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Uncertain Lifetime and Saving

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Abstract: This paper provides an explanation for the finding that a significant fraction of the elderly hold little or no savings. Instead of invoking irrational or non-optimal behavior to resolve the puzzle, it is shown that widespread low savings are consistent with a rational life-cycle model of saving. When there is uncertainty about the length of life, it is optimal for some elderly to save little and exhaust their wealth early. The characteristics of these elderly are derived. The simulation results support that uncertain lifetime can account for widespread low savings and early wealth depletion.

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I. Introduction

A common and salient finding in national surveys of the elderly is that a significant fraction of the aged have little or no liquid wealth. Although home ownership is widespread, older surveys as well as recent ones have consistently shown that a sizeable minority, generally at least 20 percent, of the elderly report little income from, and very low holdings of, liquid assets.[1] Even when home equity is included, the 1991 Survey of Income and Program Participation (SIPP) shows that 22.95 percent of people aged 65 and above had net worth less than $25,000 (Eller, 1994, Table 4). Among these elderly people, 23 percent had zero or negative net worth and 68 percent had net worth below $10,000.

Low savings may result in early wealth depletion. In an econometric analysis of the Retirement History Survey, Hurd (1989) finds a high rate of wealth decumulation among the retired elderly. The parameter estimates imply that most people will completely exhaust their savings in only a few years after retirement and will live entirely on their annuity income thereafter. For instance, one set of estimates shows that the mean and median time (after age 65) to depletion of wealth is 7 years and 5.3 years, respectively. The distribution of the wealth depletion time is very skewed: 10 percent of his sample depletes its wealth in less than 1.3 years and 90 percent in less than 14.3 years. Hurd's (1989) finding of early wealth depletion is unique in the literature. He claims that low initial wealth is the cause of the early wealth depletion in his sample.

[1] An excellent summary of the findings from various older surveys covering the period 1941-1963 can be found in Diamond (1977). For evidence from more recent surveys, see, e.g., Kotlikoff, Spivak, and Summers (1982), Hurd and Shoven (1985), and Poterba, Venti, and Wise (1994).

To reconcile widespread low savings with the standard life-cycle theory of saving, one has to assume that those elderly saved little either because their lifetime earnings were low or because they had very high discount rates. Diamond (1977) and Diamond and Hausman (1984) reject the low income argument because they find that the phenomenon of low savings is not confined to the bottom of the income distribution. The use of very high discount rates to explain low savings is not appealing because it is not informative (Thaler 1994). Furthermore, it suggests that many people are extremely myopic, which may not be really consistent with the usual notion of rational behavior. Hence, the standard life-cycle theory has not yet been able to provide a satisfactory explanation for the puzzling phenomenon.

Because of these problems, widespread low savings have often been regarded as one of the evidence against the rational life-cycle theory of saving. For example, King and Dicks-Mireaux (1982) believe that the life-cycle model is not appropriate for people with low or negative net worth. They speculate that this group of people do not plan for the future, are backward-looking rather than forward-looking, or are simply unable to manage their own financial affairs. Similarly, Bernheim and Scholz (1993) and Thaler (1994) express that individuals undersave and do not save optimally because the life-cycle saving decision is too complicated for them to solve. The extraordinary complexity of the optimization problem, the paucity of learning opportunities, and the lack of simple rules of thumb prevent individuals from behaving optimally.

The apparent inability of the life-cycle model to account for widespread low savings has affected both empirical and theoretical work. For instance, Diamond and Hausman (1984) exclude all those individuals with low wealth from the sample in their empirical analysis because they believe that the life-cycle theory is not applicable to this part of the population. Shefrin and Thaler (1988) develop a quasi-rational theory of saving that emphasizes bounded rationality and inadequate will power.

In this paper I attempt to explain widespread low savings without invoking quasi-rational, irrational, or non-optimal behavior. By incorporating uncertain lifetime and mortality risk into the traditional life-cycle model, I show that it is optimal for rational individuals to save little and entirely decumulate their assets well before the maximum lifetime. When there is uncertainty about the length of life, some individuals may choose to consume most of their income and save little because of the possibility that life may be shorter than expected. Unlike the traditional life-cycle model in which wealth is exhausted only at the very last date, I show that wealth depletion must occur before that date in a model of uncertain lifetime. My model therefore explains both widespread low savings among the elderly and Hurd's (1989) unique finding of early wealth depletion. The elderly do not undersave; they just behave optimally.

It has long been recognized that uncertainty about the length of life can reduce savings.[2] A novel contribution of this paper is to explain why and how uncertain lifetime reduces savings via wealth depletion. By showing that wealth must be depleted at some time before the maximum lifetime, I can then use the wealth depletion time to explain why some individuals hold

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[2] See, e.g., Rae (1834) and Fisher (1930). The famous passage in Rae is worth quoting: "We know not the period when death may come upon us, but we know that it may come in a few days, and must come in a few years. Why then be providing goods that cannot be enjoyed until times, which, though not very remote, may never come to us, or until times still more remote, and which we are convinced we shall never see?" (Rae, 1834, p.54)

little or no wealth. The existence of a wealth depletion time means that savings are low just before wealth is exhausted and there are no savings thereafter. The earlier the wealth is exhausted, the longer will be the period of low and zero savings. I provide some simulation results to confirm that wealth depletion can occur very early. In some cases an individual may run down his assets in just a few years after retirement.

Since the wealth depletion time is the key to explaining low and zero savings, I also investigate how it is determined. I provide some conditions to establish the existence and uniqueness of the wealth depletion time and report some comparative static results. I find that low initial wealth, low interest rate, low risk aversion, high discount factor, high post-retirement income, or poor health will give rise to early wealth depletion.

While this paper shows that uncertain lifetime can be employed successfully to explain widespread low savings, it seems to contradict Davies' (1981) renowned result that uncertain lifetime can account for the observed slow dissaving among the elderly.[3] Without relying on a bequest motive, Davies' simulations show that precautionary saving for uncertain lifetime reduces consumption (relative to what it would be if lifetime was certain), and the magnitude is large enough to account for the observed slow decumulation. In other words, Davies' result suggests that uncertain lifetime reduces dissaving because life may be longer than expected, which contradicts the argument of this paper that uncertain lifetime increases dissaving because life may be shorter than expected. To resolve this inconsistency, I conduct a simulation study employing Davies' setup and

[3] There are some discussions on whether the observed slow decumulation among the elderly is genuine; see Modigliani (1986) and Hurd (1992).

incorporating a wealth depletion time into the model (which is absent in Davies' computational procedure). In contrast to Davies' results, I find that in almost all cases, consumption under uncertain lifetime is higher than consumption under certain lifetime, which means that uncertain lifetime hastens dissaving. Thus, my simulation results do not support Davies' claim that uncertain lifetime accounts for the low rate of dissaving among the retired. To the contrary, uncertain lifetime accounts for low savings and fast decumulation. To explain the observed slow dissaving (assuming that it is a genuine phenomenon), one may need to introduce a bequest motive or other motives for precautionary saving.

