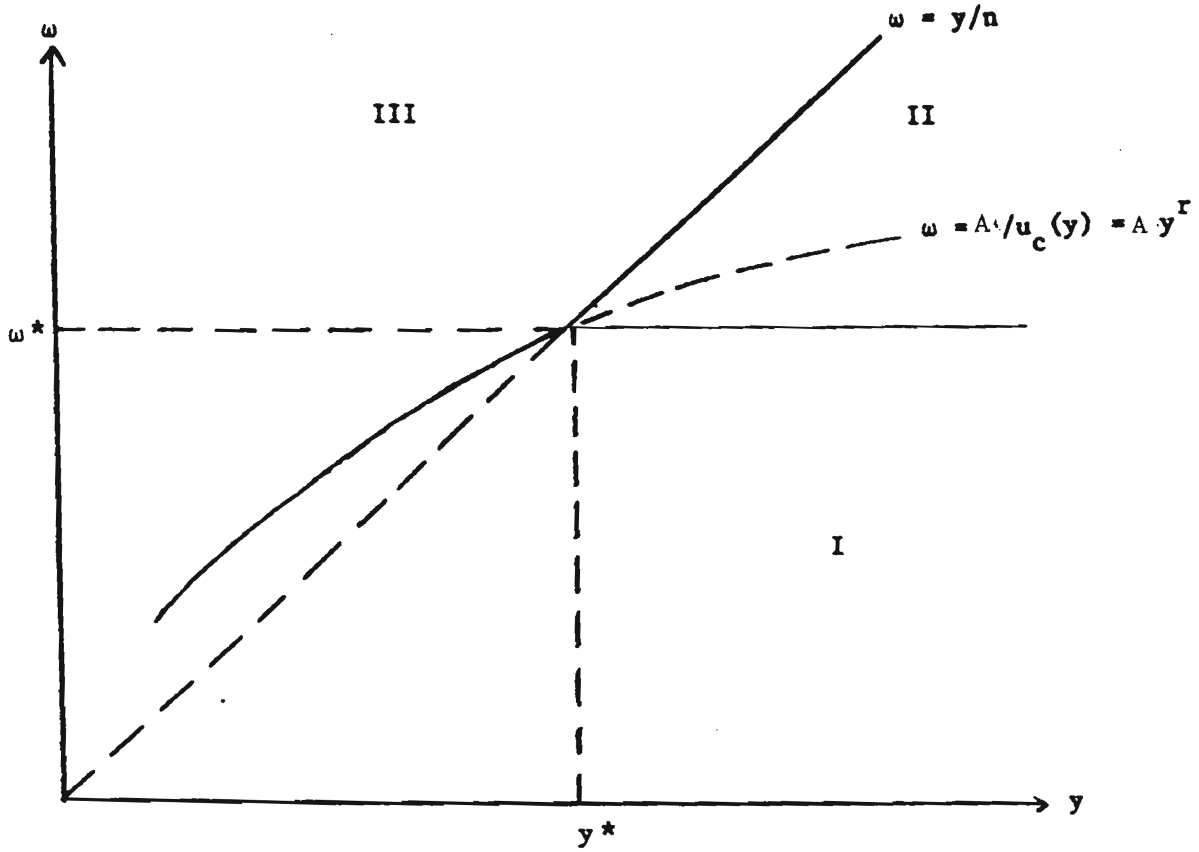
[11] Recall that the constant A is implicitly given by $A = \beta E[\mu']/(1-\beta E[1/\omega'])$.

Figure 1



$$\lambda = A/\omega,$$

$$\nu = u_c(c) - \lambda = y^{-r} - A/\omega \geq 0 \text{ and } \mu = 0.$$

In region I there is excess supply and neither the liquidity constraint nor the rationing constraint binds. Both the marginal utility of real balances and the multiplier of the rationing constraint are zero, and the marginal utility of wealth equals the marginal utility of consumption. In region II, there is still excess supply but now the liquidity constraint is binding, the marginal utility of real balances is positive and the marginal utility of wealth is less than marginal utility of consumption. In region III there is excess demand and the rationing constraint is binding. Its multiplier is positive and the marginal utility of wealth is less than the marginal utility of consumption.

Figure 2 shows iso-value curves for consumption/output, and Figure 3 shows iso-value curves for μ and ν.[12]

To further understand the nature of the solution and the relative location of the regions, consider a state corresponding to a point in region II. The consumer is liquidity constrained, consumption equals real balances and is less than capacity. There is excess supply of goods. Suppose monetary expansion is increased, but capacity is held constant. Then the point in region II moves north. Real balances increase and so does consumption. If monetary expansion increases sufficiently, consumption will hit the capacity constraint and the point enters region III. Further increases in monetary expansion leads to excess demand for goods, and the point moves into the interior of region III.

Consider also a state corresponding to a point in region I. Here neither the liquidity nor the capacity constraint is binding, and there is excess supply of goods. Why is consumption so low? One way to understand this is to note that the monetary expansion is relatively low in this region. Next period's

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[12] The iso-value curves for μ and ν are given by the equations $\omega = A/(n^{-r}\omega^{-r} - \mu)$ and $\omega = Ay^{r}/(1 - \nu y^{r})$ for region II and III, respectively.

