Rochester Center for
Economic Research

Writing Papers

Thomson, William

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Contents

1 Introduction

This is a list of recommendations for writing better papers (and to some extent, giving better seminars). There is no unique solution to the problem of writing well and these recommendations obviously reflect my personal tastes. Also, you will find that they are sometimes incompatible, or that in some circumstances they are not appropriate. This is where judgement comes in. Exercise yours. I have probably omitted some important points. I welcome your suggestions for a revised edition.

My first recommendation: Do not look at my own papers for illustrations of the principles that I enunciate below.

2 General Principles

2.1 Write so that you will not have to be read

Nobody wants to have to actually read another thirty-page paper. Convey your message efficiently.

By leafing through your paper, your reader should be able to easily spot the main results, guess most of the notation, and locate the precise statements of the crucial definitions needed to understand the hypotheses and the conclusions of each theorem.

A reader who has found your main results interesting and wants to know more should then be able to get an idea of your methods of proof by visual inspection. Glancing at the way a proof is structured, identifying the main assumptions and the known theorems on which it is based, is often quite informative.

Think about the way ***you*** read. You probably do not proceed in a linear way. Instead, you scan the paper for the main results and look around them for the explanation of the notation and terminology that you do not recognize or guess. You do not like having to search through the whole document to find what you need. You have better things to do. Your readers also have better things to do.

2.2 Show that what you did is not trivial, and that it has not been done before

In order to show that your results are important, the temptation is great to present them with the greatest generality, with big words and in gory detail. Resist it. Try instead to make your argument appear simple, and even trivial. This exercise in humility will be good for your soul. It will also give referees a warm feeling about you. Perhaps most importantly, it will help you prove your results at the next level of generality.

To show that what you do has not been done before, explain how the assumptions under which you are operating are different from the assumptions that are used in related literature, and why indeed these differences are significant, both conceptually and technically. Demonstrate your knowledge of this literature by citing the relevant articles and indicating how they pertain to your subject.

2.3 Do not forget the process by which you made your discovery

By the time your paper is finished, you will have an arbitrary number of goods and agents, general production possibilities, uncertainty... and nobody will understand it. If you read it several months later, you will not understand it either. You got to your main theorem in small steps, by first working it out in the 2-agent, 2-good, linear technologies case, with no uncertainty, and by drawing lots of diagrams. It is also by looking at simple versions of your model that your reader will understand the central ideas, and it is most likely these central ideas that will be helpful to her in her own work, not the details of proofs. Unfortunately, it is not easy to reproduce the process of discovery in a paper, but you should try. In a seminar quite a bit more can be done though. Take advantage of the informality of such occasions.

The refereeing process and publication constraints have the unfortunate effect of wiping out from an article most of what could make the main points easily understandable, and you may think that if yours does not contain at least one result that looks difficult, then it is not ready for a journal. You are rightly proud of the sophisticated reasoning that led you to your theorem, but you should nevertheless work hard to make it look simple.[1]

2.4 Do not forget your errors

There is nothing like having misunderstood something to really understand it, and there is nothing like having seriously misunderstood it to really, really understand it. Instead of being embarassed by your errors, you should cherish them. I will even say that you cannot claim to have understood something until you have a very complete understanding of the various ways in which it can be misunderstood.

Your readers are likely to be victims of the same misunderstandings as you were. By remembering where you had trouble, you will be able to anticipate where they will have trouble too, and you will give better explanations.

3 Definitions

3.1 Make it unambiguous when you are defining a new term

When you introduce a new term, make it immediately clear that indeed it is new. Do not let your reader think that you may already given the definition but she missed it, or that you are assuming her to know the definition.

Here are three possible ways of introducing a definition:

"A function is ***monotone*** if ..."

"A function is 'monotone' if ..."

"A function is said to be ***monotone*** if ..."

I prefer the first format because it is direct and the different typeface will permit an easy retrieval, if needed. Boldface is best in that regard, preferable to italics and to quotation marks, neither of which makes it stand out sufficiently.

The crucial definitions should probably be displayed separately (see the examples below).

[1] As a young economist, it is natural that you should be proud of the complicated things you do; as you get older, you will become proud of the simple things you do. (Of course, it is not because you will not be able to do the complicated things any more!)

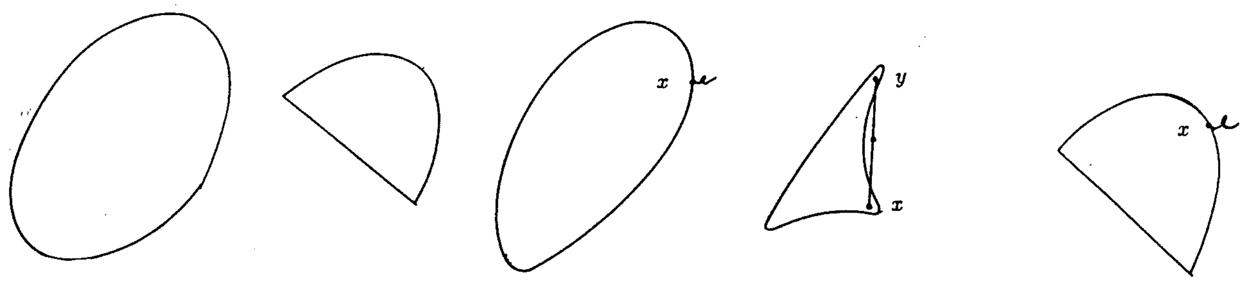


Figure 1: ***Example of convex and non-convex sets.*** (a), (b) and (c): Convex sets. (d) and (e): Non-convex sets. In examples (c) and (e), point x has been removed from the boundary.

3.2 When introducing a novel definition, give illustrative examples

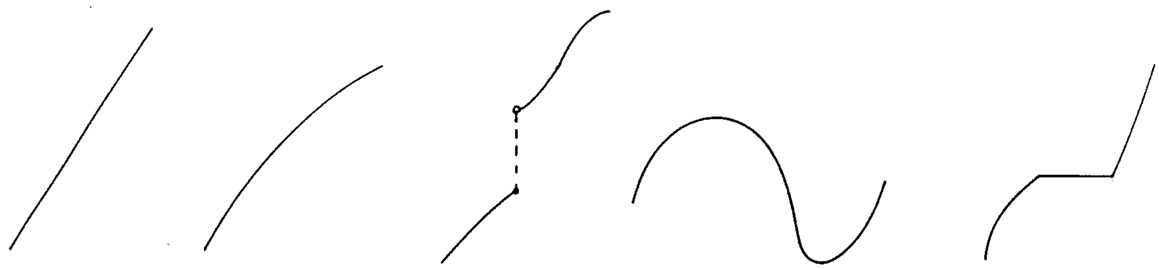
Give examples to illustrate your definitions. If the definition is a property that an object may or may not have, give examples of

1. Objects that satisfy the definition
2. Objects that do not satisfy the definition
3. Objects that satisfy the definition but almost do not
4. Objects that do not satisfy the definition but almost do

Examples in categories 3 and 4 are particularly important as they are the ones that are responsible for three-fourths of the work in the proofs. In a paper, giving a range of examples that are representative of all four categories is once again not easily achieved because of space limitations, but in seminars much more can be done. Here are a few illustrations:

Definition: A subset S of $\mathbf{R}^2$ is ***convex*** if for all x, $y \in S$ and all $t \in [0,1]$, we have $tx + (1-t)y \in S$.

Figure 1b is a better illustration of the notion of convexity than Figure 1a because it will force your reader to realize that you do not mean strict convexity. Figure 1c is a little more subtle because the set it represents



(a) (b) (c) (d) (e)

Figure 2: ***Examples of increasing and non-increasing functions.*** (a), (b) and (c): Increasing functions. (d) and (e): Non-increasing functions.

is almost non-convex (a point of the boundary has been removed; compare with Figure 1e). Figure 1d illustrates the typical way in which convexity is violated, and Figure 1e shows a non-convex set whose closure is convex.

Definition: A function $f\colon [0,1] \to \mathbf{R}$ is ***strictly increasing*** if for all t, $t' \in [0,1]$ with $t > t'$, we have $f(t) > f(t')$.

Figures 2a and 2b are a little dangerous because they may plant in your reader's mind the seed that the functions with which you will be working are linear, or perhaps concave. Figure 2c is what you need: it represents an increasing function in its full generality, with kinks, convex parts, concave parts, and discontinuities. Figure 2d is useful too as it shows a typical non-increasing function. Figure 2e is very important because makes it clear that you want more than for the function to be "non-decreasing".

Definition The continuous preference relation R defined on $[0,1]$, with asymmetric part P, is ***single-peaked*** if there is $x^* \in [0,1]$ such that for all x, $x' \in [0,1]$ with either $x < x' \leq x^*$ or $x^* \leq x' < x$, we have $x'Px$.

The graphs of the numerical representations of four preference relations are given in Figure 3. Obviously, R_2 is single-peaked and R_3 is not. But your reader may not immediately think of R_1 as being single-peaked because the peak is at a corner, or may think that R_4 is admissible, although it has a "plateau" and not a peak. Presenting these examples will be very useful to ensure that she fully perceives the boundary of your domain.

