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## SELECTION IN INSURANCE MARKETS

by

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Consider an insurance market—say, for insurance against accidental injuries. People face different levels of accident risk, and have private information about their own risk levels.

Much depends on why individual risk levels differ. If the differences are exogenous, we have the well known case of adverse selection, first modelled by Rothschild and Stiglitz (1976). In the Rothschild-Stiglitz model, the only possible equilibrium is a separating equilibrium where high-risk individuals purchase full insurance and low-risk individuals purchase partial insurance. The outcome is suboptimal in the sense that if risk levels were observable, there would be a new and Pareto-superior equilibrium.

On the other hand, if risk levels differ because of behavioral differences, then new issues arise. The most familiar of these is moral hazard, where the opportunity to buy insurance induces suboptimal precautions. Like adverse selection, moral hazard can occur even if all individuals are identical.

More recent literature deals with new issues that arise when behavioral differences are driven by differences in tastes. For example, Bond and Crocker (1991) suggest the following story: relative risk-lovers choose (observably) to smoke, thereby revealing their tastes and their propensity to engage in other (unobservable) high-risk activities. This enables insurers to separate low-risk from high-risk individuals; non-smokers benefit from smoking because their voluntary abstinence demonstrates their risk aversion and allows them to insure at lower rates.

The Bond and Crocker story suggests that low-risk individuals would prefer a world where risk differences are driven by heterogeneity of tastes, as opposed to the adverse selection world where risk differences are purely exogenous. In the current paper, I will present an argument in the opposite direction. When tastes differ, those individuals who choose more risk also tend to demand less insurance. Assuming that there exists an

equilibrium with an insurance market, I will show that this reduction in demand has two undesirable consequences. First, low-risk individuals are unambiguously worse off than they would have been if the same risk differences had arisen exogenously. Second, it becomes less likely that the equilibrium can be efficient in a sense considered by Rothschild and Stiglitz.

The natural intuition seems to go the other way. The author can attest to having heard the argument, from participants in more than one recent seminar, that when high-riskers demand less insurance, they cause less damage to the insurance market. This natural intuition has even found its way into print. Hemenway (1990) christens the phenomenon *propitious selection*, a name which I shall adopt despite arguing that it is ultimately ironic.

Under adverse selection, high-risk individuals demand more insurance than low-risk individuals, and the outcome is suboptimal. Under propitious selection, high-risk individuals demand *less* insurance than low-risk individuals. This leads Hemenway (and others) to guess that propitious selection is a sort of mirror image of adverse selection, and should therefore tend to improve market outcomes just as adverse selection tends to worsen them. When high risk individuals opt out of insurance markets, it is suggested, their existence can no longer cause market imperfections.

Although the intuition is simple, it is exactly wrong. Here is the correct intuition: In the adverse selection equilibrium, something must be done to prevent high-risk individuals from purchasing low-risk insurance contracts. The solution is to tempt high-riskers out of the low-risk market with an offer of full insurance at high-risk rates. But if high-riskers are also less risk averse, then a full insurance contract has less value to them. This makes it *more* difficult to prevent high-riskers from entering the low-risk insurance market. As a result, the low-risk contracts must be made less attractive. This is a loss to low-riskers with no offsetting gain.

The above argument works whenever both groups are risk-averse, but differ in their degree of risk-aversion. If the high-risk group is actually risk-preferring, the outcome can differ. It turns out that a *small* amount of risk preference is actually worse than a small amount of risk aversion; when the high-risk group is only slightly risk-preferring (or when

it is perfectly risk-neutral) the insurance market vanishes altogether. (That is, the only equilibrium is where zero contracts are sold.) Only high levels of risk preference can indeed cause high-riskers to drop entirely out of the market, restoring the natural intuition as expressed by Hemenway.

In Section 1 below I review the Rothschild-Stiglitz model that will be the basis for the analysis. In Section 2 I analyze propitious selection with a variety of assumptions about risk aversion and show that it is anything but propitious. Section 3 is a note on appropriate welfare criteria, and contains a proof that propitious selection equilibria are less likely than adverse selection equilibria to be efficient in the sense of being unimprovable through any system of subsidies from low-riskers to high-riskers. Section 4 is a brief concluding discussion. The appendix addresses a natural question left open from Section 2.

