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Economic theories of deterrence have primarily been built on static models. A common and serious shortcoming of the existing dynamic deterrence models is the assumption of a two-period structure that ignores recidivism. The aims of this paper are to formulate and solve a general dynamic deterrence model that incorporates recidivistic behavior, explore its implications, and derive some testable predictions. The analysis shows how the value and the intensity of engaging in illegal activity change over time, highlights the weaknesses of two-period deterrence models, and compares the deterrent effectiveness of increasing the likelihood of punishment versus the severity of punishment. Finally, the recidivistic model provides a structural foundation for the widely used stochastic-process models of crime in operations research and criminology.

Keywords: Recidivism, dynamic programming, deterrence theory, law and economics.

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# INTRODUCTION

Compliance with laws and regulations is generally achieved by the deterrent effects of punishments. With very few exceptions, economic theories of deterrence have all been confined to a static framework. Recent studies have begun to reveal the limitations and deficiencies of static deterrence models (e.g. Davis, 1988; Leung, 1991a, 1993; Nash, 1991; Polinsky and Rubinfeld, 1991).

Although the importance of dynamics has begun to be recognized in the deterrence literature, the development of the subject is still in an early stage. One major shortcoming of the existing dynamic deterrence models is the assumption of a two-period structure that can simply be described as follows. An individual engages in illegal activity in the first period. If he is caught and convicted, he will be fined or imprisoned (or both). He will then participate only in legal activity thereafter, and this concludes the second (and final) period. With the exception of Flinn (1986), this <u>illegal-legal</u> two-period structure underlies all the existing dynamic deterrence models, even though some of them may appear to be multi-period.

The fundamental problem with the two-period assumption is the exclusion of recidivistic behavior as every individual is only allowed to break the law once. While the two-period assumption is often justified on grounds of simplicity and analytical tractability, it is plainly not supported by empirical evidence. Reports on the prevalence of recidivism abound in the literature. For instance, Avi-Itzhak and Shinnar (1973) report that about 67 percent of felons released from prisons in California in 1964 were rearrested at least once within three years, and about 46 percent resulted in prison terms. Similarly, Schmidt and Witte (1988) find that, for each of the two cohorts (1978 and 1980) of releasees from the North Carolina prison system, about 35 percent returned to prison within 46 months. Similar rates of recidivism, based on different samples in different time periods, are also reported in Bureau of Justice Statistics (1987, 1989).

Recidivism is more pervasive for economic crimes. Many studies find that prisoners released or paroled for property offenses (such as burglary, theft, and fraud) have significantly higher recidivism rates than those released for violent, drug, or public-order offenses (Bureau of Justice Statistics, 1987, 1989). The two-period models leave recidivistic behavior unexplained. In addition, the ignorance of recidivism raises questions on the generality of the results obtained from the existing two-period models. Hence, modeling recidivism is important not only for the fact that recidivistic behavior is prevalent but also because it may generate new insights that cannot be gained from the existing two-period models.

The aim of this paper is to formulate a general dynamic deterrence model that allows individuals to recidivate. Using dynamic programming techniques and some recent results in Leung (1991b), I show that the complicated recidivistic decision problem that thwarts previous attempts can be made analytically tractable. Incorporating recidivism enables the model to address issues that cannot be handled by the existing models. To illustrate the usefulness of the recidivistic model, four issues are discussed in the paper. First, I examine how a repeat offender's decision to engage in illegal activity, as well as the offense rate given participation, changes with time and with the number of prior convictions. Second, I derive some comparative static results and contrast them with those of the two-period models. Third, I investigate whether an increase in the likelihood of punishment has a greater deterrent effect than an increase in the severity of punishment. Fourth, I explore the connection between the recidivistic model and the stochastic-process models of crime that have been extensively used in operations research and criminology.

The fourth issue needs some elaboration. In contrast with economics, the importance of recidivism has long been recognized in operations research and criminology, where a great deal of work has been done to model recidivistic behavior as a stochastic process.[1] These models have been widely utilized to evaluate the effectiveness of crime control policies. In these models, individuals are assumed to commit crimes according to a stochastic process (typically a Poisson process). The assumption is rarely justified in the literature; perhaps the only exception is Carr-Hill and Payne (1971). They contend that crimes can be regarded as random events with offenders as unlucky victims, therefore the occurrence of an offense can be likened to an accident. In this way, the modeling of crime commission can be drawn on an extensive and established literature in probability theory that models the occurrence of accidents as a stochastic process. Nevertheless, economists may find it difficult to accept these stochastic-process models of crime because of the assumptions that every individual follows the postulated stochastic process mechanically and does not behave purposively. These machine-like lawbreakers do not optimize as rational economic individuals. No previous attempt has been made to examine the relationship between economic models and stochastic-process models of crime.[2] In the following analysis, I will explore the connection between the two types of models by investigating what type of stochastic-process model of crime is implied by the dynamic economic model.

## I. A DYNAMIC PROGRAMMING MODEL OF DETERRENCE

Consider a risk neutral individual who contemplates participating in illegal activity at time t. The returns from crime are given by $\pi(c(t))$, where $c(t)$ is the intensity of offending. The precise meaning of the offense rate $c(t)$ depends on the type of illegal activity. It can be defined as the frequency of offending, the effort or time allocated to illegal activity, or the severity of the offense (e.g. amount of income tax evaded, value of items stolen, quantity of pollutants discharged, collusive price-fixing markup). For the purpose of a general analysis a specific interpretation of the offense rate is unnecessary and therefore will not be given here.

Let n be the number of times the offender has previously been convicted ($n = 0,1,2,3,\ldots$). Both $c(t)$ and n affect the hazard rate (instantaneous conditional probability) of detection and conviction $h(c(t),n)$. If the illegal activity is detected, the offender will be arrested. He will face two possible forms of punishment if he is convicted: a fine of $\theta(c(t),n)$ and an imprisonment term of $s(c(t),n)$.[3] The severity of punishment is allowed to depend upon the offense rate and the number of prior convictions. The returns that the offender obtains during imprisonment are given by a constant $\bar{\pi}$. After he is released, he may contemplate again whether to participate in illegal activity. The decision problem restarts again, and the main difference is that the number of convictions has been increased by one in this round.[4]

Assume that the individual faces two choices: participating in legal activity versus illegal activity, and let the returns from engaging in legal activity be $\hat{\pi}(n)$. The offender faces an infinite horizon and discounts future returns at a rate of r. There are three basic assumptions:

Assumption 1: $r \in (0,1)$ and $c(t) \in [0,1]$. $\pi$, $\theta$, h, and s are bounded and continuous functions.

Assumption 2: $\pi(0) = 0$, $\pi \geq 0$, $\pi'' < 0$, $h(0,n) = 0$, $h_1 \geq 0$, $h_{11} \geq 0$, $\theta(0,n) = 0$, $\theta_1 \geq 0$, $\theta_{11} \geq 0$, $s(0,n) = 0$, $s_1 \geq 0$, $\hat{\pi}(n+1) \leq \hat{\pi}(n)$, $\theta(c,n+1) \geq \theta(c,n)$, and $s(c,n+1) \geq s(c,n)$.

The restrictions put on $\pi$, $\theta$, h, and s in Assumption 2 are mild. Both the fine and the imprisonment term increase with the offense rate because the more serious the offense, the more severe will be the punishment. The severity of punishment is also allowed to increase with the number of prior convictions since repeat offenders usually receive heavier penalties (Greenwood 1982). There are two reasons for assuming that $\hat{\pi}(n)$ decreases with n. First, a criminal history may create a stigma effect that reduces the offender's legal returns and market opportunities. Second, participation in illegal activity requires time and effort, thus the offender foregoes his human capital investment opportunity in legal activity. In addition, the offender's human capital may depreciate because of the lack of legal market experience. These stigma and human capital effects lower the earnings from legal activity.

With these assumptions, the offender's decision problem at time $\tau$ given n prior convictions ($\tau \geq 0$, $n \geq 0$) is to choose $c(t)$, $t \in [\tau,\infty)$, to maximize the expected present value of returns:

$$V(\tau,n) = \text{Max}\ \{V_L(\tau,n), V_I(\tau,n)\}, \tag{1}$$

where $V_L$ and $V_I$ denote the legal and the illegal returns, respectively, and

$$V_L(\tau,n) = \int_\tau^\infty e^{-r(t-\tau)}\hat{\pi}(n)dt = \hat{\pi}(n)/r, \tag{2}$$

$$V_I(\tau,n) = \underset{c(.)}{\text{Max}} \left\{ \int_\tau^\infty e^{-r(t-\tau)}\pi(c(t))\frac{1-F_n(t)}{1-F_n(\tau)}dt \right. \tag{3}$$

$$+ \int_{\tau}^{\infty} e^{-r(t-\tau)} \left[ -\theta(c(t),n) + \int_{t}^{t+s(c(t),n)} e^{-r(x-t)}\tilde{\pi}dx \right.$$

$$\left. + e^{-rs(c(t),n)} V(t+s(c(t),n),n+1) \right] \frac{F_n(t)}{1-F_n(\tau)} dt \Bigg\}$$

subject to

$$F_n'(t) = h(c(t),n)[1-F_n(t)]. \tag{4}$$

Constraint (4) follows directly from the definition of the hazard rate, where $F_n'(t)$ denotes the probability density function of the time of detection and conviction.[5] The expression for $V_I(\tau,n)$ requires some elaboration. At any time t, the offender will gain $\pi(c(t))$ if the illegal activity is not detected, the conditional probability of which is given by $[1-F_n(t)]/[1-F_n(\tau)]$. The probability of arrest and conviction is given by the conditional density $F_n'(t)/[1-F_n(\tau)]$. If the offender is detected and convicted at time t, he will be fined $\theta(c(t),n)$ and imprisoned for a period of $s(c(t),n)$. After he is released at time $t+s(c(t),n)$, the decision problem restarts again. Notice that there is a discount term $e^{-rs(c(t),n)}$ on the value function $V(t+s(c(t),n),n+1)$ because imprisonment postpones the realization of future returns from time t to $t+s(c(t),n)$. Consequently, the value function at time $t+s(c(t),n)$ has to be discounted by $e^{-rs(c(t),n)}$ in order to bring it back to the present value at time t.[6]