Figure 2

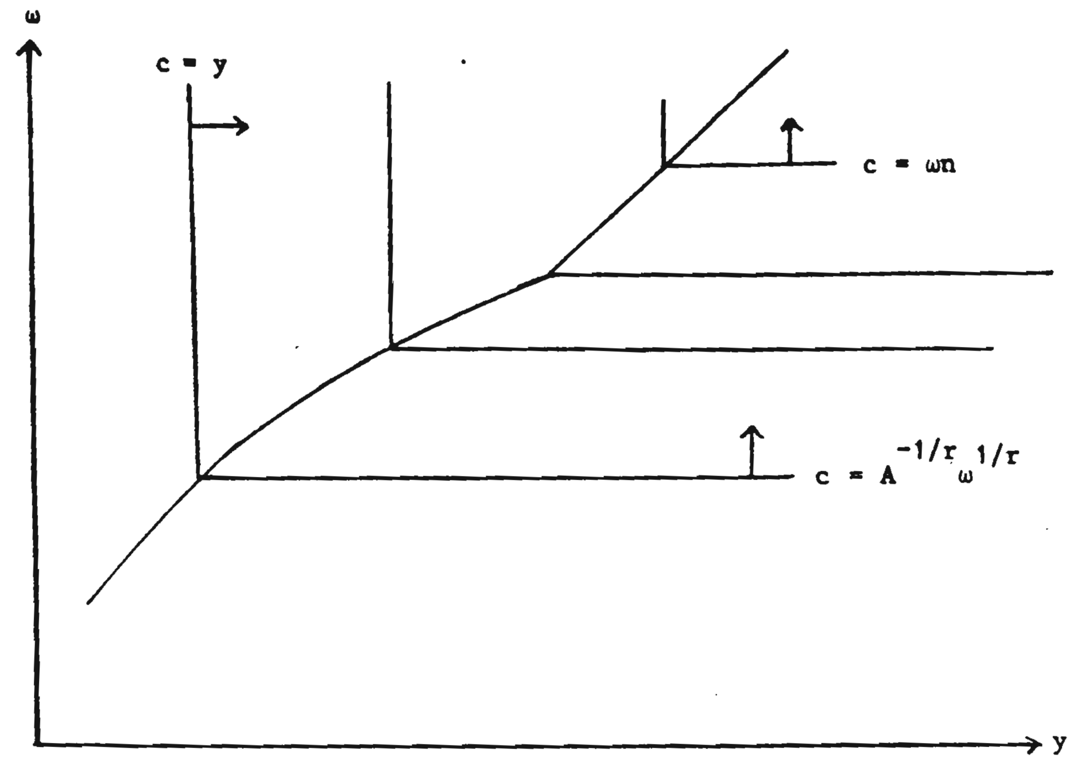
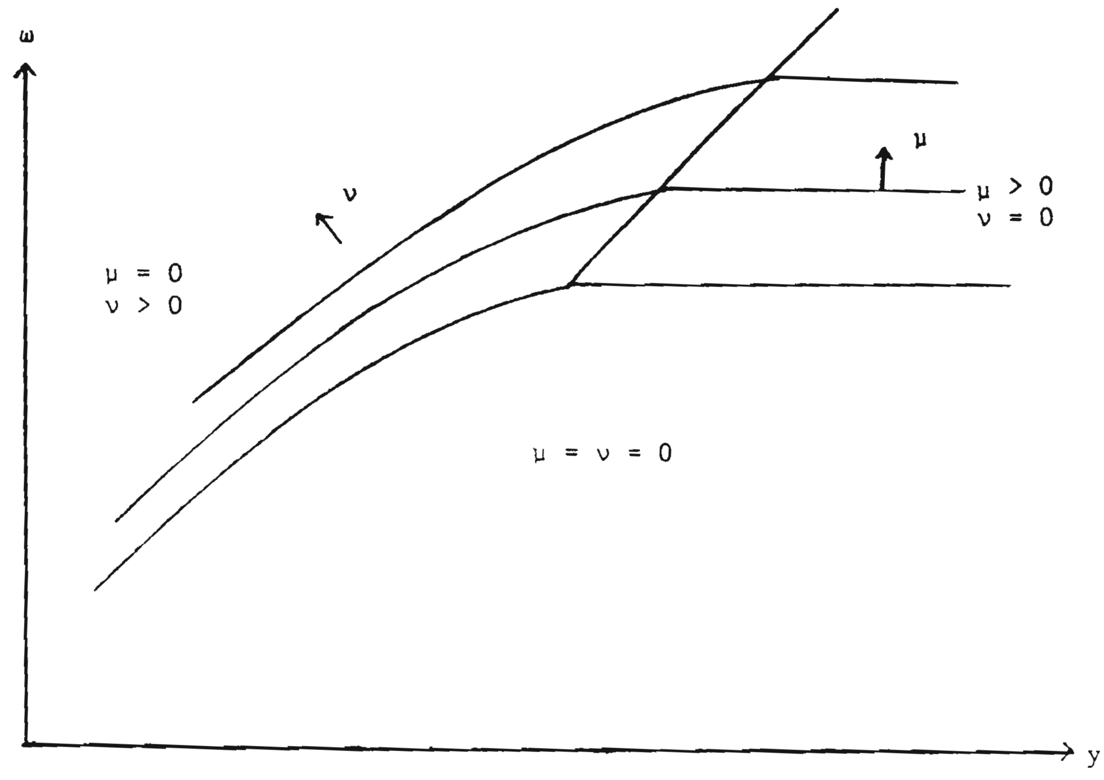


Figure 3



price level is proportional to current monetary expansion. The current price level is predetermined. Hence, it follows that the inflation rate $P'/P = \omega\bar{M}_{-1}/nP$ is relatively low. This means that the total return to holding money is relatively high, which implies that the consumer prefers to hold real balances rather than consume. This in turn means a low demand for consumption. Suppose now that monetary expansion increases. The point in region I moves north. This increases the rate of inflation and decreases the total return on holding money. This makes the consumer want to decrease real balances and increase consumption. If capacity is low, i.e. below the critical level y^*, eventually the capacity constraint will bind, and the point enters region III. If capacity is high, instead eventually the liquidity constraint will bind and the point enters region II. Real balances also increase with monetary expansion, but consumption increases more, because of a large substitution effect due to a high degree of intertemporal substitution. So eventually the liquidity constraint binds.

