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I. Introduction

A central objective of the real business cycle research program is to construct models consistent with observed fluctuations in aggregate economic variables. These models typically assume that all economic activity takes place in the market. The thesis of this essay is that, for some purposes, it is useful to also explicitly consider nonmarket activity - or *household production*.

As a factual matter, the household sector is sizable, in terms of both the labor and capital inputs used in home production, and in terms of home produced output. Consider the following evidence for the US economy:

1. Studies such as the Michigan Time Use Survey indicate that a typical married couple allocates about 25 percent of its discretionary time to work in household production activities, including cooking, cleaning, child care and so on; by comparison, the typical couple spends about 33 percent of its discretionary time working for paid compensation (see Hill 1984 or Juster and Stafford 1991 for descriptions of the time use data).

2. The post-war national income and product accounts indicate that investment in household capital, defined as purchases of consumer durables and residential structures, actually exceeds investment in market capital, defined as purchases of producer durables and nonresidential structures, by about 15 percent (see Section III for details concerning the data and more precise calculations).

3. Attempts to measure the value of the output of home production come up with numbers between 20 and 50 percent of the value of measured market GNP (see the survey by Eisner 1988).

Despite these facts, household production has only recently been incorporated into macro models. It has, however, been part of the standard paradigm in labor economics for some time (fundamental references include Becker 1965, Pollak and Wachter 1975, and Gronau 1977, 1985). In his presidential address to the American Economic Association, Becker (1988) advocates the introduction of home production into macroeconomics, and several subsequent papers have pursued this. Rios-Rull (1992) includes home production in a dynamic general equilibrium model and analyzes life cycle, business cycle, and cross-sectional wage behavior. Nosal, Rogerson and Wright (1992) show that adding home production into two models of the labor market can affect the interpretation of unemployment and underemployment. Benhabib, Rogerson and Wright (1991) and Greenwood and Hercowitz (1991) explicitly incorporate household sectors into real business cycle theory. McGrattan, Rogerson and Wright (1992) and Fisher (1992) generalize these models and estimate their structural parameters econometrically. Fung (1992) introduces money into a home production model.

This research demonstrates that there can be interesting interactions between household and market activity. In this paper, we try to communicate and extend some of the findings in these papers. Our starting point is a basic growth model, modified to include a home production function that transforms household labor and capital into home produced output, just as the usual production function transforms market labor and capital into market output. Although it entails a relatively minor increase in complexity, the addition of a household sector implies a much richer model. For example, with home production, agents must allocate their time between leisure, market work and home work, rather than simply between leisure and labor as in the standard model. Similarly, they must allocate output between consumption, investment in market capital and investment in household capital, rather than simply between consumption and investment. This increase in generality can be significant for the analysis of both long run and business cycle issues.

We calibrate the model to match certain key first moments in the data, including the amount of time and capital allocated to both market and household activity. One finding that emerges from this exercise is that models with household production can more easily reconcile the evidence on the capital stock, labor's share of income, and taxation. We then simulate several alternative specifications of the model. The standard real business cycle model can be nested within our framework, in the sense that one can choose parameter values so that Hansen's (1985) model is a special case. For the parameter values that emerge from our calibration, however, the home production model does a better job than the standard model of accounting for the following aspects of the data: 1. the volatility of output; 2. the relative volatilities of output, consumption, investment, and hours; 3. the correlation between hours and productivity; and 4. the correlation between the investments in home and market capital.

The rest of the paper is organized as follows. In Section II we lay out a household's decision problem in a dynamic model with home production, and discuss how explicitly incorporating the household sector can make a difference in a qualitative sense. In Section III we embed this decision problem into a general equilibrium setting, and introduce functional forms and parameter values. In Section IV we report the results of simulating alternative specifications of the model and compare them with the data. In Section V we present an extension of the analysis designed to better capture certain long run growth facts. Some concluding remarks are contained in Section VI. The basic message is that household production models perform reasonably well (that is, better than models without home production) along many dimensions, although there remain some deviations between facts and theory that seem worthy of further investigation.

II. The Household Problem

Consider a decision maker with preferences described by

$$(2.1) \qquad U = \sum_{t=0}^{\infty} \beta^t u(c_{Mt}, c_{Ht}, h_{Mt}, h_{Ht}),$$

where $\beta \in (0,1)$ is the discount factor. The instantaneous utility function u is defined over four arguments at each date: c_{Mt} is consumption of a market produced commodity, c_{Ht} is consumption of a home produced commodity, h_{Mt} is labor time spent in market work, and h_{Ht} is labor time spent in home work. We normalize the total amount of discretionary time available in a period to unity, and define leisure to be the time remaining after market and home work: $\ell_t = 1-h_{Mt}-h_{Ht}$. All variables are constrained to be nonnegative, although we suppress these constraints in what follows. We assume that u is continuously differentiable and concave, and that $u_1 > 0$, $u_2 > 0$, $u_3 < 0$, and $u_4 < 0$.[1]

At each date, the individual is subject to a market budget constraint that allocates total income between three uses: the purchase of the market consumption good, the purchase of household capital, and the purchase of market capital. Capital goods purchased in one period are brought forward and become usable in the next period. Household capital, k_{Ht}, is used in home production, whereas market capital, k_{Mt}, is rented to firms and used in

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[1] Following Becker (1965), Greenwood and Hercowitz (1991) assume that there is no direct disutility to labor; following Gronau (1975), Benhabib et. al. (1991) assume that there is direct disutility to labor, as we do for the most part in this paper. Excluding labor from u has the advantage of theoretical and computational parsimony, but including it can generate some additional insights. Note that there are direct measures of all three variables – market work, home work, and leisure – in the time use survey data, and measures of both market work and home work in some panel data (like the PSID).

market production. If δ_M and δ_H are the depreciation rates on the two types of capital, w_t the wage rate, and r_t the rental rate on market capital, then (ignoring taxation, for now) the budget constraint can be written

$$(2.2)\qquad c_{Mt} + k_{Mt+1} + k_{Ht+1} = w_t h_{Mt} + r_t k_{Mt} + (1-\delta_M)k_{Mt} + (1-\delta_H)k_{Ht}.$$

The relative prices of the consumption and two capital goods equal unity, as we assume they can be transformed back and forth freely at a point in time.