## 1. Adverse Selection

Assume first that there are two groups of individuals, with different (exogenous) levels of accident risk. One group has accident probability  $p$  and the other has probability  $q > p$ . Each individual is endowed with wealth  $W$ , which is reduced to  $W - D$  in case of an accident.

The indifference curve diagram in Figure 1 relates income in two states of the world, “no accident” versus “accident”. Everybody starts at the endowment point  $E$ . The two lines emerging from  $E$  represent fair odds for the two groups; their absolute slopes are  $(1 - p)/p$  and  $(1 - q)/q$ . All individuals share the same expected utility function, with the usual properties. The pictured indifference curve is for a typical high-risk individual; low-risk individuals have steeper indifference curves that are not pictured.

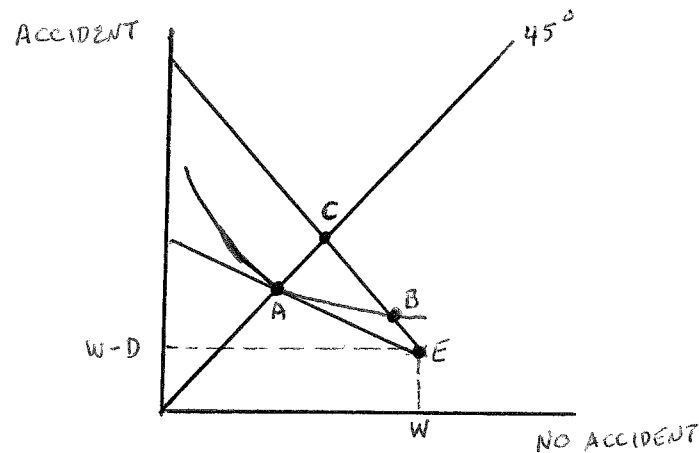


FIGURE 1

The insurance company offers a set of contracts. Each individual may choose any one of these contracts, or no contract at all. An equilibrium set of contracts is one where the insurance company earns zero profits, and no competing firm can offer a profitable contract that would lure away customers.

An easy argument (see Rothschild and Stiglitz) shows that there can be no pooling equilibrium (i.e. an equilibrium in which only one kind of contract is offered.) In fact, the only possible equilibrium is one where the company offers two contracts, allowing buyers to shift their endowments to points *A* and *B* in Figure 1. High-risk individuals (with the illustrated indifference curve) are indifferent between the two contracts and choose *A* in equilibrium. Low-risk individuals (whose indifference curves are steeper) prefer *B* and choose it. Low-riskers would prefer any point between *B* and *C*, but the company cannot offer such contracts because high-riskers would purchase them.

## 2. Propitious Selection

**2.1 Differing Risk Aversion.** Now I will perturb the model of Section 1 by assuming that risk differences, instead of being exogenous, arise from an endogenously determined choice of precautionary measures. I will continue to assume just two risk classes with probabilities of accident  $p$  and  $q$  ( $q > p$ ); people are endowed with accident probability  $q$

but can buy it down to  $p$  at a fixed cost  $C$ . Individuals are exogenously endowed with one of two different levels of risk aversion, causing one group to undertake precautionary procedures and the other not.

This setup raises issues of moral hazard that are well understood but not terribly germane to the present discussion. It is possible that the naturally low-risk group, after purchasing insurance, will lose its incentive to take precautions. In that case we have only one risk group and the selection problems disappear. Therefore I *assume* that the magnitude of  $C$  is sufficiently low that when low-riskers are permitted to purchase an equilibrium quantity of insurance, they continue to take precautions. High-riskers, who will be fully insured in equilibrium, will continue not to take precautions.

The analysis remains identical to that of Figure 1. The only change is that the pictured high-risk indifference curve must now be more nearly linear, since high-riskers now have *both* higher risk *and* less risk-aversion than low-riskers (as opposed to just higher risk as in Section 1.) This means that point  $B$ , where the indifference curve crosses the segment  $EC$ , must now lie closer to  $E$  than before. Since low-riskers would prefer to be as close as possible to  $C$  along this segment, they must be worse off in the new equilibrium. High-riskers remain fully insured as they were before.

