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Risk Aversion and International Markets:

Does Asset Trade Smooth Real Income?*

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ABSTRACT

In a world in which all goods are traded and in which international asset markets are complete, individuals are able to eliminate all idiosyncratic risk. We continue to assume complete asset markets but incorporate the existence of non-traded goods into a two-country, equilibrium model and find that, in general, individuals no longer choose to eliminate all risk in aggregate consumption. The presence of non-traded goods forces individuals to make a choice between reducing uncertainty in aggregate consumption and reducing uncertainty in the composition between traded and non-traded goods. We find that this choice depends on a comparison of the standard coefficient of relative risk aversion with a second type of risk aversion that becomes relevant only when non-traded goods are present - one that captures aversion to risk in composition. In addition, we find that the risk premium that arises in this context reflects both these types of risk aversion.

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1. Introduction

Complete international asset markets enable individuals to smooth utility perfectly across states of the world when the source of the uncertainty lies in idiosyncratic country-specific supply shocks. However, restrictions on trade in goods may alter the optimal trades in assets such that idiosyncratic risk is deliberately incompletely pooled. In order to maximize expected utility, risk-averse individuals may choose a pattern of consumption across states that causes the variance of utility levels to be increased rather than decreased.

The key factor behind this result is the individual's concern about the composition of the consumption bundle. For just as an individual can be averse to risk in the *level* of aggregate consumption, so too can the individual be averse to risk in the *composition* of aggregate consumption. We derive the distinction between these two components of risk aversion and show how aversion to risk in composition becomes important when some good must be entirely consumed within the country in which it is produced, either because trade restrictions prohibit international exchange or inherent physical characteristics create high transport costs.

The concerns over uncertainty in composition, which influence optimal risk-sharing arrangements, may also alter the risk premium that individuals are willing to pay to avoid uncertainty. Although the Arrow-Pratt measure of the risk premium can be expressed as a function of the standard coefficient of relative risk aversion in a one-good model, the risk premium that arises when both traded and non-traded goods enter utility depends on aversion to risk in composition as well as in levels. This derivation of the risk premium extends previous work on risk aversion in a many-good world.[1]

Our characterization of risk aversion for a two-good economy is formulated to pay particular attention to the effect of trade asymmetries on optimal responses in an uncertain environment. This decomposition of risk aversion, which has not previously appeared in the literature on risk in a multi-good world, draws a distinction in attitudes towards risk analogous to the components of uncertainty that arise when some goods are traded internationally, but others are not.

Non-tradable goods have been introduced into the standard two-country, two-good, equilibrium asset-pricing model to improve the explanatory power of these models for the behavior of real and nominal exchange rates, consumption correlations across countries, savings-investment correlations, and portfolio shares. Equilibrium models in which all goods are tradable are inconsistent with the observation that, on average, individuals hold a greater proportion of wealth in the assets of their own country, rather than divide wealth equally between home and foreign assets, as in the Lucas(1982) perfectly-pooled equilibrium. Stockman and Dellas(1989) find that the introduction of non-traded goods serves to increase the proportion of domestic assets held by domestic agents.[2] In this model preferences are separable so agents remain perfectly pooled in the traded good while owning all claims to the domestic non-traded good. The composition of portfolios would thus be skewed towards domestic assets. Tesar(1990) examines a model with non-traded goods and is able to explain the observed high correlation of savings and investment within countries as well as the observed low correlation of

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[1] See Stiglitz (1969), Diamond and Stiglitz (1974), Kihlstrom and Mirman (1974, 1981), and Paroush (1975).

[2] Eldor, Pines, and Schwartz(1988) derive conditions that generate portfolios that are skewed toward domestic assets in a model with trade in equities and where productivity disturbances are limited to the non-traded goods sector.

consumption across countries. While equilibrium models in which agents are perfectly pooled in all goods imply that countries' consumptions will be perfectly correlated, the presence of non-traded goods which must be entirely consumed in the country of origin creates a significant reduction in the covariance of aggregate consumption across countries. The results derived by Stockman and Dellas and by Tesar depend upon the role of aversion to risk in composition as well as aversion to risk in levels of consumption. Stockman and Dellas in effect balance the tradeoff that is created by these two aspects of uncertainty by considering the special case of separability, an assumption which restricts the aversion to risk in levels to be equal to aversion to risk in composition. Tesar permits trade over time in the form of capital accumulation, thereby relaxing this tradeoff by reducing the degree of the constraint imposed by the existence of a non-traded good. The present paper examines the underlying causes of the tradeoff arising out of the asymmetry in the international tradability of goods and derives implications for the degree to which agents use asset markets to reduce uncertainty in aggregate consumption and for the level of the risk premium.

The paper proceeds as follows. In section 2 we examine the nature of risk aversion when two goods enter utility and reveal the source of the distinction between aversion to risk in levels and aversion to risk in composition. Section 3 confronts the problem faced by a risk-averse agent in a world where the tradability of goods is asymmetric and shows wherein the two components of risk aversion influence the agent's decisions. The consideration of some special cases of preferences which are commonly found in the literature is undertaken at the conclusion of the model. In section 4 we demonstrate that the risk premium, when traded and non-traded goods enter utility, depends also on the two types of risk aversion. Section 5 offers concluding remarks.

2. Uncertainty and Preferences over Two Goods

An individual who is assumed to consume two goods may respond differently to risk than would one facing uncertainty in a single good. The presence of a second good may create an additional type of risk - that in the *composition* of goods supplied - whereas with a single commodity the uncertainty would appear only in the *level* of the sole consumption good. The altered behavior arising in the face of uncertainty in two distinct goods depends not only on the characteristics of the uncertainty itself, but also on the nature of preferences and, in particular, the agent's aversion to risk in the two-good case. It is to this issue that we now turn.

