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INVESTMENT, CAPACITY UTILIZATION AND THE REAL BUSINESS CYCLE

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1. Introduction

In the real business cycle models, of the type Kydland and Prescott (1982) and Long and Plosser (1983) developed, the cycles are generated by exogenous productivity shocks. A stylized version of the main mechanism working in these models can be described as follows. Dynamic optimizing behavior on the part of agents in the economy implies that both consumption and investment react positively to the supply shocks. Since labor marginal productivity is directly affected, employment is also procyclical, along with measures of labor productivity. The resulting capital accumulation provides a channel of persistence, even if the technology shocks are serially uncorrelated. Hence, these productivity shocks are able to generate, from a neo-classical framework, co-movements of macroeconomic variables and persistence of fluctuations that conform to those typically observed during business cycles.

A commonly perceived problem with this way of viewing the cycle is that important productivity shifts of this type do not seem to occur often in reality. The oil price shocks could be examples of apparent quantitative significance, but it is not easy to think of important others. Weather related disturbances correspond ideally to the theoretical shock of the production function, but it does not seem plausible that they play an important macroeconomic role.

In contrast with this approach in which investment *reacts* to output supply shocks, the Keynesian view of the cycle is that investment movements *generate* business fluctuations. It is the 'marginal efficiency of capital' that shifts exogenously, affecting investment demand and hence, given the disequilibrium in the labor market, also employment and output. The pure case of this type is when there is a change in the expected future marginal productivity of capital which does not affect the current production function.

When an investment shock of this type occurs in a standard neo-classical growth model, employment and output also tend to increase, but the mechanism is very different. Market clearing necessitates an adjustment of the real interest rate, which stimulates current labor effort and output through an intertemporal substitution effect on leisure. A problem with this mechanism, as discussed in Barro and King (1984), is that the intertemporal substitution effect which operates to reduce leisure also works to cut consumption. Therefore, consumption tends to move counter-cyclically, which contradicts the evidence. Labor productivity moves in the 'wrong' direction too. An expansion of labor effort given the fixed supply of capital in the short run causes labor's productivity to decline.

The present paper adopts the Keynesian view that direct shocks to investment are important for business fluctuations, but incorporates them in a neo-classical framework where the rate of capital utilization is endogenous. In contrast to the intertemporal substitution effect mentioned above, the transmission mechanism of the investment shocks works here through the optimal capacity utilization decision and the demand side of the labor market. Labor productivity shifts are therefore endogenous outcomes, rather than given exogenously. The importance of this feature for the model's predictions is that labor productivity co-moves with investment and output, and the ensuing real wage change generates a procyclical effect on consumption as well. The crucial feature of the present model, the capacity utilization decision involves Keynes' (1936) notion of 'user cost' in production.

To sharpen the distinction between this and the real business cycle models with direct shocks to the production function, no such shifts are included. The shocks to investment are modelled as current technological changes that affect the productivity of new capital goods only, leaving unchanged the productivity of the existing stock. Because of a time-to-build delay, only future capital's productivity is affected and hence technological changes correspond to investment demand shocks rather than to supply shocks, as in the models of Kydland and Prescott and Long and Plosser. This type of technological change may be more realistic than the current shock to productivity. Important technological improvements of new productive capital seem to occur quite often. As will be discussed, it is crucial for this model that the new technology does not affect directly the productivity of the existing capital stock. Realistically, this assumption does not seem to be a restriction because it is not easy to find opposite examples.

Another type of shock to investment considered is tax changes such as investment subsidies that affect the private marginal productivity of investment.

The remainder of the paper is organized as follows. An outline of the basic environment which is assumed to characterize the economy is undertaken in section 2. In section 3 the representative agent's optimization problem is presented. The impact and dynamic effects of technological change on the business cycle are analyzed in Sections 4 and 5. The next section 6 addresses the effects on the economy of an investment subsidy. Some concluding remarks are offered in the last section.

