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Dutta, Prajit K. and Ananth Madhavan

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Abstract

This paper examines the process by which private information is impounded in security prices in a market where some traders are consistently better informed about short-run asset values. We analyze equilibria where traders can adopt general non-cooperative strategies, and characterize the 'best' attainable equilibrium. Trading strategies in this equilibrium are nonlinear functions of information signals and exhibit history-dependence. In contrast to previous papers, competition among informed traders need not lead to rapid revelation of private information. Although the volatility of fundamental prices is constant, transaction prices may exhibit time-varying volatility. Finally, in contrast to linear models, the degree to which prices impound private information depends on the signal itself.

1 Introduction

A fundamental question in financial economics concerns the process by which traders' private information is impounded in security prices. The literature on rational expectations emphasizes the static aspect of the problem, namely how less informed agents infer the nature of private information from public signals in a single transaction.[1] This paper is a contribution to the recent literature which focuses on the dynamic process by which traders with private information exploit their informational advantage.

An influential paper by Kyle (1985) develops an intertemporal model of a monopolist insider who trades sequentially in a market against traders without private information. Kyle's major result is that the insider trades slowly, so that prices gradually reflect private information. More recently, papers by Foster and Vishwanathan (1991) and Holden and Subrahmanyam (1992) have generalized Kyle's (1985) model to incorporate competition among multiple insiders with long-lived private information.

The results of Foster and Vishwanathan (1991) and Holden and Subrahmanyam (1992) provide a sharp contrast to Kyle (1985); in the unique linear equilibrium, competition among even a few insiders leads to aggressive trading and causes private information to be rapidly incorporated into the security's price. As a group, the insiders would be better off if they could restrict their trades to collectively trade the monopolist volume. However, restricted trading is not incentive compatible because traders could increase their own expected profits, at others' expense, by trading larger volumes. In equilibrium, the volume of informed trading is much larger than the volume traded by a single trader, and the insiders' private information is rapidly impounded into prices.

The above results have important implications for public policy; they suggest that prices in securities markets closely approximate the idealized notion of strong-form efficiency when multiple insiders are present. Indeed, recent empirical evidence indicates that episodes of insider trading are generally associated with multiple insiders. For ex-

[1]See, for example, Grossman (1976), Grossman and Stiglitz (1980), Hellwig (1980), Diamond and Verrecchia (1981), and Admati (1985) for competitive static models and Kyle (1989) and Laffont and Maskin (1990) for imperfectly competitive static models.

ample, Meulbroek (1991) examines data on illegal insider trading obtained from the SEC and finds that an episode of insider trading typically involves multiple informed traders. Similarly, a study of insider trading in Anheuser-Busch stock by Cornell and Sirri (1992) found that these episodes involved multiple insiders.

The models of Foster and Vishwanathan (1991) and Holden and Subrahmanyam (1992) are ideally suited to the analysis of sporadic episodes of corporate insider trading based on private information concerning a specific future event. Many security markets, however, are characterized by the continued presence of traders or dealers who repeatedly obtain valuable short-lived private information regarding current asset values. For example, large money-center banks play an important role in making markets in foreign exchange, and by virtue of their predominance in trading and knowledge of customer orders, often possess informational advantages over smaller traders.[2] Similarly, large primary dealers play an important role in the secondary market for U.S. treasury bonds, and may also possess an informational advantage over smaller traders. In such markets, it is likely that trading behavior is influenced by past trades. The equilibrium strategies of traders in such markets may be very different from those of insiders with event-specific, private information.

This paper analyzes a market where imperfectly competitive traders who repeatedly receive short-lived private information signals can adopt very general, non-cooperative, trading strategies, including strategies that are nonlinear or that depend on past trades.[3] In this model, the intuition of Foster and Vishwanathan (1991) and Holden and Subrahmanyam (1992), namely that competition among informed traders leads to faster impounding of private information, still holds if we restrict our attention to linear history-independent strategies. However, in contrast to the previous literature, we demonstrate that traders with private information can support a non-cooperative equilibrium which is arbitrarily 'close' to the solution of Kyle (1985) for a wide range of plausible discount

[2]See, e.g., Lyons (1992) who provides a formal model of the foreign exchange market. Lyons notes that for each of the major currencies, four or five brokers dominate trading.

[3]While our model is one of short-term information, our results generalize to the case of long-lived information.

rates. This equilibrium is a natural object of attention in the sense that traders' strategies yield expected profits which are strictly greater than in all other equilibria, including the unique linear equilibrium. The presence of multiple informed traders need not lead to the rapid dissipation of private information.

The intuition for our result is straightforward. The continued presence of other informed traders creates incentives for each trader to restrict trading because 'large' orders may result in all traders trading more aggressively in the future, thereby lowering expected discounted profits. Each informed trader faces a trade-off between the desire for future profits and the immediate gains from a large trade, much as in repeated-game models of cartel behavior among firms. However, the trading game we consider is a dynamic game, not a repeated game, because the value of private information (measured by the discrepancy between the expectation of the asset's value given public information and the expectation given private information) is stochastic. Thus, restricted trading is not feasible for all information signals. If there is a large difference between the expected value of the security given public information and the actual value of the security, traders will trade aggressively; if prior beliefs are relatively accurate, slower trading strategies are sustainable. Paradoxically, more 'valuable' private information leads to more aggressive trading and greater revelation of private information. By contrast, in the linear equilibrium of Foster and Vishwanathan (1991) and Holden and Subrahmanyam (1992), the fraction of private information revealed in price is a constant irrespective of the magnitude of the information signal.

The nonlinear nature of the strategies considered here lead to complex price behavior. We demonstrate conditions where price volatility is serially correlated even though the volatility of fundamental prices is constant over time. This occurs even though price changes are serially uncorrelated. Periods of high price volatility, associated with restricted informed trading, alternate with periods of low price volatility, associated with more aggressive and competitive informed trading. These stochastic changes in price volatility are endogenous and are unrelated to the volatility of the underlying asset, which is a constant in our model. These results provide a justification for the use of Generalized Autoregressive Conditional Heteroskedasticity (GARCH) models for high frequency transaction data. For example, a recent study by Baillie and Bollerslev (1990) finds strong evidence of short-lived volatility patterns in foreign exchange markets using a GARCH model. They note that this evidence is consistent with more aggressive trading by informed traders at certain points in the day, as is the case here.[4]

We extend the model to consider the case where the informed traders trade more than one security. This extension is of interest because a trader's private information may be of relevance for several, closely related, assets. For example, large bond dealers typically trade bonds with different coupons, maturities, and risks, and foreign exchange dealers usually trade in several currencies. We show that our results are strengthened when traders trade in multiple assets. Intuitively, traders engage in a form of cross-subsidization across markets that enables them to sustain restricted trading in all markets.

The rest of the paper proceeds as follows: In Section 2 we establish the theoretical framework and describe the Markov equilibrium. In Section 3 we consider the case where informed traders can use general trading strategies. Section 5 contains a discussion of these results and their implications for empirical analysis. In Section 4 we discuss the effect of trading in multiple assets. Finally, Section 6 summarizes the paper. Proofs are contained in the appendix.

2 The Trading Model

2.1 The Structure of Trading

A risky security is traded in a series of sequential auctions taking place at dates $t = 1, 2, \ldots,$. After trading in auction t is complete, the security pays a dividend, denoted by d_t. A group of traders, referred to as *informed traders*, observe the dividend to be realized at the end of the trading period before trading takes place. The dividend d_t is observed by all other traders just after trading in auction t is complete. There are $N \geq 1$ informed

[4]This is also consistent with the trade concentration hypothesis of Admati and Pfleiderer (1988).

traders, indexed by $i = 1, \ldots, N$. Let q_{it} represent the order quantity of informed trader i in auction t with the convention that $q > 0$ denotes a trader purchase and $q < 0$ a trader sale. Informed traders are assumed to be risk-neutral.

In addition to informed traders, the market also contains other agents, referred to as *uninformed traders*, who have no private information. There are two types of uninformed traders in our model. The first group, whom we refer to as *liquidity or noise traders*, trade for exogenous, liquidity-motivated reasons. Let x_t denote the aggregate noise trade in auction t, where $x > 0$ ($x < 0$) denotes net purchase (sale) as before. We assume that x_t is serially uncorrelated and is symmetrically distributed with zero mean and variance σ_x^2. The second group of uninformed traders is a competitive fringe of speculative traders or market makers. These traders act as providers of liquidity, by absorbing the net order flow of informed and noise traders at competitive prices.

Let v_t be the expected present value of dividends given information about the impending dividend payment d_t. We assume for simplicity that $E[\tilde{d}_{t+1}|d_t] = d_t$, i.e., that dividends follow a random walk with zero drift. Then, it follows that $v_t \equiv d_t/(1-\beta)$, where β is the common discount factor of all traders. Under our assumption for the dividend process, the change in fundamental values, denoted by $\eta_t \equiv v_t - v_{t-1}$, is a random variable with mean 0 and variance denoted by σ_η^2. It is worth noting for the future that η_t has an interpretation as a measure of the imprecision in public information in period t, because the best public estimate of v_t prior to trading is v_{t-1}. Thus, if η_t is large in absolute value, prior public beliefs are inaccurate in period t.

The trading protocol follows Kyle (1985). Denote by p_t the security's price in auction t. As discussed above, this price is determined in an auction market by competitive market makers who absorb the net order flow at that time. The price in market t is determined by the net order flow in that market, $\sum_i q_{it} + x_t$, as well as the market makers' expectation of the present value of dividends, given their information prior to trading, v_{t-1}. We assume that the price function has a linear structure: $p_t = v_{t-1} + \lambda\left[\sum_i q_{it} + x_t\right]$, $\lambda > 0$. Note that the best estimate of v_t given public information at time t is v_{t-1} because η_t has mean zero.

A net buy (sell) imbalance indicates, to competitive market makers (or equivalently, to a fringe of speculators or liquidity providers) that the security is underpriced (overpriced) relative to prior beliefs, i.e., that $\eta_t > 0$ ($\eta_t < 0$), leading them to raise (lower) the price from their expected valuation of the security.

