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1 The origins of the axiomatic theory of resource allocation: cooperative games and fair division

Fair division is as old as mathematics. According to the Roman historian Proclus, the litigious division of land after the yearly flood of the Nile triggered the invention of geometry by the Egyptians, and the necessities of trade and commerce that of arithmetic by the Phoenicians (see Guilbaud, 1952). The modern literature on fair allocation is however very new. Its origin can be traced back to three seminal papers: on the one hand, Nash's 1950 paper on the bargaining problem and Shapley's 1953 paper on coalitional form games; on the other hand, Foley's 1967's essay introducing the no-envy test for the distribution of unproduced resources.

This essay reports (without attempting an exhaustive survey of the literature) on a body of formal research rooted in the models with which these authors were concerned: the model of bargaining, the model of coalitional opportunities, and the model of distribution of unproduced resources; and dealing axiomatically with a variety of other allocation problems. Beyond the analysis of the canonical problem of fair division of a social endowment, this program has indeed been growing at a fast rate, and it now encompasses many new problems (that may involve — for instance — production or bilateral matching), and allows for a rich configuration of property rights (including fractional ownership, in the standard Arrow-Debreu-McKenzie fashion, but also common ownership and combinations thereof). This literature displays strong conceptual and methodological unity, as it rests on a small number of general principles. We propose to refer to it under the name of **Axiomatic analysis of resource allocation.**

After providing the brief historical overview in Section 2 of the developments that have led to this literature, we constrast it in Section 3 with the Social Choice literature, the main bastion of the axiomatic approach, to which most of the volume is devoted. Section 4 reviews the main principles that have been the focus of axiomatic analysis within the context of resource allocation, and Section 5 gives a brief but hopefully suggestive summary of some of the main findings. Section 6 concludes by outlining directions for future research.

2 The origins of the axiomatic analysis of resource allocation: cooperative games and fair division

Both the axiomatic theory of bargaining (Nash, 1950) and the theory of coalitional form games (Shapley, 1953) rely on the welfarist assumption that only utility possibility sets are relevant to the analysis. These sets are assumed to appropriately summarize the opportunities available to groups of agents from exploiting unspecified resources under their control. The objective is to identify systematic methods of selecting, for each configuration of these feasible sets, one (or several) utility profile(s). It uses a number of axioms based on the shape and relative positions of these sets, or involving comparisons of configurations of the sets.

The problem of fair division (Foley, 1967)[1] has for ingredients a vector of commodities (the resources to be divided), and a profile of preference orderings; the objective here is to select particular divisions of these resources, interpreted as "fair" divisions, among the beneficiaries. Despite some similarity in the objectives of the theories of cooperative games and of fair division — for the first one it is utility that has to be divided and for the second it is physical resources — their technical foundations are very different. The cardinal measurement of individual utilities is indeed essential to the axiomatic theories of cooperative games (as formally established by Shapley, 1969), whereas the theory of fair division only relies on preferences, implicitly declaring irrelevant the agents' intensities of preferences, and in particular ruling out interpersonal comparisons of welfare; instead, the axioms it uses involve preference-based comparisons of commodity bundles (as in the no-envy test: agent i does not prefer the bundle consumed by agent j to his own). As a result, most of the solutions encountered in the study of the fair division problem (such as the Walrasian solution with equal incomes, or the egalitarian-equivalent solution) have no counterpart in the theory of cooperative games. Conversely, the solutions developed in that theory have seldom been applied to the problem of fair division; although it is always

[1] We should mention here the mathematical theory initiated by Steinhaus (1948) and further developed by Dubins and Spanier (1961) and many others (see the extensive bibliography in Brams and Taylor (1994)) dealing with the division of an atomless measure space (a piece of land) under a restrictive additivity assumption on individual preferences.

possible to associate a cooperative game with a problem of fair division,[2] this transformation has not been used very often (exceptions are Chun and Thomson (1988) and Moulin (1992a)). Thus the two literatures developed quite independently from one another for a long time (as evidenced by the 1985 survey on the fair division problem by Thomson and Varian.)[3]

Toward the middle of the eighties however, several authors extended the reach of the axiomatic methodology to the study of fair allocation in two important directions. First, while preserving the focus on models with ordinal preferences, they examined a host of microeconomic problems that never before had been examined in that light. Second, they formulated a variety of new conditions, most of which had no counterpart in cooperative game theory.