2. The Economic Environment

Consider a perfectly competitive closed economy populated by a very large number of identical households and identical firms. Aggregate output is given by the following constant-returns-to-scale production function which differs from the standard neo-classical specification solely by the inclusion of a variable rate of capital utilization:

(1) $y_t = F(k_t h_t, \ell_t)$,

where y_t is the output of the single good in period t, k_t is the capital stock (see below for a discussion about its units) at the beginning of period t, h_t is the period-t utilization rate of k_t, and ℓ_t is labor input in this period. The non-negative function F is assumed to satisfy F_k, $F_\ell > 0$ and the concavity conditions F_{kk}, $F_{\ell\ell} < 0$ and $F_{kk}F_{\ell\ell} - F_{k\ell}^2 > 0$. It is also assumed that

$F_{k\ell} > 0$ so that capital and labor services are complements in the Edgeworth-Pareto sense. This feature provides a positive link between capital utilization and labor demand.

The capital utilization decision involves Keynes' notion of 'user cost'. That is, a more intensive utilization causes a faster depreciation of the capital stock. As in Taubman and Wilkinson (1970), Merrick (1984) and Hercowitz (1986), this effect is modelled in the evolution of the capital stock as

(2) $k_{t+1} = k_t[1 - \delta(h_t)] + i_t(1 + \varepsilon_t)$,

where the non-negative depreciation function δ satisfies $\delta' > 0$, $\delta'' > 0$. Gross investment, as corresponding to the national income accounts, is i_t. Its contribution to the production capacity in t + 1, however, depends on the technological shift factor ε_t, affecting the productivity of the new capital goods. The productivity of the already installed capital stock k_t is not directly affected by the new technology. Correspondingly, k_{t+1} is a measure of the future capital stock in productivity units. (Similarly k_t would include past technological changes).

Note that this technological disturbance is very different than the usual technological shock, attached to the production function, used in the real business cycle models. The latter is a supply shock, while ε_t here affects investment and hence works as a demand shock. By substituting k_{t+1} into the production function corresponding to t + 1, it becomes clear that ε_t works as a typical shift in the future marginal efficiency of capital, that drives current investment.

An important aspect of (2) in the present context is the standard time-to-build of physical capital. This is so because the analysis focuses primarily on the effects of investment shocks during the current period, prior to the incorporation of the new investment into k_{t+1}. In this sense, the discussion is similar to the typical Keynesian treatment of investment shifts. The length of the basic period, which corresponds to the time-to-build, is thought of as non-trivial, say one year (see discussion in Kydland and Prescott (1982)).

The representative household in this economy maximizes the lifetime utility function

$$(3) \quad \sum_{t=0}^{T} \beta^{t} \underline{U}(c_t, \ell_t) \qquad\qquad 0 < \beta < 1,$$

where c_t and ℓ_t are the period-t flows of consumption and labor effort, and β is the discount factor. A finite horizon is used in order to facilitate the derivation of the comparative results below. This does not seem to be particularly restrictive, however, because T can be arbitrarily large.

The specific form of $\underline{U}$ adopted is

$$\underline{U}(c_t, \ell_t) = U(c_t - G(\ell_t)),$$

with $U' > 0$, $U'' < 0$, $G' > 0$ and $G'' > 0$. This utility function satisfies the standard properties $U_c > 0$, $U_\ell < 0$, U_{cc}, $U_{\ell\ell} < 0$, $U_{cc}U_{\ell\ell} - U_{c\ell}^2 > 0$, and it implies that the marginal rate of substitution between consumption and labor supply depends on the latter only:

$$-\frac{U_\ell(c_t,\ell_t)}{U_c(c_t,\ell_t)} = G'(\ell_t).$$

That is, labor supply is determined independently of the intertemporal consumption-savings choice, which is very convenient in obtaining results from the model. As a consequence, the intertemporal substitution effect on labor supply, common in the neo-classical macro models, is eliminated. Rather than being a drawback, this implication of the utility function has the advantage of emphasizing the alternative transmission mechanism of investment shocks being studied here. This mechanism operates on the labor demand side (via the capacity utilization decision) rather than on the labor supply side. The description of the setup is completed by the resource constraint:

$$(4) \quad y_t = c_t + i_t.$$

The analysis is carried out in a deterministic framework since for the present purposes uncertainty is an unnecessary complication.

