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Abstract

This paper introduces the concept of cotrending to trend stationary series with structural breaks. We apply it to nominal interest rates and inflation in order to test the long-run Fisher effect.

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1. Introduction

The concept of cointegration, introduced in Engle and Granger (1987) is appealing, in part, because it allows for the estimation of structural parameters without the need to impose exogeneity assumptions. Furthermore, many economic models can be tested by examining implied cointegration restrictions. However, cointegrating relationships can exist only when economic time series are integrated (or "difference stationary"). The question of whether or not macroeconomic data are difference stationary is a contentious one. Nelson and Plosser (1982) are unable to reject the null hypothesis of difference stationarity for many common time series when considered against the alternative of trend stationarity without any structural breaks, but Perron (1988) found that difference stationarity *could* be rejected for many series if the alternate trend stationary specification included "breaks" in the trend. If economic variables are more appropriately characterized as trend stationary around a trend which includes structural breaks, then the concept of cointegration cannot be applied to the evaluation of economic theories.

The purpose of this paper is to introduce the concept of cotrending to trend stationary time series with structural breaks.[1] Cotrending can fulfill a role similar to that of cointegration. Specifically, structural parameters can be estimated without exogeneity assumptions, and economic models can be tested by examining the implied cotrending restrictions. We propose a test for detecting the presence of cotrending. The $S(p_0, \ldots, p_n; q_0, \ldots, q_n)$ test, is similar to the $G(p,q)$ test for a stochastic trend described in Park (1990). It is based on testing the joint parameter restriction on trend terms which are "superfluous" under the null hypothesis of trending (when applied to an individual series) or cotrending (when applied to a linear combination of a vector time series), and it has the substantial advantage of a limiting chi-square distribution.

As an example, cotrending is applied to nominal interest rates and inflation. This application is of independent interest because the long-run Fisher effect is at the heart of most macroeconomic models. When nominal interest rates and inflation are modelled as integrated series, the proposition that nominal interest rates move one-for-one with inflation in the long run is typically rejected (see, for example, Rose (1988) and King and Watson (1992)).[2] We re-examine this proposition by modelling nominal interest rates and inflation as trend stationary with structural breaks in the deterministic components of the series. Our results still reject the proposition that nominal interest rates move one-for-one with inflation. Alternatively, we reject the hypothesis that the real interest rate is stationary.

The rest of the paper is organized as follows: Section 2 defines trending, cotrending and the $S(p_0, \ldots, p_n; q_0, \ldots, q_n)$ statistic. These concepts are applied to inflation, the nominal yield on one month U.S. treasury bills, and the ex post real yields in section 3, and a brief summary and conclusions are in section 4.

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[1]Ogaki and Park (1992) introduced cotrending for the special case of time series with linear trends and without structural breaks. Bai, Lumsdain, and Stock (1992) studied integrated time series that contain deterministic trends with structural breaks.

[2]On the other hand, Neusser (1991), using Johansen's (1988) test for cointegration, found evidence which is consistent with stationarity of the real interest rate.

2. Trending and Cotrending

If x(t) is the time series of interest, we can consider a representation with multiple structural breaks, along the lines of Park (1988).[3] If the break points occur at n distinct dates, denoted by $t = \{m_1, m_2, \ldots, m_n\}$, then x(t) can be written as:

$$x(t) = \sum_{j=0}^{n} v_j(t)'\beta_j + w(t) \tag{1}$$

$$where \quad v_0(t) = (1, t, t^2, \ldots, t^{q_0}) \quad and \quad v_j(t) = (t_{m_j}^0, t_{m_j}^1, \ldots, t_{m_j}^{q_j})$$

$$t_{m_j}^k = \begin{cases} 0 & t \le m_j \\ (t-m_j)^k & t > m_j \end{cases} \tag{2}$$

