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Matching Human Capital and Displacement

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Matching, Human Capital and Displacement

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1 Introduction

Traditional analyses of the role of human capital in labor markets have focused on theoretical extremes in which capital is either purely match-specific or perfectly transferable. In contrast, empirical estimates suggest that an intermediate situation is a more appropriate description of the data.[1] Once these extremes are abandoned, the reallocation of semi-specific capital becomes an interesting economic problem. The purpose of this paper is to develop a simple decision-theoretic model that considers this reallocation problem. In our model, the quality of a match between a firm and a worker determines the transferability of previously acquired capital.

The paper is motivated by three related set of observations. First, there is considerable evidence (see [19], for example) that the rise of wages with the length of service on a job reflects the accumulation of skills. Second, the observation that displaced workers (see [6], [7], [14], [15], [16], [18]) suffer substantial reduction in wages suggests that job skills are not perfectly transferable. Third, the length of the unemployment spell after a displacement and the wage loss after reemployment vary in fairly systematic ways with characteristics of the last job. In particular, the duration of the unemployment spell after displacement increases with the seniority in the previous job (see [14], [16]) and decreases with wage on the previous job (see [2].) Tenure in the previous job has a much stronger impact on the relative wage loss at reemployment than does overall experience ([18], [16],[15]) The percentage wage loss also is an increasing function of the wage in the previous job ([14].)

In our model, workers accumulate capital while employed. Jobs disappear at an exogenous rate and workers are displaced when jobs disappear. Displaced workers search to be matched with a new job. A worker who accepts an offer loses part of his capital. The amount of capital lost depends on the quality of the match, which varies across jobs. The problem faced by a displaced worker is to decide which matches to accept. The decision rule which solves this problem determines the wages of displaced workers when reemployed, and together with the arrival rate of new offers determines the duration of unemployment. These wages and the unemployment duration will thus depend on how much capital a worker has. This capital is, in turn,

[1] According to Addison and Portugal [2], "... previous tenure has a positive impact on postdisplacment wages...tenure on the lost job raises wages on that job by more than it does ond the *second* job."

determined by the history of prior employment spells, displacements, and the quality of the reemployment match.

We parameterize our model using a variety of empirical studies and compute the solutions to the model. We show that wage losses are higher for displaced workers with more capital, that is for high wage (and seniority) workers. We also show that the duration of unemployment increases with seniority in the previous job. Our model provides an estimate of the capital loss suffered by a worker upon displacement. The losses are sizable. For example, a worker with the equivalent of five years experience in a job loses the equivalent of 1.7 years of salary upon displacement. This loss is mostly due to the imperfect match at reemployment; income loss due to the unemployment spell following displacement accounts for only 20% of the total loss.

We view this matching model as a particularly simple way to capture costs of displacement. We depart from the theoretical extremes in which all capital is purely firm or match-specific (see [8],[9], [12]), those in which all capital is perfectly transferrable ([17],[1]). The closest antecedent to our model is [19] who estimates a model in which experience and tenure both yield wage growth. The key difference is that in our model, the quality of a displaced workers' match depends upon that worker's capital. This feature allows us to analyze the unemployment duration of displaced workers as a function of their pre-displacement experience.

The distinction between general and specific human capital has been often stressed in the theory. Though this distinction may be convenient for some purposes, we believe it is not an accurate description of employment histories. The extent to which previously accumulated human capital will be available in a new job depends on the quality of the match between this job and the worker's prior experience. We endogenize this value by explicitly modeling the re-matching problem in a search theoretic framework. In this way, the extent to which human capital is specific or not is a function of the parameters of the search problem. For example, we show that workers with higher learning abilities will be less concerned about the quality of this match and will thus lose on average a larger fraction of their previously accumulated human capital.

The paper is organized as follows. Section 2 develops the model. Section 3 provides computations of the model. An extension of the basic model is considered in Section 4. Finally the appendix contains the proof of our main proposition and details of the computational method used.

2 The Search Problem.

Consider the following infinite horizon model. A worker with capital stock k who is currently employed, produces output at a rate given by a production function $f(k)$. The capital of an employed worker grows at an exogenously given rate δ. Jobs disappear at an exogenous rate given by a Poisson process with parameter ρ. That is, the probability that a job is still in existence after a time interval of length τ is equal to $e^{-\rho\tau}$.

Once a job disappears, worker becomes unemployed and must search. While unemployed, workers produce at a rate given by $L(k)$. This production can be interpreted as the value of leisure or the output produced at an alternative job outside the industry. The human capital of unemployed workers depreciates at a rate $\lambda \geq 0$.

