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Working Paper No. 373
March 1994

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by Marcus Berliant and Hideo Konishi[(*)]

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Abstract

Much of the literature on the endogenous generation of a city employs increasing returns to scale in order to obtain agglomeration. In contrast, the model considered here focuses on the role of marketplaces or trading centers in the agglomeration of population as cities. Gains to trade in combination with transportation and marketplace setup costs suffice to endogenously generate a city or cities with one or multiple marketplaces. It is assumed that consumers are fully mobile while production functions are location–specific. The exchange of commodities takes place in competitive markets at the marketplaces, while the number and locations of the marketplaces are determined endogenously using a core concept. Unlike the standard literature of urban economics, our model can deal with differences in geography by letting the setup costs of marketplaces and the transportation system depend on location. After showing that an equilibrium exists and that equilibrium allocations are the same as core allocations, we investigate the equilibrium number and locations of marketplaces, the population distribution, and land prices. In contrast with earlier literature, the results are general in the sense that specific functional forms are not needed to obtain existence of equilibrium, equilibria are first best, and equilibria are locally unique (in our examples).

(*) Department of Economics, University of Rochester, Rochester NY, 14627–0156.
The authors wish to thank the Center for Public Financial Management, Carnegie–Mellon University, for computational support and its director, Robert P. Strauss, for general discussions. Comments and suggestions made by Masa Fujita, Tony Smith, and Jacques Thisse in the Regional Science Theory Workshop at the University of Pennsylvania were invaluable. Conversations with Jim Peck were quite helpful as well. We also thank John H. Boyd III, Richard Boylan, Graciela Chichilnisky, Charles Engel, Lionel McKenzie, Suzanne Scotchmer, Tomoichi Shinotsuka, Raghu Sundaram, William Thomson, and Bill Zame for their comments. The authors retain full responsibility for any errors remaining in the paper.

> Where human beings organize their economy around market exchange, trade between city and country will be among the most powerful forces influencing cultural geography and environmental change. The ways people value the products of the soil, and decide how much it costs to get those products to market, together shape the landscape we inhabit.
>
> Cronon (1991, p. 50)

1. Introduction

Over the past few years, increasing attention has been focused on the economics of cities. Why do cities form where they do? What are the driving forces behind the formation of cities? What roles do gains to trade and the location of marketplaces play? Is perfect competition consistent with spatial modeling?

Naturally, in order to motivate our approach to these questions, it is useful to see how the extant literature has addressed them. There are two important aspects of an explanation of why a city is formed in a particular place. These have been labeled as first and second natures of geography (Cronon (1991)). A first nature of geography is an advantage created by nature of locating at a certain place (such as a natural harbor, a river, and so on), while a second nature of geography is an advantage created by human beings of locating there.

There are many ways to interpret the notion of second nature of geography. Krugman (1991a,b) points out that increasing returns to scale from population agglomeration are important in explaining the formation of cities, and stresses the importance of the history of a city. History affects the consumers' location choices in each period, and it determines the future of a city. Krugman also introduces transportation cost and discusses the resulting location of cities. Krugman (1993a) introduces dynamics into the model, while Krugman (1993b) uses a continuum of locations and endogenously generates potential functions. Fujita and Krugman (1993) include land explicitly in the model, allow many types of manufactured goods, and obtain various types of city configurations and industrial specializations.[1] Similar models were investigated earlier by Fujita and Ogawa (1982) and Fujita (1988) as well. Jane Jacobs (1969, 1984) lends support to this entire line of argument.

Although it is limited to a specific functional form, the use of a Dixit–Stiglitz (1977) model in this line of research to explain city formation seems important, since it can generate various types of urbanization in developed (industrialized) countries. However, since the model features increasing returns to scale and monopolistic competition, there is a large indeterminacy of the set of equilibria. Therefore, from the same economic data (preferences, endowments, and so on), we can obtain many equilibrium city structures (different numbers of cities at different locations).

In this paper, we will focus on the history of the formation of a city as detailed by Cronon (1991), who analyzes the birth and the development of Chicago. Before and in the early stages of industrialization, Cronon says that the second nature of geography played a crucial role in the development of Chicago. What Cronon means by "second nature" is actually something different from increasing returns to scale due to population agglomeration.

> Each new improvement meant a shift in regional geography — a dredged harbor here, a canal or a road there — so the advantages sustaining the city came to have an ever larger human component. A kind of 'second nature,' designed by people and 'improved' toward human ends, gradually emerged atop the original landscape that nature — 'first nature' — had created as such an inconvenient jumble.
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> Cronon (1991, p.56)

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1 Ciccone and Hall (1993) find a positive relationship between productivity level and the density of economic activity. Kim (1993) provides some empirical evidence on regional concentration or specialization of industrial production in the United States. For the data from the 1920's to the present, this evidence seems to contradict the verbal arguments of Krugman (1991b) concerning industrial specialization.

What Cronon calls the "second nature" of geography is the class of commodities called local public goods. Cronon cites a dredged harbor, a canal, a road, a railroad, and especially a marketplace as examples of the second nature of geography that contributed to the agglomeration of Chicago (Cronon, 1991, pp. 56 – 62).

> The number and scale of such interregional trading connections critically determined a city's eventual position in the urban hierarchy. Cities with the greatest access to the East would become the new metropolises of their region; towns with less direct eastern ties would rely on western wholesaling centers for the bulk of their merchandise and develop only a local retail trade of their own.
>
> Cronon (1991, p. 62)

To the north of Chicago is a forest that can produce lumber, while to the south is a prairie that is most suitable for producing wheat. Since Chicago established marketplaces for these commodities, many people accessed Chicago, and as a result, Chicago became the main regional market for all of these commodities. That is, the early development of Chicago was generated from gains from trade, location–specific production, and the establishment of marketplaces; see Cronon (1991, pp. 154–155). Transportation systems, canals, and railroads played important roles in the development of Chicago as well. The Illinois and Michigan Canal made Chicago accessible to the Illinois and Mississippi Rivers (Cronon, 1991, pp. 63–64). A local railroad network reduced the cost of shipping lumber and crops and enlarged Chicago's regional market (Cronon, 1991, pp. 65–70). More globally, the Erie Canal and railroads linked Chicago with New York — the center of the eastern market containing a major harbor that imported European commodities. The Union Central Pacific Railroad connected Chicago with San Francisco on the west coast (Cronon, 1991, p. 70). These improvements, made by people, changed Chicago from the town with 'the muddy roads and shallow harbor' to 'the new metropolis of the Great

West' (Cronon, 1991, p. 63).

This paper provides a model of city formation that focuses on marketplaces and on mass transportation systems for transporting commodities among marketplaces. We make the locations of marketplaces endogenous and we impose an individual transportation cost for accessing a marketplace as well as setup costs for both marketplaces and mass transportation systems that connect marketplaces (railroads, canals, and so on). Consumers are endowed with labor and land[2] that is used either for consumption or to produce commodities; the commodities are produced using location–specific production technologies, so the amount of a commodity produced depends on the distribution of labor across locations.[3] Consumers can trade commodities and land only at marketplaces, and they pay transportation cost individually (the opportunity cost of leisure) to access marketplaces. Hence, if there is only one marketplace, then a consumer must access it if she wants to trade goods. If there is more than one marketplace, then society must build a mass transportation system to connect these marketplaces in order to transport commodities from one marketplace to another. In this case, each consumer accesses the marketplace that is most convenient. The explicit introduction of a mass transportation system to the model makes our model rich enough to explain the development of cities in the places where terminals of a mass transportation system were located, while allowing the location of such a system to be endogenous.[4] Gains to trade provide an

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[2] There is no absentee landlord.

[3] Location specific production is one of the assumptions that implies gains to trade. In the context of Chicago's rise, Cronon (1991) points to location specific production of lumber, grains, and livestock in separate chapters of his book. Of course, Chicago eventually became the main regional market for all of these commodities. In terms of a theoretical predecessor, Schweizer, Varaiya and Hartwick (1976) have location specific production.

[4] Buffalo and Los Angeles are other examples of such cities. Buffalo became a large city because it is located at the west end of the Erie Canal. Although San Diego has a better natural harbor than Los Angeles, Los Angeles is bigger than San Diego, as Los Angeles was chosen as the terminus of a transcontinental railroad.

incentive for marketplaces to coalesce. Although most of the papers in this field introduce increasing returns to scale in order to induce agglomeration, our model focuses on cities as marketplaces or trading centers following Berliant and Wang (1993).[5] That is, we introduce (local) public goods into the model instead of increasing returns to scale to obtain population agglomeration. The locations of marketplaces and consumers as well as land prices and the method of sharing the cost of setting up marketplaces and a mass transportation system are all endogenous and simultaneously determined. Since we treat marketplaces as local public goods, the collective choices made by consumers to set up marketplaces, to set up a mass transportation system, and to share the costs of these goods play an important role in the analysis.[6]

Moreover, since we let the setup costs of marketplaces and a mass transportation system be dependent on location, our model can capture aspects of the first nature of geography.[7] For example, we can allow for the following situations: one location might be more suitable than another for dredging a harbor, or it might be less costly to build a railroad between one pair of marketplaces than another. In the previous literature, including the work with increasing returns, it is usually assumed that the geography is a homogeneous plane or line. That is, the locations in the economy are physically identical ex ante. However, the role of natural harbors is certainly quite

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5 Berliant and Wang (1993) construct a model of city formation without production where the location of marketplaces is endogenous and study under what conditions a monocentric city is formed. Gains to trade, transport cost, and the setup cost of marketplaces drive the model. They start from a social welfare maximization problem and find price support for the social optimum, which generates a market equilibrium. Consumers have limited mobility, as they must reside in the area where their endowments are found.

6 The reasons why we must employ specialized rather than general models to study our questions are as follows. Aside from the notion that endogeneity of location as well as prices and quantities makes it difficult to establish precisely characteristics of equilibrium, the introduction of a spatial dimension causes some interesting problems in solving our model. As we shall see in Example 1, the nonemptiness of the core is not generally assured if there is heterogeneity in consumers.

7 Ellickson and Zame (1994) also stress the importance of the first nature of geography.

important in explaining the development of Boston and New York. Since Chicago faces on Lake Michigan, heavy lumber could easily be transported to Chicago from the north. The location–specific production technology implicitly describes part of the first nature of geography as well. Our model can capture these elements.

Although we follow Berliant and Wang (1993) in regarding marketplaces as a kind of local public good, our approach is more positive. Assuming that consumers incur costs when establishing a marketplace, we try to find where marketplaces will be located, and how the setup cost of marketplaces is shared at a core allocation. The core is a solution concept from cooperative game theory that requires that no coalition (subgroup) of consumers wants to deviate from the allocation. In our particular model, we require that no coalition of consumers wants to construct an exclusive marketplace or set of marketplaces that only they can use to increase their utility. Therefore, the core allocations are regarded as stable allocations. Using this solution concept, we can analyze the determination of the location of marketplaces and the cost–sharing of the setup costs of marketplaces and mass transportation systems simultaneously. We will establish the existence of a core allocation and characterize it in our model. As a result, we can see how cities and marketplaces are set up by consumers' cooperative actions for their mutual benefit.[8] Gains to trade derived from location–specific production give consumers an incentive to set up marketplaces. Through a core equivalence theorem, exchange at a marketplace can be interpreted as resulting from a competitive price equilibrium with a participation fee for marketplaces, given the marketplace locations. That is, given locations of the marketplaces, consumers choose their locations and consumption plans subject to their budget constraints and firms maximize their profits, while a city manager minimizes

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[8] We should note that some recent work in finance, such as Greenwood and Smith (1993), examines the dynamics of the opening of financial markets in a setting that lacks a spatial dimension.

the setup costs of marketplaces and of a mass transportation system given a price vector and charges participation fees to consumers that are used to finance the setup costs of marketplaces and a mass transportation system.

In the context of the models contained in the literature, there are three ways to define a city, or agglomeration. The definition can involve any combination of (i) the concentration of population (e.g. Baruchov and Hochman (1977) and Papageorgiou and Smith (1983)), (ii) the concentration of market or transaction activities (e.g. Stuart (1970) and Baesemann (1977)), and (iii) the concentration of employment and production (e.g. Mills (1967), Papageorgiou (1979) and Imai (1982)). Our work is concerned with all of these aspects of agglomeration. We *define* a city to be where population agglomerates, but both marketplace location and employment play roles in our model.

It is important to relate our work to the Spatial Impossibility Theorem of Starrett (1978), one of the central results in the field (see Fujita (1986) for an insightful discussion of this result). Loosely speaking, the Theorem states that if (i) there is no cost of relocation, (ii) consumers' preferences and firms' technologies are independent of location, (iii) the economy is closed, and (iv) each location has complete competitive markets, then there is no equilibrium with positive transportation cost in aggregate. In essence, under these assumptions, economic activity and agents are completely spread out over space if land is included in the model. Our model circumvents this result by dropping assumptions (ii) and (iv), employing location specific production and a setup cost for marketplaces.

Most models in the literature drop assumptions (ii) and/or (iv) as well: consumers must access a central business district, the location of which is *exogenously* given. The central business district might be the only marketplace, or the only location where goods are produced. For example, Ellickson and Zame (1991) prove very general theorems on the existence of market equilibrium and core equivalence in an exchange economy with this type of framework. In contrast, we employ location specific production and determine the locations of marketplaces endogenously. In the end, if one is to have agglomeration in equilibrium, one must dispense with one or more of the Starrett assumptions. A review of the various articles accomplishing this in different ways can be found in Berliant and Wang (1993). We note in passing that much of the literature of regional science does not violate any of the assumptions of the Starrett Theorem, and thus must rely on other forces, such as population externalities, to achieve agglomeration; see Berliant and ten Raa (1993) for further detail.