The standard measure of risk aversion involves diminishing returns to *aggregate* consumption. To capture this aspect of risk aversion when agents derive utility over two different goods, it is useful to create a composite commodity, which we do by introducing the consumption function,

$$C = C(N, T), \tag{1}$$

where we let N and T represent the two goods being consumed. We require only that this function be linearly homogeneous. This preserves a high degree of generality albeit at the cost of imposing homotheticity in demand. Utility can now be expressed as a function of aggregate consumption,

$$U = U(C), \tag{2}$$

where U is twice continuously differentiable, $U'(C) > 0$, and $U''(C) < 0$. As with the consumption function a high degree of generality is retained in the specification of the utility function so that the results derived herein are applicable to a wide range of preferences. This includes, but is not limited to, the whole class of constant elasticity of substitution (CES) functions. Special cases of both the consumption function and the utility function which have been highlighted in the literature are considered after the model is presented in section 3.

With two goods being consumed, the utility surface is three dimensional and is assumed to be strictly concave. Usually, goods are aggregated and the degree of risk aversion is captured by the degree of curvature in the utility surface as the level of aggregate consumption is varied. We illustrate this component of risk aversion in panel (a) of figure 1. In this diagram aggregate consumption is assumed to be stochastically supplied and the two equally probable outcomes, C_1 and C_2, have expected value, $\bar{C}$. The risk averseness of the individual is revealed by the disparity between the agent's level of expected utility, E{U[C(N,T)]}, and the utility of the expected level of aggregate consumption, U{E[C(N,T)]}. Clearly, the degree to which the latter exceeds the former depends upon the curvature of the utility function. The greater this difference, the greater is the agent's degree of risk aversion. This type of risk aversion is more accurately referred to as aversion to risk in the *level* of aggregate consumption.[3]

Before turning to the specific measure of this aspect of risk aversion, let us consider the curvature of the utility surface in the orthogonal direction. The strict concavity of the utility surface in this direction is illustrated for a given level of aggregate consumption by varying the composition of consumption. Panel (b) of figure 1 shows this curvature as we slice through the utility surface at a given level of consumption and thus utility. Suppose now that the ratio of goods consumed is uncertain and, in particular, assume that there are two equally probable states of the world that yield the same level of utility and thus are illustrated by points *a* and *b* on the indifference curve, $\tilde{U}$. As in panel (a), the risk averseness of the

[3] Or, equivalently, it can be called aversion to risk in the level of utility or of real income, since utility is a monotonic function of aggregate consumption and real income is synonymous with utility.

individual is captured by the curvature of the utility surface. While earlier we found that the presence of uncertainty in the level of aggregate consumption gives rise to greater utility when the expected level of aggregate consumption is received with certainty, aversion to risk in the composition of goods consumed implies that higher utility is attained when the expected ratio of goods is received with certainty; clearly point *c*, which represents this expected ratio, lies on a higher indifference curve. Once again the greater is the curvature of the utility surface, the greater is the disparity between expected utility, $\tilde{U}$, and the utility of the expected consumption bundle, $U\{C[E(N),E(T)]\}$, and the greater is the degree of aversion to risk in *composition*. Indeed, even if alternative states of the world all yield the same level of real income, there still remains a role for asset trade.

We can assess the degree of risk aversion along each of these dimensions through a closer examination of the utility surface. The standard coefficient of relative risk aversion, ρ, measures the curvature of the utility surface in the direction of varying aggregate consumption and is expressed as

$$\rho = \frac{-U''C}{U'} \quad . \tag{3}$$

Note that this is equivalent to the elasticity of the slope of the utility function with respect to C so that ρ can be defined as

$$\rho \equiv \frac{-\hat{U}'}{\hat{C}} \tag{4}$$

where a 'hat' denotes a proportional change. Panel (c) of figure 1 corresponds to the utility function drawn in panel (a) and shows the marginal utility schedule as a downward-sloping function of the level of aggregate consumption. The elasticity of this curve is ρ, the coefficient of relative aversion to risk in the *level* of aggregate consumption.[4] By analogy we can define a

[4]When the elasticity of this function is constant the utility function is characterized by constant relative aversion to risk in the level of consumption. In general, however, this elasticity need not be constant.

measure of aversion to risk in the *composition* of aggregate consumption as the elasticity of the slope of the utility surface with respect to the ratio of goods consumed when aggregate consumption is held constant. More explicitly, let this elasticity, ω, be defined as

$$\omega \equiv \frac{-(\hat{C}_T - \hat{C}_N)}{(\hat{T} - \hat{N})} \tag{5}$$

where C_T and C_N denote the derivatives of C with respect to consumption of the traded and non-traded goods, respectively. (The reciprocal of ω is a more familiar concept — it is the elasticity of substitution between the two goods.)[5] Panel (d) in figure 1 corresponds to the utility surface depicted in panel (b) and relates the slope of the indifference curve to the composition of consumption. The elasticity of this curve is ω, the coefficient of relative aversion to risk in the *composition* of aggregate consumption.

The linear homogeneity of C(N,T) simplifies the solution for ω. When C(N,T) is homogeneous of degree one, C_N and C_T are homogeneous of degree zero so that

$$C_{NN}N + C_{NT}T = 0, \text{implying} \quad C_{NN} = -\frac{T}{N}C_{NT}. \tag{6}$$

Also,

$$C_{TN}N + C_{TT}T = 0, \text{implying} \quad C_{TT} = -\frac{N}{T}C_{NT} \ (\text{since } C_{TN} = C_{NT}). \tag{7}$$

Solving for $\hat{C}_N$ and $\hat{C}_T$:

$$\begin{aligned} \hat{C}_N \equiv \frac{dC_N}{C_N} &= \frac{C_{NN}N}{C_N}\hat{N} + \frac{C_{NT}T}{C_N}\hat{T} \\ &= \frac{C_{NT}T}{C_N}\left(\hat{T} - \hat{N}\right) \end{aligned} \tag{8}$$

and, in similar fashion,

$$\hat{C}_T = -\frac{C_{NT}N}{C_T}\left(\hat{T} - \hat{N}\right). \tag{9}$$

[5]For a recent discussion of the relationship between relative risk aversion and the elasticity of substitution see Mann(1991).

