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Allocative Efficiency and the Maturity Composition of the Capital Stock

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ABSTRACT

We investigate how transaction costs, and the liquidity of secondary capital markets, affect (a) capital accumulation and growth, (b) real returns, (c) the level of financial market activity, (d) the maturity composition of the capital stock, and (e) welfare. The research proceeds on the notion that financial markets and intermediaries reconcile the desire of investors for liquidity with the fact that illiquid investments with long gestation periods will often be the technologically most productive forms of investment. We describe a dynamic general equilibrium model with production, long gestation investment projects, and transaction costs. The model delivers predictions about how improvements in transactions technologies (increased liquidity) affect (a) - (e). Our interest in these questions stems from the prominence they receive in the literature on economic development. In addition, the model delivers empirically testable predictions about the relationship between transaction costs and equilibrium returns.

Introduction

This paper is an attempt to investigate the allocative functions of liquid secondary financial markets. It sometimes appears difficult to rationalize actual financial markets and stock exchanges with the textbook role of capital markets. Market allocation of scarce capital resources to the highest available rates of return promotes efficiency, but the overwhelming majority of transactions on the exchanges simply retrade existing securities. They thus rearrange the ownership of capital, but do not directly alter its allocation in production. Absent significant new public offerings, the observed activity of capital markets appears only with difficulty to be describable as a mechanism for the efficient allocation of scarce capital resources.

It has nevertheless been widely argued that organized secondary capital markets and financial intermediation are essential to allocative efficiency and to the process of economic growth and development.[1] Our study reconciles these apparently contradictory views, while investigating the theoretical foundations of secondary asset markets. Secondary asset markets provide liquidity, and liquidity significantly reduces the cost of capital. Secondary capital markets and intermediation are essential in simultaneously satisfying two conflicting forces in the investment process:

a) long-lived physical capital or capital that takes some time to put in place will generally be more productive than shorter-lived (but liquid) capital that can be put in place rapidly.[2]

b) investors may have short horizons relative to the life of some investments or face imperfectly predictable liquidity needs that will require liquidation of some portion of their portfolios on short notice.

Secondary capital markets and intermediation allow the competing claims of productivity and liquidity to be addressed simultaneously. This paper models the implications of this observation.

The idea that investments with long gestation periods are likely to have high marginal products is, of course, an old one [Bohm-Bawerk (1891)], but one that has found relatively little reflection in modern growth theory. Kydland and Prescott (1982) argued forcefully for its importance and reintroduced it into a growth theory framework, but did not consider the possibility of it being in conflict with a desire for liquidity. Diamond and Dybvig's (1983) model of banking includes both a demand for liquidity by wealth-holders and the feature that longer-lived (and less liquid) assets are more productive, but does not consider issues related to growth.

Finally, Aiyagari and Gertler (1991) use transactions costs and liquidity requirements to partially explain the equity premium, but they also focus on pure exchange economies.

We describe a model in which longer gestation periods for capital may permit investment activity to be more productive. However, because the gestation periods of some capital may be long relative to the horizons of investors, the existence of liquid secondary asset markets is essential in order for primary offerings to raise new capital successfully. These markets thus make long-lived investments more attractive to investors, thereby reducing the cost of capital and significantly influencing both the level of investment and its maturity composition. It is in this way that the development of financial markets and exchanges is integral to allocative efficiency and to the growth process.

The latter notion, that financial market development and economic development go hand-in-hand, appears repeatedly in the literature on economic development. In much of this literature it is argued that the "shallowness" of financial markets in LDCs precludes their economies' rapid development.[3] Goldsmith (1969) put forward a significant body of empirical evidence that financial market activity and per capita income levels are strongly related, and capital market development is thought to precede the beginning of rapid real development, as asserted by Cameron (1967) or McKinnon (1973).

In order to address these issues, we develop a dynamic general equilibrium model in which there are costs associated with financial transactions. We then ask how the structure of transactions costs affects capital accumulation, real returns, the maturity composition of the capital stock, and the level of secondary market activity. We believe that there are good reasons, both empirical and theoretical, for approaching the questions we pose in this way. First, it is widely argued that transactions costs are largest in less developed economies.[4] Our approach allows us to ask whether this can explain why there is simultaneously less development of financial markets, and lower per capita capital stocks and income in these countries. Second, any questions regarding why financial markets fail to form are obviously questions about what markets exist (are active) in an economy. This is exactly the kind of issue that the theoretical literature on transactions costs in general equilibrium contexts was developed to address.[5] Such an approach has already been employed by Greenwood and Jovanovic (1990) to study the endogenous development of financial markets, although in their context liquidity provision by these markets is not an issue of concern.

Our objective, then, is to add two features to a standard neoclassical growth model: (i) investments with longer gestation periods are more productive and less liquid than those of shorter gestation (and there is an endogenous choice to be made along this dimension), and (ii) financial transactions may be costly to undertake and implement. We then want to explore how aspects of the transaction technology influence the equilibrium levels of financial market activity, the capital stock, per capita income, real returns, and the maturity composition (gestation period) of investment. The development literature suggests that increased development should be associated with reduced transaction costs, greater financial market activity, higher per capita capital stocks and income levels, and the increased use of illiquid investment technologies.

Our vehicle for investigating these issues is Diamond's (1965) overlapping generations model with production, to which we add the features just described. In the next section we outline one version of such a model in which capital production technologies and transaction costs are linear. We establish the existence of a non-trivial stationary equilibrium under standard assumptions, and also provide conditions under which that equilibrium is unique. We then use the model to analyze the consequences of improvements in transactions technologies (increased liquidity) for (a) capital accumulation and income levels, (b) equilibrium rates of return, (c) capital gestation periods, (d) welfare, and (e) the level of secondary financial market activity.

In the standard Diamond (1965) model, when there is a unique (non-trivial) stationary equilibrium, technological improvements that do not impair the marginal productivity of either capital or labor raise per capita income and capital stocks. This turns out not to be the case in our model. In particular, under conditions we describe, increasing the liquidity of secondary capital markets (improving transactions technologies) can simply increase the fraction of savings consumed by secondary market transactions, and hence can actually result in a reduction in the steady capital stock. In this situation increases in the liquidity of financial markets can also reduce steady state welfare. These results are of interest, as they indicate the possibility that "financial deepening" need not be either growth- or welfare-promoting. This seems to have sometimes been the case in various less-developed economies.[6]

While increasing the liquidity of secondary markets need not be growth-promoting, we also described conditions under which it is. We show that, under these conditions, improvements in transactions technologies (financial deepening) will result in capital deepening and will also raise the ratio of financial market transactions to total assets. Hence, across countries that differ (only) with respect to the level of transactions costs, income (the capital stock) and some measure of financial market activity will be positively correlated. There is considerable empirical evidence [Goldsmith (1969)] that this is the case. Finally, we show that financial deepening always increases the steady state equilibrium return on savings, as argued by McKinnon (1973) and Shaw (1973).

The paper concludes by suggesting several possible extensions of the analysis. A particular interest is in examining more complicated transactions cost structures that could result in both portfolio specialization and endogenously emerging intermediation.

I. The Model

A. Environment

We consider a two period lived, overlapping generations economy with production. Time is indexed by $t = 1,2,\ldots$, and at each date t a new young generation appears with N members. All young agents are identical,[7] being endowed with one unit of labor when young (which is supplied inelastically), and no labor when old. In addition, agents have no endowment of capital or consumption goods.[8]

There is a single consumption good at each date, which can either be eaten or converted into capital. Let $C_{it} \in \mathbb{R}_+$ denote period i consumption of a representative agent born at t. All agents have the common utility function $u(C_{1t},C_{2t})$, with u being twice continuously differentiable, increasing in each argument, and strictly quasi-concave.

With respect to production, there is a commonly available constant returns to scale technology for converting labor and capital into the consumption good. In particular, a labor input of L_t and a capital input of K_t produces $F(K_t,L_t)$ units of consumption at t. We again assume that F is twice continuously differentiable, increasing in each argument, and strictly concave. Finally, we let $f(k_t)$ denote the intensive production function, with k_t being the time t capital-labor ratio. f is assumed to satisfy $f(0) = 0$ and the usual Inada conditions.

Thus far the model is identical to that of Diamond (1965) in the absence of national debt. The difference we introduce is that there are assumed to be J different technologies available for converting consumption goods into capital. These technologies are indexed by $j = 1,2,\ldots,J$, and differ as follows: one unit of the consumption good invested in the j^{th} technology at t returns $R_j > 0$ units of capital at $t + j$. Thus capital technologies vary by gestation period (j), and productivity (R_j).

If an investment technology with $j > 1$ is employed, our assumptions on agents' life cycles will force agents to sell capital goods in process on secondary capital markets. In particular, "capital in the pipeline" (CIP) is not productive until it matures, so this capital must be "rolled over" by investors from inception to maturity. Our interest is in examining how the liquidity of the secondary capital markets that accomplish this affects capital accumulation, national income, equilibrium returns, and the equilibrium maturity of investments.

In order to capture the notion of the liquidity of secondary capital markets we assume that there are transactions costs associated with trade in these markets. The level of transactions costs is inversely related to the liquidity of these markets. The introduction of transactions costs will allow us to investigate formally the arguments discussed above that the efficiency of financial markets is integrally related to the degree of economic development. Or, phrased differently, if we associate reductions in transactions costs with financial deepening, we will then be prepared to investigate the relationship between financial and capital deepening.

For simplicity, we assume a proportional transactions cost structure. In particular, one unit of CIP in technology j, that has been in place h periods (is j-h periods from maturity), has a proportional transactions cost of $\alpha^{j,h}$. More specifically, a fraction $\alpha^{j,h} \epsilon [0,1)$ of the project is "used up" in the process of selling it h periods after inception.

Finally, we assume that when CIP matures it is used in the production process, and then depreciates completely. This assumption is without real loss of generality.