Unemployed workers search for new jobs. The search problem is modelled along the lines [12] (see also [13]). Job offers arrive according to a Poisson process with parameter α. Associated with each offer is a match-specific random variable denoted by s with distribution function $G(s)$. If the worker chooses to accept the offer, his capital stock in the new position is given by $k_0 + s(k - k_0)$, where $k_0 \geq 0$ is a basic level of capital which is perfectly transferable. The variable s captures the quality of the match between the worker and the job. What we have in mind is that industry specific capital is composed of an array of skills. Different jobs require different levels of specialization in the array of skills. We have chosen to capture this array by a single-dimensional variable called the capital stock.

The match specific random variable then captures how valuable this capital is to the new job. Note that our model captures cases in which all acquired capital is job-specific ($s = 0$ for all matches) and that in which all capital is transferable ($s = 1$ for all matches.) Our specification implicitly assumes that once a new job is accepted the worker loses a part of his acquired capital for all subsequent matches. These assumption of convenience allows us to summarize a worker's history by a single index, k. In section 4 we relax this assumption to some extent.

Individual workers are risk-neutral and maximize expected discounted consumption. The interest rate is denoted by r. We analyze now the decision problem of the workers. Let $V(k)$ denote the value function or the expected discounted utility of a currently employed worker and $W(k)$ the value function or the expected discounted utility of a currently unemployed worker. For a small time interval h, the value function of an employed worker is approximately

$$V(k) \doteq f(k)h + e^{-rh}\left[e^{-\rho h}V(ke^{\delta h}) + (1 - e^{-\rho h})W(ke^{\delta h})\right] \tag{2.1}$$

where h is small enough that we may ignore higher order terms: *i.e.* the probability of an unemployed worker receiving a job offer. The first term in (2.1) is the flow of output produced by the worker. The first term in the brackets is the product of the probability that the current job is not destroyed and the value conditional on its continuation. The second term is the product of the probability of job destruction and the value of an unemployed worker. Rearranging terms, dividing by h and taking limits we get

$$rV(k) = f(k) + \rho\left[W(k) - V(k)\right] + V'(k)\delta k \tag{2.2}$$

This equation gives the flow equivalent of $V(k)$ which consists of $f(k)$, the flow of output, the term in brackets which gives the capital loss due to job termination and $V'(k)\delta k$, the value of the appreciation of human capital. Solving for $V(k)$ we obtain a more convenient expression

$$(\rho + r)V(k) = f(k) + \rho W(k) + V'(k)\delta k \tag{2.3}$$

The differential equation that corresponds to $W(k)$ is similarly derived and given by

$$rW(k) = L(k) + \alpha \int \max\left\{V\left(k_0 + s(k - k_0)\right) - W(k), 0\right\} G(ds) - W'(k)\lambda k \tag{2.4}$$

where the first term is the flow of output (or leisure) while unemployed, the second term represents the value of the option of becoming reemployed and the third term the loss of value due to depreciation. As usual, a worker will accept a job only if its value exceeds the value of remaining unemployed. For each level of k, the acceptance rule is given by a reservation value $z(k)$ which solves

$$V\left(k_0 + z(k)(k - k_0)\right) = W(k) \tag{2.5}$$

All jobs with $s \geq z(k)$ are accepted and all other jobs are rejected. Equations (2.3), (2.4) and (2.5) fully describe the problem faced by a worker.

The model has implications for the duration of unemployment, the wage loss, and the value loss for workers as a function of their capital k, which we now analyze. Workers with higher k benefit relatively more from a favorable match. But the opportunity cost of rejecting an offer is also likely to be higher. We can evaluate the local behavior of $z(k)$ by totalling differentiating (2.5), obtaining

$$[z'(k)(k-k_0)+z(k)] = \frac{W'(k)}{V'(k_0+z(k)(k-k_0))} \tag{2.6}$$

It is straightforward to show that at k_0

$$\frac{W'(k_0)}{V'(k_0)} = \frac{\alpha\overline{z}}{\alpha+r} + \frac{L'(k_0)}{(\alpha+r)V'(k_0)}$$

where $\overline{z} = \int z\, G(dz)$. Since $k_0 > 0$ implies that $V'(k_0) > 0$, it follows that $\frac{W'(k_0)}{V'(k_0)} > 0$ and consequently $z'(k_0) = \infty$. More generally, the behavior of $z(k)$ depends in a complicated way on properties of f and L, the values of the parameters in the model and the size of k_0. In general there is no explicit solution for the integral-differential system given by equations (2.3), (2.4) and (2.5). But for a special case, which we now analyze, such a solution exists.

Homogeneous case. $k_0 = 0$, $f(k) = ak^\theta$ and $L(k) = bk^\theta$, $\theta \in (0,1]$.