Combining these:

$$\hat{C}_N - \hat{C}_T = C_{NT} \left(\frac{T}{C_N} + \frac{N}{C_T} \right) (\hat{T} - \hat{N}) \tag{10}$$

$$= C_{NT} \left(\frac{C}{C_N C_T} \right) (\hat{T} - \hat{N})$$

by Euler's Theorem. Thus we see by (5) that

$$\omega = \frac{C_{NT} C}{C_N C_T} \, . \tag{11}$$

A description of an individual's attitude towards risk requires both parameters, ρ and ω. The earlier literature in a multi-good environment has shown how individual attitudes towards risk depend upon ρ, but in so doing has imposed comparability in the value of ω by assuming that individuals share a common pattern (but not numbering) of indifference curves.[6] It is precisely the concern over composition, as reflected in ω, that becomes important both for the portfolio decision discussed in the following section and for the determination of the risk premium in section 4.[7]

Since it is often assumed that risk in composition can be completely avoided by trading on open markets, only the question of avoiding risk in the level of income usually arises. However, on international markets the selection of the composition of the consumption bundle may be severely restricted since goods are asymmetric in the extent to which they can be traded. We will see that as a consequence of this asymmetry, both measures of risk aversion will be involved in determining the use to which international asset markets are put: to reduce uncertainty in aggregate consumption or, perhaps, to reduce uncertainty in the composition of consumption.

[6] See Kihlstrom and Mirman (1974, 1981).

[7] The nature of ordinal preferences is allowed to differ in Stiglitz (1969). Relative prices are assumed fixed, however, and agents are permitted to exchange goods at those prices so that all uncertainty in composition can be avoided.

3. A Model with Asymmetric Trade

Consider a two-country world where each country is stochastically endowed with the same two goods, N and T. Home and foreign agents derive utility over both goods and share identical preferences. Ex ante, wealth is equal in the two countries since the expected endowments are the same, although the realizations are less than perfectly correlated. This generates trade in state-contingent commodities between the home and foreign risk-averse representative agents. If the supply of each good is perfectly negatively correlated at home and abroad, the world supply of each is constant and all variation in consumption could be eliminated if both goods could be freely traded. Although such perfect negative correlation is not to be expected, it is a useful assumption to make to highlight the effects of trade restrictions on the behavior of risk-averse individuals who in the absence of such restrictions could avoid all risk.

Suppose there are two equally probable states of the world. In state 1 the home country receives $\bar{N}_1$ and the foreign country receives $\bar{N}_1^*$, where $\bar{N}_1$ is less than $\bar{N}_1^*$. In state 2 the home country receives the larger endowment, $\bar{N}_2$ equal to $\bar{N}_1^*$, while the foreign country receives the smaller endowment, $\bar{N}_2^*$ equal to $\bar{N}_1$. The expected endowment of N equals $\bar{N}$ in both countries. These endowments are illustrated in figure 2. The endowments of T are also perfectly negatively correlated with the same expected endowment, $\bar{T}$, at home and abroad, so that ex ante (expected) wealth is the same. If T can always be traded across states, and endowments are perfectly negatively correlated, the pattern of the endowments of T does not influence consumption, although it of course affects the pattern of asset trade. The value of T used as a point of comparison is the expected value per country, $\bar{T}$.

If both goods could freely be traded, trade in contingent claims would enable each individual to consume one-half the world endowment of each good in every state of nature. This would be an example of the perfectly-pooled equilibrium studied by Lucas(1982). Both home and foreign agents would attain that level of utility given by an indifference curve passing through the common consumption point, E.[8] When N is a non-traded good, however, only T can be traded to pool risk; the consumption of N must equal the endowment of N in each state.[9] Claims to T can be traded to allow the home state-1 consumption point to move vertically along the $\bar{N}_1$ constraint line, while in state 2 the traded good can be adjusted to permit consumption along the $\bar{N}_2$ constraint line. Therefore, although asset markets are complete in that claims to all goods in all states of nature can be traded, only T can be shifted across countries. All payoffs to claims must eventually occur in units of T and consequently, in equilibrium only claims to T need be traded.

Since world supply and demand for claims to T in states 1 and 2 are equal, and these states are equal probability events, the world price of T in state 1 relative to T in state 2 equals one.[10] That is, while agents can trade in T to pool risk there must be an even exchange of the traded good across states. The possibilities are seen in figure 2 by rotating a line through the equal-division point E, allowing both negative and positive slopes. For any

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[8]Indifference curve not drawn.

[9]The nature of this trade restriction makes the problem studied here fundamentally different from previous applications of multi-dimensional risk aversion (see footnote 1). Earlier work has focused on the consumption-savings decision where the return to saving or the supply of second-period goods is uncertain (see also Sandmo(1970) and Levhari and Srinivasan(1969)). Agents in these models choose how much to save and thus choose how much of one good (current consumption) to convert into the other (future consumption). In our model the supply of one of the goods is uncertain, but no such exchange is possible; T cannot be exchanged for N. Instead, given the stochastic properties of N the agent chooses the optimal pattern of T across *states*.

[10]World demand is constant across states since preferences are identical and endowments are symmetric.

given line, the points of intersection with the vertical $\bar{N}_1$ and $\bar{N}_2$ constraint lines illustrate a pair of even trades. For example, for points F and G the home country would consume AF more than $\bar{T}$ in state 1 and an equivalent amount (BG) less than $\bar{T}$ in state 2.

To pursue this geometric representation, note that when this trade line is horizontal each agent consumes the same amount of T in both states; we refer to this as the benchmark case. As this line becomes positively sloped, the trades in T reflect an effort to reduce uncertainty in the composition of consumption, at the expense of greater uncertainty in the levels of consumption and utility. When this line becomes negatively sloped, the levels of aggregate consumption and utility become more equal in the two states, while the difference in the composition of the consumption bundles becomes greater. At the extreme there exists a unique negatively-sloped line, FEG, whose intersections lie on a common indifference curve, $\tilde{U}$. Thus, trade in claims to T alone would enable agents completely to eliminate risk in aggregate consumption and utility. Although all such risk would be removed, the level of utility would still fall short of that obtainable if perfect pooling in both goods were possible, a reflection of the strict concavity of the utility function.