We have hence demonstrated how consumption and output depend on both capacity and monetary expansion. Let us also consider share prices and interest rates. Solving (2.11e) gives the share price

$$q = A^*/\lambda \text{ where } A^* = E[\lambda' c']/\delta \tag{2.15}$$

where the rate of time preference δ is defined by $\beta = 1/(1 + \delta)$. Together with (2.11d) this gives

$$q = A^*\omega/A. \tag{2.15a}$$

The share price varies inversely with the marginal utility of wealth. Therefore it increases with monetary expansion and is independent of capacity.

As is well known, additional assets can easily be introduced and priced in this framework, even if the quantity held in equilibrium is zero. Let us consider the nominal interest rate on a one-period bond traded, and paying interest, on the asset market. (The timing of the trading of any asset, as well as timing of its dividend, must be specified. Assets traded before the goods market has closed are not perfect substitutes to assets traded after it has closed. See Svensson (1985b) for further discussion of this point). Let B_t denote the nominal bonds bought on the asset market in (at the end of) period t and carried into period t+1, and let i_{t+1} denote the sure nominal interest rate paid on the asset market in period t+1. (B_t will equal zero in equilibrium). Including $\pi_t B_t$ on the left-hand side of (2.5a) and $\pi_t(1 + i_t)B_{t-1}$ on the right-hand side of (2.5b) and maximizing over B_t gives the first-order condition $\lambda\pi = (1 + i')\beta E[\lambda'\pi']$. The nominal interest rate is hence given by

$$(2.16) \qquad 1 + i' = \lambda\pi/\beta E[\lambda'\pi'],$$

which together with (2.10b) and (2.11d) can be reformulated to

$$(2.16a) \qquad 1 + i' = 1/\beta E[1/\omega'].$$

The nominal interest rate does not depend on the realization of capacity and monetary expansion, only on the the probability distribution of monetary expansion.[13]

Let us by the real interest rate, ρ_{t+1}, denote the sure interest rate on a one period indexed bond that pays a sure unit of real wealth (cash deflated by the price level) on the asset market in period t+1. Introducing b_t for the quantity of such bonds on the left-hand side of (2.5a) and $(1+\rho_t)b_{t-1}$ on the right-hand side of (2.5b) gives the first-order condition

$$(2.17) \qquad 1 + \rho' = \lambda/\beta E[\lambda'].$$

By (2.11d) and (2.16a) we get

$$(2.17a) \qquad 1 + \rho' = (1+i')/\omega.$$

[13] That the nominal interest rate is independent of both capacity and monetary expansion may appear surprising. As shown and further discussed in Svensson (1985b), the interest rate can be written as $1+i' = E[\mu'\pi']/E[\lambda'\pi']$, which with predetermined prices simplifies to $1+i' = E[\mu']/E[\lambda']$. The nominal interest rate compensates for the absence of liquidity services μ' of bonds relative to money. The current interest rate as defined here is related to next periods liquidity services. With serially uncorrelated shocks, current shocks give no information about next period's level of liquidity services. Hence the nominal interest rate is constant.

The real interest rate varies directly with the marginal utility of wealth. Hence it decreases with monetary growth and is independent of capacity.

We have hence demonstrated that monetary policy affects the real interest and share prices. Let us next briefly contrast the results above with the situation with flexible goods prices and full capacity utilization. In this situation neither the rationing constraint nor the pricing function (2.6) applies. The goods market equilibrium condition (2.7a) holds with equality. By introducing the notation $m_t = \pi_t \bar{M}_t$ for real balances in period t, the first-order conditions and the market equilibrium conditions can be manipulated to read[14]

(2.18a) $y \leqslant m \qquad [\mu \geqslant 0]$,

(2.18b) $u_c(y) = \lambda + \mu$,

(2.18c) $\lambda m = B$, where $B = \beta E[u\ (y')m'/\omega']$, and

(2.18d) $\lambda q = B^*$, where $B^* = \beta E[\lambda'(q' + y')]$.

The equations (2.18a-c) can be solved for real balances, the marginal utility of wealth and the marginal utility of real balances as functions $m(y)$, $\lambda(y)$ and $\mu(y)$ of output y only. The rate of momentary growth has no effect on these variables. Next period's monetary growth enters into the expression for B in (2.18c), but since the states are serially independent it integrates out and B is a constant. Given $\lambda(y)$ the share price $q(y)$ can be solved for in equation (2.18d), where B^* will be a constant.

The price level $P = 1/\pi$ will be a function

(2.19) $P = \bar{M}/m(y)$.

Hence, the current price level is proportional to the current supply of money, $\bar{M} = \omega\bar{M}_{-1}$, and money is neutral in the sense that a doubling of the money stock in all periods doubles the price level in all periods. Money is also

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14 The asset-pricing equation for money referred to in the Introduction is $\lambda\pi = \beta E[(\lambda' + \mu')\pi']$, which is the first-order condition for maximizing the Lagrangean with respect to M_t. Substituting $\pi = m/\bar{M}$ and $\pi' = m'/\omega'\bar{M}$ and using (2.18b) gives (2.18c).

superneutral, in the sense that a (serially independent) increase in the rate of growth of money leads to an equal increase in the rate of inflation. This is so because real balances are independent of both the money stock, $\bar{M}_{-1}$, and the rate of growth of money, ω. (Serially correlated money shocks will, of course, not be superneutral).

The nominal interest rate is still given by (2.16) and independent of both capacity and monetary expansion. The share price is given by

$$q = B^{*}/\lambda(y), \tag{2.20}$$

independent of monetary expansion and, it can be shown, increasing in capacity, since the marginal utility of wealth can be shown to the decreasing in capacity. The real interest rate, given by (2.17) is independent of monetary expansion and decreasing in capacity. Clearly, with flexible goods prices, monetary policy - meaning serially independent shocks to the rate of growth of the money stock - has no real effects, either on consumption or on any real asset price, including the real interest rate.[15] Before further examination of the equilibrium with sticky goods prices, we first return to the pricing function (2.10).