Our main results are as follows. Under the assumptions that all consumers are identical and the setup costs for marketplaces and mass transportation systems are proportional to the number of participants in a market, the core is shown to be nonempty and equivalent to the set of competitive equilibrium allocations with participation fees for marketplaces when the number and locations of marketplaces are optimal. At a core allocation, consumers are treated equally almost surely in the utility sense. If consumers are heterogeneous, the core can be empty due to a nonconvexity caused by the cost an individual consumer incurs in accessing a market. Finally, if the setup costs for marketplaces and mass transportation systems are independent of the number of participants and all consumers are identical, the core is shown to be nonempty but the set of core allocations is larger than the set of competitive equilibrium allocations. The characteristics of the solutions are examined through simulations.[9] The numerical analysis says that as the setup cost of marketplaces goes up, the number of marketplaces decreases and population agglomeration is enhanced. The rent of land in the center of a city will increase

[9] The reason we use simulations instead of analytically deriving comparative statics to characterize equilibrium is that it is very difficult to obtain analytical results since we have many endogenous location variables as well as many (utility equating) constraints due to the free mobility of consumers.

with marketplace setup cost as well. If either preferences are asymmetric in produced goods or the production technology for marketplaces is asymmetric in produced goods, the location of marketplaces will be biased. If the setup cost of a mass transportation system is dependent on the locations of marketplaces, the equilibrium number and locations of marketplaces are affected. The last result suggests the importance of the first nature of geography in the theory of city formation.

We set up a general model and define some relevant concepts in section 2. Section 3 shows that the core can be empty in general and demonstrates the nonemptiness of the core in a special case. Section 4 characterizes core allocations using examples. Section 5 concludes. An appendix contains all of the proofs.

2. A general model

Here we will construct an abstract spatial economy that contains variants of the linear city and the monocentric city models as special cases.

<u>(a) An Overview of the Economy:</u> There are I (finite and integer) produced consumption commodities in the economy. There is a finite number of different types of locations in the economy. The *location set* is denoted by $J \subset \mathbb{R}^m$, where m is a positive integer (typically, $m = 1$ or 2) and J is finite. Each location $j \in J$ is just a point, but it contains a positive amount of homogeneous land. A consumer can choose one location from J to live in. Each consumer owns land and labor as her endowment. Although commodities can be transported physically among locations, land and labor are specific to locations. Each consumer produces commodities using production technologies available at her location, and brings her

products to a marketplace to trade goods (commodities and land). In marketplaces, other consumers are selling goods she wants. The consumer is required to spend time to go to a marketplace; the time spent depends on the location of the marketplace and her residential location. Marketplaces can be established in *a feasible marketplace location set* $D \subset \mathbb{R}^m$. The set D is assumed to be compact.[10] If we use a linear city model, then D is an interval in $\mathbb{R}$. If we use a monocentric city type model, then D is a subset of $\mathbb{R}^2$. More than one marketplace can be established if needed. In such a case, marketplaces must be connected by a mass transportation system to get commodity flows between them. To establish a marketplace and a mass transportation system, a coalition of consumers has to pay costs depending on the size of the coalition using the marketplace system. Figures 1 and 2 illustrate examples of our economy when m = 1. In the examples, J = {0, .33, .67, 1} and D = [0, 1]. In locations {0, .33}, commodity 1 is produced, and in locations {.67, 1}, commodity 2 is produced. In each $j \in J$, there is a certain (possibly different) amount of land. In Figure 1, there is only one marketplace, located at {.5}. At equilibrium, a commodity will have the same price at each marketplace. If a consumer is living at location {0}, she travels from {0} to {.5} in order to trade. If, instead, a consumer is living at location {.33}, she only travels from {.33} to {.5} in order to trade. In Figure 2, there are two marketplaces with locations {.167} and {.833}. If a consumer is living at {0}, she accesses the marketplace {.167}, since the travel cost to that marketplace is lower. On the other hand, if a consumer is living at {1}, she accesses the marketplace at {.833}. The

10 There is an asymmetry between the cardinality of consumer locations (finite) and potential marketplace locations (a continuum). If we modelled both as a continuum, complications would arise in both the simulations and in core equivalence, aside from much more technical proofs. If both were finite, odd locations of marketplaces would occur in the simulations simply because marketplace locations that would otherwise be optimal might be omitted from the set of potential marketplace locations. The latter case is covered by our model.

two marketplaces are connected by a mass transportation system that supplies commodities to each marketplace. If this transportation system did not exist, only commodity 1 would be available at the marketplace {.167}, and only commodity 2 would be available at the marketplace {.833}. A consumer chooses her location j from J taking into account, for each location, the wage, rent, and commuting cost to the closest marketplace.

(b) Marketplaces: The location of a marketplace can be anywhere in D. No marketplace requires land: a marketplace is a point $d \in D$. The locations of the marketplaces are denoted by $\{d_1, d_2, \cdots, d_k\} = \mathbf{d} \subset D$. Let K be the collection of finite sets in D ($\emptyset \in K$). If there is only one marketplace, all consumers have to access it to trade with each other. If there is more than one marketplace, then each consumer will access the marketplace that is most convenient. We assume that the transportation cost of commodities between marketplaces is negligible, while to access marketplaces consumers have to use their time endowments. This assumption appears reasonable, but we can introduce costs of transporting mass quantities of commodities between marketplaces with little change in the analysis that follows (but notation would become quite complicated). Instead, we wish to focus on the trade–off between the number of marketplaces (with a set–up cost) and the (leisure) cost of consumer access to them.

(c) Consumers: There is a continuum of consumers. The set of consumers is denoted $A = [0, 1]$, and a representative element of A is a. The consumers form an atomless measure space $(A, \mathscr{A}, \nu)$, where $\mathscr{A}$ is the Borel sigma algebra of A and ν is Lebesgue measure on A. By definition, $\nu(A) = 1$. Until Theorem 3, we will allow for heterogeneous types of consumers.

<u>(d) Individual Transportation:</u> Consumers access marketplaces to trade commodities with others. They pay a cost to bring their products to marketplaces. To keep the model simple, there are no physical or monetary transport costs; there is only the cost of time to travel to a market. We assume that travel cost (round trip travel time) from location j to a marketplace d is represented by $\tilde{\delta}_j(d)$, where $\tilde{\delta}_j: D \rightarrow \mathbb{R}_+$ is a continuous function. Note that this function captures Cronon's first nature of geography in accessing a marketplace, since travel cost can vary with the terrain between a consumer and a market. Travel cost is assumed to be independent of the quantity of goods transported. Since a consumer accesses a marketplace that is most convenient, time to travel to a marketplace in marketplace structure **d** for a consumer residing at $j \in J$ is given by $\delta_j(\mathbf{d})$, where $\delta_j: K \rightarrow \mathbb{R}_+$ is such that $\delta_j(\mathbf{d}) = \min_{d \in \mathbf{d}} \tilde{\delta}_j(d)$. The Euclidean (round trip) distance from location j to the closest marketplace given marketplace structure $\mathbf{d} \in K$ is an example of $\delta_j(\mathbf{d})$; i.e., $\delta_j(\mathbf{d}) = 2 \min_{d \in \mathbf{d}} \|j - d\|$.[11]

<u>(e) Mass Transportation among Marketplaces:</u> When there is more than one marketplace, it is necessary to build a mass transportation system to have commodity flows among them. One can imagine the situation where there is a railroad station in front of each marketplace. Commodities traded in a marketplace are either brought by individual traders or transported from other marketplaces by the mass transportation system. For simplicity, we assume that there is no marginal transportation cost to bring commodities from one marketplace to another.[12] On the

11 We could replace the Euclidean norm with any other norm as well. For other types of norms, see Beckmann and Thisse (1986, pp. 59–62).

12 We could introduce a marginal transportation cost in the mass transportation system. Although the arguments below are not affected much, this change would introduce differences in commodity price vectors across markets. In such a case, notation would become very messy, and each consumer's choice over which marketplaces to access would depend on the price vector in each marketplace as well as the travel cost from her location to each marketplace.

other hand, the cost of building a mass transport system depends on the marketplace structure $\mathbf{d} \in K$.

(f) The Trading Set: It is assumed that each consumer must reside in exactly one location in J. Since consumers can choose their locations freely, consumption sets are nonconvex, and even disconnected. This fact does not depend on the finiteness of the location set J. Even if J is a convex subset of $\mathbb{R}^m$, the trading set is always nonconvex. This point can be seen easily in the following example. No convex combination of one unit of land in Boston and one unit of land in Philadelphia will generate one unit of land in New York. There is a further complication. Since production is location–specific, we cannot treat labor as a homogeneous good, since it has a different effect on output at different locations. Thus, the usual definition of a labor endowment does not make sense. To avoid these difficulties, we treat labor in each location as a different good, and use a trading set instead of a consumption set (McKenzie (1959)). *A trading set* is a consumption set where the endowment point is normalized to the origin. Let $\Omega = \Omega_c \times \Omega_\ell \times \Omega_L \times \Omega_J = \mathbb{R}^I \times \mathbb{R}^{2J} \times J$ be the potential trading set where $\Omega_c = \mathbb{R}^I$, $\Omega_\ell = \mathbb{R}^J$, and $\Omega_L = \mathbb{R}^J$ denote potential commodity, leisure, and land trading sets, and $\Omega_J = J$ denotes the potential location choice set, respectively. Consumers' trading sets with no transportation costs are represented by the closed–valued measurable correspondence $X\colon A \twoheadrightarrow \Omega$. For simplicity, we let $X(a) = \cup_J X_j(a)$, where $\cup_J X_j(a)$ is the union of $X_j(a)$ for $j \in J$. We define $X_j(a) = \mathbb{R}_+^I \times H_j(a) \times L(a) \times \{j\}$, where $H_j(a)$ and $L(a)$ are a type a consumer's leisure trading set when she chooses location j and land trading set, respectively. The specification of the trading set implies that no consumer is endowed with produced commodities. The j–th axis of $H_j(a) \subset \mathbb{R}^J$ is $[-T(a), 0]$ and the other axes are all $\{0\}$, where $T(a)$ is the leisure endowment of a type a consumer. By this we mean that if a consumer chooses location j, she can consume leisure or supply labor only at j. The land trading set is $L(a) = \{L \in \mathbb{R}^J : L \geq -b(a), b(a) \geq 0\}$, where b(a) denotes the initial endowment (vector) of land belonging to consumer a (in all locations).[13] A typical element of X(a) is denoted (c, ℓ, L, j), representing commodity consumption, net leisure consumption, net land consumption, and location choice, respectively. Note that $-\ell$ denotes labor supply plus transportation cost. To show core equivalence (Theorem 1) and the existence of equilibrium (Theorem 2), it is convenient to modify the trading sets. Let $\bar{X}(\mathbf{d}, a) = \cup_J \bar{X}_j(\mathbf{d}, a)$, where $\bar{X}_j(\mathbf{d}, a) = \mathbb{R}_+^I \times \bar{H}_j(\mathbf{d}, a) \times L(a) \times \{j\}$, where the j–th axis of $\bar{H}_j(\mathbf{d}, a)$ is $[-T(a), -\delta_j(\mathbf{d})]$; i.e., $\bar{X}_j(\mathbf{d}, a)$ is obtained by truncating $X_j(a)$ at the transportation cost from j to $\mathbf{d}$, $\delta_j(\mathbf{d})$. We call $\bar{X}(\mathbf{d}, a)$ the *truncated trading set* (see also Berliant and Fujita (1992)). Let $\tilde{X}(\mathbf{d}, a) = \cup_J \tilde{X}_j(\mathbf{d}, a)$, where $\tilde{X}_j(\mathbf{d}, a) = \mathbb{R}_+^I \times \tilde{H}_j(\mathbf{d}, a) \times L(a) \times \{j\}$, where the j–th axis of $\tilde{H}_j(\mathbf{d}, a)$ is $[-T(a) + \delta_j(\mathbf{d}), 0]$; i.e., $\tilde{X}_j(\mathbf{d}, a)$ is obtained by translating $\bar{X}_j(\mathbf{d}, a)$ by $\delta_j(\mathbf{d})$. We call $\tilde{X}(\mathbf{d}, a)$ the *translated trading set.*

(g) Preferences: The *preference relation* $R(a) \subset X(a) \times X(a)$ is assumed to be a complete preorder. R(a) is closed in $X(a) \times X(a)$. To get the *preference relation on the translated trading set* $R(\mathbf{d}, a) \subset \tilde{X}(\mathbf{d}, a) \times \tilde{X}(\mathbf{d}, a)$, restrict R(a) to $\bar{X}(\mathbf{d}, a) \times \bar{X}(\mathbf{d}, a)$ and translate the preference relation by $\delta_j(\mathbf{d})$. Denote by $\succ_a$ ($\succ_a^{\mathbf{d}}$) and $\succsim_a$ ($\succsim_a^{\mathbf{d}}$) the strict and weak preference relations, respectively, induced by R(a) on $X(a) \times X(a)$ ($R(\mathbf{d}, a)$ on $\tilde{X}(\mathbf{d}, a) \times \tilde{X}(\mathbf{d}, a)$). The space of admissible preferences is denoted by $\mathscr{R}$ and endowed with the topology of closed convergence (Hildenbrand (1974)). We assume that $R: A \to \mathscr{R}$ is a measurable mapping, where $\mathscr{R}$ is endowed with the Borel σ–algebra.

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13 We define vector inequalities as follows: Let $x, x' \in \mathbb{R}^n$. Then $x \geq x'$ if $x_i \geq x_i'$ for all i = 1, ⋯, n, $x > x'$ if $x \geq x'$ and $x \neq x'$, and $x >> x'$ if $x_i > x_i'$ for all i = 1, ⋯, n.