3. The Representative Agent's Optimization Problem

The decision-making of consumer-workers and firms in competitve equilibrium can be summarized by the outcome of the following "representative" agent's dynamic programming problem

$$(5)\quad V^{t}(k_t;\underline{\varepsilon}_t) = \max_{c_t,k_{t+1},h_t,\ell_t} [\underline{U}(c_t,\ell_t) + \beta V^{t+1}(k_{t+1};\underline{\varepsilon}_{t+1})],$$

subject to

$$(6)\quad c_t = F(k_t h_t,\ell_t) - \frac{k_{t+1}}{1+\varepsilon_t} + \frac{k_t}{1+\varepsilon_t}[1-\delta(h_t)],$$

$$[\text{with } \underline{\varepsilon}_t \equiv (\varepsilon_t,\varepsilon_{t+1},\ldots,\varepsilon_T)]$$

where the transition equation (6) is obtained by substituting the production function (1) and the capital evolution equation (2) into the resource constraint (4).[1] The solution to the above programming problem is characterized by the following three efficiency conditions--in addition to (6)

$$(7)\quad U'(c_t - G(\ell_t))/(1+\varepsilon_t) = \beta V_k^{t+1}(k_{t+1};\underline{\varepsilon}_{t+1})$$
$$= \beta U'(c_{t+1}-G(\ell_{t+1}))[F_k(k_{t+1}h_{t+1},\ell_{t+1})h_{t+1}+(1-\delta(h_{t+1}))/(1+\varepsilon_{t+1})] \qquad \text{for } t=0,\ldots,T-1$$

$$(8)\quad F_k(k_t h_t,\ell_t) = \delta'(h_t)/(1+\varepsilon_t) \qquad \text{for } t=0,\ldots,T$$

$$(9)\quad F_\ell(k_t h_t,\ell_t) = G'(\ell_t) \qquad \text{for } t=0,\ldots,T.$$

The first equation (7) is a standard optimality condition governing investment. The left-hand side of this equation represents the loss in current utility which is realized when an extra unit of current investment is undertaken. The right-hand side portrays the discounted future utility obtained from an extra unit of investment today. Note that an increase in the investment technological shift factor, $(1+\varepsilon_t)$, reduces the utility cost of an extra unit of capital accumulation in this period. This occurs because a given increase in future output can now be obtained with a lower amount of current investment.[2] The next equation (8) characterizes efficient capital utilization. It states that capital should be utilized at the rate, h_t, which sets the marginal benefit of increased capital utilization in terms of extra current output equal to its marginal cost in terms of replacing the resulting depreciated capital. Finally, equation (9) sets the marginal product of labor equal to the marginal disutility of working, measured in terms of consumption. Again, given the form of the utility function adopted, the latter depends only upon current labor effort, and thus is determined independently of the agent's intertemporal consumption-savings decision. The advantage of this characteristic is that the system of equations (6)-(9) is recursive in the sense that (8) and (9) jointly determine h_t and ℓ_t, while then given these solutions equation (7)--in conjunction with (6)--determines the intertemporal allocation, which amounts here to specifying values for k_{t+1} and c_t.

4. Impact Effects of Investment Shocks

An analysis of the impact effect on output, hours worked, capacity utilization, productivity, investment and consumption of a shift in the technology factor, $(1+\varepsilon_t)$, governing the marginal productivity of newly produced capital will now undertaken. In the spirit of Long and Plosser (1983) this shift in the technology factor is restricted to be purely temporary in nature--that is ε_t is not associated with similar movements in the ε_{t+j}'s, for $j > 0$--so as to emphasize the natural channel of persistence emerging from the model's propagation mechanism, and to focus on what may be viewed as business cycle factors rather than long-run or growth ones.

The main practical advantage of the present framework relative to a standard neo-classical model with constant capacity utilization, is its ability to generate procyclical productivity and possibly procyclical consumption in response to investment shocks. In the standard model, such shocks work through the intertemporal substitution effect: by generating a higher real interest rate labor supply and hence output increase. This implies a movement along a given labor demand schedule, however, and hence lower labor productivity. With respect to consumption, the same mechanism depressing the demand for leisure reduces also the demand for consumption.

Given the structure of the optimality conditions (7)-(9), the effect on the aggregate supply variables, h_t, ℓ_t, y_t and productivity, can be calculated from (8) and (9) only. Note that these two equations involve only ε_t and not its future values. Hence, for those variables it does not matter whether the technological change is temporary or persistent.