B. Trade

We assume that young agents at t sell their labor to producers in a competitive labor market. One unit of labor at t earns the real wage rate w_t. In addition, producers rent capital in a competitive rental market, paying the rental rate r_t at t. In order to keep the model as similar to the Diamond (1965) model as possible, we assume that there are no transactions costs in factor markets. Thus the conventional factor pricing relationships obtain, and

(1) $$w_t = f(k_t) - k_t f'(k_t) \equiv w(k_t)$$

(2) $$r_t = f'(k_t).$$

After earning the wage income w_t, a young agent at t makes a savings decision and a portfolio choice. Let S_t denote savings by this agent, measured in units of CIP. This saving can be invested in new investment projects, or can be used to purchase CIP which has yet to mature. Let $\theta_t^{j,h}$ be a portfolio weight, which indicates the fraction of time t savings invested in technology j capital that has been in place h periods. Thus $\theta_t^{j,0}$ is new investment in technology j, while $\theta_t^{j,j-1}$ is investment in technology j capital that will mature next period, etc.

Let $P_t^{j,h}$ denote the time t price (in units of period t consumption) of one unit of CIP in technology j that is j-h periods from maturity. Since agents initiate new projects with consumption goods, $P_t^{j,0} = 1 \ \forall$ t,j. In addition, mature capital is simply rented in factor markets. As one unit of CIP in technology j produces R_j units of rentable capital on maturity, $P_t^{j,j} = r_t R_j$; i.e., $P_t^{j,j}$ is just the rental value of capital at t. For j > 1 and 0 < h < j, $P_t^{j,h}$ will have to be determined.

We assume (without loss of generality) that transactions costs are borne by sellers of CIP. Thus a young agent at t chooses S_t and $\theta_t^{j,h}$ to maximize $u(C_{1t}, C_{2t})$, subject to

(3) $$C_{1t} + \sum_{j=1}^{J} \sum_{h=0}^{j-1} \theta_t^{j,h} P_t^{j,h} S_t \leq w_t$$

(4) $$C_{2t} \leq \sum_{j=1}^{J} \sum_{h=0}^{j-1} \theta_t^{j,h} P_{t+1}^{j,h+1} S_t (1 - \alpha^{j,h+1})$$

non-negativity, and

$$\sum_{j=1}^{J} \sum_{h=0}^{j-1} \theta_t^{j,h} = 1. \tag{5}$$

If technology j is in use at all dates, then $\theta_t^{j,h} > 0$ holds $\forall$ h = 0,1,...,j-1. This obviously requires that the return to holding technology j CIP must be equated for all possible times to maturity; i.e.,

$$(1 - \alpha^{j,h+1})\, P_{t+1}^{j,h+1}/P_t^{j,h} = (1 - \alpha^{j,h})\, P_{t+1}^{j,h}/P_t^{j,h-1},\ \forall\, t,\ \forall\, h=1,\dots,j\text{-}1. \tag{6}$$

Similarly, if technologies j and ℓ are in use at all dates, then

$$(1 - \alpha^{j,h+1})\, P_{t+1}^{j,h+1}/P_t^{j,h} = (1 - \alpha^{\ell,m+1})\, P_{t+1}^{\ell,m+1}/P_t^{\ell,m};\ \forall\, t,\ \forall\, h=0,\dots,j\text{-}1,\ \forall\, m=0,\dots,\ell\text{-}1. \tag{7}$$

Let γ_t denote this common (gross) rate of return between t and t+1. Then from (6), if technology j is in use,

$$P_{t+1}^{j,h+1}/P_t^{j,h} = \gamma_t/(1 - \alpha^{j,h+1})\ ;\ \forall\, t,\ \forall\, h=0,\dots,j\text{-}1. \tag{8}$$

When rates of return on all capital investments in use are equated, obviously young agents are individually indifferent regarding portfolio composition. Hence for each young agent only the real value of savings $\tilde{S}_t \equiv \Sigma\Sigma\, \theta_t^{j,h} P_t^{j,h} S_t$ is determinate, and

$$\tilde{S}_t \equiv \operatorname{argmax} u(w_t - \tilde{S}_t, \gamma_t \tilde{S}_t) \equiv s(w_t, \gamma_t). \tag{9}$$

We assume throughout that $s_1(w_t, \gamma_t) \geq 0$ and $s_2(w_t, \gamma_t) \geq 0$, so that savings is non-decreasing both in income, and the rate of return.

II. Stationary Equilibrium

We now describe the determination of equilibrium prices, capital stocks, and rates of return, focussing throughout on stationary equilibria. Therefore we drop time subscripts in the remainder of the paper.

Recall that

$$p^{j,j} = rR_j \tag{10}$$

$$p^{j,0} = 1,$$

and write

$$\text{(11)} \qquad p^{j,j} = p^{j,0} \prod_{h=0}^{j-1} (p^{j,h+1}/p^{j,h}).$$

Then if technology j is in use, substitution of (8) and (10) into (11) yields

$$\text{(12)} \qquad rR_j = (\gamma)^j \left[\prod_{h=0}^{j-1} (1 - \alpha^{j,h+1})\right]^{-1},$$

where $(\gamma)^j$ denotes (the stationary value of) γ raised to the j^{th} power. Define

$$\text{(13)} \qquad \tilde{R}_j \equiv R_j \prod_{h=0}^{j-1} (1 - \alpha^{j,h+1}).$$

Therefore, from (12), if technology j is in use,

$$\text{(14)} \qquad \gamma = (r\tilde{R}_j)^{1/j} = [f'(k)\tilde{R}_j]^{1/j},$$

where k is the steady state capital-labor ratio.

Evidently, if j* is an equilibrium choice of project length,

$$\text{(15)} \qquad [f'(k)\tilde{R}_{j*}]^{1/j*} \geq [f'(k)\tilde{R}_j]^{1/j} \; \forall \, j$$

must hold. Let

$$\text{(16)} \qquad M(k) \equiv \{j \,|\, j=1,\ldots,J; \text{ j maximizes } [f'(k)\tilde{R}_j]^{1/j}\}.$$

As j* need not be unique, M(k) gives the set of return maximizing project lengths as a function of the capital stock. These are the project lengths in use when the capital stock is k.

The equilibrium capital-labor ratio is determined as follows. For each project length in use [$j^* \in M(k)$], $\theta^{j*,0}s(w,\gamma)$ new projects are initiated at each date. Each such new project yields $\tilde{R}_{j*}$ (net of transactions costs) units of capital j* periods later. Thus the capital stock at each date is the sum of maturing projects;

$$(17) \qquad k = \sum_{j^* \epsilon M(k)} \tilde{R}_{j^*} \theta^{j^*,0} s(w,\gamma),$$

since only technologies in M(k) are in use.

Finally, $\forall j^* \epsilon M(k)$, the market in CIP must clear for $h=1,...,j^*-1$. The time t demand for j^* period projects with j^*-h periods to maturity in real terms is $\theta^{j^*,h} s(w,\gamma)$. The supply of such projects, measured in current consumption goods, is $P^{j^*,h} \theta^{j^*,0} s(w,\gamma) \prod_{\ell=0}^{h-1}(1-\alpha^{j^*,\ell+1})$, since $\prod_{\ell=0}^{h-1}(1-\alpha^{j^*,\ell+1})$ of the initial investment has been consumed by the transactions technology h periods after initiation. Thus the market for CIP clears if

$$(18) \qquad \theta^{j^*,h} s(w,\gamma) = P^{j^*,h} \theta^{j^*,0} s(w,\gamma) \prod_{\ell=0}^{h-1}(1-\alpha^{j^*,\ell+1})$$

holds $\forall j^* \epsilon M(k)$, $\forall h = 1,...,j^*-1$. In addition, (5) becomes

$$(19) \qquad \sum_{j^* \epsilon M(k)} \sum_{h=0}^{j^*-1} \theta^{j^*,h} = 1.$$

Equations (17) - (19) and $j^* \epsilon M(k)$ constitute the steady state equilibrium conditions.

A. Characterization of Equilibrium

Using equation (10) and

$$P^{j,h} = P^{j,0} \prod_{\ell=0}^{h-1} (P^{j,\ell+1}/P^{j,\ell})$$

in (18) gives

$$(20) \qquad \theta^{j^*,h} = \theta^{j^*,0} \prod_{\ell=0}^{h-1} [P^{j^*,\ell+1}(1 - \alpha^{j^*,\ell+1})/P^{j^*,\ell}] \equiv \theta^{j^*,0} (\gamma)^h, \forall j^* \epsilon M(k), \forall h=0,...,j^*-1.$$

Therefore (19) can be written as

$$(21) \qquad \sum_{j^* \epsilon M(k)} \sum_{h=0}^{j^*-1} \theta^{j^*,h} = \sum_{j^* \epsilon M(k)} \theta^{j^*,0} \sum_{h=0}^{j^*-1} (\gamma)^h = 1.$$

Furthermore since

$$\sum_{h=0}^{j^*-1} (\gamma)^h = (1 - \gamma^{j^*})/(1 - \gamma) = [1 - \tilde{R}_{j^*} f'(k)]/\{1 - [\tilde{R}_{j^*} f'(k)]^{1/j^*}\},$$

(21) is equivalent to

$$(21') \qquad \sum_{j^* \epsilon M(k)} \theta^{j^*,0}[1 - \tilde{R}_{j^*} f'(k)]/\{1 - [\tilde{R}_{j^*} f'(k)]^{1/j^*}\} = 1.$$

Equation (17) is essentially a definition of a stationary equilibrium. It describes a self-reproducing capital stock. To characterize a solution to (17), note that the terms on the right-hand side are constants ($\tilde{R}_{j^*}$) or functions of k. $\theta^{j^*,0}$ depends on γ by (21), but γ is itself a function of k by (14), leading to $\theta^{j^*,0}$ as a function of k by (21$'$). The arguments of s are γ (a function of k) and w, which is a function of k by (1). Hence, if there were only a single maturity $j^* = j$ in use, we could rewrite the right-hand side of (17) by substituting the appropriate functions of k for w, $\theta^{j^*,0}$, and γ. Thus we define

$$(22) \qquad G_j(k) \equiv \tilde{R}_j s[w(k), (\tilde{R}_j f'(k))^{1/j}]/ \sum_{h=0}^{j-1} [\tilde{R}_j f'(k)]^{(h/j)}$$

$$= \tilde{R}_j s[w(k), (\tilde{R}_j f'(k))^{1/j}] \; [1 - (\tilde{R}_j f'(k))^{1/j}]/[1 - \tilde{R}_j f'(k)]; \; j = 1,\dots,J.$$