The price set by rational competitive market makers or speculators takes the form $p_t = v_{t-1} + E[\eta_t | \sum_i q_{it} + x_t]$. We restrict market makers to a linear price adjustment rule for analytical simplicity. However, as we show in detail below, a linear price adjustment rule provides the correct conditional expectation of value in a Kyle-market if market makers believe there is a single informed trader. We show that such a rule is "almost-optimal" in our model in the sense that $E[v_t] = v_{t-1} + \lambda [\sum_i q_{it} + x_t] + \epsilon$, where ϵ is arbitrarily small for reasonable choices of discount rates.

After trading in the t^{th} auction is complete, the dividend, d_t, is paid, informed traders observe the value of d_{t+1}, market makers update their price quotation schedule, and the trading process in period $t+1$ begins afresh. In period t, trader i, whose share inventory is I_{it} has expected cash flows of $d_t(I_{it} + q_{it}) - E[p_t q_{it}]$. Inventory adjusts according to the transition rule $I_{i,t+1} = I_{it} + q_{it}$. There are no short sale restrictions or capital constraints, so I_{it} can be negative.

2.2 Trading Strategies and Inventory

A *strategy* for an informed trader is a decision rule to select the order quantity q_{it} as a function of the past trades, past dividends, inventory levels, and the current expected value of the security, v_t. The trader is risk neutral and chooses a trading strategy to maximize the discounted sum of trading profits, given the current state. The triplet (η_t, v_t, I_t) constitutes the state variable in our problem; the state variable summarizes the relevant information required to describe the future evolution of the system. Omitting the subscript i, the discounted expected payoffs, for any strategy, can be written as $d_t(I_t + q_t) - E[p_t q_t] + \beta E[V(\eta_{t+1}, v_{t+1}, I_{t+1}; q_t)]$, where the last term is the expected payoff from future trades. Note that the expected payoff from future trades may depend on the current trade, i.e., the trading strategy may be history-dependent.

A vector of strategies forms a *Nash equilibrium* in the first auction if each player's strategy maximizes expected discounted profits conditional upon the strategies of all other players being correctly conjectured. The vector of strategies is *subgame perfect* if the continuation strategies, given any history of trades and dividends, is a Nash equilibrium at every auction. A subgame perfect equilibrium is a *Markov Perfect Equilibrium* if the decision rules are Markov, i.e, if q_t depends only on the current state (η_t, v_t, I_t), and not on past trades.

Conditioning on three possible state variables would greatly complicate the analysis. Intuition suggests that the actual levels of asset value v_t and current inventory I_t affect the value of a particular strategy, but not the equilibrium choices made by the trader. It is the change in asset value implied by the dividend innovation, rather than the actual level of the dividend, that is relevant in selecting the order quantity. Similarly, with unlimited short-sales, share inventory should have no affect on the optimal choice of order quantity. The following lemma formalizes this intuition. More precisely, Lemma 1 establishes that the set of trading strategies that constitute subgame perfect equilibria in time t do not depend on the trader's inventory level or the current value of the asset; the only relevant state variable is the value innovation η_t. Of course, the expected discounted wealth corresponding to the equilibrium strategies depends additively on the value of share inventory.

Lemma 1 *If a particular trading strategy vector constitutes an equilibrium for the state (η_t, v_t, I_t) in period t, then it is an equilibrium strategy vector for the state (η_t, v_t', I_t').*

Lemma 1 allows us to ignore the current level of inventory and the asset value in our subsequent analyses.

2.3 Markov Perfect Equilibria

We begin our analysis of trading by describing the familiar linear equilibrium with Markov trading strategies that has been the focus of the recent literature. In this equilibrium, trader i trades a fixed proportion of the innovation in the asset's fundamental value, η_t.

Proposition 1 *In auction t, there exists a Markov equilibrium where the optimal trading strategies of informed traders are linear functions of the price deviation:*

$$q_{it}^{*}(\eta_t) = \frac{\theta_N}{N}\eta_t \tag{1}$$

where:

$$\theta_N = \frac{N}{\lambda(N+1)}. \tag{2}$$

The equilibrium characterized by Proposition 1 can be shown to be the unique linear Markov equilibrium. The linear equilibrium forms an important benchmark for our later analyses, and hereafter we will refer to it as the *Markov Perfect Equilibrium* (MPE). The price function $p_t = v_{t-1} + \lambda_N \left(\sum_{i=1}^{N} q_{it} + x_t\right)$ underlying this equilibrium can be shown to be consistent with rational expectations by market makers. If $\lambda_N = \frac{\sqrt{N}}{N+1}\left(\frac{\sigma_\eta}{\sigma_x}\right)$, and the distributions of x and η are normal, the price is the expected value of the asset. Note that θ_N is the combined trading intensity with N traders, and is an increasing function of N.

A well-known property of the MPE is that the fraction of private information aggregated in prices is a constant which depends only on the number of traders, N.[5] This result holds independently of the choice of λ. The fraction of private information revealed in prices increases rapidly with the number of traders. In the Kyle (1985) model, where $N = 1$, the fraction of private information reflected in prices through trading is 50%; by contrast, when $N = 4$, this fraction rises to 80%.

In the MPE, expected profits are maximized when there is only one informed trader. Competition lowers profits because informed agents reveal a larger fraction of their private information by trading more aggressively. Traders could collectively earn higher profits

[5]The measure of information aggregation is the variance of the asset's value conditional upon order flow divided by the unconditional variance of the asset's value.

by restricting their trades, but no single trader will find it optimal to trade less because the benefits are largely external.

We now consider alternative equilibria based on non-Markov strategies that may support more complex trading behavior.

3 Dynamic Strategies

In the Markov equilibrium strategies examined in the previous section, informed traders systematically ignore any information other than the current innovation η_t in determining the size of their transactions. This assumption is reasonable if the model is interpreted as describing a one-time episode of insider trading, but may not be so reasonable for a market where a group of dealers or traders repeatedly trade with one another. In such a situation, it is natural to examine strategies (non-cooperative, of course) where traders restrict their trades to keep profits high. This behavior could be supported by the threat to trade more aggressively upon a perceived departure from the proposed strategy. We refer to such strategies as *non-Markov strategies.*

For the moment, we assume that any deviations from the equilibrium strategy in auction t are observable at the time of trade in auction $t+1$. In other words, we assume full disclosure *ex post* of trading information.[6] For some market structures, e.g., the primary market for U.S. treasury bonds, this assumption is reasonable. For some markets, however, *ex post* trade information is hard to obtain. We treat this more difficult case in Section 3.3.

3.1 The Optimal Non-Cooperative Equilibrium

As described above, it is quite natural to expand the class of admissible equilibria to allow history-dependent, possibly nonlinear, trading strategies. An immediate problem, of course, is that there may be many such equilibria and we are faced with the task of selecting an economically reasonable one. In this paper, we focus attention on that sub-

[6] If current trades were observable, our conclusions would be strengthened.

game perfect equilibrium whose trading strategies produce the highest expected profits to the privately informed traders among all possible equilibria. We refer to the perfect equilibrium corresponding to these optimal trading strategies as the *Optimal Non-Cooperative Equilibrium (ONCE)*.[7]

In general, it is very difficult to characterize the optimal non-cooperative trading strategies in all but the simplest games because they can theoretically be very complicated functions of past trades and states.[8] Fortunately, we are able to obtain a complete description of the optimal equilibrium for this model. The optimal strategies are intuitive and take a relatively simple form. If the absolute value of the innovation η is below a critical level, say η_1, informed traders collectively trade the monopoly volume. If the absolute innovation is above the critical amount, informed traders trade a quantity strictly between the monopoly volume and the volume under the Markov equilibrium. Moreover, the larger is $|\eta|$, the closer is the ONCE trading volume to the Markov trading volume. Any departure from this strategy by any trader results in all traders switching to the equilibrium strategies associated with the lowest expected discounted profits. It is this threat of switching to the "worst" possible equilibrium that supports restricted trading close to the monopoly level of Kyle (1985) if the value innovation is sufficiently small. Formally:

Proposition 2 *The ONCE trading strategy is fully described by a constant $\eta_1 < \infty$ and a strictly increasing function θ^* (on $\mathcal{R}_+$) such that*

$$q_i(\eta_t) = \begin{cases} \frac{\theta_1}{N}\eta_t & |\eta_t| \leq \eta_1 \\ \frac{\theta^*(|\eta_t|)}{N}\eta_t & |\eta_t| > \eta_1 \end{cases}$$

where $\theta_1 < \theta^*(\cdot) < \theta_N$.

[7] This equilibrium is sometimes referred to as the "second-best" equilibrium; the first-best or Pareto optimal allocation may not be attainable as an equilibrium if agents act non-cooperatively. The optimal non-cooperative equilibrium is a natural object of attention for two reasons: (1) Some evolutionary models show that agents learn to achieve the best incentive-compatible outcomes, (2) Institutions may emerge to exploit arbitrage opportunities if the equilibrium is not optimal.

[8] There are very few papers containing an explicit characterization of the ONCE. See, however, Rotemberg and Saloner (1986) and Abreu (1988).

Upon deviation from this trading rule, traders switch to the equilibrium strategies associated with the lowest expected discounted profits.[9]

The optimal strategy is both nonlinear and history-dependent, but the underlying economics is intuitive. An important paper by Abreu (1988) demonstrates that any equilibrium strategy in a multi-period game can be described in two parts: (1) a *normal trading rule* which informed traders follow provided no trader has deviated in the past, and (2) a *fallback rule* which is followed if some trader has not traded according to the normal trading rule in the past. A Markov equilibrium is characterized by the property that the normal and fallback trading rules are identical, i.e., it ignores trading history. On the other hand, an equilibrium trading strategy which does condition on past trades distinguishes between these rules. The optimal strategy turns out to have exactly this form.

The "normal" trading strategy is depicted in figure 1. The intuition for Proposition 2 is straightforward. An informed trader will deviate from a proposed trading rule of $\frac{\theta}{N}\eta$ only if the immediate increase in expected profits, denoted by $D(\theta, \eta)$, exceeds the future expected loss from the change in other traders' strategies. The key point is that although the expected future loss is *independent* of the current innovation η, the increase in current expected profits, $D(\theta, \eta)$, is not. Indeed, $D(\theta, \eta)$ is in fact increasing in θ as well as $|\eta|$. As a result, if the innovation is sufficiently large, the benefits from deviating from the monopoly intensity θ_1 outweigh the costs in terms of lower future profits. Thus, unlike a repeated game, the monopoly volume cannot be supported independent of the state variable η, as demonstrated by the proposition.