The new allocation problems include the following: public decision when monetary compensations are possible, provision of a public good (and of an excludable public good), production of a private good under increasing marginal cost and under decreasing marginal cost, one-commodity models of fair division with single-peaked preferences, fair allocation of indivisible goods when monetary compensations are possible, matching models when monetary compensations are, or are not, possible.

The new properties consist of a variety of monotonicity conditions, of lower bounds on individual welfare levels (which, technically if not conceptually, can be loosely seen as generalizing "individual rationality" or "participation" constraints), and of dual notions of upper bounds. In addition, they include incentive properties that had been the object of extensive analysis in the theory of abstract social choice and in the theory of mechanism design, strategyproofness and various notions of implementability. All of these properties, and more, are reviewed in Section 4.

[2]By using a canonical representation of individual preferences. See, however, Roemer (1986a) for a discussion of the informational loss incurred in this transformation.

[3]It is interesting that the no-envy test has also been studied in the context of abstract social choice, Austen-Smith (1979), Suzumura (1981a,b, 1982, 1983, 1994, this volume), Gekker (1991).

3 Constrasting mainstream social choice and axiomatic models of resource allocation

Almost half a century after their initial formulation, the two principal currents of axiomatic analysis in economic theory are still largely independent of each other. They are social choice theory on the one hand and cooperative game theory on the other. To be sure, certain key ideas have played a role in both literatures,[4] but the focus and the messages of the two literatures have been quite different.

In social choice theory, the focus is commonly on obtaining a complete ranking of the set of feasible alternatives as a function of the profile of individual preferences. This ranking is interpreted as a social preference relation. In contrast, the theory of cooperative games is devoted to the construction of allocation rules, that is, ways of selecting for each admissible problem a unique feasible alternative, (or at least a "small" subset of the set of feasible alternatives). This alternative is interpreted as a fair compromise. It is of course much less difficult to design a reasonable allocation rule that a full-fledged procedure of aggregation of preferences.[5] Accordingly, social choice theory contains a large proportion of impossibility theorems (theorems stating the non-existence of aggregation procedures satisfying a list of desirable properties); in contrast, the theory of cooperative games is replete with possibilities and characterizations (theorems stating that a certain allocation rule is the only one to satisfy a certain list of axioms).

Consider now the axiomatic investigations of resource allocation. As their counterparts in the theory of cooperative games, their focus is on the search for allocation rules, no attempt being made to obtain a complete ranking of the entire feasible set. Unlike in that theory, which was developed entirely in an abstract space of utility profiles (as required by the welfarist postulate that treats the actual consumption bundles as ethically irrelevant to the evaluation of profiles of welfares of the members of society), the models of

[4]An early example is the condition of independence of irrelevant alternatives that Nash formulated in this analysis of the bargaining problem, a property that is closely related to the rationalizability of a choice function (see e.g. Theorem 11.8 in Moulin (1988)). A more recent example is Maskin's monotonicity mentioned again at the end of Section 4.

[5]Of course, the choice function of the social choice model determines an allocation rule as well, but the point is that in bargaining/cooperative games there is no attempt at rationalizing this rule by means of a social preference relation.

resource allocation take full account of the microeconomic structure of the problems to be solved. Their description may include data on the nature of the goods — whether they are private goods or public goods for instance — on endowments, whether social or individual, on features of production possibilities such as returns to scale, and on features of individual preferences. This descriptive richness permits a great deal of flexibility at two levels.

First, properties of allocation rules can be formulated directly in terms of the physical attributes of the economy (e.g. the no-envy test is based on comparing individual bundles; the stand-alone test is based on technological opportunities), whose relevance can therefore be directly recognized and discussed. Secondly, the detailed mathematical structure of microeconomic models allows a host of variations on each general principle. For instance, consider the general property of monotonicity of an allocation rule: monotonicities with respect to endowments, technologies or productivities can now be formulated and studied.

Note that social choice theory itself has recently developed in a similar direction, widening its framework by incorporating information about economic environments (as described in Le Breton's essay in this volume), thus gaining the same flexibility (and at the risk of producing specialized results; see below). But as its objective has mainly remained to obtain complete rankings of sets of feasible alternatives, its conclusions have so far remained largely negative (again, see Le Breton).

When moving from the abstract, unstructured, sets of alternatives of social choice theory to finely grained microeconomic models of resource allocation, the risk is that what may appear at first sight like a minor variation in a model (for instance, imposing the requirement of normality of goods) may dramatically affect the results (turning a possibility into an impossibility, or vice versa), the theory producing an amorphous mass of special results that could not be organized in any particular way. Luckily, this has not happened.