<u>(h) Production:</u> In our economy, the production structure is location–specific. Land and labor are used to produce consumption commodities. *The production set that is available at location j* is denoted by $Y_j \subset \mathbb{R}^I \times \mathbb{R}^{2J}$; Y_j is a closed convex cone with vertex at the origin. We assume $Y_j \cap \mathbb{R}_+^I \times \mathbb{R}_+^{2J} = \{0\}$, and $\mathbb{R}_-^I \times \mathbb{R}_-^{2J} \subset Y_j$. For any $j \in J$, there exists $i \in I$ such that $Y_j^i = \mathbb{R}_-$, where Y_j^i is the projection of Y_j on the i–th axis. This implies that for any location j there is at least one commodity which is not producible. Such an assumption excludes an autarkic equilibrium. To produce commodities, labor at j is assumed to be essential. Denote *the aggregate production set* by $Y = \Sigma_J Y_j$, which is also assumed to be a closed, convex cone with vertex at the origin, and $Y \cap (\mathbb{R}_+^I \times \mathbb{R}_+^{2J}) = \{0\}$. These assumptions imply a constant returns to scale technology. In this specification, we did not explicitly exclude the possibilities that (i) inputs (such as raw materials) are transported among locations (there is trade in inputs), and (ii) produced commodities are used as inputs for the production of other commodities (there is trade in intermediate goods). Although these possibilities do not cause any mathematical problems in our proofs, they cause some problems in the interpretation of the model.[14] Hence, we will

[14] The problem is how to introduce the transportation cost of inputs and intermediate goods. If it is costless to transport these goods from one location to another, then it is asymmetric to assume that consumers need to transport final products (commodities) to marketplaces, and that this transportation is costly. In this case, for symmetry, consumers should be able to obtain commodities without incurring a cost to access a marketplace. On the other hand, if transporting inputs and intermediate goods is costly, then their transportation cost functions must be introduced into the model. Although this can be done using the transportation technology sets provided by Schweizer, Varaiya, and Hartwick (1976), these transportation technologies must be treated as correspondences from the set of marketplace structures to the commodity space if inputs and intermediate goods are transported via marketplaces and a mass transportation system. Of course, final products (commodities) should be transported in the same way for symmetry. In fact, the latter approach is very rich, and it could explain industrial specialization in certain areas; for example, we could explain why steel factories are located in the areas close to coal mines. We conjecture that it is possible to generalize our model to deal with the approach involving positive transportation costs for all goods, but it requires very complicated notation and arguments. Thus, for simplicity, we assume that inputs and intermediate goods are not transportable in this paper.

exclude these cases for now; finding assumptions to exclude these possibilities is quite easy, while adjusting the model to account for these possibilities is easy but technical and cumbersome. We will return to this point in the conclusion of the paper.

(i) A Topology on the Set of Marketplace Structures: Here we induce a topology on K that is used to define continuity of functions and correspondences on K. It is defined as follows. We can partition $K = \cup_{\ell=0}^{\infty} K_\ell$, where K_ℓ is the set of elements of D with cardinality ℓ. We can induce a topology on K_ℓ using a metric defined in the following manner. Let $\mathscr{S}(\mathbf{d})$ be the set of permutations of **d**, and let $s(\mathbf{d})$ be a representative element of $\mathscr{S}(\mathbf{d})$. For any $\mathbf{d}, \mathbf{d}' \in K_\ell$, the metric distance between **d** and **d′** is defined by $\min_{s(\mathbf{d}) \in \mathscr{S}(\mathbf{d})} \sum_{k=1}^{\ell} \|s_k(\mathbf{d}) - d'_k\|$, where $s_k(\mathbf{d})$ and d'_k denote the k–th elements of $s(\mathbf{d})$ and **d′**, respectively. Taking the disjoint union of the spaces K_ℓ ($\ell = 0,1,2,\ldots$), we obtain a topological space $(K, \mathscr{O})$, where $\mathscr{O}$ is the topology on K thus defined. Note that the function δ^j is continuous on the topological space $(K, \mathscr{O})$. From now on, the topology induced on K is always taken to be $\mathscr{O}$.

(j) The Setup Cost of Marketplaces: To establish a marketplace, consumers must pay costs depending proportionally on the number (measure) of consumers who use the marketplace. Specifically, we assume that for each measurable coalition $S \in \mathscr{A}$, the commodity bundles required to establish marketplaces at $\mathbf{d} \in K$ are given by the set $Q^M(\mathbf{d}, S) = \nu(S) \times \mathscr{Q}^M(\mathbf{d})$, where $\mathscr{Q}^M: K \twoheadrightarrow \mathbb{R}_+^I \times \mathbb{R}_+^{2J}$ is closed–valued and convex–valued, and satisfies (i) $\mathscr{Q}^M(\mathbf{d}) \subset \mathbb{R}_+^I \times \mathbb{R}_+^{2J}$ and $0 \notin \mathscr{Q}^M(\mathbf{d})$ (positive cost) and (ii) $\mathscr{Q}^M(\mathbf{d}) + \mathbb{R}_+^I \times \mathbb{R}_+^{2J} \subset \mathscr{Q}^M(\mathbf{d})$ (a free disposal property) for any $\mathbf{d} \in K$. We call $\mathscr{Q}^M$ *the input requirement correspondence for establishing a marketplace structure.* Note that this correspondence captures Cronon's first nature of geography in setting up a marketplace: the setup cost of marketplaces can depend on the number and locations of marketplaces. As a special case, we can let $\mathcal{Q}^M(d) = \sum_{d\in\mathbf{d}} \tilde{\mathcal{Q}}^M(d)$, where $\tilde{\mathcal{Q}}^M(d)$ denotes the input requirement set for setting up a marketplace at $d \in D$.

An important assumption is that the setup cost of marketplaces is proportional to the size of a coalition (as is the setup cost of a mass transportation system; see (k) below). This assumption is used only for proving a core equivalence theorem, and is not related to the nonemptiness of the core. Note that we do not assume anything about cost allocation among consumers ex ante.

(k) The Setup Cost of a Mass Transportation System: When there is more than one marketplace, they must be connected by a mass transportation system. The cost results, for example, from building stations in front of marketplaces and from constructing railroads between them. We define the setup cost of a mass transportation network in the following manner: $Q^T(\mathbf{d}, S) = \nu(S) \times \mathcal{Q}^T(\mathbf{d})$, where $\mathcal{Q}^T: K \twoheadrightarrow \mathbb{R}^I_+ \times \mathbb{R}^{2J}_+$ is *the input requirement correspondence for establishing a mass transportation system.* Again we assume that $\mathcal{Q}^T: K \twoheadrightarrow \mathbb{R}^I_+ \times \mathbb{R}^{2J}_+$ is continuous, closed–valued, and convex–valued, and satisfies (i) $\mathcal{Q}^T(\mathbf{d}) \subset \mathbb{R}^I_+ \times \mathbb{R}^{2J}_+$ and $0 \notin \mathcal{Q}^T(\mathbf{d})$ (positive cost), and (ii) $\mathcal{Q}^T(\mathbf{d}) + \mathbb{R}^I_+ \times \mathbb{R}^{2J}_+ \subset \mathcal{Q}^T(\mathbf{d})$ (a free disposal property).

The specification of $\mathcal{Q}^T$ is general enough to capture Cronon's first nature of geography in setting up a mass transportation system: $\mathcal{Q}^T(\mathbf{d})$ can be dependent on the number of marketplaces, the locations of marketplaces, and the distances between marketplaces. In the railroad example, $\mathcal{Q}^T(\mathbf{d})$ depends on the cost of building stations in each marketplace as well as the cost of building a railroad connecting marketplaces; the latter is dependent on the distances among marketplaces and the geographical characteristics of areas between them. Using our specification, many methods for measuring the distance between marketplaces are captured. For example, let $\mathcal{Q}^T(\mathbf{d}) = Q^s \times (\#\mathbf{d}) + Q^r \times \tau(\mathbf{d})$, where Q^s is the input requirement set to produce one station, Q^r is the input requirement set to build one mile of railroad, and $\tau(\mathbf{d})$ is the distance among the marketplaces (there is no first nature of geography in this example). Here we can use the following specifications of the distances between marketplaces, which make $\tau(\mathbf{d})$ a continuous function on $(K, \mathcal{O})$: (i) $\min_{d' \in D} \sum_{d \in \mathbf{d}} \|d - d'\|$, or (ii) $\min_{s(\mathbf{d}) \in \mathcal{S}(\mathbf{d})} \sum_{k=1}^{\#\mathbf{d}-1} \|s_{k+1}(\mathbf{d}) - s_k(\mathbf{d})\|$. In case (i), the marketplaces are connected radially, while in case (ii) the marketplaces are connected by a piecewise linear curve.

(l) The Total Setup Cost of Marketplaces and a Mass Transportation System: For notational simplicity, we combine the two types of setup costs into one. The total setup cost of marketplaces and a mass transportation system is denoted $Q(\mathbf{d}, S) = Q^M(\mathbf{d}, S) + Q^T(\mathbf{d}, S)$.

(m) An Economy: An economy $\mathcal{E}$ is a list $((A, \mathcal{A}, \nu); X, R; Y, Q, J, D)$.

(n) Feasible Allocations: *A consumption allocation* in $\mathcal{E}$ is a Borel integrable function $f: A \to \Omega$ such that $f(a) \in X(a)$ a.e. in A. The collection of consumption allocations is denoted by $\mathcal{H}$. A list $(\mathbf{d}, f)$ is called an *allocation.* Given $(\mathbf{d}, f)$, let $\tilde{f}(\mathbf{d}, a)$ be such that if $\text{proj}_J\, f(a) = j$ then $\tilde{f}_j^{\ell}(\mathbf{d}, a) = f_j^{\ell}(a) - \delta_j(\mathbf{d})$ and other elements of $\tilde{f}(\mathbf{d}, a)$ are the same as those of $f(a)$, where $f_j^{\ell}(a)$ denotes consumer a's net leisure consumption at j, and $\text{proj}_{(\cdot)}(*)$ is the projection operation of (*) onto $(\cdot)$. This is a consumption allocation in the translated trading set. An allocation $(\mathbf{d}, f)$ in $\mathcal{E}$ is *feasible* if $(\mathbf{d}, f)$ satisfies:

(i) $$\tilde{f}(\mathbf{d}, a) \in \tilde{X}(\mathbf{d}, a) \quad \text{a.e. in } A,$$

(ii) $$\int_A \text{proj}_C\, \tilde{f}(\mathbf{d}, a)\, d\nu(a) \in Y - Q(\mathbf{d}, A),$$

where $C = \Omega_c \times \Omega_\ell \times \Omega_L$, and hence $\text{proj}_C\, \tilde{X}(\mathbf{d}, a) = \cup_J \{\mathbb{R}_+^I \times H_j(\mathbf{d}, a) \times L(a)\}$.

The collection of feasible allocations is denoted by $\mathscr{F}$.[15]

<u>(o) Individually Rational Allocations:</u> An allocation (**d**, f) in $\mathscr{E}$ is *individually rational* if $f(a) \succsim_a (0, j)$ for any $j \in J$ a.e. in A.

<u>(p) Coalitional Deviations:</u> Let $S \in \mathscr{A}$ be a coalition which is trying to deviate from a feasible allocation (**d**, f). *A feasible coalitional deviation* in $\mathscr{E}$ *given the locations of marketplaces* $\mathbf{d}' \in K$ is a list of $\mathbf{d}'$, S, and an allocation $g: A \to \Omega$ that satisfies the following condition:

$$\int_S \mathrm{proj}_C\, \tilde{g}(\mathbf{d}', a)\, d\nu(a) \in Y - Q(\mathbf{d}', S).$$

We say $(\mathbf{d}', S, g)$ *improves upon* (**d**, f) if $g(a) \succ_a f(a)$ a.e. in S.

<u>(q) Core Allocations:</u> *A core allocation* in $\mathscr{E}$ *given the locations of marketplaces* **d** is a feasible, individually rational allocation (**d**, f) such that there is no feasible coalitional deviation given locations of marketplaces **d** that improves upon f. The collection of core consumption allocations in $\mathscr{E}$ given locations of marketplaces $\mathbf{d} \in K$ is denoted *Core*(**d**). A *core allocation* in $\mathscr{E}$ is a feasible, individually rational allocation (**d**, f) such that for any $\mathbf{d}' \in K$ there is no feasible coalitional deviation given $\mathbf{d}'$ that improves upon f. The collection of core allocations in $\mathscr{E}$ is denoted by *Core*.

<u>(r) An Equilibrium with Participation Fees:</u>[16] Due to the setup cost of marketplaces,

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[15] Notice that our feasibility concept is dependent on the marketplace structure, since there cannot be more than two marketplace structures simultaneously. We assume that goods cannot be transferred from one coalition to another coalition without merging into one coalition. This is a basic assumption in cooperative game theory. The meaning of this assumption in our setting is that without sharing the same marketplace structure, consumers cannot transfer goods to others.

[16] Our 'participation fees' are a different concept from the 'participation price system' used in Wooders (1993). Wooders analyzes a local public good economy (a coalition structure

the usual Walras equilibrium concept itself is not useful in this model. We will use a modified Walras equilibrium that includes a city manager in the equilibrium concept. *An equilibrium with participation fees* in $\mathscr{E}$ *given the locations of marketplaces* is a list (**d**, f, y, q, p) where $\mathbf{d} \in K$, $y \in Y$, $q \in Q(\mathbf{d}, A)$, and $p \in \Delta$ (Δ is the unit simplex in $\mathbb{R}^I_+ \times \mathbb{R}^{2J}_+$) such that:

(i) $p \cdot y \geq p \cdot y'$ for any $y' \in Y$,

(ii) $p \cdot q \leq p \cdot q'$ for any $q' \in Q(\mathbf{d}, A)$,

(iii) $\int_A \text{proj}_C \, \tilde{f}(\mathbf{d}, a) \, d\nu(a) = y - q$,

(iv) $p \cdot \{\text{proj}_C \, \tilde{f}(\mathbf{d}, a)\} \leq - p \cdot q$ a.e. in A,

(v) $\tilde{f}(\mathbf{d}, a) \succsim_a^{\mathbf{d}} x \;\; \forall\, x \in \tilde{X}(\mathbf{d}, a)$ s.t. $p \cdot (\text{proj}_C \, x) \leq - p \cdot q$ a.e. in A.