Performing the standard comparative statics exercise on (8) and (9) yields

$$(10) \quad \frac{dh_t}{d\varepsilon_t} = -\delta'(t)[F_{\ell\ell}(t) - G''(t)]/[(1+\varepsilon_t)^2\,\Omega(t)] > 0$$

$$(11) \quad \frac{d\ell_t}{d\varepsilon_t} = F_{k\ell}(t)k_t\,\delta'(t)/[(1+\varepsilon_t)^2\,\Omega(t)] > 0$$

with $\Omega(t) \equiv [F_{kk}(t)k_t - \delta''(t)/(1+\varepsilon_t)][F_{\ell\ell}(t) - G''(t)] - k_tF_{k\ell}(t)^2 > 0$, where $\Omega(t)$ is the Jacobian associated with this system of equations, which is positive in value following from the concavity of $F(\cdot)$, and the convexity of $\delta(\cdot)$ and $G(\cdot)$.

The interpretation of these results is that ε_t reduces the cost of capital utilization and hence induces a higher h_t. Since $F_{k\ell} > 0$, labor's marginal productivity (labor demand) increases, resulting in a higher level of employment.

Given that k_t is predetermined, (10) and (11) immediately imply a positive output effect. The increase in capital utilization implies that labor productivity also rises. Using (10) and (11) it is easy to establish that the marginal product of labor, $F_\ell(k_t h_t, \ell_t)$, moves upwards. Specifically, one finds

$$\frac{dF_\ell(t)}{d\varepsilon_t} = [F_{k\ell}(t)k_t\delta'(t)G''(t)]/[(1+\varepsilon_t)^2\Omega(t)] > 0.$$

Given the constant-returns-to-scale assumption, the average product of labor, $F_\ell(k_t h_t, \ell_t)/\ell_t$, also must rise. This can be shown as follows. The marginal product increases if and only if the capital to labor services ratio, $k_t h_t/\ell_t$, also increases, since $F_\ell(k_t h_t, \ell_t) = F(k_t h_t/\ell_t, 1) - (k_t h_t/\ell_t)F_k(k_t h_t/\ell_t, 1)$. This is relevant since the average product $F(k_t h_t, \ell_t)/\ell_t$ can be expressed as a strictly increasing function of $k_t h_t/\ell_t$: $F(k_t h_t, \ell_t)/\ell_t = F(k_t h_t/\ell_t, 1)$. Therefore, average productivity also moves procyclically.

The impact effects of the investment shock, ε_t, on next period's capital stock, k_{t+1}, and current consumption, c_t, are easily deduced by displacing the system of equations (6) and (7) while making use of the first-order conditions (8) and (9). The results of this routine procedure are:

$$(12)\quad \frac{dk_{t+1}}{d\varepsilon_t} = \frac{-U'(t)}{[U''(t)+\beta(1+\varepsilon_t)^2 V^{t+1}_{kk}(t+1)]} + i_t \frac{U''(t)}{[U''(t)+\beta(1+\varepsilon_t)^2 V^{t+1}_{kk}(t+1)]} > 0,$$

and

$$(13)\quad \frac{dc_t}{d\varepsilon_t} = \frac{F_{\ell}(t)k_t F_{k\ell}(t)\delta'(t)}{(1+\varepsilon_t)^2\,\Omega(t)} + \frac{U'(t)/(1+\varepsilon_t)}{[U''(t)+\beta(1+\varepsilon_t)^2 V_{kk}^{t+1}(t+1)]}$$

$$+\, i_t \frac{\beta(1+\varepsilon_t)V_{kk}^{t+1}(t+1)}{[U''(t)+\beta(1+\varepsilon_t)^2 V_{kk}^{t+1}(t+1)]} \gtrless 0.$$

Note that the above expressions involve the second derivative of the period-t+1 value function. In the current finite horizon setting it is possible to establish that this derivative is negative (and continuous) so that $V_{kk}^{t+1}(t+1) < 0$ (See appendix A). As can be seen from (12), the technology factor, ε_t, has two effects on the period-t+1 capital stock. The first term illustrates the positive substitution effect that an increase in the productivity of newly produced capital has on the period-t+1 capital stock. The second-term represents the income effect associated with the shock, which is positive if $i_t > 0$. A given desired level for next period's capital stock can now be obtained with a lower level of current investment. Consumption smoothing agents will utilize part of this savings in current resource utilization to increase the future stock of capital.