Conjecture that the solution is of the form $V(k) = vk^\theta$ and $W(k) = wk^\theta$. Equation (2.3) is now given by $(r+\rho)vk^\theta = ak^\theta + \rho wk^\theta + \delta\theta vk^\theta$ which implies

$$v = \frac{a+\rho w}{r+\rho-\delta\theta} \tag{2.7}$$

Equation (2.5) is now given by $vz(k)^\theta k^\theta = wk^\theta$ so $z(k)$ is independent of k and satisfies $z(k) = z = \left(\frac{w}{v}\right)^{\frac{1}{\theta}}$. Using this equation (2.4) can be written as

$$(r+\lambda\theta)w = b + \alpha\int_{\left(\frac{w}{v}\right)^{\frac{1}{\theta}}} \left(vs^\theta - w\right) G(ds) \tag{2.8}$$

To analyze the equilibrium it is convenient to divide both equations by w and use $z = \left(\frac{w}{v}\right)^{1/\theta}$, obtaining

$$z^{-\theta} = \frac{\frac{a}{w} + \rho}{r + \rho - \delta\theta} \tag{2.9}$$

and

$$r + \lambda\theta - \frac{b}{w} = \alpha \int_z \left(z^{-\theta} s^{\theta} - 1 \right) G(ds) \tag{2.10}$$

These two equations are depicted in Figure 1. Curve M_1 corresponds to equation (2.9) and curve M_2 to equation (2.10).The signs of the slopes as well as the intercepts can be easily derived from these equations. There is a unique solution to the agent's problem. Using Figure 1, some comparative static results can be easily derived.

An increase in a shifts the M_1 curve upwards, resulting in a decrease in z and an increase in w. Similar results are obtained from an increase in δ. This result implies that those that face better opportunities in the market or learn the specific skills faster will have lower reservation values, therefore having shorter unemployment spells. Furthermore, those agents with higher growth rates of capital δ will, on average, have higher wages at the time of displacement and because of the lower reservation value will exhibit larger percentage wage losses but faster wage growth after reemployment. This is consistent with the findings obtained from the Displaced Workers survey reported in Horvath [5] mentioned above.

It is easy to show that an increase in ρ moves the M_1 curve downwards, leading to a higher reservation value but lower w. An increase in r shifts both curves downwards, with an ambiguous effect on z. An increase in b has the opposite effect of an increase in a, leading to a higher reservation value. This effect is similar to an increase in unemployment benefits. An increase in λ shifts the M_2 curve downwards, resulting in lower z. Finally, an increase in the arrival rate α shifts the M_2 curve upwards, leading also to higher z.

The homogeneous case is useful as a benchmark and to highlight the comparative statics of the search model. But several empirical facts seem counter to its predictions. In first place, since reservation values are independent of k, specific experience per se has no effect on reemployment behavior. The empirical studies above indicate that seniority in the previous job has a positive effect on unemployment duration after displacement. Secondly, the linearity of f implies that the rate of growth of wages $\frac{f'(k)\delta k}{f(k)} = \delta$, which is independent of seniority. Topel [19] and others have established that rate of wage growth decreases with seniority. Finally, the model implicitly assumes that workers with no specific capital ($k = 0$) will never acquire any positive amount and their wages will always be zero.

If wage growth is decreasing in k and $k_0 = 0$, the above comparative statics results suggest that workers with higher k will have lower reservation values, i.e. $s(k)$ will be decreasing. Some insights can be obtained from local analysis of the converse case.

Suppose that $f(k) = a + g(k)$, where $g(k)$ is a homogeneous function. Note that this implies that wage growth $\frac{f'(k)k}{f(k)} = \frac{g'(k)k}{(a+g(k))}$ is increasing in k. Solving $V(k)$ forward we get

$$\begin{aligned} V(k) &= \int e^{-(r+\rho)t} \left[f(e^{\delta t}k) + \rho W(e^{\delta t}k) \right] dt \\ &= \frac{a}{r+\rho} + \int e^{-(r+\rho)t} \left[g(e^{\delta t}k) + \rho W(e^{\delta t}k) \right] dt \end{aligned}$$

It can be shown that for small a, $V(k) \doteq c(a) + vk^\theta$ and $W(k) \doteq d(a)(1 - G(z))$ where $g(k) = k^\theta$, $c(a) > d(a) > 0$ and v, w, z are the value functions and optimal stopping rule corresponding to the case where $a = 0$. In this case and using equation (2.6)

$$\begin{aligned} z'(k) &\doteq W'(k)k - V'(z(k)k)z(k)k = \theta \left[wk^\theta - v(z(k)k)^\theta \right] \\ &> \theta \left[d(a)wk^\theta - c(a)v(z(k)k)^\theta \right] = 0. \end{aligned}$$

Consequently, workers with higher k will have higher reservation values.