The consumer's maximization problem reveals the importance of the two measures of risk aversion discussed in section 2. Agents at home and abroad will exchange claims to the traded good to maximize expected utility, and the pattern of asset trade will depend directly on the comparison between these coefficients $-\rho$, which reflects aversion to risk in the level of aggregate consumption, and ω, which reflects aversion to risk in the composition of consumption. The home agent chooses consumption of the traded good in states 1 and 2 to maximize expected utility. Since the probability of each state is one-half, we can express expected utility as:

$$E[U(C)] = \tfrac{1}{2}\Big\{U\big[C(N_1,T_1)\big] + U\big[C(N_2,T_2)\big]\Big\} \tag{12}$$

The entire endowments of N_1 and N_2 must be consumed, so $N_1 = \bar{N}_1$ and $N_2 = \bar{N}_2$, and expected utility depends on the choice of T_1:[11,12]

$$\frac{dE[U(C)]}{dT_1} = \tfrac{1}{2}\ \big[U'(1)C_T(1) - U'(2)C_T(2)\big] \tag{13}$$

The home agent's decision to increase or decrease consumption of the traded good in state 1 (relative to $\bar{T}$), and either to smooth utility across states or to increase its variance, depends on the sign of the term in brackets. Let us consider the two cases illustrated in figure 2: (i) Agents trade T to equalize composition across states, allowing the home agent to consume at point H in state 1, while in state 2 the home agent would consume at point D, and vice versa for the foreign agent. (ii) In the second extreme case individuals trade claims to T to equalize aggregate consumption and utility across states. That is, the home agent would consume at points F and G in states 1 and 2, respectively. Treating these cases in more detail:

(i) We can evaluate equation (13) at points H and D where proportions consumed are kept the same, so that $C_T(1)$ equals $C_T(2)$ although U(1) is less than U(2). The curvature of the utility function ensures that $U'(1) > U'(2)$ so that

$$\frac{dE[U(C)]}{dT_1} > 0 \ .$$

Therefore, expected utility would be raised by an increase in consumption of the traded good in state 1 and a decrease in consumption of T in state 2. In figure 2 the consumption points would shift upwards from point H and downwards

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[11] Since the relative price of T_1 in terms of T_2 equals one (by the symmetry conditions), we use the budget constraint to replace T_2 by $(\bar{T}_1 + \bar{T}_2) - T_1$.

[12] Utility over aggregate consumption in state i is abbreviated as U(i), and aggregate consumption over traded and non-traded goods in state i is abbreviated as C(i) where i= 1,2.

from D and towards more equal utility across states. This brings consumption levels closer together at the expense of creating uncertainty in the composition of consumption.

(ii) The levels of aggregate consumption and utility in states 1 and 2 are equalized in the second case through appropriate trades in T. Given endowments of the non-traded good, trades in T place consumption at points F and G in figure 2. Since the levels of consumption and, thus, utility are the same in the two states, U'(1) equals U'(2). However, equalization of the levels of consumption is achieved by creating differences in the composition of consumption across states. In particular,

$$\frac{T_1}{N_1} > \frac{T_2}{N_2},$$

which implies that $C_T(1) < C_T(2)$. Consequently,

$$\frac{dE[U(C)]}{dT_1} < 0.$$

Expected utility can be raised by moving away from perfect smoothing of the level of utility across states. The individual reduces consumption of the traded good in state 1 below F, which makes consumption levels less equal, in order to make the composition of consumption more equal.

The analysis of the maximization of expected utility from an initial position either of equal aggregate consumption or of equal composition of consumption reveals that in general neither extreme would be chosen by a utility-maximizing individual. Now consider the situation where agents are perfectly pooled in the traded good. When $T_1 = T_2$ the sign of $dE[U(C)]/dT_1$ is less obvious, and depends on the comparison of $U'(1) \cdot C_T(1)$ with $U'(2) \cdot C_T(2)$. On the one hand, aggregate consumption in state 1, C_1, is less than aggregate consumption in state 2, C_2, so that $U'(1) > U'(2)$. On the other hand, $T_1 = T_2$ implies that

$$\frac{T_1}{N_1} > \frac{T_2}{N_2},$$

so that $C_T(1) < C_T(2)$. As a result, this argument still leaves the optimal trades in T ambiguous.

The key to resolving this ambiguity is to ask how the expression $U'(C)\cdot C_T$ changes from an initial point at which consumption of both commodities is equated across the two states (point E in figure 2) to a state in which N_1 and N_2 differ. From point E,

$$\frac{\partial \{U'[C(N,T)]\cdot C_T(N,T)\}}{\partial N} = U'\cdot C_{TN} + C_T\cdot U''\cdot C_N = \frac{U'\cdot C_T\cdot C_N}{C}\cdot(\omega - \rho). \qquad (14)$$

This term is positive if and only if ω is greater than ρ. If the ranking of ω and ρ does not change with the move to N_1 and N_2, the condition $\omega > \rho$ suffices to establish that

$$U'(2)\cdot C_T(N_2,T) > U'(1)\cdot C_T(N_1,T) ,$$

in which case the perfect pooling of the traded good is not optimal. Instead, T_1 should be lowered and T_2 should be raised by a comparable amount. That is, if individuals are more averse to risk in composition than to risk in consumption levels ($\omega>\rho$), expected utility is only maximized by allowing a greater difference in aggregate consumption levels between states than would be generated by perfect pooling of the traded good. Consequently, in a world in which some goods are not traded and when there is uncertainty in the supply of these goods, there is no a priori reason to expect individuals to use the traded good to smooth the level of aggregate consumption over states.[13,14]

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[13]In their analysis of the transmission of monetary disturbances across countries, Svensson and van Wijnbergen (1989) compare the intertemporal elasticity of substitution with the intratemporal elasticity of substitution; if our model were cast in a multiperiod setting with time-separable preferences, our analysis of ω and ρ would correspond to their discussions of elasticities.

[14]In a recent paper, Michael Pakko finds that the ranking of aversion to risk in the level of aggregate consumption and in the composition of consumption also determines the optimal risk-sharing arrangements when the asymmetry lies not in the tradability of goods, but in the taste patterns of the two countries.

Special Cases

Further insight into the distinction between aversion to risk in the level of consumption and aversion to risk in the composition of consumption can be obtained by considering three possibilities for the consumption function: the extremes of fixed coefficients and linear indifference curves and the special intermediate case of a Cobb-Douglas consumption function. We start with the latter.