Finally, I discuss why only a significant fraction, but not all, of the elderly hold little or no wealth. I show that individuals with a bequest motive or an actuarially fair insurance do not have to deplete their wealth before the maximum lifetime, thus they can keep their savings for a long period of time even after retirement. The analysis therefore provides a list of probable characteristics of those elderly who have little or no wealth. They are most likely to be individuals who have no bequest motive, no life insurance, low initial wealth, low interest rates, low risk aversion, high discount rates, high post-retirement income, and poor health.

The paper is organized as follows. Section II presents the model and proves that wealth must be exhausted before the maximum lifetime. Conditions for the existence and uniqueness of the wealth depletion time as well as comparative statics are derived in Section III. Section IV provides some simulations to substantiate the theory. Section V compares the findings with Davies' results. Section VI explores why only a significant fraction of the elderly hold little or no wealth. Section VII concludes the paper.

II. Uncertain Lifetime and Wealth Depletion

Consider an individual whose lifetime T is a random variable distributed over $[0,\bar{T}]$, $0 < \bar{T} < \infty$. At each time $t \in [0,\bar{T}]$, the individual consumes $c(t)$ and saves $S(t)$. He earns $E(t)$ for $t < 65$, retires at age 65, and receives a constant and positive Social Security benefit M for $t \geq 65$. His income stream $m(t)$ can therefore be expressed as

$$m(t) = (1-\theta)E(t)I(t) + M[1 - I(t)], \tag{1}$$

$t \in [0,\bar{T}]$, where θ denotes the rate of Social Security tax on earnings and $I(t)$ is an indicator function that equals 1 if $t < 65$ and 0 otherwise.[4]

Let $\Omega(t)$ and $\pi_t(t)$ denote the individual's survival probability and hazard rate of death (mortality hazard) at time t, respectively, then $\Omega(t) = \exp[-\int_0^t \pi_x(x)dx]$, $t \in [0,\bar{T}]$. Following Hubbard and Judd (1987), the Social Security benefit is assumed to be actuarially fair in the sense that

$$M\int_{65}^{\bar{T}} \Omega(t)e^{-jt}dt = \theta\int_0^{65} \Omega(t)E(t)e^{-jt}dt, \tag{2}$$

where j denotes the interest rate (a constant). Assume that there is no bequest motive and life insurance is not available. The individual has utility function $g(c)$ and discounts the future at a fixed rate α. Following Yaari (1965), I assume that $\Omega(0) = 1$, $\Omega(\bar{T}) = 0$, $\pi_t(t) > 0$ for $t \in (0,\bar{T})$, $\lim_{t\to\bar{T}} \pi_t(t) = \infty$, $g'(c) > 0$, and $g''(c) < 0$. The individual's decision problem is to

$$\max_{c(.)} \int_0^{\bar{T}} \Omega(t)e^{-\alpha t}g[c(t)]dt \tag{3}$$

[4] One can easily incorporate pensions into the model without affecting the essential results. The income stream will become $m(t) = (1-\theta-\delta)E(t)I(t) + (M+P)[1 - I(t)]$, where δ is the percentage of payroll deduction, $\delta E(t)$ is the individual's contribution to private pension funds, and P is the annuity income received from the funds after retirement.

subject to

$$c(t) \geq 0, \tag{4}$$

$$S(t) \geq 0, \tag{5}$$

$$S'(t) = jS(t) + m(t) - c(t), \tag{6}$$

$$S(\overline{T}) = 0. \tag{7}$$

The individual faces two nonnegativity constraints: (4) and (5). The inequality in (5) has been known as the wealth, liquidity, or borrowing constraint.[5] The Hamiltonian for this problem is given by

$$H^A = \Omega(t)e^{-\alpha t}g[c(t)] + \lambda(t)[jS(t)+m(t)-c(t)] + \eta(t)c(t) + \mu(t)S(t), \tag{8}$$

where $\lambda(t)$, $\eta(t)$, and $\mu(t)$ are the multipliers for (6), (4), and (5), respectively. Let $c^*(t)$ denote the optimal solution, the necessary conditions are given by:

$$\partial H^A/\partial c(t) = \Omega(t)e^{-\alpha t}g'[c^*(t)] - \lambda(t) + \eta(t) = 0, \tag{9}$$

$$\partial H^A/\partial S(t) = -\lambda'(t) = j\lambda(t) + \mu(t), \tag{10}$$

$$\eta(t) \geq 0,\ \eta(t)c^*(t) = 0, \tag{11}$$

$$\mu(t) \geq 0,\ \mu(t)S(t) = 0. \tag{12}$$

Solving (10),

$$\lambda(t) = \lambda(z)e^{-j(t-z)} - \int_z^t e^{-j(t-w)}\mu(w)dw, \tag{13}$$

for any z and t such that $0 \leq z < t \leq \overline{T}$. Substituting (13) into (9),

$$\Omega(t)e^{-\alpha t}g'[c^*(t)] + \eta(t) = \lambda(z)e^{-j(t-z)} - \int_z^t e^{-j(t-w)}\mu(w)dw, \tag{14}$$

where $\lambda(z) = \Omega(z)e^{-\alpha z}g'[c^*(z)] + \eta(z)$. If $c^*(t) > 0$ and $S(t) > 0$, then $c^*(t)$ is continuously differentiable at t and (14) implies that

$$c^{*\prime}(t) = [\pi_t(t) + \alpha - j]g'[c^*(t)]/g''[c^*(t)]. \tag{15}$$

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[5] Debts in the form of collateralized loans (such as home mortgages) are allowed. The borrowing constraint only requires that net savings be nonnegative, i.e., debts cannot exceed the total value of tradable assets. Even if negative net savings are allowed, the essential results of the paper will still hold as long as there is a fixed lower bound on savings, i.e., $S(t) \geq D$, where D is a fixed nonpositive finite number.