Here, the setup cost of marketplaces and a mass transportation system is financed by poll taxes on market participants ($p \cdot q$). A city manager minimizes the setup costs of marketplaces and the mass transportation system given p. The collection of equilibrium allocations with participation fees in $\mathscr{E}$ given the locations of marketplaces $\mathbf{d} \in K$ is denoted $E(\mathbf{d})$.

(s) Envy Free Allocations: A consumption allocation $f \in \mathscr{H}$ is *envy free in* S if the following condition is satisfied a.e. for $a \in S$: $f(a) \succsim_a f(a')$ a.e. for $a' \in A$. The collection of (not necessarily feasible) allocations in $\mathscr{E}$ that are envy free in A is denoted by *F*.

(t) Pareto Efficient Allocations: Let $\succsim_A$ be such that for $f, f' \in \mathscr{H}$, $f \succsim_A f'$ iff $f(a) \succsim_a f'(a)$ a.e. in A. A feasible allocation $(\mathbf{d}, f) \in \mathscr{F}$ is *Pareto efficient*, if for any

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economy), and her 'participation price' for a community is a payment made by a consumer to join that community. On the other hand, in this paper we consider an integrated economy (a grand coalition economy), and our 'participation fees' are used to finance the setup costs of marketplaces and a mass transportation system for the whole economy. Hence, our participation fees are more like tax payments to the central government.

$(\mathbf{d}', f') \in \mathscr{F}$, it is not the case that $f' \succsim_A f$ with $f'(a) \succ_a f(a)$ for a a.e. in some S, $\nu(S) > 0$. The collection of Pareto efficient allocations in $\mathscr{E}$ is denoted by *P*.

3. The Core

We will demonstrate the nonemptiness of *Core* allocations in our economy. It is convenient to establish a couple of preliminary results in order to prove the nonemptiness of and characterize the *Core*. The strategy for this section is as follows: First, we establish the equivalence between *Core*(**d**) and *E*(**d**), and the nonemptiness of *Core*(**d**) in a general model. After that, we give an example in which there are two types of consumers and the *Core* is empty. Finally, assuming identical consumers in the economy, we prove the nonemptiness of the *Core*. All of the proofs are given in the appendix. First we define a couple of assumptions.

Local Nonsatiation: Given $\mathbf{d} \in K$, for any $x \in \tilde{X}(\mathbf{d}, a)$, for any $\epsilon > 0$, there exists $x' \in \tilde{X}(\mathbf{d}, a)$ such that $\|x - x'\| < \epsilon$ and $x' \succ_a^{\mathbf{d}} x$, a.e. in A.

This assumption is standard, and says that for any consumption plan, there is a better one nearby.

The Boundary Condition: If $(c, \ell, L, j) \in X(a)$ satisfies $c_i = 0$ for some $i \in I$, or $\ell_j = -T(a)$, or $L_j = -b_j(a)$, then for any $(c', \ell', L', j') \in X(a)$, $(c', \ell', L', j') \succsim_a (c, \ell, L, j)$, a.e. in A.

This assumption says that any interior consumption plan is at least as good as any boundary consumption plan.

Theorem 1. For all $\mathbf{d} \in K$, *Core*($\mathbf{d}$) = $E(\mathbf{d})$ under local nonsatiation and the boundary condition.

Remark. The proof of the theorem is a modification of Hildenbrand's (1974). However, the *boundary condition* is an unusual assumption in the context of core equivalence theorems. This assumption is used in two ways. First, it gives incentives to consumers to participate in markets. Second, it is used to prove that a minimum expenditure equilibrium is an equilibrium with participation fees. Due to the lack of convexity of trading sets, a minimum expenditure equilibrium is not even a quasi–equilibrium. Hence, neither *irreducibility* nor *monotonicity of preferences* helps in proving core equivalence (and nonemptiness of $E(\mathbf{d})$), since there are multiple wealth levels that have consumption plans without cheaper points. Assumption C.3 together with C.2 in Wooders (1980) in the context of a local public good economy is very close to our boundary condition.

Since $Core \subset \bigcup_{\mathbf{d} \in K} Core(\mathbf{d})$ by definition, Theorem 1 implies that any core allocation is represented by an equilibrium with participation fees for some location of marketplaces. Therefore, we can analyze a core allocation using the notion of equilibrium with participation fees. The following corollary is a consequence of this argument.

Corollary 1. Let $S \in \mathscr{A}$ be a set of consumers in which each consumer has the same preference relation and trading set with $\nu(S) > 0$. Under the assumptions of Theorem 1, at any core allocation consumers are envy free a.e. in S.

Corollary 1 is important, since it establishes equal treatment of identical consumers (in the almost everywhere sense) at core allocations. Next, we prove the nonemptiness of $E(\mathbf{d})$ and $Core(\mathbf{d})$. We need a couple more assumptions.

Interiority: There exists $j \in J$ such that:

$$\text{int}(Y - Q(\mathbf{d}, A)) \cap \int_A \text{proj}_C \tilde{X}_j(\mathbf{d}, a)\, d\nu(a) \neq \emptyset.$$

This assumption says that the set of feasible allocations with non–zero consumption is nonempty.

Monotonicity in Commodity Consumption: For any $i \in I$, for any $j \in J$, for any $(c, \ell, L, j) \in X(a)$ such that $c >> 0$, $\ell_j > -T(a)$, $L_j > -b(a)$, if $c' > c$ then $(c', \ell, L, j) \succ_a (c, \ell, L, j)$ a.e. in A.

Remark. Monotonicity in commodity consumption implies local nonsatiation in the interior of trading sets for any $\mathbf{d} \in K$.

Theorem 2. For all $\mathbf{d} \in K$ such that $(Y - Q(\mathbf{d}, A)) \cap \text{proj}_C \tilde{X}(\mathbf{d}, a) \neq \emptyset$ a.e. in A, $E(\mathbf{d}) \neq \emptyset$ under the boundary condition, monotonicity in commodity consumption, and interiority.

Remark. Each of these three assumptions plays an important role in the proof. Due to disconnected consumption sets, we do not have continuity of budget relations. Interiority and monotonicity in commodity consumption assure positive commodity prices, which implies that (location–specific) price vectors are nonzero at any location. All three properties are used to prove that individual demand correspondences have closed graphs. Completeness and transitivity of $\succsim_a$ are also important in this part of the proof (see Konishi (1993)). We cannot use a *dispersed wealth distribution* type of assumption (see Mas–Colell (1977) or Yamazaki (1978) for a definition), due to Example 1 below. Note that in this theorem we allow for the possibility of empty locations. Monotonicity in commodity consumption assures that there exists a commodity with a positive price at every location.

Corollary 2. For all $\mathbf{d} \in K$ such that $(Y - Q(\mathbf{d}, A)) \cap \mathrm{proj}_C\, \tilde{X}(\mathbf{d}, \mathbf{a}) \neq \emptyset$ a.e. in A, *Core*(**d**) $\neq \emptyset$ under the boundary condition and monotonicity in commodity consumption.

Remark. Interiority is not needed to prove *Core*(**d**) $\neq \emptyset$.

Unfortunately, there is an unpleasant example related to the nonemptiness of *Core*. If consumers are not homogeneous then the *Core* can be empty, although it is not difficult to show that *Core*(**d**) is nonempty.

Example 1.[17] Let $D = [0, 1]$. Let location 1 be at $\{0\}$, and let location 2 be at $\{1\}$. There are two types of consumers (1 and 2); the measure of each type is 1/2. There is no land in the economy, and each type of consumer owns 2 units of leisure as endowment regardless of location choice. To set up a market, a coalition S must pay $\nu(S)/2$ units of location–specific labor in *both* locations. The cost to set up two marketplaces is twice as much as the cost to set up one marketplace (there is no setup cost for a mass transportation system). There is no first nature of geography in the example. The individual (marginal) transportation cost is one. Each

17 This example is not a counterexample in the strict sense, as it violates the *boundary condition*. We employ this example for expositional purposes. Using the same method as Example 1, we can construct a formal counterexample by employing the two types of preferences used in the numerical examples in the next section.

commodity is produced using a location–specific production function $y_i + \ell_i \leq 0$ for i = 1, 2, where y_i denotes the output of commodity i. Consumers have Leontief–type utility functions: type 1 and type 2 consumers' utility functions are $u^1(c_1, c_2) = \min \{2c_1, c_2\}$, and $u^2(c_1, c_2) = \min \{c_1, 2c_2\}$, respectively. That is, neither type of consumer obtains utility from leisure consumption. In this economy, the *Core* is empty.

To see this, first notice that there can be only one marketplace in the economy. If there is no marketplace in the economy, then no one can get positive utility. If there are two marketplaces, then even if there is no individual transportation cost, again no one can get positive utility due to the setup costs of marketplaces and of the mass transportation system. Therefore, if there exists a core allocation, it must have only one marketplace. Denote the location of the marketplace by $d \in D$, which denotes the distance from location 1. If $d = 1/2$, then $y_1 + y_2 = 1/2$, and the production allocation is determined by the population distribution over locations. If d takes a different value, then the aggregate production set changes (see Figure 3). As a result, taking an envelope of production possibility frontiers over d, we obtain the curve AB. Each point on AB corresponds an element $d \in D$. By calculation, we can show that type 1 and 2 consumers' ideal locations for the marketplace are $d^1 = 5 - \sqrt{21}$ and $d^2 = -4 + \sqrt{21}$, respectively. That is, by allocating the population properly, type i consumers can attain their most preferred consumption vector C^i by forming a homogeneous coalition consisting of consumers of only their type distributed over both production locations (see Figure 4). Since d^1 and d^2 are different, for any $d \in D$, at least one type of consumer will deviate to improve upon the original allocation. Therefore, the core is empty. In fact, any convex combination of C^1 and C^2 is infeasible; see, for example, the point E in Figure 4. How about the case where each group forms its own marketplace and no consumer accesses the marketplace established by the other group? Unfortunately, even in this case, some type 1 consumers living in location 2 want to access the marketplace d^2, since it is closer to location 2. Then both types of consumers use both marketplaces, and they need to pay doubled setup costs. Therefore, without assuming exclusion in the use of marketplaces, we cannot obtain a stable allocation.[18]

The problem that this example suggests is that if there are transportation costs, the production possibility sets with endogenous location of marketplaces are nonconvex in general. This is the source of the trouble. Although we assume Leontief preferences in this example to make the point clear, it is not essential to the robustness of the example.

Due to this unpleasant counterexample, our theorem on the nonemptiness of the core requires identical consumers. In a strange way, this aids in justifying the use of a continuum of consumers in this model. Berliant (1985) points out that, under some conditions, the use of both land and a continuum of consumers in a model creates problems with consistency of the model. When all consumers are identical, however, the arguments of Papageorgiou and Pines (1990) apply. Although these arguments are not entirely convincing (see Berliant and ten Raa (1991)), they do provide some justification for our framework. One more assumption is needed before we can state our main result.

Boundedness of Feasible Marketplace Structures: There exists ℓ, finite and integer,

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[18] In our model, an allocation is a list consisting of two elements: a marketplace structure d and a consumption allocation f (see footnote 15). With our solution concepts, it is not possible to split the economy into two separate, self sufficient units at a core allocation. The reason is that land must be traded at marketplaces, and every consumer owns land at every location (if all consumers are identical). Readers might think that consumers could do better than joining the grand coalition by separating into several self sufficient coalitions. However, since we do not have explicit externalities from population agglomeration, we can always find an allocation under the grand coalition that is at least as good as an allocation derived from a structure with multiple coalitions for almost all consumers.

such that for all $\ell' \geq \ell$, for all $\mathbf{d} \in K_{\ell'}$, $(Y - Q(d, A)) \cap \int_A \bar{X}(d, A)\, d\nu(a) = \emptyset$.

Remark. This assumption says that all the resources in the economy can only generate a finite number of marketplaces. Since there is only a finite number (#J) of locations for consumers, the optimal number of marketplaces is always less than or equal to #J if the total marketplace setup cost increases by adding one more marketplace. In such a case this assumption is trivially satisfied.

Theorem 3. Suppose that consumers are identical a.e. in A. Then, *Core* $\neq \emptyset$ under the boundary condition, monotonicity in commodity consumption, and boundedness of feasible marketplace structures.

The following is a characterization of *Core*, which is proved by using an argument in the proof of Theorem 3.

Corollary 3. If all consumers are identical a.e. in A, *Core* $= F \cap P$.

In our model, marketplace and mass transportation setup costs have been assumed to be proportional to the size of the coalition using it. This is an extreme case. The other extreme case is that irrelevant of the size of coalitions, the marketplace and mass transportation setup costs are fixed. In such a case, it is harder for coalitions to deviate from a proposed allocation. Thus, *the Core in the proportional setup cost case* is a subset of *the Core in the fixed setup cost case.* Hence, nonemptiness of *the Core in the fixed setup cost case* is easy to show (see Scarf (1986)).

Corollary 4. Suppose that consumers are identical a.e. in A. If marketplace setup cost is not a function of the population using the marketplaces and mass transportation system, then $Core \neq \emptyset$, but the equal treatment property does not hold since $Core(\mathbf{d}) \neq E(\mathbf{d})$.