The individual is also subject to the home production constraint at each date,

$$(2.3)\qquad c_{Ht} = g(h_{Ht}, k_{Ht}, z_{Ht}).$$

The home production function, g, yields consumption of the home good as a function of the time spent in home work and the household capital brought into the period, plus a stochastic term z_{Ht} representing technological change. We assume that g is increasing and concave in labor and capital. Note that (2.3) implies there are no uses for home produced output other than consumption – it cannot be sold or transformed into capital, for example, the way that market-produced output can. This is a key asymmetry between the market and home sectors: only the former can produce capital.[2]

We now define a reduced form utility function, by substituting the home production constraint into the momentary utility function and maximizing

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[2] The appropriate decision making unit in reality is a household or family, which may of course consist of more than one individual. This implies that it may be possible to consume a home cooked meal, for example, without actually cooking it. At the level of abstraction adopted here, the family is taken to be one single-minded decision making unit with no internal bargaining or disagreement. This may not be particularly realistic, but it does make things simpler. Lundberg and Pollack (1991) discuss bargaining within the family and provide references to the related literature.

with respect to time spent in home work, taking the values of the other variables as given:

$$(2.4) \qquad V(c_M, h_M, k_H, z_H) \equiv \max_{h_H} u[c_M, g(h_H, k_H, z_H), h_M, h_H].$$

It is straightforward to show that the reduced form utility function is continuous, increasing in c_M and k_H, decreasing in h_M, and concave in its first three arguments. Hence, V defines a well-behaved preference ordering over (c_M, h_M, k_H), conditional on z_H.

Now consider the following two individual decision problems:

$$(2.5) \qquad \max \sum_{t=0}^{\infty} \beta^t u(c_{Mt}, c_{Ht}, h_{Mt}, h_{Ht})$$

s.t. (2.2) and (2.3);

$$(2.6) \qquad \max \sum_{t=0}^{\infty} \beta^t V(c_{Mt}, h_{Mt}, k_{Ht}, z_{Ht})$$

s.t. (2.2).

Given the definition of V, problems (2.5) and (2.6) yield identical solutions for the variables c_{Mt}, h_{Mt}, k_{Ht}, and k_{Mt}. This implies the following observational equivalence result: for any model with momentary utility function u and home production function g, there is an alternative model with utility function V that makes no explicit reference to home production, except that k_{Mt} enters V directly, and delivers identical predictions for all variables that are traded in the market.[3]

[3] A converse result also holds: for any model without home production but with home capital entering the utility function directly, there is a model

One can conclude from this that there is nothing we can do with a home production model, in terms of explaining market quantities, that cannot be done, in principle, with a model that does not explicitly include home production. In practice, however, the home production approach provides us with direction and discipline in the specification of functional forms and parameter values, something that is obviously critical in the quantitative-theoretic real business cycle framework. Moreover, the interpretation of the results can hinge on whether we explicitly include home production, as can be seen from the following discussion.

Suppose that the momentary utility function is $u = \log(C) + A\cdot\log(\ell)$, where $C = C(c_M, c_H)$ is a composite of the two consumption goods and $\ell = 1-h_M-h_H$ is leisure. In the following cases, the reduced form utility function can be obtained analytically.

Case 1: $C = c_M + c_H$ and $g = a_0k_H + a_1h_H$ implies

$$V = \log[c_M + a_0k_H + a_1(1-h_M)].$$

Case 2: $C = c_M^a c_H^{1-a}$ and $g = a_0k_H + a_1h_H$ implies

$$V = a\log(c_M) + (1-a+A)\log[a_0k_H + a_1(1-h_M)].$$

Case 3: $C = c_M^a c_H^{1-a}$ and $g = k_H^{\eta}h_H^{1-\eta}$ implies

$$V = a\log(c_M) + (1-a)\eta\log(k_H) + [(1-a)(1-\eta)+A]\log(1-h_M).$$

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with a utility function of the form (2.1) and a well-behaved home production function that delivers the same predictions for all of the market variables.

Notice how V can be very different, starting from the same preferences over C and ℓ, depending on the degree of substitutability between the two types of consumption in the utility function and between the two inputs in the home production function. Case 3 delivers a reduced form that is equivalent to a model that has the same utility function, $u = \log(C) + A \cdot \log(\ell)$, and ignores the home production process — that is, a model that sets $C = c_M$ and $\ell = 1-h_M$ — but then simply adds the $\log(k_H)$ term.[4] Case 1, on the other hand, yields a reduced form in which c_M, k_H and $1-h_M$ are perfect substitutes — very different from the underlying utility function. Case 2 is intermediate, in the sense that $1-h_M$ and k_H are perfect substitutes while c_M enters separably.

An interesting feature of Case 1 is that although leisure as measured by $\ell = 1-h_M-h_H$ is a normal good according to u, the wealth effect on the quantity $1-h_M$ is identically zero according to V. That is, an increase in wealth leads to a reduction in h_M+h_H but no change in h_M. A version of this functional form is the one used by Greenwood, Hercowitz and Huffman (1988) in their business cycle model, and it has desirable properties resulting from the fact that it implies a large labor supply elasticity because the wealth effect on h_M is zero.[5] Without considering the underlying home production model, however, one could question a choice of functional form that implies a zero wealth effect on h_M. In particular, how can it be reconciled with the fact that h_M has displayed no trend growth over long time periods despite large increases in market productivity and real wages?

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[4] This is what has often been done in the literature on consumer durables; see the discussion in Macklem (1989), for example.

[5] Devereaux, Gregory and Smith (1992) also show how these preferences can be used to reconcile the puzzle that observed cross-country consumption correlations are lower in the data than predicted by conventional open-economy real business cycle models.

To understand this, note that this long run fact implies that the substitution and wealth effects on market hours offset each other (see King, Plosser and Rebelo 1987, for example), and therefore the wealth effect must be negative. Starting with a zero wealth effect specification appears to be inconsistent with this observation. However, in a home production model with the above specification, market hours will be constant as long as productivity in the home and productivity in the market increase at the same rate on average. That is, a home production model can generate a balanced growth path with no trend in h_{Mt} and h_{Ht} even though the reduced form utility function implies a zero wealth effect on h_{Mt}. The growth in home productivity shows up in the reduced form as trend growth in the marginal utility of leisure. Of course, one can assume directly that preferences are changing over time in a particular way; but incorporating home production suggests an arguably more palatable interpretation.[6]

Another implication is that whenever the home technology is stochastic we end up with what look like preference shocks in the reduced form utility function V. This can improve the performance of business cycle models along some dimensions.[7] Positive shocks to the marginal utility of leisure or the home technology reduce labor supplied to the market, which tends to induce a negative relation between market hours and productivity. This counteracts

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[6] This analysis bears on another issue in real business cycle theory. The indivisible labor model of Rogerson (1984, 1988) as used by Hansen (1985) and several others has the following property: unemployed workers enjoy greater utility than their employed counterparts, not only for the particular functional forms used in those studies, but for any utility function that implies leisure is a normal good. If we incorporate home production, however, the model need not have this implication; see Nosal et. al. (1992) for details.