Although the dynamic programming problem appears intractable, it can be converted into a simpler problem. For convenience, let $\tilde{\pi}$ be normalized to zero.[7] Since time $\tau$ enters the problem only through the discount term, the optimal control problem is autonomous. Furthermore, the problem has an infinite horizon, therefore the value function does not depend on time $\tau$ explicitly, i.e., $V(\tau,n)$ can be replaced by $V(n)$. Without loss of generality, let $\tau = 0$ and $F_n(0) = 0$. In addition, by virtue of (4), $F_n'(t)$ in (3) can be replaced by $h(c,n)[1-F_n(t)]$. As a result of these arguments and simplifications, problem (1)-(4) can be reduced to:

$$(5)\qquad V(n) = \text{Max}\ \{V_L(n), V_I(n)\},$$

$$(6)\qquad V_L(n) = \hat{\pi}(n)/r,$$

$$(7)\qquad V_I(n) = \text{Max}_{c(.)} \int_0^\infty e^{-rt}[\pi(c)-\theta(c,n)h(c,n) + e^{-rs(c,n)}V(n+1)h(c,n)][1-F_n(t)]dt.$$

The decision problem (5)-(7) indicates that after each conviction, the individual compares the returns $V_L$ and $V_I$, and decides whether to engage in illegal activity again.[8] The following proposition forms the basis of the rest of the analysis.

PROPOSITION 1. Under Assumption 1,

(a) there exists a unique, bounded, and continuous function V satisfying the functional equation (7), and

(b) the dynamic programming problem (7) is equivalent to

$$(8)\qquad V_I(n) = \underset{c}{\text{Max}} \left\{ \frac{\pi(c) - \theta(c,n)h(c,n) + e^{-rs(c,n)}V(n+1)h(c,n)}{r + h(c,n)} \right\}.$$

PROOF: See Appendix.

Proposition 1a establishes the existence of the value function, while Proposition 1b shows that the original continuous time optimal control problem can be substantially simplified to a more tractable dynamic programming problem that does not explicitly involve the time variable.[9] It also implies that the optimal solution $c(t)$, $t \in [0,\infty)$, of problem (7) is a constant that does not depend on time t. Thus, the offense rate will not rise or fall with time during the interval between convictions.

As (8) reveals, the dynamics of the model are generated from three sources: $\theta(c,n)$, $h(c,n)$, and $s(c,n)$. This means that (8) will degenerate into a static model (i.e., $V(n)$ will be the same as $V(n+1)$ and there is no difference between the present and the future) only if $\theta$, h, and s do not depend on n. Therefore, the model is more general than Flinn's (1986) model which relies exclusively on imprisonment to generate the dynamics.

## II. IMPLICATIONS

### A. Recidivism

For any $n > 0$, an offender will recidivate if $V_L(n) < V_I(n)$. Let $\Gamma(c,n) = [\pi(c)-\theta(c,n)h(c,n)]/[r+(1-e^{-rs(c,n)})h(c,n)]$ and consider the condition

(C1): If $\Gamma(c,n) \geq 0$, then $\Gamma(c,n) \geq \Gamma(c,k)$ for all $k \geq n$.

Proposition 2 describes how recidivism changes over time.

PROPOSITION 2: Under Assumptions 1 and 2, if (C1) is satisfied, then $V_I(n) \geq V_I(n+1)$ and $V(n) \geq V(n+1)$. These two inequalities will be strict if $V_I(n) > V_L(n)$ and if the inequality in (C1) is strict.

PROOF: See Appendix.

The condition $V_I(n) > V_L(n)$ means that the offender continues to participate in illegal activity after n convictions. This condition is clearly needed, for if the offender desists from crime after p convictions, then $V(n) = V_L(p) = \hat{\pi}(p)/r$ for all $n \geq p$. Hence, $V(n) = V(n+1)$ for $n \geq p$ and the inequality cannot be strict.

Proposition 2 describes the entry and exit behavior of the offender's criminal career. It shows that the value of participating in illegal activity decreases with the number of prior convictions and indicates whether and when a potential offender will engage in illegal activity. As the number of prior records increases, the expected gain from illegal activity diminishes and the likelihood of recidivism decreases. Whether the offender will desist from crime depends on the slopes of $V_L(n)$ and $V_I(n)$.

Figures 1 and 2 describe two main possibilities. In Figure 1, the curve AC describes the relationship between V(n) and n, and BC describes the declining legal returns $\hat{\pi}(n)/r$.[10] The offender will quit the criminal career when the two curves meet at C (where n = p). The number of convictions will stay at p permanently since the offender will never participate in illegal activity again. Clearly, it is possible that the offender will never quit the criminal career. This is described by the curves AA′ and BB′ in Figure 2. In this case, $V_I(n)$ always stays above $V_L(n)$ so that there does not exist any p such that $V_I(p) \leq \hat{\pi}(p)/r$. The individual becomes a chronic offender in this case.

Figures 1 and 2 here

Proposition 2 shows that if the offender desists from crime, it will be a permanent quit. In other words, once the offender exits from the illegal sector, he will never participate in illegal activity again because the returns from doing so are lower than the legal returns. Temporary quits are not implied by the model, unless there are unforeseen stochastic changes that alter the difference between $V_L$ and $V_I$.

Given that $\theta(.,n)$ and $s(.,n)$ increase with n (Assumption 2), a sufficient condition for (C1) to hold is that $h(.,n)$ increases with n. Several arguments can be put forth to justify a positive relationship between $h(.,n)$ and n. First, if an offender had been arrested before, law enforcement agencies would have more information on the offender and subsequently could monitor him more closely. Likewise, private citizens may help to detect a repeat offender because they know more about the offender. These public and private information effects increase the probability of apprehension for recidivists. Second, law enforcement agencies may also learn more about the offender's habits as the number of convictions increases. Third, law enforcement agencies may pursue more tenaciously offenders who repeatedly defy the laws. Again, actions taken by private citizens will also reinforce the last two factors because they too may learn more about the offender's habits and strive tenaciously to convict repeat offenders. For example, a company which had been convicted of producing fraudulent products will be watched more carefully by individual consumers, special consumer groups, communities, and perhaps rival companies.

Although the previous three considerations (information, learning, and tenacity) suggest that n has a positive effect on $h(.,n)$, there is a learning effect which works in the opposite direction. As the number of convictions rises, an offender's experience in evading detection may increase, making it more difficult to arrest him. Theory cannot ascertain whether this negative effect dominates the positive effect, so the issue can only be resolved empirically. The evidence to date seems to support the view that $h(.,n)$ increases with n.[11]

Even if $h(.,n)$ decreases with n, the conclusions of Proposition 2 still hold as long as (C1) is satisfied. This result is intuitively reasonable, for if $h(.,n)$ does not fall too fast with n, then the value of participating in illegal activity should decline with n because the severity of punishment rises with n. The inequality in (C1) provides an explicit condition to show how fast $h(.,n)$ can decrease with n. Although the expression $\Gamma(c,n)$ may not have a convenient interpretation, one can replace it by a stronger but more interpretable condition:

(C2): Both $\theta(c,n)h(c,n)$ and $(1-e^{-rs(c,n)})h(c,n)$ increase with n.

It is easy to check that (C2) implies (C1). (C2) is more interpretable than (C1) because $\theta(c,n)h(c,n)$ is the expected fine and $(1-e^{-rs(c,n)})h(c,n)$ can be regarded as the expected penalty from imprisonment. Since the offender's returns after incarceration are given by $e^{-rs(c,n)}V(n+1)$, the incarceration costs him $V(n+1)-e^{-rs}V(n+1) = (1-e^{-rs})V(n+1)$, because he would have gotten $V(n+1)$ had he not been incarcerated. Hence, $1-e^{-rs(c,n)}$ can be treated as a loss factor which measures the penalty from imprisonment. Condition (C2) accords with the intuition that if the expected penalties (fine and imprisonment) increase with n, then the value of engaging in illegal activity will diminish with n.

The model suggests that law enforcement and rehabilitative agencies can make crime commission less attractive to ex-offenders by raising their alternative legal opportunities. This may be accomplished indirectly by providing them with training and educational programs to increase their earning capabilities or more directly by giving them income subsidies. In any case, the alternative legal returns must be raised to at least $V_I$.[12]

Proposition 2 is concerned with the participation decision, the next proposition examines the intensity decision. An additional condition is needed:

$$(C3): \ -hs_{11} + rhs_1^2 - 2h_1s_1 + h_{11} \leq 0.$$

PROPOSITION 3: Under Assumptions 1 and 2, if $c_n^*$ is an interior solution to the maximization problem (8) and (C3) is satisfied, then $c = c_n^*$ must satisfy the equation

$$(9) \qquad \pi'(c) - \theta_1(c,n)h(c,n) - \theta(c,n)h_1(c,n) - rs_1(c,n)e^{-rs(c,n)}V(n+1)h(c,n) + e^{-rs(c,n)}V(n+1)h_1(c,n) - V_I(n)h_1(c,n) = 0.$$

PROOF: See Appendix.