3. Equilibrium Goods Prices[16]

In this Section we shall derive the pricing policy of firms, and, more specifically, explain what determines the constant n in (2.10). We shall do this by introducing differentiated products and monopolistic competition along the lines of Dixit and Stiglitz (1977).

Let there be a continuum of firms. The set of firms is represented by the unit interval, and each firm is indexed by j, $0 \leq j \leq 1$. Each firm produces a

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[15] In Svensson (1985b) monetary transfers are distributed after the goods market is closed, rather than before it opens. Then current monetary transfer cannot be used to purchase current goods. For that case real balances will depend also on the rate of growth of money, and money is no longer superneutral, even with independent shocks.

[16] I received crucial help from Elhanan Helpman in both formulating and solving the pricing problem.

unique differentiated product. Hence we let j refer to both firm j and its product. Let the firms face perfectly correlated economy-wide shocks to their capacity. Then we can let y denote the capacity of each firm as well as the aggregate capacity of the economy (since $\int_{j=0}^{1} y dj = y$).

Let y_j, P_j and Q_j be firm j's actually sold output, the nominal price of its product, and its nominal stock market value, respectively. These are related by the asset-pricing equation[17]

$$(3.1) \qquad \lambda\pi Q_j = \beta E[\lambda'\pi'(Q_j' + P_j'y_j')].$$

Exploiting stationarity, that P_j' is predetermined, and that from (2.10) and (2.11d) $\lambda'\pi'/\lambda\pi = 1/\omega'$,[18] we can solve (3.1) to get

$$(3.2) \qquad Q_j = P_j'E[y_j'/\omega']/\delta,$$

where the rate of time preference δ is defined from $\beta = 1/(1+\delta)$. We assume that firm j takes as given the price level in each period and the market discount factors $\lambda'\pi'/\lambda\pi\delta = 1/\omega'\delta$, and chooses the price of its product P_j' in order to maximize its stock market value given by (3.2). It obviously then remains to specify how the expression $E[y_j'/\omega']$, the expectation of the firm's output discounted by the gross rate of growth of money, depends on the price P_j'.

Let us therefore consider the consumer's preferences for the firms' differentiated products. We choose to represent this along the lines of Dixit and Stiglitz (1977). We consider consumption c in the utility function u(c) in (2.3) as being aggregate real consumption, more precisely given by the subutility function

$$(3.3) \qquad c = [\int_{j=0}^{1} c_j^{(\sigma-1)/\sigma} dj]^{\sigma/(\sigma-1)}, \quad \sigma > 1.$$

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[17] Note that whereas λ and λ' in (2.8e) is the marginal utility of real wealth in period t and t+1, respectively, $\lambda\pi$ and $\lambda'\pi'$ is the marginal utility of nominal wealth in period t and t+1, respectively.

[18] Note that the discount factor of one unity of cash in state $s' = (\omega', y')$ next period in terms of one unit of cash this period is simply $\beta\lambda'\pi'/\lambda\pi = \beta/\omega'$, which is independent of output. The nominal present value of a sure unit of cash next period is consequently equal to $\beta E[1/\omega']$, which should equal one over one plus the nominal interest rate. Indeed it does, as is seen in (2.16).

That is, aggregate consumption is a CES subutility function of consumption c_j of product j, $0 \leqslant j \leqslant 1$, where the constant elasticity of substitution σ is above unity. We identify the price level P with the corresponding price index

$$(3.4a) \qquad P = [\int_{j=0}^{1} P_j^{1-\sigma} dj]^{1/(1-\sigma)} .$$

It follows, by a standard derivation, that the demand for product j obeys

$$(3.4b) \qquad c_j = (P_j/P)^{-\sigma} c.$$

Let us now specify that the actual output of firm j fulfills

$$(3.5) \qquad y_j' = \min (y', c_j').$$

In situations with full capacity and excess demand, that is in region III in Figure 1, the firm can sell all output up to capacity independent of the price P_j', $y_j' = y' \leqslant c_j'$. In situations of excess capacity, that is in the "interior"[19] of region I and II in Figure 1, the firm cannot sell all its capacity y' and its output y_j' is constrained by demand c_j', $y_j' = c_j' < y'$. Furthermore, firm j then perceives demand as given by (3.4), where aggregate consumption c is given by the solution to (2.14), which we denote $c(s,n)$, a function of the state $s = (y,\omega)$ and the given (economy-wide) n. The firm takes P', n and the function $c(s,n)$ as given, but affects c_j' by choosing P_j'. Introducing the relative price $p' = P_j'/P'$, we can hence define the function

$$(3.6) \qquad g(p',n) = E[y_j'/\omega'] = \\ = \int_{III} (y'/\omega') dF(s') + \int_{IUII} p'^{-\sigma} (c(s',n)/\omega') dF(s'),$$

where the integration over the region I and II, IUII, refers to the interior of the region.

We now realize from (3.2) that maximization of the stock market value of firm j implies that the price P_j' is chosen such that the elasticity of (3.6) with respect to p' equals minus unity. Furthermore, in equilibrium all firms

[19] By the "interior" of region I and II we refer to the points not on the borderline to region III

choose the same price, hence the relative price p' equals unity, $p' = P_j'/P' = 1$.[20]

Hence we have the equilibrium condition

$$g_p(1,n)/g(1,n) = -1, \tag{3.7}$$

which implies

$$-\sigma\int_{IUII}(c(s',n)/\omega')dF(s')/g(1,n) = -1. \tag{3.8}$$

We see that the price elasticity of expected discounted sales (3.6) equals the weighted average of the elasticity at full capacity (which is zero) and the elasticity at excess capacity (which is the negative of σ). Let us finally use (3.6) to rewrite (3.8) as

$$\int_{III}(y/\omega)dF(s) - (\sigma - 1)\int_{IUII}(c(s,n)/\omega)dF(s) = 0, \tag{3.9}$$

where we for simplicity have dropped the prime. This is the equation that determines n, given the solution c(s,n) and the regions I, II and III in (2.14). (Note that the regions also depend on n). We assume that (3.9) gives rise to a unique n.