We have shown the following: Given  $p$  and  $q$  (the low-risk and high-risk accident probabilities) and given the tastes of the low-risk population, low-riskers are worse off under a market equilibrium that is made imperfect by “propitious” selection than they are under a market equilibrium that is made imperfect by ordinary adverse selection.

Clearly the more risk-averse we make the high-riskers, the worse off the low-riskers will be. The less risk averse they are, the more nearly linear the indifference curve will be and the closer point  $B$  moves to point  $E$ .

**2.2 Risk Neutrality.** Suppose that the low-risk group is risk-averse and the high-risk group is risk-neutral. The risk-averse group finds it worthwhile to take accident precautions and the risk-neutral group does not. In this case, no low-risk insurance at all can be available in equilibrium.

Figure 1 is again the relevant picture, except that the high-risk indifference curve



is now perfectly linear and corresponds with the segment AE. Points B and E coincide and the equilibrium requires that low-risk individuals remain at their endowment point, purchasing no insurance.

The point here is that the opportunity to purchase actuarially fair insurance is worth exactly zero to the risk-neutral parties. If any contract at all is offered for sale above the segment AE, then they will happily give up their full-insurance option to purchase it. Thus no such contract can be offered and the low-risk insurance market disappears completely.

**2.3. Risk Preference.** Risk-preferrers might want to purchase negative amounts of insurance (so that they receive additional income in the event that no accident occurs). I assume this is impossible because it would eliminate the incentive to report accidents. Thus I will consider only insurance contracts that are issued in positive quantities.

If the high-riskers are risk-preferring, their indifference curves are concave. The three panels of Figure 2 show three possible configurations. Here the points A, C and E, and the budget lines, are as in Figure 1. In Panel A (a small amount of risk preference) there can be no low-risk insurance offered. In Panel B (a middling amount of risk preference) partial low-risk insurance is offered. (Low-riskers are able to move to point B.) Only in Panel C (a great deal of risk preference) can the low-risk group be fully insured.