Proposition 3 characterizes the solution of the dynamic programming problem.[13] To study how $c_n^*$ changes with n, consider first a simple case in which $\theta(c,n) = \theta$ (a constant), $s(c,n) = 0$, and $h(c,n) = ch(n)$. Assume that $V(n) = V_I(n)$ (the offender has not yet desisted from crime), then (9) becomes $\pi'(c_n^*) = [\theta + V(n) - e^{-rs}V(n+1)]h(n)$. Differencing this equation yields

$$c_{n+1}^* - c_n^* = \{[\theta+V(n+1)-V(n+2)]h(n+1) - [\theta+V(n)-V(n+1)]h(n)\}/\pi''(z),$$

where z lies between $c_{n+1}^*$ and $c_n^*$. This expression suggests that the sign of $c_{n+1}^* - c_n^*$ is indeterminate. Even if one assumes $h(n+1) > h(n)$, $c_{n+1}^* - c_n^*$ will still depend on the curvature of V(n). For instance, if $V(n+2)-V(n+1) \leq V(n+1)-V(n)$ (i.e. V(n) is "concave"), then $c_{n+1}^* - c_n^* \leq 0$ (since $\pi'' < 0$). Unfortunately, the model does not yield an unambiguous sign for $[V(n+2)-V(n+1)] - [V(n+1)-V(n)]$. From this simple example one can infer that the sign of $c_{n+1}^* - c_n^*$ is also ambiguous in the general case. Thus, even though V(n) and $V_I(n)$ fall with n, $c_n^*$ may rise with n.

B. Comparative Statics

Equations (8) and (9) can be used to study the effects of shifts in the functions h, $\theta$, s, and $\hat{\pi}$ on $V_I(n)$ and $c_n^*$. Consider first a parallel shift in $\theta(c,n)$, i.e. a shift in $\theta(c,n)$ (upward or downward) without altering the slope $\theta_1(c,n)$ at each c.[14] Applying the envelope theorem to (8),

$$\partial V_I(n)/\partial\theta = (-h+e^{-rs}[\partial V(n+1)/\partial\theta]h)/(r+h). \tag{10}$$

Applying the implicit theorem and differentiating (9) with respect to $\theta(c,n)$, one obtains

$$M(\partial c_n^*/\partial\theta) = h_1 + rs_1e^{-rs}[\partial V(n+1)/\partial\theta]h - e^{-rs}[\partial V(n+1)/\partial\theta]h_1 + [\partial V_I(n)/\partial\theta]h_1, \tag{11}$$

where $M = \pi'' - \theta_{11}h - 2\theta_1h_1 - \theta h_{11} + e^{-rs}V(n+1)(-hs_{11}+rhs_1^2-2h_1s_1+h_{11}) - V_I(n)h_{11}$. Substituting (10) into (11) and simplifying,

$$M(\partial c_n^*/\partial\theta) = rh_1/(r+h) + re^{-rs}[\partial V(n+1)/\partial\theta][s_1h - h_1/(r+h)]. \tag{12}$$

The effects of parallel shifts in s and h on $V_I(n)$ and $c_n^*$ can also be derived analogously. It can be shown that

$$\partial V_I(n)/\partial s = e^{-rs}[-rV(n+1) + \partial V(n+1)/\partial s]h/(r+h), \tag{13}$$

$$M(\partial c_n^*/\partial s) = re^{-rs}([\partial V(n+1)/\partial s] - rV(n+1))[s_1h - h_1/(r+h)], \tag{14}$$

$$\partial V_I(n)/\partial h = (-\theta + e^{-rs}[V(n+1) + [\partial V(n+1)/\partial h]h] - V_I(n))/(r+h), \tag{15}$$

$$M(\partial c_n^*/\partial h) = \theta_1 - \theta h_1/(r+h) + re^{-rs}[\partial V(n+1)/\partial h][s_1h - h_1/(r+h)] + e^{-rs}V(n+1)[rs_1 + h_1/(r+h)] - V_I(n)h_1/(r+h). \tag{16}$$

For a parallel upward shift in $\hat{\pi}$, there are two cases to consider. In Figure 2, if BB′ is shifted upward, but still lies below AA′ at every n, then the offender will still not desist from crime at any n. Hence the shift in $\hat{\pi}$ will not have any effect on $V_I(n)$ and $c_n^*$ at all. For the case in Figure 1 in which the offender quits at n = p, $V(p) = \hat{\pi}(p)/r$. It can be shown that for any $n < p$,

(17) $$\partial V_I(n)/\partial\hat{\pi} = e^{-rs}[\partial V(n+1)/\partial\hat{\pi}]h/(r+h),$$

(18) $$M(\partial c_n^*/\partial\hat{\pi}) = re^{-rs}[\partial V(n+1)/\partial\hat{\pi}][s_1 h - h_1/(r+h)].$$

One can use the fact that $\partial V(p)/\partial\hat{\pi} = 1/r$ to iterate (17) backward and obtain for each $n < p$ an explicit expression for $\partial V_I(n)/\partial\hat{\pi}$ that does not contain $\partial V(n+1)/\partial\hat{\pi}$. The following proposition follows readily from (10) - (18).

PROPOSITION 4: Under Assumptions 1 and 2, $V_I(n)$ is decreasing in $\theta$ and s, and increasing in $\hat{\pi}$. If (C1) holds, then $V_I(n)$ is also decreasing in h. If $s_1 h \leq h_1/(r+h)$ and (C3) is satisfied, then $\partial c_n^*/\partial\theta \leq 0$, $\partial c_n^*/\partial s \leq 0$, and $\partial c_n^*/\partial\hat{\pi} \geq 0$.

PROOF: See Appendix.

As expected, an increase in fine, imprisonment, or the hazard rate of arrest reduces the value of illegal activity. As a result, the offender may quit his criminal career earlier. An increase in legal returns, however, raises the value of illegal activity. This seemingly counter-intuitive result does not imply that the offender will engage in crime for a longer period of time. From (17), $\partial V_I(p-1)/\partial\hat{\pi} = e^{-rs}h/[r(r+h)] < 1/r$, hence iterating (17) backward yields $\partial V_I(n)/\partial\hat{\pi} < 1/r$ for all $n < p$. Since $\partial V_L(n)/\partial\hat{\pi} = 1/r > \partial V_I(n)/\partial\hat{\pi}$ for all $n < p$, hence the increase in the value of legal activity is larger than the increase in the value of illegal activity, so the offender may quit crime earlier.

Although parallel shifts in h, $\theta$, s, and $\hat{\pi}$ produce unambiguous effects on the participation decision, the effects on the intensity decision are more difficult to determine. Proposition 4 shows that an increase in $\theta$, s, or $\hat{\pi}$ will reduce the offense rate if the inequality $s_1 h \le h_1/(r+h)$ holds. This condition will be satisfied if the length of incarceration is independent of the offense rate (i.e. $s_1 = 0$). If the condition is not met, then an increase in fine, imprisonment, or legal returns may result in a higher offense rate. This is intuitively possible because given that the decision to engage in crime has been made, an offender may commit more serious offenses to compensate for the higher expected punishment or the higher foregone legal earnings. This leads to an important policy tradeoff. An increase in penalty reduces the value of illegal activity, so that fewer people will commit crimes, other things being equal. On the other hand, for those who will commit crimes, the increase in penalty may increase their offense rates. It is therefore possible that increasing the penalty may result in a higher total crime rate.

Even if $s_1 h \le h_1/(r+h)$, the sign of $\partial c_n^*/\partial h$ is still indeterminate. For example, if $h_1 = 0$ and $s_1 = 0$, then (16) becomes $M(\partial c_n^*/\partial h) = \theta_1$, which implies that $\partial c_n^*/\partial h \le 0$. On the other hand, if $\theta_1 = 0$, $s_1 = 0$, $V(n) = V_I(n)$, and $\partial V(n+1)/\partial h = 0$ (say when $n = p-1$), then (16) becomes $M(\partial c_n^*/\partial h) = -\theta h_1/(r+h) + [e^{-rs}V(n+1) - V(n)]h_1/(r+h)$, thus $\partial c_n^*/\partial h \ge 0$. Hence the effect of a shift in h on $c_n^*$ could go either way.

Equations (10) - (18) can also be utilized to highlight the differences between the recidivistic model and the existing two-period deterrence models. Consider a prototypical two-period deterrence model in which $V(0) = \text{Max}\,\{V_L(0), V_I(0)\}$, $V_L(0) = \hat{\pi}(0)/r$, and $V(1) = V_L(1) = \hat{\pi}(1)/r$. A potential offender compares the legal returns $V_L(0)$ and illegal returns $V_I(0)$. If $V_I(0)$ is larger than $V_L(0)$, then he will engage in illegal activity. If he is caught and convicted, he will never engage in illegal activity again and will earn the legal returns $\hat{\pi}(1)/r$ thereafter. This illegal-legal transition underlies the structure of all the two-period deterrence models in the literature. Notice that in this model, $V(0) \geq V(1)$ follows directly from the assumption that $\hat{\pi}(0) \geq \hat{\pi}(1)$. Since $\partial V(1)/\partial\theta = 0$, $\partial V(1)/\partial s = 0$, and $\partial V(1)/\partial h = 0$, one can modify (10), (12) - (18) to obtain

(19) $$\partial V_I(0)/\partial\theta = -h/(r+h),$$

(20) $$M(\partial c_n^*/\partial\theta) = rh_1/(r+h),$$

(21) $$\partial V_I(0)/\partial s = -e^{-rs}V(1)h/(r+h),$$

(22) $$M(\partial c_n^*/\partial s) = -r^2e^{-rs}V(1)[s_1h - h_1/(r+h)],$$

(23) $$\partial V_I(0)/\partial h = [-\theta + e^{-rs}V(1) - V(0)]/(r+h),$$

(24) $$M(\partial c_n^*/\partial h) = \theta_1 - \theta h_1/(r+h) + e^{-rs}V(1)[rs_1 + h_1/(r+h)] - V_I(0)h_1/(r+h),$$

(25) $$\partial V_I(0)/\partial\hat{\pi} = e^{-rs}h/r(r+h),$$

(26) $$M(\partial c_n^*/\partial\hat{\pi}) = e^{-rs}[s_1h - h_1/(r+h)],$$

where the arguments $c_0^*$ and $n = 0$ are suppressed for simplicity.