[7] For example, Bencivenga (1991) introduces preference shocks directly, while Christiano and Eichenbaum (1992) incorporate innovations to government spending that can be interpreted as taste shocks.

the positive relation between hours and productivity induced by shocks to the market technology. Models with shocks to both technologies can generate patterns of hours versus productivity closer to the data than those with only market shocks. Of course, to make these arguments precise in a quantitative sense we need a fully specified general equilibrium model; this is what we provide in the next section.

III. The General Equilibrium Model

This section specifies and calibrates the general equilibrium model with household production. There is a large number of identical infinite-lived agents, with instantaneous utility function specified by

$$u = b\log(C_t) + (1-b)\log(\ell_t), \tag{3.1}$$

where

$$C_t = [ac_{Mt}^e + (1-a)c_{Ht}^e]^{1/e} \tag{3.2}$$

and $\ell_t = 1-h_{Mt}-h_{Ht}$. The elasticity of substitution between c_{Mt} and c_{Ht} is given by $1/(1-e)$. For simplicity, the two types of work are always assumed to be perfect substitutes in what follows.

There is a representative firm with a constant returns to scale technology described by the market production function,

$$f(h_{Mt},k_{Mt},z_{Mt}) = k_{Mt}^{\theta}(z_{Mt}h_{Mt})^{1-\theta}. \tag{3.3}$$

All households have access to the home production function

$$(3.4) \qquad g(h_{Ht},k_{Ht},z_{Ht}) = k_{Ht}^{\eta}(z_{Ht}h_{Ht})^{1-\eta}.$$

Here, θ and η are the capital share parameters and z_{Mt} and z_{Ht} represent labor augmenting technological change. Technical change occurs as follows: $z_{Mt} = \lambda^t\tilde{z}_{Mt}$ and $z_{Ht} = \lambda^t\tilde{z}_{Ht}$, where λ^t is a deterministic component, and $\tilde{z}_{Mt}$ and $\tilde{z}_{Ht}$ are stochastic processes with

$$(3.5) \qquad \log(\tilde{z}_{Mt+1}) = \rho_M\log(\tilde{z}_{Mt}) + \varepsilon_{Mt+1}$$

$$(3.6) \qquad \log(\tilde{z}_{Ht+1}) = \rho_H\log(\tilde{z}_{Ht}) + \varepsilon_{Ht+1}.$$

We assume that $|\rho_M|$ and $|\rho_H|$ are less than unity. The innovations ε_{Mt} and ε_{Ht} are independent and identically distributed over time, with standard deviations σ_M and σ_H and contemporaneous correlation γ.

For reasons that will become apparent in the calibration exercise, it is important to include taxation. We therefore assume that each period the government levies proportional taxes on labor and capital income (net of depreciation) at the constant rates τ_h and τ_k, transfers a lump sum T_t back to individuals, and consumes the surplus. Hence, government consumption is given by

$$(3.7) \qquad G_t = w_t h_{Mt}\tau_h + r_t k_{Mt}\tau_k - \tau_k\delta_M k_{Mt} - T_t,$$

where $\delta_M k_{Mt}$ is a depreciation allowance. For simplicity, we assume from now on that all revenue is rebated as a lump sum back to consumers, so that G_t = 0 in what follows.[8]

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[8] More generally, G could enter the utility function and we could assume

Feasibility implies market output, $y_t = f(h_{Mt}, k_{Mt}, z_{Mt})$, is allocated across market consumption c_{Mt}, investment x_t, and government spending G_t:

$$y_t = c_{Mt} + x_t + G_t. \tag{3.8}$$

Investment augments the capital stock according to the law of motion

$$k_{t+1} = (1-\delta_M)k_{Mt} + (1-\delta_H)k_{Ht} + x_t. \tag{3.9}$$

The aggregate stock can be divided between market and household capital at a point in time according to $k_t = k_{Mt} + k_{Ht}$. Although capital can be freely transformed between its two uses, it may depreciate at different rates in its two uses. Investment in each of the two capital goods is defined residually by[9]

$$x_{Mt} = k_{Mt+1} - (1-\delta_M)k_{Mt} \tag{3.10}$$

$$x_{Ht} = k_{Ht+1} - (1-\delta_H)k_{Ht}. \tag{3.11}$$

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that G in the model mimics government spending in the data (either its stochastic properties, or at least its average). Note, however, that if we assume government consumption is a perfect substitute for market consumption in the utility function, then a model with $G \neq 0$ generates exactly the same statistics as a model with $G = 0$, except for the fact that c_M will change one-for-one to offset changes in G.

[9] Although capital is freely mobile between home and market at a point in time, in the experiments that we conducted it is rare that any capital physically moves between sectors, since typically gross (if not net) investment in each is positive. Hence, free mobility seems to play little role. What is important, however, is that capital does not have to be committed to either sector until the shocks have been observed. Greenwood and Hercowitz (1991) assume that capital does have to be allocated in advance, which has some advantages in terms of the results. We adopt the specification in the text for simplicity.

A competitive equilibrium for this economy is defined in the usual manner.[10] The representative firm solves a sequence of static problems at each date: maximize instantaneous profit Π_t, where

$$(3.12) \qquad \Pi_t = f(h_{Mt}, k_{Mt}, z_{Mt}) - w_t h_{Mt} - r_t k_{Mt},$$

taking as given w_t, r_t, and z_{Mt}. The representative consumer maximizes EU where U is given by (2.1), subject to a budget constraint modified to include taxes

$$(3.13) \qquad c_{Mt} + x_{Mt} + x_{Ht} = w_t(1-\tau_h)h_{Mt} + r_t(1-\tau_k)k_{Mt} + \delta_M \tau_k k_{Mt} + T_t$$

and the home production constraint (2.3), taking as given stochastic processes for $\{w_t, r_t, T_t\}$ and the shocks. Given stochastic processes for the exogenous variables (technology shocks and policy variables) and initial capital, an equilibrium is a set of stochastic processes for prices $\{w_t, r_t\}$ and quantities $\{c_{Mt}, c_{Ht}, h_{Mt}, h_{Ht}, k_{Mt}, k_{Ht}\}$ that solve both the producer and the consumer problems.