$G_j(k)$ is the level of capital stock (of maturity j) provided by savings decisions and the financial sector as a function of the wage rate, and of the marginal product of capital, $f'(k)$, or equivalently, of the yield on investment $\gamma = [f'(k)\tilde{R}_j]^{1/j}$. Then (17) can be alternatively expressed as

$$(23) \qquad k = \sum_{j^* \epsilon M(k)} \theta^{j^*,0}([1 - \tilde{R}_{j^*} f'(k)]/\{1 - [\tilde{R}_{j^*} f'(k)]^{1/j^*}\}) \; G_{j^*}(k).$$

In view of (21$'$) and $\theta^{j^*,0}[1 - \tilde{R}_{j^*} f'(k)]/\{1 - [\tilde{R}_{j^*} f'(k)]^{1/j^*}\} \geq 0$, (23) asserts that the equilibrium per capita capital stock k equals a convex combination of the values $G_{j^*}(k)$. In order to express (23) more compactly, we define the correspondence G(k) by

$$(24) \qquad G(k) \equiv CH\{G_{j^*}(k) \,|\, j^* \epsilon M(k)\},$$

where CH denotes convex hull. Then (23) can be written as

(25) $\qquad k \in G(k)$.

Equation (25) is now the sole equilibrium condition.

It will evidently be useful to know more about the correspondence $G(k)$. First, since each $G_j(k)$ is a continuous function, $G(k)$ is single-valued if $M(k)$ is single-valued. Second, from (24) it is apparent that G is upper hemi-continuous and convex-valued. Third, it is straight-forward to show that k can be bounded by some finite value k_{max}. Thus, by Kakutani's fixed point theorem an equilibrium exists satisfying $k^* \in G(k^*)$. However, since $0 \in G(0)$, we would like to provide conditions under which a non-trivial stationary equilibrium capital-labor ratio ($k^* > 0$) exists. We now turn our attention to this problem, which requires a sharper characterization of the correspondence G.

In order to provide such a characterization, we begin by stating four lemmas. The proof of each appears in the appendix.

<u>Lemma 1</u>. Suppose that $j,\ell \in M(\hat{k})$ for some $\hat{k}$, and that $j > \ell$. Then there exist values $\varepsilon_1, \varepsilon_2 > 0$ such that $\{\ell\} = M(k)\ \forall\ k \in (\hat{k} - \varepsilon_1, \hat{k})$ and $\{j\} = M(k)\ \forall\ k \in (\hat{k}, \hat{k} + \varepsilon_2)$.

Lemma 1 asserts that $M(k)$ fails to be single-valued only at (finitely many) isolated points, and hence that $G(k)$ fails to be single-valued only at those same points. Moreover, it follows from lemma 1 that if $j > \ell$ and $j \in M(k')$ for some $k' > 0$, then $\ell \notin M(k)$ for any $k > k'$. Loosely speaking, then, lemma 1 asserts that the equilibrium maturity length is non-decreasing in k.

<u>Lemma 2</u>. For k sufficiently near zero, $\{1\} = M(k)$. For k sufficiently large, $\{J\} = M(k)$.

<u>Lemma 3</u>. Let $j,\ell \in M(\hat{k})$ for some $\hat{k}$ and let $j > \ell$. Then

(26) $\qquad G_j(\hat{k}) < G_\ell(\hat{k})$.

Lemma 4. For each $j=1,\ldots,J$,

$$\lim_{k\to\infty} G_j(k)/k = 0.$$

Lemma 4 asserts that each function $G_j(k)$ eventually lies below the 45° line in Figure 1, and hence that $G_J(k)$ does.

It follows from lemmas 1-4 that the correspondence G(k) has the appearance depicted in Figure 1. Evidently, if we assume that

$$\text{(a.1)} \qquad \lim_{k\downarrow 0} G_1(k)/k > 1,$$

then there exists a non-trivial stationary equilibrium.[9] (a.1) and lemmas 1-4 characterize the correspondence G(k). For k near zero, the only maturity in use is 1 and G(k) lies above the 45° line. As k increases, there will be values $\hat{k}$ that are switch points between maturities (say ℓ,j). At these, $G(\hat{k})$ is set valued as the capital stock is allocated between $G_\ell(\cdot)$ and $G_j(\cdot)$. As k increases beyond $\hat{k}$, $G(\cdot)$ switches downward to longer maturities; for $j > \ell$, we have $G_j(\hat{k}) < G_\ell(\hat{k})$. Thus G is a sequence of continuous segments, linked by vertical declines with increasing maturity. Finally, for k sufficiently large, G(k) lies below the 45° line.

It would further appear, say, from Figure 2, that there is considerable scope for the existence of multiple stationary equilibria. In fact this is less the case than might be suspected. We now state

Proposition 1. Suppose that (a) $s(w,\gamma)/w$ is non-increasing in w (i.e., the income elasticity of savings does not exceed one); and (b) f displays an elasticity of substitution, denoted σ, satisfying $\sigma \geq 1$. Then if (a.1) holds there exists a unique non-trivial stationary equilibrium.

The proof is given in the appendix.

B. Secondary Market Transactions

The value of per capita purchases in secondary capital markets, measured in units of current consumption, is just per capita savings by young agents less per capita investment in new projects. Let $\rho(k^*)$ denote the real value of these purchases in equilibrium. Then

$$(27) \qquad \rho(k^*) = s(w,\gamma) - \sum_{j^*} \theta^{j^*,0} s(w,\gamma) = s(w,\gamma)[1 - \sum_{j^*} \theta^{j^*,0}] \geq 0$$

where the last inequality is strict if $\{1\} \neq M(k^*)$. From (21),

$$(28) \qquad \sum_{j^* \in M(k)} \theta^{j^*,0} [1 - (\gamma)^{j^*}]/(1 - \gamma) = 1.$$

Solving (28) for $\Sigma\theta^{j^*,0}$, substituting the result into (27), and using (17) yields that

$$(29) \qquad \rho(k^*) = \gamma s(w,\gamma) - rk^* = [\tilde{R}_{j^*} f'(k^*)]^{1/j^*} s\{w(k^*), [\tilde{R}_{j^*} f'(k^*)]^{1/j^*}\} - k^*f'(k^*).$$

Equation (29) describes how the volume of secondary market transactions depends on the equilibrium capital stock (k^*), as well as its maturity composition (j^*). We will be interested in examining how the liquidity of secondary capital markets and the volume of secondary market activity are related.

III. The Comparative Statics of Changes in Transactions Costs

We now wish to investigate how reductions in transactions costs (which can be regarded as increasing the liquidity of secondary capital markets, or as an aspect of financial deepening) affect (i) the steady state equilibrium levels of the capital stock, per capita income, and the return on savings, (ii) steady state welfare, and (iii) the level of secondary capital market activity. Unambiguous results along these dimensions require that there be a unique non-trivial steady state equilibrium: therefore for the remainder of the section we impose the conditions of proposition 1;

(a.2) $\qquad s(w,\gamma)/w$ is non-increasing in w,

(a.3) $\qquad \sigma \geq 1$.

In addition, it will be useful to have a single parameter which controls the transactions cost structure. We therefore assume that (a.4) $\tilde{R}_j = \tilde{R}_j(\beta)$; $j=1,\ldots,J$.

β is a scalar parameter; for concreteness we assume that increases in β represent reductions in transactions costs. Thus $\tilde{R}'_j(\beta) \geq 0 \; \forall \, j$.

Definitive results on the consequences of a change in the transactions cost parameter require some assumptions on the functions $\tilde{R}_j(\beta)$. First, since there are no transactions costs associated with one period length projects we assume that, $\forall \, \beta$,

(a.5) $\tilde{R}'_1(\beta) = 0$.

Second, since longer-lived projects involve more transactions than shorter-lived projects, we assume that a change in β has a larger proportional impact on projects of longer maturities. Our specific technical assumption is that, $\forall \, \ell$, j,

(a.6) $\tilde{R}'_j(\beta)/j\tilde{R}_j(\beta) > \tilde{R}'_\ell(\beta)/\ell\tilde{R}_\ell(\beta)$

whenever $j > \ell$. In section IV we show that some obvious transactions cost structures satisfy (a.6).

A. Capital Stock, Income, and Returns

There are two ingredients in investigating how a change in β affects the steady state equilibrium capital stock: (a) the effect of a change in β on the correspondence G(k) if G(k) is single valued [i.e., if M(k) is single-valued], and (b) the effect if G(k) is not single-valued. We begin with case (a).

If G(k) is single-valued, then there exists a $j \in M(k)$ such that $\{G_j(k)\} = G(k)$. The following lemma then establishes how G(k) is affected by a change in β:

Lemma 5. An increase in $\tilde{R}_j$ increases $G_j(k)$, $\forall \, j$, $\forall \, k$.

Lemma 5 is proved in the appendix. It has the implication that, if $\{j\} = M(k)$ and $\tilde{R}'_j(\beta) > 0$, then an increase in β shifts G(k) upwards in figure 3.