Equations (19), (21), (23), and (25) illustrate that the effects of shifts in h, $\theta$, s, and $\hat{\pi}$ are qualitatively similar to those of the recidivistic model. However, the sign of $\partial c_n^*/\partial\theta$ is now unambiguously nonpositive, while the signs of $\partial c_n^*/\partial s$ and $\partial c_n^*/\partial\hat{\pi}$ still depend on $s_1h - h_1/(r+h)$.[15] The sign of $\partial c_n^*/\partial h$ remains indeterminate.

There are two main differences between the two-period model and the recidivistic model. First, the inequality $V(0) \geq V(1)$ holds trivially in the two-period model, whereas some condition such as (C1) is needed to ensure that $V(n) \geq V(n+1)$ in the recidivistic model. The inequality $V(n) \geq V(n+1)$ plays an essential role in establishing some of the implications of the recidivistic model. Second, V(1) is exogenously given in the two-period model, while V(n+1) is endogenously determined in the recidivistic model (except for V(p) if the offender desists from crime at n = p). The offender has to ponder the effects of his decisions not only on the current gains but also on the future returns. By ignoring future considerations, the implications of the two-period model can be very misleading. For instance, the two-period model predicts that an increase in fine will unambiguously reduce the offense rate. The recidivistic model, however, reveals that this prediction is not robust when recidivism is allowed.

## C. Certainty versus Severity of Punishment

A frequently discussed issue in deterrence theory is the relative deterrent effectiveness of a change in the probability of detection and conviction versus a change in the severity of punishment. It is widely believed that an increase in the probability has a greater deterrent effect than an increase in the severity.[16] Becker (1968) develops an economic model to show that this widespread presumption necessarily implies that offenders are risk preferrers. Subsequent works by Brown and Reynolds (1973) and Heineke (1975), however, show that Becker's result is sensitive to his assumptions. They demonstrate that the presumption in general does not imply anything about the offenders' attitudes toward risk. Economic theory has not yet provided a satisfactory explanation for the presumption.

Some advance has been made by Davis (1988). He argues that if one considers the timing of rewards and punishments, the presumption can be explained irrespective of the offender's attitude toward risk. The main problems with Davis' analysis, however, are that he assumes a two-period horizon and ignores recidivism, and uses Becker's (1968) notion of "compensating changes" to derive the result. As Brown and Reynolds (1973) have pointed out, the device "compensating changes" (compensating an increase in the probability of detection by an equal percentage reduction in the fine so as to leave the expected income from the offense unchanged) is both unappealing and arbitrary. They show that one can immediately infer criminal risk preferences from compensating changes since the device amounts to the standard method of determining risk preferences by altering variances while leaving means unchanged. If the device is used, then Becker's analysis on the implication of the presumption on risk preferences will simply be redundant. The arbitrariness of the device is also revealed in the way the expected income is held constant in Davis (1988).

By considering recidivistic behavior and without using compensating changes, the dynamic model developed here can provide an explanation for the greater deterrent effectiveness of a change in the probability than in the severity of punishment. Consider the more relevant case in which the individual engages in illegal activity, i.e. $V(n) = V_I(n)$. Following Brown and Reynolds (1973), the elasticities of $V(n)$ with respect to $\theta$ and h will be used to compare the relative deterrent effectiveness of a change in the probability versus a change in the severity of punishment. Let $\epsilon_\theta$ and $\epsilon_h$ denote the elasticities $-[\theta/V(n)][\partial V(n)/\partial\theta]$ and $-[h/V(n)]/[\partial V(n)/\partial h]$, respectively.

PROPOSITION 5: Under Assumptions 1 and 2, if $V(n) = V_I(n)$ and (C1) is satisfied, then $\epsilon_h \geq \epsilon_\theta$.

PROOF: See Appendix.

Proposition 5 shows that the offender's value of engaging in illegal activity is more responsive to a change in the hazard rate of detection and conviction than a change in the severity of punishment. Hence, an increase in the certainty of punishment has a greater deterrent effect than an increase in the severity of punishment. One of the most important insights gained from the recidivistic model, which cannot be obtained from all the existing two-period models, is to uncover the conditions under which the result will hold. In a two-period model, (19) and (23) imply that

$$\epsilon_h - \epsilon_\theta = [V(0) - e^{-rs}V(1)]h/[(r+h)V(0)],$$

which shows that $\epsilon_h$ is unambiguously greater than $\epsilon_\theta$. On the other hand, condition (C1) is needed for $\epsilon_h \geq \epsilon_\theta$ to hold in the recidivistic model. If (C1) is not satisfied, the inequality $\epsilon_h \geq \epsilon_\theta$ may be reversed.[17] This possibility illustrates one of the weaknesses of ignoring recidivistic behavior in the existing two-period models because they cannot expose the importance of the relationship between V(n) and V(n+1).

## D. Stochastic-Process Models of Crime

Proposition 1b shows that the optimal offense rate that solves the dynamic programming problem is a constant. This implies that from the nth conviction to the (n+1)st conviction, the optimal offense rate $c_n^*$ does not vary with time. Similarly, the optimal offense rate $c_{n+1}^*$ is a constant from the (n+1)st conviction to the (n+2)nd conviction.[18] As h(c,n) only depends on c and n, the hazard rate of detection and conviction also does not vary with time between any two consecutive convictions. The time between two consecutive convictions is therefore exponentially distributed because the exponential distribution is the only distribution with a constant hazard rate. Ignoring the time spent in incarceration, this implies that the crime process is a <u>Markov jump (point) process</u>.[19] The state space is the set of the number of convictions $\{0,1,2,3,\ldots\}$, and a jump occurs when there is a conviction.[20] Therefore the stochastic-process model of crime implied by the recidivistic model is a Markov-jump-process model. In contrast with all the stochastic-process models of crime in the literature, the Markov-jump-process model obtained here is derived from an underlying economic optimizing behavioral model.

Most of the stochastic-process models of crime in operations research and criminology are Poisson-process models, and therefore they are just a special case of the Markov-jump-process model. This is because a Poisson process, which requires the hazard rate to be the same at any time $t \in [0,\infty)$, is a special case of the Markov jump process which only requires the hazard rate to be constant during the time between any two jumps (see Hoel et al., 1972; Breiman, 1968; Gikhman and Skorokhod, 1968). Hence, the recidivistic model provides a behavioral and structural foundation for the Poisson-process models of crime. The results also indicate how the parameters of the behavioral model are connected to the parameters of the stochastic-process models.

There is a subtle difference between the two types of models, however. In a typical Poisson-process model of crime in operations research and criminology, it is assumed that the <u>crime process</u> (the process of crimes committed by an individual) follows a Poisson process and an arrest is determined at random for each crime. Thus, the <u>arrest process</u> is a <u>thinned</u> version of the crime process in that only a subset of the crimes results in arrest (Lehoczky, 1986). As long as the probability of arrest does not depend upon the number of crimes committed and the number of previous arrests (a strong assumption), the arrest process (thinned Poisson process) will still be a Poisson process.[21] This Poisson arrest process is the basis of the Poisson-process model of crime. In contrast, the recidivistic model assumes that only the arrest process is stochastic. An individual commits crime over time and faces a random arrest process governed by the hazard rate of detection. Although the two models may be observationally equivalent (since the crime process, unlike the arrest process, is unobservable), the underlying behavioral assumptions are fundamentally different. This arises from the ways non-economists and economists approach the problem.

Non-economists tend to focus exclusively on the number of crimes. In contrast, economists are more concerned with offense rates such as the time allocated to illegal activity, the amount of tax evaded, the number of illegal immigrants hired. In many applications, the economic approach is more appropriate because the number of crimes may not be a good measure of criminality. For example, suppose a cartel is engaged in price fixing and charges a high price over a long period of time (say, until the crime is detected). In this case, the number of crimes committed is one, but the number is not very useful. The offense rate (the markup above the marginal cost of production) is clearly a more important measure of criminality (see Block et al. (1981) for an example of illegal price fixing in the bread industry). Hence, the Markov-jump-process model derived here, which is based on a more appropriate concept of offense rate and does not require the strong assumption that the probability of detection to be independent of the number of prior convictions, is better than the Poisson-process models of crime in operations research and criminology.

The Markov-jump-process model is empirically testable since one can check whether the time between convictions is exponentially distributed. Available evidence supports this prediction. For example, in Stollmack and Harris' (1974) study on releasees from correctional programs and Nash's (1991) work on violations of consumer protection regulations, the exponential distribution fits the data reasonably well.