4 An overview of the axioms for resource allocation

In reviewing the most often studied properties relevant to our subject, we point out their connections to properties analyzed in abstract social choice theory, when such connection exists. As we will see, many do not have counterparts in abstract social choice.

1. Efficiency: This is the well-known requirement of Pareto-optimality, undoubtedly the single most important axiom throughout normative economics.

2. Symmetry properties: Here, we have the very minimal requirement of "non-dictatorship" (an agent is a dictator if the allocation rule always selects an allocation that is first in his ranking), the basic requirement of "equal treatment of equals" (two agents with identical characteristics should receive the same bundle or the same welfare level), and the slightly more restrictive requirement of "anonymity" (the allocation rule is invariant under renamings of the agents).

Efficiency and symmetry properties are standard in abstract social choice theory.

3. Fairness properties: Next we have a series of properties expressing in various ways that the distribution of resources is fair.

3a. The "no-envy" property (at the chosen allocation, every agent prefers his assigned bundle to that of any other agent), originally discussed by Foley (1967), Kolm (1972) and Varian (1974), has played a central role in the microeconomic literature on fair division in the last decades. A related but less demanding property is "no-domination" (the bundle of no agent dominates, commodity by commodity, that of any other agent).

3b. "Egalitarian-equivalence" (there exists some reference bundle such that every agent is indifferent between that bundle and his assigned bundle) was proposed by Pazner and Schmeidler (1978). It is the main alternative to fairness as no-envy (see Section 5).

3c. Next, we have a variety of tests of fairness known as "welfare bounds". Given a resource allocation problem and a profile of individual preferences, a welfare lower bound for a given agent is a welfare level that must be guaranteed to him by the solution: this welfare level depends upon the social endowment and on this agent's characteristics, but it does not depend on the other agents' characteristics, (in particular their preferences). Similarly, a welfare upper bound places a cap on the agent's welfare. Several welfare upper bounds depending on the same data as listed above (the social endowment and the agent's own characteristics) have been considered (Moulin

(1991), (1992b)). A familiar lower bound in the classical fair division problem is the welfare level at equal division (no agent prefers an equal split of the goods to his assigned bundle). For more general allocation problems, define the "unanimity welfare" of an agent as the welfare that she would reach in an economy with the same resources (social endowment and technology) but where all agents would have identical preferences to hers, and where the chosen allocation is efficient and gives the same welfares to all agents (Moulin (1990c)). For some problems, such as fair division or cooperative production under increasing marginal cost, the unanimity welfare is a feasible lower bound; in other problems, such as cooperative production under decreasing marginal cost, or provision of a public good, it is a feasible upper bound.

A different kind of welfare bound (related to the population monotonicity property explained below) is the familiar "stand-alone" test of the cost sharing literature. In a cooperative production problem with increasing returns, the test is a welfare lower bound. It is computed by giving the agent free access to the technology (assuming the absence of any other agent). If returns to scale are decreasing, the test turns into an upper bound. Several other welfare bounds have been introduced and discussed, both in the cooperative production and in the fair division problem (Moulin (1991), (1992a), Maniquet (1994), Fleurbaey and Maniquet (1994)).

None of these conditions have counterparts in abstract social choice theory. Conceptually, however, one could think of the notions of rights that have been the object of numerous studies in that theory (see the Suzumura and Seidl essays in the present volume) as distant relatives of the notions of guarantees and welfare lower bounds just described. The symmetric notions of welfare upper bounds can perhaps be understood as formalizations of dual notions of "obligations".

4. Monotonicity properties: The various parameters entering the description of the problems under study are usually taken from spaces endowed with order structures, and a number of restrictions can be formulated on the way allocation rules should respond to changes in parameters that can be evaluated with respect to these orders. In many situations, an increase in a parameter unambiguously is socially desirable, in the sense that it permits a Pareto improvement. "Monotonicity" says that such an increase causes all agents to gain. Conversely, if the increase makes the welfare levels initially chosen infeasible, it should cause all agents to lose. Note that such a requirement is most meaningful when imposed on single-valued, or essentially single-valued (this means single-valued up to Pareto-indifference) allocation rules, but formulations for multi-valued allocation rules are certainly possible by appropriate choice of the quantifiers. Next, we present a list of examples, illustrating the richness in applications of this general idea.