Current consumption is affected in three ways by a movement in the contemporaneous investment technology parameter [c.f.(13)]. The second term which is negative, illustrates the intertemporal substitution effect associated with the improved productivity of newly produced capital. In a market economy, the increase in capital's productivity tends to raise the current real interest rate, which operates to dissuade current consumption and promote capital accumulation. The income effect associated with this technological change, which was explained above, works to raise current consumption and is represented by the third term. The standard macroeconomic presumption is that the intertemporal substitution generated by such technological shift will dominate the income effect, a situation ensured if the initial level of investment is small enough. The new element that the present model introduces is the first term, which has to do with the intratemporal margin of substitution between consumption and leisure. This effect may be interpreted as follows. A higher utilization rate increases the marginal productivity of labor--$F_{k\ell} > 0$. In a market economy this increases labor demand and hence the real wage, which generates a substitution effect, away from leisure and towards consumption. Hence, the present model provides a channel by which both consumption and investment can possibly react procyclically.

Finally, the impact effect on gross investment, i_t, is given by

$$(14)\quad \frac{di_t}{d\varepsilon_t} = \frac{dk_{t+1}}{d\varepsilon_t} - \frac{i_t}{(1+\varepsilon_t)} + \delta'(t)k_t \frac{dh_t}{d\varepsilon_t} .$$

The first two terms loosely represent opposite "substitution" and "income" type effects. If the initial i_t is relatively small, the substitution effect will clearly dominate. Here there is another positive effect on gross investment coming from the additional depreciation term $\delta'(t)k_t dh_t/d\varepsilon_t$.

The results obtained so far depend crucially upon the assumption that the technological shift parameter pertains only to newly produced capital goods. Suppose alternatively that it applies both to newly produced, i_t, and existing capital, $[1-\delta(h_t)]k_t$. Then, equation (2) governing the evolution of capital becomes

$$k_{t+1} = k_t[1-\delta(h_t)](1+\varepsilon_t) + i_t(1+\varepsilon_t),$$

and the transition equation (6) now appears as

$$c_t = F(k_t h_t, \ell_t) - \frac{k_{t+1}}{1+\varepsilon_t} + k_t[1-\delta(h_t)].$$

While the form of the efficiency conditions (7) and (9) characterizing the optimal choices for k_{t+1} and ℓ_t remain unchanged, equation (8) specifying the optimal level for h_t is significantly altered to

$$F_k(k_t h_t, \ell_t) = \delta'(h_t).$$

Since the productivity term, ε_t, no longer enters the system of equation (8) and (9) now, the positive effects of a technological shift on h_t, ℓ_t, y_t and productivity, in addition to the procyclical effect on consumption, are all lost. This result obtains since it no longer pays to depreciate "off" old capital through higher levels of capacity utilization.

5. Dynamic Effects of Investment Shocks

Since the shift in the technology factor, ε_t, affecting the productivity of newly produced capital is temporary, the only channel through which persistence can be generated is k_{t+1}. In the standard paradigm, a higher k_{t+1} implies more capital services, which directly tends to prolong the initial effects. In the present model, where the utilization is endogenous, higher capital does not obviously mean higher capital services. Hence, to see whether there are prolonged output effects, it is necessary to calculate how k_{t+1} affects the decisions at t+1, and in particular capacity utilization.

From the optimality conditions (8) and (9) corresponding to period t+1, it follows that

$$(15)\quad \frac{dh_{t+1}}{dk_{t+1}} = \{-F_{kk}(t+1)h_{t+1}[F_{\ell\ell}(t+1)-G''(t+1)]+F_{k\ell}(t+1)^{2}h_{t+1}\}/\Omega(t+1) < 0,$$

and

$$(16)\quad \frac{d\ell_{t+1}}{dk_{t+1}} = [\delta''(t+1)F_{k\ell}(t+1)h_{t+1}]/(1+\epsilon_{t+1})\Omega(t+1) > 0,$$

with the signs of the above expressions following from the facts that Ω, $F_{k\ell} > 0$ and F is concave. The optimal rate of utilization declines since the higher k_{t+1} reduces the marginal productivity of capital services. However, this is only a partial offsetting. The optimal flow of capital services $k_{t+1}h_{t+1}$, increases:

$$(17)\quad \frac{d(k_{t+1}h_{t+1})}{dk_{t+1}} = h_{t+1} + k_{t+1}\frac{dh_{t+1}}{dk_{t+1}}$$

$$= -h_{t+1}\delta''(t+1)[F_{\ell\ell}(t+1) - G''(t+1)]/[(1+\epsilon_{t+1})\Omega(t+1)] > 0.$$

From (16) and (17) it follows that $dy_{t+1}/dk_{t+1} > 0$. The effects will persist also beyond t+1 because from equation (7) and the first-order conditions (8) and (9) updated one period it transpires that[3]

$$(18)\quad \frac{dk_{t+2}}{dk_{t+1}} = \frac{U''(t+1)\{(1+\epsilon_{t+1})F_k(t+1)h_{t+1}+[1-\delta(t+1)]\}}{[U''(t+1) + \beta(1+\epsilon_{t+1})^{2}V^{t+2}_{kk}(t+2)]} > 0.$$

The sign of this effect is based again on $V_{kk} < 0$. (See appendix A.) Correspondingly, output will be higher also in t+2,...,T.