## III. CONCLUSION

Economic theories of deterrence have primarily been built on static models. Of the few dynamic deterrence models existed in the literature, a major shortcoming is the assumption of a two-period structure that ignores recidivism. Although "few criminal justice issues have matched recidivism in stirring public opinion and in engaging the attention of criminal justice professionals" (Bureau of Justice Statistics, 1989), the modeling of recidivistic behavior has virtually been ignored in economics. The aim of this paper has been to formulate and solve a general dynamic deterrence model that incorporates recidivism. It demonstrates how the complicated recidivistic problem can be brought into a dynamic programming framework, from which a set of useful implications can be derived. In addition to formulating and solving the recidivistic model, the paper makes four contributions. It shows how the value and the intensity of engaging in illegal activity change over time, highlights the deficiencies of two-period models, establishes the conditions under which an increase in the certainty of punishment will have a greater deterrent effect than an increase in the severity of punishment, and provides a structural foundation for the widely used stochastic-process models of crime in operations research and criminology. The model also generates some testable predictions. For example, it shows that the time between convictions is exponentially distributed.

There are many issues which can further be explored. As in Flinn (1986), Nash (1991), Polinsky and Rubinfeld (1991), and many others, the model assumes risk neutrality. It would be useful to study the effects of risk aversion or risk preference on the implications of the recidivistic model. However, this is technically a difficult problem. If utility is defined on consumption, then lifetime discounted utility will be the same as lifetime discounted consumption for a risk neutral offender. Since lifetime discounted consumption equals lifetime discounted income (wealth), the offender's utility maximization problem becomes a wealth maximization problem. Because of this equivalence, the offender's decision problem can be formulated as the wealth maximization problem described by equations (1) - (3). If the offender is not risk neutral, then he cannot just maximize wealth because his lifetime discounted utility is no longer equivalent to his wealth. In modeling the offender's utility maximization problem, it is now necessary to consider whether lending and borrowing are allowed. If saving is not permitted, then the offender has to consume all his returns (legal or illegal) at each moment. By reinterpreting $\pi$ as a utility function, (1)-(3) can be modified to handle this case. However, a major problem arises in modeling the fine $\theta$ because the offender does not have any savings to pay the fine if he is caught. It appears that the only solution is to make the unattractive assumption that $\theta = 0$. In the more realistic case in which saving is permitted, the offender not only has to choose the offense rate but also the rate of consumption at each instant. Hence, relaxing the risk neutrality assumption will give rise to a series of problems and the analysis will become much more complicated. These problems do not arise in a static deterrence model because consumption must be equal to wealth, as there is only one period in the model.

In the discussion on certainty versus severity of punishment, the deterrent effect is measured in terms of the value of illegal activity. A more complete analysis would include the effects of altering the certainty versus the severity of punishment on the offense rate. The combined impact of punishment on the participation <u>and</u> the intensity of offending has not yet been investigated in the literature.[22]

Another issue also deserves attention. From a general equilibrium point of view, the recidivistic model only deals with the supply side (the supply of offenses). It would be useful to model the demand side and then investigate the equilibrium properties. Many policy debates can only be properly addressed by a dynamic general equilibrium model, especially for those policies that involve time and other dynamic considerations. The best example is selective incapacitation since imprisonment occurs over time. The existing general equilibrium literature either uses static models and therefore ignores incapacitation (e.g. Neher, 1978; Balkin and McDonald, 1981; Usher, 1986; Furlong, 1987), or makes ad hoc assumptions on the supply side (Ehrlich, 1982). Using the results established in this paper, it is possible to develop a more complete general equilibrium analysis of crime.

## APPENDIX: PROOFS

PROOF OF PROPOSITION 1:

(1a) Let $C(Z_+)$ ($Z_+$ = the set of nonnegative integers) be the space of bounded and continuous functions $v: Z_+ \rightarrow R$, with the sup norm $\|v\| = \sup_{x \in Z+} |v(x)|$. Define an operator T on $C(Z_+)$ by $Tv = \text{Max}\ (\hat{\pi}(n)/r,\ \text{Max}_{c(.)}\ J)$, where J is given by $J = \int_0^\infty e^{-rt}[\pi(c)-\theta(c,n)h(c,n)+e^{-rs(c,n)}vh(c,n)][1-F_n(t)]dt$. Clearly, $T: C(Z_+) \rightarrow C(Z_+)$. Let w be a real nonnegative number. Since $e^{-rs(c(t),n)} \leq 1$, $T(v+w) \leq \text{Max}\{\hat{\pi}(n)/r,\ \text{Max}_c[J + w\int_0^\infty e^{-rt}F_n'(t)dt]\}$. Let $D(r) = \int_0^\infty e^{-rt}F_n'(t)dt$. Clearly, $D(0) = \int_0^\infty F_n'(t)dt \leq 1$, and $D'(r) < 0$ for $r > 0$. It follows that $D(r) < 1$ for $r > 0$. Furthermore, $e^{-rt}$ is strictly convex in t for $r > 0$. By Jensen's inequality, $D(r) > \exp[-r\int_0^\infty tF_n'(t)dt] \geq 0$. Hence, $0 < D(r) < 1$ for $r > 0$. Let $\beta$ be a real number such that $D(r) < \beta < 1$, then $T(v+w) \leq \text{Max}\{\hat{\pi}(n)/r,\ \text{Max}_c(J + \beta w)\} = \text{Max}\{\hat{\pi}(n)/r,\ (\text{Max}_c\ J) + \beta w\} \leq Tv + \beta w$. Hence for any given $r > 0$, there exists a positive real number $\beta < 1$ such that $T(v+w) \leq Tv + \beta w$. It is easy to check that T is a monotone operator. It follows from Blackwell's theorem that T is a contraction mapping with modulus $\beta$, and so the functional equation (7) has a unique bounded and continuous fixed point V.

(1b) The expression $\pi(c(t)) - \theta(c(t),n)h(c(t),n) + e^{-rs(c(t),n)}V(n+1)h(c(t),n)$ inside the integral in (7) is a function of c(t) and n only, and does not depend on t explicitly. It is easy to verify that the model is a member of the Kamien-Schwartz class of models (Kamien and Schwartz, 1971; Leung, 1991b). Therefore the theorem in Leung (1991b) can be applied to solve the optimization problem (7), and it follows that the optimal solution c(t) is a constant for all t. As a result, the right-hand side of (7) can be integrated to obtain the right-hand side of (8).

PROOF OF PROPOSITION 2:

If $V_I(n) \geq V_I(n+1)$, then $V(n) \geq V(n+1)$ (since $\hat{\pi}(n) \geq \hat{\pi}(n+1)$). Hence, it suffices to prove $V_I(n) \geq V_I(n+1)$. There are two possibilities: the offender does not desist from crime (a chronic offender) or he desists at some point in time. The proofs for these two cases are slightly different.

*Case (i)*. Suppose that the offender never desists from crime. Since $V(n+1) = V_I(n+1)$, therefore $V_I(n) - V_I(n+1) = V_I(n) - V(n+1)$ $= \mathrm{Max}_c \{\pi(c)-\theta(c,n)h(c,n)-[r+h(c,n)(1-e^{-rs(c,n)})]V(n+1)\}/[r+h(c,n)]$. Hence,

$$\text{(A1)}\qquad V_I(n) - V_I(n+1) \geq \{\pi(c)-\theta(c,n)h(c,n)-[r+h(c,n)(1-e^{-rs(c,n)})]V(n+1)\}/[r+h(c,n)],$$

for any $c \in [0,1]$. Thus, $V_I(n) - V_I(n+1) \geq 0$ if a $c \in [0,1]$ can be found such that the right-hand-side of (A1) is nonnegative. Consider

$$\text{(A2)}\qquad \bar{c}_n = \underset{c\in[0,1]}{\mathrm{argmax}}\ \Gamma(c,n),$$

where $\Gamma(c,n) = [\pi(c)-\theta(c,n)h(c,n)]/[r+(1-e^{-rs(c,n)})h(c,n)]$. Clearly, such a $\bar{c}_n$ exists because $\Gamma(c,n)$ is continuous in c and c lies in a compact set [0,1]. Now it remains to show that this particular $\bar{c}_n$ will yield the desired inequality for (A1), i.e.,

$$\text{(A3)}\qquad \pi(\bar{c}_n)-\theta(\bar{c}_n,n)h(\bar{c}_n,n)-[r+(1-e^{-rs(\bar{c}_n,n)})h(\bar{c}_n,n)]V(n+1) \geq 0.$$

Since the offender never desists from crime, $V(n) = V_I(n)$ for all $n \geq 0$. Thus, substitute $V(n)$ for $V_I(n)$ and iterate the functional equation (8),

$$\begin{aligned}\text{(A4)}\qquad V(n) = \mathrm{Max}_{\{c\}}\ \{\ &\Phi(c_n,n) + \Phi(c_{n+1},n+1)D(c_n,n)\\ &+ \Phi(c_{n+2},n+2)D(c_{n+1},n+1)D(c_n,n)\\ &+ \Phi(c_{n+3},n+3)D(c_{n+2},n+2)D(c_{n+1},n+1)D(c_n,n) + \ldots\ \},\end{aligned}$$

where $\Phi(c_k,k) = [\pi(c_k)-\theta(c_k,k)h(c_k,k)]/[r+h(c_k,k)]$, and $D(c_k,k) = e^{-rs(c_k,k)}h(c_k,k)/[r+h(c_k,k)]$. Using the product symbol $\Pi$, (A4) can be expressed more compactly as

(A5) $$V(n) = \text{Max}_{\{c\}} \{\Phi(c_n,n) + \Sigma_{i=n}^{\infty} [\Phi(c_{i+1},i+1) \Pi_{j=n}^{i} D(c_j,j)]\}.$$