It now remains to describe the effect of a change in β on the correspondence G when G(k) is <u>not</u> single-valued. If G(k) consists of two or more elements, then there exist values ℓ and j with $\{\ell,j\} \subset M(k)$. In this event define the capital-labor ratio $\hat{k}_{\ell,j}$ by

$$\{\ell,j\} \in M(\hat{k}_{\ell,j}) \tag{30}$$

and $0 < \hat{k}_{\ell,j} < \infty$. Thus $\hat{k}_{\ell,j}$ is that capital-labor ratio which yields ℓ and j as equilibrium project lengths. Lemma 1 implies that for any pair (ℓ,j), there exists at most one value $\hat{k}_{\ell,j}$ satisfying (30): if one exists it is defined by

$$[\tilde{R}_j(\beta)f'(\hat{k}_{\ell,j})]^{1/j} \equiv [\tilde{R}_\ell(\beta)f'(\hat{k}_{\ell,j})]^{1/\ell}. \tag{31}$$

The following lemma then states how G(k) is affected by a change in β if G(k) is <u>not</u> single-valued:

<u>Lemma 6</u>. Suppose that $\{\ell,j\} \subset M(\hat{k}_{\ell,j})$ and that $j > \ell$. Then

$$d\hat{k}_{\ell,j}/d\beta < 0.$$

Lemma 6 is proved in the appendix. It asserts that a reduction in transactions costs reduces $\hat{k}_{\ell,j}$, for all ℓ,j such that $\hat{k}_{\ell,j}$ exists.

Lemmas 5 and 6 now allow us to infer how a change in transactions costs affects the entire correspondence G. In figure 3 the solid (dashed) locus represents a high (low) transactions cost economy. A change in transactions costs does not affect the correspondence G(k) when $\{1\} = M(k)$, since by assumption $\tilde{R}_1'(\beta) = 0$. If $\{j\} = M(k)$ for some $j = 2,\ldots,J$, then $\tilde{R}_j'(\beta) > 0$, and a reduction in transactions costs shifts G(k) upwards. Finally, if $\{\ell,j\} = M(\hat{k}_{\ell,j})$ for some ℓ,j and $j > \ell$, then $G(\hat{k}_{\ell,j})$ is a vertical segment. Lemma 6 implies that this vertical segment shifts to the left with an increase in β, as shown in the figure.

As is apparent from figures 3, 4, and 5, a reduction in transactions costs (an increase in β) has an ambiguous effect on the steady state per capita capital stock (and hence on per capita income). There are three general possibilities for a "small" change in β.

Case 1. $\{j^*\} = M(k)$ both before and after the change in transactions costs, and $j^* > 1$. This situation is depicted in figure 3. Evidently in this case a reduction in transactions costs leads to a higher steady state capital stock; this corresponds to the event that financial deepening leads to capital deepening.

Case 2. $\{\ell, j\} \subset M(k)$ both before and after the change in β. This case is depicted in figure 4. Here a reduction in transactions costs actually causes a reduction in the steady state capital stock. We establish below that this situation occurs when a reduction in transactions costs simply results in a portfolio shift from less to more transactions (and transactions cost) intensive investments, without channeling additional resources into capital accumulation.

Case 3. $\{1\} = M(k)$, both before and after the change in transactions costs. This event is depicted in figure 5; evidently a change in β has no effect on the steady state capital stock. This situation arises when secondary capital markets are undeveloped, so that no transactions costs are incurred in any event.

The possibility, illustrated in case 2, that financial deepening need not lead to capital deepening is more than simply a theoretical curiosity. McKinnon (1973) and Shaw (1973), for instance, argued that financial deepening would generally lead to increased investment and capital formation. However, in practice, attempts to stimulate financial deepening in developing countries have met with mixed success. Taylor (1980), van Wijnbergen (1983, 1985), Buffie (1984), and Diaz-Alejandro (1985) even assert that attempts at financial deepening have often been detrimental to capital accumulation. This is, in fact, what occurs in figure 4, where improvements in the liquidity of secondary capital markets simply increase activity in those markets (see below) without resulting in capital deepening.

It remains to consider the consequences for the equilibrium return to savings of a change in β. As before, if the change in β is not large enough to affect M(k) in equilibrium, there are three possibilities. We consider each in turn.

Case 1. $\{j^*\} = M(k)$, $j^* > 1$. In this case the equilibrium (gross) rate of return on savings is given by $[\tilde{R}_{j^*}(\beta)f'(k)]^{1/j^*}$. We now state

Lemma 7. Suppose that $\{j^*\} = M(k)$. Then an increase in β (weakly) increases $[\tilde{R}_{j^*}(\beta)f'(k)]^{1/j}$.

Lemma 7 is proved in the appendix. Thus, in this case, an increase in β does not reduce the steady state equilibrium rate of return.

Case 2. $\{\ell,j\} \subset M(k)$. In this event an increase in β reduces k (which equals $\hat{k}_{\ell,j}$). The equilibrium return on saving is given by $[\tilde{R}_\ell(\beta)f'(\hat{k}_{\ell,j})]^{1/\ell} = [\tilde{R}_j(\beta)f'(\hat{k}_{\ell,j})]^{1/j}$. Since at least one of the values $\tilde{R}_\ell(\beta)$ and $\tilde{R}_j(\beta)$ must rise, the return on savings does as well.

Case 3. $\{1\} = M(k)$. In this situation a change in β affects neither $\tilde{R}_1$ nor k. Thus the equilibrium rate of return, which equals $\tilde{R}_1f'(k)$, is unaffected by β.

To summarize:

Proposition 2. The steady state equilibrium rate of return is not reduced by a reduction in transactions costs; it is increased if M(k) is not single-valued, or if $\{1\} \neq M(k)$ and either (a.2) or (a.3) holds as a strict inequality.

The result that financial deepening raises the return on savings is widely asserted in the development literature; for instance by Goldsmith (1969), McKinnon (1973), and Shaw (1973).

B. Welfare

In this section we consider how a small change in β [small enough to leave M(k) unaffected] affects the steady state equilibrium level of agents' utilities. As before there are three possibilities.

Case 1. $\{j^*\} = M(k), j^* > 1$. Here lemma 7 implies that $\gamma = [\tilde{R}_{j^*}(\beta)f'(k)]^{1/j^*}$ rises (weakly) with a reduction in transactions costs. Furthermore, as in figure 3, the steady state capital stock rises, and so therefore does the real wage rate w(k). Thus, by the envelope theorem, the steady state welfare level $u\{w(k) - s[w(k),\gamma], \gamma s[w(k),\gamma]\}$ necessarily increases when transactions costs are reduced.

Case 2. $\{\ell,j\} \subset M(k), j > \ell$. In this case, as before, the steady state welfare level is given by

$$U = u\{w(\hat{k}_{\ell,j}) - s[w(\hat{k}_{\ell,j}), \gamma], \gamma s[w(\hat{k}_{\ell,j}), \gamma]\},$$

since $\hat{k}_{\ell,j}$ is the steady state capital-labor ratio. Then, by the envelope theorem,

$$\frac{dU}{d\beta} = u_1(\cdot)\{w'(\hat{k}_{\ell,j})d\hat{k}_{\ell,j}/d\beta + [s(\cdot)/\gamma]d\gamma/d\beta\}. \tag{32}$$

Moreover, $\hat{k}_{\ell,j}$ satisfies (31), so that

$$(33) \qquad f'(\hat{k}_{\ell,j}) \equiv [\tilde{R}_j(\beta)]^{\ell/(j-\ell)}/[\tilde{R}_\ell(\beta)]^{j/(j-\ell)}.$$

Differentiating (33) and using $w'(k) = -kf''(k)$ yields

$$(34) \qquad w'(\hat{k}_{\ell,j})d\hat{k}_{\ell,j}/d\beta = -kf'(k)\{[\tilde{R}'_j(\beta)/j\tilde{R}_j(\beta)] - [\tilde{R}'_\ell(\beta)/\ell\tilde{R}_\ell(\beta)]\}[j\ell/(j-\ell)].$$

In addition, γ is given by

$$(35) \qquad \gamma = [\tilde{R}_j(\beta)f'(\hat{k}_{\ell,j})]^{1/j}.$$

Substituting (33) into (35) gives

$$(36) \qquad \gamma = [\tilde{R}_j(\beta)/\tilde{R}_\ell(\beta)]^{1/(j-\ell)}.$$

From (36) it follows that

$$(37) \qquad d\gamma/d\beta = \gamma\{[\tilde{R}'_j(\beta)/\tilde{R}_j(\beta)] - [\tilde{R}'_\ell(\beta)/\tilde{R}_\ell(\beta)]\}/(j-\ell).$$

Equations (32), (34), and (37) imply that $dU/d\beta \le 0$ holds iff

$$(38) \qquad kf'(k)\{\ell[\tilde{R}'_j(\beta)/\tilde{R}_j(\beta)] - j[\tilde{R}'_\ell(\beta)/\tilde{R}_\ell(\beta)]\} \ge s[w(k),\gamma]\{[\tilde{R}'_j(\beta)/\tilde{R}_j(\beta)] - [\tilde{R}'_\ell(\beta)/\tilde{R}_\ell(\beta)]\}$$

Equation (38) can easily be satisfied. For instance

Lemma 8. Suppose that

$$(39) \qquad \tilde{R}'_j(\beta)/(j-1)\tilde{R}_j(\beta) \ge \tilde{R}'_\ell(\beta)/(\ell-1)\tilde{R}_\ell(\beta)$$

whenever $j > \ell$, and that

$$(40) \qquad kf'(k)/w(k) \ge s[w(k),\gamma]/w(k)$$

always holds. Then $dU/d\beta \le 0$.

The lemma follows from straight-forward algebraic manipulation.

Equation (39) will be satisfied for some obvious transactions cost structures (such as those considered in section IV). Equation (40) will hold whenever the ratio of capital's share to labor's share is at least as great as the savings rate of young agents. It is easy to produce examples satisfying this condition. Thus it can easily happen that reductions in transactions costs result in reductions in steady state welfare.

Case 3. $\{1\} = M(k)$. In this case a reduction in transactions costs affects neither k nor γ, and consequently has no welfare effects.

In summary, just as a reduction in transactions costs can have ambiguous consequences for the steady state equilibrium capital stock, it can have ambiguous consequences for steady state welfare. It therefore seems appropriate to develop an empirical criterion for determining when an increase in β will reduce the capital stock (and possibly welfare). We now consider this issue.