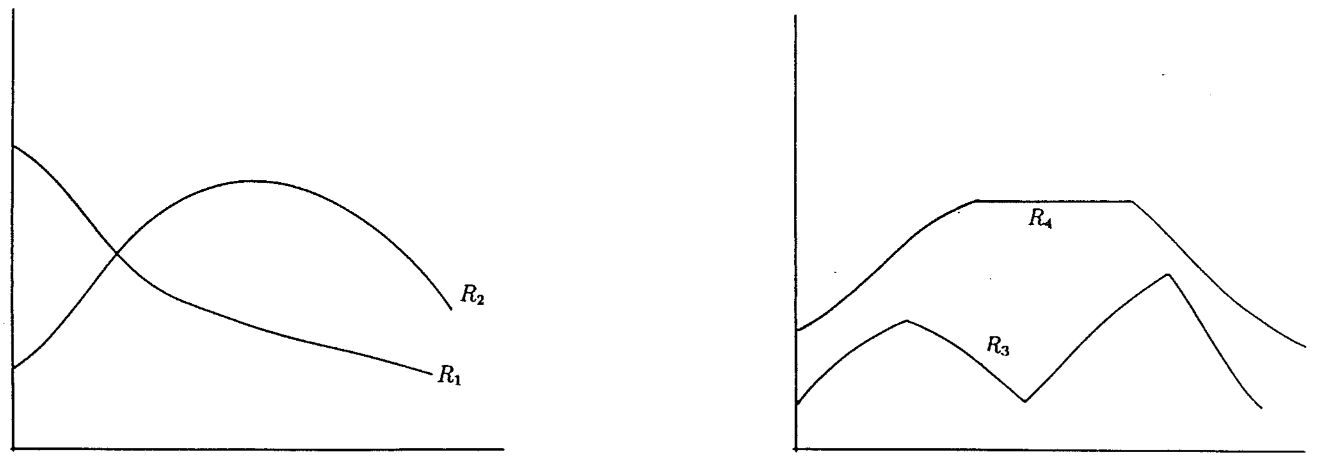


Figure 3: ***Examples of single-peaked and of non single-peaked preference relations*** (a) Single-peaked preferences. (b) Not single-peaked.

3.3 Write definitions in logical sequences

Define terms in such a way that the definition of each new concept only involves terms that have already been defined, instead of asking your readers to wait until the end of the paragraph for everything to be clarified.

For instance, state the dimensionality of the commodity space before you introduce consumers or technologies. In the standard model, a consumer is no more than a preference relation defined over a subset of that space, together with an endowment vector in the space. A technology is simply a subset of the space. In each case, it is therefore natural to specify the space, that is, the number of goods, first.

Do not write: "$\mathcal{E}$ is the class of monotone preferences, where by monotone is meant that for all x, $y \in \mathbb{R}^{\ell}_{+}$ with $x \geq y$, we have $x \succeq y$, ℓ being the dimensionality of the commodity space."

Instead write: "Let $\ell \in \mathbb{N}$ be the number of goods. The preference relation $\succeq$ on $\mathbb{R}^{\ell}_{+}$ is ***monotone*** if for all x, $y \in \mathbb{R}^{\ell}_{+}$ with $x \geqq y$, we have $x \succeq y$. Let $\mathcal{E}$ be the class of monotone preferences."

As another example, do not write:

Definition The social choice correspondence $F: \mathcal{R}^n \to A$ is ***Maskin-monotonic*** if for all R, $R' \in \mathcal{R}^n$, and all $a \in F(R)$, if for all $i \in N$, $L(a, R) \subseteq L(a, R')$, then $a \in F(R')$, where $L(a, R)$ is the lower contour set of the preference relation R_i at a, with R and R' being profiles of preference relations defined over A, some alternative space, and Maskin being an economist at Harvard.

Instead write:

Definition Let Maskin be an economist at Harvard. Let A be a set of alternatives. Given R_i, a preference relation defined over A, and a, an alternative in A, let $L(a, R_i)$ be the lower contour set of R_i at a. The social choice correspondence $F: \mathcal{R}^n \to A$ is ***Maskin-monotonic*** if for all R, $R' \in \mathcal{R}^n$, and all $a \in F(R)$, if for all $i \in N$, $L(a, R) \subseteq L(a, R')$, then $a \in F(R')$.

It is even better to first introduce the basic notation — it will probably be used in other definitions and in the proofs — and then to write the definition in a way that highligths the essential idea:

Let A be a set of alternatives. Given R_i, a preference relation defined over A, and a, an alternative in A, let $L(a, R_i)$ be the lower contour set of R_i at a. Let $\mathcal{R}$ be a class of admissible preference relations defined over A. A ***social choice correspondence*** is a mapping from $\mathcal{R}^n$ into A.

Definition The social choice correspondence $F: \mathcal{R}^n \to A$ is ***Maskin-monotonic*** if for all R, $R' \in \mathcal{R}^n$, and all $a \in F(R)$, if for all $i \in N$, $L(a, R) \subseteq L(a, R')$, then $a \in F(R')$.

You may also want to display the hypothesis and the conclusion:

Definition The social choice correspondence $F: \mathcal{R}^n \to A$ is ***Maskin-monotonic*** if for all R, $R' \in \mathcal{R}^n$, and all $a \in F(R)$, if

$$\text{for all } i \in N, L(a, R) \subseteq L(a, R')$$

then

$$a \in F(R')$$

If the hypotheses and the conclusions are simple enough, as they are in this example, displaying them may not be needed however.

3.4 Find good names for the concepts you use

When you introduce a definition, you need to spend some time finding a good name for it, a term or a phrase that suggests its content. If you use a multi-word expresssion, do not worry too much about its length. Your priority is that it should be clear to the reader which concept you are designating. In any case, you can also use abbreviated forms of the expressions. A good way of doing this is as follows:

"An allocation $z \in Z$ is (Pareto)-***efficient*** if there is no allocation $z' \in Z$ that all agents weakly prefer and at least one agent strictly prefers". Later on, you can simply talk about "efficient allocations".

For an example taken from the theory of consistency, "A solution is "(Davis-Maschler)-***consistent*** if for all games ...". Here too, in the rest of the text, you can speak of "*consistent* solutions".

Unless you use several notions of efficiency or consistency, in which case you obviously need to distinguish between them by means of different expressions, the shorter expression will be unambiguous and slightly easier to use.

Actually, I do not think that long expressions are much of a problem in a text. In a seminar, however, they may be. For that reason, you should try to find relatively short ones. Alternatively, you can use the long but descriptive expression a few times, and when you think that the concept has been absorbed by your audience, tell them that "From here one, I will only use the following shorter expression:"

Keeping in mind that a given condition may have different interpretations depending upon the context, it is preferable to use neutral expressions that would be appropriate for the various possible applications, rather than expressions that are too intimately linked to the particular set-up to which your paper pertains.

For instance, the requirement that an allocation rule be monotonic with respect to an agent' s endowment can be seen from the strategic viewpoint; it will make it unprofitable for the agent to destroy some of the resources he controls. Alternatively, it may be seen from the perspective of fairness; the agent should derive some benefit from an increase in the resources he has earned. Instead of phrases taken from game theory or from the theory of fair allocation however, use a neutral expression such as "monotonicity", (or "endowment monotonicity" if you also discuss monotonicities with respect to other parameters). And let your readers decide which interpretation they prefer.

3.5 Separate formal definitions from their interpretations

A formal model can often be given several interpretations. It is therefore of great value to separate its formal description from the interpretation you intend in your particular application.

For example, first write:

Definition Let $\mathcal{V}^n$ be a class of n-person coalitional form games. A ***solution on*** $\boldsymbol{\mathcal{V}^n}$ is a function that associates with every game $v \in \mathcal{V}^n$ a point $x \in \mathbf{R}^n$ such that $\sum x_i = v(N)$.

Then explain: "If F is a solution on $\mathcal{V}^n$, v is a game in $\mathcal{V}^n$, and i is an agent in N , the number $F_i(v)$ is usually interpreted as the "value to player i of being involved in the game v", that is, the amount that the player would be willing to pay to have the opportunity to play it. Alternatively, it can be interpreted as the amount that an impartial arbitrator would recommend the player should receive."

The advantage of this separation is that it will help your reader, (and even yourself), discover the relevance of your results to other situations that she (and you) had not thought about initially. To pursue the example I just gave, the theory of coalitional form games is also the theory of cost allocation. Some of your readers are not interested in abstract games, but only in applications; others do not care for the applications. You can catch the attention of all by first giving general definitions and then pointing out the distinct possible interpretations of your model.

Another example is the class of ***bankruptcy problems***. A bankuptcy problem is simply a point in an $(n+1)$-dimensional Euclidean space whose coordinates satisfy a certain inequality: the sum of the first n numbers is greater than the last number. The first n coordinates are interpreted as the claims of n claimants on the net worth of a bankrupt firm, this worth being given as the last number. The inequality means that there is not enough to satisfy all the claims (this is why we call this a model of bankruptcy). The class of bankuptcy problems is mathematically identical to an interesting class of taxation problems: there, the first n coordinates are the incomes of taxpayers and the last number is the amount that has to be collected; the same inequality is imposed but its interpretation is different. It means that the sum of the incomes should be sufficient to cover the cost of the project.

3.6 When you introduce a piece of notation, tell your reader what kind of mathematical object it designates

When you introduce a piece of notation, specify right away what kind of mathematical object it is, whether it is a point in a vector space, a set, a function ...

Do not write, "A pair (p, x) is a ***Walrasian equilibrium*** if..." Instead, after having defined the simplex $\Delta^{\ell-1}$ and the allocation space X, write "A pair $(p, x) \in \boldsymbol{\Delta^{\ell-1}} \times \boldsymbol{X}$ is a ***Walrasian equilibrium*** if ..."

Similarly, do not write, "The function φ is ***strategy-proof*** if ...", but "The function $\varphi: \mathcal{R}^n \rightarrow Z$ is ***strategy-proof*** if ..."

Indicating explicitly the nature of the objects that you introduce is especially important if the reader is not likely to be familiar with the concept this piece of notation designates. If you write "A triple $(\pi, x, y) \in \Delta^{(\ell-1)n} \times \mathbf{R} \times \mathbf{R}^{\ell}$ is a Lindahl equilibrium if ...", you are helping the reader become aware of the fact that π has components indexed by agents (these are the Lindahl individualized prices).

3.7 When you define a concept, indicate what the concept depends on

Do not write "The function f is ***differentiable*** at x if blah, blah, blah of x". Since what follows "if" depends on x, you should write "The function f is ***differentiable at x*** (including "at x" in the expression in boldface italics) if blah, blah, blah of x". Then, you can continue and say "The function f is ***differentiable*** if it is differentiable at x for all x in its domain".