In order to calibrate the model, we need to derive some properties of the balanced growth path (that is, the equilibrium path to which the economy converges when $z_{Mt} = z_{Ht} = \lambda^t$ for all t). When $z_{Mt} = z_{Ht} = \lambda^t$, given the initial conditions, the equilibrium converges to a path where $h_{Mt} = h_M$ and $h_{Ht} = h_H$ are constant, while all other endogenous variables grow at rate λ,

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[10] Due to the presence of distorting taxes, equilibrium allocations are not generally Pareto optimal, so we have to work with the equilibrium directly rather than the social planner's problem. The discussion here is not intended to be particularly rigorous. Greenwood and Hercowitz (1991) define a recursive competitive equilibrium for the model more carefully, along the general lines of the chapter by Hansen and Prescott in this volume. The solution procedure that we use is described in detail in McGrattan (1991).

so that $y_t = y\lambda^t$ for some constant y, $c_{Mt} = c_M\lambda^t$ for some constant c_M, and so on. To describe this in more detail, substitute the budget and home production constraints into the consumer's objective function and then differentiate to obtain the first order conditions:

$$(3.14) \qquad h_{Mt}: \qquad u_1(t)w_t(1-\tau_h) = - u_3(t)$$

$$(3.15) \qquad h_{Ht}: \qquad u_2(t)g_1(t) = - u_4(t)$$

$$(3.16) \qquad k_{Mt}: \qquad u_1(t)[r_t(1-\tau_k)+1-\delta_M+\delta_M\tau_k] = u_1(t-1)/\beta$$

$$(3.17) \qquad k_{Ht}: \qquad u_1(t)(1-\delta_H) + u_2(t)g_2(t) = u_1(t-1)/\beta$$

where the notation $\xi(t)$ means that a function ξ is evaluated at its arguments as of date t.

If we use the first order conditions for the firm problem, $w_t = f_1(t)$ and $r_t = f_2(t)$, then given our functional forms the above expressions can be simplified to yield

$$(3.18) \qquad abc_M^{e-1}C^{-e}y(1-\theta)(1-\tau_h) = (1-b)h_M/\ell$$

$$(3.19) \qquad (1-a)bc_H^eC^{-e}(1-\eta) = (1-b)h_H/\ell$$

$$(3.20) \qquad \theta(1-\tau_k)y/k_M = \lambda/\beta - 1 + \delta_M(1-\tau_k)$$

$$(3.21) \qquad \eta(1-a)c_H^ec_M^{1-e}/ak_H = \lambda/\beta - 1 + \delta_H$$

Additionally, equations (3.10) and (3.11) imply

$$x_M/k_M = \lambda - 1 + \delta_M \tag{3.22}$$

$$x_H/k_H = \lambda - 1 + \delta_H. \tag{3.23}$$

We now proceed to choose parameter values, setting some numbers based on *a priori* information and setting the others according to the balanced growth conditions. Since we interpret the period as one quarter, we set λ = 1.005 in order to match the quarterly growth rate of output in our data.[11] The discount factor is set so that the annual real rate of return on assets in the model is about 6 percent, which yields $\beta = 0.9898$. We set the labor income tax rate to $\tau_h = 0.25$, the average value in the series in McGrattan et. al. (1992), which is based on the definitions in Joines (1981). The effective tax rate on capital income is more controversial, and there is a wide range of estimates in the literature. For example, the series in McGrattan et. al. implies τ_k is about 0.50 on average, while Feldstein, Dicks-Mireaux and Poterba (1983) estimate τ_k is between 0.55 and 0.85 in the period 1953-1979.

We use the mean of the Feldstein, Dicks-Mireaux and Poterba estimates, and set $\tau_k = 0.70$. This is higher than some other studies in the real business cycle literature, but two reasons suggest that it is the right number for our purposes. First, given that we are trying to model both market and nonmarket investment, we want τ_k to capture all forms of government regulation, interference, or any other institutional disincentive to invest in market capital, and not only direct taxation. Second, the capital's share coefficient in the market production function, θ, which is

[11] Table 1 below reports exact parameter values; in the text, we round off most parameters to a few digits.

calibrated below, turns out to be sensitive to the choice of the capital income tax rate. Setting $\tau_k = 0.70$ implies a value for θ that is consistent with independent evidence from the national income accounts (we will return to this issue in what follows).

We now use (3.18)-(3.23) to match the following six observations: the two capital/output ratios, the two investment/output ratios, and labor hours in the two sectors. The postwar national income and product accounts yield $k_M/y = 4$, $k_H/y = 5$, $x_M/y = 0.118$, and $x_H/y = 0.135$, on average, where home capital is measured by consumer durables plus residential structures and market capital is measured by producer durables plus nonresidential structures. Averaging data from the 1971 and 1981 time use surveys, we find $h_H = 0.25$ and $h_M = 0.33$ for a typical household, where these numbers are defined as fractions of discretionary time (24 hours per day minus personal care, which is mainly sleep). These six observations determine δ_M, δ_H, θ, η, and two of the three preference parameters a, b and e.

The system (3.18)-(3.23) has a simple recursive structure. Equations (3.22) and (3.23) yield $\delta_M = 0.0247$ and $\delta_H = 0.0218$, which we approximate by setting the two depreciation rates to a common value of $\delta = 0.0235$. Equation (3.20) yields $\theta = 0.29$, and then (3.21) yields $\eta = 0.32$.[12] The value $\theta = 0.29$ is also exactly what we compute from the national income and product accounts.[13] Three preference parameters remain to be specified, a, b and e, but we only have two equations left. In what follows, we consider

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[12] It looks as though one needs to know the parameter a in order to determine η from (3.21); however, a can be eliminated from (3.21) using other conditions.