The worst equilibrium trading strategy can also be derived analytically. It has properties which are the exact opposite of the ONCE strategy. For small value innovations, trading is so aggressive that expected profits are negative, whereas for larger innovations, trading resembles the MPE. The trades for small value innovations are sustainable because there are fewer profitable deviations when $|\eta_t|$ is small. Overall, this strategy yields the

[9] If the future is completely discounted, i.e., $\beta = 0$, then the only possible equilibrium is the Markov equilibrium. Hence, for "small" β, η_1 is zero.

lowest expected discounted profits, and are strictly less than the MPE payoffs. Formally, the trading strategy is: trade $\hat{\theta}\eta_t$ when $|\eta_t| < \eta_2$, (where the optimal deviation against $\hat{\theta}$ yields zero profit) and trade $\tilde{\theta}(\eta_t)\eta_t$ when $|\eta_t| > \eta_2$ (where the function $\tilde{\theta}(\cdot)$ is decreasing in η_t and $\hat{\theta} > \tilde{\theta}(\eta_t) > \theta_N$). Moreover, when $|\eta_t| > \eta_2$, the punishment phase lasts exactly one period, whereas for small $|\eta|$, it continues with an expected continuation value in between the payoffs to the best and worst equilibria. Since the derivation and intuition are similar to that of the ONCE, we do not develop these arguments in detail here.

Similar trade-offs also arise in Rotemberg and Saloner (1986), who examine price wars in oligopolies with Bertrand competition and constant marginal costs. They examine a game consisting of a series of static models rather than a dynamic game as we do here.

3.2 An Example

To better understand the nature of Proposition 2, we analyze a trading strategy that approximates the optimal strategy: if $|\eta_t| \leq \bar{\eta}$ then each informed trader trades θ_1/N, so that collectively the traders trade at the single-trader level. If, however, $|\eta_t| > \bar{\eta}$, then traders switch to the linear MPE strategy, and each trader trades θ_N/N. Further, upon any deviation from this strategy in the previous auction, all traders switch to the Markov strategy forever. We refer to such a trading strategy as a *trigger trading strategy.*[10] This strategy is a special limiting case of the ONCE strategy.

We first establish the existence of equilibrium with trigger strategies, and then use our results to further explore the nature of the ONCE strategies.

Proposition 3 *Every trigger trading strategy is an equilibrium trading strategy provided the discount factor β is bounded below by $\bar{\beta}(\bar{\eta})$, where $\bar{\beta}(\bar{\eta}) < 1$ is a constant. If equilibrium exists, then the switch point $\bar{\eta}$ is strictly less than the switch point η_1 under the ONCE strategy. Further, expected profits with trigger strategies are strictly larger than the expected profits in the Markov Perfect Equilibrium.*

[10]There is an extensive literature on the nature of trigger strategies in repeated games, but the known results on trigger strategies have limited applicability here because ours is not a repeated game.

Proposition 3 demonstrates the existence of a trigger strategy equilibrium provided traders are sufficiently patient. This equilibrium closely resembles, but does not necessarily coincide with, the optimal equilibrium and yields higher expected profits than the MPE. The expected profits to the traders increase with the switch point $\bar{\eta}$ because the traders collectively obtain monopoly profits in more states of the world. However, it is easy to show that as the switch point $\bar{\eta}$ increases, the lower bound on the discount factor β rises, so that restricted trading is not possible in all states of the world.

Proposition 3 also implies that the critical value innovation at which trading intensities increase above the monopolist level is at least as large in the optimal equilibrium, i.e., $\eta_1 \geq \bar{\eta}$. This means that under the ONCE strategies, the Kyle (1985) equilibrium *at least as often* as under the trigger strategy. This result provides a way to assess how often trading behavior is as described by the single-period Kyle equilibrium for various discount rates.

Consider the following numerical example based on the strategy above where each trader trades θ_1/N for pricing errors $|\eta| \leq \bar{\eta}$ and θ_N/N otherwise. Suppose that $\sigma_\eta = 0.1$, $N = 7$, and the critical bound for η is ten standard deviations from the mean, i.e., $\bar{\eta} = 10\sigma_\eta$. Suppose that η is distributed normally with mean zero. Hence, the probability of an error sufficiently large to trigger a switch from the monopolist trading intensity is essentially zero. In this unlikely event, trader's adopt the MPE trading strategy. It is straightforward to show numerically that the Kyle (1985) trading strategy is sustainable if traders have discount rates under 0.4427%. This discount rate is very reasonable if the trading horizon is a day or less; if the time between auctions is interpreted as a day, the corresponding annualized rate is over 400%. The upper bound increases rapidly as $\bar{\eta}$ declines. For example, when $\bar{\eta} = 3\sigma_\eta$ (which implies that approximately 99.74% of the time trading resembles the Kyle single-trader equilibrium) the corresponding upper bound on the discount rate applicable to the time between auctions is 4.78%.

This argument shows that for plausible discount rates, irrespective of the values of market depth (i.e., λ) and the level of noise trading, (as measured by σ_x^2), the single trader equilibrium can be sustained for virtually all η values. Our numerical estimates are also relatively insensitive to the choice of N and σ_η. As the number of traders falls, the critical upper bound on the discount rate increases, showing that less patience is required to maintain the slow trading equilibrium. When $N = 4$, the critical discount rate is 0.6838%, corresponding to a discount factor of 0.9932. The strategy is more natural than the linear strategy in the sense that expected profits are strictly greater than under the linear MPE strategy. These profits arise from the fact that less information is aggregated in prices by less aggressive trading. In the example, the fraction of private information revealed in prices is 50%, as opposed to 88% under the linear MPE.

Finally, the example provides a rationale for our assumption that the market maker's price quotation function is linear as in the Kyle equilibrium. In the Kyle model, the conditional expectation of the asset's value is a linear function of order flow for an appropriately chosen value of λ. Hence, market makers price linearly if they believe that multiple informed traders act as a monopolist insider. At low discount rates, this conjecture is essentially correct.

3.3 Equilibrium Without Trade Disclosure

We now relax our assumption that there is complete *ex post* trade disclosure. To focus attention on the effect of trading information, we consider a case where there is no trade reporting, even after execution. As trading is *ex post* anonymous, informed traders are unable to detect departures from the proposed (normal) trading strategy with certainty. For technical reasons, we assume, however, that there is an exogenous chance, possibly extremely remote, that the trade will be revealed to other traders. Formally, with probability $\epsilon > 0$ traders obtain trading information about the previous round of trade. We do not consider the case of $\epsilon = 0$, i.e., the case in which informed traders are completely unable to detect deviation. An equivalent assumption (one for which the intuition is identical) is that trade sizes are a discrete rather than continuous variable (with $\epsilon = 0$).

If traders have been unable to detect deviation, they have to rely on observed prices to form a "guess" about the likelihood of a departure from the proposed (normal) strategy. Motivated by the characterization of the ONCE and the trigger strategy, we consider the following trading strategies: if no deviation was detected and prices in the last auction were "normal," trade $\frac{\theta_1}{N}\eta$ if $|\eta| \leq \bar{\eta}$ and trade $\frac{\theta_N}{N}\eta$ if $|\eta| > \bar{\eta}$. On the other hand, if the price movement at the last auction was "abnormal," trade $\frac{\theta_N}{N}\eta$ and trade according to this rule (regardless of η) for the next T auctions. Finally, if a deviation was detected, trade $\frac{\theta_N}{N}$ in all subsequent auctions. A price in the last auction is defined to be "normal" if $\left|p_{t-1} - E(p_{t-1}|\text{trigger})\right| \leq \rho$. So, such a strategy is completely specified by the three parameters $(\bar{\eta}, \rho, T)$. The following proposition demonstrates conditions under which a trigger strategy equilibrium of this type exists:

Proposition 4 *For any $\bar{\eta} > 0$, $\rho > 0$, $T < \infty$, there is $\beta^* < 1$, such that there exists an equilibrium with trigger strategies $(\bar{\eta}, \rho, T^*)$ for all $\beta \geq \beta^*$.*

The proposition shows that the basic intuition of the model carries through to the more complicated case where trades are not reported, even *ex post*. Proposition 4 states that there exist trigger strategy equilibria provided the discount factors are bounded below by some critical number. These strategies can support an equilibrium closer to the highest profit monopolist solution of Kyle (1985). Thus, in contrast to the Holden-Subrahmanyam model, competition among informed traders need not lead to almost complete revelation of private information. The difference in results arises only because of the nature of the admissible trading strategies.

4 Multiple Securities

Large traders generally trade more than one security. For example, bond dealers trade a variety of bonds differentiated by their coupon rates, maturities, issuer, and risk. The same factors (e.g., size or research capabilities) that give a trader an information advantage in one security may confer information advantages in related assets. In this section we investigate the optimal pattern of insider trading when informed traders meet in more than one security's market.

Consider two securities, whose respective valuations, v_{1t} and v_{2t}, evolve according to a random walk; $v_{jt} = v_{jt-1} + \eta_{jt}$, where (η_{1t}, η_{2t}) are distributed with a correlation coefficient ρ. A group of N informed traders has information on η_{jt} prior to trading in the t-th auction. As before, v_t becomes common knowledge after the t-th trading round and $p_{jt} = v_{jt-1} + \lambda \left[\sum_i q_{it}^j + \tilde{x}_t^j\right]$, where q_{it}^j and $\tilde{x}_t^j$ are the insider and liquidity trades in the j-th market. Note that for simplicity we take the depth parameter λ to be the same in both markets.

The principal result of this section demonstrates that informed traders "pool" their trades by trading at an identical intensity in both markets. This implies that the combined expected discounted profits from both markets is higher if traders' strategies consider them jointly than if they are viewed as independent. Intuitively, suppose traders consider the two markets separately and, without loss of generality, $|\eta_1| > |\eta_2|$. Proposition 2 implies that informed traders trade more aggressively in the first market (i.e., $\theta_1 > \theta_2$), sacrificing potential profits. This is unavoidable because a deviation from the monopoly trading level is profitable in the first market but not in the second. Note that expected future losses from deviating from the restricted (monopoly) trading level, Δ, are identical in both markets.