Similarly, for an example taken from the theory of implementation, speak of a ***monotonic transformation of agent i's preferences at x_i***, and not just of a ***monotonic transformation.***

For a final example drawn from the theory of cooperative games, do not speak of the ***reduced game of a game v*** since for the reduction operation to be well-defined, you need to specify a subset of the initial set — you should of course clearly indicate this initial set of players — and some initial payoff vector. If N is the initial set of players, and $\mathcal{V}^N$ the class of games in which they may be involved, speak of the ***reduced game of*** $v \in \mathcal{V}^N$ ***with respect to the subgroup*** $N' \subset N$ ***and the payoff vector*** $x \in \mathbb{R}^N$.

3.8 If $f\colon \mathbb{R} \to \mathbb{R}$ is a function, $f(x)$ is the value taken by the function when the argument is x.

So $f(x)$ cannot be differentiable, or concave ... These terms apply to the function and not to their values. Designate the function simply by f (this is better than $f(\cdot)$. Similarly $u_i(x_i)$ is not agent i's utility function; u_i is!

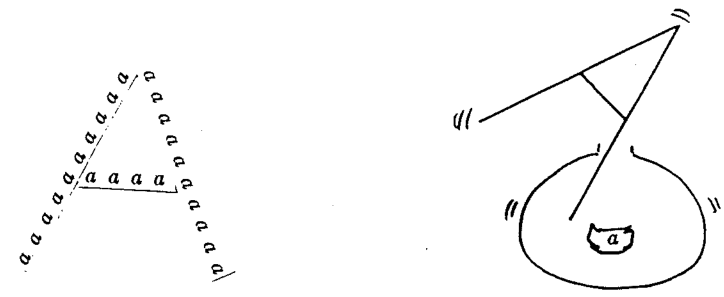
4 Notation

4.1 Choose notation that is easily recognizable

When you see a man walking down the street with a baguette under his arm and a beret on his head, you do not have to be told he is a Frenchman. You know he is. You can right away invest him with all the attributes of Frenchness, which will greatly facilitate the way you think about him. You can guess his children's names — Jacques or Maurice — and chuckle at his supposed admiration for Jerry Lewis.

Similarly, if Z designates a set, call its members z and z'..., perhaps x, y and z, but certainly not b, or ℓ. Upon encountering z and z', your reader will immediately know what space they belong to, how many components they have, that these components will be called z_i and z'_j ... If Φ is a family of functions, reserve the notation φ and $\tilde{\varphi}$, (perhaps ψ or even f) for members of the family, but certainly not α or m.

A set contains its elements, so designate it by a bigger letter than its elements. Writing "$a \in A$" makes sense; "$A \in a$" is unbelievable.



(a) (b)

Figure 4: ***Use notation that make sense.*** (a) Lots of a's fit in A. (b) If you insist on fitting A in a, you will break something.

4.2 Respect the few conventional uses of some of the letters of the alphabet that are almost universally accepted

Do not use ϵ to designate just any quantity. Keep ϵ for small quantities or quantities that you will make go to zero. Designate a generic agent by i, his preference relation by R_i, his utility function by u_i, and his endowment by ω_i. A production set is Y. Prices are p, quantities q...

4.3 State your conventions for vector inequalities

Do not let the reader guess or infer from the context what your inequality symbols mean. Define them in the text or in a footnote[2] the first time you use them. You can also give them in a preliminary section of notation.

4.4 Choose mnemonic notation for variables

If you have no problem remembering what x and ξ are, congratulations! But you have been working on your paper for several months. Unfortunately, what you call x is what your reader has been calling m since graduate school.

[2] $x \geqq y$ means $x_i \geq y_i$ for all i; $x \geq y$ means $x \geqq y$ and $x \neq y$; $x > y$ means $x_i > y_i$ for all i.

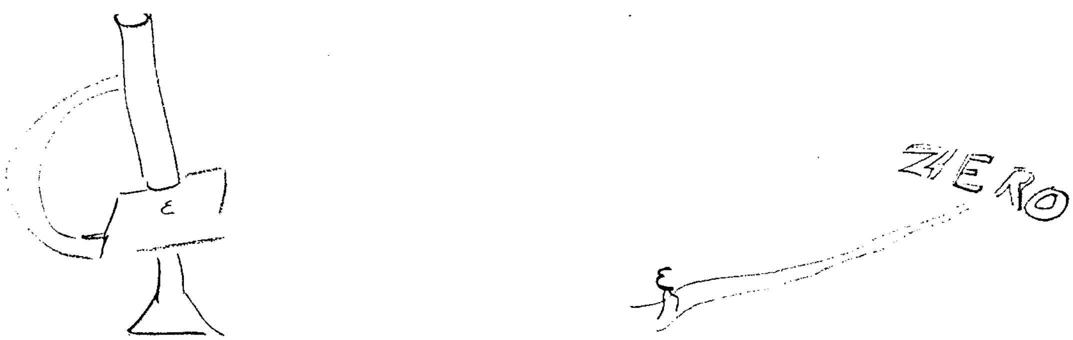


Figure 5: ***The notation ϵ rarely designates large quantities.*** The notation ϵ designates a small quantity or a quantity that goes to zero.

Designate time by t, land by ℓ, alternatives by a, mnemonic notation by $mn\ldots$. (and make sure that no two concepts in your paper start with the same letter!)

4.5 Choose mnemonic abbreviations for assumptions and properties

Avoid designating your assumptions and properties by numbers, letters, or letter-number combinations.

On page 10, where you state your first theorem, it is virtually impossible to remember what "Assumptions $A1 - A3$ and $B1 - B4$" are, but the fact that "Assumptions ***Diff***, ***Mon***, and ***Cont***" refer to differentiability, monotonicity, and continuity will be obvious to a reader who starts there. Choose these abbreviations carefully: If you write ***Con***, we may not know whether you mean continuity or convexity — so, write ***Cont*** or ***Conv***. The cost to you is one extra strike on your keyboard. But it will save us from searching at the beginning of the paper to find which property was intended. Admittedly, the possibility of naming each assumption in a way that will suggest its content does not always exist, especially in technical fields.

In axiomatic analyses, many authors refer to axioms by numbers or abbreviations, but I do not see any advantage to that. The argument that it saves space is not very convincing given that you will not shorten a 20 page paper by more than 5 lines, and it certainly does not save time to your reader:

if different typeface is used for the axioms, which I strongly recommend, (for instance, italics), each of the axioms stands out from the rest of the text and it is perceived globally, as a unit: it is not read syllable by syllable. An alternative way to achieve this visual separation of the axioms from the text is to capitalize them.

Certainly, do not use abbreviations in a section heading.

4.6 Do not use notation that you can't pronounce or draw on the board

Many people have trouble recognizing a number of the capitalized script letters, especially when they are hand-drawn on the board. Avoid them. If you have trouble distinguishing between Greek letters, avoid them too. (Actually, it seems to me that you should know them. Get your Greek classmates to coach you.)

If you are Japanese or Korean, do not use ℓ and r in the same paper. If you are Greek, avoid Greek letters, since you will find it difficult to mispronounce them correctly. If you are French, eliminate all words containing the "th" sound or beginning with the letter "h" (je plaisante, voyons!).

If you can't say "substitutability", assume that the goods are complements instead, or give up on demand theory. If you have trouble with "heteroscedasticity", econometrics is not for you.

4.7 Do you really need all these subscripts and superscripts?

Avoid multiple subscripts and superscripts. If you have only two agents, call their consumption bundles x and y, with coordinates x_k and y_k (instead of x_1 and x_2, with coordinates x_{1k} and x_{2k}). In a seminar, watch out for the sliding superscripts that end up as subscripts.

4.8 Do not bother introducing a piece of notation if you will use it only once or twice

There is no point in introducing a new piece of notation if you will hardly ever use it. How many times should a concept be used to justify introducing a notation for it? Twice? Three times? I will let you decide. Certainly, do not bother introducing a piece of notation if you never use it!

Do not define in footnotes notation that is likely to be unfamiliar to your reader, and that you will use later in the main body of the paper.

4.9 A sequence is not a set

Do not write $\{x^k\} \subseteq X$. Do not write $\{x^k\} \in X$ either.

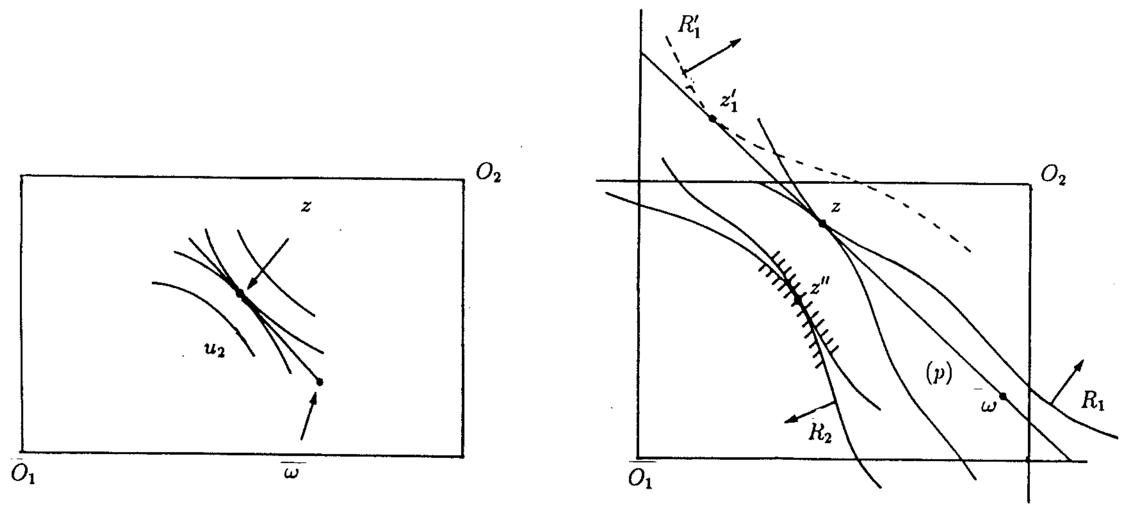
5 Figures

Figures are very important to lighten the text and to convey the main concepts. Take the time to make a few.