[13] To compute θ from the national income accounts, we subtract proprietor's income from total income, as is standard, and also subtract the service flow attributed to the housing stock from output since this is household and not market output. The result is $\theta = 0.29$ in our sample.

several alternative values of e, which is the parameter that determines the elasticity of substitution between c_M and c_H, and for each we solve for the values of a and b from (3.18) and (3.19). As e varies a and b will change, but the values of δ_M, δ_H, θ, and η will not.

Finally, we need to specify the parameters describing the stochastic elements of the model. As in much of the literature we set $\rho_M = 0.95$ and set σ_M so that the innovation in $(\tilde{z}_{Mt})^{1-\theta}$ has a standard deviation of 0.007. We then set $\rho_M = \rho_H$ and $\sigma_M = \sigma_H$, so that the home shock mimics the market shock. This leaves γ, which is the correlation between the innovations ε_{Mt} and ε_{Ht}. Unfortunately there is little independent evidence to guide us in choosing this parameter. In what follows, as with the preference parameter e, we report the results of experiments with different values of γ.

To summarize, all of the parameters except e and γ have been set. The parameter e measures agents' willingness and the parameter γ measures agents' incentive to move economic activity between the home and market. Higher values of e mean that agents are more willing to substitute consumption of one sector's output for that of the other. Lower values of γ mean that the technology shocks more frequently take on different values across sectors and this implies a greater incentive to move resources across sectors. As will be shown in the next section, changing either the willingness or incentive to substitute between the home and market can affect the implications of the model for business cycles.

To close this section we return to the interaction between taxes and household production. Consider a model without taxation under the standard assumption that the entire capital stock enters into the market production function, so that k_M/y is about 9. Then, calibrating the model as we did above, we find $\theta = 0.34$, which is close to the value implied by the national income accounts and typically used in the real business cycle literature.[14] However, zero taxes are clearly counterfactual. If we set $\tau_k = 0.70$, then in order to get $k_M/y = 9$ we need to set $\theta = 0.66$, which seems far too high. Even a more conservative tax rate of $\tau_k = 0.50$ implies $\theta = 0.48$, which still seems too high. Intuitively, when capital income is taxed we must assume the marginal product of capital is big in order to get agents to accumulate a stock as large as $k_M/y = 9$, and θ is the key parameter governing this marginal product. In a home production model we do not interpret all capital as market capital; therefore, k_M/y is 4 rather than 9. This in combination with taxation implies $\theta = 0.29$, which is just what we observe in our data.

IV. Simulation Results

Table 1 lists some summary statistics for the U.S. economy, and for several versions of the model to be described below.[15] We focus on the following statistics: the standard deviation (in percent) of y; the standard deviations relative to y of x, c_M, h_M, and w; the correlation between h_M and w; and the correlation between x_M and x_H. The variable w can either be interpreted as the real wage or, equivalently, as the average product of

[14] Depending on details, such as how one treats proprietors' income, the national income accounts indicate that θ could be anywhere between 0.25 and 0.43 (see Christiano 1988, for example). Prescott (1986) argues for $\theta = 0.36$, while, as indicated earlier, we find $\theta = 0.29$.

[15] The U.S. data are quarterly and are from the period 1947:1-1987:4. Often in the literature, only data after 1955 are considered, presumably to eliminate the effect of the Korean war; summary statistics are similar in the two periods (see Hansen and Wright 1992). We take logarithms and detrend using the Hodrick-Prescott filter (see Cooley and Prescott in this volume) before computing statistics, both for the U.S. data and for data generated by the models. The notes to the table provide more details.

hours worked in the market, since the wage equals the marginal product in equilibrium and the marginal product is proportional to average product with a Cobb-Douglas technology. Investments in the two capital stocks are defined by letting market capital be producer structures plus equipment and letting home capital be residential structures plus consumer durables. Total investment is the sum. Consumption is defined to include nondurables plus services minus the service flow imputed to the housing stock. Market output is defined to be consumption plus investment and government spending. Market hours are from the household survey.

In Model 1 we set $e = 0$, implying that the elasticity of substitution between c_M and c_H is unity. We also set the correlation between the shocks ε_M and ε_H to $\gamma = 2/3$, as in Benhabib et. al. (1991) (although when $e = 0$ the value of γ does not matter for the results). Except for minor details, Model 1 is the base model in Greenwood and Hercowitz (1991), and is designed to minimize the role of household production. This can be seen from Case 3 in the examples analyzed in Section II, which is the current specification with $e = 0$. Recall that in this case the reduced form utility function is

$$V = a\log(c_M) + (1-a)\eta\log(k_H) + [(1-a)(1-\eta)+A]\log(1-h_M).$$

If $\eta = 0$, this reduces to the standard utility function that ignores home production. Hence, the home production model replicates the results of the standard model exactly when $e = \eta = 0$. Even if $\eta > 0$, when $e = 0$ the home production model generates results that are close to the standard model.

As is well known, the statistics generated by the standard model, and therefore the results generated by Model 1, differ from the data along several dimensions. First, output is less volatile in the model than in the data. Second, in the model, investment is too volatile and consumption is not volatile enough relative to output. Third, in the model, hours worked are not volatile enough relative to either output or productivity. Fourth, hours worked and productivity are highly positively correlated in the model but not in the data. Fifth, the two investment series are positively correlated in the data but not in the model. See the Kydland chapter in this volume, Hansen and Wright (1992), and Benhabib et. al. (1991) for discussions. Although the results generated by Model 1 are perhaps somewhat better than the prototypical real business cycle model, such as Hansen's (1985) divisible labor model, the model still differs from the data along the five dimensions listed above.

In Model 2 we set $e = \gamma = 2/3$. This corresponds to a situation where consumers are much more willing to substitute between c_M and c_H than in Model 1. Notice first that, in comparison to Model 1, the volatility of output in Model 2 has increased. Second, the relative volatility of investment has decreased and that of consumption has increased. Third, hours have become more variable relative to output and to productivity. Fourth, the correlation between hours and productivity has decreased slightly, although not very much. Fifth, the correlation between the two investment series is lower. As found in Benhabib et. al. (1991), increasing the value of e moves the model in the right direction vis a vis the data, except for the correlation between x_M and x_H.