4. City Formation: Examples

In this section, we will characterize core allocations using numerical methods. We do not use comparative statics for this purpose for the reason given in footnote 9. The computations use a nonlinear programming package, GAMS/MINOS version 2.25. The basic model is the one that is described in Section 2(a) and in Figures 1 and 2. Locations are one dimensional and contained in the unit interval [0, 1]. There are four types of land, J = {0, 0.33, 0.67, 1}. For convenience, we call these types of land a, b, c, and d, respectively. Commodity 1 is produced in a and b while commodity 2 is produced in c and d. The feasible location set for marketplaces is D = [0, 1]. The leisure endowment T is set at 2, and the land endowment is 1 at each location (for each consumer). Individual travel cost to a marketplace is the opportunity cost of leisure, and is the distance from a consumer's residence to the marketplace. The preferences of consumers are represented by a Cobb–Douglas type utility function, $U(c, \ell, L, j) = (c_1)^{\alpha_1}(c_2)^{\alpha_2}(2 - \ell_j)^{\alpha_3}(1 - L_j)^{\alpha_4}$, where $\alpha_1 + \alpha_2 + \alpha_3 + \alpha_4 = 1$. The production function at j is simple, and is denoted by $y_j = (-\ell_j) \times m_j$, where y_j and m_j denote the production level at j and the population (measure) at j, respectively. The setup cost of a marketplace is represented by a kind of composite good of commodities 1 and 2. To produce one marketplace, we need $\bar{q}$ units of the composite commodity. That is, the level of $\bar{q}$ represents the setup cost of a marketplace. The production function that produces the composite good is denoted $\bar{q} = (c_1)^{\gamma}(c_2)^{1-\gamma}$. That is, we only use the two commodities that are outputs to produce the composite good. In the first three examples, we do not introduce a setup cost for a mass transportation system, and we assume that if there are k marketplaces then the cost of producing marketplaces is k times the cost of producing one marketplace. We will use the following sets of parameters. (Case–I): $\alpha_i = 0.25$ for i = 1, 2, 3, 4, and $\gamma = 0.5$. (Case–II): $\alpha_1 = 0.2$, $\alpha_2 = 0.3$, $\alpha_3 = \alpha_4 = 0.25$, and $\gamma = 0.5$. We tried many values for $\bar{q}$ ($\bar{q}$ = 0.01, 0.05, 0.1, 0.2, 0.3, 0.4, 0.5); it is a very important parameter in our model, since marginal travel cost is normalized to one.

(Case–I): The results are summarized in the following table:

$\bar{q}$	#	location	population				output				rent			
.01	4	a,b,c,d	.250	.250	.250	.250	.34	.34	.34	.34	.024	.024	.024	.024
.05	2	b,c	.194	.306	.306	.194	.23	.43	.43	.23	.015	.027	.027	.015
.1	1	.5	.188	.312	.312	.188	.20	.40	.40	.20	.013	.026	.026	.013
.2	1	.5	.179	.321	.321	.179	.20	.44	.44	.20	.011	.023	.023	.011
.3	1	.5	.167	.333	.333	.167	.20	.48	.48	.20	.008	.021	.021	.008
.4	1	.5	.150	.350	.350	.150	.19	.52	.52	.19	.006	.019	.019	.006
.5	1	.5	.125	.375	.375	.125	.17	.59	.59	.17	.004	.016	.016	.004

Table 1[19]

Commodities 1 and 2 are treated symmetrically in this case. One immediate observation is that when $\bar{q}$ is small ($\bar{q}$ = 0.01 or 0.05) there are many marketplaces (4 and 2, respectively), but if $\bar{q}$ becomes larger ($\bar{q} \geq 0.1$), there is only one marketplace. The population in the middle (b, c) increases as $\bar{q}$ increases. The rents in the center area relative to those in the outer area also increase with $\bar{q}$. The reason why the absolute values of rents go down as $\bar{q}$ increases is that the

19 The locations of marketplaces when $\bar{q}$ = 0.05 can be any of {a, c}, {a, d}, {b, c}, or {b, d}. We choose {b, c} only because of the nice contrast with other parameter values. The same applies to Table 2.

production technology to produce marketplaces uses commodities but not land as inputs. As $\bar{q}$ goes up, more of the commodity production is used for marketplace production. Hence, commodities become scarce and the relative prices of land go down as $\bar{q}$ increases. One interesting feature is that when there are two marketplaces ($\bar{q}$ = 0.05) we still have agglomeration of consumers, although we also have a symmetric situation within each area producing a commodity, {a,b} and {c,d}.[20]

(Case–II): The results are summarized in the following table.

$\bar{q}$	#	location	population				output				rent			
.01	4	a,b,c,d	.224	.224	.276	.276	.30	.30	.38	.38	.020	.020	.028	.028
.05	2	b,c	.180	.268	.333	.220	.20	.36	.45	.25	.014	.025	.035	.020
.1	1	.67	.156	.273	.350	.221	.15	.32	.50	.26	.010	.022	.029	.016
.2	1	.67	.150	.287	.355	.209	.15	.36	.53	.26	.009	.020	.026	.013
.3	1	.67	.140	.304	.362	.193	.15	.40	.57	.25	.007	.019	.022	.010
.4	1	.66	.128	.327	.374	.172	.14	.48	.61	.24	.005	.017	.019	.007
.5	1	.62	.109	.357	.393	.142	.13	.52	.65	.20	.003	.015	.017	.004

Table 2

The main difference from Case–I is the location of marketplaces when the number of marketplaces is one. Since consumers prefer commodity 2 to commodity 1 in Case–II, more of commodity 2 should be produced. This requires that a larger portion of consumers live in c and d. Then, to save individual transportation cost, the location of marketplaces is biased to the right. Therefore, in the cases $\bar{q}$ = 0.1, 0.2, 0.3, the location of the marketplace is c (0.67). Consider next the cases when $\bar{q}$ = 0.4 or 0.5. The location of the marketplace is pulled toward b a little because of

20 This is because in this example there are only two locations producing each commodity. If we add more locations between a and b and between c and d, the locations of marketplaces converge to symmetric positions when there is no mass transportation system setup cost.

a 'symmetric' marketplace production function ($\gamma = 0.5$). As $\overline{q}$ becomes larger, the economy is required to produce more of both types of commodity. In such a case, a larger and larger portion of consumers is required to live in a or b. Then, if the location of the marketplace is very biased, the total transport bill is high. This is the source of the pulling power of the location of the marketplace.

We give a couple more examples. The next case (Case–III) is one where consumers' preferences are symmetric ($\alpha_1 = \alpha_2 = \alpha_3 = \alpha_4 = 0.25$), while the technology for production of a marketplace is asymmetric ($\gamma = 0.3$). We only provide the core allocation for $\overline{q} = 0.1$. The location of the only marketplace is at 0.605, and the distributions of population, output, and rent are as follows: 0.174, 0.295, 0.327, 0.205 (population); 0.168, 0.359, 0.446, 0.228 (output); 0.012, 0.025, 0.027, 0.014 (rent). Comparing this result with the case ($\overline{q} = 0.1$) in Table 1, we can say that if the technology for marketplace production is asymmetric, the location of the marketplace is biased toward the region that produces the commodity that is needed more in the production of marketplaces.

Finally, we focus on the setup cost for a mass transportation system. In the next two examples, we assume that the setup cost for marketplaces ($\overline{q}$) is constant independent of the number of marketplaces, while the setup cost of a mass transportation system depends on the locations of marketplaces and the distances among them. To produce a mass transportation system, the same composite commodity as in marketplace production is used. To focus on the first nature of geography in setting up a mass transportation system, we assume $\alpha_i = 0.25$ for all i, and $\gamma = 0.5$. To provide marketplaces, consumers must pay $\overline{q}$ independent of the number of marketplaces. In (Case–IV), the setup cost of a mass transportation system in terms of the composite good is $3 \times \overline{q}$ per unit distance. In (Case–V), the setup cost for a mass transportation system in terms of the composite good is $9 \times \overline{q}$ per unit distance if it is built in the interval [0, 0.5], and is $0.3 \times \overline{q}$ per unit distance otherwise. The reader can imagine a situation in which the interval [0, 0.5] is mountainous, where it is costly to build a railroad, while the interval (0.5, 1] is a prairie, where it is less costly to build a railroad. For these two cases, we investigated only the value $\bar{q}$ = 0.05. In (Case–IV), we get two marketplaces {b, c}, while population, output, and rent distributions are exactly the same as those in the case $\bar{q}$ = 0.05 in Table 1. For (Case–V), we obtain three marketplaces {0.5, c, d}, while population, output, and rent distributions are: 0.183, 0.295, 0.261, 0.261 (population); 0.187, 0.375, 0.357, 0.357 (output); 0.015, 0.028, 0.022, 0.022 (rent). Aside from the setup cost of a mass transportation system, (Case–IV) and (Case–V) are the same, so we can see that the first nature of geography matters for the equilibrium number and locations of marketplaces and population agglomeration, as well as the rent distribution.

5. Conclusion

We have seen in this paper how gains to trade, location–specific production and the setup costs of marketplaces and a mass transportation system can be used to generate agglomeration and city formation.[21] In this paper, we assumed that there is no input nor intermediate good trade to avoid complications (see Footnote 14). Commodities are produced only by labor and the land available at each location. Commodities are transported from residences to marketplaces by individual consumers. However, this specification is unrealistic in industrialized economies. Raw materials as well as intermediate goods are traded over locations. Fortunately, our model is

[21] Although our model is based on gains to trade derived from the exchange of commodities, we could alternatively make the simple assumption that consumers must access marketplaces.

rich enough to capture these features, but only with further complications. We can introduce transportation technologies for moving goods between production or residential locations and a marketplace, in addition to mass transportation technologies for moving goods among marketplaces, following Schweizer, Varaiya, and Hartwick (1976). These transportation technologies are described as production set correspondences from the set of marketplace structures (K) to the commodity space. By adding transportation technologies to location–specific production technologies, we can obtain net location–specific production set correspondences from the set of marketplace structures to the commodity space. Using this method, we can introduce input and intermediate good trade into our model consistently without changing the basic arguments.

The model we have used can be considered complementary to the models of city formation based on increasing returns to scale from population agglomeration. In a model with increasing returns to scale, the history of a city is important in the development of a city. An important question that should be asked is: what determines the initial population distribution in the economy? The answer to this question gives us not only the initial state of the economy, but also determines the resulting development of cities over time through the mechanisms described by models employing increasing returns. Our model can reduce the "indeterminacy" problem of the increasing returns model by determining the initial population distribution and city structure. In essence, our model is driven by the same forces that drive classical international trade theory, while the models used in the spatial increasing returns literature are driven by the same forces that drive the new international trade theory.

Although we can say that our model, in combination with increasing returns models, has the potential to explain the history of a city and the resulting city structure, we have not investigated the dynamics of the development of cities explicitly. Completion of a theory of city formation seems to be an important project.[22] At the very least, it can be said that unlike the previous literature, our model can account for Cronon's first nature of geography in explaining where cities locate. An interesting variation of our model would make population endogenous by adding immigration. Although the setup costs for marketplaces and a mass transportation system are rather abstract in our model, we might be able to obtain more results by specifying these costs explicitly. These ideas seem worthy of further investigation.

22 Palivos and Wang (1993) provide a dynamic model of city formation based on an endogenous growth framework.

Appendix

Proof of Theorem 1. First note that without setting up marketplaces, by the boundary condition, consumers will obtain the worst possible outcome. Hence, *Core*(**d**) and *E*(**d**) are always individually rational. Second, we prove *E*(**d**) ⊂ *Core*(**d**). Suppose (**d**, f) ∈ *E*(**d**) but (**d**, f) ∉ *Core*(**d**). Then there exists a coalition S ∈ $\mathscr{A}$, $\nu(S) > 0$, and there exists an allocation (**d**, g) such that (i) $\tilde{g}(\mathbf{d}, a) \succ_a^{\mathbf{d}} \tilde{f}(\mathbf{d}, a)$ a.e. in S, and (ii) $\int_S \mathrm{proj}_C\, \tilde{g}(\mathbf{d}, S)\, d\nu(a) \in Y - Q(\mathbf{d}, S)$. By (i), $p\cdot(\mathrm{proj}_C\, \tilde{g}) > p\cdot(y - q) = -\, p\cdot q$ a.e. in S. This implies $p\cdot(\int_S \mathrm{proj}_C\, \tilde{g}(\mathbf{d}, a)\, d\nu(a)) > \max_{y \in Y} p\cdot(Y - Q(\mathbf{d}, S)) = \nu(S) \times \{\max_{y \in Y} p\cdot(Y - Q(\mathbf{d}, A))\}$, which contradicts (ii). Hence *E*(**d**) ⊂ *Core*(**d**).

Next, we will prove the most difficult part of the theorem, *Core*(**d**) ⊂ *E*(**d**). First, we will demonstrate the existence of a unique individual profit function. Let $\Delta^* = \Delta \cap Y^*$, where Y^* is denotes the dual cone of Y. Given $p \in \Delta^*$, let $\Pi(\mathbf{d}, S, p) = \max p\cdot y$ s.t. $y \in Y - Q(\mathbf{d}, S)$, and let $\pi(\mathbf{d}, p) = \max p\cdot y$ s.t. $y \in Y - Q(\mathbf{d}, A)$.[23] Note that $\Pi(\mathbf{d}, S, p) = \int_S \pi(\mathbf{d}, p)\, d\nu$, since $Q(\mathbf{d}, S) = \nu(S) \times Q(\mathbf{d}, A)$. Therefore, $\pi(\mathbf{d}, p)$ is a Radon–Nikodym derivative of $\Pi(\mathbf{d}, S, p)$, which is unique up to ν–equivalence.