C. Secondary Market Transactions

The level of secondary market activity, in real terms, is described by equations (27) and (29). Evidently, if $M(k^*) = \{1\}$, $\rho(k^*) = 0$. Thus for the remainder of this section we consider only the case where $\{1\} \neq M(k^*)$.

Our objective is to describe how the real volume of secondary market transactions is affected by changes in transactions costs. As before, we will consider only small enough changes in β to leave $M(k^*)$ unaffected. Again the analysis involves two possibilities.

Case 1. $\{j^*\} = M(k^*)$, $j^* > 1$. In this case we can use the equilibrium condition

$$(41) \qquad k^* = G_{j^*}(k^*) = \tilde{R}_{j^*}s[w(k^*), (\tilde{R}_{j^*}f'(k^*))^{1/j^*}][1-(\tilde{R}_{j^*}f'(k^*))^{1/j^*}]/[1-\tilde{R}_{j^*}f'(k^*)]$$

as follows. Multiply both sides of (41) by $f'(k^*)$, use the relation $\gamma = [\tilde{R}_{j^*}f'(k^*)]^{1/j^*}$, and substitute the result into (29) to obtain

$$(42) \qquad \rho(k^*) = \gamma s(w,\gamma)[\gamma-(\gamma)^{j^*}]/[1-(\gamma)^{j^*}] \equiv \gamma s(w,\gamma)Q_{j^*}(\gamma).$$

Now we have established that an increase in β increases both w and γ. Thus $\gamma s(w,\gamma)$ rises with a reduction in transactions costs. It is also straightforward to show that $Q'_{j*} \geq 0$. Therefore we have

Lemma 9. Let $\{j^*\} = M(k^*)$, with $j^* > 1$. Then a reduction in transactions costs increases the level of secondary market activity.

We can say more than this, however. The ratio $\rho(k^*)/s(w,\gamma)$ gives the fraction of savings devoted to purchases in secondary capital markets. We have

Lemma 10. Suppose that $M(k^*)$ is a singleton. Then the ratio $\rho(k^*)/s(w,\gamma)$ (weakly) increases with a reduction in transactions costs.

Proof. From (42),

$$\rho(k^*)/s(w,\gamma) = \gamma Q_{j*}(\gamma). \tag{43}$$

This is increasing in γ, and $d\gamma/d\beta \geq 0$. □

Thus in an economy with a unique equilibrium project length, improvements in the liquidity of financial markets will result in an increase in the ratio of secondary capital market activity (which can be thought of as financial transactions) to total savings. In addition, these increases in liquidity (financial deepening) will lead to capital deepening. In short, when β increases the ratio of financial transactions to total assets will rise, as will the per capita capital stock and income level. Goldsmith (1969) has noted the strongly positive observed cross-country correlation between these objects.

Case 2. $\{\ell,j\} \subset M(k^*)$, with $j > \ell$. In this case $k^* = \hat{k}_{\ell,j}$, and from (29),

$$\rho = \rho(\hat{k}_{\ell,j}) = [\tilde{R}_j(\beta)f'(\hat{k}_{\ell,j})]^{1/j} s\{w(\hat{k}_{\ell,j}), [\tilde{R}_j(\beta)f'(\hat{k}_{\ell,j})]^{1/j}\} - \hat{k}_{\ell,j}f'(\hat{k}_{\ell,j}). \tag{44}$$

$d\rho/d\beta$ is not easily signed; however it is possible to show that as β is increased (transactions costs are reduced), the fraction of savings consumed by secondary capital market purchases increases. More specifically,

Lemma 11. The ratio $\rho(\hat{k}_{\ell,j})/s(\cdot)$ increases with an increase in β.

Lemma 11 is proved in the appendix. It asserts that, in the economy depicted in figure 4, a reduction in transactions costs results in a reduction in the fraction of savings devoted to new capital formation (as previously).

The empirical implications of the analysis are now very different, however. An increase in β (enhanced liquidity of secondary markets) raises the ratio of financial transactions to total assets, as before, but also results in a reduced capital stock and income level. This appears to be contrary to observation [Goldsmith (1969)], suggesting that case 1 is the empirically relevant situation. This result also suggests an empirical test for when case 1 (or 2) obtains: case 1(2) results in a positive (negative) correlation between $\rho/s(w,\gamma)$ and per capita income across regimes that differ with respect to the liquidity of secondary capital markets.

IV. An Example

We now illustrate our results with an example that makes possible a complete characterization of the correspondence M(k). The example assumes a constant proportional transactions cost structure; that is

$$\text{(45.a)} \qquad \alpha^{j,0} = \alpha^{j,j} = 0; \ \forall j.$$

$$\text{(45.b)} \qquad \alpha^{j,h} = \alpha \in [0,1) \ \forall j, \ \forall h = 1,\dots,j-1.$$

Then, if $\beta \equiv 1 - \alpha$,

$$\text{(46)} \qquad \tilde{R}_j(\beta) = R_j\beta^{j-1}.$$

It is easily established that these $\tilde{R}_j(\beta)$ functions satisfy (a.5), (a.6), and equation (39).

Under the assumptions of equation (45), it is straightforward to characterize M(k). In particular, $j^* \in$ M(k) iff

$$\text{(47)} \qquad [\tilde{R}_{j^*}f'(k)]^{1/j^*} \geq [\tilde{R}_j f'(k)]^{1/j}, \qquad \forall j = 1,\dots,J.$$

From (47) it follows that $1 \in$ M(k) iff

$$\text{(48)} \qquad f'(k) \geq \max_{j\geq 2} (\tilde{R}_j)^{1/(j-1)}/(\tilde{R}_1)^{j/(j-1)}.$$

Similarly, $J \in M(k)$ iff

$$(49) \qquad f'(k) \leq \min_{j<J} (\tilde{R}_J)^{j/(J-j)}/(\tilde{R}_j)^{J/(J-j)}.$$

Finally, $j^*(\neq 1,J) \in M(k)$ iff

$$(50) \qquad \min_{j<j^*} (\tilde{R}_{j^*})^{j/(j^*-j)}/(\tilde{R}_j)^{j^*/(j^*-j)} \geq f'(k) \geq \max_{j>j^*} (\tilde{R}_j)^{j^*/(j-j^*)}/(\tilde{R}_{j^*})^{j/(j-j^*)}.$$

With one additional assumption it is possible to completely characterize $M(k)$. We now assume that

$$(a.7) \qquad R_{j+2}/R_{j+1} < R_{j+1}/R_j;\ j=1,2,\ldots,J-2.$$

Obviously (a.7) and (45) imply that

$$(51) \qquad \tilde{R}_{j+2}/\tilde{R}_{j+1} < \tilde{R}_{j+1}/\tilde{R}_j;\ j=1,\ldots,J-2.$$

Then it follows that

Proposition 3. Equation (51) implies

$$(52) \qquad \max_{j>j^*} (\tilde{R}_j)^{j^*/(j-j^*)}/(\tilde{R}_{j^*})^{j/(j-j^*)} = (\tilde{R}_{j^*+1})^{j^*}/(\tilde{R}_{j^*})^{j^*+1}$$

$$(53) \qquad \min_{j<j^*} (\tilde{R}_{j^*})^{j/(j^*-j)}/(\tilde{R}_j)^{j^*/(j^*-j)} = (\tilde{R}_{j^*})^{j^*-1}/(\tilde{R}_{j^*-1})^{j^*}.$$

Moreover, $\forall\, j = 2,\ldots,J-1$,

$$(54) \qquad (\tilde{R}_{j+1})^{j}/(\tilde{R}_j)^{j+1} < (\tilde{R}_j)^{j-1}/(\tilde{R}_{j-1})^{j}.$$

Proposition 3 is proved in the appendix. It has the following immediate corollary.

Corollary. (a) Suppose that (a.7) holds. Then $1 \in M(k)$ iff

$$(55) \qquad f'(k) \in [\tilde{R}_2/(\tilde{R}_1)^2, \infty),$$

$J \in M(k)$ iff

(56) $$f'(k) \in [0, (\tilde{R}_J)^{J-1}/(\tilde{R}_{J-1})^J],$$

and j^* ($\neq 0,J$) $\in M(k)$ iff

(57) $$f'(k) \in [(\tilde{R}_{j^*+1})^{j^*}/(\tilde{R}_{j^*})^{j^*+1}, (\tilde{R}_{j^*})^{j^*-1}/(\tilde{R}_{j^*-1})^{j^*}].$$

Moreover, each interval on the right-hand side of (57) is non-empty.

Proposition 3 and its corollary indicate that, for each $j \in \{1,...,J\}$ there exists a non-empty closed interval I_j such that $j \in M(k)\ \forall\, k \in I_j$. The union of these intervals is $\mathbb{R}_+$. Thus $\cup_k M(k) = \{1,2,...,J\}$, so all project lengths can conceivably be employed.

It is also possible to produce different assumptions that result in substantially different properties for M(k). For instance, if (a.7) is replaced by

(a.8) $$R_{j+2}/R_{j+1} > R_{j+1}/R_j;\ j=1,...,J\text{-}2,$$

then it is possible to show that $1 \in M(k)$ iff

(58) $$f'(k) \in [(\tilde{R}_J)^{1/(J-1)}/(\tilde{R}_1)^{J/(J-1)}, \infty)$$

while $J \in M(k)$ iff

(59) $$f'(k) \in [0, (\tilde{R}_J)^{1/(J-1)}/(\tilde{R}_1)^{J/(J-1)}].$$

If $j \neq 1$, J, then $j \notin M(k)$ for any k. As these examples illustrate, M(k) can assume a variety of possibly configurations. These seem to be limited only by lemmas 1 and 2.

V. Conclusions

We have presented a model in which secondary financial markets perform an important allocative function. In particular, they allow investments with long lives or long gestation periods to be undertaken while simultaneously addressing the liquidity requirements of investors. We have also shown how the liquidity of these markets affects (a) capital accumulation, income and returns, (b) welfare, and (c) the level of financial market activity.