The following Proposition, which is proved in the appendix, gives a result for the case of decreasing wage growth.

Proposition 1 *Suppose $\frac{f'(k)k}{f(k)}$ is decreasing in $k, L(k) = 0$ and $\lambda = 0$. Then $z(k)$ is decreasing in k.*

This implies that workers with higher seniority exhibit shorter unemployment spells, contrary to the above cited evidence. But this conclusion relies on the assumption that $k_0 = 0$. When $k_0 > 0$ and wage growth is decreasing, one may reasonably conjecture that there will exist some $k^* > k_0$ such that $z'(k) > 0$ for $k < k^*$ and $z'(k) < 0$ for $k > k^*$. The precise value of k^* depends in more complicated ways on the wage-seniority profile.

3 Computation

In this section, we compute the value functions and the optimal acceptance rule for a suitably parameterized version of the economy described in Section 2. We choose parameter values and functional forms based on a variety of empirical studies. We view this exercise as a tentative first step in asking whether this simple decision-theoretic model is consistent with quantitative observations on wage losses and duration of unemployment.

The wage function was obtained as follows. Topel [19] provides estimates of wage growth as a function of total labor market experience and years of seniority. Let s_{it} and e_{it} denote, respectively, seniority and experience for worker i in period t. Consider a worker who enters the labor market in period 0, so that $s_{i0} = e_{i0} = 0$. Given δ and k_0, and provided the employment relation is not terminated, the worker will attain capital $k > k_0$ in period t, where $k = e^{\delta t} k_0$ or $t = \frac{\ln(k) - \ln(k_0)}{\delta}$. Therefore $\frac{f'(k)\delta k}{f(k)}$ is the wage growth of a worker with $s_{it} = e_{it} = \frac{\ln(k) - \ln(k_0)}{\delta}$. Using the estimated wage growth functions and given a fixed value for $f(k_0)$, the wage function is determined. The value chosen for $f(k_0)$ is simply a scaling factor with no consequence for the decision rules and relative values[2].

Similarly, one can show that the particular choice of δ is of no consequence as far as f is chosen in the way described above. Specifically, δ just determines the units of measurement of k. In all the computations reported below we set $\delta = 0.04$. Several values for λ between 0 and 0.06 were tried, with almost no change in results. The values reported below correspond to the case $\lambda = 0$. The value of leisure was set to zero for all k. This makes our estimates of the cost of job loss to individuals an upper bound.

In setting the value for the job loss rate ρ, two alternative approaches were used. The first one is to use information on number of displaced workers from the Displaced Workers Survey. According to Horvath [5] 10.8 million workers 20 years of age and over lost jobs because of plant closings or employment cutbacks between January 1981 through January 1986. This gives approximately 2 million per year, roughly 2 percent of the labor force, suggesting a

[2]Topel used a polynomial of degree four for the wage growth function. It provides a good fit for the most frequent years of experience (from 0 thru 10). However, the behavior beyond that period is erratic. For our wage function we decided to keep the wage growth constant at the 10 year level (approximately 3 %) for experience levels beyond 10.

value $\rho = 0.02$. It has been argued that this number underestimates the true value, since workers that have been reemployed tend to forget displacement lapses. A much higher number is obtained considering the flows of displacement from the PSID data; as reported in [18], which range from a low of roughly 2 percent in 1968 to 11 percent in 1982-83. This last figure is consistent with findings in [3], which indicate that the typical worker at any point in time is employed on a job that has lasted 9 years. This suggests a value for ρ of approximately 10 percent[3], which is the one used in the computations below.[4]

For the distribution function $G(s)$ of the quality of matches, we considered a uniform distribution between zero and some upper value $\bar{s} \leq 1$. This upper bound was chosen to obtain average wage losses at reemployment of approximately 20%, which is consistent with the estimates provided in [18]. To set the arrival rate α, the value for mean unemployment duration for displaced workers of 18.3 weeks (Horvath, Table 8) was used. After experimenting with the model, this figure was more appropriately matched setting $\alpha = 6$, an average of 1 wage offer every two months.

The method of computation is described in the Appendix.

Tables 1–3 provide general results obtained from the computational exercise. Though the wage function displays decreasing growth rates, the value of $s(k)$ is increasing. This result is a consequence of our assumption that the worker's capital stock cannot fall below the undepreciable stock k_0. This result implies that the duration of unemployment is increasing in cumulative experience. This can be seen more clearly in Table 2, where average weeks of unemployment range from almost 9 weeks for a worker with 1 year of cumulative experience to 21 weeks for one with 10 years of cumulative experience.