Cobb-Douglas Consumption Function

The Cobb-Douglas consumption function exhibits the property of unitary elasticity of substitution along the indifference curve. Recall the definition of ω given in (5). The inverse of ω is the elasticity of substitution between traded and non-traded goods, σ, and each equals unity in the Cobb-Douglas case. With this simplification we can focus on the properties of the utility function in order to determine whether or not an agent would trade to smooth utility.

Consider a very simple CES utility function,

$$U(C) = \alpha C^n , \tag{15}$$

where α is a constant and n is less than one for concavity. The coefficient of relative aversion to risk in the level of aggregate consumption, ρ, is (1-n). When n lies between zero and one, ρ is less than one. Thus, ω exceeds ρ so that utility maximization via asset trade — allowing agents to shift traded goods over states — results in a greater degree of uncertainty in the level of utility than in the case of perfect pooling of the traded good.[15]

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[15]When n is less than zero the utility function must be modified to ensure that utility rises with consumption. We can write $U = A - \alpha C^n$ where A is a positive constant greater than α and C is constrained to values greater than one. With n negative, ρ is greater than unity and thus exceeds ω in the Cobb-Douglas case; levels of utility become less uncertain than in the perfect-pooling case.

Alternatively, in the case of logarithmic utility where

$$U(C) = \alpha \, log \; C \, , \tag{16}$$

ρ equals one. Thus, if the consumption function is Cobb-Douglas, ρ equals ω and the aversion to risk in levels just offsets the aversion to risk in composition. Given these attitudes towards risk, agents choose to eliminate uncertainty in the consumption of the traded good regardless of the pattern of consumption in the non-traded good. This corresponds to a utility function that is separable in traded and non-traded goods.

Fixed Coefficients

The case of fixed coefficients in the consumption function demonstrates most forcefully the role of aversion to risk in composition. Figure 3 illustrates the case where agents' preferred consumption ratio is $\bar{T}/\bar{N}$. Initially, consider the trades in claims to T that eliminate uncertainty in the level of aggregate consumption and utility. Across states utility is equalized when consumption in state 1 occurs at point F in figure 3 and in state 2 at point G. While uncertainty in the level of utility is eliminated, it is clear that by reducing consumption of the traded good in state 1 and increasing it in state 2, expected utility will be raised. A continual rotation of the trade line counterclockwise around point E reveals that optimal consumption of T across states occurs at points H and D. By moving consumption in state 1 from point F to H, the level of utility in state 1 remains unchanged while the shift from G to D raises utility in state 2. This extreme specification of the consumption function drives agents to eliminate uncertainty in the composition of consumption regardless of the degree of aversion to risk in the level of aggregate consumption.

Linear Indifference Curves

At the opposite extreme, linear indifference curves reflect the absence of aversion to risk in the composition of consumption, ω=0. In this case if individuals are at all averse to risk in the level of aggregate consumption, claims to T will be traded until aggregate consumption and utility are equalized across states. In figure 2 we can imagine a family of straight-line indifference curves. Consumption and utility are equalized across states and expected utility is maximized when T is traded until consumption points in states 1 and 2 lie on the indifference curve that passes through point E. This corresponds to the level of utility that the individuals receive when both T and N are traded on commodity markets, since the absence of aversion to risk in composition eliminates the effect on expected utility of the constraint on commodity trade.

4. The Risk Premium with Traded and Non-traded Goods

Attitudes towards composition may create incentives for countries to arrange trades that will increase, rather than decrease, uncertainty in the aggregate. These compositional concerns may also alter the risk premium that agents would be willing to pay to avoid risk. To see this, we contrast the risk premium that arises in two environments that are distinguished by the presence or absence of compositional uncertainty.[16] The comparison of these premia, moreover, will reveal the optimal pattern of traded goods consumption that countries would arrange using international asset markets.

The Arrow-Pratt measure of the risk premium is derived in the context of a one-good world and captures the amount an individual is willing to pay to

[16]Initially, we postulate stochastic allocations of T and N that cannot be altered.

receive expected utility with certainty. Exactly the same expression obtains in our two-good world if T and N were perfectly positively correlated. Let N represent consumption and assume (i) that N has the same stochastic properties as in section 3, and (ii) that T always varies directly with N so that $\bar{N}_1=\bar{N}-\varepsilon$ and $\bar{T}_1=\bar{T}-\varepsilon$, while $\bar{N}_2=\bar{N}+\varepsilon$ and $\bar{T}_2=\bar{T}+\varepsilon$.[17] Then the risk premium, p, is implicitly defined by

$$U(\bar{N} - p) \quad = \quad E\{U(N)\} \quad = \tfrac{1}{2}U(\bar{N}-\varepsilon) + \tfrac{1}{2}U(\bar{N}+\varepsilon) \tag{17}$$

where $\bar{N}$ is the expected value of the random variable N. For small risks the premium is approximated precisely as in a one-good model so that the following familiar expressions appear:

$$p \;\cong\; \tfrac{1}{2}\left[\frac{-U''}{U'}\right]\varepsilon^2 \quad = \quad \tfrac{1}{2}\left[\frac{-U''\bar{N}}{U'}\right]\frac{\varepsilon^2}{\bar{N}} \quad . \tag{18}$$

A proportionate measure of the risk premium is obtained by expressing p relative to $\bar{N}$:[18]

$$r_1 \equiv \frac{p}{\bar{N}} \quad \cong \quad \tfrac{1}{2}\left[\frac{-U''\bar{N}}{U'}\right]\left[\frac{\varepsilon}{\bar{N}}\right]^2 \tag{19}$$

Since N, T, and C all vary by the same amount, the term in the first set of brackets is equivalent to the coefficient of relative aversion to risk in the level of consumption, ρ. This expression indicates that greater uncertainty (larger ε) raises the risk premium by an amount that is proportional to ρ.

Now consider the premium an individual would be willing to pay if N varied as before, but T were constant across states. Utility is then expressed as a function of aggregate consumption, where aggregate consumption becomes a function of the two goods, N and T, and retains those properties described in section 2 (eq.(1)). In this case the risk premium is implicitly defined by

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[17]Since the variation in aggregate consumption is exactly the same as in N or T, we could derive the risk premium using T or N as proxies, or C itself.