Our interest in these issues stems, in part, from the prominence they received in the development literature. Many have argued that increasing the efficiency of financial markets is essential in the growth process, while others have noted that not all instances of financial deepening seem to have desirable consequences. Interestingly, our model reconciles these views by showing how each possibility can arise. It also provides conditions under which financial deepening will lead to capital deepening, as well as conditions under which it will not. However, the analysis does indicate that improvements in the efficiency of secondary capital markets will raise the equilibrium return on savings.

We conclude by discussing some possible extensions of the analysis. One would be to examine the dynamical properties of non-stationary equilibria. For small J this would be a tractable undertaking. Moreover, it would permit us to analyze the dynamical relationship between financial market activity and real activity.

Second, it would be interesting to consider the consequences of taxing or subsidizing financial market activity. In the U.S. there have been recent proposals to tax certain kinds of financial transactions, presumably with the idea that these promote economic instability and are not of value from a growth perspective. In many developing countries, on the other hand, "financial liberalizations" are often proposed as being conducive to growth. These liberalizations typically involve either subsidizing, or reducing the taxation of various financial market activities. The model of the previous sections can easily be modified to discuss these questions by allowing transactions costs to have a tax/subsidy component. Our analysis suggests a potentially interesting characterization of when tax/subsidy schemes (such as financial liberalizations) will be growth-promoting and when they will not.

Third, it is possible to introduce national debt into the model [as in Diamond (1965)], either with or without a balanced government budget. It would then be possible to examine how government debt and/or budget deficits affect not just capital accumulation, but the level of secondary capital market activity as well.

Perhaps the most interesting potential extension would be to allow for transactions cost structures that give rise to financial intermediation and portfolio specialization. In particular, our analysis has implicitly assumed that it is not possible for some agents (intermediaries) to finance capital accumulation, and to issue liabilities with lower transactions costs than long-term capital investments. If this assumption is relaxed, then the model permits a role for intermediaries. These intermediaries will hold long-maturity illiquid assets and issue short-term liquid liabilities, as do real world financial intermediaries. Moreover, such an extension would permit an analysis of how intermediation affects capital formation, and dynamical versions would permit an analysis of how increases in financial intermediation and capital deepening occur through time.

This would also allow a theory of the endogenous formation and development of intermediation. In particular, for capital-labor ratios with $\{1\} = M(k_t)$, there is no need for intermediaries as no transactions costs are being borne. As k_t grows, so that longer-lived investments are brought into use, intermediaries that economize on transactions costs would be expected to emerge. Of some interest in this context is the possibility of "low-level development traps," in which economies "get stuck" in equilibria with little or no intermediation and low per capita income levels. This should be possible even when "better" steady state equilibria also exist.[10]

Finally, if the transactions cost structure admits some non-convexities, the model will suggest specialization of portfolios as well as the presence of intermediation. This will permit investigation of how increasing development and increasing financial specialization are related.

The presence of intermediaries with illiquid, long-maturity assets and liquid short-term liabilities indicates that problems associated with a "mismatch" of maturities between assets and liabilities may be possible. If agents are allowed to be slightly longer-lived and face random liquidity needs [as in Diamond-Dybvig (1983)], bank panics will be a real possibility. How such panics affect the economy would also be an interesting topic for investigation.[11]

APPENDIX

A.1. Proof of Lemma 1

The proof proceeds in two parts. We first prove: (a) if $j,\ell \in M(\hat{k})$ and $m \notin M(\hat{k})$, then there exists an interval $(\hat{k}-\varepsilon_1, \hat{k}+\varepsilon_2)$ such that $m \notin M(k)$, $\forall\, k \in (\hat{k}-\varepsilon_1, \hat{k}+\varepsilon_2)$. We then establish: (b) $\{\ell\} = M(k)\ \forall\, k \in (\hat{k}-\varepsilon_1, \hat{k})$, and $\{j\} = M(k)\ \forall\, k \in (\hat{k}, \hat{k}+\varepsilon_2)$.

Proof of (a). Since $j,\ell \in M(\hat{k})$ and $m \notin M(\hat{k})$, we have from equation (15) that

$$\text{(A.1)} \qquad [\tilde{R}_j f'(\hat{k})]^{1/j} > [\tilde{R}_m f'(\hat{k})]^{1/m}$$

$$\text{(A.2)} \qquad [\tilde{R}_\ell f'(\hat{k})]^{1/\ell} > [\tilde{R}_m f'(\hat{k})]^{1/m}.$$

It then follows from continuity that we can choose values ε_1, $\varepsilon_2 > 0$ such that, for all $k \in (\hat{k}-\varepsilon_1, \hat{k}+\varepsilon_2)$,

$$\text{(A.3)} \qquad [\tilde{R}_j f'(k)]^{1/j} > [\tilde{R}_m f'(k)]^{1/m}$$

$$\text{(A.4)} \qquad [\tilde{R}_\ell f'(k)]^{1/\ell} > [\tilde{R}_m f'(k)]^{1/m}.$$

This establishes (a).

Proof of (b). Define the function $\delta_{j,\ell} : \mathbf{R}_+ \to \mathbf{R}$ by

$$\text{(A.5)} \qquad \delta_{j,\ell}(k) \equiv [\tilde{R}_j f'(k)]^{1/j} - [\tilde{R}_\ell f'(k)]^{1/\ell}.$$

Then, since $j,\ell \in M(\hat{k})$, equation (15) implies that $\delta_{j,\ell}(\hat{k}) = 0$. Moreover,

$$\text{(A.6)} \qquad \delta'_{j,\ell}(k) = \{(1/j)[\tilde{R}_j f'(k)]^{1/j} - (1/\ell)[\tilde{R}_\ell f'(k)]^{1/\ell}\}\ f''(k)/f'(k).$$

Since $\delta_{j,\ell}(\hat{k}) = 0$ and $j > \ell$, it follows that $\delta'_{j,\ell}(\hat{k}) > 0$. Thus it is possible to choose values ε_1, $\varepsilon_2 > 0$ such that (A.3) and (A.4) hold $\forall\, m \notin M(\hat{k})\ \forall\, k \in (\hat{k}-\varepsilon_1, \hat{k}+\varepsilon_2)$, and such that

$$\text{(A.7)} \qquad [\tilde{R}_\ell f'(k)]^{1/\ell} > [\tilde{R}_j f'(k)]^{1/j}$$

$\forall\, k \in (\hat{k}-\varepsilon_1, \hat{k})$, while $\forall\, k \in (\hat{k}, \hat{k}+\varepsilon_2)$,

(A.8) $$[\tilde{R}_j f'(k)]^{1/j} > [\tilde{R}_\ell f'(k)]^{1/\ell}. \qquad \square$$

A.2. Proof of Lemma 2

$\{1\} = M(k)$ iff $\delta_{1,\ell}(k) > 0\ \forall\, \ell \neq 1$. From (A.5) this condition obtains iff

(A.9) $$f'(k) > (\tilde{R}_\ell)^{1/(\ell-1)}/(\tilde{R}_1)^{\ell/(\ell-1)}$$

$\forall\, \ell = 2,\ldots,J$. But (A.9) holds for all k sufficiently close to zero, by the Inada condition.

Similarly $\{J\} = M(k)$ iff $\delta_{J,\ell}(k) > 0\ \forall\, \ell \neq J$. From (A.5), this condition is satisfied iff

(A.9′) $$f'(k) < (\tilde{R}_J)^{\ell/(J-\ell)}/(\tilde{R}_\ell)^{J/(J-\ell)}$$

$\forall\, \ell = 1,\ldots,J-1$. But again, satisfaction of (A.9′) for large enough k is guaranteed by the Inada condition. $\square$

A.3. Proof of Lemma 3

Since $j,\ell \in M(\hat{k})$, $\delta_{j,\ell}(\hat{k}) = 0$. In addition, by definition $G_j(\hat{k}) < G_\ell(\hat{k})$ iff

(A.10) $$\tilde{R}_j s[w(\hat{k}), (\tilde{R}_j f'(\hat{k}))^{1/j}]/\sum_{h=0}^{j-1} [\tilde{R}_j f'(\hat{k})]^{h/j} < \tilde{R}_\ell s[w(\hat{k}), (\tilde{R}_\ell f'(\hat{k}))^{1/\ell}]/\sum_{h=0}^{\ell-1} [\tilde{R}_\ell f'(\hat{k})]^{h/\ell}.$$

But $\delta_{j,\ell}(\hat{k}) = 0$ implies that $(\tilde{R}_j f'(\hat{k}))^{1/j} = (\tilde{R}_\ell f'(\hat{k}))^{1/\ell}$, so that (A.10) reduces to

(A.11) $$\tilde{R}_\ell \sum_{h=0}^{j-1} [\tilde{R}_j f'(\hat{k})]^{h/j} > \tilde{R}_j \sum_{h=0}^{\ell-1} [\tilde{R}_\ell f'(\hat{k})]^{h/\ell}.$$

Substituting $[\tilde{R}_j f'(\hat{k})]^{1/j} = [\tilde{R}_\ell f'(\hat{k})]^{1/\ell}$ into (A.11) yields

(A.12) $$\tilde{R}_\ell \sum_{h=0}^{j-1} [\tilde{R}_\ell f'(\hat{k})]^{h/\ell} > \tilde{R}_j \sum_{h=0}^{\ell-1} [\tilde{R}_\ell f'(\hat{k})]^{h/\ell}.$$

Moreover,

$$\text{(A.13)} \qquad \tilde{R}_j = [\tilde{R}_\ell f'(\hat{k})]^{j/\ell}/f'(\hat{k}),$$

holds. Substituting (A.13) into (A.12), we obtain

$$\text{(A.14)} \qquad \sum_{h=0}^{j-1} [\tilde{R}_\ell f'(\hat{k})]^{h/\ell} > [\tilde{R}_\ell f'(\hat{k})]^{(j-\ell)/\ell} \sum_{h=0}^{\ell-1} [\tilde{R}_\ell f'(\hat{k})]^{h/\ell} = \sum_{h=j-\ell}^{j-1} [\tilde{R}_\ell f'(\hat{k})]^{h/\ell},$$

which obviously holds, establishing the desired relation. □

A.4. Proof of Lemma 4

From equation (22) it is apparent that

$$\text{(A.15)} \qquad G_j(k)/k \leq \tilde{R}_j s[w(k), (\tilde{R}_j f'(k))^{1/j}]/k \leq \tilde{R}_j w(k)/k$$

holds $\forall$ k, $\forall$ j=1,...,J. But then, $\forall$ j,

$$\text{(A.16)} \qquad \lim_{k\to\infty} G_j(k)/k \leq \lim_{k\to\infty} \tilde{R}_j w(k)/k.$$

Furthermore, by L'Hospital's rule, $\forall$ j

$$\lim_{k\to\infty} \tilde{R}_j w(k)/k = 0,$$

establishing the result. □

A.5. Proof of Proposition 1

The existence of a non-trivial stationary equilibrium follows immediately from (a.1) and lemmas 1-4 (see figure 1). We now establish uniqueness of the non-trivial steady state equilibrium.