for $j = 1, \ldots, n$. w(t) is a stationary random variable with zero mean[4]. In this representation, we adopt the conventions that the q_j -th component of β_j is nonzero and that there is no j-th term when $q_j=-1$. A series is said to be *trending of order* $(q_0, \ldots, q_n)$, denoted $x(t) \sim T(q_0, \ldots, q_n)$, if it has a representation as in equation (1). As an example, consider a series with a single break point which exhibits a linear trend prior to the break and a quadratic trend after the break. In this case, $x(t) \sim T(1,2)$. The components of a vector series are said to be *cotrending of order* $(d_0, \ldots, d_n; q_0, \ldots, q_n)$, denoted $x(t) \sim CT(d_0, \ldots, d_n; q_0, \ldots, q_n)$, if (i) all components of x(t) are $T(q_0, \ldots, q_n)$; (ii) there exists a vector α such that $z(t) = \alpha'x(t) \sim T(q_0-d_0, \ldots, q_n-d_n)$. The vector α is called *the cotrending vector*.

A cotrending vector can be consistently estimated by ordinary least squares (OLS), which will be called a cotrending regression, under certain conditions. Suppose that the first element of a cotrending vector α is nonzero. Normalize α by setting its first element to be one. Let y(t) be the first element of x(t), and z(t) the rest of x(t). Furthermore, assume that z(t) is $T(q_1, \ldots, q_n)$. Consider an OLS regression of y(t) onto z(t). Suppose that the largest q_i $(i=1,\ldots,n)$ is greater than or equal to one and that the trending functions in z(t) are linearly independent. Then the normalized cotrending vector is consistently estimated by this regression and the estimator has an asymptotic normal distribution. The estimators converge at a rate faster than the square root of the sample size (the actual rate depends on the order of trending). A proof can be obtained along the lines of West (1988) by ignoring the trend components in his unit root nonstationary case.

For testing, we assume that α is known for simplicity. The test for the null hypothesis that the components of a vector series x(t) are $CT(d_0, \ldots, d_n; q_0, \ldots, q_n)$ with a known cotrending vector α is based on the following OLS

[3] In this paper, we assume that the break points are known and are not stochastic. Perron (1991) develops a test for unknown break points. One interpretation of our model is that break points are stochastic, perhaps along the lines of Hamilton (1988,1989), but they are known to the econometrician (for example, because of the Federal Reserve Board's announcements about changes in their policy rules). If the break points are not known to the econometrician with certainty, they need to be estimated, as in Garcia and Perron (1991).

[4] It is the "non-trend" component of the series.

regression:

$$z(t) = \sum_{j=0}^{n} v_j(t)'\beta_j v_j(t)'\beta_j + \sum_{j=0}^{n} s_j(t)'\gamma_j + w(t) \tag{3}$$

$$where\ v_0(t) = (1,t,t^2,\dots,t^{p_0})',\ s_0(t) = (t^{p_0+1},\dots,t^{q_0})',\ v_j(t) = (t_m^0, t_m^1,\dots,t_m^{p_j})',\ and\ s_j(t) = (t_m^{p_j+1},\dots,t_m^{q_j})'$$

z(t) is a scalar random variable. Under the null hypothesis, $s_i(t)$ is a vector of superfluous trends (i=1, ..., n). By convention, there is no $s_i(t)$ term if $p_i=q_i$, and there is no $v_i(t)$ term when $p_i=-1$.

To test the null hypothesis, we take z(t) = α'x(t) and $p_i=q_i-d_i$ for i=0, ... ,n. Then $\gamma_1= \cdots =\gamma_n=0$ under the null. This motivates a chi-square test for the restrictions $\gamma_1= \cdots =\gamma_n=0$ which is similar to Park's G(p,q) test. Specifically, define:

$$S(p_0,\dots,p_n;q_0,\dots,q_n) = \frac{RSS_R - RSS_{UR}}{\hat{\omega}^2} \tag{4}$$

where RSS_{UR} is the residual sum of squares from the OLS regression in equation (3), and RSS_R is that from the regression with the restriction $\gamma_1= \cdots =\gamma_n=0$. The long run variance $\hat{\omega}^2$ is estimated from the residual in the restricted regression.[5] Under the same conditions imposed on w(t) in (3) as Park's conditions on the disturbances for the application of the G(p,q) tests, these statistics have asymptotic χ^2 distributions with the degrees of freedom being equal to the number of restrictions, as long as m increases with T in such a way that m/T converges to a constant. The proof of this result is similar to the one employed by Park (1990) for the G(p,q) test, or it can be constructed as a special case of the generalized method of moments.