[18]Expressing p relative to $\bar{N}$ frees the discussion of the size of risk and risk premia from any particular units. Instead, one can consider risk and risk premia in percentages.

$$U[C(\bar{N}-p,\bar{T})] \;=\; E\{U[C(N,\bar{T})]\} \;=\; \tfrac{1}{2}U[C(\bar{N}-\varepsilon,\bar{T})] + \tfrac{1}{2}U[C(\bar{N}+\varepsilon,\bar{T})] \;. \qquad (20)$$

The risk premium when T is constant becomes a function of $\bar{N}$, $\bar{T}$, and the level of risk in N, ε. This function can be approximated using a second-order Taylor-series expansion evaluated at $\bar{N}$, $\bar{T}$, and ε=0:

$$p(\bar{N},\bar{T},\varepsilon) \;\cong\; p(\bar{N},\bar{T},0) \;+\; \left.\frac{\partial p(\bar{N},\bar{T},\varepsilon)}{\partial\varepsilon}\right|_{\varepsilon=0}\cdot\varepsilon \;+\; \tfrac{1}{2}\cdot\left.\frac{\partial^2 p(\bar{N},\bar{T},\varepsilon)}{\partial\varepsilon^2}\right|_{\varepsilon=0}\cdot\varepsilon^2 \qquad (21)$$

To simplify this expression for the risk premium we first note that $p(\bar{N},\bar{T},0) = 0$. Then we differentiate (20) with respect to ε to find

$$-U'[C(\bar{N}-p,\bar{T})]\cdot C_N(\bar{N}-p,\bar{T})\cdot\frac{\partial p(\bar{N},\bar{T},\varepsilon)}{\partial\varepsilon}$$
$$= \tfrac{1}{2}\{-U'[C(\bar{N}-\varepsilon,\bar{T})]\cdot C_N(\bar{N}-\varepsilon,\bar{T}) \;+\; U'[C(\bar{N}+\varepsilon,\bar{T})]\cdot C_N(\bar{N}+\varepsilon,\bar{T})\} \;. \qquad (22)$$

The right hand side of (22) goes to zero in the limit as ε goes to zero, so that the second term in (21) also vanishes. This leaves the second-order effects which are found by differentiating (22):

$$-U'[C(\bar{N}-p,\bar{T})]\cdot C_N(\bar{N}-p,\bar{T})\cdot\left.\frac{\partial^2 p(\bar{N},\bar{T},\varepsilon)}{\partial\varepsilon^2}\right|_{\varepsilon=0} \;=\; U''[C(\bar{N},\bar{T})]\cdot C_N^2 \;+\; U'[C(\bar{N},\bar{T})]\cdot C_{NN} \qquad (23)$$

Given the solutions for ρ and ω provided in equations (3) and (11), respectively, these second-order effects can be simplified as:

$$\frac{\partial^2 p}{\partial\varepsilon^2}\cdot\bar{N} \;=\; \Theta_N\cdot\rho \;+\; \Theta_T\cdot\omega \;\equiv\; \gamma \quad , \qquad (24)$$

which is a weighted average of the two types of risk aversion where the weights are the expected consumption shares, $\Theta_N = C_N\bar{N}/C$ and $\Theta_T = C_T\bar{T}/C$. Inserting this solution into the Taylor-series expansion and expressing the risk premium relative to $\bar{N}$ we find

$$r_2 \equiv \frac{p(\bar{N},\bar{T},\varepsilon)}{\bar{N}} \;\cong\; \tfrac{1}{2}\left[\gamma\right]\left[\frac{\varepsilon}{\bar{N}}\right]^2 \;. \qquad (25)$$

The decomposition of risk aversion that was found useful for the portfolio decision also serves to reveal the underlying forces involved in the determination of the risk premium. Uncertainty in N by itself introduces two types of risk that are of relevance to risk aversion: (i) Uncertainty in N creates uncertainty in aggregate consumption and, through this effect, serves to lower expected utility. The importance of this effect depends both on the weight which N represents in aggregate consumption, Θ_N, and on the measure of relative aversion to risk in income levels, ρ. (ii) Uncertainty in N creates uncertainty in the proportion of T relative to N, altering the composition of consumption, thereby bringing the ω coefficient of relative aversion to risk in composition into play. The weight this term receives is the share of T in aggregate consumption, Θ_T. The expression for γ in equation (24) brings these two components of risk aversion together.

The comparison of the risk premium in these two environments reveals the significance of aversion to risk in composition and indicates the optimal behavior of T across states. In contrasting these risk premia, it must be recognized that while the variance in N is the same in the two cases the variance in aggregate consumption is significantly lower when T is constant across states. Let us assume that the relative aversion to risk in levels, ρ, takes on the same value for the representative individual in either environment. One might conclude that given the lower variance in C the risk premium in the latter environment would be lower than in the case where T and N move together. Although the risk in real income is lower, however, risk in composition has been introduced. We can see by the solution for r_2 that the relative magnitude of the two premia will depend on the ranking of ω and ρ. If the individual does not care about composition (ω=0), then clearly the risk premium ought to be smaller since uncertainty in real income is lower. Even if the agent cares *less* about risk in composition than in aggregate consumption ($\omega<\rho$), the premium in the latter case will be lower. If, however, the individual cares more about composition than about the level of real income, then more would be paid to avoid the situation wherein composition is uncertain even though uncertainty in the level of real income is lower. Thus,

r_2 exceeds r_1 when ω is greater than ρ.

This ranking of premia signals the type of uncertainty that is of lesser importance to the agent, and thereby indicates the use to which asset markets would be put. When the asymmetry between T and N is present (i.e., the supply of N is uncertain, but the supply of T is not), international asset trade becomes a way to convert T from a safe good into a risky good and, in particular, enables countries to *create* the desired pattern of risk in T. This pattern depends upon the ranking of r_1 and r_2. Thus, trades in claims to T would be arranged to create a *negative* covariance between T and N if r_2 is less than r_1, since this ranking reflects a greater desire to reduce uncertainty in aggregate consumption even at the expense of greater uncertainty in composition. On the other hand, asset markets would be used to create a *positive* covariance between T and N if r_2 exceeds r_1, which reveals a greater concern towards risk in composition. In the knife-edge case where the two components of risk aversion are just equal, the risk premium will be the same in both environments. This indicates that the effect of the reduction in the variance in C (when T is constant) is just offset by the addition of variance in composition and, as a consequence, there is nothing to be gained by creating variance in T. It is clear by this discussion that the difference between the risk premium derived when T and N move together - a situation that mimics a one-good world - and that derived with explicit recognition of uncertainty in the composition of consumption reflects the comparison between the two components of risk aversion and reveals the nature of risk one would prefer to have embodied in T.