Benhabib et. al. (1991) set e and γ more or less arbitrarily. Another approach is to estimate the model using maximum likelihood techniques, as in McGrattan et. al. (1992). This procedure yields $e = 0.4$ and $\gamma = 0$ (after rounding), which we use in Model 3. These parameter values correspond to a situation where, as compared to Model 2, consumers are less willing to substitute between the two sectors but there is more of an incentive to do so. Notice that Models 2 and 3 yield similar results. This illustrates the interaction between assuming that individuals are more willing to substitute (a higher value of e) and assuming they have greater incentives to do so (a lower value of γ): raising e for a given γ is very similar to lowering γ for a given e.[16]

Although neither Model 2 or 3 does well in terms of the correlation between h_M and w, this is a statistic that can in principle be matched by introducing home production. Intuitively, the standard model with shocks only to the market technology is driven by a shifting labor demand curve, so simulations trace out in (h_M, w) space a stable upward sloping labor supply curve and yield a correlation between the two variables close to unity. What is needed is a second shock to shift labor supply, such as a preference or home technology shock.[17] Home technology shocks change the amount people are willing to work in the market at a given wage, shifting the labor supply curve and reducing the hours-productivity correlation. In Models 2 and 3 this effect is present but small. Increasing the standard deviation of the home technology can reduce the correlation between hours and productivity much more, however; see Hansen and Wright (1992) for further discussion.

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[16] One might think that the parameter values from McGrattan et. al. (1992) would do even better than indicated by the results in Table 1 since, after all, they were estimated by fitting the model to the aggregate time series. Several points are relevant in this regard. First, the model in that paper differs from the one here in certain respects, such as the fact that it includes *stochastic* taxation and government consumption. Second, although we use the same e and γ, some of the other parameter values are different. Finally, the likelihood function takes into account aspects of the time series other than the small number of second moments computed from filtered data considered in Table 1; for example, estimation trades off the fit at business cycle frequencies against the fit at other frequencies.

[17] Christiano and Eichenbaum (1992) argue for preference shocks, which (again) they identify with changes in government spending. The idea is that as long as government spending is less than a perfect substitute in utility for private consumption an increase in G entails a negative wealth effect which shifts labor supply. Stochastic tax shocks, as in McGrattan (1990) or Braun (1990), can have similar effects in terms of shifting labor supply.

We now turn to the correlation between x_M and x_H, which the above models do not capture well at all. The problem is that in times of high relative market productivity agents want to move inputs out of the home and into the market (since, in particular, that is where they can build capital in order to spread the effects of a temporary productivity rise into the future). The movement of resources between the two sectors is part of what makes a home production model work: the reallocation of hours from nonmarket to market labor, rather than exclusively from leisure to labor as in the standard model, increases the volatility of h_M for a given technology shock. But it also leads to a problem: How can we make agents want to invest in both market and home capital at the same time, especially when the home and market labor inputs are moving in opposite directions over the cycle?

Greenwood and Hercowitz (1991) approach the problem by assuming a more general home production function,

$$(4.1) \qquad g(h_H, k_H, s_H) = [\eta k_H^{\psi} + (1-\eta)(z_H h_H)^{\psi}]^{1/\psi}$$

($\psi = 0$ reduces to the Cobb-Douglas case we have considered so far). They also assume that the shocks z_H and z_M are highly correlated, so that when a positive technology shock hits the market it also hits the home. When a positive shock arrives, since z_H is labor augmenting, it is possible to move hours out of the home and into the market and still end up with more *effective* hours in the home. That is, $z_H h_H$ can increase while h_H decreases. Thus, effective hours in home production can increase during upswings in market activity and, depending on ψ, this can imply a desire to increase capital in the home.

Model 4 uses the technology in (4.1) with $\psi = -1/2$, $\gamma = 0.99$, $e = 2/3$, and otherwise keeps the parameters as described above. As can be seen, this does generate a positive correlation between x_M and x_H. However, it requires a high correlation between the shocks, and if the two shocks are very highly correlated the model does not entail frequent incentives to substitute between household and market activity. Therefore, generating a positive correlation between x_M and x_H involves sacrificing at least part of the other improvements that can be achieved by introducing home production. It is not obvious how to resolve this tension. Additionally, the U.S. data display a clear phase shift, with investment in household capital leading investment in market capital. Building a model that better accounts for these phenomena remains to be done.

Let us summarize the findings from these experiments. With e = 0, the model generates second moments that are similar to but somewhat better than those of a standard model without home production. By increasing e for a given γ we can affect the volatility of output, investment, consumption and hours in the right direction. A similar effect can be obtained by decreasing γ for a given e. These results do not require a large home shock, and we point out that the model performs about as well if the home technology is nonstochastic.[18] However, the larger the home shock the better the resulting correlation between hours and productivity. The correlation between investments in the two sectors can also be improved by considering a more general home technology, although this tends to reduce the impact of home production along other dimensions.[19]

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[18] This is because, even if the home technology is nonstochastic, shocks to the market production function obviously still induce relative productivity differentials between the sectors.

[19] Combining indivisible labor with home production may be interesting, since this should increase the volatility of hours, and help with the correlation between hours and productivity, given the second shock. Combining home production with stochastic tax or government spending shocks may have similar effects.

V. An Extension

In this section we briefly discuss an extension of the framework that is capable of replicating the following fact: over time, the relative prices of consumer durables and producer durables have declined while expenditures on these items have remained roughly constant. Therefore, we do not actually observe balanced growth in the data, since the stocks of both producer and consumer durables are growing faster than the other series. Following Greenwood, Hercowitz and Krusell (1992), we address this by assuming that technological change is embodied in the form of new capital goods – in our context, in the home sector as well as the market sector. We also allow capital utilization to be a variable factor of production, and assume that the cost to using capital more intensely is that it depreciates more quickly. We will not completely solve the model sketched here and we have not attempted calibration or simulation; the discussion is only meant to suggest future research topics.

Let the market and home technologies be described by the following functional forms,

$$(5.1) \qquad f(h_M, \mu_M k_M) = (\mu_M k_M)^{\theta} h_M^{1-\theta}$$

$$(5.2) \qquad g(h_H, \mu_H k_H) = (\mu_H k_H)^{\eta} h_H^{1-\eta},$$

where μ_M and μ_H are capital utilization rates. Higher utilization results in increased depreciation: $\delta_M = \delta_M(\mu_M)$ and $\delta_H = \delta_H(\mu_H)$, where $\delta_M' > 0$ and $\delta_H' > 0$. Notice that these technologies are deterministic. The shocks enter via the relative prices of capital goods, so that

(5.3) $$k_{Mt+1} = [1-\delta(\mu_M)]k_{Mt} + x_{Mt}z_{Mt}$$

(5.4) $$k_{Ht+1} = [1-\delta(\mu_H)]k_{Ht} + x_{Ht}z_{Ht}.$$

For example, a high value of z_{Mt} indicates a favorable rate at which market output can be transformed into capital next period. The shocks follow the usual processes: $z_{Mt} = \lambda^t\tilde{z}_{Mt}$ and $z_{Ht} = \lambda^t\tilde{z}_{Ht}$, where $\tilde{z}_{Mt}$ and $\tilde{z}_{Ht}$ are given by (3.5) and (3.6).