Wage losses are higher for workers with higher k and so are value losses. The last column of Table 1 gives the average value losses measured in terms of yearly wages. For example, a worker with 5 years of cumulative experience loses in average an equivalent to 1.7 year of wages (at the wage rate received

[3]An alternative formulation could make ρ a decreasing function of k. This can be easily introduced in the model. We chose not to follow this approak for lack of appropriate information to estimate the ρ function.

[4]The qualitative results do not change when $\rho = 0.02$ and the quantitative results are very similar.

when displaced). These numbers indicate that the average private costs of displacement are substantial.

The value loss comes from three sources: i) the lack of wage earning while unemployed; ii) the lack of learning while unemployed and iii) the loss of firm specific capital at reemployment. Table 3 gives the share of each of the latter component. According to these results, more than 80% of the value loss of a worker with median tenure can be attributed to the capital loss. This suggests that unemployment benefits provide little insurance against displacement.

The implications of the model are consistent with the positive effect of seniority on unemployment duration. Without any additional source of variation, wages and seniority are almost perfectly correlated in the model, so nothing significant can be said about the independent effect of earnings prior to displacement. As suggested before, variation can be introduced through heterogeneity in learning abilities (δ) in the population. Tables 4 and 5 provide the computed values for $\delta = 0.02$, which represents an asymptotic wage growth half the size of the original calculations. For the same years of specific experience, workers with lower δ will have lower k and, consequently, lower wages. A comparison of tables 4 and 1 shows that they will also have lower average percentage wage losses, which is consistent with the evidence form the Displaced Workers Survey. Table 6 shows that, holding the stock of human capital (and thus the wage) constant, unemployment durations will be longer for the slow learning workers. Given that such group of workers must in average have a longer tenure to have reach the same k, it follows that this source of heterogeneity could also account for the positive association between the length of the unemployment spells and tenure in the displaced job found in the data.

The empirical evidence on wage profiles suggests that returns to seniority are higher after a displacement occurs, so that the gap between pre and post-displacement wages narrows down quite rapidly after reemployment [18],[7]. Given the decreasing returns to tenure, our model can partly explain this fact. The following section discusses an extension to the model that can provide further insights on this matter.

4 A simple two state model

In the above model we assumed that workers totally lose the *unmatched* component of specific capital upon reemployment. This has the unrealistic feature that an unemployed worker who takes a job (with $z < 1$) which terminates immediately after, is back to unemployment with a lower capital stock (zk).

An alternative formulation is presented here where it is not lost but the worker takes time to adapt to the new job. In order to do so we must distinguish between the total accumulated capital of a worker k and the level of capital which has been adapted to the current job, x. Thus $x \leq k$ and for the case of general human capital $x = k$. As before, the rate of growth of specific capital s is given by δ, but we now allow the growth rate δ to depend on the cumulative capital k and the specific x. In particular the rate of growth of x is given by a function $d(x, k)$. Furthermore, we assume k does not grow until x reaches it. ¿From that point on, both grow at the same rate so the distinction disappears. This is clearly a simplification which intends to capture the idea that there is a period of adjustment to a new job. This model allows this period to be determined endogenously and depend on the previous experience of a worker.

With this modification, the model is now a two state model. But for the specification we now define, it can be transformed into a single state system. Assume:

$$d(x, k) = \delta \left[\frac{\max(k, x)}{x} \right]$$

Note that this formulation implies that the growth rate of s is decreasing as a function of seniority but increasing in the level of previous experience. For a homogeneous wage function, the same properties apply to wage growth. Consistent with the above definition, when x exceeds k it becomes the new level of cumulative experience. Thus, a worker who loses his job having accumulated expertise x and with previously accumulated capital k, will start his job search with a value $k' = \max(k, x)$. In contrast to the model in the previous section, this formulation assumes workers never totally lose their previously acquired experience.

Let $U(x, k)$ denote the value of an employed worker with state x, k and let $V(k) = U(k,k)$. Equations (2.3), (2.4) and (2.5) are now replaced by

$$(\rho + r)U(x,k) = f(x) + \rho W(k) + U_1(x,k)\delta k \tag{4.1}$$

$$\begin{array}{rcl} rW(k) & = & L(k) + \alpha \int_{z(k)} \left(U\left(k_0 + x(k-k_0), k\right) - W(k)\right) \\ & & dG(z) - W'(k)\lambda k \end{array} \tag{4.2}$$

$$U\left(k_0 + z(k)(k-k_0), k\right) = W(k) \tag{4.3}$$

Consider now the special case in which the function f is linear and $k_0 = 0$. Without loss of generality let $f(k) = k$. All our earlier results extend easily to the homogeneous case. Conjecture the following solution:

$$\begin{array}{rcl} W(k) & = & wk \\ V(k) & = & vk \\ U(zk,k) & = & h(z)k \end{array} \tag{4.4}$$