Since $\Phi(c_k,k) = \Gamma(c_k,k)[1-D(c_k,k)]$, (A5) can be rewritten as

(A6) $$V(n) = \text{Max}_{\{c\}}\{\Gamma(c_n,n)[1-D(c_n,n)] + \Sigma_{i=n}^{\infty} [\Gamma(c_{i+1},i+1)[1-D(c_{i+1},i+1)]\Pi_{j=n}^{i} D(c_j,j)]\}.$$

Assumption 2 implies that $\pi'(0) > 0$. Since $\pi(0) = \theta(0,n)h(0,n) = 0$, and $\theta_1(0,n)h(0,n)+\theta(0,n)h_1(0,n) = 0$, hence for each $n > 0$, there exists some $c \in [0,1]$ such that $\pi(c) > \theta(c,n)h(c,n)$. Thus, $\Gamma(\bar{c}_n,n) > 0$ for all $n > 0$. Combining this with (C1),

(A7) $$\Gamma(c,n+k) \leq \Gamma(\bar{c}_n,n)$$

for any $c \in [0,1]$ and any $k \geq 0$. Since $0 \leq D(c_k,k) < 1$ for any $k \geq 0$, substituting (A7) into (A6) yields

(A8) $$V(n) \leq \Gamma(\bar{c}_n,n)\{\text{Max}_{\{c\}} \{[1-D(c_n,n)] + \Sigma_{i=n}^{\infty} [[1-D(c_{i+1},i+1)]\Pi_{j=n}^{i} D(c_j,j)]\}\}.$$

In addition, $[1-D(c_n,n)] + \Sigma_{i=n}^{\infty} [[1-D(c_{i+1},i+1)]\Pi_{j=n}^{i} D(c_j,j)]$ $= \{1-D(c_n,n) + [1-D(c_{n+1},n+1)]D(c_n,n) + [1-D(c_{n+2},n+2)]D(c_{n+1},n+1)D(c_n,n) + [1-D(c_{n+3},n+3)]D(c_{n+2},n+2)D(c_{n+1},n+1)D(c_n,n) + \ldots\} \leq 1$. Thus, (A8) implies

(A9) $$V(n) \leq \Gamma(\bar{c}_n,n) = [\pi(\bar{c}_n)-\theta(\bar{c}_n,n)h(\bar{c}_n,n)]/[r+(1-e^{-rs(\bar{c}_n,n)})h(\bar{c}_n,n)].$$

As $\bar{c}_n$ may not be an optimal solution for the dynamic programming problem (8), substituting $\bar{c}_n$ into the right-hand-side of (8) yields

(A10) $$V(n) \geq [\pi(\bar{c}_n)-\theta(\bar{c}_n,n)h(\bar{c}_n,n)+e^{-rs(\bar{c}_n,n)}V(n+1)h(\bar{c}_n,n)]/[r+h(\bar{c}_n,n)].$$

Combining (A9) and (A10), (A3) is obtained. This completes the proof. If $\Gamma(c,n+k) < \Gamma(c,n)$, then the inequality (A7) will be strict. It follows that the inequality (A9) will also be strict, hence $V(n) > V(n+1)$.

*Case (ii)*. Suppose there exists a finite integer $p > 0$ such that $V_L(p) \geq V_I(p)$. In this case, the offender will desist from crime at $n = p$ because Case (i) has proved that if he continues to engage in illegal activity, $V_I(n) \leq V_I(p)$ for all $n > p$. Since $V(n) = \hat{\pi}(p)/r$ for $n \geq p$, it remains to prove that $V_I(n) \geq V_I(n+1)$ for $0 \leq n \leq p-1$. The proof is essentially the same as Case (i) except that a different argument is needed to show that $V(n) \leq \Gamma(\bar{c}_n,n)$. Since the offender desists at $n = p$, $V(n) = V_I(n)$ for $0 \leq n \leq p-1$, and (A6) has to be modified as follows. For $0 \leq n \leq p-2$,

$$\text{(A11)} \qquad V(n) = \text{Max}_{\{c\}}\{\Gamma(c_n,n)[1-D(c_n,n)] + \Sigma_{i=n}^{p-2}\,[\Gamma(c_{i+1},i+1)[1-D(c_{i+1},i+1)]\Pi_{j=n}^{i}\,D(c_j,j)] + [\hat{\pi}(p)/r][1-D(c_{p-1},p-1)]\Pi_{j=n}^{p-2}\,D(c_j,j)]\}.$$

Notice that $p \geq 2$ is implicitly assumed in (A11). The proof for the case $p = 1$ is trivial, since $V(0) = \text{Max}\{\hat{\pi}(0)/r, V_I(0)\} \geq \hat{\pi}(0)/r \geq \hat{\pi}(1)/r = V(1)$. Apply (A7) to replace every $\Gamma(.,.)$ in (A11) by the maximum $\Gamma(\bar{c}_n,n)$,

$$\text{(A12)} \qquad V(n) \leq \text{Max}_{\{c\}}\,\{\Gamma(\bar{c}_n,n)[1-D(c_n,n) + \Sigma_{i=n}^{p-2}\,[1-D(c_{i+1},i+1)]\Pi_{j=n}^{i}\,D(c_j,j)] + [\hat{\pi}(p)/r][1-D(c_{p-1},p-1)]\Pi_{j=n}^{p-2}\,D(c_j,j)\}$$
$$= \text{Max}_{\{c\}}\,\{\Gamma(\bar{c}_n,n) - \Gamma(\bar{c}_n,n)D(c_{p-1},p-1)\Pi_{j=n}^{p-2}\,D(c_j,j) + [\hat{\pi}(p)/r][1-D(c_{p-1},p-1)]\Pi_{j=n}^{p-2}\,D(c_j,j)\}.$$

It follows from (A12) that $V(n) \leq \Gamma(\bar{c}_n,n)$ if it can be shown that $\Gamma(\bar{c}_n,n) \geq \hat{\pi}(p)/r$. Let $c_{p-1}^{*}$ denote the optimum c that solves (8) when $n = p-1$, then

$$V_I(p-1) = \{\pi(c_{p-1}^{*}) - \theta(c_{p-1}^{*},p-1)h(c_{p-1}^{*},p-1) + [\exp(-rs(c_{p-1}^{*},p-1))]V(p)h(c_{p-1}^{*},p-1)\}/[r+h(c_{p-1}^{*},p-1)]$$
$$\geq \hat{\pi}(p-1)/r,$$

since the offender desists at $n = p$. From this inequality,

$$V(p) \geq \{[r+h(c_{p-1}^{*},p-1)][\hat{\pi}(p-1)/r]-\pi(c_{p-1}^{*})+\theta(c_{p-1}^{*},p-1)h(c_{p-1}^{*},p-1)\}/ \{[\exp(-rs(c_{p-1}^{*},p-1))]h(c_{p-1}^{*},p-1)\}.$$

Since $V(p) = \hat{\pi}(p)/r$ and $\hat{\pi}(p) \leq \hat{\pi}(p-1)$, therefore

$$\hat{\pi}(p)/r \geq \{[r+h(c^*_{p-1},p-1)][\hat{\pi}(p)/r]-\pi(c^*_{p-1})+\theta(c^*_{p-1},p-1)h(c^*_{p-1},p-1)\}/$$
$$\{[\exp(-rs(c^*_{p-1},p-1))]h(c^*_{p-1},p-1)\}.$$

Rearranging this inequality yields

$$\text{(A13)} \quad [\pi(c^*_{p-1})-\theta(c^*_{p-1},p-1)h(c^*_{p-1},p-1)]/\{r+[1-\exp(-rs(c^*_{p-1},p-1))]h(c^*_{p-1},p-1)\}$$
$$\geq \hat{\pi}(p)/r.$$

The left-hand side of (A13) is just $\Gamma(c^*_{p-1},p-1)$, which by (C1) is less than or equal to $\Gamma(\bar{c}_n,n)$ (since $p-1 \geq n$). Thus, $\Gamma(\bar{c}_n,n) \geq \hat{\pi}(p)/r$. Consequently, $V(n) \leq \Gamma(\bar{c}_n,n)$ and the rest of the proof in Case (i) follows.

PROOF OF PROPOSITION 3:

Let $U(c,n) = \pi(c) - \theta(c,n)h(c,n) + e^{-rs(c,n)}V(n+1)h(c,n)$. The first-order and second-order necessary conditions of (8) are given by $[r+h(c^*_n,n)]U_1(c^*_n,n) - U(c^*_n,n)h_1(c^*_n,n) = 0$, and $M = [r+h(c^*_n,n)]U_{11}(c^*_n,n) - U(c^*_n,n)h_{11}(c^*_n,n) \leq 0$, respectively. These can be expressed more simply as

$$\text{(A14)} \qquad U_1(c^*_n,n) - V_I(n)h_1(c^*_n,n) = 0,$$
$$M = U_{11}(c^*_n,n) - V_I(n)h_{11}(c^*_n,n) \leq 0,$$

since $V_I(n) = U(c^*_n)/[r+h(c^*_n,n)]$. Equation (9) is identical to (A14) when $c = c^*_n$. It remains to verify that $c^*_n$ is indeed a maximizer. The second-order sufficient condition will be satisfied if $M < 0$. For convenience, the arguments $c^*_n$ and $n$ are suppressed. Straightforward calculation yields

$$\text{(A15)} \quad M = \pi'' - \theta_{11}h - 2\theta_1h_1 - \theta h_{11} + e^{-rs}V(n+1)(-hs_{11} + rhs_1^2 - 2h_1s_1 + h_{11})$$
$$- V_I(n)h_{11}.$$

It follows from Assumption 2 and (C3) that $M < 0$, and the second-order sufficient condition is satisfied.