To do so it will be useful to begin with two preliminary results.

Result 1. Define the function $h : \mathbf{R}_+ \to \mathbf{R}_+$ by

(A.17) $$h(x) \equiv x(1-x^{1/j})/(1-x).$$

Then $h'(x) \geq 0 \; \forall \, x \in \mathbb{R}_+$.

Proof. Differentiating (A.17) yields

(A.18) $$h'(x) = \eta(x)/(1-x)^2,$$

where

(A.19) $$\eta(x) \equiv 1 + (1/j)x^{(j+1)/j} - [(j+1)/j]x^{1/j}.$$

Evidently, $\eta(1) = 0$. Moreover,

(A.20) $$\eta'(x) = [(j+1)/j^2](x-1)x^{(1-j)/j}.$$

Thus

(A.21) $$\eta'(x) \leq 0; \; x \leq 1$$

$$\eta'(x) \geq 0; \; x \geq 1$$

holds. It follows that $\eta(x) \geq 0 \; \forall \, x \in \mathbb{R}_+$, and hence that $h'(x) \geq 0 \; \forall \, x \in \mathbb{R}_+$. □

Result 2. For all $j=1,\ldots,J$, the equation

(A.22) $$k = G_j(k)$$

has at most one solution.

Proof. Using the definition of $G_j(k)$ in (22), rewrite (A.22) as

(A.23) $$kf'(k)/w(k) = s[w(k), (\tilde{R}_j f'(k))^{1/j}] \; h[\tilde{R}_j f'(k)]/w(k),$$

where h is defined in (A.17). Assumption (a) of the proposition (along with $s_2 \geq 0$) implies that $s[w(k), (\tilde{R}_j f'(k))^{1/j}]/w(k)$ is a non-increasing function of k. Similarly, Result 1 implies that $h[\tilde{R}_j f'(k)]$ is a non-increasing function of k. Finally, the assumption that the elasticity of substitution σ satisfies

$$\text{(A.24)} \qquad \sigma \equiv -f'(k)[f(k)-kf'(k)]/kf(k)f''(k) \geq 1$$

$\forall$ k implies that $kf'(k)/w(k)$ is a non-decreasing function of k. Thus (A.23) [and hence (A.22)] has at most one solution. □

We are now prepared to prove the remainder of proposition 1. By result 2 each $G_j(k)$ crosses the 45° line in figure 1 at most once. By lemma 4, each $G_j(k)$ lies below the 45° line for large enough values of k. Thus each $G_j(k)$ crosses the 45° lines from above (if at all). Therefore, if $k \epsilon G(k)$ and M(k) is a singleton, G(k) also crosses the 45° line from above. If $k \epsilon G(k)$ and M(k) is not a singleton, then k corresponds to a vertical portion of G, and lemma 3 implies that G(k) crosses the 45° line from above. Thus $\forall$ $k > 0$ satisfying $k \epsilon G(k)$, G(k) intersects the 45° line from above, and there can be at most one non-trivial solution to the equilibrium condition $k \epsilon G(k)$. □

A.6. <u>Proof of Lemma 5</u>

We wish to consider (with reference to figure 1) the vertical shift in the term

$$\text{(A.25)} \qquad G_j(k) = \tilde{R}_j s[w(k),(\tilde{R}_j f'(k))^{1/j}]\, [1-(\tilde{R}_j f'(k))^{1/j}]/[1-\tilde{R}_j f'(k)] = s[w(k),(\tilde{R}_j f'(k))^{1/j}]\, h[\tilde{R}_j f'(k)]/f'(k)$$

associated with an increase in $\tilde{R}_j$, where the function h is as defined in (A.17). This vertical shift is given by the term

$$s(\cdot)h'[\tilde{R}_j f'(k)] + (1/j)[\tilde{R}_j f'(k)]^{(1-j)/j}\, h[\tilde{R}_j f'(k)]s_2(\cdot).$$

Since $s_2 \geq 0$, result 1 implies that this term is non-negative. Moreover, as is clear from the proof of result 1, it is positive whenever $\tilde{R}_j f'(k) \neq 1$, and it is positive even there if $s_2 > 0$. □

A.7. Proof of Lemma 6.

Differentiating (31) gives

(A.26) $$\{(1/j)[\tilde{R}_j(\beta)f'(\hat{k}_{\ell,j})]^{1/j} - (1/\ell)[\tilde{R}_\ell(\beta)f'(\hat{k}_{\ell,j})]^{1/\ell}\}(f''/f')d\hat{k}_{\ell,j}/d\beta$$

$$= [\tilde{R}_\ell(\beta)f'(\hat{k}_{\ell,j})]^{1/\ell}[\tilde{R}'_\ell(\beta)/\ell\tilde{R}_\ell(\beta)] - [\tilde{R}_j(\beta)f'(\hat{k}_{\ell,j})]^{1/j}[\tilde{R}'_j(\beta)/j\tilde{R}'_j(\beta)].$$

Thus (31), (a.6), and $j > \ell$ imply that $d\hat{k}_{\ell,j}/d\beta < 0$. □

A.8. Proof of Lemma 7

Using $\{j^*\} = M(k)$ and (A.23), write the equilibrium condition as

(A.27) $$kf'(k)/w(k) = s[w(k),(\tilde{R}_{j^*}f'(k))^{1/j^*}]\ h[\tilde{R}_{j^*}f'(k)]/w(k).$$

Using (a.2) and (a.3), it is easy to show that (A.27) gives $\tilde{R}_{j^*}f'(k)$ as a (weakly) increasing function of k, say

(A.28) $$\tilde{R}_{j^*}f'(k) = \psi(k).$$

Since an increase in β increases k, in this case, it follows that $\tilde{R}_{j^*}f'(k)$ must (weakly) increase in order to satisfy (A.28). □

A.9. Proof of Lemma 11

From (43),

(A.29) $$\rho(\hat{k}_{\ell,j})/s(w,\gamma) = \gamma - [\hat{k}_{\ell,j}f'(\hat{k}_{\ell,j})/w(\hat{k}_{\ell,j})][w(\hat{k}_{\ell,j})/s(w,\gamma)].$$

From equation (37), $d\gamma/d\beta > 0$, while from (34), $d\hat{k}_{\ell,j}/d\beta < 0$. Moreover, these facts and assumption (a.2) imply that $w/s(w,\gamma)$ (weakly) declines when β increases, and assumption (a.3) implies that $\hat{k}_{\ell,j}f'(\hat{k}_{\ell,j})/w(\hat{k}_{\ell,j})$ (weakly) declines when β increases. Thus an increase in β increases $\rho/s(w,\gamma)$. □

A.10. Proof of Proposition 3

Let $j^* \in \{1,...,J\}$ be given. We show that

(a) $(\tilde{R}_j)^{j^*/(j-j^*)}/(\tilde{R}_{j^*})^{j/(j-j^*)}$ is non-increasing in j, for $j > j^*$.

(b) $(\tilde{R}_{j^*})^{j/(j^*-j)}/(\tilde{R}_j)^{j^*/(j^*-j)}$ is non-increasing in j, for $j < j^*$.

(52) and (53) then follow.

Proof of (a): It suffices to show that

$$(\text{A.30}) \qquad (\tilde{R}_j)^{j^*/(j-j^*)}/(\tilde{R}_{j^*})^{j/(j-j^*)} \geq (\tilde{R}_{j+1})^{j^*/(j+1-j^*)}/(\tilde{R}_{j^*})^{(j+1)/(j+1-j^*)}$$

holds, $\forall\, j \in \{j^*+1,\ldots,J\}$. To establish (A.30), take natural logarithms of both sides to obtain

$$(\text{A.31}) \qquad (j+1-j^*)[j^* \ln \tilde{R}_j - j \ln \tilde{R}_{j^*}] \geq (j-j^*)[j^* \ln \tilde{R}_{j+1} - (j+1) \ln \tilde{R}_{j^*}].$$

Rearranging terms in (A.31) yields

$$(\text{A.32}) \qquad \ln (\tilde{R}_j/\tilde{R}_{j^*}) \geq (j-j^*) \ln (\tilde{R}_{j+1}/\tilde{R}_j).$$

We now observe that

$$(\text{A.33}) \qquad \tilde{R}_j/\tilde{R}_{j^*} = (\tilde{R}_j/\tilde{R}_{j-1})(\tilde{R}_{j-1}/\tilde{R}_{j-2}) \cdots (\tilde{R}_{j^*+1}/\tilde{R}_{j^*})$$

$$\geq (\tilde{R}_j/\tilde{R}_{j-1})^{(j-j^*)},$$

where the last inequality follows from (51). But (A.33) implies that (A.32) holds if

$$(\text{A.34}) \qquad \tilde{R}_j/\tilde{R}_{j-1} \geq \tilde{R}_{j+1}/\tilde{R}_j.$$

However (A.34) is implied by (51). □

(b) is established via identical reasoning.