PROPOSITION 1: There exists a $t^* \in [0,\overline{T})$ such that, for all $t \in [t^*,\overline{T}]$, $S(t) = 0$ and $c^*(t) = m(t)$.

PROOF: Choose a time $k \in [65,\overline{T})$ such that $\pi_t(t) + \alpha - j > 0$ for all $t > k$, then $c^{*\prime}(t) < 0$ and $S''(t) = jS'(t) - c^{*\prime}(t)$ for $t \in (k,\overline{T}]$. This implies that $S'(t) \leq 0$ for $t \in (k,\overline{T}]$ because, if $S'(t) > 0$ for some $t \in (k,\overline{T}]$, then $S''(t) > 0$ so that $S(\overline{T})$ can never reach zero, contradicting (7). Now suppose the proposition is not true, then there must exist an $h \geq k$ such that $S(t) > 0$ for all $t \in [h,\overline{T})$, thus $\mu(t) = 0$ for $t \in [h,\overline{T})$ and $\int_h^{\overline{T}} e^{-j(\overline{T}-w)}\mu(w)dw = 0$. Since $m(\overline{T}) = M > 0$, (5), (6), and (7) imply that $c^*(\overline{T}) > 0$, hence $\eta(\overline{T}) = 0$ and $g'[c^*(\overline{T})] < \infty$. It follows from $\Omega(\overline{T}) = 0$ and (14) that $\int_h^{\overline{T}} e^{-j(\overline{T}-w)}\mu(w)dw = \lambda(h)e^{-j(\overline{T}-h)} > 0$, contradiction. Hence, there exists a $t^* \in [0,\overline{T})$ such that $S(t) = 0$ for all $t \in [t^*,\overline{T}]$, and $c^*(t) = m(t)$ follows.

The proposition states that there must be a wealth depletion time t^* before $\overline{T}$. The individual will completely deplete his wealth at t^* and will not hold any savings thereafter. During this period of zero savings, consumption will be equal to income at each instant.[6]

It is possible to offer an intuitive interpretation for Proposition 1. If (4) and (5) are never binding, (9) gives the usual optimality condition that $\Omega(t)e^{-\alpha t}g'[c^*(t)] = \lambda(t) = \lambda(0)e^{-jt}$, i.e., the expected discounted marginal utility of consumption, $\Omega(t)e^{-\alpha t}g'[c^*(t)]$, is equal to the marginal value of savings $\lambda(t)$. Since $\Omega(\overline{T}) = 0$, $\Omega(t)e^{-\alpha t}g'[c^*(t)]$ will eventually decline to zero as t approaches $\overline{T}$, while $\lambda(t)$ will fall to its minimum $\lambda(0)e^{-j\overline{T}} > 0$. Hence, there exists a time τ such that $\Omega(t)e^{-\alpha t}g'[c^*(t)] < \lambda(0)e^{-jt}$, $t \in [\tau,\overline{T}]$. The breakdown of the equality between $\Omega(t)e^{-\alpha t}g'[c^*(t)]$ and $\lambda(0)e^{-jt}$ means that the individual should try to reduce consumption so as to restore the equality between the two margins. Decreases in

[6] The assumptions that α, j, and M are constant can all be relaxed, however, the proof is considerably more complicated, see Leung (1993).

consumption, however, cannot raise the expected discounted marginal utility of consumption indefinitely because the marginal utility of consumption is bounded above. Consumption should therefore stay as low as possible, but it cannot be lower than income at the end of life, otherwise savings will be strictly positive and the terminal condition $S(\overline{T}) = 0$ will be violated. Hence, beginning at some time t^*, consumption will be equal to income because it is the lowest feasible level that satisfies the terminal condition, and savings will be zero thereafter. The equality between the two margins can then be restored because $\mu(w) > 0$ for some w (so that $S(w) = 0$) allows $\lambda(t)$ to be lowered from $\lambda(0)e^{-jt}$ to $\lambda(0)e^{-jt} - \int_0^t e^{-j(t-w)}\mu(w)dw$, hence $\lambda(t)$ can decline to zero as t approaches $\overline{T}$.

Both the proof and the interpretation show that the key factor that causes wealth depletion is uncertain lifetime (the presence of $\Omega(t)$ in (9) and the fact that $\Omega(\overline{T}) = 0$). As one ages, the survival probability decreases. Thus, one should expect that uncertain lifetime has a stronger impact on the saving decision of the elderly. If some elderly believe that they have a low survival probability, then they will deplete their wealth before the maximum lifetime because they may die soon and the assets have no value to them after they die.

Some individuals believe that they have a low survival probability because of poor health. The effect of health on the savings of the elderly should receive more emphasis. There is an empirical literature which shows that poor health is the main reason for early retirement (see Clark, Kreps, and Spengler (1978) for a survey). If poor health has a strong impact on the decision to retire, it should also have some impact on the saving decision. Unfortunately, there is surprisingly little work relating the wealth and health of the elderly. The evidence to date is consistent with the theory, e.g., Diamond and Hausman (1984, Table 10) find that poor health has a significantly negative impact on savings.

III. Determination of t^* and Comparative Statics

Although uncertain lifetime drives wealth depletion, there are many factors that affect when it will take place. To study how t^* is determined, assume for simplicity that $\eta(t) = 0$ and $\mu(t) = 0$ for $t \in [0,t^*]$. As $c^*(t^*) = m(t^*)$, (14) implies that $\lambda(0) = \Omega(t^*)e^{(j-\alpha)t^*}g'[m(t^*)]$ and $g'[c^*(t)] = g'[m(t^*)]\Omega(t^*)e^{(j-\alpha)t^*}/[\Omega(t)e^{(j-\alpha)t}]$, $t \in [0,t^*]$. Since $g'' < 0$, the inverse $(g')^{-1}$ exists. Thus,

$$c^*(t) = (g')^{-1}\left[\frac{g'[m(t^*)]\Omega(t^*)e^{(j-\alpha)t^*}}{\Omega(t)e^{(j-\alpha)t}}\right], \qquad t \in [0,t^*]. \tag{16}$$