5. Concluding Remarks

Our examination of the nature of risk aversion that characterizes a standard two-good, strictly concave utility function reveals the underlying forces that determine the use of international asset markets when some goods cannot be exchanged in international trade. In the absence of such asymmetries in the extent to which goods may be traded on international markets, we know that purely idiosyncratic risk can be eliminated if asset markets are complete. Even when claims to all goods in all states of the world can be exchanged on international asset markets, however, restrictions on the physical exchange of goods force the individual to make a tradeoff between reducing uncertainty in the composition of consumption or reducing uncertainty in the level of aggregate consumption.

Attitudes towards these two types of risk are captured by the coefficient of relative aversion to risk in the level of aggregate consumption and by the coefficient of relative aversion to risk in the composition of aggregate consumption. We find that a direct comparison of the magnitude of these measures of risk aversion determines whether agents deviate from perfect pooling in the traded good and in which direction. If the aversion to risk in composition exceeds the aversion to risk in levels, then agents' use of asset markets, in their efforts to maximize expected utility, will generate greater variance in the level of aggregate consumption over states.

The influence of aversion to risk in composition also appears in the determination of the risk premium, and its ommission would result in a misspecification of the level of the premium. The comparison of the risk premium, when composition as well as aggregate consumption is uncertain, with that derived under the assumption that all goods move precisely together reflects the relative importance of aversion to risk in composition. The difference between these two measures of the risk premium reveals the use to which international trade in contingent claims could be put to create the desired type of riskiness in the traded good. Rather than reducing uncertainty, it may very well be the case that international asset trade will create additional uncertainty in consumption, but in so doing would raise welfare.