Now suppose traders consider both markets in their decision making. This introduces a "cross-subsidy" possibility given that the potential future losses are those arising in the two markets combined. Intuitively, traders can revert to treating the two markets independently in the future, so that the combined future losses are at least 2Δ. If traders trade more aggressively in the unprofitable market (increase θ_2), the profits to deviating in this market fall. Hence, this trade can be sustained by the threat of losing, say, $\Delta - \epsilon$, where $\epsilon > 0$. Correspondingly, the traders can trade less aggressively in the profitable market (decrease θ_1) because the threatened loss is now $\Delta + \epsilon$. Note that marginal profits are higher in market 1, and hence decreased trading in that market is (potentially) more profitable than the losses incurred from increased trading in market 2.

Proposition 5 makes precise our intuition and shows that the cross-subsidization continues until $\theta_1 = \theta_2$. This argument is formalized in the following result:

Proposition 5 *In the ONCE, informed traders trade the monopoly volumes in both markets provided $\eta_1^2 + \eta_2^2$ is less than some critical level, say $\bar{\eta}$. Above this level, the volume traded is in between the monopoly and Markov volumes. Furthermore, the trading intensity is always the same in the two markets and this is true for all η_1, η_2. Finally, this common trade intensity, say θ^*, depends only on $\eta_1^2 + \eta_2^2$ and increases in $\eta_1^2 + \eta_2^2$.*

Corollary 1 *Suppose the correlation coefficent $\rho < 1$. Then, for all η_1, η_2, $\overline{V}(\eta_1, \eta_2) > \overline{V}_1(\eta_1) + \overline{V}_2(\eta_2)$, where $\overline{V}_i(\eta_i)$ is the ONCE profits in market i, when the markets are separate, and $\overline{V}$ is the ONCE profit with the markets viewed jointly.*

The intuition for our result is similar to that of Bernheim and Whinston (1990), who examine strategic interaction between firms in multiple product markets. However, our results differ for theirs in that they did not characterize the ONCE strategy (i.e., they established Corollary 1 but no analog of Proposition 5).

5 Empirical Implications

In this section, we examine asset price dynamics under the optimal strategies and their implications for empirical analyses of security returns. We focus primarily on the conditional expectation, variance, and autocorrelations of observed returns, and the relation between these moments and trading volume, but also discuss the informativeness of prices.

We first examine whether prices are unconditionally unbiased predictors of the asset's value. Let $u_t \equiv v_t - p_t$ denote the pricing error in period t. At time $t-1$, both p_t and v_t are random variables, and the pricing error is also random. However, after trading in period t, an econometrician who observes transaction prices, dividends, and quotations can infer the realized pricing error. For this reason, u_t is a natural object of attention. It is straightforward to show that the expected pricing error is zero, i.e., that $E_{t-1}[u_t] = 0$, irrespective of the previous trading history. To see this, note that $E_{t-1}[v_t - p_t] = E_{t-1}[v_t - v_{t-1} - \lambda(\theta(\eta_t)\eta_t + x_t)] = 0$, because $E_{t-1}[\eta_t(1 - \lambda\theta(\eta_t))] = 0$

as $\theta(\cdot)$ is a symmetric function for all t.[11] When trading information is available following market clearing, the equilibrium in this case is arbitrarily close to the one-period Kyle (1985) single-trader equilibrium for reasonable discount rates. Thus, the equilibrium shares the same properties. In particular, if λ is appropriately chosen, the price is the conditional expectation of the asset's value in virtually all states.

It is also interesting to examine the statistical properties of changes in transaction prices because price quotations may be unobservable to an econometrician. Unlike the pricing error, however, transaction-to-transaction returns need not have a conditional expectation of zero because of the public observes the latest dividend after trading. To see this, let $R_t \equiv p_t - p_{t-1}$ represent the price change or total return in period t, and denote by Φ_t the public information set immediately after trading at time t.[12] Further, let $Q_t = \sum_i q_{it}$ represent the aggregate volume of informed trading. Then, $E[R_{t+1}|p_t] = E[\eta_t + \lambda(Q_{t+1} - Q_t) + \lambda(x_{t+1} - x_t)|\Phi_t] = \eta_t - \lambda(Q_t + x_t) = u_t$. Thus, the expected price change in period $t+1$ is the realized pricing error in period t, i.e., u_t. If this error were unobservable, the expected return would be zero, but because dividends are paid after trading, the pricing error, and hence the expected return is (almost surely) non-zero.

The results above also imply that price changes also exhibit serial correlation. To see this, note that $\text{Cov}[R_{t+2}, R_{t+1}|\Phi_t] = E[(R_{t+2} - E[R_{t+2}])(R_{t+1} - E[R_{t+1}]|\Phi_t] = E[\lambda((Q_{t+1} + x_{t+1})(\eta_{t+1} + \lambda(Q_{t+2} - Q_{t+1}) + \lambda(x_{t+2} - x_{t+1})|\Phi_t] = \lambda E[Q_{t+1}\eta_{t+1}(1 - \lambda\theta(\eta_{t+1}))] - \lambda^2\sigma_x^2$. The expectation $E[Q_{t+1}\eta_{t+1}(1 - \lambda\theta(\eta_{t+1}))] > 0$, because Q_{t+1} and η_{t+1} always have the same sign. Thus, the serial correlation in returns may be positive or negative depending on the magnitude of the noise trading, measured by σ_x^2.

This result appears puzzling since successive changes in fundamental value are independent (i.e., $\text{Cov}[v_{t+2} - v_{t+1}, v_{t+1} - v_t|\Phi_t] = E[\eta_{t+1}\eta_t] = 0$). However, there is an intuitive explanation for our findings. Consider the case where the volatility in noise trading is high, and suppose that the order flow originating from noise traders in period $t+1$ is

[11] Note that the functional form of θ itself changes stochastically, but this does not alter our argument.

[12] Formally, the information set is the σ-algebra generated by the history of trading and dividends, including v_t and p_t.

positive. A net buy imbalance would tend to induce a positive change in price. Further, conditional on a positive realization of a noise trade, the expected pricing error, u_{t+1}, in period $t+1$ is negative. Formally, $E[u_t|x_t > 0] = E[v_t - v_{t-1} - \lambda(Q_t + x_t)] = E[\eta_t(1 - \lambda\theta(\eta))|x_t > 0] + \lambda E[x_t|x_t > 0] = \lambda E[x_t|x_t > 0] > 0$, where we have used the fact that the informed traders' strategies are independent of the realization of the noise trade. However, our previous results show that the expected price change from period $t+1$ to $t+2$ is the realized pricing error in period $t+1$, which is, on average negative. As the volatility of noise trading increases further, successive price changes exhibit increasingly negative serial correlation. Thus, noise trades induce pricing errors that are subsequently reversed; these reversals give rise to negative serial correlation if the magnitude of noise trading is large.

Next, consider the volatility of prices. It is useful to establish a reference point, which we take as the volatility of fundamental prices. In our model, this benchmark is represented by σ_η^2, i.e., the variance of $v_t - v_{t-1}$, which is a constant. We now examine the volatility of transaction prices, beginning with the case where trades are observed *ex post*. In this case, the conditional variance of returns is given by $\sigma^2[R_{t+1}|\Phi_t] = \lambda^2(E[Q_{t+1}^2] + \sigma_x^2)$. As $E[Q_{t+1}^2] = E[(\theta(\eta_{t+1})\eta_{t+1})^2]$ is independent of past trades and innovations, because no trader will depart from the proposed ONCE trading rule if deviations are observed. Hence, the volatility of returns is a constant independent of previous volumes or price movements. The variance of prices is an increasing function of the variance in fundamental prices, the trading intensities of informed traders, and the volatility of noise trades.

A related question concerns the informativeness of prices, i.e., what fraction of private information is impounded in prices. The pricing error, u_t, in any period may be large simply because of a large value innovation. A more direct measure of informativeness is provided by the ratio of the pricing error to the innovation in fundamental values, $\psi_t \equiv u_t/\eta_t = (1 - \lambda\theta(\eta_t))$. This measure lies strictly between 0 and 1; if $\psi_t = 0$, prices reflect all private information, and if $\psi_t = 1$, no private information is impounded in price. In the single-period Kyle model, $\psi_t = 0.5$, i.e., half the private information is reflected in prices.

In our model, however, this is not the case. Paradoxically, more 'valuable' private information leads to more aggressive trading and greater revelation of private information. To see this, suppose that if $|\eta_t|$ is large, informed trading volume, measured by $\theta(\eta_t)|\eta_t|$ is also large, since $\theta(\cdot)$ is an increasing function of $|\eta|$, and consequently realized trading volume is also high. Recall that the pricing error is $u_t = \eta_t(1 - \lambda\theta(\eta_t))$. Large shocks naturally give rise to higher pricing errors and larger volumes, but in this case, the fraction of the innovation impounded in price, measured by $\psi_t = \frac{u_t}{\eta_t}$, is smaller. In other words, although higher volumes are associated with larger price movements (as has been extensively documented in the empirical literature), the informativeness of prices is greater when volume is high. By contrast, in the linear MPE, the trading intensity θ is a constant, so that price informativeness is also constant.

When traders cannot observe previous volumes *ex post*, their strategies are history-dependent, and it need not be the case that price volatility is constant. Indeed, we show below that the volatility of the pricing error is stochastic and serially correlated. When trades are unobservable traders restrict their trading intensities (i.e., by trading $\theta(\eta) < \theta_N$) until a perceived departure from the predicted price change, upon which traders increase their trading intensities. The possibility of switches in trading intensities suggests that the variance of the pricing error is not a constant, but depends systematically on previous prices and volumes. Under the restricted trading regime, both volume and price variability are low. However, upon a switch to a higher trading intensity regime, triggered by an abnormal price movement, we observe larger trading volumes and greater price variability for the following T periods, as shown above. Because informed traders cannot observe deviations, they condition on observed price changes. In order to obtain a representation of the factors triggering these "regime shifts," we need to characterize the conditions under which restricted trading breaks down.