3. An Examination of the Ex Post Real Yield on Treasury Bills

One possible formulation of the long-run Fisher relation is to say that the ex ante real interest rate is stationary. This would mean that any existing trends in the inflation rate are eliminated by trends in the nominal interest rate. If we make the additional (reasonable) assumption that the forecasting error for inflation is stationary,

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[5]It is estimated as the spectral density of the residuals at frequency zero, using the Quadratic Spectral kernel and the automatic bandwidth procedure of Andrews (1990).

then the long-run Fisher relation can be examined by testing whether or not the ex post real interest rate is stationary. The inflation measures are the continuously compounded one month growth rate in the implicit deflator for Personal Consumption Expenditures on nondurables and nondurables plus services for the period from February 1959 to December 1990. Both series are from the CITIBASE database. The nominal yield is the continuously compounded yield-to-maturity on one month U.S. treasury bills (based on the average of the bid and ask spread) from the Government Bond File of the Center for Research in Security Prices.

Many previous authors have tested inflation rates and nominal yields for difference stationarity versus the alternative of trend stationarity without a structural break in the trend. Typically, difference stationarity is not rejected by the data.[6] Table 1 contains the results of $S(1,2;q_0,q_1)$ tests for $(q_0,q_1) \in \{(2,3),(3,4),(4,5),(5,6)\}$. These tests explicitly allow for a break in the deterministic trend in October 1979, a date which was chosen based on prior research (see Huizinga and Mishkin (1986) and Hamilton (1989)) and exogenous information about Federal Reserve operating procedures. All of the tests are consistent with the hypothesis that these series are T(1,2). For example, the S(1,2;3,4) test for the treasury yield has a value of 6.833. This statistic is based on the Wald test for coefficient values of zero on the second and third order trend in the first sub-period and the third and fourth order terms in the second sub-period. The statistic has an asymptotic χ_4^2 distribution, and its p-value is 0.145 for the null hypothesis of stationarity. In contrast to the prior literature, these results support the hypothesis that the behavior of these series is adequately characterized by stationary fluctuations about a trend which includes a break point in October 1979.

So, the hypothesis of a single trend break appears to be an adequate univariate description of the inflation rates and yield series examined above, but is it an adequate multivariate representation? Just as any reasonable theoretical model would impose the joint restriction of cointegration on nominal yields and inflation rates (i.e. ex post real yields are not integrated), the trend break model should impose cotrending restrictions on these series. We will consider the following two possibilities: (i) (yld, $\boldsymbol{\pi}$) are CT(1,3;1,2) with $\boldsymbol{\alpha} = (1,-1)$, or (ii) (yld, $\boldsymbol{\pi}$) are CT (1,2;1,2). In the first model, the effect of cotrending is to remove the trend break entirely. This makes the ex post real rate a process which exhibits stationary fluctuations around a constant mean. The second model assumes that the effect of the co-movements in the deterministic trends is to remove the existence of the higher order trend in the second sub-period, but the ex post real rate is allowed to exhibit a shift in its mean.