PROOF OF PROPOSITION 4:

Consider first the case for $\theta$. If the offender desists from crime at $n = p$, then $V(p) = \hat{\pi}(p)/r$, so that $\partial V(p)/\partial\theta = 0$. One can then iterate (10) backward to prove that $\partial V_I(n)/\partial\theta \le 0$, hence $V_I(n)$ is decreasing in $\theta$ for any $n < p$. If the offender never desists from crime, then $V(n) = V_I(n)$ for all n. In this case one can employ standard dynamic programming techniques (e.g. Stokey and Lucas, 1989, Theorem 4.7) to show that the contraction maps decreasing functions of $\theta$ to decreasing functions of $\theta$. Hence the fixed point $V_I(n)$ is decreasing in $\theta$ and $\partial V(n)/\partial\theta \le 0$. The proofs for the other cases are similar and are therefore omitted. Condition (C1) ensures that $V(n) - e^{-rs}V(n+1) \ge 0$ and hence the right-hand side of (15) is nonpositive. The proofs for the signs of $Zc_n^*/\partial\theta$, $\partial c_n^*/\partial s$, and $\partial c_n^*/\partial\hat{\pi}$ follow directly from inspecting (12), (14), and (18).

PROOF OF PROPOSITION 5:

From (10) and (15),

(A16) $$[\partial V(n)/\partial\theta]\theta - [\partial V(n)/\partial h]h$$
$$= \{\{[\partial V(n+1)/\partial\theta]\theta - [\partial V(n+1)/\partial h]h\}e^{-rs} + [V(n)-e^{-rs}V(n+1)]\}h/(r+h).$$

First, consider the case where the offender never desists from crime. Define a <u>truncated</u> value function $V^m(n)$ $(0 < m < \infty)$:

(A17) $$V^m(n) = \text{Max}_{\{c\}} \{\Phi(c_n,n) + \Sigma_{i=n}^m [\Phi(c_{i+1},i+1) \Pi_{j=n}^i D(c_j,j)]\}$$

for $n < m$, and define $V^m(n) = 0$ for all $n \ge m$. Let $Q^m(n) = [\partial V^m(n)/\partial\theta]\theta - [\partial V^m(n)/\partial h]h$, and $R^m(n) = V^m(n)-e^{-rs}V^m(n+1)$. It is easy to verify that (A16) also holds for the truncated case (with V replaced by $V^m$). Hence,

(A18) $$Q^m(n) = [e^{-rs}Q^m(n+1) + R^m(n)]h/(r+h).$$

It is easy to check that Proposition 2 remains valid when the value function is truncated, hence $R^m(n) \geq 0$ for $n \leq m$. Since $Q^m(m) = 0$, it follows from induction on (A18) that $Q^m(n) \geq 0$ for all $n \leq m$. The proof will be completed if it can be shown that $\lim_{m\to\infty} Q^m(n) = Q(n) = [\partial V(n)/\partial\theta]\theta - [\partial V(n)/\partial h]h$. This amounts to proving that $\lim_{m\to\infty} \partial V^m(n)/\partial\theta = \partial V(n)/\partial\theta$ and $\lim_{m\to\infty} \partial V^m(n)/\partial\gamma = \partial V(n)/\partial\gamma$. Differentiate (A17) with respect to $\theta$ and use the envelope theorem,

(A19) $$\partial V^m(n)/\partial\theta = -\{G(c_n^m,n) + \Sigma_{i=n}^m [G(c_{i+1}^m,i+1)\ \Pi_{j=n}^i\ D(c_j^m,j)]\},$$

where $G(c_k^m,k) = \theta(c_k^m,k)h(c_k^m,k)/[r+h(c_k^m,k)]$, and $c_k^m$ $(k = n,n+1,\ldots,m)$ denote the optimal offense rate for the truncated problem (A17). Since $\theta$ and h are bounded functions, let $\bar\theta = \sup\ \theta < \infty$ and $\delta = \sup\ \{h/[r+h]\} < 1$. Since $e^{-rs} \leq 1$, (A19) implies that $|\partial V^m(n)/\partial\theta| \leq \bar\theta\delta[1 + \delta + \delta^2 + \ldots + \delta^{m-n}] \leq \bar\theta\delta/(1-\delta)$ (a constant and therefore $\partial V^m(n)/\partial\theta$ is Lebesgue integrable). It follows from the dominated convergence theorem that

(A20) $$\lim_{m\to\infty} \int_a^b [\partial V^m(n)/\partial\theta]d\theta = \int_a^b [\lim_{m\to\infty} \partial V^m(n)/\partial\theta]d\theta.$$

By the fundamental theorem of calculus,

(A21) $$\int_a^b [\partial V^m(n)/\partial\theta]d\theta = V^m(n)|_{\theta=b} - V^m(n)|_{\theta=a},$$

(A22) $$\int_a^b [\partial V(n)/\partial\theta]d\theta = V(n)|_{\theta=b} - V(n)|_{\theta=a},$$

where $V(n)|_{\theta=b}$ denotes that V(n) is evaluated at $\theta = b$. The contraction property implies that $\lim_{m\to\infty} V^m(n)|_{\theta=b} = V(n)|_{\theta=b}$, and $\lim_{m\to\infty} V^m(n)|_{\theta=a} = V(n)|_{\theta=a}$, thus (A20), (A21), and (A22) imply that $\lim_{m\to\infty} \partial V^m(n)/\partial\theta = \partial V(n)/\partial\theta$. A similar argument shows that $\lim_{m\to\infty} \partial V^m(n)/\partial h = \partial V(n)/\partial h$, thus $Q(n) \geq 0$. As $\epsilon_h - \epsilon_\theta = Q(n)/V(n)$ $(V(n) > 0)$, the result follows immediately.

If the offender desists from crime at $n = p$ for some $p > 0$, then the proof is even simpler since the value function has a natural truncation point at $m = p-2$ (see (A11)) and there is no need to use the above limiting argument. The induction on (A18) is sufficient.

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## NOTES

1. See e.g. Avi-Itzhak and Shinnar (1973), Stollmack and Harris (1974), Shinnar and Shinnar (1975), Blumstein and Nagin (1978), Chaiken and Rolph (1980), Maltz and Pollock (1980), Greenwood (1982), Tierney (1983), Lehoczky (1986), Greenberg (1991), and the reviews in Cohen (1978) and Blumstein et al. (1986).

2. This deficiency has also been recognized by Manski (1978), who offers several other reasons to emphasize the importance of finding a connection between behavioral models and stochastic models of crime.

3. Instead of imprisonment, the punishment for some types of crime (such as corporate or white-collar crimes) may be in the form of a temporary suspension of the offender's license to operate his business. The model can easily handle this case since $s(c,n)$ can also be interpreted as the duration of suspension.

4. The hazard rate $h(c,n)$ is employed to model the decision problem because the time of detection and conviction is a random variable. As the decision to engage in illegal activity is made at each point in time, taking the past as given, it is therefore conditional in nature. Since the hazard rate describes the instantaneous probability of arrest at time $t$ given the offender has not yet been arrested by time $t$, it is appropriate to model the effects of $c$ and $n$ directly through the hazard rate.

5. For convenience it is assumed that there is no time lag between detection and conviction. This does not imply that detection must result in conviction because conviction can also be modeled as a random variable. For example, let $\mu(c(t),n)$ be the hazard rate of detection and $\nu(c(t),n)$ be the probability of conviction, then assuming independence, the hazard rate of detection and conviction will be given by $h(c(t),n) = \mu(c(t),n)\nu(c(t),n)$. It is easy to check that the analysis will go through with this modification.

6. This can be proved rigorously by iterating $V(t+s(c(t),n),n+1)$ explicitly and noting the difference between the discount terms (at time $t+s(c(t),n)$ versus at time $t$).

7. The following argument shows that the normalization is innocuous. Since $\int_t^{t+s(c(t),n)} e^{-r(x-t)}\tilde{\pi}dx = [1-e^{-rs(c(t),n)}]\tilde{\pi}/r$, integration by parts shows that (3) can be expressed as $V_I(\tau,n) = \tilde{\pi}/r + \mathrm{Max}_{c(\cdot)} \int_\tau^\infty e^{-r(t-\tau)}\{[\pi(c(t))-\tilde{\pi}][1-F_n(t)]/[1-F_n(\tau)] + [-\theta(c(t),n) + e^{-rs(c(t),n)}[V(t+s(c(t),n),n+1)-\tilde{\pi}/r]]F_n'(t)/[1-F_n(\tau)]\}dt$. Subtracting $\tilde{\pi}/r$

from both sides of this expression yields $W(\tau,n) = \text{Max}\ \{W_L(\tau,n), W_I(\tau,n)\}$, where $W(\tau,n) = V(\tau,n)-\tilde{\pi}/r$, $W_L(\tau,n) = V_L(\tau,n)-\tilde{\pi}/r$, and $W_I(\tau,n) = V_I(\tau,n)-\tilde{\pi}/r$. It follows that $\tilde{\pi}$ can be treated as baseline returns since all the other returns ($\pi$, V, $V_L$, and $V_I$) can be defined relative to $\tilde{\pi}$. Hence, to economize the use of symbols, $\tilde{\pi}$ is normalized to zero so that the V symbols for the value functions can be maintained.