Replacing these definitions in equation (4.1) and eliminating k we obtain the following differential equation for $h(z)$

$$(r+\rho)h(z) = z + \rho w + h'(z)\delta \tag{4.5}$$

It follows immediately that $v = h(1)$ and that

$$v = \frac{\alpha + \rho w}{r + \rho - \delta} \tag{4.6}$$

By the linearity of U and W in k, the stopping rule $z(k) = z^*$ is independent of k and satisfies

$$h(z^*) = w \tag{4.7}$$

Finally, equation (4.2) is now given by

$$rw = b + \alpha \int_{h^{-1}(w)} (h(z) - w)G(dz) \tag{4.8}$$

Using $v = h(1)$ equation (4.6) can be solved explicitly for z with parameters that involve v and w. Using this solution, equations (4.6) and (4.8) provide a nonlinear system which can be used to solve for v and w.

To characterize the solution, a method similar to that used for the homogeneous case in the previous section will be used. First note that equation

(4.6) is identical to (2.7) when $\theta = 1$. In turn, equation (4.8) is similar to (2.8), where vz is replaced by $h(z)$. Solving for the function h as described above, it is easy to show that $\frac{h(z)}{w}$ is decreasing in w and also in $\frac{w}{v}$. Hence, Figure 1 can be used to prove existence and uniqueness of an optimal solution. It is easy to show that $h(z) > vz$. This implies that curve M_2' corresponding to equation (4.8) will be to the right of curve M_2. Consequently, this model leads to higher w, v and $\frac{w}{v}$. Using equation (4.6) it is easy to show that percentage value loss $\frac{v-w}{v}$ is decreasing in w. Combined with the previous result, this implies, as expected, lower private cost of displacement in model 2. It can also be established that the reservation value z^* will be lower and consequently unemployment spells shorter. The comparative statics of this model are similar to those described in the previous section and are thus omitted.

The model described here can be easily adapted to allow for on the job search by workers recently reemployed. For this purpose one may assume that employed workers get draws for z, with a possibly lower arrival rate. With this addition, implications about rates of turnover of displaced workers could be obtained.

5 Final Remarks

In this paper, we have developed a model in which human capital is neither purely match specific nor purely transferable.Our model stresses the idea that the efficient reallocation of semi-specific capital is central to the reemployment process. The paper provides a simple framework to study this reemployment problem.

The model is consistent with a wide range of observations relating the length of unemployment spells and wage losses for displaced workers to their previous employment history. The model thus provides a simple structure to organize the empirical evidence on the reallocation of displaced workers and in particular, for the accounting of the private costs of displacement.

The comparative statics of the model also suggests some further potential applications. For example, our results suggest differences in learning abilities (as reflected by values of δ) as a potentially important source of heterogeneity. In particular, the evidence on displaced workers suggests that education has a negative effect on the duration of unemployment. A plausible hypothesis is that education raises the growth rate of capital accumulation, δ. As we have seen, as δ increases workers become less picky and the duration of unemployment declines.

The framework is could also be used to address a variety of issues related to sectoral shocks and the associated reallocation of workers (see [10]). For example, the model could be used to analyze the effects of increases in the pace of sectoral reallocation. An increase in the rate of job loss would be one way of modeling this exercise. We expect that workers would become less picky so that the duration of unemployment spells would decline.

There are many interesting extensions of our basic model. We have abstracted completely from life-cylce considerations. Introducing finite lifetimes or mandatory retirement would provide testable implications regarding the age of displaced workers and the characteristics of their re-matching process. Another simplifying assumption has been the constant hazard rate for job termination. This could also be modified in accordance with the data. Finally, our model abstracts from general equilibrium considerations. For example, we have considered the process of job arrivals and the distribution of matches as exogenously given. Integrating the structure provided here into a general equilibrium model is obviously an important extension.

6 Appendix

6.1 Proof of Proposition 1

Lemma 2 $\frac{\partial z^*(k)}{\partial k} \overset{s}{=} W'(k) - V'(z(k)k)z(k)$.

Proof. Follows immediately by differentiation and applying the envelope theorem. ■

Lemma 3 *Suppose* $\frac{V'(s)s}{V(s)}$ *is strictly increasing in* s *for all* $s < k$. *Then* $\frac{W'(k)k}{W(k)}$ *will also be strictly increasing in* k *and* $\frac{\partial z^*(k)}{\partial k} > 0$.