As $S(t^*) = 0$, $S(0) = \int_0^{t^*} e^{-jt}[c^*(t)-m(t)]dt$. Substituting (16) into this budget constraint yields

$$S(0) = \int_0^{t^*} e^{-jt}\left\{(g')^{-1}\left[\frac{g'[m(t^*)]\Omega(t^*)e^{(j-\alpha)t^*}}{\Omega(t)e^{(j-\alpha)t}}\right] - m(t)\right\} dt. \tag{17}$$

Therefore, t^* is solely determined by (17). Clearly, (17) does not furnish an explicit solution for t^*. Given $S(0)$, $\Omega(t)$, α, $g(c)$, j, and $m(t)$, one can use (17) to solve for t^* numerically. The following proposition offers some conditions to guarantee the existence and uniqueness of t^*. Let

$$H(x) = S(0) - \int_0^{x} e^{-jt}\left\{(g')^{-1}\left[\frac{g'[m(x)]\Omega(x)e^{(j-\alpha)x}}{\Omega(t)e^{(j-\alpha)t}}\right] - m(t)\right\} dt,$$

$A(x) = -\pi_x(x) - \alpha + j + g''[m(x)]m'(x)/g'[m(x)]$, and $B(x) = \int_0^x e^{-jt}\{g'[c^*(t)]/g''[c^*(t)]\}dt$, $x \in [0,\bar{T}]$. Using the fact that $\partial(g')^{-1}(y)/\partial x = (\partial y/\partial x)/g''[(g')^{-1}(y)]$, one can check that $H'(x) = -A(x)B(x)$.

PROPOSITION 2: If $S(0) \geq 0$, then there exists a t^*, $0 \leq t^* < \overline{T}$, that satisfies (17) (if $S(0) > 0$, then $0 < t^* < \overline{T}$). In addition, if there is at most one $\epsilon \in [0,\overline{T})$ such that $A(\epsilon) = 0$ (a sufficient condition for this to hold is that $A'(x) < 0$ for all $x \in [0,\overline{T})$), then t^* is unique and $A(t^*) < 0$.

PROOF: If $S(0) \geq 0$, then $H(0) \geq 0$. As $\Omega(\overline{T}) = 0$, $g' > 0$, and $g'' < 0$, hence $(g')^{-1}(0) = \infty$ and $H(\overline{T}) = S(0) - \int_0^{\overline{T}} e^{-jt}\{(g')^{-1}(0) - m(t)\}dt = -\infty$. Thus, there exists a $t^* \in [0,\overline{T})$ such that $H(t^*) = 0$. If $S(0) > 0$, then $t^* \in (0,\overline{T})$. Since $H'(x) = -A(x)B(x)$, $B(x) < 0$, and there is at most one $\epsilon \in [0,\overline{T})$ such that $A(\epsilon) = 0$, $H(x)$ can therefore have at most one critical point in $[0,\overline{T})$. It follows from $H(0) \geq 0$ and $H(\overline{T}) = -\infty$ that the t^* that solves $H(t^*) = 0$ is unique and $A(t^*) < 0$.

One can employ (17) to study how t^* varies with $S(0)$, α, θ, j, and other parameters of the model. In the rest of the paper it will be assumed that $t^* > 0$ and t^* is unique, therefore $A(t^*) < 0$.

PROPOSITION 3:

(i) $\partial t^*/\partial S(0) > 0$, $\partial t^*/\partial \alpha < 0$, and $\partial t^*/\partial j > 0$.

(ii) Suppose $\Omega(t) = \exp[-\int_0^t \phi\pi_x(x)dx]$, where ϕ is a shift parameter and $\phi\pi_x(x)$ is the hazard rate of death at time x, then $\partial t^*/\partial\phi < 0$.

PROOF: (i) Differentiating (17) with respect to $S(0)$, one obtains $1 = A(t^*)B(t^*)[\partial t^*/\partial S(0)]$. Since $A(t^*) < 0$ and $B(t^*) < 0$, $\partial t^*/\partial S(0) > 0$. Similarly, differentiating (17) with respect to α, $A(t^*)B(t^*)[\partial t^*/\partial\alpha] = t^*B(t^*) - \int_0^{t^*} e^{-jt}t\{g'[c^*(t)]/g''[c^*(t)]\}dt < 0$, hence $\partial t^*/\partial\alpha < 0$. Differentiating (17) with respect to j, one obtains $A(t^*)B(t^*)[\partial t^*/\partial j] = \int_0^{t^*} e^{-jt}t[c^*(t)-m(t)]dt - \int_0^{t^*} e^{-jt}(t^*-t)\{g'[c^*(t)]/g''[c^*(t)]\}dt$. Using (6) and integration by parts, one can verify that $\int_0^{t^*} e^{-jt}t[c^*(t)-m(t)]dt = \int_0^{t^*} e^{-jt}S(t)dt > 0$. Thus, $\partial t^*/\partial j > 0$. (ii) Differentiating (17) with respect to ϕ yields $A(t^*)B(t^*)[\partial t^*/\partial\phi] = B(t^*)\int_0^{t^*} \pi_x(x)dx - \int_0^{t^*} e^{-jt}\{g'[c^*(t)]/g''[c^*(t)]\}[\int_0^t \pi_x(x)dx]dt$, hence $\partial t^*/\partial\phi < 0$.

The comparative static results are intuitively reasonable. Increases in initial wealth delay the wealth depletion time. The more impatient the individual, the sooner the wealth will be exhausted. When interest rates are low, the returns to savings are small, so savings are exhausted earlier. An individual with a higher mortality hazard will run down his savings earlier because he may not live long enough to enjoy his assets. Other things being equal, healthier people save more than the less healthy.

If the utility function is of the CRRA (constant relative risk aversion) form, several more results can be obtained. Let $g(c) = c^{1-\gamma}/(1-\gamma)$, then (17) becomes

$$S(0) = \int_0^{t^*} e^{-jt} \left\{ m(t^*) \left[\frac{\Omega(t)e^{(j-\alpha)t}}{\Omega(t^*)e^{(j-\alpha)t^*}} \right]^{1/\gamma} - m(t) \right\} dt. \tag{18}$$

PROPOSITION 4: Suppose $g(c) = c^{1-\gamma}/(1-\gamma)$, $\gamma > 0$ ($g(c) = \log(c)$ if $\gamma = 1$).

(i) If $\Omega(t)e^{(j-\alpha)t} > \Omega(t^*)e^{(j-\alpha)t^*}$ for $t \in [0,t^*)$, then $\partial t^*/\partial\gamma > 0$.

(ii) If $t^* > 65$, then $\partial t^*/\partial\theta < 0$.