5.1 The Edgeworth box

The Edgeworth box is an extremely useful expositary device. We use it to introduce most of the concepts of equilibrium theory, welfare economics, implementation ..., and to give the main ideas of proofs. A common mistake is to draw it as a simple rectangle, and this has the unfortunate consequence of obscuring boundaries issues. The ***feasible set*** is of course adequately represented by the rectangle. However, many properties of allocation rules involve information outside of the rectangle and a number of allocation rules depend on such information, even though they only take values within the feasible set.

Starting from two copies of the two-dimensional commodity space, each containing relevant information about the preferences and the endowment of one of two agents, agent 1 and agent 2, the Edgeworth box is constructed by rotating agent 2's consumption space 180 degrees and sliding it so that the two endowment points meet. When that happens, the rectangle has the right size. Then, the two pairs of axes extend beyond the rectangle; so do many of the indifference curves, and if your purpose is to explain the notion of a Walrasian equilibrium, the budget sets. In fact, if prices are not equilibrium prices, agents may well maximize their preferences on their budget sets at points that are not in the Edgeworth box (Agent 1 in Figure 6b at z_1' if his



(a) (b)

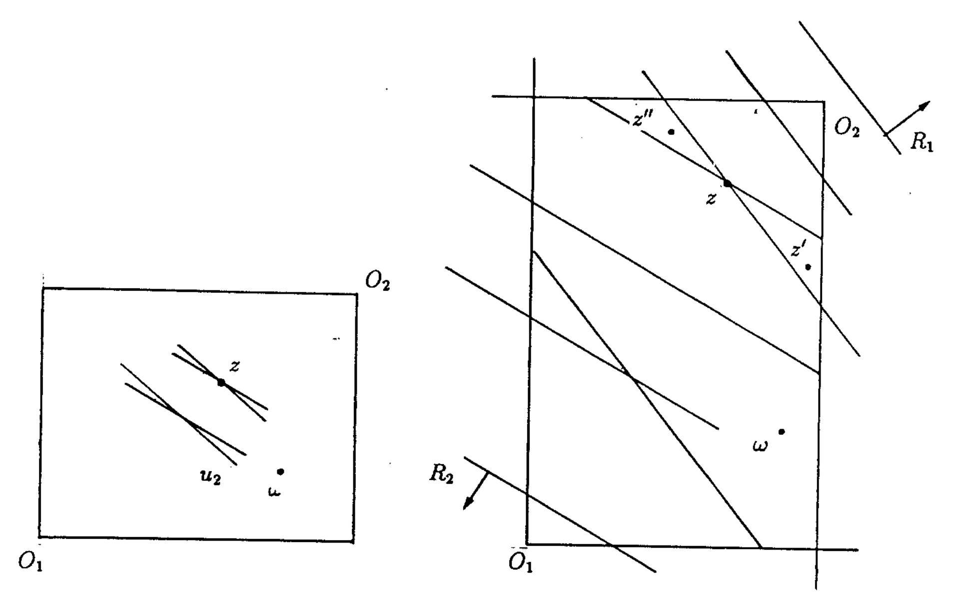
Figure 6: ***The wrong and the right ways to draw an Edgeworth box.*** (a) Do you call that an Edgeworth box? (b) ***That's*** an Edgeworth box.

preferences are R_1').[3]

Label a few sample indifference curves for each agent. If you assume convexity of preferences, as you probably will, and if in fact you draw the indifference curves strictly convex, it will be unambiguous who owns which indifference curve. If you do not make that assumption — you may very well work with linear preferences or non-convex preferences — it will not always be so clear. In Figure 7, where indifference curves are linear, you help the reader remember that agent 1 is the one with the relatively greater affinity for good 1 (his indifference curves are steeper) by (i) representing one of his indifference curves in a region that could not be part of agent 2's consumption space since it would correspond to negative consumptions of some of the goods (this is the one to the North-East of O_2), (ii) labelling a few of his indifference curves with the notation R_1, and (iii) indicating the direction of increasing preferences with arrows.

I recently became curious whether Edgeworth himself would pass my "Edgeworth box" test and I looked up his *Mathematical Psychics* (1881). Figure 8a is the closest to an Edgeworth box that I found (on p.28), and

[3]Remember that it is one of the merits of the Walrasian allocation rule that agents need not know the aggregate feasibility constraints when solving their individual maximization exercises. In order to talk about Maskin-monotonicity, you also need to be careful about boundaries and to draw indifference curves that extend beyond the rectangle.



(a) (b)

Figure 7: ***The wrong and the right ways to draw Edgeworth boxes in the case of linear preferences.*** (a) That is not an Edgeworth box any more than Figure 6a was. (b) Draw indifference curves that extend beyond the feasible set (Is this the first Edgeworth box in the history of economic analysis that is taller than it is wide?)

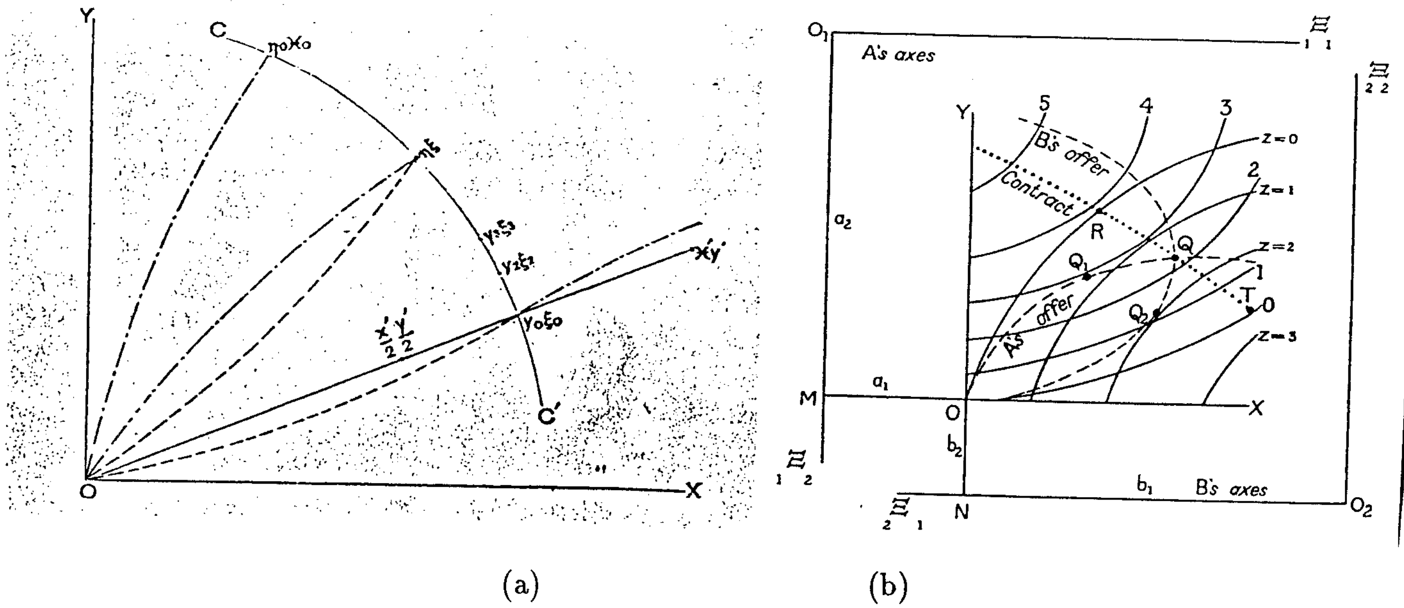


Figure 8: ***Edgeworth's and Bowley's boxes.*** (a) Edgeworth's box. (b) Bowley's box.

according to all the secondary sources that I consulted, there is nothing in Edegworth's other works that remotely resembles an Edgeworth box.[4]

Incidentally, when you explain what a Walrasian equilibrium is, you should not draw your indifference curves convex. Convexity becomes relevant only when you need to prove existence. There is a general principle here: ***introduce the basic concepts of your theory without imposing the extra assumptions that you will need to invoke for the rest of the analysis.***

Label your figures as completely as possible. Label the allocations, the

[4] V. J. Tarascio ("A correction: on the genealogy of the so-called Edgeworth-Bowley Diagram", in the *Western Economic Journal* 10, (1972), 193-97) and W. Jaffé ("Edgeworth's contract curve: Part 1. A propaedeutic essay in clarification" in *History of Political Economy*, 343-59) had looked this up before me: Jaffé writes "It may come as a surprise, to those who rely exclusively on secondary sources of information on past analytic achievements, to learn that nowhere in Edgeworth's published writing is there anything resembling what is so frequently referred to as an "Edgeworth box diagram." To my knowledge, the earliest adumbration of a true box diagram is found on page 288 (Fig. 16) of the fifth instalment of Vilfredo Pareto's article, "Considerationi sui principii fondamentali dell'economia pura," which appeared in October 1893, twelve years after the publication of Edgeworth's *Mathematical Psychics.*" As far as Bowley is concerned, there is one box in *The Mathematical Groundwork of Economics* (1924) (Figure 8b) but it does not quite meet my standards either, and it should not meet yours! I haven't found Pareto's box yet. Will it pass the test?

supporting prices, a few indifference curves (some redundancy might be useful), the endowments. To indicate efficiency of an allocation, it is sometimes convenient to shade the upper contour sets in the neigborhood of that allocation (see allocation z'' in Figure 6b). On the other hand, avoid unnecessary arrows such as the ones pointing to ω and z in Figure 6a. You can most often position your labels close to the items they designate without creating ambiguities. Use arrows only if the Figure would get too crowded, in particular if the label is too long.

5.2 The Kolm triangle

Many of the concepts of the theory of public good allocation can be explained graphically by means of the Kolm triangle, the counterpart of the Edgeworth box. Learn to use it. You will not regret it.