In sum, the full equilibrium is given by (2.14) and (3.9), which together determine c, λ, μ, and ν as functions of s, as well as the constant n in the pricing function (2.10), for given parameters of the model (including the probability distribution of the states, F(s)).

We note that (3.9) implies that equilibrium requires that underutilization occurs with positive probability. If there were full utilization in all states, the price elasticity of expected discounted sales would be zero, and each firm would perceive that it can increase its market value by increasing the nominal price of its output. The price level would increase (n would fall) and real balances would fall until demand would be restricted such that there would be excess capacity in some states.

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[20] Also, all differentiated goods are consumed in the same quantity and $c_j = c(s,n)$, $0 \leqslant j \leqslant 1$.

We also see that there must be full utilization with positive probability. If there were underutilization in all states, firms would perceive the price elasticity of expected discounted sales to be minus σ, less than minus unity, and it would pay to lower prices until real balances would be so high as to increase demand to capacity in some states.

4. Monetary Policy and Welfare[21]

Now that we have specified the nature of the equilibrium, including the endogenous determination of goods prices, we can go on and explore the role of monetary policy. Let us first note the obvious circumstance that in this rudimentary model, what is labelled monetary policy is actually a combination of monetary and fiscal policy, namely net transfers financed by money creation. So far we have specified the equilibrium for an arbitrary given joint probability distribution $F(y,\omega)$ of capacity and monetary expansion. It is clear that for a given probability distribution, realizations of monetary expansion, even serially independent, have an effect on output and hence consumption and welfare, since they may determine in which region in Figure 2 the state is, and hence whether or not there is underutilization of resources.

We have also seen, in discussing the first-order condition for firms' stock market value maximization, that for any probability distribution there will be underutilization in some states with positive probability. Hence, average utilization is always less than full, and there is a corresponding welfare loss, in terms of expected utility, compared to an economy with flexible goods prices and always full utilization. Let us here note that average utilization and welfare depends on the degree of competition by firms as measured by the elasticity of substitution in consumption between differentiated products, σ. If this elasticity approaches its minimum (infinum, to be precise) allowed in (3.3), unity, it follows from the first-order condition (3.9) that

[21] I benefited from specific help by Jörgen Weibull on this section.

the probability of full utilization approaches zero. The absolute value of the price elasticity of the demand facing each firm approaches unity in the states with underutilization, and firms are content with setting so high prices as being almost always in such states. For the case when capacity and monetary expansion are independent distributed, as further discussed below, the consequence is that consumption approaches the minimum capacity level, and average consumption and welfare approach their minimum. If instead the elasticity of substitution approaches infinity, the differentiated products become perfect substitutes in consumption and competition becomes perfect. Then the elasticity of demand is so high in states with underutilization, that it pays for firms to lower prices until real balances and demand is so high that these states with underutilization do not occur. Then consumption is always equal to capacity, and welfare is at its maximum. In this model, competition is indeed good.[22]

Let us next consider a situation when monetary growth is not given by a fixed joint probability distribution with output, but is subject to choice by a government. Suppose the government wants to maximize consumers' expected utility. Let us consider two cases; first, when the government must choose the current rate of monetary growth before it knows current capacity, and, second, when the government can choose the rate of growth after it has observed current output.

In the first case, since shocks are serially independent there is no point in choosing anything but either a constant or a randomized rate of monetary growth. Let us here mainly discuss the choice of a constant rate of monetary

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[22] This result apply in the two limits for the elasticity of substitutions, unity and infinity. More precisely, they apply for σ arbitrary close to, but not equal to, the two limits. Also, it does not follow, without further restriction, that average consumption and welfare vary monotonically with the elasticity of substitution between differentiated products.

growth. The case with a randomized rate of growth is discussed below and in the Appendix.

Hence, the government chooses a constant rate of monetary expansion,

(4.1) $\omega = \omega^0 \geqslant \beta$

(if $\omega^0 < \beta$ no equilibrium exists). Let capacity be serially independent and distributed according to the given distribution function G(y). It can be shown that region I is empty and that the solution (2.14) for given n is

(4.2) $c = y$ for $y \leqslant \omega^0 n$ (region III), and

$c = \omega^0 n$ for $y > \omega^0 n$ (region II).

The condition (3.9) can then be written

(4.3) $\int_{y \leqslant \omega^0 n} y dG(y) - (\sigma - 1)\omega^0 n \int_{y > \omega^0 n} dG(y) = 0.$

It is clear from (4.3) that there will be a critical level of capacity y^0, which depends on σ and the probability distribution G(y) but is independent of ω^0, such that n fulfills

(4.4) $\omega^0 n = y^0.$

Hence from (4.4) we see that the consumer's expected utility is

(4.5) $Eu(c) = \int u(\min(y, y^0))dG(y).$

It follows that ω^0 does not affect expected utility. For each rate of growth ω^0, firms adjust and choose prices such that real balances $\omega n = y^0$ are unaffected. Monetary policy can increase neither welfare nor the utilization of resources.

With regard to asset prices, since $\lambda = A/\omega^0$ is constant, we get from (2.15)

(4.6) $q = E[c']/\delta = E[\min(y, y^0)]/\delta,$

and the value of the firms is constant and independent of ω^0. The nominal interest rate is given by

(4.7) $1 + i' = \omega^0/\beta,$

and is increasing in ω^0. The real interest rate is, since λ is constant, equal to the rate of time preference,

(4.8) $$1 + \rho' = 1/\beta = 1 + \delta.$$

Clearly, the deterministic monetary policy has no effect on real asset prices.