Proof. $W(k)$ satisfies

$$rW(k) = \alpha \int_{z^*(k)}^{1} [V(sk) - W(k)]\, dG(s)$$

so

$$r = \alpha \int_{z^*(k)}^{1} [\frac{V(sk)}{W(k)} - 1]\, dG(s)$$

Applying the Envelope theorem this identity implies that

$$\begin{aligned} 0 &= \int_{z^*(k)}^{1} \left(\frac{\partial}{\partial k}\frac{V(sk)}{W(k)}\right) dG(s) \\ &\overset{s}{=} \int_{z^*(k)}^{1} V(sk) \left(\frac{V'(sk)\, sk}{V(sk)} - \frac{W'(k)k}{W(k)}\right) dG(s) \end{aligned} \tag{6.9}$$

Now suppose by way of contradiction that $\frac{\partial}{\partial k}\frac{W'(k)k}{W(k)} \leq 0$. First note that $W'(k)k > V'(z^*(k)k)z^*(k)k$, for otherwise equation 6.9 would not hold. Also note that 6.9 defines implicitly $z^*(k)$. Since $\frac{V'(sk)sk}{V(sk)}$ is strictly increasing in k, our contradiction assumption implies that the term in brackets will decrease for all s as k increases. It can also be shown that under our assumption,

$$\int_{z^*(k)}^{1} V'(sk)\, s \left(\frac{V'(sk)\, sk}{V(sk)} - \frac{W'(k)k}{W(k)}\right) dG(s) \geq 0.$$

Thus, for equation 6.9 to hold as k increases, it must be the case that $z^{*\prime}(k) < 0$, which by Lemma 2 contradicts the fact that $W'(k)k > V'(z^*(k)k)z^*(k)k$. ■

Lemma 4 $V'(k)k/k$ *is increasing in* k *if and only if* $\frac{V'(k)k}{V(k)} - \frac{f'(k)k+\rho W'(k)k}{f(k)+\rho W(k)} >$ 0.

Proof. First note that

$$\partial \frac{V'(k)k}{V(k)} / \partial k \overset{s}{=} \left(\frac{\partial V'(k)k}{\partial k} \right) V(k) - V'(k)V'(k)k. \tag{6.10}$$

But

$$\left(\frac{\partial V'(k)k}{\partial k} \right) V(k) = \frac{1}{\delta} \left[-f'(k) - \rho W'(k) + (\rho + r) V'(k) \right] V(k) \tag{6.11}$$

and

$$V'(k)V'(k)k = \frac{1}{\delta} \left[-f(k) - \rho W(k) + (\rho + r) V(k) \right] V'(k). \tag{6.12}$$

Replacing 6.11 and 6.12 in equation 6.10 the proof is completed.

Proof of Proposition. From lemma 3 it follows that $z(k)$ is increasing if $\frac{V'(k)k}{V(k)}$ is increasing in k. ¿From lemma 4 it follows that the latter holds iff $\frac{V'(k)k}{V(k)} - \frac{f'(k)k}{f(k)} > 0$. When $\rho = 0$, $V(k) = \int e^{-rt} f(k(t))\, dt$ and $V'(k)k = \int e^{-rt} f'(k(t))k(t)\, dt$, where $k(t) = e^{\delta t}k$. It follows that

$$\frac{V'(k)k}{V(k)} = \frac{\int e^{-rt} f'(k_t) k_t \, dt}{\int e^{-rt} f(k_t) \, dt}.$$

The numerator and denominator correspond, respectively, to the integrals of $f'(k(t))k(t)$ and $f(k(t))$ with respect to an exponential distribution with coefficient r. Letting $g(t) = f(k(t))$ and $h(t) = \frac{f'(k(t))k(t)}{f(k(t))}$ then $\frac{V'(k)k}{V(k)} = \frac{\int h(t)g(t)\, e^{-rt} dt}{\int g(t)\, e^{-rt} dt}$. If $\frac{f'(k)k}{f(k)}$ for all k, then both h and g will be increasing functions of t and by Lemma 10 in [4] $\frac{V'(k)k}{V(k)} > \frac{f'(k)k}{f(k)}$ so $z'(k) > 0$. ■

6.2 Method of Computation.

Equations (2.3), (2.4) and (2.5) define an integro-differential system of equations. The following iterative scheme was followed to compute solutions. For a fixed W, equation (2.3) defines an ordinary differential equation for V. For large k, a boundary value can be derived as follows. Asymptotically wage growth is constant and thus V homogeneous. Let g denote the asymptotic wage growth, which in our computations was set to 3 percent for all cumulative experience levels greater than 10 years. Hence, for $k > \overline{k} = e^{10\delta}k_0$, the wage is given by $\alpha k^{g/\delta}$, where the constant α is obtained by setting $f(\overline{k}) = \alpha\overline{k}^{g/\delta}$. Using the results for the homogeneous wage case, for large k, $V(k) = vk^{g/\delta}$, where v is uniquely determined. This defines a boundary condition which was used to solve for V backwards.