Let (**d**, f) ∈ *Core*(**d**). Define a correspondence ψ from K × (A, $\mathscr{A}$, ν) into Ω by $\psi(\mathbf{d}, a) = \{x \in \tilde{X}(\mathbf{d}, a): x \succ_a^{\mathbf{d}} \tilde{f}(\mathbf{d}, a)\}$. Then a standard argument shows ψ is measurable (Hildenbrand, 1974, p. 134). The local nonsatiation property of preferences and the integrability of f imply $\int_S \mathrm{proj}_C\, \psi(\mathbf{d}, a)\, d\nu(a) \neq \emptyset$ by a measurable selection theorem (Hildenbrand, 1974, D.II.2 Theorem 1, p. 54). Let $Z = \cup \{\int_S \mathrm{proj}_C\, \psi(\mathbf{d}, a)\, d\nu(a) - Y + Q(\mathbf{d}, S): S \in \mathscr{A}\ \nu(S) > 0\}$. We claim $0 \notin Z$.

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23 In the appendix, 'y' sometimes represents an element of Y – Q(**d**, A) (the net production set) instead of Y (the private production set). This is just for notational simplicity, and the reader should be able to distinguish between these two by the context of usage.

Suppose $0 \in Z$. Then there exist $S \in \mathcal{A}$, $\nu(S) > 0$, and $\tilde{g}(\mathbf{d}, \cdot) \in \psi(\mathbf{d}, \cdot)$ s.t. $\int_S \text{proj}_C\, \tilde{g}(\mathbf{d}, a)\, d\nu(a) \in Y - Q(\mathbf{d}, S)$. Then $(\mathbf{d}, g, S)$ can improve upon $(\mathbf{d}, f)$ since $\tilde{g}(\mathbf{d}, a) \succ^{\mathbf{d}}_a \tilde{f}(\mathbf{d}, a)$ a.e. in S. Hence, $0 \notin Z$. Note that $Q(\mathbf{d}, S) = \tau(\mathbf{d}) \times \int_S Q\, d\nu(a)$. By Vind's lemma (Hildenbrand, 1974, D.II.4 Proposition 5, p. 62), Z is convex. By a separation theorem, there exists $p \in \mathbb{R}^I \times \mathbb{R}^{2J}$ s.t. $p \neq 0$, $0 \leq p\cdot z$ for all $z \in Z$. In fact, $p > 0$, for if not there exists $z \in Z$ s.t. $p\cdot z < 0$ by the free disposal property in Q. Moreover, $p\cdot Z \geq 0$ implies $p \in Y^*$. Therefore, without loss of generality, we can normalize $p \in \Delta^*$.

Hence, for every $S \in \mathcal{A}$, $\nu(S) > 0$, we have $p\cdot y \leq p\cdot x$ for every $y \in Y - Q(\mathbf{d}, S)$ and $x \in \int_S \text{proj}_C\, \psi(\mathbf{d}, a)\, d\nu(a)$. Then we obtain $p\cdot y \leq p\cdot x$ for every $y \in Y - Q(\mathbf{d}, S)$ and $x \in \int_S \text{proj}_C\, \psi(\mathbf{d}, a)\, d\nu(a)$. Hence, $p\cdot(Y - Q(\mathbf{d}, S)) \leq \Pi(\mathbf{d}, S, p) \leq \inf(p\cdot\int_S \text{proj}_C\, \psi(\mathbf{d}, a)\, d\nu(a))$. By interchangeability of infimum and integration (Hildenbrand, 1974, D.II.4 Proposition 6, p. 63), we obtain $\int_S \pi(\mathbf{d}, p)\, d\nu = \Pi(\mathbf{d}, S, p) \leq \int_S \inf\{p\cdot(\text{proj}_C\, \psi(\mathbf{d}, a))\}\, d\nu(a)$. Hence, $\pi(\mathbf{d}, p) \leq p\cdot x$ for every $x \in \text{proj}_C\, \psi(\mathbf{d}, a)$ a.e. in A.

Now we claim $p\cdot(\text{proj}_C\, \tilde{f}(\mathbf{d}, a)) = -\min p\cdot Q(\mathbf{d}, A)$ a.e. in A. Since $\int_S \text{proj}_C\, \psi(\mathbf{d}, a)\, d\nu(a) \neq \emptyset$, $\Pi(\mathbf{d}, S, p) < \infty$ and $\pi(\mathbf{d}, p) < \infty$. This implies $\pi(\mathbf{d}, p) = -\min p\cdot Q(\mathbf{d}, A)$, since Y is a convex cone with vertex at the origin. Actually the minimum exists, since the feasible $Q(\mathbf{d}, A)$ are bounded and feasibility is satisfied under p. Local nonsatiation implies $p\cdot(\text{proj}_C\, \tilde{f}(\mathbf{d}, a)) \geq \pi(\mathbf{d}, p)$ a.e. in A. Suppose that there exists $S \in \mathcal{A}$, $\nu(S) > 0$, s.t. $\int_S \pi(\mathbf{d}, p)\, d\nu < p\cdot\int_S \text{proj}_C\, \tilde{f}(\mathbf{d}, a)\, d\nu(a)$. Then there exists $\bar{y} \in Y$ s.t. $\int_S \text{proj}_C\, \tilde{f}(\mathbf{d}, a)\, d\nu(a) \in \{\bar{y}\} - Q(\mathbf{d}, S)$, and $p\cdot\bar{y} > 0$, which contradicts profit maximization by firms. Therefore, $p\cdot(\text{proj}_C\, \tilde{f}(\mathbf{d}, a)) = \pi(\mathbf{d}, p)$ a.e. in A.

So far, we have proved that $\tilde{f}$ attains a minimum expenditure a.e. in A, i.e., $(\mathbf{d}, f, y, q, p)$ is an expenditure minimizing equilibrium (see Hildenbrand (1968)). However, a minimum expenditure consumption plan is not in general the most preferred plan in the budget set. To prove that an expenditure minimizing equilibrium is an equilibrium with participation fees, we need to make use of the boundary condition and local nonsatiation. Consider first the set of $a \in A$ such that $\inf p\cdot(\mathrm{proj}_C\ \tilde{X}_j(d, a)) \geq \pi(d, p)$ for all $j \in J$. Then, $p\cdot(\mathrm{proj}_C\ \tilde{f}(d, a)) = \min p\cdot(\mathrm{proj}_C\ \tilde{X}_j(d, a))$, which implies by the boundary condition that $\tilde{f}(d, a)$ is a maximal element in the budget set as long as $\tilde{f}(d, a)$ is in the budget set. Next consider the set of $a \in A$ such that there exists $j \in J$ such that $\inf p\cdot(\mathrm{proj}_C\ \tilde{X}_j(d, a)) < \pi(d, p)$. By the boundary condition, $\mathrm{proj}_J\ \tilde{f}(d, a) \in \{j' \in J: \inf p\cdot(\mathrm{proj}_C\ \tilde{X}_{j'}(d, a)) < \pi(d, p)\}$. We claim that $\tilde{f}(d, a)$ is a maximal element in the budget set. To show this, let $B_j(d, a, p) = \{x \in \tilde{X}_j(d, a): p\cdot(\mathrm{proj}_C\ x) \leq \pi(d, a)\}$, and let $\mathring{B}_j(d, a, p) = \{x \in \tilde{X}_j(d, a): p\cdot(\mathrm{proj}_C\ x) < \pi(d, p)\}$. Without loss of generality, let $j = \mathrm{proj}_J\ \tilde{f}(d, a)$. Suppose that there exist $j' \in J$ and $x' \in B_{j'}(d, a, p)$ s.t. $x' \succ_a^d \tilde{f}(d, a)$. Then $j' \in \{j'' \in J: p\cdot(\mathrm{proj}_C\ \tilde{X}_{j''}(d, a)) > \pi(d, p)\}$ by the boundary condition. Pick a convergent sequence $\{x_k'\}_{k=1}^{\infty} \subset \mathring{B}_{j'}(d, a, p)$ s.t. $x_k' \to x'$. Since $R(d, a)$ is a closed complete preorder, $P^a(d, a)$ is open. Therefore, there exists an integer $\bar{k}$ s.t. for any $k > \bar{k}$, $x_k' \succ_a^d \tilde{f}(d, a)$, a contradiction. Hence, $\tilde{f}(d, a)$ is a maximal element in the budget set. This proves $Core(d) = E(d)$.//

<u>Proof of Corollary 1.</u> By the definition of *Core* and $Core(d)$, $Core \subset \cup_K Core(d)$. From Theorem 1, $Core \subset \cup_K E(d)$. The proof is completed by applying the definition of $E(d)$.//

<u>Proof of Theorem 2.</u> This proof is a combination of the proofs of Hildenbrand (1974, II.2.2 Theorem 2, p. 151) and Hildenbrand (1974, II.4.2 Theorem 2, p. 219), greatly complicated by the use of disconnected consumption sets.

A standard argument proves that the feasible *average* translated trading set and feasible production set are bounded. Truncate the *net* production set (the total setup cost is subtracted) $Y - Q(\mathbf{d}, A)$ by a compact hypercube in C that contains the feasible production set in its interior. Denote the net truncated production set by $\hat{Y}(\mathbf{d})$. Next truncate *individual* translated trading sets in the following way: $\tilde{X}^k(\mathbf{d}, a)$ is such that $\text{proj}_C\, \tilde{X}^k_j(\mathbf{d}, a) = \text{proj}_C\, \tilde{X}_j(\mathbf{d}, a) \cap (Y + ke)$, for $j \in J$ and $k = 1, 2, \cdots$, where $e = (1, 1, \cdots, 1) \in C$. Since $\mathbf{d} \in K$ is feasible, $\tilde{X}^k(\mathbf{d}, a) \neq \emptyset$ a.e. in A for all k.

Let $\Delta^* = \Delta \cap Y^*$, where Y^* denotes the dual cone of Y. Clearly, Δ^* is compact and convex. Let $\eta(\mathbf{d}, \cdot)\colon \Delta^* \twoheadrightarrow \hat{Y}(\mathbf{d})$ be such that $\eta(\mathbf{d}, p) = \{y \in \hat{Y}(\mathbf{d})\colon p \cdot y \geq p \cdot y'$ for any $y' \in \hat{Y}(\mathbf{d})\}$. Then, $\eta(\mathbf{d}, \cdot)$ has a closed graph, and is compact– and convex–valued. Also let $\pi(\mathbf{d}, \cdot)\colon \Delta^* \to \mathbb{R}$ be such that $\pi(\mathbf{d}, p) = p \cdot \eta(\mathbf{d}, p)$. Then, $\pi(\mathbf{d}, \cdot)$ is a continuous function.

Let us move to the consumer sector. Let $B^k_j(\mathbf{d}, a, p) = \{x \in \text{proj}_C\, \tilde{X}^k(\mathbf{d}, a)\colon p \cdot x \leq \pi(\mathbf{d}, p)\}$, $B^k(\mathbf{d}, a, p) = \cup_J\, B^k_j(\mathbf{d}, a, p)$, and $B^k_V(\mathbf{d}, a, p) = \cup_V\, B^k_j(\mathbf{d}, a, p)$ for $V \subset J$ s.t. $V \neq \emptyset$. These denote budget relations in some (or all) locations. Let $\varphi^k\colon K \times A \times \Delta^* \twoheadrightarrow \tilde{X}^k(\mathbf{d}, a)$ be such that $\varphi^k(\mathbf{d}, a, p) = \{x \in B^k(\mathbf{d}, a, p)\colon x \succsim^{\mathbf{d}}_a x'$ for any $x' \in B^k(\mathbf{d}, a, p)\}$. φ denotes consumer a's demand relation when the consumption set is truncated by $Y + ke$. Since $(Y - Q(\mathbf{d}, A)) \cap \text{proj}_C\, \tilde{X}(\mathbf{d}, a) \neq \emptyset$ a.e. in A, there exists $j \in J$ such that $\hat{Y}(\mathbf{d}) \cap \text{proj}_C\, \tilde{X}^k_j(\mathbf{d}, a) \neq \emptyset$ for all k a.e. in A. Since $p \in \Delta^*$, there exists $j \in J$ such that $B^k_j(\mathbf{d}, a, p) \neq \emptyset$ a.e. in A.

Let $\mathring{\Delta} = \{p \in \Delta^*\colon p^c >> 0\}$, where p^c is commodity price vector. Eventually, we will show that the equilibrium price relative to k is in $\mathring{\Delta}$. Given $a \in A$, $\mathbf{d} \in K$, $p \in \mathring{\Delta}$, we partition J into three sets:

$$J_>(\mathbf{d}, a, p) = \{j \in J\colon \pi(\mathbf{d}, p) > \min p \cdot [\text{proj}_C\, \tilde{X}^k_j(\mathbf{d}, a)]\},$$
$$J_=(\mathbf{d}, a, p) = \{j \in J\colon \pi(\mathbf{d}, p) = \min p \cdot [\text{proj}_C\, \tilde{X}^k_j(\mathbf{d}, a)]\},$$
$$J_<(\mathbf{d}, a, p) = \{j \in J\colon \pi(\mathbf{d}, p) < \min p \cdot [\text{proj}_C\, \tilde{X}^k_j(\mathbf{d}, a)]\}.$$

Since $\mathbf{d}$ is feasible, $J_>(\mathbf{d}, a, p) \neq \emptyset$ a.e. in A. Given $a \in A$, $\mathbf{d} \in K$, $j \in J$, we partition $\mathring{\Delta}$ into three sets in a similar manner:

$$\mathring{\Delta}_{>}(d, a, j) = \{p \in \mathring{\Delta}: \pi(d, p) > \min p\cdot[\mathrm{proj}_C\ \tilde{X}^k_j(d, a)]\},$$
$$\mathring{\Delta}_{=}(d, a, j) = \{p \in \mathring{\Delta}: \pi(d, p) = \min p\cdot[\mathrm{proj}_C\ \tilde{X}^k_j(d, a)]\},$$
$$\mathring{\Delta}_{<}(d, a, j) = \{p \in \mathring{\Delta}: \pi(d, p) < \min p\cdot[\mathrm{proj}_C\ \tilde{X}^k_j(d, a)]\}.$$

Obviously $\mathring{\Delta}_{>}(d, a, j)$ is open relative to $\mathring{\Delta}$. It is easy to show $B_j(d, a, \cdot)$ is continuous on $\mathring{\Delta}_{>}(d, a, j)$.