We also note that when the elasticity of substitution σ approaches unity, the critical level of capacity approaches the minimum capacity level,[23] and consumption will in the limit equal the minimum capacity level in all states. Welfare is then at its minimum. When the elasticity of substitution approaches infinity, it follows from (4.3) that consumption in the limit will equal capacity in all states, and welfare will be at its maximum.

Let us also consider the case when the government has perfect information in the sense of being able to choose ω conditional upon current capacity y. Consider the following monetary policy,

(4.9) $$\omega = \omega^0 \text{ for } y \leqslant y^0, \text{ and}$$
$$\omega = \omega^0 + (1 - \varepsilon)\omega^0(y - y^0)/y^0 \text{ for } y > y^0,$$

where y^0 is given by (4.3) and (4.4) and $\varepsilon > 0$ is a small constant. This is illustrated in Figure 4. Region III is to the left of the vertical line $y = y^0$, region II to the right. For the case of any constant $\omega^0 \geqslant \beta$, firms choose n such that the ray $\omega n = y$ goes through the point (y^0, ω^0) and hence (4.4) is fulfilled. Then $c = y$ for $y \leqslant y^0$. Choosing monetary policy according to (4.9) implies that the first order condition for firms (4.3) still holds for the same y^0 and $n = y^0/\omega^0$. Firms are in region III and II with unchanged probabilities, and the expected demand elasticity they face does not change. But now consumption fulfills

(4.10) $$c = y \quad \text{for } y \leqslant y^0 \text{ and}$$
$$c = y - \varepsilon(y - y^0) \text{ for } y > y^0.$$

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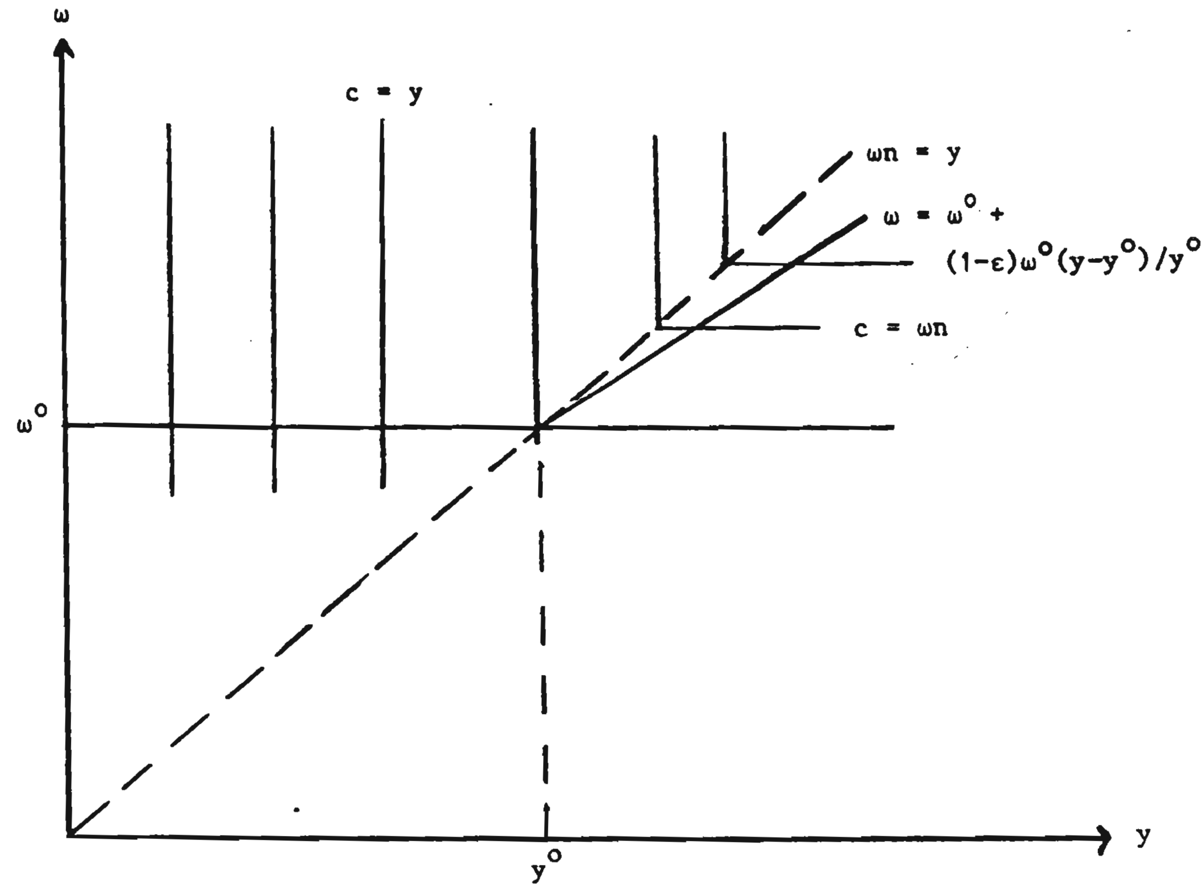
[23] That is, the infinum of the support of y.

Figure 4



If ε is chosen very small, consumption is very close to capacity for all capacity levels. Although firms have excess capacity, they are very close to full capacity. Hence, when monetary policy can be chosen conditional upon current output, it has an effect and can indeed improve utilization and welfare to be arbitrary close to their maximum (suprenum, to be precise).

With monetary policy according to (4.9), real asset prices again depend on the realization of monetary growth, as given by (2.15a) and (2.17a).