4a. Endowments, social or individual, can be compared according to the usual vector ordering. "Social endowment monotonicity" is the requirement that all agents benefit from an increase in the social endowment. In fair division, the social endowment is the bundle of commodities to be distributed (Thomson (1983), Roemer (1986a,b), Moulin and Thomson (1988), Chun and Thomson (1988), Geanokoplos and Nalebuff (1988)). In cooperative production problems, the technology can be viewed as a social resource, and we obtain the axiom of "technological monotonicity" (all agents benefit from an improvement in the technology: Moulin (1987a,b), Roemer (1986), Moulin and Roemer (989), Roemer and Silvestre (1987, 1993)).

4b. "Individual endowment monotonicity" is the requirement on an allocation rule that an agent benefits from an increase in his endowment (Aumann and Peleg (1974); Thomson (1978, 1987)); "no-negative effects on others" says that no other agent is hurt by such an increase (Thomson (1979a)); in production economies, "productivity monotonicity" says that no agent is hurt by an increase in his productivity; "input monotonicity" is the requirement that an agent benefits from an increase in the amount of input that he contributes to production (Thomson, 1987b).

4c. Sizes of populations can of course be directly compared: when an increase in the population places greater strains on the resources available, we obtain the requirement of "population-monotonicity", which says that all agents initially present should lose; when it is a bonus, that is, when the arrival of the new agents is accompanied by a "sufficient" increase in resources, it says that all agents initially present should gain (Thomson (1983), Chichilnisky and Thomson (1987), Thomson (1991b), Tadenuma and Thomson, (1993), Alkan (1994), Bevia (1992), (1993); see Thomson (1992b) for a survey).

Many of the conditions presented above can be understood as minimal fairness conditions, but they often have an incentive interpretation as well. In situations when agents have control over resources, some of these conditions will give them the incentive not to withhold or destroy them.

Monotonicity axioms have played an important role in the axiomatic theory of bargaining. There, the strongest monotonicity axiom says that an enlargment (in the inclusion sense) of the set of feasible utility profiles should benefit all agents (Kalai (1977), Thomson and Myerson (1980)): it corresponds to our social endowment monotonicity. Similarly, population monotonicity was first formulated in the context of the axiomatic bargaining context (Thomson (1983)).

Note that none of the monotonicity axioms is meaningful in an abstract social choice framework.

5. Welfare-domination under "replacement" of one parameter value by another. Next, we consider properties having to do with simply replacing some of the data defining the problem at hand by other data taken from the admissible space. When this space does not have an order structure, it is in general not possible to tell whether the change will permit a Pareto-improvement or whether it will necessarily be accompanied by a Pareto deterioration. However, the requirement of solidarity among agents is still meaningful: it says that a replacement of this data affects all agents in the same direction. Also, even when an order structure does exist on the space of admissible data, one may not want to limit oneself to changes that can be evaluated in the order, but impose instead the requirement of welfare domination of one allocation by the other for **any** replacement of the data within its domain. A primary example of a space that is not endowed with a natural order structure is the space of preferences. The condition of "welfare-domination under preference-replacement" is obtained as an application of the general idea just outlined by focusing on all agents whose preferences are fixed: the change affects all of them in the same direction (Moulin (1987), Sprumont (1993), Thomson (1992b, 1993, 1994)). See Thomson (1990b) for a general formulation of the "replacement principle", and Sprumont (1993) and Sprumont and Zhou (1994) for applications in which several data (preferences and resources) change simultaneously.

This sort of conditions, not relying on particular structures, are in principle applicable to social choice theory. However, precisely because of this lack of structure, they are unlikely ever to be ever. It is because of the additional economic structure available in the models reviewed here that they are satisfied by interesting allocation rules.

6. Consistency and its converse: Next are other requirements, which like population-monotonicity, link choices across societies of different cardinalities. This time however, they are independence conditions. Note that they are applicable to multi-valued allocation rules with no difficulty.

6a. "Consistency" says that each allocation chosen by a solution for some economy is in agreement with the choices made by the solution for the "reduced economy" obtained from the original one by imagining the departure of some of the agents with their assigned bundles. This property and its dual presented next, have been the object of a considerable amount of attention in the last several years. The applications of consistency to resource allocation problems include Thomson (1988), Tadenuma and Thomson (1991), (1993), Bevia (1992), (1993), Moulin and Shenker (1994), Thomson and Zhou (1993), Young (1993), Sasaki and Toda (1992), Toda (1993a, b, c); for a survey, see Thomson (1994).

6b. "Converse consistency" permits the opposite operation, to deduce the desirability of an outcome for some problem from the desirability of its restrictions to subgroups for the associated reduced problems these subgroups face.