For simplicity, ignore taxation. Then the equilibrium can be found as the solution to the planner's problem of maximizing EU, where U is given by (2.1), subject to the home production constraint (2.3) and

(5.5) $$c_{Mt} + x_M + x_H = f(h_M, \mu_M k_M).$$

To describe the deterministic growth path, consider the case where $z_{Mt} = z_{Ht} = \lambda^t$ for all t. Then the first order conditions imply

(5.6) $$h_{Mt}: \quad u_1(t)f_1(t) = - u_3(t)$$

(5.7) $$h_{Ht}: \quad u_2(t)g_1(t) = - u_4(t)$$

(5.8) $$k_{Mt}: \quad u_1(t)f_2(t)\mu_{Mt} + u_1(t)[1-\delta_M(\mu_{Mt})]/z_{Mt} = u_1(t-1)/\beta z_{Mt-1}$$

(5.9) $$k_{Ht}: \quad u_2(t)g_2(t)\mu_{Ht} + u_1(t)[1-\delta_H(\mu_{Ht})]/z_{Ht} = u_1(t-1)/\beta z_{Ht-1}$$

(5.10) $$\mu_M: \quad f_2(t) = \delta_M'(\mu_{Mt})/z_{Mt}$$

(5.11) $$\mu_H: \quad g_2(t)u_2(t)/u_1(t) = \delta_H'(\mu_{Ht})/z_{Ht}.$$

Assume that the instantaneous utility function is given by (3.1) with e = 0 — that is, with a unitary elasticity of substitution between c_M and c_H. Then it is straightforward, if somewhat tedious, to verify that the model has an "almost balanced" growth path with the following properties. First, h_{Mt}, h_{Ht}, μ_{Mt} and μ_{Ht} are all constant. Second, y_t, c_{Mt}, x_{Mt} and x_{Ht} all grow at the same rate, which is $\lambda^{\theta/(1-\theta)}$. Third, k_{Mt} and k_{Ht} both grow at the rate $\lambda^{1/(1-\theta)}$, which is greater than the growth rate of market output because $\theta < 1$. Fourth, c_{Ht} grows at the rate $\lambda^{\eta/(1-\theta)}$, which can be greater than or less than the growth rate of market output, depending on whether η is greater than or less than θ. We conclude that this model is capable of rationalizing the observation that the growth rates of the capital stocks exceed the growth rates of other market variables. It also has a home sector that can either grow or shrinks relative to the market, depending on parameter values.

Although we have not done so, it may be worth pursuing the business cycle implications of this structure. Suppose, for example, that the shocks z_{Mt} and z_{Ht} are highly correlated. Then positive shocks reduce the cost of acquiring both consumer and producer durables and therefore tend to increase both investments (see 5.8 and 5.9). To the extent that home consumption goes up by more than market consumption, because market output is being invested rather than consumed, x_H will go up by less than x_M (see 5.9). There will also be an increase in capacity utilization and hours worked in the market sector to take advantage of the lower cost of putting capital in place. Furthermore, positive shocks encourage greater capital utilization in order to accelerate the depreciation of the existing stock, since replacing it is now cheaper (see 5.10 and 5.11). Of course, this discussion is only meant to be suggestive, and the net quantitative effects generated by such a model remain to be seen. We leave exploration of this to future research.

VI. Conclusion

We have argued that home production is empirically sizable, and further suggested that there may be interesting interactions between the household and market sectors. We have shown how to incorporate home production into an otherwise standard real business cycle model and how to calibrate it. With reasonable parameter values it is possible to replicate first moment properties of the U.S. data, including the observed allocation of capital and time to both market and home production. In terms of the dynamic properties of the model, it does a good job of accounting for the standard features of observed business cycles. There do remain deviations between the theory and data, such as some aspects of the behavior of the two investment series. As should be expected, the exact results depend on the willingness and incentive to substitute between the home and market sectors and on the functional form of the home technology. There is unfortunately not a lot of independent evidence on the parameters dictating these features of the model, and it would seem worthwhile to try to uncover more such information from micro studies.

In summary, we have tried in this chapter to illustrate both the strengths and the weaknesses of incorporating home production into macroeconomic models, and to make suggestions for future research projects.

Table 1: Summary Statistics

	σ_y	σ_x/σ_y	σ_{c_M}/σ_y	σ_{ℓ_M}/σ_y	σ_w/σ_y	σ_{ℓ_M}/σ_w	$c(h_M,w)$	$c(x_M,x_H)$
Data	1.96	2.61	0.54	0.78	0.73	1.06	-0.12	0.30
Model 1	1.36	2.82	0.41	0.41	0.60	0.68	0.96	-0.09
Model 2	1.60	2.34	0.61	0.52	0.52	1.00	0.86	-0.82
Model 3	1.59	2.44	0.53	0.48	0.53	0.91	0.95	-0.75
Model 4	1.21	2.95	0.38	0.39	0.62	0.63	0.95	0.50

Notes:

Data are y = gross national product minus gross housing product, x_M = fixed nonresidential private investment, x_H = private residential investment plus personal consumption expenditure on durable goods, $x = x_M + x_H$, c_M = personal consumption of nondurables plus services minus gross housing product, h_M = manhours of employed labor force (household survey), $w = y/h_M$. All series are quarterly and are from the period 1947:1-1987:4. Nominal variables are deflated into 1982 dollars. The series were divided by population, logged, and detrended using the Hodrick-Prescott filter. The statistic σ_j is the standard deviation of series j (expressed as a percent), and $c(j,j')$ is the correlation between series j and j'. Model statistics are sample means over 50 simulations, each the same length as our data.