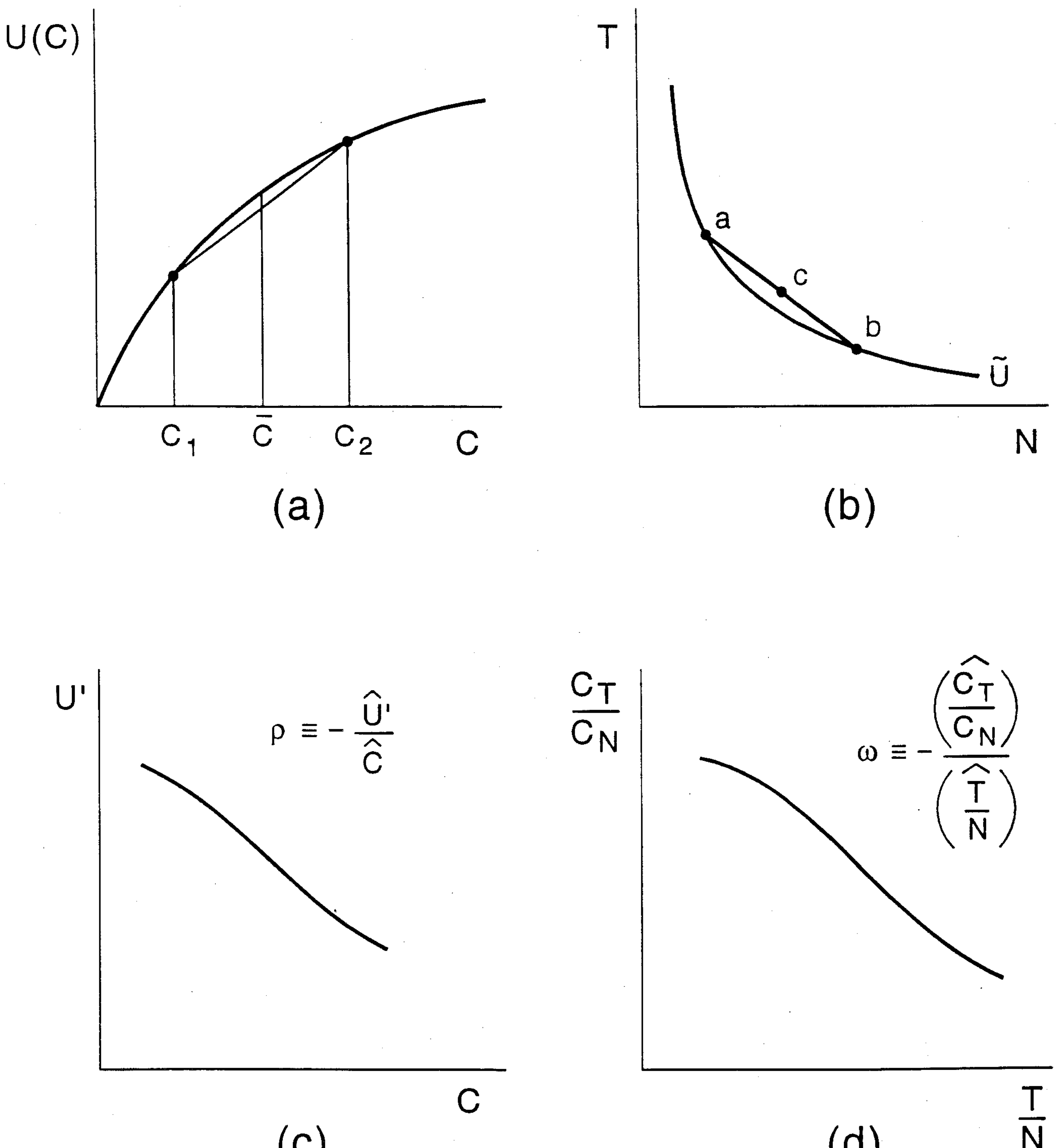

Figure 1

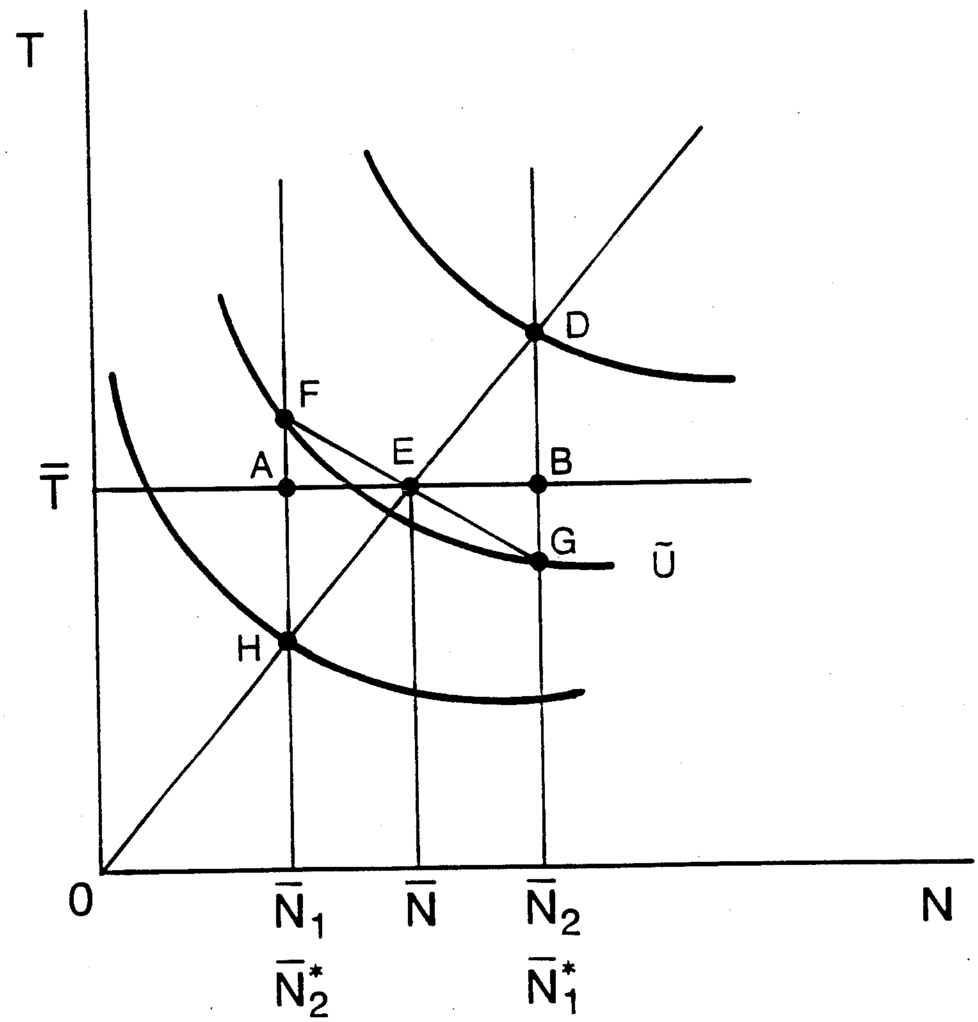

Figure 2

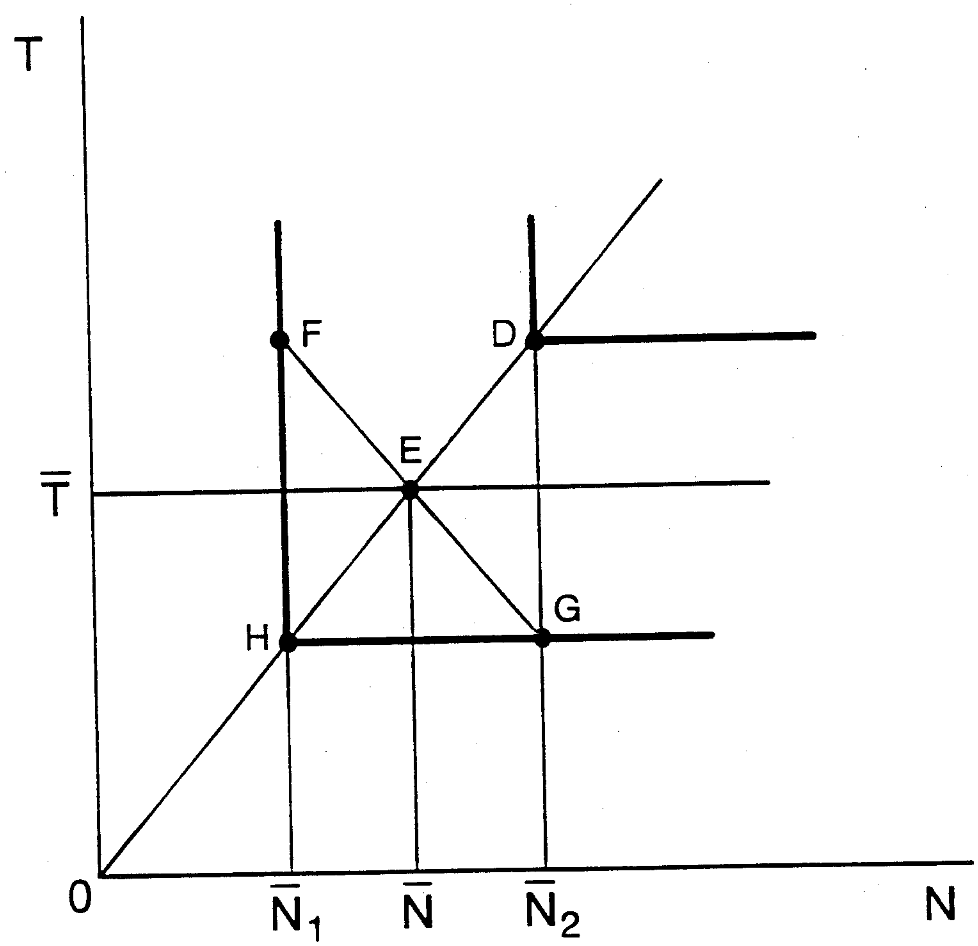

Figure 3

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