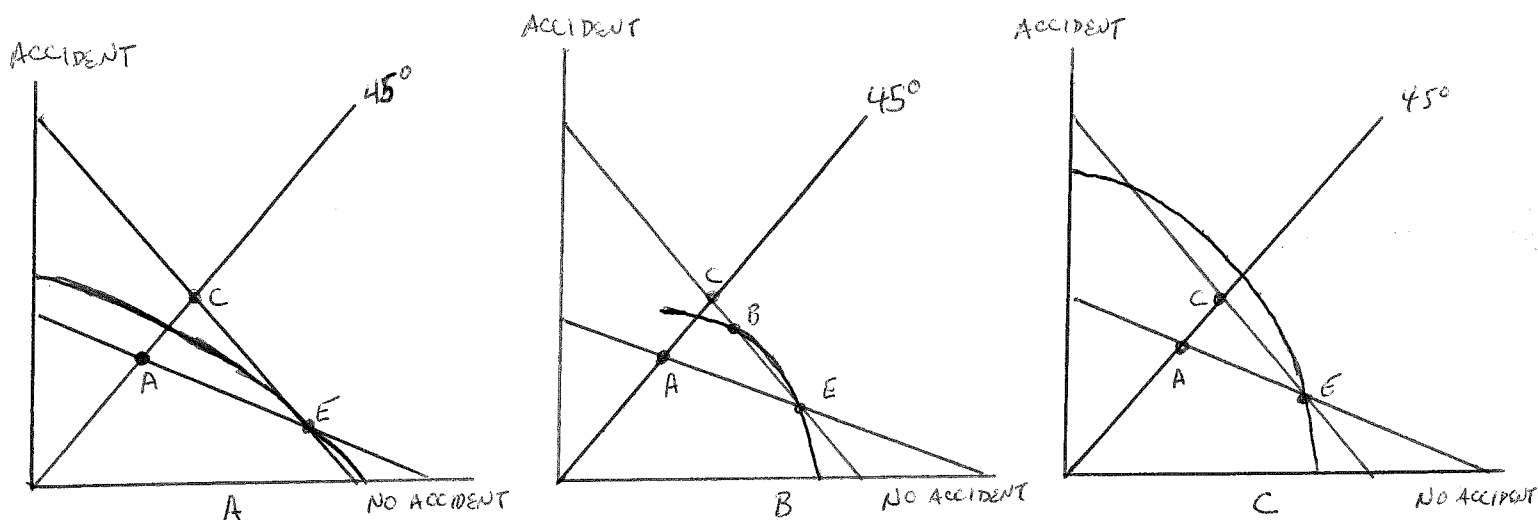
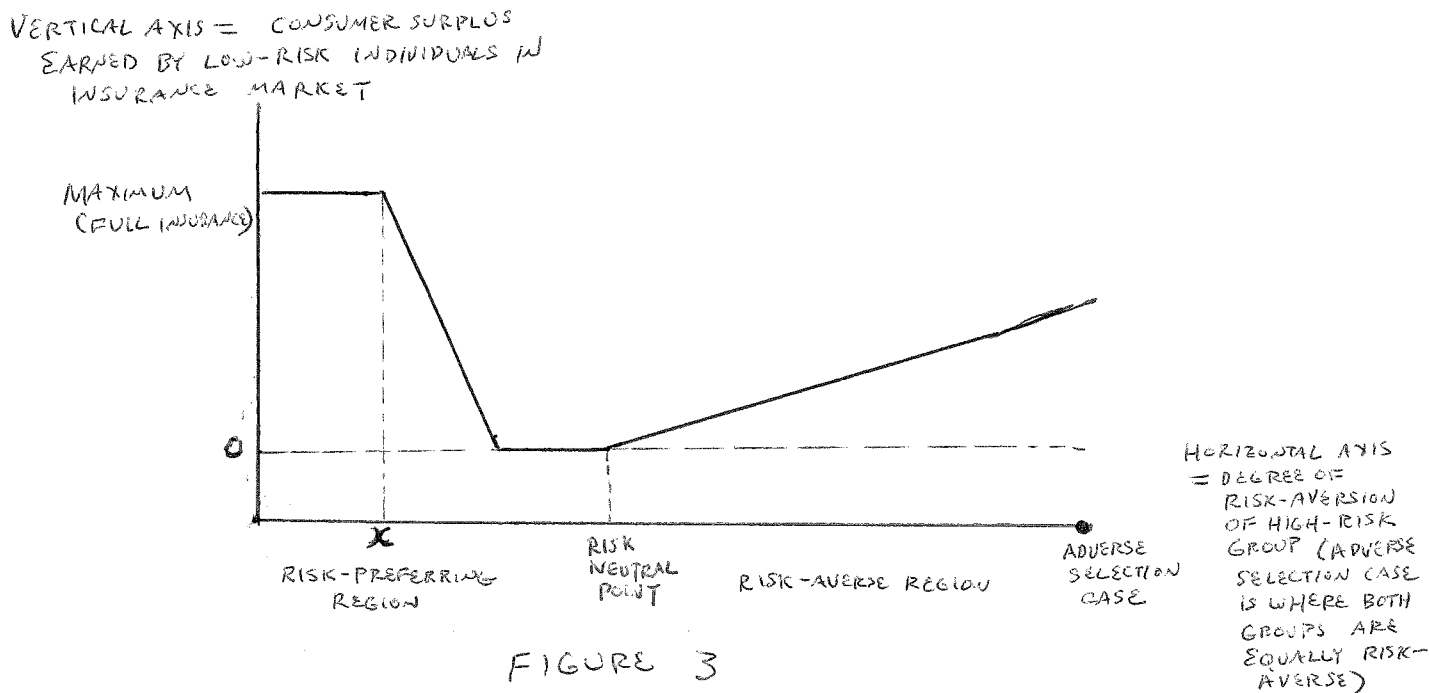


FIGURE 2

**2.4. Summary.** Figure 3 shows the welfare consequences for low-risk buyers when

high-risk buyers show varying degrees of risk preference. The right-hand point on the horizontal scale represents the situation where high-risker's risk preference is the same as low-risker's risk preference; in other words the two groups have identical tastes. This is the classical adverse selection case in which differences in risk level result from something other than tastes.



One can ask the quantitative question: How risk-preferring must the high-risk group be before the low-risk group can be offered full insurance? In other words, one wants to identify the point  $x$  in Figure 3, in terms of exogenous parameters. An approximate answer is given in the appendix.