6. Investment Subsidies

The present model can also be used to gain some insight about the impact of investment subsidies (or tax credits) on the economy's general equilibrium. Consider now a government in the economy considered above which imposes a lump sum tax η_t to finance a gross-investment subsidy at the rate θ_t. The private sector budget constraint is now

$$c_t = F(k_t h_t, \ell_t) - i_t(1-\theta_t) - \eta_t .$$

Substituting i_t from the capital evolution equation (2) (with ε_t set to zero) yields

$$(19) \quad c_t = F(k_t h_t, \ell_t) - k_{t+1}(1-\theta_t) + k_t[1-\delta(h_t)](1-\theta_t) - \eta_t .$$

Assuming as before that the shift is temporary, the private sector maximizes an expression as in (5)--with $\underline{\theta}_t \equiv (\theta_t, \theta_{t+1}, \ldots, \theta_T)$ replacing $\underline{\varepsilon}_t$--subject to (19). The form of the optimality conditions associated with k_{t+1}, h_t and ℓ_t remain exactly the same as in (7), (8) and (9) with $(1-\theta_t)$ substituting for $1/(1+\varepsilon_t)$.[4]

However, one cannot draw the conclusion that a shift in the subsidy rate θ_t generates, in general, the same type of effects as the technology shock ε_t. This is so because the government's budget constraint implies that $\eta_t = \theta_t\{k_{t+1} - [1-\delta(h_t)]k_t\}$ and hence equation (19) will not represent the true law of motion facing society. The economy's law of motion is obtained by substituting the government's budget constraint into equation (19) to obtain

$$(20) \quad c_t = F(k_t h_t, \ell_t) - k_{t+1} + [1 - \delta(h_t)]k_t .$$

Thus, the economy's general equilibrium will not in general be characterized as a simple solution to the representative agent's concave programming problem--that is, by the analogues to (7), (8), (9) in the current situation plus the private sector's law of motion (19)--since the constraint facing the individual is not the same as that facing society. However, in the situation when the initial vector of investment subsidies is zero, or formally where $\underline{\theta}_t = (\theta_t, \theta_{t+1}, \ldots, \theta_T) = \underline{0}$, equations (19) and (20) will coincide. Hence, the previous line of analysis remains valid for investigating the impact of shifts in investment subsidies around the point $\underline{\theta}_t = \underline{0}$.

For the variables h_t, ℓ_t, y_t and labor productivity both systems react identically to shifts in ϵ_t and θ_t since h_t and ℓ_t are determined solely by equations (8) and (9) which are the same in both cases--except with $(1-\theta_t)$ replacing $1/(1+\epsilon_t)$ in the latter situation. Starting from the initial positions of $\underline{\epsilon}_t = 0$ and $\underline{\theta}_t = \underline{0}$, the effects on c_t and i_t differ, however. In the investment subsidy case there is no longer an income effect (the terms involving i) on consumption and investment. This is obviously so since the imposition of a subsidy does not, in and of itself, increase society's future productivity. Hence, the comparative statics results here differ from the previous ones only in predicting a more procyclical investment and a less procyclical consumption. The persistance effects remain the same.

7. Concluding Remarks

This paper addressed the macroeconomic effects of direct shocks to investment in a framework where the investment decision affects the optimal utilization rate of the already installed capital stock. The shocks considered take the form of technological changes that affect the productivity of new capital goods, or changes in taxes relevant for investment decisions, such as investment subsidies.

The results in the paper imply that a variable capacity utilization rate may be important for the understanding of business cycles. It provides a channel by which investment shocks generate a higher utilization rate of the existing capital stock and hence higher labor demand.[5] This mechanism stands in contrast to the intertemporal substitution effect which works on labor supply.