If the market is in equilibrium, deviations occur only by accident, i.e., if large imbalances give the appearance of abnormal trading by one of the informed traders. The switch in trading intensities is, in equilibrium, triggered when the abnormal price movement caused by a large noise trade exceeds a critical level. Formally, the probability of a switch from the restricted trading regime in period t is $\kappa \equiv \text{Prob}\,[|p_t - E[p_t|\text{ trigger }|] = \text{Prob}\,[\lambda|x_t| > \rho] = \text{Prob}\,[|x_t| > \lambda^{-1}\rho]$. A regime shift is thus characterized by abnormally large volume, since it is triggered by a high $|x_t|$. Our previous results show that this abnormal volume is associated with higher volatility and larger pricing errors. If the price movement in period t is large, the expected price movement in the next period is also large. Thus, the expected absolute pricing error (or volatility) in period $t+1$ is positively related to the realized volatility in period t, even though the successive pricing errors are serially uncorrelated. Similarly, aggregate trading volume tend to be serially correlated, with periods of high volume following sharp price movements, as documented in the empirical literature.

This result provides an economic justification for the use of Generalized Autoregessive Conditional Heteroskedasticity (GARCH) models for high frequency transaction data. Such models are often preferred over standard regression techniques in modelling asset prices and foreign exchange where there time-varying volatility. Our model also provides some new hypotheses for such empirically motivated investigations. In our model, a switch in trading intensities is associated with abnormally large price movements and volume, suggesting that the process is better modeled conditioning jointly on both volume and prices.

6 Conclusions

We examine the process by which traders' private information is impounded in security prices, modeling trading as a dynamic game between strategic agents with private information. Papers by Foster and Vishwanathan (1991) and Holden and Subrahmanyam (1992) suggest that, unlike Kyle (1985), prices in securities markets should approximate the idealized notion of strong-form efficiency when there is competition among informed traders. This conclusion is particularly important from the viewpoint of public policy since recent empirical evidence indicates that most episodes of insider trading are associated with multiple informed traders, but is based on the assumption that traders' strategies are not conditional functions of past prices and trades.

In a market where some traders consistently receive private information signals, however, it seems reasonable to consider trading strategies that do not ignore potentially valuable information about past prices and trades. Further, there is no reason, *per se* to restrict attention to linear equilibria. We examine general non-cooperative trading strategies, including strategies that are nonlinear or that depend on past trades. In contrast to the previous literature, we demonstrate that traders with private information can support a non-cooperative equilibrium which is arbitrarily 'close' to the solution of Kyle (1985) for very reasonable discount rates. This equilibrium is natural in the sense that traders' strategies yield expected profits which are strictly greater than in all other equilibria, including the unique linear equilibrium. This suggests that the presence of multiple traders is not a guarantee of highly efficient markets.

The strategies in the optimal equilibrium are surprisingly intuitive. For 'small' value innovations (i.e., where the deviation between public and private information is small), the informed traders total trading quantity is the quantity traded by the Kyle (1985) monopolist insider. For larger innovations, restricted trading is not sustainable, and trading intensity increases. If the innovation is very large, the equilibrium quantities closely resemble the Nash equilibrium quantities of Foster and Vishwanathan (1991) and Holden and Subrahmanyam (1992). Paradoxically, more 'valuable' private information leads to more aggressive trading and greater revelation of private information. By contrast, in the linear equilibrium, the fraction of private information revealed in price is a constant irrespective of the magnitude of the information signal.

The nonlinear nature of the strategies considered here lead to complex price behavior. We demonstrate conditions where price volatility is serially correlated even though the volatility of fundamental prices is constant over time. This occurs even though price changes are serially uncorrelated. Periods of high price volatility and high volume follow periods of low price volatility and lower volume. The shift to a high volume and high volatility regime is generally triggered by an abnormal price movement. These stochastic changes in price volatility are endogenous and are unrelated to the volatility of the underlying asset, which is a constant in our model. These results provide a justification for the use of Generalized Autoregressive Conditional Heteroskedasticity (GARCH) models for high frequency transaction data.

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Appendix

Proof of Lemma 1:

We first provide a formal description of a result that allows us to focus on equilibrium strategies that are only a function of the value innovations $\{\eta_t\}$. Let $B(\eta_t, v_t, I_t)$ be the set of equilibrium payoffs.

Then, the equilibrium payoff correspondence $B(\eta_t, v_t, I_t)$ can be decomposed as follows:

$$B(\eta_t, v_t, I_t) = W(\eta_t) + \{v_t I_t\} \tag{A.1}$$

where the correspondence $W(\eta_t)$ is the set of equilibrium payoffs that depend on η_t alone, characterized as:

$$\begin{aligned} W(\eta_t) &= \Big\{ u \in \mathcal{R}^N : \text{ there exists } w^j(\cdot) \in W(\cdot), j = 1, 2, \theta(\cdot), \text{ such that} \\ u_i &= \Big(1 - \lambda\theta(\eta_t)\Big)\frac{\theta(\eta_t)}{N}\eta_t^2 + \beta \int w_i^1(\eta_{t+1}) dF(\eta_{t+1}) \\ &\geq \left(1 - \lambda\frac{(N-1)}{N}\theta(\eta_t) - \lambda\frac{\theta'}{N}\eta_t\right)\frac{\theta'}{N}\eta_t^2 + \beta\int w_i^2(\eta_{t+1}) dF(\eta_{t+1}), \forall i, \theta'. \Big\} \end{aligned}$$

To verify this claim, first note that an adaptation of the self-generation argument of Abreu, Pearce, and Stachetti (1990) for repeated games to our model shows that the equilibrium payoff *set* satisfies the following Bellman-like equation:*

$$\begin{aligned} B(v_t, \eta_t, I_t) &= \Big\{ u \in \mathcal{R}^N : \exists \nu^j(\cdot) \in B(\cdot)\,, \quad j = 1, 2, q_i\,, \quad i = 1, \ldots N \quad \text{s.t.} \\ u_i &= -E[p_t q_i] + (1-\beta) v_t (I_t + q_i) + \beta E \nu_i^1 (v_{t+1}, \eta_{t+1}, I_t + q_i) \\ &\geq -E[p_t q_i'] + (1-\beta) v_t (I_t + q_i') + \beta E \nu_i^2 (v_{t+1}, \eta_{t+1}, I_t + q_i')\,, \quad \forall q_i', i \Big\}\,. \end{aligned} \tag{A.2}$$

We now show, by employing the Bellman equation (A.2), that the equilibrium payoff set $B(v_t, \eta_t, I_t)$ actually has the separable structure claimed above. We do this by the following (standard) recursive argument: we assume a continuation value set with the separable structure and prove that this structure is maintained in an iteration. So, suppose that $B(v_t, \eta_t, I_t) = W_0(\eta_t) + \{v_t I_t\}$ and define the Bellman operator:

$$T\Big[W_0(\eta_t) + \{v_t I_t\}\Big] = \Big\{ u \in \mathcal{R}^N : \exists w^j(\cdot) \in W_0(\cdot), J = 1, 2,\ \theta \quad \text{s.t.} \tag{A.3}$$

$$\begin{aligned} u_i = E\Big[&-(v_{t-1} + \lambda\theta\eta_t + \lambda x_t)\frac{\theta}{N}\eta_t + (1-\beta)\left(I_t + \frac{\theta}{N}\eta_t\right) v_t \\ &+ \beta\int w_i^1(\cdot) dF + \beta v_{t+1}\left(I_t + \frac{\theta}{N}\eta_t\right)\Big] \end{aligned} \tag{A.4}$$

$$\begin{aligned} \geq E\Big[&-\left(v_{t-1} + \lambda\frac{N-1}{N}\theta\eta_t + \lambda\frac{\theta'}{N}\eta_t + \lambda x_t\right)\frac{\theta'}{N}\eta_t \\ &+ (1-\beta)\left(I_t + \frac{\theta'}{N}\eta_t\right) v_t + \beta\int w^2(\cdot) dF \\ &+ \beta v_{t+1}\left(I_t + \frac{\theta'}{N}\eta_t\right)\Big]\,, \quad \forall i, \theta' \Big\} \end{aligned} \tag{A.5}$$

*Note that our game is, formally speaking, a Markovian or dynamic game; Dutta and Sundaram (1992) show that the arguments of Abreu, Pearce, and Stachetti (1990) continue to hold for this more general class of games.

where the i-th trader's trade q_i has been rewritten as $\frac{\theta}{N}\eta_t$. Note that (A.5) simply says that this trade is incentive-compatible if there is no alternative trade $\frac{\theta'}{N}\eta_t$ which yields the trader higher lifetime profits when the continuation values for the two trades are, respectively, $E[\int w_i^1(\cdot)dF + v_{t+1}\left(I_t + \frac{\theta}{N}\eta_t\right)]$ and $E[\int w_i^2(\cdot)dF + v_{t+1}\left(I_t + \frac{\theta'}{N}\eta_t\right)]$. Recall that by our assumption, $E(v_{t+1}|v_t) = v_t$ and $E(x_t) = 0$. Hence the incentive constraint says: θ is incentive-compatible if

$$-(v_{t-1} + \lambda\theta\eta_t)\frac{\theta}{N}\eta_t + (1-\beta)\left(I_t + \frac{\theta}{N}\eta_t\right)v_t + \beta Ew_i^1 + \beta\left(I_t + \frac{\theta}{N}\eta_t\right)v_t \geq \tag{A.6}$$
$$-\left(v_{t-1} + \lambda\frac{N-1}{N}\theta\eta_t + \lambda\frac{\theta'}{N}\eta_t\right)\frac{\theta'}{N}\eta_t + (1-\beta)\left(I_t + \frac{\theta'}{N}\eta_t\right)v_t + \beta Ew_i^2 + \beta\left(I_t + \frac{\theta'}{N}\eta_t\right)v_t\,.$$

Upon collecting terms, substituting $v_t - v_{t-1} \equiv \eta_t$ and rearranging, the incentive constraint turns out to be

$$(1-\lambda\theta)\frac{\theta}{N}\eta_t^2 + \beta Ew_i^1 \geq \left(1 - \lambda\frac{N-1}{N}\theta - \lambda\frac{\theta'}{N}\right)\frac{\theta'}{N}\eta_t^2 + \beta Ew_i^2\,. \tag{A.7}$$

Note that the incentive constraint is independent of v_t and I_t. Furthermore, the Bellman equation (A.3) has now become:

$$\begin{aligned} T\Big[W_0(\eta_t) + \{v_t I_t\}\Big] &= \Big\{u \in \mathcal{R}^N : \exists w^j(\cdot) \in W_0(\cdot)\,, \quad j = 1,2, \text{and} \theta \quad \text{s.t.} \\ u_i &= (1-\lambda\theta)\frac{\theta}{N}\eta_t^2 + v_t I_t + \beta Ew_i^1 \\ &\geq \left(1 - \lambda\frac{N-1}{N}\theta - \lambda\frac{\theta'}{N}\right)\frac{\theta'}{N}\eta_t^2 + v_t I_t + \beta Ew_i^2\,, \quad \forall\theta', i\Big\}\,. \end{aligned} \tag{A.8}$$

Clearly, $T[W_0(\eta_t) + \{v_t I_t\}] = W_1(\eta_t) + \{v_t I_t\}$ for some correspondence $W_1(\cdot)$. Furthermore $W_1(\cdot)$ is non-empty valued since, at every η_t, there is a Nash equilibrium θ_N which, for $w^1 = w^2$, satisfies the incentive constraint (A.7).