### 3. Welfare Analysis

Under ordinary adverse selection, tastes are assumed to be homogeneous. By contrast, propitious selection requires heterogeneous tastes. Therefore the two phenomena require different populations and it makes no sense to apply the ordinary Pareto criterion in ranking outcomes.

Under either type of selection, the high-risk population is fully insured in equilibrium. Therefore it seems natural to confine one's welfare analysis to the low-risk population,

whose tastes *can* be held constant across the two models.

The natural question is: Suppose that you are a low-risk individual. Would you prefer to live in a world where risk levels differ exogenously (the adverse selection case) or in a world where tastes differ exogenously (the propitious selection case)? The answer derived in Section 2 is unambiguous: By this criterion, adverse selection is preferable to “propitious” selection.

In this section, we will analyze the welfare properties of propitious selection from a different point of view, by showing that there is a sense in which propitious selection equilibria are less likely to be constrained-Pareto-efficient (with appropriate break-even constraints) than are adverse selection equilibria. I thank Eric Bond for suggesting that I investigate this question.

Rothschild and Stiglitz (1976) point out that adverse selection equilibria can be inefficient in the following sense: We can envision a single insurance company offering two different contracts, one to high-riskers and one to low-riskers, which jointly break even though the high-risk contract loses money; that is, the low-riskers effectively subsidize the high-riskers. Such subsidies can not exist in competitive equilibrium, yet they be Pareto improvements over competitive equilibrium. Rothschild and Stiglitz give a condition for competitive equilibrium to be efficient in the sense that no such Pareto improvement is possible.

Continue to let  $p$  and  $q$  represent the low-risk and high-risk accident probabilities; let  $\gamma$  represent the fraction of the population that is high-risk, and let  $U$  be the utility function shared by all. Then the Rothschild-Stiglitz condition for efficiency is

$$\frac{\gamma(q-p)}{p(1-p)} > \frac{U'(Y)(U'(Z) - U'(X))}{U'(X)U'(Z)} \quad (1)$$

where  $X$  and  $Z$  are the incomes of low-risk individuals in the two states of the world and  $Y$  is the income of the (fully insured) high-risk individuals, at the appropriately constrained optimum.

Rothschild and Stiglitz describe the derivation of this inequality as “straightforward but tedious”. To generalize to the case where low-risk and high-risk individuals have

different utility functions  $U$  and  $V$  is equally straightforward, but thanks to the advent of modern symbolic computation packages, no longer tedious. The generalized inequality is

$$\frac{\gamma(q(1-p)V'(Z)U'(X) - p(1-q)U'(Z)V'(X))}{p(1-p)} > V'(Y)(U'(Z) - U'(X)) \quad (2)$$

which reduces to (1) in the adverse selection case  $U = V$ .

The propitious selection case is the one where  $V$  is more nearly linear than  $U$ ; to save notation I will restrict to the case where  $V$  is perfectly linear, so that  $V'$  is constant. Then (2) becomes

$$\frac{\gamma(q(1-p) - p(1-q)\frac{U'(Z)}{U'(X)})}{p(1-p)} > \frac{U'(Z) - U'(X)}{U'(X)} \quad (3)$$

It is an easy exercise in algebra (using  $U'(Z) > U'(X)$  and  $U'(Z) > U'(Y)$ , both of which must hold at the optimum) to show that (3) always implies (1). In other words, the condition for equilibrium to be efficient under propitious selection is more stringent than the same condition under adverse selection. Roughly, this says that propitious selection equilibria are *less* likely to be efficient than are adverse selection equilibria. This negative feature of propitious selection is *in addition* to the negative features discussed in Section 2.

#### 4. Concluding Observations

I have shown in Section 2 that propitious selection, as opposed to adverse selection, worsens the separation problem and leads to a worse equilibrium outcome (in those cases where an equilibrium exists).

It is possible to envision alternative mechanisms for propitious selection, but they all seem to founder on the same point. For example, an alternative setup might involve some fixed cost to purchasing insurance. Only the most risk-averse individuals are willing to incur this fixed cost, so those who take high risks are excluded from the market.

This setup doesn't work either. Figure 4 illustrates the high-risk and low-risk indifference curves through the endowment point  $E$ . In the vision just described, only one contract is offered and only low-riskers accept it.