It remains to establish (54). However, upon rearranging terms in (54), it is apparent that (54) is equivalent to (51). □

A.11. Proof of Corollary

From equations (48) and (52), $1 \in M(k)$ iff

$$f'(k) \geq \tilde{R}_2/(\tilde{R}_1)^2. \tag{A.35}$$

But this is exactly (55). Similarly, from equations (49) and (53), $J \in M(k)$ iff

$$f'(k) \leq (\tilde{R}_J)^{J-1}/(\tilde{R}_{J-1})^J. \tag{A.36}$$

But this is exactly (56). Finally, equations (50), (52), and (53) imply that $j^* \in M(k)$ iff

$$(\tilde{R}_{j^*})^{j^*-1}/(\tilde{R}_{j^*-1})^{j^*} \geq f'(k) \geq (\tilde{R}_{j^*+1})^{j^*}/(\tilde{R}_{j^*})^{j^*+1}. \tag{A.37}$$

But this is exactly (57). □

Figure 1

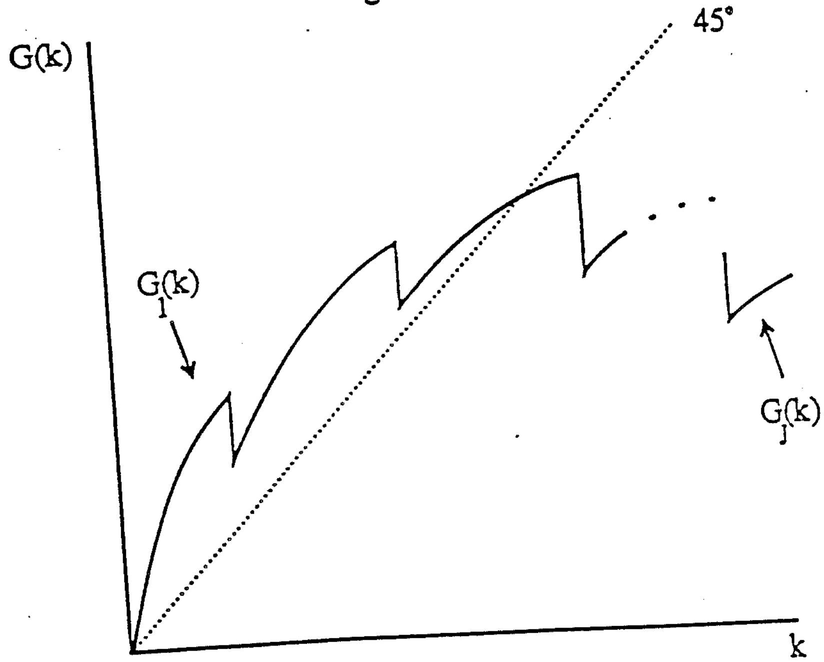


Figure 2

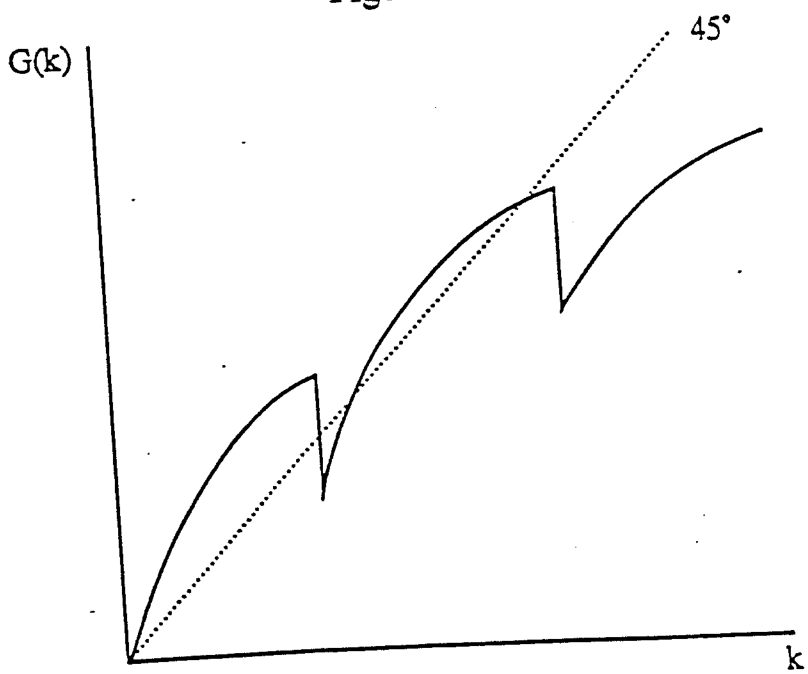
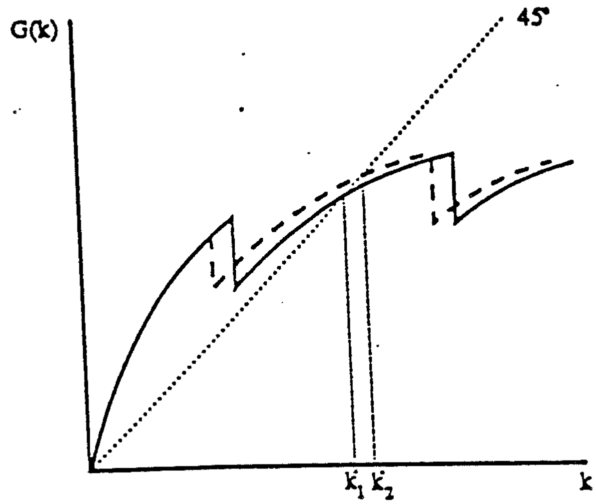
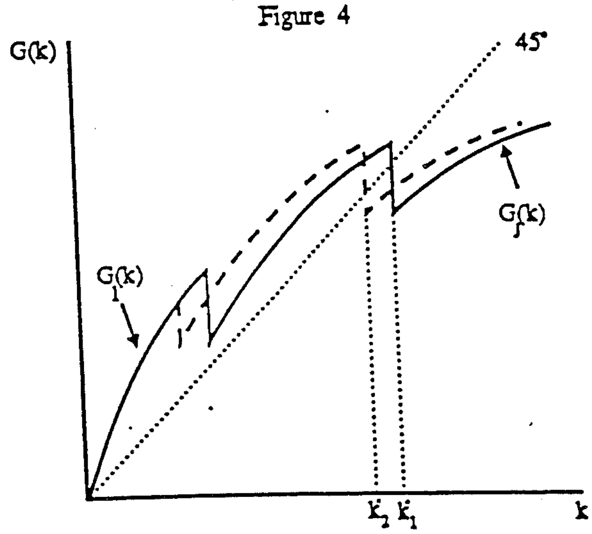


Figure 3



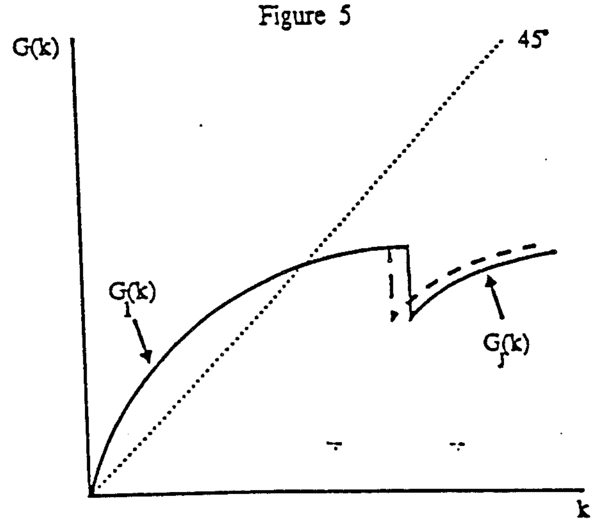
A Reduction in Transactions Costs

Figure 4



Reductions in Transactions Costs

Figure 5



Reductions in Transactions Costs

Notes

1. See, for instance, Bagehot (1873), Patrick (1966), Gurley and Shaw (1967), Cameron (1967), Goldsmith (1969), McKinnon (1973), and Shaw (1973). For recent formalizations of this argument see Starr (1987), Greenwood and Jovanovic (1990), Bencivenga and Smith (1991), or Cooley and Smith (1991). [Parente and Prescott (1991) can also be given an interpretation along these lines.] For a survey of empirical evidence see The World Development Report 1989.

2. An example of the former type of investment might be a steel mill, while inventories are an example of the latter. It is often argued very specifically that inventory investment is "excessive" when intermediation and secondary markets are attenuated, as for instance by Patrick (1966) and McKinnon (1973).

3. See, for instance, Patrick (1966), McKinnon (1973), and Shaw (1973).

4. See, for instance, McKinnon (1973) or Shaw (1973).

5. This is quite explicit, as in Foley (1970), Hahn (1971, 1973), Starrett (1973), Heller and Starr (1976), or Townsend (1978).

6. See, for instance, Galbis (1979), van Wijnbergen (1983, 1985), Buffie (1984), Diaz-Alejandro (1985), or Khatkhate (1988).

7. Allowing for heterogeneity creates no problems, but also does not introduce any additional substantive issues.

8. Except for the initial old. Since our focus is on steady state equilibria, we omit a description of initial conditions.

9. Observe that (A.1) is the standard assumption made to guarantee the existence of a non-trivial stationary equilibrium in the Diamond (1965) model. [See, for instance, Azariadis (1988).]

10. Such a possibility is illustrated by Cooley and Smith (1991) in an endogenous growth model with no transactions costs.

11. Simons (1948) argued that all capital formation should be equity financed, because high withdrawal demand could force socially costly liquidation of capital projects financed by bank lending. The extensions we contemplate would permit the first analysis of this issue, which underlies the entire "Chicago Plan of Bank Reform" discussed by Friedman (1960).

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