8. The model assumes that the individual participates either in legal activity or in illegal activity. Carr-Hill and Stern (1979) argue that such a dichotomous choice between legal and illegal activities may not be useful for some types of crime. For example, they maintain that a substantial proportion of property offenses and minor thefts are committed by people with full-time legal jobs. The pursuit of these illegal activities is to increase total wealth. To deal with this possibility, the model can be modified as follows. Let $V(n) = \text{Max}_{c(.)} \int_0^\infty e^{-rt}[\pi(c) + \hat{\pi}(n) - \theta(c,n)h(c,n) + e^{-rs(c,n)}V(n+1)h(c,n)][1-F_n(t)]dt$. The individual obtains $\pi(c)$ from illegal activity, along with $\hat{\pi}(n)$ from legal activity. In fact, this problem becomes even simpler than (5)-(7) as it is no longer necessary to distinguish between $V_L$ and $V_I$. Although this problem will not be analyzed below, one can verify that all the main results of the paper apply to this case as well.

9. The transformation from (7) to (8) was first proved by Kamien and Schwartz (1971) in a different and much simpler setup. Since then, many economists have used Kamien and Schwartz's proof to justify similar transformations in other contexts (e.g. Davis, 1988). However, Kamien and Schwartz's proof is unwarranted because it relies on an unverified transversality condition for an infinite horizon control problem. For details and a solution to the problem, see Leung (1991b).

10. Strictly speaking, the curves AC and BC should be step functions since n only takes on integer values. They are drawn as continuous curves simply for convenience.

11. See e.g. Maguire et al. (1988) and Schmidt and Witte (1984, 1988, 1989). However, as pointed out by a referee, there is a subtle problem in interpreting these empirical findings. Although these authors find that n has a significant positive effect on the hazard rate of rearrest, they do not fully control for c in their hazard estimations. As a result, these studies may not offer conclusive evidence for the view that h(c,n) increases with n, holding c constant. Nevertheless, Maguire et al. (1988, section 4.1) are aware of this problem, so they restrict their sample to offenders who were rearrested for felonies because misdemeanors vary widely in seriousness. In this way they can partially control for the variation in c. One can also interpret the explanatory variable TSERVD (time served in prison) in Schmidt and Witte (1984, 1988, 1989) as a partial control for c because the sentenced length of incarceration is a good proxy for the seriousness of the offense. Unfortunately, they define TSERVD as the time served for the <u>previous</u> offense, not for the <u>current</u> offense (i.e. the offense that led to the offender's subsequent return to prison). Hence TSERVD is a good proxy for the seriousness of the current offense only if the intensities of successive offenses are highly correlated. On the other hand, one can argue that using the time served for the current offense as a

proxy is problematic because it is an endogenous variable. Furthermore the usual explanatory variables (demographic and past criminal history) employed in the hazard estimations (e.g. Schmidt and Witte, 1984, section 3.3) should be able to provide a fairly good control for the seriousness of the current offense. This is a subtle empirical problem which remains to be resolved.

12. This implication is supported by Rauma and Berk's (1987) empirical findings. In a 5-year follow-up study of a California program started in 1978, Rauma and Berk (1987) find that recidivism among ex-offenders can be reduced by providing unemployment compensation immediately after their release from prison.

13. As demonstrated in the proof in Appendix, the purpose of condition (C3) is to guarantee that the second-order condition of the maximization problem (8) is satisfied. Notice that if $V(n) = V_I(n)$ and (C1) or (C2) holds, then $e^{-rs}V(n+1) - V_I(n) = e^{-rs}V(n+1) - V(n) \leq 0$ (by Proposition 2), so (C3) can be replaced by the weaker condition (C4): $-hs_{11} + rhs_1^2 - 2h_1s_1 \leq 0$. All three terms in (C4) are related to the slope of s. The expression arises from the nonlinear (exponential) effect of imprisonment on future returns. If $s_1 = 0$, i.e. the length of imprisonment is independent of the offense rate, then (C4) will be satisfied.

14. More formally, let $\Theta(c,n) = \lambda+\theta(c,n)$, where $\lambda$ is a shift parameter, and replace every $\theta$ in (8) and (9) by $\Theta$. The parallel shift in the function $\theta$ described in the text is equivalent to a change in the parameter $\lambda$ because $\partial\Theta(c,n)/\partial\lambda = 1$ and $\partial\Theta_1(c,n)/\partial\lambda = 0$ (i.e. $\Theta_1(c,n) = \theta(c,n)$). The expressions for $\partial V(n)/\partial\theta$ and $\partial c_n^*/\partial\theta$ in the text can be shown to be qualitatively equivalent to $\partial V(n)/\partial\lambda$ and $\partial c_n^*/\partial\lambda$, respectively. Since adding a new parameter to each of the functions h, $\theta$, s, and $\pi$ will make the notations unduly heavy, the simpler convention used in the text is adopted.

15. In fact, none of the existing two-period deterrence models considers imprisonment. In these models, $s = 0$ is assumed. As $s_1 = 0$, the condition $s_1h \leq h_1/(r+h)$ is automatically satisfied.

16. There is some evidence supporting this general belief. For example, Ross' (1984) analysis of drunk driving in Europe and North America suggests that an increase in the certainty of punishment is more deterrent effective than an increase in the severity.

17. A key step in the proof of Proposition 5 is the induction on (A18) (see Appendix), which crucially rests on the inequality $V(n) \geq e^{-rs}V(n+1)$. This in turn relies on the validity of Proposition 2. For example, if (C1) does not hold, then (A7) in Appendix may not hold and Proposition 2 cannot be established. As a result, the proof of Proposition 5 fails and it is possible that $\epsilon_\theta > \epsilon_h$. In particular, if $V(n) < e^{-rs}V(n+1)$, then the induction on (A18) yields $Q^m(n) < 0$ for all $n \leq m$. It follows from the rest of the proof of Proposition 5 that $\epsilon_\theta > \epsilon_h$.

18. Notice, of course, that the two optimal offense rates ($c_n^*$ and $c_{n+1}^*$) may differ, but the key is that the optimal offense rate does not vary with time between any two consecutive convictions.

19. A Markov jump process is a Markov process of which the sample paths are constant except for isolated jumps, and right-continuous. See Hoel et al. (1972), Gikhman and Skorokhod (1968), or Breiman (1968) for details.

20. A jump process is pure (or non-explosive) if $\lim_{n\to\infty} \tau_n = \infty$, where $\tau_n$ is the time at which the nth jump occurs. For the model here, a sufficient condition for the Markov jump process to be pure is that $\sup h(c,n) < \infty$; see Breiman (1968, p.337) for a proof. This condition is satisfied here by virtue of Assumption 1.

21. Mathematically, if the crime process is Poisson with mean $\lambda$ and the probability of arrest for each crime is q (q is independent of the number of previous arrests and the number of crimes committed), then the arrest process will be Poisson with mean $\lambda q$. This is a standard result in the theory of compound distributions, see Feller (1968, p.287) for a proof.

22. The possible tradeoff between participation and intensity may be formally analyzed in the following way. Assume that the legal returns of each individual are distributed randomly with a cumulative distribution function H(.) and that individuals are otherwise identical. Then, the expected offense rate at $n = 0$ is $E(c) = 0.[1-H(V_I(0))] + c_0^*H(V_I(0)) = c_0^*H(V_I(0))$, because individuals with $V_L(0)$ below $V_I(0)$, the probability of which is $H(V_I(0))$, will engage in illegal activity with offense rate $c_0^*$. For those with $V_L(0)$ higher than $V_I(0)$, the offense rate is zero. The social objective is to minimize E(c), and $\partial E(c)/\partial\theta = [H(V_I(0))][\partial c_0^*/\partial\theta] + c_0^*[H'(V_I(0))][\partial V_I(0)/\partial\theta]$. A similar expression for $\partial E(c)/\partial h$ can be obtained. Since theoretical comparisons between the two expressions do not seem to yield any tractable results, numerical simulations may be needed to solve the problem.

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Figure 1. A non-chronic offender

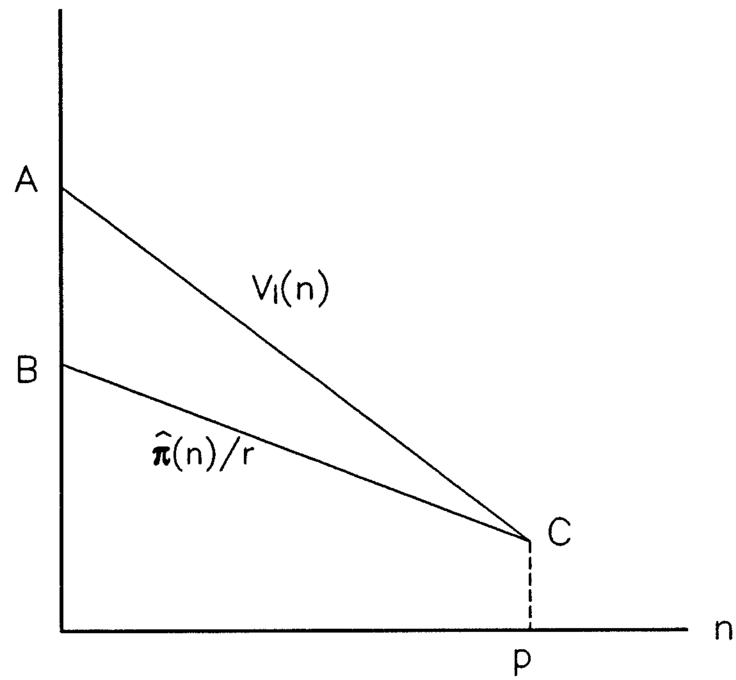


Figure 2. A chronic offender