(iii) Suppose $m(t) = (1-\theta)\xi E(t)I(t) + M[1-I(t)]$, where ξ is a shift parameter and $\xi E(t)$ is the earnings before age 65, then $\partial t^*/\partial\xi < 0$.

PROOF: For brevity, let $H(t,t^*) = \Omega(t)e^{(j-\alpha)t}/[\Omega(t^*)e^{(j-\alpha)t^*}]$.
(i) Differentiating (18) with respect to γ, one obtains $A(t^*)B(t^*)[\partial t^*/\partial\gamma] = [m(t^*)/\gamma^2]\int_0^{t^*} e^{-jt}H(t,t^*)^{1/\gamma}\log H(t,t^*)dt$. As $H(t,t^*) > 1$, so $\partial t^*/\partial\gamma > 0$.
(ii) Differentiating (18) with respect to θ yields $A(t^*)B(t^*)[\partial t^*/\partial\theta] = \int_0^{t^*} e^{-jt}[\partial m(t)/\partial\theta]dt - [\partial m(t^*)/\partial\theta]\int_0^{t^*} e^{-jt}H(t,t^*)^{1/\gamma}dt$. Differentiating (2) with respect to θ, one obtains $\partial M/\partial\theta = M/\theta$ and therefore $\partial m(t)/\partial\theta = -E(t)I(t) + (M/\theta)[1-I(t)]$. Since $t^* > 65$, $m(t^*) = M$ and $\partial m(t^*)/\partial\theta = M/\theta$. Thus, $A(t^*)B(t^*)[\partial t^*/\partial\theta] = \int_0^{t^*} e^{-jt}[-E(t)I(t)+(M/\theta)(1-I(t))]dt - (M/\theta)\int_0^{t^*} e^{-jt}H(t,t^*)^{1/\gamma}dt$. Since $S(0) = \int_0^{t^*} e^{-jt}[c^*(t)-m(t)]dt = M\int_0^{t^*} e^{-jt}H(t,t^*)^{1/\gamma}dt - \int_0^{t^*} e^{-jt}[(1-\theta)E(t)I(t)+M(1-I(t))]dt$, combining these two equations gives $A(t^*)B(t^*)[\partial t^*/\partial\theta] = -[S(0) + \int_0^{t^*} e^{-jt}E(t)I(t)dt]/\theta < 0$,

hence $\partial t^*/\partial \theta < 0$ follows. (iii) Similar to the proof of (ii), one can verify that $\partial M/\partial \xi = M/\xi$ and $\partial m(t)/\partial \xi = m(t)/\xi$, $t \in [0,t^*]$, hence $A(t^*)B(t^*)[\partial t^*/\partial \theta] = -S(0)/\xi$, implying $\partial t^*/\partial \xi < 0$.

More risk averse individuals hold on to their wealth for a longer period of time. Increases in Social Security tax will raise the post-retirement income, resulting in earlier wealth depletion. Individuals with a higher income profile than others will exhaust their wealth sooner because their post-retirement income (Social Security benefit or private pension) is also higher. Although the last two predictions may appear counter-intuitive, the key factor in understanding them is the post-retirement income. The predictions are actually reasonable because individuals with a higher post-retirement income can afford to deplete their savings earlier.

IV. Evidence

Proposition 1 shows that, when the date of death is uncertain, it is optimal for a rational individual to exhaust all his savings before the maximum lifetime. If the wealth depletion time occurs early enough, then the model can account for the finding of little or no savings among a significant fraction of the elderly. Propositions 3 and 4 reveal that low initial wealth and high discount rates are not the only causes of early wealth depletion. Other things being equal, the higher the mortality hazard and post-retirement income, or the lower the interest rate and risk aversion, the sooner the wealth will be exhausted.

To investigate practically how early wealth is depleted, I conduct a simulation study using the conventional CRRA utility function. Since the focus of the paper is on the elderly, the initial date in the model is taken to be 65. As in (1), income after age 65 is assumed to be a constant M.[7] With these assumptions, one can check that (18) becomes

$$S(65)/M = \int_{65}^{t^*} e^{-j(t-65)} \left\{ \left[\frac{\Omega(t)e^{(j-\alpha)t}}{\Omega(t^*)e^{(j-\alpha)t^*}} \right]^{1/\gamma} - 1 \right\} dt. \qquad (19)$$

Following the convention in the literature, j is set at 0.03. As there is no clear consensus on the values of γ and α, several values of γ and α are examined. Assuming that mortality follows the Gompertz Law (see Leung (1993)), I set $\Omega(t) = \exp[-0.00093\phi(\exp(0.087t) - 1)]$, $\phi = 1, 2$. Two values of ϕ are used to study how t^* varies with healthiness. An individual with $\phi = 2$ has twice the hazard rate of death than an individual with $\phi = 1$.

Given these assumptions, t^* can be solved numerically from (19).[8] Table 1 reports the simulation results for three different values of S(65)/M.[9] Several observations are in order. First, it is clear that wealth depletion can occur very early. In some cases it happens only a few years after retirement. In the extreme case where $\gamma = 0.1$ and $\alpha = 0.1$, even a relatively wealthy elderly with S(65)/M = 10 will deplete his assets at age 68.[10] Second, the magnitude of t^* varies appreciably with the values of γ, α, ϕ,

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[7] The assumption of a constant stream of income after retirement is a good approximation, see, e.g., the evidence in Diamond and Hausman (1984) and Hurd (1989). Furthermore, it simplifies the calculations and minimizes the assumptions one needs to put on the age-income profile after retirement.

[8] The solution t^* is unique because, for all $t \in [65,\overline{T}]$, $A(t) = -0.00008091\exp(0.087t) - \alpha + 0.03$, hence $A'(t) < 0$ and the condition in Proposition 2 (restricted to the interval $[65,\overline{T}]$) is satisfied.

[9] Empirical evidence suggests that the average wealth to income ratio at retirement, S(65)/M, is about 5, see, e.g., King and Dicks-Mireaux (1982), Diamond and Hausman (1984), and Hurd (1989).