We can illustrate the effect of the contract *inclusive of fixed costs* by a point in the figure, which must lie in the shaded area below the high-risk indifference curve in order to exclude high-riskers from the market. It must also lie above the low-risk indifference curve in order to appeal to low-riskers and on the low-risk fair-odds line through  $E$  (also pictured) by the zero profits condition. A glance at the figure reveals that there are no such points.

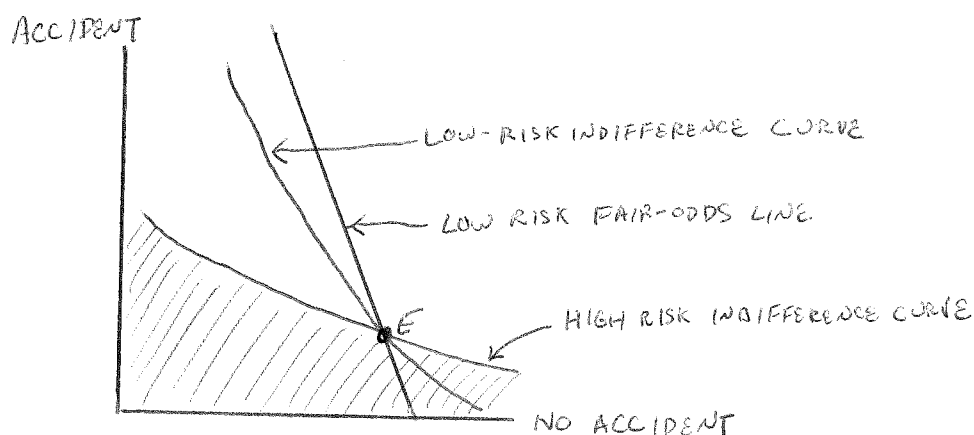


FIGURE 4

Similar considerations show that no system of state-dependent taxes and subsidies can solve the problem.

### Appendix. Degrees of Risk Preference

In this appendix I will address a question left over from Section 2. Suppose that the high-risk group is risk-preferring. How risk-preferring must they be before the low-risk group is offered partial insurance? How much more risk-preferring must they be before the low-risk group is offered full insurance?

Denote the high-riskers' utility-of-wealth function by  $V$ , and define their *coefficient of risk preference* at wealth level  $W$  by

$$\alpha = \frac{V''(W)}{V'(W)}.$$

$\alpha$  is positive by the assumption of risk-preference.

Continue to write  $W$  for endowed wealth and  $D$  for the cost of an accident. Then in Figure 2, the absolute slope of the indifference curve through point  $E$  is

$$\frac{1-q}{q} \cdot \frac{V'(W)}{V'(W-D)}$$

whereas the absolute slope of the line segment  $EC$  is

$$\frac{1-p}{p}.$$

In order to avoid the disappearance of the insurance market ala Figure 2A, the indifference curve must be steeper than the line segment, i.e. we must have

$$\frac{1-q}{q} \cdot \frac{V'(W)}{V'(W-D)} > \frac{1-p}{p}. \quad (4)$$

Linearly approximating  $V'(W-D)$  by  $V'(W) - DV''(W)$  and doing some algebra, (4) becomes (approximately)

$$D\alpha > \frac{q-p}{q-qp} \quad (5)$$

which is the (approximate) necessary and sufficient condition for an insurance market to exist in the face of risk preference.

In other words: when there is risk preference, an insurance market can exist only if either the coefficient of risk preference or the cost of accidents is large, in the sense specified by equation (5).

We can derive a similar condition for the survival of *full* insurance, made possible by the complete exit of risk-preferrers from the insurance market. In order for full insurance contracts to survive, the picture must be as in Figure 2C, which requires the high-risk indifference curve to pass above the point  $C = (W - pD, W - pD)$ . Thus we need

$$V(W - pD) < (1-q)V(W) + qV(W - D).$$

Taking a quadratic approximation to  $V$ , this becomes

$$D\alpha > \frac{2(q-p)}{q-p^2} \quad (6)$$

which is the (approximate) necessary and sufficient condition for propitious selection to be truly propitious. Again, either the degree of risk preference or the cost of accidents must be large.

Of course the conditions (5) and (6) are approximations and should be taken with appropriate grains of salt.

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