The cotrending tests are in table 2; panel (A) presents results for tests of CT(1,3;1,2), while panel (B) contains

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[6]The interested reader is referred to Rose (1988), Mishkin (1991), Perron and Garcia (1991) and Schwert (1987). We performed Augmented Dickey-Fuller tests, Phillips and Perron tests and Park's G and J tests and reached similar conclusions using our data series and sample period. These results are available upon request.

tests of the hypothesis that inflation and yields ar CT(1,2;1,2). They consist of S(p,q) tests applied to the ex post real rate. They are conceptually similar to testing for cointegration with a known cointegrating vector by applying a unit root test to the "residual." Both sets of tests strongly reject the null hypothesis of cotrending. For example, the S(0,0;2,2) test assumes the null hypothesis of constant but different means in the pre- and post-October 1979 period. These statistics have asymptotic χ_4^2 distributions. For the ex post real rate using the inflation rate for nondurables, S(0,0;2,2) is 18.74 with a p-value of 0.0009. The S(0,0;2.2) test for the rate using nondurables plus services is 19.05 with a p-value of 0.0008. This means that, while a single break might be an adequate univariate representation, either cointegrated stochastic trends or multiple break points are needed to capture the movements of the ex post real yield. In other words, by imposing joint restrictions on the deterministic trends, we are able to reject the single trend break model for inflation and nominal treasury yields.

4. Conclusions

In this paper we have introduced the concepts of trending and cotrending for characterizing the joint movements in economic time series exhibiting structural breaks. The S(p,q) test was presented as a test for trending and cotrending. As in the G(p,q) test described in Park (1990), it is based on testing the joint coefficient restrictions on trend terms which are "superfluous" under the null hypothesis. It has the considerable advantage of a limiting χ^2 distribution.

As an example, these concepts were applied to the joint movements of inflation and the nominal yield on one month treasury bills for the period from February 1959 to December 1990, allowing for a single break in the deterministic trend functions in October 1979. For each of these time series, the single break model without a stochastic trend is not rejected by the data. The basic model of cotrending, however, is strongly rejected for this data set. Thus, our test results for the trend break model are consistent with those of Rose (1988) and King and Watson (1992) for the stochastic trend model: real interest rates are not stationary.

Table 1: Stationarity Tests for Inflation and Nominal Yields
Assuming a Trend Break in October 1979
February 1959 to December 1990

Series	S(1,2;2,3)	S(1,2;3,4)	S(1,2;4,5)	S(1,2;5,6)
π_{ND}	2.305	3.054	4.373	7.477
	(0.316)	(0.549)	(0.626)	(0.486)
π_{NDS}	4.241	7.426	9.265	11.680
	(0.120)	(0.115)	(0.159)	(0.166)
YLD1M	3.846	6.833	11.467	13.729
	(0.146)	(0.145)	(0.075)	(0.089)

π_{ND} is the continuously compounded one month growth rate in the implicit deflator for Personal Consumption Expenditures on nondurable goods. π_{NDS} is the continuously compounded one month growth rate in the implicit deflator for Personal Consumption Expenditures on nondurables plus services. Both of these series are from the CITIBASE directory. YLD1M is the nominal yield-to-maturity on one month U.S. treasury bills. It is from the CRSP Government Bond File. The $S(1,2;q_0,q_1)$ tests are described in section 2, and they test the null hypothesis of stationarity about a deterministic trend which includes a break in October 1979. The statistics are asymptotically distributes as chi-squared with $(q_0-1)+(q_1-2)$ degrees of freedom. P-values are reported in parentheses.

Table 2: Cotrending tests of Nominal Yields and Inflation
February 1959 to December 1990

Panel (A): Testing for CT(1,3;1,2)

Series	S(0,-1;1,0)	S(0,-1;2,1)	S(0,-1;3,2)
YLD1M - π_{ND}	21.19	22.69	27.02
	(2.51E-5)	(1.45E-4)	(1.44E-4)
YLD1M - π_{NDS}	24.31	25.49	29.08
	(5.26E-6)	(4.01E-5)	(5.88E-5)

Panel (B): Testing for CT(1,2;1,2)

Series	S(0,0;1,1)	S(0,0;2,2)	S(0,0;3,3)
YLD1M - π_{ND}	12.40	18.74	25.74
	(0.002)	(0.001)	(0.000)
YLD1M - π_{NDS}	15.76	19.05	31.39
	(0.000)	(0.001)	(0.000)