But now the argument can be iterated to yield a sequence $W_{n+1}(\eta_t) + \{v_t I_t\} \equiv T\Big[W_n(\eta_t) + \{v_t I_t\}\Big]$. The limit of this sequence of correspondences is the equilibrium payoff set.† The claim is proved.

The lemma follows immediately, because the incentive constraint (A.7) is independent of v_t and I_t. Hence, the sustainable trades in period t are independent of v_t, I_t as well. The argument now repeats for all t. ∎

Proof of Proposition 1

Suppose that each trader trades $q_t = \frac{\theta}{N}\eta_t$. From the incentive constraint (A.7), and the fact that in a Markov equilibrium $w^1 = w^2$, it follows that θ' is incentive-compatible if and only if $(1-\lambda\theta)\frac{\theta}{N} \geq \max_{\theta'}(1 - \lambda\theta\frac{(N-1)}{N} - \lambda\frac{\theta'}{N})\frac{\theta'}{N}$. Note that deviation profits are maximized for $\theta' = \frac{N}{2\lambda}(1 - \lambda\theta\frac{(N-1)}{N})$. In equilibrium, $\theta' = \theta$, and this yields $\theta_N = \frac{N}{\lambda(N+1)}$. ∎

Proof of Proposition 2:

Following Abreu (1988), any equilibrium trading strategy in a dynamic game can be described in two parts: (1) a normal trading rule which informed traders follow provided no trader

†That such a limit exists and is the equilibrium payoff set, is shown in Dutta and Sundaram (1992).

has deviated in the past, and (2) a fallback rule which is followed if some trader has not traded according to the normal trading rule in the past. Without loss of generality, the fallback rule can always be taken to be the worst equilibrium trading rule in the game.‡ Further, from this point on, we invoke Lemma 1 and ignore inventory levels in our analysis.

Let $\overline{V}(\eta)$ denote the (infinite-horizon) payoffs to an informed trader in the best (symmetric) equilibrium if the value innovation is η; likewise let $\underline{V}(\eta)$ denote the (infinite-horizon) payoffs to the worst (symmetric) equilibrium, i.e., $\overline{V}(\eta) = \max W(\eta)$ and $\underline{V}(\eta) = \min W(\eta)$. Proposition 3 demonstrates that best and worst equilibria are distinct for large discount factors. Suppose that $\Delta \equiv \beta \int \left[\overline{V}(\eta) - \underline{V}(\eta)\right] dF(\eta)$, where F is the distribution of η.

The proposition is proved by way of the following lemma:

Lemma 2 *The optimal equilibrium payoffs satisfy*

$$\overline{V}(\eta) = \max_{\theta \in \varphi(\eta)} \left\{(1 - \lambda\theta)\frac{\theta}{N}\eta^2\right\} + \beta \int \overline{V}(\cdot) dF(\cdot) \tag{A.9}$$

where $\varphi(\eta)$ is the set of incentive-compatible trades at η, and is defined by

$$\varphi(\eta) \equiv \left\{\theta : \left[(1 - \lambda\theta)\frac{\theta}{N} - \frac{1}{4\lambda}\left(1 - \lambda\frac{N-1}{N}\theta\right)^2\right]\eta^2 \geq -\Delta\right\} . \tag{A.10}$$

Proof of Lemma 2: Consider any equilibrium of the trading game which is ϵ-optimal, i.e., equilibrium payoffs are at least $\overline{V}(\eta) - \epsilon$. Suppose that in this equilibrium each trader trades, say, $\frac{\hat{\theta}}{N}\eta$ in the current auction. We first show that $\hat{\theta}$ is an incentive-compatible trade in the sense of (A.10).

Denote the anticipated lifetime equilibrium profits $w(\eta')$, if the next-period's price-dispersion is η'. Further, in the event of picking a trade $\frac{\tilde{\theta}}{N} \neq \frac{\hat{\theta}}{N}$, denote the anticipated lifetime (equilibrium) profits $w(\eta'|\tilde{\theta})$. By the definition of incentive-compatibility in equation (A.7),

$$\begin{aligned}(1 - \lambda\hat{\theta})\frac{\hat{\theta}}{N}\eta^2 + \beta\int w(\eta')\, dF(\eta') \quad &\geq \quad \left(1 - \lambda\frac{N-1}{N}\hat{\theta} - \lambda\frac{\tilde{\theta}}{N}\right)\frac{\tilde{\theta}}{N}\eta^2 \\ &\quad + \beta\int w(\eta'|\tilde{\theta})\, dF(\eta') .\end{aligned} \tag{A.11}$$

Since $w(\eta')$ and $w(\eta'|\theta)$ are themselves equilibrium payoffs, it follows, again by definition, that $\overline{V}(\eta') \geq w(\eta')$ and $w(\eta'|\tilde{\theta}) \geq \underline{V}(\eta')$. Hence,

$$\begin{aligned}\left[(1 - \lambda\hat{\theta})\frac{\hat{\theta}}{N} - \left(1 - \lambda\frac{N-1}{N}\hat{\theta} - \lambda\frac{\tilde{\theta}}{N}\right)\frac{\tilde{\theta}}{N}\right]\eta^2 \quad &\geq \quad \beta\int\left[w(\eta'|\tilde{\theta}) - w(\eta')\right]dF(\eta') \\ &\geq \quad \beta\int\left[\underline{V}(\eta') - \overline{V}(\eta')\right]dF(\eta') \\ &\geq \quad -\Delta .\end{aligned} \tag{A.12}$$

‡See, e.g., Abreu (1988). Note also that we restrict attention to symmetric equilibria.

Equation (A.12) holds for all trading quantities $\frac{\tilde{\theta}}{N} \neq \frac{\hat{\theta}}{N}$. It can be shown that the trade $\frac{\tilde{\theta}}{N}$ which maximizes trader i's immediate profits is $\frac{\tilde{\theta}}{N} = \frac{1}{2\lambda}\left(1 - \lambda \frac{N-1}{N}\theta\right)$. Substituting this value of $\frac{\tilde{\theta}}{N}$ into equation (A.12) yields:

$$\left[(1-\lambda\hat{\theta})\frac{\hat{\theta}}{N} - \frac{1}{4\lambda}\left(1-\lambda\frac{N-1}{N}\hat{\theta}\right)^2\right]\eta^2 \geq -\Delta. \tag{A.13}$$

We have shown that $\hat{\theta}$ is incentive-compatible in the sense of equation (A.10), i.e., that $\hat{\theta} \in \varphi(\eta)$. Further, since this equilibrium is ϵ-optimal, it follows that:

$$\begin{aligned}
\overline{V}(\eta) - \epsilon \;&\leq\; (1-\lambda\hat{\theta})\frac{\hat{\theta}}{N}\eta^2 + \beta\int w(\eta')\,dF(\eta') \\
&\leq\; (1-\lambda\hat{\theta})\frac{\hat{\theta}}{N}\eta^2 + \beta\int \overline{V}(\eta')\,dF(\eta') \\
&\leq\; \max_{\theta\in\varphi(\eta)}\left\{(1-\lambda'\theta)\frac{\theta}{N}\right\}\eta^2 + \beta\int \overline{V}(\eta')\,dF(\eta')\,.
\end{aligned} \tag{A.14}$$

The last inequality follows because $\hat{\theta} \in \varphi(\eta)$. Since (A.14) holds for all $\epsilon > 0$, we have actually proved that:

$$\overline{V}(\eta) \leq \max_{\theta\in\varphi(\eta)}\left\{(1-\lambda\theta)\frac{\theta}{N}\right\}\eta^2 + \beta\int \overline{V}(\eta')\,dF(\eta')\,. \tag{A.15}$$

We now demonstrate that the opposite inequality holds as well. Consider any $\hat{\theta} \in \varphi(\eta)$ and form the following strategy: trade $\frac{\hat{\theta}}{N}\eta$ in the current auction and from the next auction onwards, trade according to an ϵ-optimal equilibrium strategy, i.e., if next auction's innovation is η', trade according to a strategy which is ϵ-optimal from η'. If any player currently trades $\frac{\tilde{\theta}}{N} \neq \frac{\hat{\theta}}{N}$, trade with the worst non-cooperative equilibrium strategy in the continuation. It is easy to see that the suggested trading strategy forms an equilibrium if and only if $\hat{\theta} \in \varphi(\eta)$. By definition therefore,

$$\overline{V}(\eta) \geq (1-\lambda\hat{\theta})\frac{\hat{\theta}}{N}\eta^2 + \beta\int \overline{V}(\eta')\,dF(\eta') - \beta\epsilon\,, \quad \forall\hat{\theta}\in\varphi(\eta)\,. \tag{A.16}$$

Since equation (A.16) holds for all $\epsilon > 0$, we have proved that:

$$\overline{V}(\eta) \geq \max_{\theta\in\varphi(\eta)}(1-\lambda\theta)\frac{\theta}{N}\eta^2 + \beta\int \overline{V}(\eta')\,dF(\eta') \tag{A.17}$$

Combining (A.15) and (A.17), the lemma is proved. ∎

We continue now with the proof of Proposition 2. To facilitate exposition, define $k(\theta) \equiv (1-\lambda\theta)\frac{\theta}{N} - \frac{1}{4\lambda}\left(1-\lambda\frac{N-1}{N}\theta\right)^2$. Note that by definition of a Nash equilibrium, $k(\theta_N) = 0$. Further, from the form of the function $k(\theta)$, it is straightforward to see that it is a quadratic function (which is symmetric around θ_N) with $k(\theta) < 0$, for all $\theta \neq \theta_N$.