[10] Notice that low values of γ and α are not implausible. For example, Lawrance (1991) finds that estimates of γ can be as low as 0.625 and estimates of α higher than 0.1. Some of Hansen and Singleton's (1983) estimates of γ are even lower, e.g., 0.164.

and S(65)/M. Consistent with the predictions of Propositions 3 and 4, t^* increases with γ and S(65)/M, and decreases with α and ϕ. Third, one cannot infer from t^* alone the characteristics of the individual. For example, an individual whose t^* is 70 can be relatively rich ($S(65)/M = 10$, $\gamma = 0.1$, $\alpha = 0.05$) or poor ($S(65)/M = 1$, $\gamma = 1$, $\alpha = 0.1$). Hence, Hurd's (1989) claim that low initial wealth is the cause of all the early wealth depletion in his sample may be overly simplistic. Other factors such as poor health, low risk aversion, or high post-retirement income could also be responsible.

In sum, the simulation results demonstrate that a rational life-cycle model can account for the observed widespread low savings and early wealth depletion. When lifetime is uncertain, it is optimal for some elderly to hold little or no wealth. According to the theory, these elderly will tend to have low risk aversion, high discount rates, poor health, low initial wealth, low interest rates, or high post-retirement income.[11]

V. A Comparison with Davies (1981)

The results in the previous two sections appear to be at odds with Davies (1981). In a widely cited study, Davies proposes that uncertain lifetime can account for the observed slow dissaving among the elderly. By numerically comparing consumption under certain lifetime with consumption

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[11] The analysis so far has treated home equity no different from other financial assets. Although it is commonly believed that elderly households do not appear to decumulate their housing wealth, there is some recent evidence proving otherwise. Sheiner and Weil (1992) find that households do reduce significantly their home equities as they age: about 58 percent of households will not leave behind a house when the last member dies. In addition, most households do not keep the money received from selling the houses. Sheiner and Weil argue strongly that previous studies have underestimated the decumulation rate of housing wealth. Their findings provide further support for the theory proposed in this paper.

under uncertain lifetime, he finds that the latter is significantly lower than the former, which implies that uncertain lifetime reduces the rate of decumulation. In other words, his finding suggests that uncertain lifetime reduces dissaving, which is contrary to the theme of this paper that uncertain lifetime increases dissaving.

A critical problem in Davies' analysis is that he does not take into account the wealth depletion time in his calculations, despite the fact that his model is the same as the one analyzed in Section II. As shown in Proposition 1, there must be a wealth depletion time before the maximum lifetime. To study whether Davies' results are robust to the addition of a wealth depletion time, I conduct a numerical analysis using exactly his setup except with one simplifying assumption. As in the previous section, the focus of this paper is on the retired, therefore the initial date is set at 65 and the post-retirement income M is assumed to be a constant.

Let $\tilde{c}(t)$ denote the optimal consumption in a model of certain lifetime. As M is a constant, one can follow the derivation of eq. (19) in Davies to show that, under certain lifetime,

$$\tilde{c}(t)/M = \frac{S(t)/M + (1 - e^{-j[E(T|t)-t]})/j}{(1 - e^{-(j-\Delta)[E(T|t)-t]})/(j-\Delta)}, \quad 65 \le t \le \bar{T}, \tag{20}$$

where $\Delta = (j-\alpha)/\gamma$ and $E(T|t) = t + \int_t^{\bar{T}} [\Omega(x)/\Omega(t)]dx$ is the expected lifetime (life expectancy) at time t given that the individual is alive at t.[12] As in the previous section, I assume that $\Omega(t) = \exp[-0.00093(\exp(0.087t) - 1)]$, $j = 0.03$, and $\bar{T} = 120$.

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[12] As will be shown below, the advantage in considering the ratio $\tilde{c}(t)/M$ instead of $\tilde{c}(t)$ in (20) is that it is not necessary to assume any specific values for S(t) and M in the computations; only the ratio S(t)/M needs to be specified.

Under uncertain lifetime, (16) implies that the ratio of consumption to income is given by

$$c^*(t)/M = \left[\frac{\Omega(t)e^{(j-\alpha)t}}{\Omega(t^*)e^{(j-\alpha)t^*}} \right]^{1/\gamma}, \quad 65 \le t \le t^*. \tag{21}$$

To compare $c^*(t)$ and $\bar{c}(t)$, one needs to make some assumptions on the evolution of $S(t)$. Following Davies, the ratio $c^*(t)/\bar{c}(t)$ is calculated in the following way. For each set of parameter values specified for $(\gamma, \alpha, S(65)/M)$, I use (21) to calculate $c^*(t)/M$, $t = 65, 66, \ldots, t^*$, where t^* is taken from Table 1. Using these values of $c^*(t)/M$, I obtain the corresponding ratio of savings to income, $S(t)/M$, by means of the formula

$$S(t)/M = [S(65)/M]e^{j(t-65)} + \int_{65}^{t} e^{j(t-z)}[1 - c^*(z)/M]dz.$$

Substituting the value of $S(t)/M$ into (20), I obtain $\bar{c}(t)/M$. Dividing (21) by (20) gives the ratio $c^*(t)/\bar{c}(t)$. This computational procedure is exactly identical to Davies' (1981, p.572) except that it takes into account the wealth depletion time t^* and exploits the formulation of the ratios $c^*(t)/M$ and $\bar{c}(t)/M$. It is easy to see that the procedure only requires an initial value for the ratio $S(65)/M$; neither $S(65)$ nor M needs to be specified.

Table 2 gives some of the simulation results. For brevity, only the case $S(65)/M = 5$ (with all four values of α) as well as the case $S(65)/M = 10$ and $\alpha = 0.03$ are reported.[13] Contrary to Davies' results, Table 2 shows that with few exceptions, $c^*(t)$ is notably higher than $\bar{c}(t)$ for all t before t^*. Thus, the rate of dissaving is generally higher under uncertain lifetime than under certain lifetime. For example, when $\gamma = 4$, $\alpha = 0.01$ (which are closest to Davies' favorable parameter values), and $S(65)/M = 5$, $c^*(t)$ stays

[13] Table 2 reports the ratios up to age t^*-1 for the case $S(65)/M = 5$. For $S(65)/M = 10$ and $\alpha = 0.03$, I only report the ratios (if applicable) up to the t^*-1 of the case $S(65)/M = 5$ and $\alpha = 0.01$.

above $\tilde{c}(t)$ for all $t \in [65,88]$. Even when t^* is as large as 92 (when $\gamma = 4$, $\alpha = 0.03$, $S(65)/M = 10$), $c^*(t)$ is always higher than $\tilde{c}(t)$ and the ratio $c^*(t)/\tilde{c}(t)$ increases monotonically with t. In some extreme cases (e.g., $\gamma = 0.1$, $\alpha = 0.01$, $S(65)/M = 5$), $c^*(t)$ is higher than $\tilde{c}(t)$ by about 400 to 900 percent. The ratios $c^*(t)/\tilde{c}(t)$ in Table 2 are almost always substantially larger than those in Davies. When there is a wealth depletion time, consumption under uncertain lifetime has to be higher than consumption under certain lifetime so that savings can be run down at a faster rate.