The Kolm triangle gives the set of feasible allocations of a two-agent economy with two goods, one private good and one public good, under the assumption that the public good is produced from the private good by operating a linear technology. If the agents' endowments of the private good and public good are $\omega_1 = (\omega_{1x}, 0)$ and $\omega_2 = (\omega_{2x}, 0)$, and units of measurement are chosen so that each unit of the private good allows the production of one unit of the public good, a feasible allocation is a list $(x_1, x_2, y) \in \mathbf{R}^3_+$ such that $x_1 + x_2 + y = \omega_{1x} + \omega_{2x}$. The set of feasible allocations can be put in one-to-one correspondence with the points of an equilateral triangle of height $\omega_{1x} + \omega_{2x}$ (Figure 9).

6 Language

6.1 Do not assume that your readers are familiar with the definitions you use

There is rarely complete agreement on definitions in the literature. Even apparently standard terms are understood differently by different people.

"Core", "public goods", and "incentive compatibility", are just examples of terms that are common enough. Nevertheless, you should define them carefully. A term such as "rationality" is often used in formal developments without a definition being given. Do not make such a mistake.

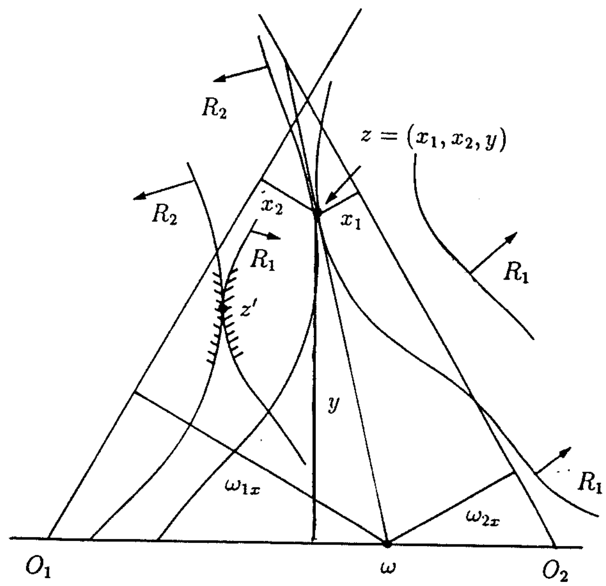


Figure 9: ***The Kolm triangle.*** In order to use the Kolm triangle, you need to represent preferences in slanted axes. In the figure, the point $z = (x_1, x_2, y)$ is a Lindahl allocation: x_1, the distance of z to the left side of the triangle, is agent 1's consumption of the private good; x_2, the distance of z to the right side of the triangle, is agent 2's consumption of the private good; y, the distance of z to the basis of the triangle is the level of the public good. The allocation z' is an efficient allocation.

6.2 Do not use several terms or phrases to designate the same concept

Refer to a given concept by only one phrase even if you have several natural choices. Make one and stick to it. Perhaps indicate in parenthesis next to your definition, or in a footnote, the other terms that appear in the literature. When you first discuss the general idea, you may still use different terms in order to vary language and avoid repetitions, but as soon as the concept has been given a formal mathematical definition and baptized, you should only use its name.

The terms "game", "game form", "mechanism" are used by different authors to designate the same concept. Then specify which one you will use: "A ***game form***[5] is a pair $(S, h)\dots$"

The terms "preference ordering", "preference relation", "utility", "utility function" are used interchangeably by some. But you should not.

In areas where language has not settled yet, you may have more choices. Do not take that as a license to go back and forth between several terms. Instead, think of it as your chance to help steer terminology in the right direction.

Do not hesitate to challenge dominant terminology and usage if you find it inadequate. If your paper is a follow-up on someone's published work (as it almost certainly will be), do not feel compelled to use the same notation or language if it was not well chosen.

For instance, there is no reason why the term "fair" should be used to designate allocations that are both envy-free ***and*** efficient. In common language, the term makes not reference to efficiency. The term "endowment" suggests resources that are owned "initially", prior to exchange, so the expression "initial endowment" is somewhat redundant. Just refer to agents' endowments. If, as Shapley, you do not like "block", (as used in the definition of the core) follow his advice and block "block". If you disagree with Shapley, be bold and block him.

[5]The terms "game" or "mechanism" are sometimes used.

6.3 Do not use the term "vector" unless you will perform vector space operations

A "vector" is an element of a vector space. If you are talking about some collection of objects taken from some set, the appropriate terms are "lists", "ordered lists", or "profiles".

The notation $(R_1, \ldots, R_n)$ designates an ordered list of preference relations (or a preference profile), not a vector of preference relations (you will probably not compute $(R_1 + R_2)/2)$ and $(s_1, \ldots, s_n)$ designates a strategy profile. The latter may of course be a strategy vector. For instance, in a game form designed to implement a solution to a public goods problem, a strategy may be a vector of public good levels, and the outcome function may select the average of the announced vectors. Consumptions bundles are usually vectors. You often have to compute averages of consumptions or multiply them by two.

6.4 Avoid long sentences

A good way to avoid ambiguities and grammatical errors, and to force yourself to write definitions in a logical sequence, is to mainly write one-clause sentences:

"Let (S, h) be a game form. Let $\mathcal{R}^n$ denote a class of admissible profile of preferences. Given $R \in \mathcal{R}^n$, a ***Nash equilibrium of the game*** $(\boldsymbol{S, h, R})$ is a point $s \in S$ such that for all $i \in N$, and all $s'_i \in S_i$, we have $h(s'_i, s_i) R_i h(s)$. If $s \in S$ is an equilibrium, $h(s)$ is its corresponding ***equilibrium outcome.*** Let $\boldsymbol{E(S, h, R)}$ designate the set of equilibrium outcomes of the game (S, h, R). The game form (S, h) ***implements the correspondence*** $\varphi\colon \mathcal{R}^n \longrightarrow Z$ if for all preference profiles $R \in \mathcal{R}^n$, we have $E(S, h, R) = \varphi(R)$.

You might think that your chance of a Nobel prize in literature will not increase much by this staccato style. Yet I could name several grammatically impaired writers, who had never used subordinate or relative clauses, and nevertheless got to make the trip to Stockholm!

6.5 To weed out gallicisms, nipponisms, sinicisms ...

get the help of a native gardener.

Get a good dictionary. I have a recommendation: it is ***The American Heritage Dictionary of the English Language***, (Houghton Mifflin). Although it is one of the few not to invoke Webster's name, I think it is vastly superior to any of its comparably priced competitors to be found in college bookstores.

6.6 Be consistent in your writing style

Do not switch back and forth between first person singular, first person plural, and passive forms.

If you write: "In section 3, ***I show*** that equilibrium exists. In Section 4, ***we establish*** uniqueness. To prove these results, ***it is assumed*** that preferences are strictly convex and have infinitely differentiable numerical representations. ***Section 5 concludes.***" your readers will think you need psychiatric help. Are your "I" or "we"? Is it because those assumptions are embarassing that you hide behind the passive form? (Believe me, we have all made embarassing assumptions!) And why do you let Section 5 conclude when you did all the work?

The passive form is found awkward by me and our advice here is to have it replaced. "I" is perhaps too personal. Between "I" and "we", I choose "we", but if you choose "I", I will respect your choice.[6]

Similarly, do not travel back and forth between present and future tenses. Do not write "First, ***I prove*** existence. Then ***I will apply*** the theorem to exchange economies. ***I conclude*** with open questions." In most cases, using the present tense throughout, even in describing past literature, is just fine.[7]

6.7 Choose the sex of your economic agents once and for all

Flip a coin. If it is a boy, rejoice! If it is a girl, rejoice! And don't subject them to sex change operations from paragraph to paragraph. Two-person games are great for sexual equality. Make one player male and the other

[6]As a reader, I rather like the "I" form, which is more engaging, but I am not comfortable using it in formal papers. I use "I" here only because of the informal style that I have chosen for this paper.

[7]Grammarians call that the "narrative (story-telling) present".

female. This will actually facilitate talking about the game and it will help your reader keep in his/her mind (sorry! I meant her mind) which player you intended. And it will save you from "he or she", "him or her", "his or her"!

6.8 Do not start a sentence with a piece of mathematical notation

Journal editors will red-pencil you if you start a sentence with a piece of mathematical notation. I agree with them that it does not look good, especially if the notation is lower case. "x designates an allocation" is not pretty. "I is the set of individuals" is not as bad because I is uppercase (but what a grammatical provocation!!).[8] "The variable x designates an allocation" is what editors will insist on.

6.9 Be consistent in your choice of running indices

If $N = \{1, \ldots, n\}$, do not write interchangeably "for all $i \in N$", "for all $i \in \{1, \ldots, n\}$", "for all $i = 1, \ldots, n$". Pick one formula and stick to it.

In most situations, the quantification on the set of agents is clear and you can skip it altogether, and simply write "for all i". This helps limiting the numbers of symbols. In general though, it is good to indicate membership explicitly. For instance, instead of "There exists z for which ..." write "There exists $z \in Z$ for which ...". Therefore, for consistency of style and esthetic reasons, when everything else is explicity quantified, it bothers me a little bit not to see membership indicated for the set of agents, even if it unambiguous where they come from. So instead of "For all i such that ..." I would write "For all $i \in N$ such that ...",

[8] "I am the set of individuals" does sound a little pretentious though!

7 Writing proofs

7.1 The optimal ratio of mathematics to English in a proof varies from reader to reader, but there is a consensus on a middle range

A proof written entirely in English is often not precise enough and too long; a proof written entirely in mathematics is impossible to understand, (unless you are a digital computer of course). Modern estimation techniques have shown that the optimal ratio of mathematics to English in a proof lies in the interval [52%, 63.5%]. Choose the point in that interval that is right for you and stick to it.

The theorems themselves should be stated in as simple English as possible. The reader who wants to know more than the probably somewhat informal description of the results that you gave in your introduction, but does not have much more time to invest in your work, will be able to gain a much more precise understanding of your contribution at a very small cost. I admit that this is sometimes difficult to achieve and for technical papers it is probably impossible, but you should try.

7.2 State your assumptions in order of decreasing plausibility or generality

When you introduce your assumptions, start with the least controversial ones, and write them in order of decreasing plausibility.