From the definition it follows that, for all η, the set of incentive-compatible trades, $\varphi(\eta)$, is an interval centered around θ_N, i.e., $\varphi(\eta) \equiv [\theta^*(\eta), \theta_*(\eta)]$, where $k\big(\theta^*(\eta)\big)\eta^2 = k\big(\theta_*(\eta)\big)\eta^2 = -\Delta$ and $\theta^*(\eta) < \theta_N < \theta_*(\eta)$. Note also that as η increases, the set of incentive-compatible trades shrinks, as shown in Figure 2.

The objective function in the right-hand side of (A.9) is seen to be a quadratic function in θ. By definition, this function reaches a maximum at θ_1, and it declines on either side of θ_1. This last fact, coupled with the properties of φ, tells us:

(a) if $\theta_1 \in \varphi(\eta)$, the constrained optimum is at θ_1. On the other hand, if $\theta_1 \notin \varphi(\eta)$, the constrained optimum is at $\theta^*(\eta)$.

(b) there is an interval $[-\eta_1, \eta_1]$ for which $k(\theta_1)\eta^2 \geq -\Delta$ and hence the constrained optimum is θ_1. Whenever $|\eta| > \eta_1$, the ONCE trade is $\frac{\theta^*(\eta)}{N}\,\eta$. The proposition is proved. ∎

Proof of Proposition 3

Using proposition 1, we note that under the MPE strategy, $\frac{\theta_N}{N}$, the trader's profit when the value innovation is η is:

$$\pi_N = \frac{1}{(N+1)^2\lambda}\eta^2\,. \tag{A.18}$$

A trader's profit in the current auction, when trading the monopoly volume under the value innovation η is:

$$\pi_N = \frac{1}{4N\lambda}\eta^2\,. \tag{A.19}$$

Let $V_s(\eta)$ denote the value function corresponding to playing the trigger strategy, and $V_N(\eta)$ the Markov profits. It is clear that

$$\int V_S(\eta')dF(\eta') - \int V_N(\eta')dF(\eta') = \frac{\beta}{1-\beta}\left[\frac{1}{4N\lambda} - \frac{1}{(N+1)^2\lambda}\right]\xi$$

where:

$$\zeta = \int_{|\eta|\leq\bar{\eta}} \eta^2 dF(\eta). \tag{A.20}$$

Now consider a trader who chooses to deviate from the proposed strategy when all others play as above. Clearly, the optimal deviation if other traders use intensity θ_N is just θ_N, since this is the Nash solution. However, if other traders play θ_1/N, trader i can profit in the short-run by trading at a higher intensity. The optimal deviation strategy for trader i is easily computed as:

$$q_i^d = \frac{N+1}{4N\lambda}\eta \tag{A.21}$$

and the corresponding single-period profit is:

$$\pi^d(\eta) = \left(\frac{N+1}{2N}\right)^2\frac{\eta^2}{2\lambda} \tag{A.22}$$

for $|\eta| \leq \bar{\eta}$. Evidently, the trigger strategy is sustainable as equilibrium only if

$$\frac{1}{4\lambda}\left[\left(\frac{N+1}{N}\right)^2\frac{1}{2} - \frac{1}{N}\right]\eta^2 \leq \frac{\beta}{1-\beta}\left[\frac{1}{4N\lambda} - \frac{1}{(N+1)^2\lambda}\right]\xi\,, \quad |\eta| \leq \bar{\eta}\,.$$

Clearly this holds for high discount factors.

Recall that $\Delta \equiv \int \left[\overline{V}(\eta)-\underline{V}(\eta)\right] dF(\eta)$ is the difference between the best and worst attainable equilibria. By definition then, $\Delta \geq E[V_S(\eta)] - E[V_N(\eta)]$, where $V_S(\eta)$ and $V_N(\eta)$ are the value functions corresponding to the trigger strategy and the MPE strategy, respectively. Clearly, if the threat of moving to the MPE strategy is sufficient to support a trigger strategy with critical bound $\bar{\eta}$, the threat of giving up Δ in present value terms can support a higher bound η_1.

The second part follows immediately because the expected profits under the trigger strategy in any state are either the single-trader MPE profits or the (lower) MPE profits, and the probability that $|\eta| < \bar{\eta}$ is positive. ∎

Proof of Proposition 4:

Suppose that $|\eta| \leq \bar{\eta}$ and trader i trades an amount $\frac{\hat{\theta}}{N}\eta$, where $\hat{\theta} \neq \theta_1$. In that case one of three things happens: i) with probability ϵ this deviation is detected (and traders adopt the MPE strategy forever), ii) the deviation goes undetected but $\left|p_t(\hat{\theta}) - E(p_t|\theta_1)\right| > \rho$ (and traders follow the MPE strategies for T periods) or iii) the deviation goes undetected and $\left|p_t(\hat{\theta}) - E(p_t|\theta_1)\right| \leq \rho$ (and then, depending on η_t, θ_1 or θ_N is played). Clearly, option iii) is preferable, to the deviant trader, to option ii).

We first show that Prob $\left(\left|p_t - E(p_t|\theta_1)\right| \leq \rho\right)$ is maximized when $\hat{\theta} = \theta_1$. Notice that since $p_t = v_{t-1} + \lambda(Q_t + x_t)$, $p_t - E(p_t|\theta) = \lambda x_t$, if the quantity traded is $\frac{\theta_1}{N}\eta$. Hence, Prob $\left(\left|p_t - E(p_t|\theta_1)\right| \leq \rho\right) =$ Prob $\left(|\lambda x| \leq \rho\right) = G\left(\frac{\rho}{\lambda}\right) - G\left(-\frac{\rho}{\lambda}\right)$ where G is the distribution of noise trades.

However, if trader i trades $\frac{\hat{\theta}}{N}$, then Prob $\left(\left|p_t(\hat{\theta}) - E(p_t|\theta_1)\right| \leq \rho\right) = G\left(\frac{\rho}{\lambda} - \frac{1}{N}(\hat{\theta} - \theta_1)\eta\right) - G\left(-\frac{\rho}{\lambda} - \frac{1}{N}(\hat{\theta} - \theta_1)\eta\right) \leq G\left(\frac{\rho}{\lambda}\right) - G\left(-\frac{\rho}{\lambda}\right)$. The last inequality follows from the symmetry around zero of the distribution G.

A corollary of the above argument then is that, in the event of non-detection, the highest future payoffs are achieved by trading $\frac{\hat{\theta}}{N} = \frac{\theta_1}{N}$. If on the other hand, deviation is in fact detected, then the future payoffs are the expected payoffs of playing Markov forever. This is clearly strictly less than the future payoffs to switching back and forth between θ_1 and θ_N (which is available if today's trade is $\frac{\theta_1}{N}\eta$). In other words, not deviating has a strictly higher lifetime payoff, say $E[V_S]$, rather than $E[V_N]$, from the next auction onwards. Hence, not deviating implies an increased future payoff that is at least $(E[V_S] - E[V_N])\epsilon$.

Deviating in the current auction yields an incremental payoff of $-k(\theta_1)\eta^2$. So, deviation is unprofitable if

$$k(\theta_1)\eta^2 \geq \frac{\beta}{1-\beta}(E[V_N] - E[V_S])\epsilon \,. \tag{A.23}$$

It is clear that if (A.23) holds with $\eta = \bar{\eta}$, then is holds for all $|\eta| < \bar{\eta}$. Furthermore, since $\frac{\beta}{1-\beta} \to \infty$, as $\beta \to 1$, clearly (A.23) holds for some $\beta^* < 1$. ∎

Proof of Proposition 5

Let $\overline{V}$ (respectively $\underline{V}$) denote the combined profits in the best symmetric (respectively, worst symmetric) equilibrium of the multiple security model. Let $\Delta = \beta \int_{\eta'} \left[\overline{V}(\eta_1', \eta_2') - \underline{V}(\eta_1', \eta_2')\right] dF$, where F denotes the joint distribution on (η_1', η_2'). Note, incidentally, that since the two markets can always be treated separately, $\overline{V} \geq \overline{V}_1 + \overline{V}_2$ (respectively, $\underline{V} \leq \underline{V}_1 + \underline{V}_2$) where $\overline{V}_j$ (respectively, $\underline{V}_j$) is the profits in the j-th market in the best (respectively, worst) equilibrium.

By arguments completely analogous to those used in proving Lemma 2, we can show

Lemma 3 *The optimal equilibrium payoffs satisfy*

$$\overline{V}(\eta_1, \eta_2) = \max_{\theta_1, \theta_2 \in \varphi(\eta_1, \eta_2)} \left\{ (1 - \lambda\theta_1)\frac{\theta_1}{N}\eta_1^2 + (1 - \lambda\theta_2)\frac{\theta_2}{N}\eta_2^2 \right\} + \beta \int \overline{V}(\cdot)dF(\cdot)$$

where $\varphi(\eta_1, \eta_2)$ *is the set of incentive compatible trades at* (η_1, η_2) *and is defined by*

$$\varphi(\eta_1, \eta_2) \equiv \left\{ (\theta_1, \theta_2) : \sum_j \left[(1 - \lambda\theta_j)\frac{\theta_j}{N} - \frac{1}{4\lambda}\left(1 - \lambda\frac{N-1}{N}\theta_j\right)^2 \right] \eta_j^2 \geq -\Delta \right\} .$$

Writing, as before, $-k(\theta_j) = \frac{1}{4\lambda}\left(1 - \lambda\frac{N-1}{N}\theta_j\right)^2 - (1 - \lambda\theta_j)\frac{\theta_j}{N}$ and $\pi(\theta_j) = (1 - \lambda\theta_j)\frac{\theta_j}{N}$, we can rewrite the problem more compactly as

$$\max_{\theta_1, \theta_2} \pi(\theta_1)\eta_1^2 + \pi(\theta_2)\eta_2^2 \tag{A.24}$$

subject to

$$-k(\theta_1)\eta_1^2 - k(\theta_2)\eta_2^2 \leq \Delta . \tag{A.25}$$

The following claim is straightforward:

Claim *Without loss of generality, we can restrict attention to* $\theta_j \in [\theta_1, \theta_N]$, *i.e., to* θ_j *that lie between the monopoly and Markov intensities.*

Proof: Consider $\hat{\theta} < \theta_1$. Then, there is $\tilde{\theta} = \theta_1 + (\theta_1 - \hat{\theta})$ with the property that $\pi(\hat{\theta}) = \pi(\tilde{\theta})$. Furthermore, given that $-k(\cdot)$ is a strictly decreasing function on the appropriate range, $k(\tilde{\theta}) > k(\hat{\theta})$. So, if $\hat{\theta}$ was incentive compatible (together with, say θ_2), then so is $\tilde{\theta} - \epsilon$ (together with θ_2), for "small" ϵ. Given the fact that $\pi(\cdot)$ is strictly decreasing it then follows that $\pi(\tilde{\theta} - \epsilon) + \pi(\theta_2) > \pi(\hat{\theta}) + \pi(\theta_2)$. This yields the desired contradiction.