All models use $\lambda = 1.004674$, $\beta = 0.9898$, $\tau_h = 0.25$, $\tau_k = 0.70$, $\delta_M = \delta_H = 0.0235$, $\theta = 0.2944$, $\eta = 0.3245$, a and b determined so that $h_M = 0.33$ and $h_H = 0.25$, $\rho_M = \rho_H = 0.95$, and $\sigma_M = \sigma_H$ determined so that the innovation in $z_M^{1-\theta}$ has standard deviation 0.007. Model 1 sets $e = 0$, $\gamma = 2/3$. Model 2 sets $e = 2/3$, $\gamma = 2/3$. Model 3 sets $e = 0.40$, $\gamma = 0$. Model 4 sets $e = 2/3$, $\gamma = 0.99$, and uses a CES home production function with $\psi = -0.5017$.

References

Gary S. Becker (1965) "A Theory of the Allocation of Time," *Economic Journal* 75, 493-517.

Gary S. Becker (1988) "Family Economics and Macro Behavior," *American Economic Review* 78, 1-13.

Valerie R. Bencivenga (1991) "An Econometric Study of Hours and Output Variation with Preference Shocks," *International Economic Review*, in press.

Jess Benhabib, Richard Rogerson and Randall Wright (1991) "Homework in Macroeconomics: Household Production and Aggregate Fluctuations," *Journal of Political Economy* 99, 1166-87.

R. Anton Braun (1990) "The Dynamic Interaction of Distortionary Taxes and Aggregate Variables in Postwar U.S. Data," working paper, University of Virginia.

Lawrence J. Christiano (1988) "Why Does Inventory Investment Fluctuate so Much?" *Journal of Monetary Economics* 21, 247-80.

Lawrence J. Christiano and Martin Eichenbaum (1992) "Current Real Business Cycle Theory and Aggregate Labor Market Fluctuations," *American Economic Review* 82, 430-450.

Michael B. Devereaux, Allan W. Gregory and Gregor W. Smith (1992) "Realistic Cross-Country Consumption Correlations in a Two-Country, Equilibrium, Business Cycle Model," *Journal of International Money and Finance* 11, 3-16.

Robert Eisner, (1988) "Extended Accounts for National Income and Product," *Journal of Economic Literature* 26, 1611-84.

Martin S. Feldstein, Louis Dicks-Mireaux and James Poterba (1983) "The Effective Tax Rate and the Pretax Rate of Return," *Journal of Public Economics* 21, 129-158.

Jonas D. M. Fisher (1992) "Relative Prices, Complementarities and Co-movement," manuscript, Northwestern University.

Siu C. Fung (1992) "Inflation, Taxation and Home Production in a Real Business Cycle Model," manuscript, University of Western Ontario.

Jeremy Greenwood and Zvi Hercowitz (1991) "The Allocation of Capital and Time Over the Business Cycle," *Journal of Political Economy* 99, 1188-1214.

Jeremy Greenwood, Zvi Hercowitz and Gregory Huffman (1988) "Investment, Capacity Utilization and the Real Business Cycle," *American Economic Review* 78, 402-417.

Jeremy Greenwood, Zvi Hercowitz, and Per Krusell (1992) "Macroeconomic Implications of Investment-Specific Technological Change", Institute for Empirical Macroeconomics Discussion Paper 76.

Reuben Gronau (1977) "Leisure, Work and Home Production – the Theory of the Allocation of Time Revisited," *Journal of Political Economy* 85, 1099-1123.

Reuben Gronau (1986) "Home Production - a Survey," in *Handbook of Labor Economics*, edited by Orley C. Ashenfelter and Richard Layard. Amsterdam: North Holland.

Gary D. Hansen (1985) "Indivisible Labor and the Business Cycle," *Journal of Monetary Economics* 16, 309-327.

Gary D. Hansen and Edward C. Prescott (1991) "Recursive Methods for Computing Equilibria of Real Business Cycle Models," this volume.

Gary D. Hansen and Randall Wright (1992) "The Labor Market in Real Business Cycle Theory," *Federal Reserve Bank of Minneapolis Quarterly Review.*

Martha S. Hill "Patterns of Time Use," in *Time, Goods and Well-Being*, edited by F. Thomas Juster and Frank P. Stafford. Ann Arbor: University of Michigan Press.

Douglas H. Joines (1981) "Estimates of Effective Marginal Tax Rates on Factor Incomes," *Journal of Business* 54, 191-226.

F. Thomas Juster and Frank P. Stafford (1991) "The Allocation of Time: Empirical Findings, Behavioral Models, and Problems of Measurement", *Journal of Economic Literature.*

Robert G. King, Charles Plosser and Sergio Rebelo (1987) "Production, Growth and Cycles: Technical Appendix," manuscript, University of Rochester.

Kydland, Finn E. (1992) "The Labor Market in Real Business Cycle Theory," this volume.

Tiff Macklem (1989) "Durable and Non-durable Consumption Goods," manuscript, Ottawa, Bank of Canada.

Ellen R. McGrattan (1990) "The Macroeconomic Effects of Distortionary Taxation," Institute for Empirical Macroeconomics Discussion Paper 37, forthcoming in *Journal of Monetary Economics.*

Ellen R. McGrattan (1991) "Notes on Computing Competitive Equilibria in Linear Models," forthcoming in *Journal of Economic Dynamics and Control.*

Ellen R. McGrattan, Richard Rogerson and Randall Wright (1992) "Estimating the Stochastic Growth Model with Household Production," manuscript, Federal Reserve Bank of Minneapolis.

Ed Nosal, Richard Rogerson and Randall Wright (1992) "The Role of Household Production in Models of Involuntary Unemployment and Underemployment," *Canadian Journal of Economics*, 25, 507-520.

Robert A. Pollack and Michael L. Wachter (1975) "The Relevance of the Household Production Function and Its Implications for the Allocation of Time," *Journal of Political Economy* 83, 255-77.

Robert A. Pollak and Shelly Lundberg (1991) "Separate Spheres Bargaining and the Marriage Market," manuscript, University of Washington.

José-Victor Rios-Rull (1992) "Working in the Market, Home Production, and the Acquisition of Skills: A General Equilibrium Approach," *American*

Economic Review, forthcoming.

Richard Rogerson (1984) "Topics in the Theory of Labor Markets," Unpublished Ph.D. dissertation, University of Minnesota.

Richard Rogerson (1988) "Indivisible Labor, Lotteries and Equilibrium", *Journal of Monetary Economics* 21, 3-16.