For utility functions, do not write

$A1$ – u_i is strictly concave

$A2$ – u_i is bounded

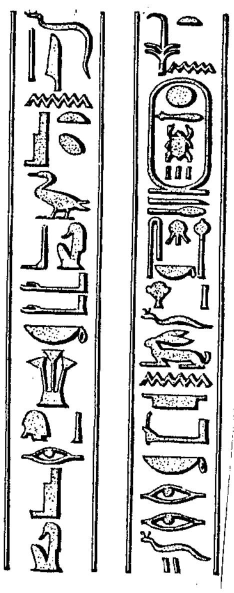
$A3$ – u_i is continuous

Instead, write

$A1$ – u_i is continuous

$A2$ – u_i is bounded

$A3$ – u_i is strictly concave



(a)

All I have to to is deduce, from what I know of you, the way your mind works. Are you the kind of man who would put the poison into his own glass, or into the glass of his enemy? [...] Now a great fool [...] would place the the wine in front of his own goblet, because he would know that only another great fool would reach first for what he was given. I am clearly not a great fool, so I will clearly not reach for your wine [...] We have now decided the poisoned cup is most likely in front of you. But the poison is powder made from iocane and iocane comes only from Australia and Australia, as everyone knows, is peopled with criminals and criminals are used to having people not trust them, and I don't trust you, which means that I can clearly not choose the wine in front of you [...] But again, you must have suspected I knew the origins of iocane, so you would have known I knew about the criminals and criminal behavior, and therefore I can clearly not choose the wine in front of me [...] You have beaten my Turk, which means you are exceptionally strong, and exceptionally strong men are convinced that they are too powerful ever to die, too powerful even for iocane powder, so you could have put it in your own cup, trusting on your strength to save you; thus I can clearly not choose the wine in front of you [...] But you also bested my Spaniard, which means that you must have studied, because he studied many years for his excellence, and if you can study, you are clearly more than simply strong; you are aware of how mortal we all are, and you do not wish to die, so you would have kept the poison as far from yourself as possible; therefore I can clearly not choose the wine in front of me". (William Golding: "The Princess Bride", Ballentine Books, NY 1973.)

(b)

Proof: This follows from the inclusion $\varphi \subset P$, Part (i) of Proposition 1, and Lemma 1 applied to φ. Q.E.D.

(c)

Figure 10: ***The ratio of mathematics to English in a proof should be in the interval [52%, 63.5%].*** (a) This proof has too much math. The density of mathematical symbols makes it virtually impossible to understand. (b) This game-theoretic proof has too much English; it is not precise enough and it is too long (and not surprisingly, two paragraphs down, the character who produced it is dead). (c) This proof is just right, said Goldilocks, and that is the one she read. It is indeed pleasantly short and clean.

7.3 Group your assumptions in categories

Introduce your assumptions in groups of related assumptions.

For a general equilibrium model,
  $A1 - A5$ pertain to consumers,
  $B1 - B6$ pertain to firms.
For a game,
  $A1 - A3$ pertain to the structure of the game,
  $B1 - B2$ pertain to the behavior of the players.

7.4 Divide proofs in steps or cases, clearly identified

Divide your proofs into meaningful conceptual units. Use paragraphs to indicate the steps. Number them: Step 1, Step 2 ... Even better, if the proof is sufficiently complex, give each step a title indicating its content. Use indentations and double indentations to indicate structure:

Step 1. The domain of the correspondence φ is compact.

Claim 1a. The domain is bounded.

Claim 1b. The domain is closed.

Step 2. The correspondence φ is upper semi-continuous.

If a proof is long, you may have to number the statements that appear in it and that you use repeatedly. Then, you can refer to them by numbers. Unfortunately, this quickly increases the complexity of the proof, (or rather, how complex it looks). If you do this, make sure that you only number the essential statements. For instance, if you end a sentence by establishing a statement that is used as hypothesis in your next sentence, and if it not used elsewhere, there is no need to number it.

7.5 Save on mathematical symbols

Do not introduce mathematical symbols that are not necessary.

For instance, the bounds of summation or integration are often unambiguous. There is then no need to indicate them. Do not write $\sum_{i=1}^{n} x_i$, $\sum_{i \in N} x_i$,

$\sum_i x_i$, $\sum_N x_i$, $\sum_{i=1,\ldots,n} x_i$ when, in most cases, $\sum x_i$ is perfectly clear. (I assure you, when you write $\sum x_i$, your readers will be unanimous in assuming that you are summing over i.)

7.6 Do not collapse two or three similar statements into one by indicating the variants in parenthesis

Consider the following definition:

"The function $f\colon \mathbf{R} \to \mathbf{R}$ is decreasing (increasing; strictly increasing) if for all x, $y \in \mathbf{R}$, with $x > y$, $f(x) \leqq f(y)$ (respectively $f(x) \geqq f(y)$; $f(x) > f(y)$).

The only way to be sure we understand this triple definition is to read it three times (once for decreasing, once for increasing, and once for strictly increasing), and yet it is pretty simple. More complicated statements in that format cannot be understood at all.

I also have a lot of trouble with "and\or" (or is it "or\and"?).

7.7 Do not put quantifiers in the middle of a sentence in English

A sentence such as

"Blah, blah, blah, $\forall x \ldots$ blah, blah, blah $\exists y \ldots$blah, blah, blah."

does not look good. Instead, pull out the mathematical statements from the English text and display them on separate lines, as follows:

"Blah, blah,..., blah, blah,

$$\forall x \ldots, \exists y \ldots$$

blah, blah, blah."

Of course, the quantifications should be unambiguous. Remember also that taking the negation of a properly written mathematical statement, with no hidden quantifications, is a trivial operation.

The only mathematical symbols that do not bother me in a text in English are $\geqq$, $\in$, and $\subseteq$, (and the other symbols of the same kind such as the strict inequalities, the strict inclusions ...), read as prepositions or verbs.

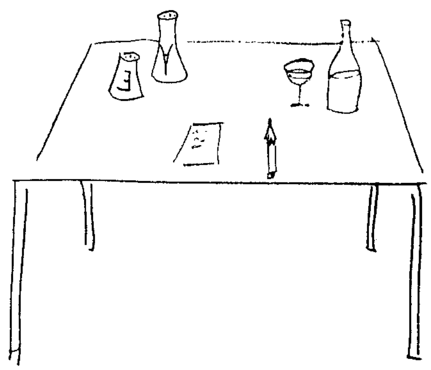


Figure 11: ***Quantifiers as a spice.*** Sprinkle your proofs with quantifiers. They will taste better.

"Blah, blah, blah, since $x \geqq y$, and $x \in A$, and therefore, blah, blah, blah, f is continuous", is OK.

$\exists$ situations where it is convenient to quantify once and $\forall$[9]. For instance, you can open your proof by stating: "In what follows, S denotes an arbitrary element of Σ". Then two paragraphs down, the requirement that the function $F: \Sigma \to \mathbf{R}^2$ satisfies $F(S) > 0$ for all $S \in \Sigma$, can simply be written as:

Positivity: $F(S) > 0$.[10]

It will be understood as:

Positivity: For all $S \in \Sigma$, $F(S) > 0$.

[9]See the problem with starting a sentence with a piece of mathematical notation (5.8)! When I wrote earlier that you should not put quantifiers in the middle of a sentence in English, I should have added: do not put them at the beginning or at the end either!

[10]Or "$F > 0$". By the way, do not place your footnote markers at the end of mathematical expressions, as they will look like exponents. Placing them beyond the punctuation mark, as the typographical convention requires, and as I have done here, helps, although logic would sometimes dictate that the marker be attached to a word inside the clause (or the sentence) that ends with the punctuation mark. Compare with the marker for the previous footnote. Its position does not create any ambiguity, and I am sure that you do not think that my intention was to raise the existential quantifier to any power, but still it does not look pretty. The same problem arises with quotation marks. I have written "$F > 0$". This is according to logic and against the rule, which is to write "$F > 0$." Given that quotation marks look like double prime, I admit that placing them after the period is better and that is what I should have done.

7.8 Gather all the conditions needed for a conclusion before the conclusion instead of distributing them on both sides

Do not write "If A and B, then D since C." or "If A and B, then D. This is because C." Instead, write "If A, B and C then D."

Especially for long statements, it helps to visually separate the hypotheses from the conclusions by "then", "we have", "it follows that"... If you write "Since A, B, C and D," we will not be sure whether you mean "Since A, then B, C and D," or "Since A and B, then C and D."

Mathematical statements usually look better when all the quantifications appear together, preferably at the beginning, instead of being distributed on both sides of the predicate. For instance, instead of "For all $x \in X$, we have $x_i > y_i$ for all $i \in N$," write "For all $x \in X$ and for all $i \in N$, we have $x_i > y_i$."

7.9 A certain amount of redundancy is useful, but do not overdo it

A certain amount of redundancy in your explanations is useful, but do not overdo it.[11]

For instance, giving an informal description of the main steps of a proof is not strictly necessary but it might be quite helpful. If you do however, do not do it within the proof itself. Do it outside of the proof, which should remain concise, and preferably before so as to better prepare us for the hard parts.

Similarly, when you state a difficult definition, it will help to give an informal explanation in addition to the formal statement. Here too, do that before the formal statement, not after it, as it will greatly facilitate grasping the formal statement. It will also save your readers[12] frustation: it is

[11]By that I mean that it is sometimes helpful to explain an argument in several different ways, but that you should not explain the same things in ***too many*** different ways. (You have to agree, this footnote is redundant).

[12]Did you notice that I sometimes refer to "your reader" (in the singular), sometimes to "your readers" (in the plural), sometimes to "us", your readers? This is an example of an inconsistency of style that should be avoided. Just like this "should be avoided" since I have throughout addressed you, my reader; therefore, I should have written, "that you

indeed quite annoying to spend time trying to figure out the meaning of a complicated definition when it is first given, only to discover two paragraphs down that you were willing to help after all. Same thing with figures by the way. If you have provided a figure to help us follow a proof, thank you very much, but why didn't you say so right at the beginning, so that we could identify on the figure the variables as you first introduced them and follow your argument on it? This is especially important because it is very hard to control where figures end up (my computer seems to always make those kinds of decisions), and a figure illustrating a proof might very well appear on the page that follows the proof instead of next to it.