On the other hand suppose that $\hat{\theta} > \theta_N$. Then there is $\tilde{\theta} = \theta_N - (\hat{\theta} - \theta_N)$ with the property that $k(\tilde{\theta}) = k(\hat{\theta})$. Furthermore, $\pi(\tilde{\theta}) > \pi(\hat{\theta})$. Again we have a contradiction and hence the claim is proved. ∎

Note for future use that on $[\theta_1, \theta_N]$, π is strictly decreasing and concave whereas $-k$ is strictly decreasing and convex. Let $\Big(\theta_1, f(\theta_1)\Big)$ denote the combination of trading intensities which satisfy the incentive constraint exactly; $k(\theta_1)\eta_1^2 + k\Big(f(\theta_1)\Big)\eta_2^2 = -\Delta$. Straightforward calculus reveals that

$$f'(\theta_1) = -\left(\frac{\eta_1}{\eta_2}\right)^2 \frac{k'(\theta_1)}{k'(\theta_2)} \tag{A.26}$$

and from the properties of $k(\cdot)$ it follows that on the relevant region, f is a decreasing, convex function.

An isoprofit curve $g(\theta_1)$ is defined by $\pi(\theta_1)\eta_1^2 + \pi\Big(g(\theta_1)\Big)\eta_2^2 = c$, where c is some profit level. Then,

$$g'(\theta_1) = -\left(\frac{\eta_1}{\eta_2}\right)^2 \frac{\pi'(\theta_1)}{\pi'(\theta_2)} \,. \tag{A.27}$$

Hence, g is a decreasing concave function.

From (A.26) and (A.27) it is easy to check that $g' > f' \Leftrightarrow \frac{k'(\theta_1)}{\pi'(\theta_1)} > \frac{k'(\theta_2)}{\pi'(\theta_2)}$. The properties of k and π then imply that this last inequality holds if and only if $\theta_1 < \theta_2$. By the same arguments, $g' < f' \Leftrightarrow \theta_1 > \theta_2$. Collecting all this we see that profits are maximized by making θ_1 and θ_2 as close to each other as is feasible given the incentive constraint. Since $\theta_1 = \theta_2 = \theta_N$ is always incentive-compatible, there are equal trading intensities which solve the incentive constraint exactly. So, the common ONCE trading rule θ^* is characterized by

$$\theta^*(\eta_1, \eta_2) = \begin{cases} \theta_1 & \text{if } -k(\theta_1)(\eta_1^2 + \eta_2^2) \leq \Delta \\ k^{-1}\left(\frac{\Delta}{\eta_1^2+\eta_2^2}\right) & \text{if } -k(\theta_1)(\eta_1^2 + \eta_2^2) > \Delta \,. \end{cases}$$

The proposition is proved. ∎

Proof of Corollary 3

Suppose that $\rho < 1$. By the proposition above, informed traders do strictly better than with separate markets whenever $\eta_1 \neq \eta_2$. Hence, as long as $\eta_1 = \eta_2$ is not a probability one event, $\int \overline{V} dF > \int \overline{V}_1 dF_1 + \int \overline{V}_2 dF_2$. By Lemmas 2 and 3, this implies that $\overline{V}(\eta_1, \eta_2) > \overline{V}_1(\eta_1) + \overline{V}_2(\eta_2)$, for all (η_1, η_2). The corollary is proved. ∎

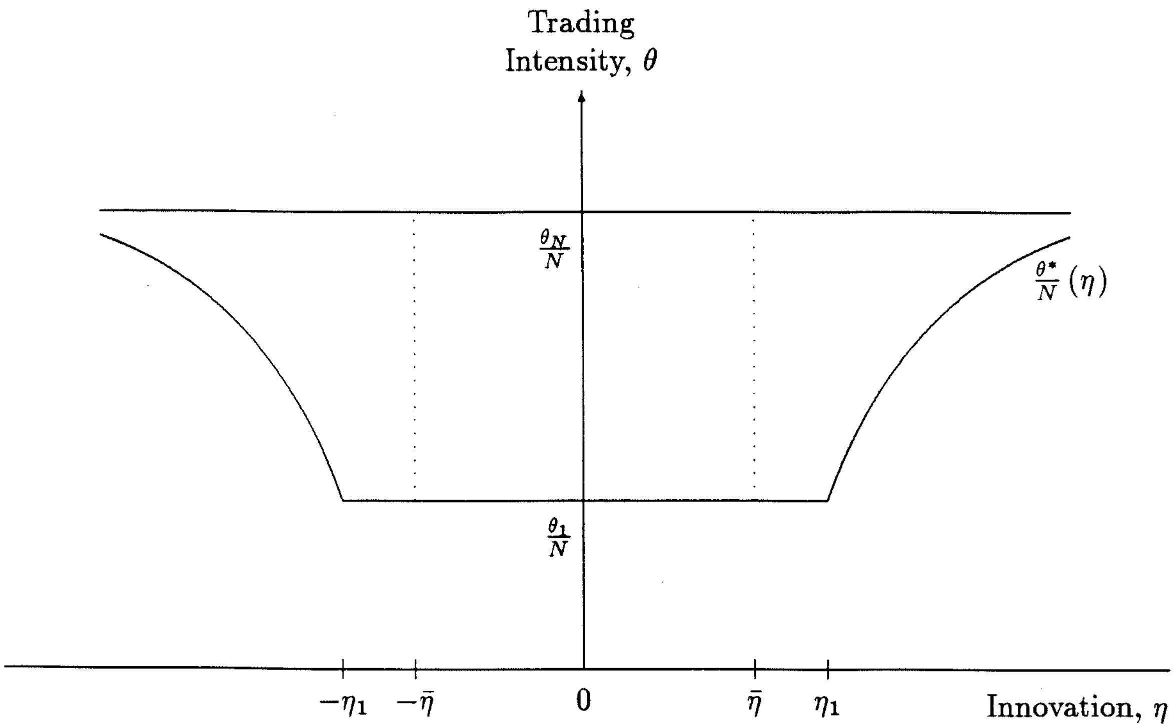

Figure 1: The Optimal Non-Cooperative Trading Strategy.

The figure shows the monopoly trading intensity, θ_1, the MPE intensity, θ_N, and the ONCE intensity, $\theta^*(\eta)$, as a function of the state, η.

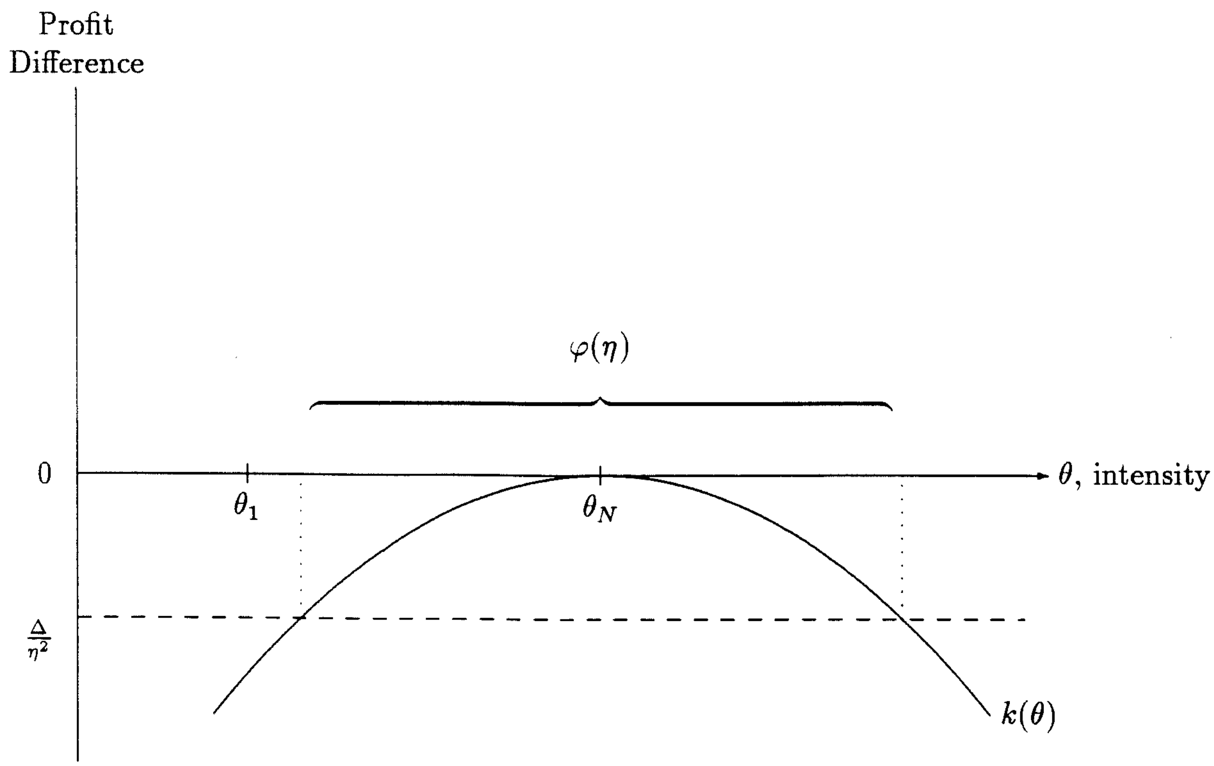

Figure 2: The Incentive Compatible Correspondence.