YLD1M is the nominal yield-to-maturity on a one month U.S treasury bill. It is from the CRSP Government bond file. π_{ND} (π_{NDS}) is the one month continuously compounded growth rate in the implicit deflator for Personal Consumption Expenditures on nondurables (nondurables plus services). Inflation series are taken from the CITIBASE directory. The $S(p_0,p_1;q_0,q_1)$ is described in section 2 of the text. Panel (A) assumes stationarity about a constant mean, and panel (B) assumes stationarity about a mean which exhibits a discrete break in October 1979. These statistics are asymptotically distributed as chi-squared with $(q_0-p_0)+(q_1-p_1)$. P-values are reported in parentheses.

REFERENCES

Andrews, D.W.K., 1991, "Heteroskedasticity and Autocorrelation Consistent Covariance Matrix Estimation," *Econometrica 59*, pages 817-858.

Bai, J., R.L. Lumsdain and J. Stock, 1992, "Testing for and Dating Breaks in Integrated and Cointegrated Time Series," paper presented at the 1992 North American Summer Meeting of the Econometric Society.

Engle, R.F. and C.W.J. Granger, 1987, "Co-integration and Error Correction: Representation, Estimation, and Testing," *Econometrica 55*, pages 251-276.

Garcia, R. and P. Perron, 1991, "An Analysis of the Real Interest Rate Under Regime Shifts," Manuscript, Princeton University.

Hamilton, J.D., 1988, "Rational-Expectations Econometric Analysis of Changes in Regime: An Investigation of the Term Structure of Interest Rates," *Journal of Economic Dynamics and Control 12*, pages 385-423.

Hamilton, J.D., 1989, "A New Approach to the Economic Analysis of Nonstationary Time Series and the Business Cycle," *Econometrica 57*, pages 357-384.

Huizinga, J. and F.S. Mishkin, 1986, "Monetary Policy Regime Shifts and the Unusual Behavior of the Real Interest Rate," *Carnegie-Rochester Conference on Public Policy 24*, pages 231-274.

Johansen, S., 1988, "Statistical Analysis of Cointegrating Vectors," *Journal of Economic Dynamics and Control 12*, pages 231-254.

King, R.G. and M.W. Watson, 1992, "Testing Long-Run Neutrality, " Manuscript, University of Rochester and Northwestern University.

Mishkin, F.S., 1991, "Is the Fisher Effect for Real?: A Reexamination of the Relationship Between Inflation and Interest Rates," NBER Working Paper No. 3632.

Nelson, C.R. and C.I. Plosser, 1982, "Trends and Random Walks in Macroeconomic Time Series," *Journal of Monetary Economics 10*, pages 129-162.

Neusser, K., 1991, "Testing the Long-Run Implications of the Neoclassical Growth Model," *Journal of Monetary Economics 27*, pages 3-37.

Ogaki, M. and J.Y. Park, 1992, "A Cointegration Approach to Estimating Preference Parameters," Working Paper, University of Rochester.

Park, J.Y., 1990, "Testing for Unit Roots and Cointegration by Variable Addition," in *Advances in Econometrics, Volume 8* (JAI Press, Inc.), pages 107-133.

Perron, P., 1989, "The Great Crash, the Oil Price Shock, and the Unit Root Hypothesis," *Econometrica 57*, pages 1361-1401.

Perron, P., 1991, "A Test for Changes in a Polynomial Trend Function for a Dynamic Series," Manuscript, Princeton University.

Rose, A.K., 1988, "Is the Real Interest Rate Stable?," *Journal of Finance 43*, pages 1095-1112.

Schwert, G.W., 1987, "Effects of Model Specification On Tests of Unit Roots in Macroeconomic Data," *Journal of Monetary Economics 20*, pages 73-103.

West, K.D., 1988, "Asymptotic Normality When Regressors Have a Unit Root," *Econometrica 56*, pages 1397-1417.