Because of the variable capacity utilization the model predicts the Keynesian type of result of less than "full capacity equilibrium". Unlike in the Keynesian model, however, the labor market always clears and partial capacity utilization is socially optimal.

Appendix A

It will be demonstrated here that the value function, V(•), is strictly concave and continuously twice differentiable in the capital stock k. The proof proceeds by induction in a manner similar in part to that outlined in Lucas, Prescott, and Stokey. Suppose that the period-t+1 value function, $V^{t+1}(k_{t+1},\underline{\varepsilon}_{t+1})$, is strictly concave and continuously twice differentiable in k_{t+1}. The period-t value function is defined by the following equation

$$\text{(A1)}\quad V^{t}(k_t;\underline{\varepsilon}_t) = \max_{k_{t+1},h_t,\ell_t} [U(z(k_{t+1},h_t,\ell_t,k_t,\underline{\varepsilon}_t))+\beta V^{t+1}(k_{t+1};\underline{\varepsilon}_{t+1})]$$

[recall $\underline{\varepsilon}_t = (\varepsilon_t,\varepsilon_{t+1},\ldots,\varepsilon_T)$]

where $z(k_{t+1},h_t,\ell_t,k_t;\varepsilon_t) \equiv F(k_t h_t,\ell_t) - [k_{t+1} - k_t(1-\delta(h_t))]/(1+\varepsilon_t) - G(\ell_t)$. Note that U(z(•)) is a strictly concave function in k_{t+1}, h_t, ℓ_t and k_t.

Next let k'_{t+1}, h'_t, and ℓ'_t be the optimal solutions for the three decisions corresponding to the initial capital stock k'_t and likewise let k''_{t+1}, h''_t, and ℓ''_t be the solutions associated with the capital stock k''_t and define the following convex combinations: $\tilde{k}_t=\phi k'_t+(1-\phi)k''_t$, $\tilde{k}_{t+1}=\phi k'_{t+1}+(1-\phi)k''_{t+1}$, $\tilde{h}_t=\phi h'_t+(1-\phi)h''_t$, and $\tilde{\ell}_t=\phi\ell'_t+(1-\phi)\ell''_t$ for $0 < \phi < 1$. It is easy to see that the chain of inequalities given below holds:

$$V^{t}(\tilde{k}_t;\underline{\varepsilon}_t) \geq U(z(\tilde{k}_{t+1},\tilde{h}_t,\tilde{\ell}_t,\tilde{k}_t;\varepsilon_t)) + \beta V^{t+1}(\tilde{k}_{t+1};\underline{\varepsilon}_{t+1})$$

[by the definition of $V^t(t)$]

$$> \phi[U(z(k'_{t+1},h'_t,\ell'_t,k'_t;\varepsilon_t)) + \beta V^{t+1}(k'_{t+1};\underline{\varepsilon}_{t+1})]$$

$$+ (1-\phi)[U(z(k''_{t+1},h''_t,\ell''_t,k''_t;\varepsilon_t)] + \beta V^{t+1}(k''_{t+1};\underline{\varepsilon}_{t+1})]$$

[by the concavity of $U(t)$ and $V^{t+1}(t+1)$]

$$= \phi V^t(k'_t;\underline{\varepsilon}_t) + (1-\phi)V^t(k''_t;\underline{\varepsilon}_t)$$

[by the definition of $V^t(t)$]

Thus, the period-t value function, $V^t(k_t;\underline{\varepsilon}_t)$ is strictly concave in k_t.

Next, it is easy to establish that $V^t(k_t;\underline{\varepsilon}_t)$ is continuously twice differentiable as well. This occurs since an application of the Implicit Function Theorem on the set of first-order conditions (7), (8), and (9) for period t characterizing the optimal solutions for k_{t+1}, h_t, and ℓ_t--after substituting out for c_t in (7) using (6)--guarantees that the obtained policy functions $k_{t+1} = k^t(k_t,\underline{\varepsilon}_t)$, $h_t = h^t(k_t,\underline{\varepsilon}_t)$, and $\ell_t = \ell^t(k_t,\underline{\varepsilon}_t)$ are continuously differentiable in k_t given that $V_k^{t+1}(k_{t+1};\underline{\varepsilon}_{t+1})$ is continuously differentiable in k_{t+1}. Utilizing this fact, simple differentiation of both sides of (A1) while applying the envelope theorem reveals that $V_k^t(k_t;\underline{\varepsilon}_t)$ is continuously differentiable in k_t since

$$V_k^t(k_t;\underline{\varepsilon}_t) = \beta U'(z(k_{t+1},h_t,\ell_t,k_t;\varepsilon_t))[F_k(k_t,\ell_t)h_t + (1-\delta(h_t))/(1+\varepsilon_t)],$$

with the right-hand side of this expression being continuously differentiable in k_t. Thus, $V_{kk}^t(k_t;\underline{\varepsilon}_t)$ is continuous in k_t.