Table 2 illustrates that, if one follows Davies' computational procedure, uncertain lifetime cannot account for the observed slow decumulation among the elderly. Instead it shows the contrary that uncertain lifetime raises the rate of dissaving beyond what it would be if lifetime was certain. This result is consistent with the ones in the previous sections: uncertain lifetime reduces the desire to provide for the future. To explain the slow decumulation among the retired (assuming that it is a genuine observation), one may have to turn to other factors that mitigate impatience, e.g., self-control, habit, bequest motive (Fisher, 1930).

VI. Bequest Motive and Life Insurance

The analysis so far has only explained why it is optimal for some elderly to have little or no wealth. To give a complete account for the empirical finding, one has to explain why it happens to only a significant fraction, but not all, of the elderly. An immediate answer, which follows from Propositions 3 and 4, is that most elderly may have high risk aversion, low discount rates, good health, high initial wealth, high interest rates, or low post-retirement income. To add to this list of factors, this section shows that there are other reasons for not depleting one's wealth early.

Suppose the individual has a bequest motive, then the decision problem in Section II becomes

$$\max_{c(.)} \int_0^{\bar{T}} \{\Omega(t)e^{-\alpha t}g[c(t)] + \pi(t)\beta(t)\varphi[S(t)]\}dt \tag{22}$$

subject to (4) and (6),

where $\beta(t)$ is a subjective weighing function for bequests, $\pi(t)$ is the probability density function of lifespan T, and $\varphi[S(t)]$ is the utility derived from leaving a bequest of S(t) (see Yaari (1965)). The Hamiltonian for this optimal control problem is

$$H^B = \Omega(t)e^{-\alpha t}g[c(t)]+\pi(t)\beta(t)\varphi[S(t)]+\lambda(t)[jS(t)+m(t)-c(t)]+\eta(t)c(t). \tag{23}$$

The necessary optimality conditions are

$$\partial H^B/\partial c(t) = \Omega(t)e^{-\alpha t}g'[c^*(t)] - \lambda(t) + \eta(t) = 0, \tag{24}$$

$$\partial H^B/\partial S(t) = -\lambda'(t) = \pi(t)\beta(t)\varphi'[S(t)] + j\lambda(t), \tag{25}$$

$$\eta(t) \geq 0, \ \eta(t)c^*(t) = 0. \tag{26}$$

Solving (25),

$$\lambda(t) = \lambda(\bar{T})e^{j(\bar{T}-t)} + \int_t^{\bar{T}} e^{j(z-t)}\pi(z)\beta(z)\varphi'[S(z)]dz. \tag{27}$$

By inspecting (24)-(27), one can see that it is no longer necessary for savings to be depleted before $\bar{T}$. The following example verifies that this is indeed the case.

EXAMPLE: Let $g(c) = c^{1-\gamma}/(1-\gamma)$ $(\gamma > 0)$, $\varphi(S) = S$, $\alpha = 0$, $\beta(t) = 1$, $j = 0$, $\Omega(t) = 1-t$, and $\bar{T} = 1$. Assume that $S(0) \geq 1$. One can verify that $c^*(t) = [(1-t)/(2-t)]^{1/\gamma}$, $t \in [0,1]$. Since $\int_0^t c^*(w)dw < 1$ for all $t \in [0,1]$, $S(t) = S(0) + \int_0^t m(w)dw - \int_0^t c^*(w)dw > 0$ for all $t \in [0,1]$.

It is easy to check that, whenever utility depends on savings (be it from bequest, status, or other motives), then wealth depletion may never occur. As savings generate utility, it is intuitively reasonable that individuals will maintain positive savings throughout the lifetime.

Now suppose that the individual has no bequest motive but life insurance is available. As in Yaari (1965), the individual seeks to

$$\underset{c(.)}{\text{Max}} \int_0^{\bar{T}} \Omega(t)e^{-\alpha t}g[c(t)]dt$$

subject to $c(t) \geq 0$ as well as the lifetime budget constraint

$$S(0) = \int_0^{\bar{T}} e^{-\int_0^t r(x)dx}[c(t)-m(t)]dt, \tag{28}$$

where $r(t)$ is the interest rate on actuarial notes. Let $\underline{Y} = \int_0^{\bar{T}} \{\exp[-\int_0^t r(x)dx]\}m(t)dt$ and $W(t) = \int_0^t \{\exp[-\int_0^z r(x)dx]\}c(z)dz$, then (28) is equivalent to the set of constraints

$$W'(t) = \{\exp[-\int_0^t r(x)dx]\}c(t), \ W(0) = 0, \text{ and } W(\bar{T}) = S(0) + \underline{Y}. \tag{29}$$

Let $\psi_1(t)$ and $\psi_2(t)$ be the multipliers, then the Hamiltonian is

$$H^c = \Omega(t)e^{-\alpha t}g[c(t)] + \psi_1(t)c(t) + \psi_2(t)\{\exp[-\int_0^t r(x)dx]\}c(t). \tag{30}$$

The necessary conditions are given by

$$\partial H^c/\partial c(t) = \Omega(t)e^{-\alpha t}g'[c(t)] + \psi_1(t) + \psi_2(t)\exp[-\int_0^t r(x)dx] = 0, \tag{31}$$

$$\partial H^c/\partial W(t) = -\psi_2'(t) = 0, \tag{32}$$

$$\psi_1(t) \geq 0, \ \psi_1(t)c(t) = 0. \tag{33}$$

Assume that the interest rate on actuarial notes is fair, i.e.,

$$r(t) = j + \pi_t(t), \tag{34}$$

then $\exp[-\int_0^t r(x)dx] = \exp\{-\int_0^t [\pi_x(x) + j]dx\} = \Omega(t)e^{-jt}$. Thus, (31) becomes

$$\Omega(t)\{e^{-\alpha t}g'[c(t)] + \psi_2(t)e^{-jt}\} + \psi_1(t) = 0. \tag{35}$$

As $\Omega(t) \neq 0$ for $t \in [0,\bar{T})$, (33) and (35) give

$$\alpha(t)g'[c(t)] + \psi_2(t)e^{-jt} = 0, \qquad t \in [0,\bar{T}), \tag{36}$$

whenever $c(t) > 0$. The survival probability $\Omega(t)$ does not appear in (36), so the arguments leading to Proposition 1 do not apply here. Thus, there is no wealth depletion time in this model. The availability of an actuarially fair insurance essentially removes the impact of uncertain lifetime from the individual's decision problem.