7.10 Be specific about which assumptions, or which parts of assumptions, you need for each step

Do not write "The above assumptions imply that f is increasing" if you need only ***some*** of the above assumptions to prove that f is increasing. Instead, write "Assumptions 3 and 4 imply that f is increasing", or even better, if you do not need part (i) of assumption 4, write "Assumptions 3 and part (ii) of Assumption 4 together imply that f is increasing".

Do not write "A and B imply C and D," if in fact "A implies C and B implies D." At a very small additional cost, you can be much more precise.

7.11 If you have several results that are variants of each other, state them in the same format so as to make their relation to each other immediate

If you first state

Theorem 1 *Under A, B, and C, then D and E.*

do not write your next theorem, which differs from Theorem 1 in that C is replaced by C' and E is replaced by $\tilde{E}$, as

Theorem 2 *Suppose A and B. In addition, consider the class of economies satisfying C'. Then D. Also, $\tilde{E}$ holds.*

should avoid".

Instead, use a paralell[13] format:

Theorem 2 *Under A, B, and C', then D and $\tilde{E}$.*

The relation between Theorems 1 and 2 will then be obvious, and your reader will discover it by simply scanning your paper. By choosing a different format, you are forcing her to actually read, and spend time making the comparisons, hypothesis by hypothesis and conclusion by conclusion, that are needed for a good understanding of how the theorems are related.

7.12 Verify the independence of your hypotheses and for each of them check whether you could proceed without it

Do not write "Under assumptions A, B, and C, then D," if A and B together imply C, or if A and B together imply D.

Having put together a toy for my daughter, I discovered some parts left in the box. Either these were replacement parts, or I had done something wrong (I will not tell you which). Similarly, after QED, look in the box for stranded hypotheses. You might have made a mistake, but you might also be pleasantly surprised to find that you can actually prove your theorem without differentiability. Wouldn't you be thrilled to discover that your result applies to Banach lattices (which you did not even know existed two weeks ago), while you thought you were working in boring n-dimensional Euclidean space?

Sometimes, you will be unable to prove that a certain hypothesis is necessary for the proof although you will be unable to conclude without it either. This is an uncomfortable situation that should keep you up late at night.

A given hypothesis may actually be the conjunction of several more elementary ones. Then, try to proceed without each of the components in turn. For instance, if you have shown that "Under compactness of the set X, conclusion C holds", do not only check that without compactness, C might not hold anymore. Instead, check whether "Under boundedness, C holds" and whether "Under closedness, C holds".

[13]This incorrect spelling of paralell (Darn, I did it again!) is an unfortunate consequence of my having finally mastered the spelling of A. Mas-Colell's name, (the name for which, in my estimation, the ratio of occurences of incorrect to correct spellings is the highest in the profession). Do spell names correctly!

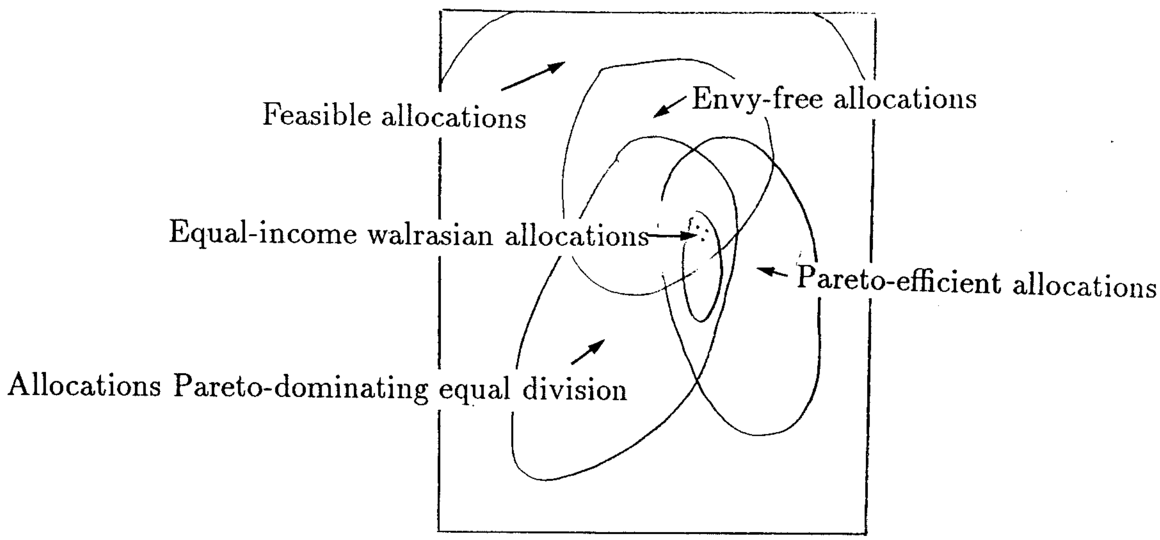


Figure 12: ***How to indicate logical relations between concepts.*** The set of feasible allocations is so large in relation to the set of Pareto-efficient allocations that its bubble does not even fit in the page. There are continua of Pareto-efficient allocations and of envy-free allocations but typically a finite number of Walrasian allocations.

7.13 Indicate logical relations between assumptions and groups of assumptions

Designate assumptions by names that help keep these logical relations in mind. *Strong monotonicity* should imply *monotonicity*, a condition that in turn should imply *weak monotonicity.*

If you have many conditions, and many logical relations between them, it is helpful to present these relations in the form of diagrams.

The best way to do this is by means of Venn diagrams, each bubble representing the set of objects satisfying one of the conditions.

When you draw two partially overlapping bubbles associated with conditions 1 and 2, it is because you have identified:

1. At least one object satisfying condition 1 but not condition 2
2. At least one object satisfying condition 2 but not condition 1
3. At least one object satisfying both.

You can also use a diagram of arrows and crossed arrows. The advantage of bubbles is that by drawing them of appropriate size, you can convey additional information about the relative strength of conditions. If condition

A is much stronger than condition B, draw a much smaller bubble for A. If you prove a theorem under assumption B, which is weaker than assumption A used in some previous literature, your reader will certainly want to know how significant your weakening is. You need to give her some sense of it.

A disadvantage is that for the diagram not to be misleading, you need to figure out ***all*** of the logical relations between your conditions. This is also the advantage: you need to figure out ***all*** of the logical relations between your conditions! You will not regret doing the work. When you use arrows, and you do not link conditions, you unambiguously tell your reader that you do not know how they are related. That option does not exist with Venn diagrams.

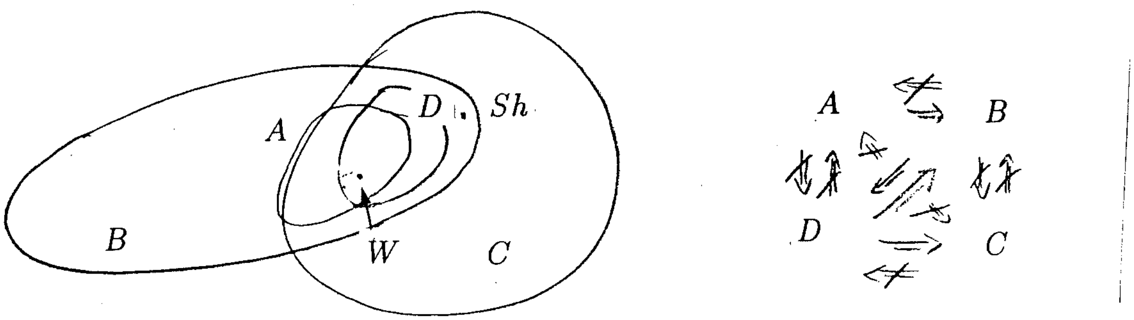
When you use Venn diagrams, you can sometimes draw the bubbles in a way that suggests some of the structure of the sets they designate: if the set is convex, draw a convex bubble; if it is defined by a system of linear inequalities, give it a polygonal boundary; if it is a lattice, draw it as a diamond ...

7.14 Make sure that there are objects satisfying all the assumptions that you are imposing

Have at least one example of an object satisfying all of the assumptions that you are imposing. After stating that you will work with economies satisfying Assumptions 1 — 10, give an example of an economy that does satisfy all of these assumptions (try Cobb-Douglas; it will probably work). If the class of objects satisfying your assumptions is empty, ***any*** statement you will make about all of the objects in the class will be mathematically correct, but of limited usefulness.

7.15 If you prove that "A and B together imply C," do not limit yourself to that statement.

If you prove that "A and B together imply C," you should find out whether similar statements hold with A replaced by the closely related conditions A', A^0, and $\tilde{A}$, or B replaced by B' and B^*, or C replaced by C^0. Knowing statement P is not enough. Discover as many statements as possible that are close to P and are also true, and statements that are close to P but are



(a) (b)

Figure 13: ***Venn diagrams convey much more information than arrows.*** The two diagrams seem to convey the same information about logical relations but the Venn diagram (a) allows you to show that "few" objects satisfy condition A but not condition C, whereas many satisfy condition B but not condition A. It also allows you to place individual objects, such as the Walrasian rule or the Shapley value, in the appropriate places. (b) I made this diagram of arrows deliberately messy to strengthen my claim that Venn diagrams are better than diagrams of arrows, but even if I had been fair, bubbles would have looked better.