To sum up, we have found that when monetary expansion is chosen before capacity is observed, and then restricted to be constant and independent of capacity, the rate of monetary expansion has no effect on utilization, consumption and welfare. Neither does it have any effect on real asset prices. However, as mentioned above, it is certainly possible to choose a stochastic rate of monetary growth, although of course with a probability distribution independent of output. It turns out that it cannot be excluded that sometimes such a randomized monetary policy improves welfare. Some preliminary calculations are reported in the Appendix. The conclusion is that a high elasticity of substitution between differentiated products and a low capacity variance relative to (the square of) the mean contribute to making a constant monetary policy better than a randomized.

When current monetary expansion can be chosen conditional upon current capacity, it is possible to improve utilization, consumption and welfare to be arbitrary close to full utilization and maximum welfare.

These results correspond to the two extreme cases when government has either no or complete information about current capacity. A more relevant situation may be imperfect information about current capacity. This can be represented by assuming that capacity y cannot be directly observed, but there exists some variable x that can be observed and that is jointly distributed with capacity. The correlation between x and capacity is then a measure of the degree of information about capacity. The problem is then to choose monetary expansion as a possibly randomized function of the observed variable x so as to maximize welfare. The results above refer to the cases zero and perfect correlation, but it would clearly be interesting to have results for the intermediate case.

6. Conclusions

We have developed a monetary asset-pricing general equilibrium model with sticky goods prices and characterized its equilibria. Goods prices are chosen by monopolistically competitive firms so as to maximize their stock market values. We have shown that underutilization of resources, "unemployment", occurs in some states. For a given (independently distributed) monetary policy, we have seen that the degree of competition between firms affect the utilization of resources. When competition is almost perfect, there is almost full utilization of resources, and when competition is close to its minimum, so is utilization.

We have also seen that the efficacy of monetary policy depends crucially on the amount of information available to the monetary authority. With no information about current capacity, a deterministic monetary policy has no effect on output and real asset prices. We have noticed the intriguing possibility that a randomized monetary policy, distributed independently from capacity, sometimes deteriorates, sometimes improves expected utility relative to a deterministic policy. With perfect information about current capacity, monetary policy can bring about full utilization of resources. We have noted the interesting problem of characterizing the optimum monetary policy when there is some, but not complete, information about current capacity.

It deserves to be emphasized that the analysis above does not take into account any welfare losses when consumers are rationed. A framework with a representative consumer is clearly inadequate for dealing with the inevitable welfare losses connected with any realistic rationing scheme with heterogenous consumers. Furthermore, the analysis disregards any distortion caused by non-zero nominal interest rates. The optimum monetary policy is here directed exclusively towards increasing utilization of resources and is not characterized by a zero nominal interest rate, in contrast to Friedman (1969).

The paper is motivated by the belief that the monetary asset-pricing models have contributed considerably to our understanding of the relation between monetary and real phenomena but that they exaggerate the variability of the price level. I believe the contribution of this paper, if any, is mainly methodological, in that it shows one way of constructing a monetary asset-pricing general equilibrium model that has a sticky price level and gives an important role to monetary policy. It also shows how to construct the price level as the individual decisions of monopolistically competitive firms that maximize their stock market values, and that excess capacity in some states then is a necessary outcome. As in Svensson (1985b), the explicit equilibria are not too complicated, and the model should be possible to apply to a variety of specific issues.

The paper is also another demonstration that it is possible to combine ideas from the literature emphasizing rational expectations and flexible prices with ideas from the so-called fix-price or disequilibrium literature. At least this is so on a superficial level - rational expectations can be combined with optimally chosen sticky prices. On a more fundamental level, problems remain, though. What explains the information constraint according to which firms must decide on goods prices before they know the current state? Why is it that it takes time to implement a price change, or why can the current state only be observed with a lag? It would, of course, be more than satisfactory to have sticky nominal prices be, say, the outcome of some optimal contract.

The general limitations of representative-agent asset-pricing models are well-known and remain urgent areas for future research. The specific limitations of this paper include the reliance on serially independent shocks.

Appendix

1. Sticky Goods Prices and Intertemporal Elasticity of Substitution below Unity

With intertemporal elasticity of substitution below unity, $r > 1$, the solution to (2.11) is the following. Region I is defined by $y \geq y^*$ and $\omega^* \leq \omega \leq Ay^r$ and there

(A.1a) $$c = A^{-1/r}\omega^{1/r} \leq \min(y, \omega n),$$
$$\lambda = A/\omega \quad \text{and} \quad \mu = \nu = 0.$$

Region II is given by $\omega \leq \min(y/n, \omega^*)$, and there

$$c = \omega n \leq y,$$
(A.1b) $$\lambda = A/\omega,$$
$$\mu = u_c(c) - \lambda = n^{-r}\omega^{-r} - A/\omega \geq 0 \text{ and } \nu = 0.$$

Region III, finally, is given by $\omega \geq \max(y/n, Ay^r)$. There

$$c = y \leq \omega n,$$
(A.1c) $$\lambda = A/\omega,$$
$$\nu = u_c(c) - \lambda = y^{-r} - A/\omega \geq 0 \text{ and } \mu = 0.$$

The solution is illustrated in Figure A.1.