Now, we claim $\varphi^k(d, a, \cdot)$ has a closed graph in $\mathring{\Delta}$. Since $B^k(d, a, \cdot)$ is not continuous in general due to disconnected consumption sets, we need several steps to prove this. First note that if $j \in J_{>}(d, a, p)$, there exists $x \in B^k_j(d, a, p)$ such that $x \succsim^d_a x'$ for $x' \in B^k_{j'}(d, a, p)$ s.t. $j' \in J_{=}(d, a, p)$ by the boundary condition. Hence if $J_{>}(d, a, p) \neq \emptyset$, then $\mathrm{proj}_J\ \varphi^k(d, a, p) \in J_{>}(d, a, p)$.

Second we show that if $J_{>}(d, a, p) \neq \emptyset$, then for any $p \in \mathring{\Delta}_{=}(d, a, j)$, for any sequence $\{p^s\}_{s=1}^{\infty} \to p$ such that $\{p^s\}_{s=1}^{\infty} \subset \mathring{\Delta}_{>}(d, a, j)$, there exists an integer $\bar{s}$ such that for any $s \geq \bar{s}$, $j \notin \mathrm{proj}_J\ \varphi^k(d, a, p^s)$. To show this, let $x \in B^k_j(d, a, p)$ and $x' \in B^k_{j'}(d, a, p)$ where $j' \in J_{>}(d, a, p)$. From local nonsatiation and the boundary condition, $x' \succ^d_a x$. Since $P(d, a)$ is open, the statement is proved.

Third, let $\mathring{\Delta}_{>}(d, a, V) = (\cap_V\ \mathring{\Delta}_{>}(d, a, j))\backslash(\cup_{J\backslash V}\ \mathring{\Delta}_{>}(d, a, j))$, where $\backslash$ is set subtraction. Then, $\mathring{\Delta}$ is partitioned by $\mathring{\Delta}_{>}(d, a, V)$'s for $V \subset J$ s.t. $V \neq \emptyset$. Let $\varphi^k_V(d, a, V) = \{x \in B^k_V(d, a, p): x \succsim^d_a x'$ for any $x' \in B^k_V(d, a, j)\}$. Since $B^k_V(d, a, \cdot)$ is continuous in $\mathring{\Delta}_{>}(d, a\ V)$, $\varphi^k_V(d, a, \cdot)$ has a closed graph in $\mathring{\Delta}_{>}(d, a, V)$ for any $V \subset J$ (Hildenbrand, 1974, II.1.2 Corollary 2, p. 104). Since $R(d, a)$ is a complete preorder, $\varphi^k(d, a, p) = \varphi^k_V(d, a, p)$ for any $p \in \mathring{\Delta}_{>}(d, a\ V)$, for any $V \subset J$ (see Konishi, 1993, Lemma 2). Hence, $\varphi^k(d, a, \cdot)$ has a closed graph in $\mathring{\Delta}_{>}(d, a, V)$ for any $V \subset J$.

Fourth, let $\{p^s\}_{s=1}^{\infty} \subset \mathring{\Delta}_{>}(d, a, V)$ be such that $j \in V$ and $p^s \to p \in \mathring{\Delta}\backslash\mathring{\Delta}_{>}(d, a, j)$. Then, if $x^s \to x$ such that $x^s \in \varphi^k(d, a, p^s)$ for each $s = 1, 2, \cdots$, then for s large enough $j \notin \mathrm{proj}_J x^s$ by the argument in the second step above.

These four steps above prove that $\varphi^k(d, a, \cdot)$ has a closed graph on $\mathring{\Delta}$.

Now, let $\Psi^k(d, p) = \int_A \text{proj}_C\ \varphi^k(d, a, p)\ d\nu(a)$. Since $\varphi^k(d, \cdot, p)$ is measurable and integrably bounded, $\Psi^k(d, p) \neq \emptyset$ (Hildenbrand, 1974, D.II.4 Theorem 2, p. 62). Furthermore, $\Psi^k(d, \cdot)$ has a closed graph on $\mathring{\Delta}$ (Hildenbrand, 1974, D.II.4 Proposition 8, p. 73), and is convex-valued (Hildenbrand, 1974, D.II.4 Theorem 3, p. 62).

Let $Z^k = \int_A \text{proj}_C\ X^k(d, a)\ d\nu(a) - \hat{Y}(d)$. Then, Z^k is compact and convex. Let $\Delta^s = \{p \in \Delta^*: p_i^c \geq 1/s$ for all $i \in I\}$. Clearly, Δ^s is compact and convex. For s large enough, $\Delta^s \neq \emptyset$, and $\Delta^s \subset \mathring{\Delta}$. Let $\zeta^{ks}: \Delta^s \twoheadrightarrow Z^k$ be such that $\zeta^{ks}(p) = \Psi^k(d, p) - \eta(d, p)$. Then ζ^{ks} has a closed graph, and is convex-valued. Let $\theta^{ks}: Z^k \twoheadrightarrow \Delta^s$ be such that $\theta^{ks}(z) = \{p \in \Delta^s: p\cdot z \geq p'\cdot z$ for any $p' \in \Delta^s\}$. Then, θ^{ks} has a closed graph, and is convex-valued. Let $\xi^{ks} = \zeta^k \times \theta^{ks}: \Delta^s \times Z^k \twoheadrightarrow \Delta^s \times Z^k$. Then ξ^{ks} has a fixed point by Kakutani's theorem. Hence, there exists a list $(d, f^{ks}, y^{ks}, p^{ks})$ such that $(d, f^{ks}) \in \mathscr{F}$, $y^{ks} \in \hat{Y}(d)$, $p^{ks} \in \Delta^s$ satisfying the following conditions: (i) $\tilde{f}^{ks}(d, a) \in \varphi^k(d, a, p^{ks})$ a.e. in A, (ii) $y^{ks} \in \eta(d, p^{ks})$, and (iii) $\int_A \text{proj}_C\ \tilde{f}^{ks}(d, a)\ d\nu(a) - y^{ks} \leq 0$.

Pick a sequence of fixed points $\{(d, f^{ks}, y^{ks}, p^{ks})\}_{k=1}^{\infty}$. From (i) and (iii), $\int_A \text{proj}_C\ \tilde{f}^{ks}(d, a)\ d\nu(a) \in \hat{Y}(d)$ for each k. Then, without loss of generality, we can assume $\int_A \text{proj}_C\ \tilde{f}^{ks}(d, a)\ d\nu(a) \to x^s$, $y^{ks} \to \bar{y}^s$, and $p^{ks} \to p^s$. Since $\eta(d, \cdot)$ is closed, $\bar{y}^s \in \eta(d, p^s)$. From Fatou's lemma in several dimensions (Hildenbrand, 1974, D.II.4 Lemma 3, p. 69), there exists f^s such that $(d, f^s) \in \mathscr{F}$ such that (i) $\tilde{f}^s(d, a)$ is an accumulation point of $\{\tilde{f}^{ks}(d, a)\}_{k=1}^{\infty}$ a.e. in A, (ii) $\int_A \text{proj}_C\ \tilde{f}^s(d, a)\ d\nu(a) \leq x^s$.

Since $\tilde{f}^{ks}(d, a) \in \varphi^k(p^{ks})$ a.e. in A, and since P(d, a) is open, $\tilde{f}^s(d, a) \in \varphi(d, a, p^s)$ a.e. in A, where $\varphi(d, a, p) = \{x \in B(d, a, p): x \succsim_a^d x'$ for any $x' \in B(d, a, p)\}$. Let $y^s = \int_A \text{proj}_C\ \tilde{f}^s(d, a)\ d\nu(a)$. We will prove that $y^s \in \eta(d, p^s)$. To accomplish this, let $z^s = y^s - \bar{y}^s$. Since local nonsatiation of preferences is implied by our assumptions, $p^s\cdot(\text{proj}_C\ \tilde{f}^s(d, a)) = \pi(d, p^s)$ a.e. in A. Hence $p^s\cdot y^s = p^s\cdot\bar{y}^s$.

We show $y^s \in Y - Q(\mathbf{d}, A)$. It suffices to show $z^s \leq 0$. Since $z^s = (\lim_{k\to\infty} \int_A \mathrm{proj}_C\, \tilde{f}^{ks}(\mathbf{d}, a)\, d\nu(a) - \bar{y}^s) - (\lim_{k\to\infty} \int_A \mathrm{proj}_C\, \tilde{f}^{ks}(\mathbf{d}, a)\, d\nu(a) - \int_A \mathrm{proj}_C\, \tilde{f}^s(\mathbf{d}, a)\, d\nu(a))$, $z^s \leq 0$ by Fatou's lemma in several dimensions and the free disposal property of $Y - Q(\mathbf{d}, A)$. Hence $(\mathbf{d}, f^s, y^s, p^s)$ is an equilibrium with participation fees relative to s.

Now, what is left to show is that for s large enough, $p_i^s > 1/s$ for all $i \in I$. If there exists such an s, $(\mathbf{d}, f^s, y^s, p^s) \in \mathcal{E}(\mathbf{d})$ holds. Suppose not. From interiority and monotonicity in commodity consumption, we can find s such that $z_i^s > 0$ for some $i \in I$ (Hildenbrand, 1974, II.2.2 Lemma 1, p. 150, and II.1.3 Proposition 6, p. 119). This contradicts $z^s \leq 0$ for all s.//

<u>Proof of Corollary 2.</u> From the boundary condition and the feasibility of **d**, a.e. in A consumer a has no incentive to deviate from the grand coalition, which proposes $(\mathbf{d}, f) \in \mathcal{E}(\mathbf{d})$. Hence $(\mathbf{d}, f) \in \mathit{Core}(\mathbf{d})$ by Theorem 1. Theorem 2 proves $\mathcal{E}(\mathbf{d}) \neq \emptyset$ if interiority is satisfied. If interiority is not satisfied, then by the boundary condition, a.e. in A consumer a can only consume a least preferred consumption plan. Hence, there is no incentive to deviate from such an allocation, and $\mathit{Core}(\mathbf{d}) \neq \emptyset$.//

<u>Proof of Theorem 3.</u> First note that $Y \cap \mathrm{proj}_C\, X(a) \neq \emptyset$, i.e., there exists a feasible allocation ($\emptyset \in K$). Note also that by the assumption that the set of feasible marketplace structures is bounded, the set of feasible marketplace structures $\mathrm{proj}_K\, \mathcal{F}$ is compact. To show this, it is enough to recall that the function δ^j is a continuous function on $(K, \mathcal{O})$, and recall the assumptions on Q, Y, and X. Denote the feasible marketplace structures by $\hat{K}$.

Since we assume that consumers are identical almost everywhere, we drop a from the notation for preference relations and consumption sets. Since $\succsim$ is a continuous complete preorder, there exists a continuous utility representation of $\succsim$, $u: X \to [0, 1]$.

By the definition of F, we can naturally define $\tilde{u}$: $F \to [0, 1]$ such that if $u(f(a)) = u^*$ a.e. in A, then $\tilde{u}(f) = u^*$. Clearly, there exists $\sup\{\tilde{u}(f): f \in \text{proj}_X F \cap \text{proj}_X \mathcal{F}\} = \bar{u}$. Then, there exists a sequence $\{(d^s, f^s)\}_{s=1}^{\infty}$ such that $\tilde{u}(f^s) \to \bar{u}$. Since the feasible average consumption set is bounded and $\hat{K}$ is compact, there exists a convergent subsequence of $\{(d^s, \int_A \text{proj}_C f(a)^s \, d\nu(a))\}_{s=1}^{\infty} \to (d, \bar{x})$. Since $\hat{K}$ is compact, $\mathbf{d} \in \hat{K}$. Since $Y - Q(\mathbf{d}, A)$ is closed, $\bar{x} \in Y - Q(\mathbf{d}, A)$. From Fatou's lemma in several dimensions, there exists $(\mathbf{d}, f) \in \mathcal{F}$, such that (i) $\int_A \text{proj}_C f \, d\nu \leq \bar{x}$, and (ii) f(a) is an accumulation point of $\{f^s(a)\}_{s=1}^{\infty}$ a.e. in A. Since $\mathbf{d}^s \to \mathbf{d}$, $\#\mathbf{d}^s = \#\mathbf{d}$. Since δ_j is continuous and X is closed, $f(a) \in \bar{X}(\mathbf{d}, a)$, which is a truncated trading set. From closedness of $\succsim$, $(\mathbf{d}, f) \in F$ so $(\mathbf{d}, f) \in F \cap \mathcal{F}$. From the continuity of u, $\tilde{u}(f) = \bar{u}$. Hence $(\mathbf{d}, f)$ is a maximal element of $F \cap \mathcal{F}$ in $\succsim_A$.