Actually consistency (and its converse) originate in the theory of cooperative games (Harsanyi (1959), Davis and Maschler (1965), and in the theory of bargaining (1987)). Here, the exchange of ideas between the various strands of literature has been extensive.[6] Note finally that the two consistency properties are not meaningful in the abstract social choice framework since the feasible set should have certain decomposability properties for it to be possible to speak of an agent's "component" of the chosen allocation.

7. Strategic properties: Finally, we have the "incentive-compatibility" requirements.

7a. When each agent has private information about some of the characteristics of the economy (such as his preferences, his endowment, or his

[6] In bargaining theory, consistency has led to characterizations of the Nash solution (Lensberg, (1987)) and of the egalitarian solution (Thomson and Lensberg (1989). In cooperative games, several versions of consistency have been formulated, and they have led to characterizations of the nucleolus (Sobolev (1975)), the Shapley value (Hart and Mas-Colell (1989)), the core (Peleg (1985)), and other solutions (Moulin (1985), Tadenuma (1992)).

productivity), the problem is to find allocation rules giving all participants the incentive to truthfully reveal what they know; an allocation rule with this property is called "strategyproof". The difficult problem of identifying all strategyproof rules in various resource allocation problems is the subject of Barberà's essay in this volume. We stress that strategyproofness is an integral part of the axiomatic program because it is a desirable property that an agent's selfish interest not conflict with the demands of the allocation rule.

7b. Unfortunately, it often turns out that the requirement of strategyproofness so severely restricts the set of allocation rules as to leave only unacceptable ones from the point of view of efficiency or distribution.[7] Then one may look for the less demanding requirement of implementability . An allocation rule is "implementable" if there is a game form such that, for each admissible preference profile, the set of allocations obtained at the equilibria of this game form coincides with the set of allocations that the rule would have chosen for the economy on the basis of the true preferences. The greater the extent to which information is privately held, the more demanding the implementability requirement (see Dutta's survey on implementability in this volume).

Note that the study of strategyproofness and implementation has been a major component of the abstract social choice literature. Apart from a few exceptions, until the early nineties, most studies of strategyproofness had indeed taken place in that context. The history of implementation is different, as this literature has developed in parallel fashion in abstract and economic models, and multiple bridges between the two strands have been established.

We should also note that the properties central to the analysis of strategic issues, such as "Maskin-monotonicity", have led to the formulation of other properties, such as "local independence", (it says that the choice should not be affected by changes in preferences that leave unchanged the marginal rates of substitutions at the bundles initially chosen). These properties have in turn been the object of studies of independent interest (Nagahisa (1992), Nagahisa and Suh (1993)), but they are part of the program that we are describing.

[7] A well-known example is the voting model (pure public good with unrestricted preferences) where the only strategyproof voting rules are either dictatorial or leave only two outcomes to choose from (see Barberà's essay).

5 What have we learned?

As is common in axiomatic analysis, most of the results are of one of two kinds: impossibility results, stating the incompatibility of a certain set of axioms, and characterization results, stating that a certain allocation rule is the only rule to satisfy a certain list of axioms.

From the existing literature, we already know that there is no shortage of either kind of results. We start with impossibilities.

A typical set of results reveals the difficulty of combining the no-envy test with other requirements of fairness. For instance, no Pareto-optimal solution to the classical fair division problem meets the no envy test and satisfies any one of the following properties: egalitarian-equivalence (Daniel (1975)), or population monotonicity (Moulin (1991)), or social endowment monotonicity (Moulin and Thomson (1988)). When some of the (unproduced) goods to distribute are indivisible, further incompatibilities arise (Tadenuma and Thomson (1991), Thomson (1994)). Similarly, in the case of cooperative production (of a private or public good) the no-envy test is incompatible with the stand-alone test (Moulin (1990)); this especially robust incompatibility extends to the fair division problem when monetary compensations are possible (Moulin (1992)).

In many cases, an impossibility result forces us to make some hard ethical choices. For instance strategyproofness is often incompatible with efficiency (Pareto-optimality): this is true in the classical fair division problem (Hurwicz (1972), Zhou (1991)), in cooperative production of private or public goods (Moulin and Shenker (1992)), and in many other models (an exception is the one-dimensional fair division model described below). Another instance is the classical fair division problem, where social endowment monotonicity proves incompatible with no domination, as well as with the equal division lower bound (Moulin and Thomson (1988)).