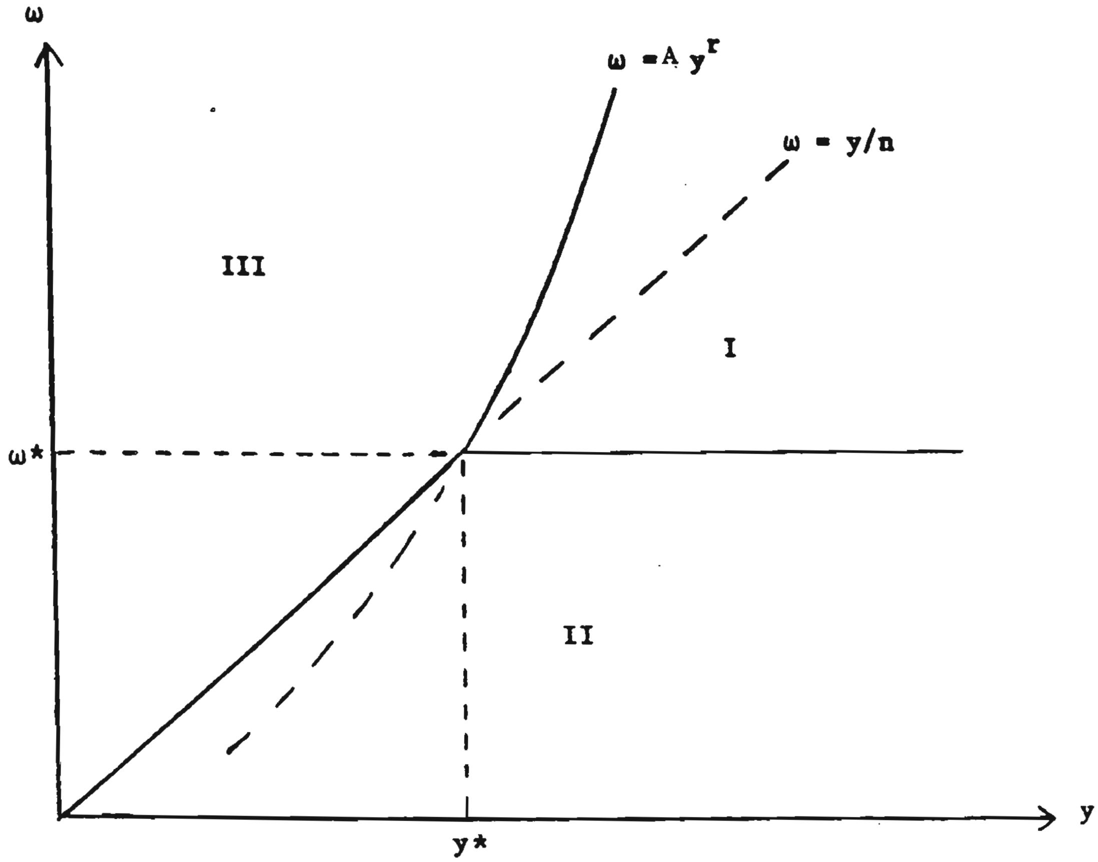
2. A Deterministic versus Stochastic Monetary Policy

Let capacity y be uniformly distributed on the interval $[a, b]$, $0 < a < b$. Let $h(\omega)$ be the density function of monetary expansion. Consider the problem of choosing the density function $h(\omega)$ so as to maximize

(A.2a) $$v = \int[\int_a^b u(\min(y, \omega n))dy]h(\omega)d\omega$$

subject to the constraint

Figure A.1



(A.2b) $\qquad \int[\int_a^{\omega n}(y/\omega n)dy - (\sigma - 1)\int_{\omega n}^{b}dy]h(\omega)d\omega = 0.$

The objective function (A.2a) corresponds to (4.5) and the constraint (A.2b) to (4.3). Clearly, (A.2a) and (A.2b) can be written

(A.3a) $\qquad v = E[R(\omega n)]$ and

(A.3b) $\qquad E[S(\omega n)] = 0,$

respectively, where the functions $R(\omega n)$ and $S(\omega n)$ are given by

(A.4a) $\qquad R(\omega n) = U(\omega n) - U(a) + u(\omega n)(b - \omega n)$ and

(A.4b) $\qquad S(\omega n) = \omega n/2 - a^2/2\omega n - (\sigma - 1)(b - \omega n),$

where $U(\omega n) = \int^{\omega n}u(y)dy$.

Restrict for convenience the expected value of ω to be unity, $E[\omega] = 1$, and consider a second-order Taylor approximation around $\omega = 1$. Then we have

(A.5a) $\qquad v = E[R(\omega n)] = R(n) + R''(n)n^2Var[\omega]/2$ and

(A.5b) $\qquad E[S(\omega n)] = S(n) + S''(n)n^2Var[\omega]/2 = 0,$

where $Var[\omega]$ denotes the variance of ω. Differentiating (A.5a) and (A.5b) around $Var[\omega] = 0$, we get

(A.6) $\qquad dv = nR'(n)[nR''(n)/R'(n) - nS''(n)/S'(n)]dVar[\omega]/2.$

Hence, whether welfare improves or deteriorates locally when the deterministic constant monetary expansion is modified to be random with a small variance depends on the sign of the term in brackets, more precisely on the relative elasticities of $R'(n)$ and $S'(n)$. These elasticities are

(A.7a) $\qquad nR''(n)/R'(n) = r - n/(b - n)$ and

(A.7b) $\qquad nS''(n)/S'(n) = - 2a^2/[(2\sigma - 1)n^2 + a^2].$

Let us consider when the bracketed term in (A.6) is likely to be negative, that is when a deterministic monetary expansion is locally better than a stochastic one. For $\omega = 1$ we can by (4.4) identify n with y^0 so we know that $a \leq n \leq b$. We then consider the following cases.

(i) σ approaches unity. Then n approaches a and the bracketed term equals $r - a/(b - a) + 1$ which is negative if $b/a < (2 + r)/(1 + r)$. For $0 \leqslant r \leqslant 1$, we have $3/2 \leqslant (2 + r)/(1 + r) \leqslant 2$, and we conclude that the bracketed term is negative if $b/a \leqslant 3/2$, that is if the variance of y is sufficiently small relative to its mean.

(ii) σ approaches infinity. Then n approaches b- and the bracketed term approaches minus infinity.

(iii) b approaches a. Then n again approaches b- and the bracketed term approaches minus infinity.

We interpret these results as indicating that for sufficiently high elasticity of substitution between differentiated products, and for a sufficiently low variance of capacity relative to its (squared) mean, a deterministic constant monetary expansion is better than a stochastic one.

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