Although few people purchase life annuities even though life insurance markets are quite well developed (Friedman and Warshawsky, 1988), most elderly may insure against uncertain lifetime through risk pooling within the family (Kotlikoff and Spivak, 1981). If family risk pooling operates as efficient as an actuarially fair insurance, then individuals do not have to deplete their wealth. Even if family insurance is less than perfect, it may be strong enough to put off the wealth depletion time.

From these results, one can then add bequest motive and life insurance to the list of factors that may explain why a majority of the elderly do not deplete their wealth early.

VII. Conclusion

The finding that a significant fraction of the elderly hold little or no wealth has been cited as evidence against the standard life-cycle theory. Many economists express that the rational life-cycle model is not applicable to this group of people because they undersave and do not save optimally. Theories based on irrational and non-optimal behavior have been proposed to explain the puzzling finding. This paper demonstrates that there is no need to abandon the rational life-cycle model. By incorporating uncertain lifetime into the model, one can account for widespread low savings and early wealth depletion. When there is no bequest motive and life insurance is not available, it is optimal for some elderly to save little and exhaust their wealth early.

The approach of this paper is methodologically novel because it resolves the puzzle of widespread low savings via wealth depletion. It first establishes formally the existence of a wealth depletion time and then employs the comparative static results to infer the probable characteristics of those elderly who save little and exhaust their wealth early.

The theory proposed in this paper is also testable. It predicts that individuals with no bequest motive, no life insurance, low risk aversion, low initial wealth, low interest rates, high discount rates, high post-retirement income, or poor health are more likely to save little and deplete their wealth early. This list of factors provides a useful basis for testing the rational life-cycle theory against the quasi-rational theories proposed by Shefrin and Thaler (1988), Bernheim and Scholz (1993), and Thaler (1994).

TABLE 1

WEALTH DEPLETION TIME t^*

γ	α	t^*					
		$\phi = 1$ S(65)/M			$\phi = 2$ S(65)/M		
		1	5	10	1	5	10
4	0.10	74	82	87	73	80	84
	0.05	77	86	90	75	82	86
	0.03	79	87	92	75	83	86
	0.01	81	89	93	76	84	87
1	0.10	70	74	76	69	73	75
	0.05	71	77	80	70	74	77
	0.03	73	79	82	71	75	78
	0.01	75	81	84	72	77	79
0.5	0.10	68	71	73	68	70	72
	0.05	70	73	76	69	72	74
	0.03	71	75	69	69	73	75
	0.01	73	78	80	70	74	76
0.1	0.10	66	67	68	66	67	68
	0.05	67	69	70	67	68	69
	0.03	68	70	71	67	68	69
	0.01	69	72	73	68	69	70

TABLE 2

RATIO OF CONSUMPTIONS $c^*(t)/\bar{c}(t)$

t	α: 0.1	0.05	0.03	0.01	0.03
	S(65)/M = 5				S(65)/M = 10
	γ = 4				
65	1.059	1.073	1.038	1.039	1.027
66	1.067	1.083	1.044	1.045	1.036
67	1.070	1.089	1.048	1.050	1.042
68	1.073	1.097	1.051	1.055	1.050
69	1.076	1.105	1.056	1.060	1.058
70	1.078	1.113	1.060	1.066	1.068
71	1.080	1.123	1.065	1.073	1.079
72	1.082	1.134	1.070	1.081	1.093
73	1.083	1.145	1.076	1.089	1.108
74	1.083	1.158	1.082	1.099	1.126
75	1.081	1.172	1.088	1.110	1.147
76	1.077	1.187	1.095	1.122	1.172
77	1.070	1.203	1.101	1.135	1.201
78	1.060	1.221	1.108	1.149	1.236
79	1.046	1.239	1.114	1.165	1.278
80	1.026	1.257	1.119	1.182	1.329
81	1.000	1.276	1.122	1.201	1.391
82		1.293	1.124	1.221	1.468
83		1.307	1.122	1.241	1.565
84		1.317	1.115	1.261	1.690
85		1.319	1.102	1.280	1.854
86			1.081	1.296	2.079
87				1.306	2.400
88				1.306	2.890

TABLE 2 (cont'd)

			$\gamma = 1$		
65	1.171	1.281	1.331	1.322	1.346
66	1.176	1.297	1.352	1.339	1.375
67	1.165	1.303	1.365	1.351	1.397
68	1.148	1.308	1.379	1.363	1.421
69	1.123	1.311	1.392	1.375	1.448
70	1.090	1.309	1.404	1.387	1.477
71	1.047	1.304	1.415	1.398	1.508
72	0.993	1.292	1.424	1.409	1.543
73	0.927	1.273	1.429	1.417	1.581
74		1.245	1.429	1.423	1.623
75		1.205	1.422	1.425	1.668
76		1.152	1.406	1.422	1.715
77			1.377	1.411	1.764
78			1.333	1.389	1.812
79				1.355	1.854
80				1.305	1.884
			$\gamma = 0.5$		
65	1.059	1.278	1.478	1.719	1.669
66	1.020	1.264	1.484	1.744	1.716
67	0.953	1.230	1.474	1.761	1.749
68	0.871	1.187	1.458	1.776	1.782
69	0.777	1.132	1.434	1.789	1.817
70	0.673	1.065	1.399	1.798	1.851
71		0.986	1.352	1.801	1.884
72		0.896	1.291	1.797	1.913
73			1.215	1.783	1.935
74			1.124	1.755	1.945
75				1.709	1.935
76				1.641	1.897
77				1.548	1.819
			$\gamma = 0.1$		
65	0.539	1.658	3.009	9.310	3.381
66	0.276	1.531	2.845	8.773	3.342
67		1.259	2.531	8.069	3.126
68		0.934	2.133	7.261	2.797
69			1.687	6.349	2.359
70				5.356	1.849
71				4.322	

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