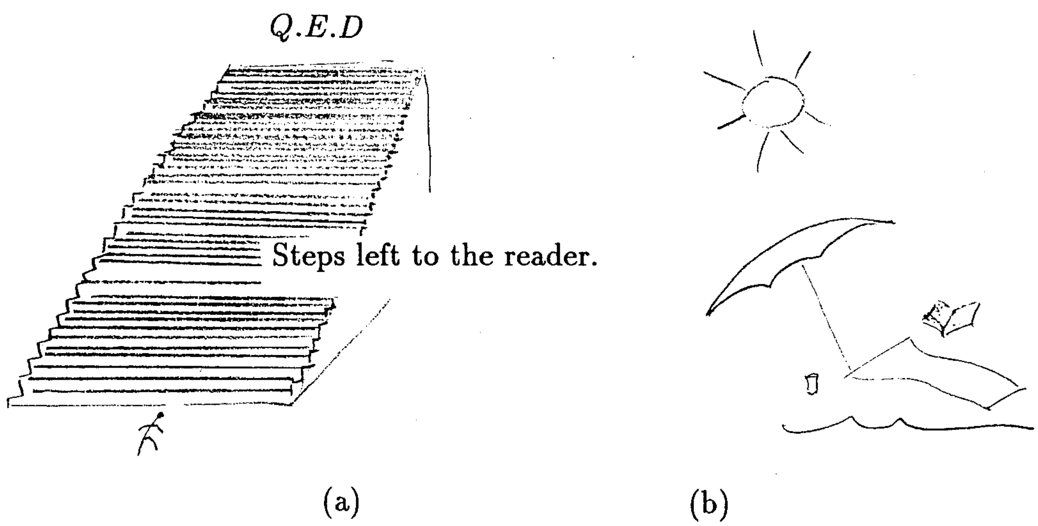


Figure 14: ***A reader to whom you leave too many steps will pick up something else to read.*** Do not leave steps to your reader. If you had the choice, would you go to (a), the reading of a proof where many steps are left to the weary reader, or to (b), the reading of a cheap novel where virtually no steps are ever left to the reader.

not true. You might want to indicate as a remark the main variants of your theorem but you should probably keep to yourself most of them.

It is equally useful to understand the multiplicities of statements around the one you are proving that could be true but are not, as the statement that you are proving. Maybe it is more useful.

7.16 Do not leave too many steps to the reader

Give complete arguments. Some steps in a proof may involve standard manipulations and detract from your main point. Perhaps they should not be in the text. Then, put them in an appendix. Your reader may not be familiar with a derivation that ***you*** have seen and done hundreds of time. Just having the option of checking the length of a step and recognizing the names of familiar theorems on which it is based will be helpful to her, even if she does not actually read all the details.

7.17 If you think a step is obvious, look again

Do not think that your mistakes necessarily occurred in the hard parts of your proofs, (I should say, what you think are the hard parts of your proofs). They may very well have hidden in (what you think are) the easy parts, taking advantage of your overconfidence. After completing your paper, search for the "Clearlys" and "Obviouslys" and make sure that what you claimed was clear and obvious is, if not clear and obvious, at least true.[14]

7.18 After stating an "if and only if theorem", do not refer to the "if part" and the "only if" part, or the "sufficiency part" and the "necessity part"

Most people will not know which direction you actually mean when you refer to the "if" direction of a theorem, or to the "necessity part". Take the time to restate the result in each direction as you discuss it. I have seen some of the greatest economists being confused about what direction was intended (in my personal pantheon, they are people whose approach to economics cannot be described as "literary").

It is a great unsolved mystery of neuroscience that the same person can prove the fanciest theorems in abstract spaces and yet have trouble understanding some very elementary operations. Remember that. After all, don't you sometimes call your relatives in England when it is 3 *a.m.* there after having carefully calculated that it would be 3 *p.m.*? You might have trouble with such very simple calculations, and yet you brilliantly passed exams where many more of your intellectual powers were being tested.

7.19 Do not hesitate to explain very simple things

It is often worth it explaining very simple things, especially in seminars where you will not have the time to explain the complicated ones in any detail.

Would you guess that most of your professors really do not know what a marginal rate of substitution is? But it is true! To most of us, a sentence such as "Agent 1's marginal rate of substitution at z is greater than agent 2's"

[14]Do not deduce from this however that simply searching and deleting the "Clearlys" and "Obviouslys" will necessarily eliminate all of your mistakes!

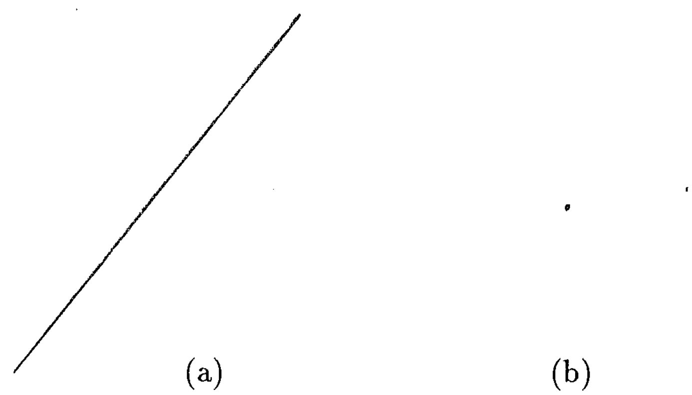


Figure 15: ***Even simple pictures can be of great help in understanding proofs.*** (a) Picture of a line. (b) Picture of a point.

only means that the two agents' indifference curves through z have different slopes at z. We just hope that which it is will be clear when we really need to know. Of course, we would never admit it in public, and I most certainly would never put such a confession in writing, for fear of being forever shunned by my colleagues!

7.20 Most of your message can be conveyed in pictures

Even simple pictures can be of tremendous help in making your oral presentations more vivid and perhaps memorable. This option is not as available in papers. Of course, a picture is not a substitute for a proof (but it can be used to give the main idea of the proof, and thereby cut down by half (probably much more than that, actually) the amount of time your reader will spend figuring it out).

7.21 Numerical examples are not always useful

It is commonly thought that numerical examples provide easy introductions to complicated proofs. That is true only if the examples are carefully specified.

The general algebraic expression has in fact the advantage of reminding us of the logic of the argument. If, to fix ideas, you choose $x_1 = 1$ and

$x_2 = 6$, the number 7 will refer to the sum $x_1 + x_2$ but it might be helpful to remember that: so write "$1+6$" instead, or "$7(=1+6)$". The expression $x_1 + x_2$ is probably preferable. In a three-person game, write the number of coalitions is $2^3 - 1$; we do not care whether that number is equal to 7.

Also, by using numerical examples instead of algebraic notation, you lose track of units of measurements. It makes it harder to check the correctness of expressions.

When you vary a parameter, as a result of which agent 1's income goes from 5 to 7 and agent 2's income from 5 to 8, it will soon be difficult to remember which are the initial incomes, which are the final incomes, and who it is whose income is 8. If you use well-chosen algebraic notation, for instance by calling the incomes I_1 and I_2 before the change and I_1' and I_2' after the change, your reader cannot be confused.

If you insist on using numbers, choose them so that whatever operations you perform on them do not turn them into monsters. If you will divide x_1 by 2, choose x_1 even; if you will take its square root, do not choose $x_1 = 10$. Actually, I take this back. It depends: if the incomes are 5 and 7 initially, and they are cut in half, they will be 5/2 and 7/2 after the change and the fractions will make it easier to remember that they are the new ones. If they were even, you would be tempted to perform the division and again, the new incomes would be hard to tell apart from the old ones.

In filling a payoff matrix, take all payoffs to be integers between 0 and 9 so that you do not need to separate them by commas.

More useful than numerical examples are examples with a small number of agents, a small number of goods, no production. Then you can save on subscripts, you can use an Edgeworth box, you can appeal to the intermediate value theorem instead of to a general fixed point theorem.

7.22 If you want to name your agents, do it in a way that helps

If you think numbering your agents from 1 to 4 is too dry in describing an example, try real names but choose them carefully so as to make it easy to remember who is who. Naming them Bob and Carol, Ted and Alice will be cute but counterproductive. Ted most certainly does not belong in this group. Also, they should be ordered alphabetically: Alice and Bob, Carol and Dwayne are your four consumers.

By the way, in a seminar, avoid cultural references that are obscure to too large a fraction of your audience. But by all means, do not avoid cultural references altogether for fear that they may be not be understood to some of your audience.[15]

8 Conclusion

If you follow all the above recommendations, not only will you be pleased with yourself, your seminar audiences enlightened, your classmates impressed, your parents proud of you, but, most importantly, your adviser will be happy.

I readily admit that any one of the recommendations does not amount to much, and you could ask "What is the big deal if I do not follow it"? You are probably right about each one of them individually. However, when added together, small imperfections will take your paper over the line that separates those that can be understood from those that cannot (there is an archimedian principle at work here). You will lose your readers or your seminar audiences much earlier than necessary. In fact, I am sure that you too will be confused.

Do not fool yourself: none of your readers will take the time to fully understand your whole paper, and a large fraction of your seminar audience will not have the faintest idea of what you are talking about when you are half-way through. So, every bit will help in keeping the attention of a few a little longer.

Another thing: if you are used to certain notational conventions, or terminology, or ways of structuring a proof, they almost necessarily seem the best to you, and perhaps the only ones worth considering. You have to be open-minded and genuinely experiment with other formulations. Only then

[15]Once, I referred to Bob and Carol, Ted and Alice in a seminar in which I discussed matching theory, and a member of the audience commented that I was showing my age! I was unfortunately not quick enough — showing my age once again — to reply that by understanding that I was showing my age, and remarking on it, he was showing his! He was right though. I recently asked the students in my graduate class whether they understood the allusion. Not one of them did. And yet, "Bob & Carol & Ted & Alice" (it's a movie) came out only yesterday (26 years ago, to be precise)! From now on, I will not use this example.

you can decide what is best. The first few times you use a new term or a new format or a new style, they will appear and sound strange to you. Give them a chance.

Let time elapse beween revisions. If your paper is so familiar to you that you essentially know it by heart, you will never discover your mistakes. You need to let it sit for a while in a drawer. When you pick it up again, it will have a freshness that will allow you to better see where it can be improved.

When after many drafts, your paper has become a smooth and shiny pebble that fits snugly in the palm of your hand, treat yourself to some Belgian chocolates (if you have found these recommendations useful, save me one!).