Given this constructed V, equations (2.4) and (2.5) define an integro-differential system to solve for W. It is easy to see that this is an initial value problem. Using the fact that $s(k_0) = 0$ equation (2.4) implies that $W(k_0) = (\frac{\alpha}{(r+\alpha)})V(k_0)$. This provides the boundary condition needed to solve for W. Letting W_n and V_n denote the values obtained after n iterations, the procedure stopped when $\|V_n - V_{n-1}\| + \|W_n - W_{n-1}\| < \epsilon$, using the sup norm applied to a finite grid.

The maximum k was set at $e^{20\delta}k_0$, an amount corresponding to 20 years of uninterrupted experience, $k_0 = 1$ and a grid of 800 points uniform in the log scale was used. The tolerance level ϵ was set to 0.01, which represents approximately just 3 thousandths of the value functions. W was initialized to the zero function. This guarantees monotone convergence. The rate of convergence was geometric.

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Table 1. Wage and Value Losses

Experience	Wages	V(k)	Wage Loss	Value Loss	Value Loss/ wages
1	1.12	29.35	7.8%	2.8%	0.7
2	1.23	30.07	13.0%	4.5%	1.1
3	1.34	30.77	16.2%	5.9%	1.4
4	1.44	31.45	18.7%	7.2%	1.6
5	1.53	32.11	20.2%	8.2%	1.7
6	1.62	32.76	21.1%	9.1%	1.8
7	1.70	33.41	21.6%	10.0%	2.0
8	1.76	34.05	21.8%	10.8%	2.1
9	1.83	34.70	21.6%	11.6%	2.2
10	1.89	35.37	21.5%	12.4%	2.3

Table 2. Duration of Unemployment

Experience	s(k)	Hazard rate	Average Weeks Unemployed
1	0.00	5.98	8.7
2	0.09	4.93	10.5
3	0.17	4.02	12.9
4	0.20	3.61	14.4
5	0.23	3.24	16.0
6	0.25	2.99	17.4
7	0.27	2.81	18.5
8	0.28	2.68	19.4
9	0.29	2.52	20.6
10	0.30	2.45	21.2

Table 3. Value Losses

Experience	Total Value Loss [2]	Value Loss at 'eemployment [4	[4]/[3]
1	2.8%	1.9%	0.66
2	4.5%	3.4%	0.74
3	5.9%	4.5%	0.76
4	7.2%	5.6%	0.78
5	8.2%	6.5%	0.79
6	9.1%	7.3%	0.80
7	10.0%	8.1%	0.81
8	10.8%	8.8%	0.82
9	11.6%	9.4%	0.82
10	12.4%	10.2%	0.83

Table 4. Wage and Value Losses

(δ=.02)

Experience	*Wages*	*V(k)*	*Wage Loss*	*Value Loss*	*Value Loss/ wages*
2	1.12	23.88	7.5%	3.5%	0.7
4	1.23	24.67	12.4%	5.7%	1.1
6	1.34	25.43	15.6%	7.3%	1.4
8	1.44	26.16	18.0%	8.8%	1.6
10	1.53	26.87	19.5%	10.0%	1.8
12	1.62	27.56	20.4%	11.1%	1.9
14	1.70	28.25	20.9%	12.1%	2.0
16	1.76	28.93	21.1%	13.0%	2.1
18	1.83	29.64	21.0%	13.8%	2.2
20	1.89	30.39	20.9%	14.9%	2.4

Table 5. Duration of Unemployment

(δ=.02)

Experience	*s(k)*	*Hazard rate*	*Average Weeks Unemployed*
2	0.06	5.26	9.9
4	0.15	4.22	12.3
6	0.21	3.43	15.2
8	0.24	3.11	16.7
10	0.27	2.80	18.6
12	0.28	2.59	20.1
14	0.30	2.45	21.2
16	0.30	2.35	22.2
18	0.32	2.21	23.5
20	0.32	2.16	24.0

Table 6. Heterogeneous Learning

	Average Weeks Unemployed		*Expected Wage loss*	
Human Capital (1)	(δ=.04)	(δ=.02)	(δ=.04)	(δ=.02)
1	8.7	9.9	7.8%	7.5%
2	10.5	12.3	13.0%	12.4%
3	12.9	15.2	16.2%	15.6%
4	14.4	16.7	18.7%	18.0%
5	16.0	18.6	20.2%	19.5%
6	17.4	20.1	21.1%	20.4%
7	18.5	21.2	21.6%	20.9%
8	19.4	22.2	21.8%	21.1%
9	20.6	23.5	21.6%	21.0%
10	21.2	24.0	21.5%	20.9%

(1) Measured in years of experience of high learning agent

Figure 1. Optimal Search