Now we will prove that $(\mathbf{d}, f) \in$ *Core*. Suppose not. Then, there exist $(\mathbf{d}', g) \in \mathcal{F}$ and $S \in \mathcal{A}$, $\nu(S) > 0$, such that $g(a) \succ f(a)$ a.e. in S. Since $\mathbf{d}' \in \text{proj}_K \mathcal{F}$, *Core*$(\mathbf{d}') \neq \emptyset$ by Corollary 2. Let $(\mathbf{d}', h) \in$ *Core*$(\mathbf{d}')$. Then, (g, S) cannot improve upon h. This implies that there exists $S' \in \mathcal{A}$, $S' \subset S$, $\nu(S') > 0$, such that $h(a) \succsim g(a)$ a.e. in S'. Since $h \in F$ by Corollary 1, $h(a) \succ f(a)$ a.e. in A by transitivity of $\succsim$. This contradicts $\tilde{u}(f) = \bar{u}$. Hence $(\mathbf{d}, f) \in$ *Core*.//

<u>Proof of Corollary 3.</u> Since *Core* $\in P$, *Core* $\subset F \cap P$ (Corollary 1). Let $(\mathbf{d}, f) \in F \cap P$. Then, $\tilde{u}(f) = \max\{\tilde{u}(f'): (\mathbf{d}', f') \in F \cap \mathcal{F}\}$, which implies $(\mathbf{d}, f) \in$ *Core* by the argument in the proof of Theorem 3.//

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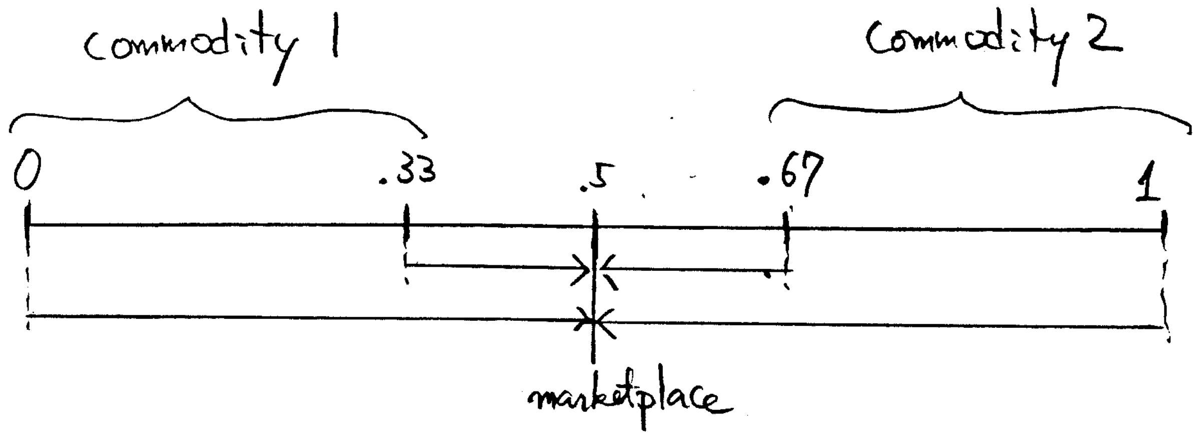


D: closed interval $[0, 1]$

J: set of locations for consumers

$\{0, .33, .67, 1\}$

Figure 1

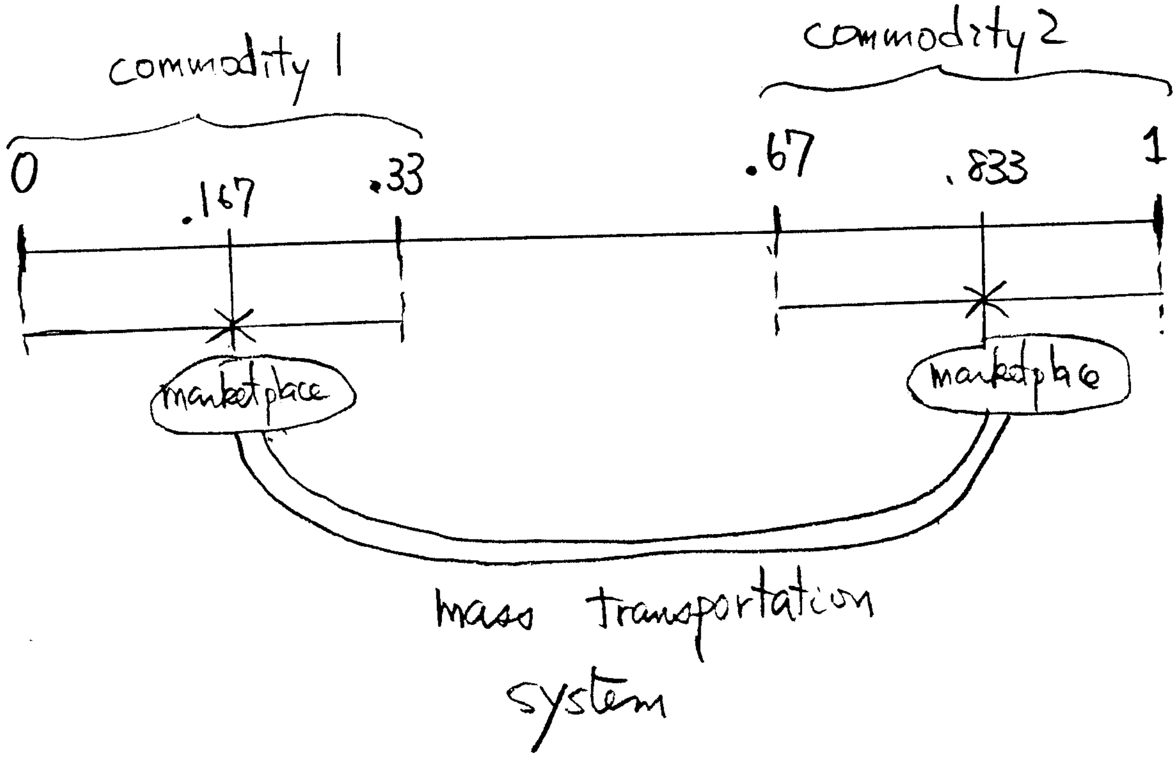
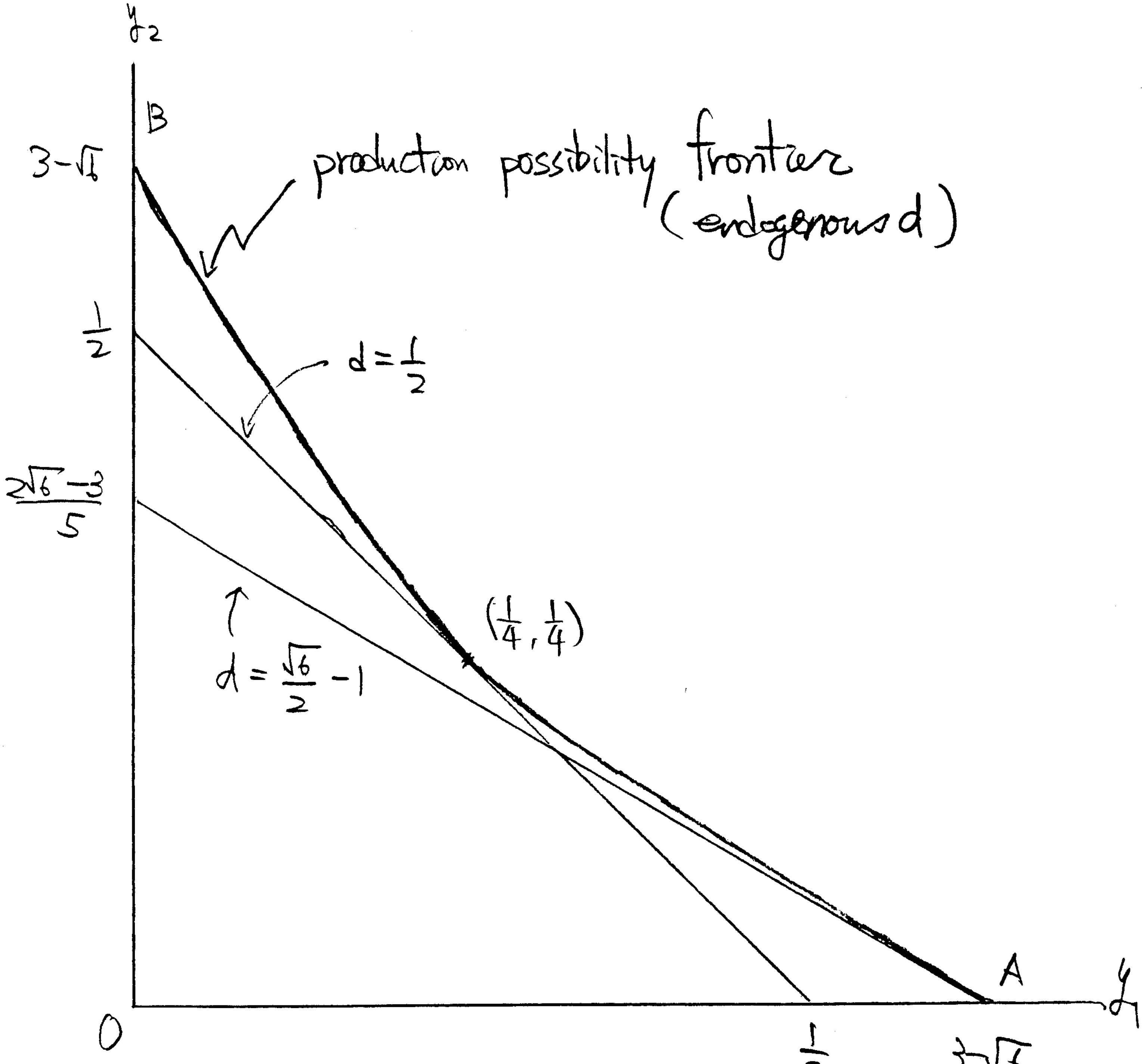

commodity 1
commodity 2
0
.167
.33
.67
.833
1
marketplace
marketplace
mass transportation system

Figure 2



y_2
B
$3-\sqrt{6}$
production possibility frontier (endogenous d)
$\frac{1}{2}$
$d=\frac{1}{2}$
$\frac{2\sqrt{6}-3}{5}$
$(\frac{1}{4},\frac{1}{4})$
$d=\frac{\sqrt{6}}{2}-1$
A
y_1
0
$\frac{1}{2}$
$3-\sqrt{6}$

Figure 3

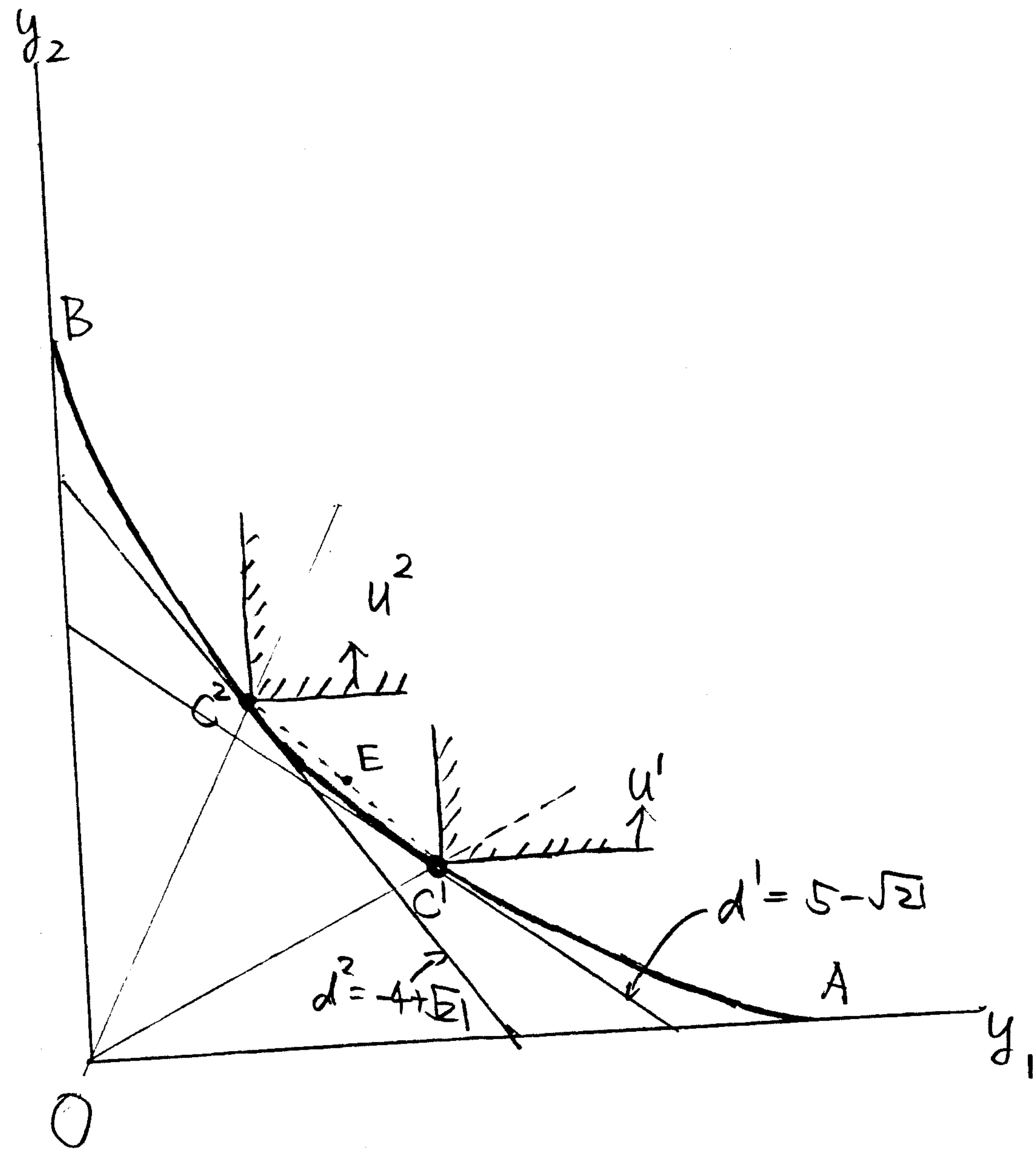


Figure 4