We turn to the "positive side" of axiomatic analysis, namely the characterization results. For a few remarkable models, virtually all directions from which the problem has been attacked have led to the same allocation rules or the same narrow family of rules. Examples are the following:

1. For the one-dimensional public good model with single-peaked preferences, a family of solutions known as the "augmented median voter solution" has been characterized on the basis of strategyproofness (Moulin,

(1980), (1984), Barberà, Gül and Stachetti (1993), Ching (1993); see also Barberà's essay in this volume); some narrow subfamilies have resulted from imposing population-monotonicity (Ching and Thomson (1992)), welfare-domination under preference-replacement (Thomson (1993)), or consistency (Moulin (1984)).

2. For the one-dimensional private good model with single-peaked preferences, a rule known as the "uniform rule" has emerged out of considerations of strategyproofness (Sprumont (1991), Ching (1992), (1994)), but consistency (Thomson (1994b)), implementation (Thomson (1992c)), and versions of resource-monotonicity (Thomson (1994a)), population-monotonicity (Thomson (1991b)), and welfare-domination under preference-replacement (Thomson (1992b)), have also led to it.

3. For the model of allocation of an indivisible good when monetary compensations are possible, a certain selection from the no-envy solution has come out of considerations of population-monotonicity (Tadenuma and Thomson (1993)), welfare-domination under preference-replacement (Thomson (1994)), and consistency (Tadenuma and Thomson (1993)).

In most resource allocation problems, however, several plausible allocation rules can be proposed. In many interesting cases, particular rules have been given axiomatic characterizations. Consider the classical fair division problem and the competitive solution with equal incomes. It has been characterized in more than one way: by means of no-envy (in models with a continuum of small, individually negligible agents (Varian (1974), Zhou (1992)); by means of Maskin-monotonicity (Thomson (1979b)), or the local independence property mentioned at the end of Section 3 (Nagahisa (1992)); by means of consistency (Thomson (1988), Thomson and Zhou (1993)). Similarly the Lindahl solution for the provision of a public good has been given several alternative characterizations.

Another family of allocation rules, inspired by the notion of egalitarian-equivalence (see Section 3), stand out as plausible alternatives to the competitive solution with equal incomes. They are not easy to characterize in the classical fair division problem but in the cooperative production problem with one input and one (private or public) output, the powerful property of technological monotonicity, combined with various welfare lower (or upper) bounds, characterizes a handful of simple and natural allocation rules

(Moulin (1987a,b), Roemer and Silvestre (1988), Maniquet (1994), Fleurbaey and Maniquet (1994)).

6 Directions for further research

We close with a short presentation of what we perceive to be interesting questions for further research.

1. The study of inefficient allocation rules has received very little attention. Yet, when we insist on strategyproofness, or on certain combinations of axioms such as no-envy and stand-alone tests, we must content ourselves with inefficient rules. What rules, then, are the least inefficient among those satisfying those requirements? A particularly interesting problem is the description of the full set of strategyproof allocation rules. We understand fairly well the structure of strategyproof rules in the fair division problem (Barberà and Jackson (1992)), in the cooperative production of a single private good (Moulin and Shenker (1992)), and of a single public good (Moulin (1994), Serizawa (1992)). Yet, much remains to be done (see Barberà's survey in this volume)).

2. For a given set of properties, it is often the case that incompatibilities can be overcome by narrowing down the domain of admissible preferences: common restrictions include quasi-linear preferences and normality of goods. We do not have a systematic way of measuring the trade-off between the choice of the domain of admissible preferences (how large can it be?) and the choice of axioms that can be jointly met. An example of this trade-off in the context of fair division is in Moulin (1992a).

3. For a given domain, in order to better identify the tradeoffs between the basic properties, it is useful to formulate parametric forms of each property measuring the extent to which the property is satisfied. Economic models lend themselves to the formulation of such quantified versions of basic requirements. For instance, partial versions of no-envy and of the monotonicity conditions are easy to define (Moulin and Thomson (1988) propose such definitions in the classical fair division problem). We should then try to identify relationship between these parameters, and allow a precise tracing out of the boundary between what can be achieved and what cannot (an

example is Thomson (1987c)). (For studies with similar objectives in the abstract social choice, see Campbell and Kelly's essay in this volume.)

4. Models with a continuum of agents have been the object of relatively few studies, even if we extend our reach to the axiomatic theory of cooperative games: Thomson and Zhou (1993) Dubey and Neyman (1984), Winter and Wooders (1994), Diamantaras (1991), Sprumont and Zhou (1994). It seems that much more could be done.

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