All that remains to be established is that the terminal value function, $V^T(k_T)$, is strictly concave and continuously twice differentiable. This is easy since $V^T(T)$ is defined simply by

$$V^T(k_T) = \max_{h_T, \ell_T} U(F(k_T h_T, \ell_T) - k_T(1-\delta(h_T)) - G(\ell_T))$$

By following the above procedure it is easy to show that $V^T(k_T)$ is both strictly concave and continuously twice differentiable.

Footnotes

[1] The functions $F(\cdot)$, $\delta(\cdot)$, $U(\cdot)$ and $G(\cdot)$ are all assumed to be continuously twice differentiable.

[2] An alternative interpretation is that the marginal product *currently* produced capital has risen. This can be seen by substituting the capital evolution equation (2) into the period-t+1 production function to get $y_{t+1} = F(i_t(1+\epsilon_t)h_{t+1} + k_t[1-\delta(h_t)]h_{t+1}, \ell_{t+1})$. Thus, ϵ_t plays the role of a Harrod technological shock on the newly produced component of the period-t+1 capital stock, or i_t.

[3] In general it does not happen that $dk_{t+2}/dk_{t+1} < 1$, as perhaps might be expected. There is no real need for concern, however, for two reasons. First, utilizing (6), (16), and (17), it is easy to show that $dc_{t+j}/dk_{t+j} > 0$ for $j \geq 1$. Hence, consumption increases in all periods dated t+1 or later in response to a rise in the period-t+1 capital stock, so that the consumption-smoothing postulate is not violated even in the circumstances where $dk_{t+j+1}/dk_{t+j} > 1$. Second, around a steady-state of an infinite horizon version of the model $dk_{t+j+1}/dk_{t+j} < 1$. Furthermore, it can be shown that the steady-state of such a model is unique and stable. This second line of reasoning assumes, though, that the second derivative of the value function, V_{kk}, is continuous for the infinite horizon problem. This is something which, unlike the finite horizon problem adopted in the text, has proven elusive to establish to date, given the usual assumptions on tastes and technology--see Lucas, Prescott and Stokey.

[4] Strictly speaking the vector $\underline{\eta}_t \equiv (\eta_t, \eta_{t+1}, \ldots, \eta_T)$ should also be entered in the period-t value function as a separate argument. Thus, equation (7) will now contain the term $V_k^{t+1}\ (k_{t+1};\ \underline{\theta}_{t+1},\ \underline{\eta}_{t+1})$.

[5] Another mechanism by which investment decisions can positively affect labor demand is modelled in Scarth (1984), where adjustment costs of new investment involve labor.

References

Barro, R.J. and R.G. King, "Time-Separable Preferences and Intertemporal Substitution Models of the Business Cycle," The Quarterly Journal of Economics, November 1984.

Hercowitz, Z., "The Real Interest Rate and Aggregate Supply," Journal of Monetary Economics, September 1986.

Keynes, J.M., The General Theory of Employment, Interest and Money, McMillan (London) 1936.

Kydland, F.E. and E.C. Prescott, "Time-to-Build and Aggregate Fluctuations," Econometrica, November 1982.

Long, J.B. and C.I. Plosser, "Real Business Cycles," Journal of Political Economy, February 1983.

Lucas, R.E., E.C. Prescott, and N.L. Stokey, "Dynamic Programming Analysis," Unpublished, Department of Economics, University of Minnesota, Undated.

Merrick, J.J., "The Anticipated Real Interest Rate, Capital Utilization and the Cyclical Pattern of Real Wages," Journal of Monetary Economics, January 1984.

Scarth, W.M., "Adjustment Costs and Aggregate Supply Theory," Canadian Journal of Economics, November 1984.

Taubman, P. and M. Wilkinson, "User Cost, Output and Unexpected Price Change," in Edmund Phelps, ed., Microeconomic Foundations of Employment and Inflation Theory, New York, 1970.

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