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Working Paper No. 429
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Rochester Center for Economic Research
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Forthcoming,
Carnegie-Rochester Conference Series on Public Policy

Abstract

Was 1974 a watershed? It saw an increase in the rate of technological change in the production of new equipment. It was the start of a sharp rise in income inequality. It signaled the beginning of the productivity slowdown. Were these phenomena related? Could they have been the result of an Industrial Revolution associated with the introduction of information technologies?

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1 Introduction

Did 1974 mark the beginning of a new industrial revolution? Was this the start of an era of rapid investment-specific technological progress associated with the development of information technologies (IT)? Did this increase in the pace of technological advance lead to a rise in income inequality? Is the productivity slowdown related to these phenomena?

A simple story is told here that connects the rate of technological progress to the level of income inequality and productivity growth. The idea is this. Imagine that a leap in the state of technology occurs and that this jump is incarnated in the form of new machines, such as information technologies. Suppose that the adoption of new technologies involves a significant cost in terms of learning and that skilled labor has an advantage at learning. Then the advance in technology will be associated with an increase in the demand for skill needed to implement it. Hence the skill premium will rise and income inequality will widen. In the early phases the new technologies may not be operated very efficiently due to a dearth of experience. Productivity growth may appear to stall as the economy undertakes the (unmeasured) investment in knowledge needed to get the new technologies running closer to their full potential. The coincidence of rapid technological change, widening inequality, and a slowdown in productivity growth is not without precedence in economic history.

1.1 *The Information Age*

Figure 1 illustrates the decline in the price of equipment over the postwar period. The price of equipment fell faster after 1974 than before it, as the following regression equation shows:

$$\begin{aligned}\ln(price) = \; & \underset{(34.8)}{64.9019} - \underset{(34.4)}{0.0327}\, time + \underset{(4.63)}{14.9231}\, D_{74} \\ & - \underset{(4.59)}{0.0075}\, time \times D_{74} - \underset{(2.35)}{0.0634}\, I_{74}, \text{ with } R^2 = 0.995 \text{ and D.W.} = 1.29,\end{aligned} \tag{1}$$

where the numbers in parenthesis are t-statistics, D_{74} is a dummy variable for the period after 1974, and I_{74} is a single year dummy variable for 1974. If the decline in the price of new

Figure 1: Price of New Equipment

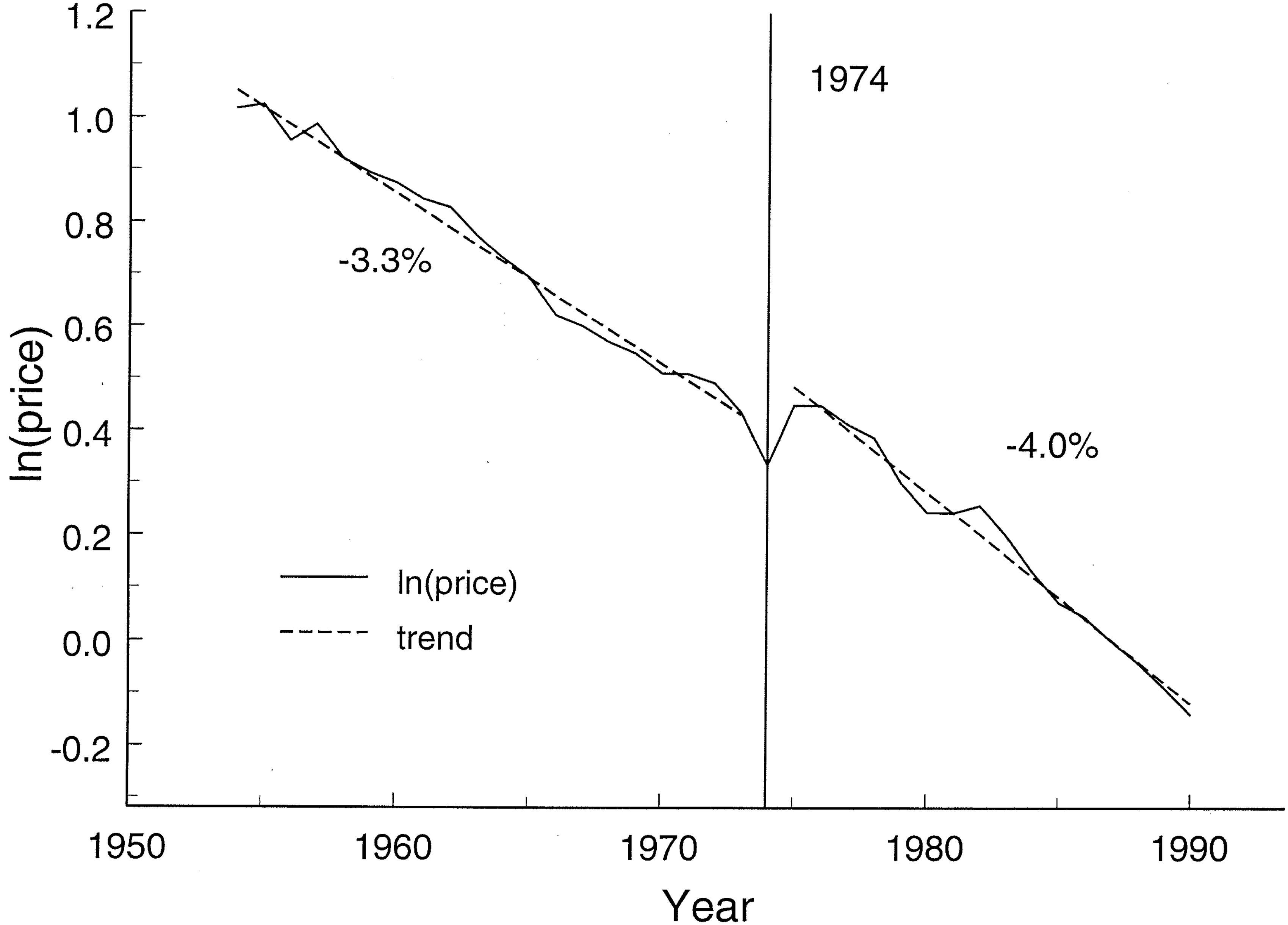
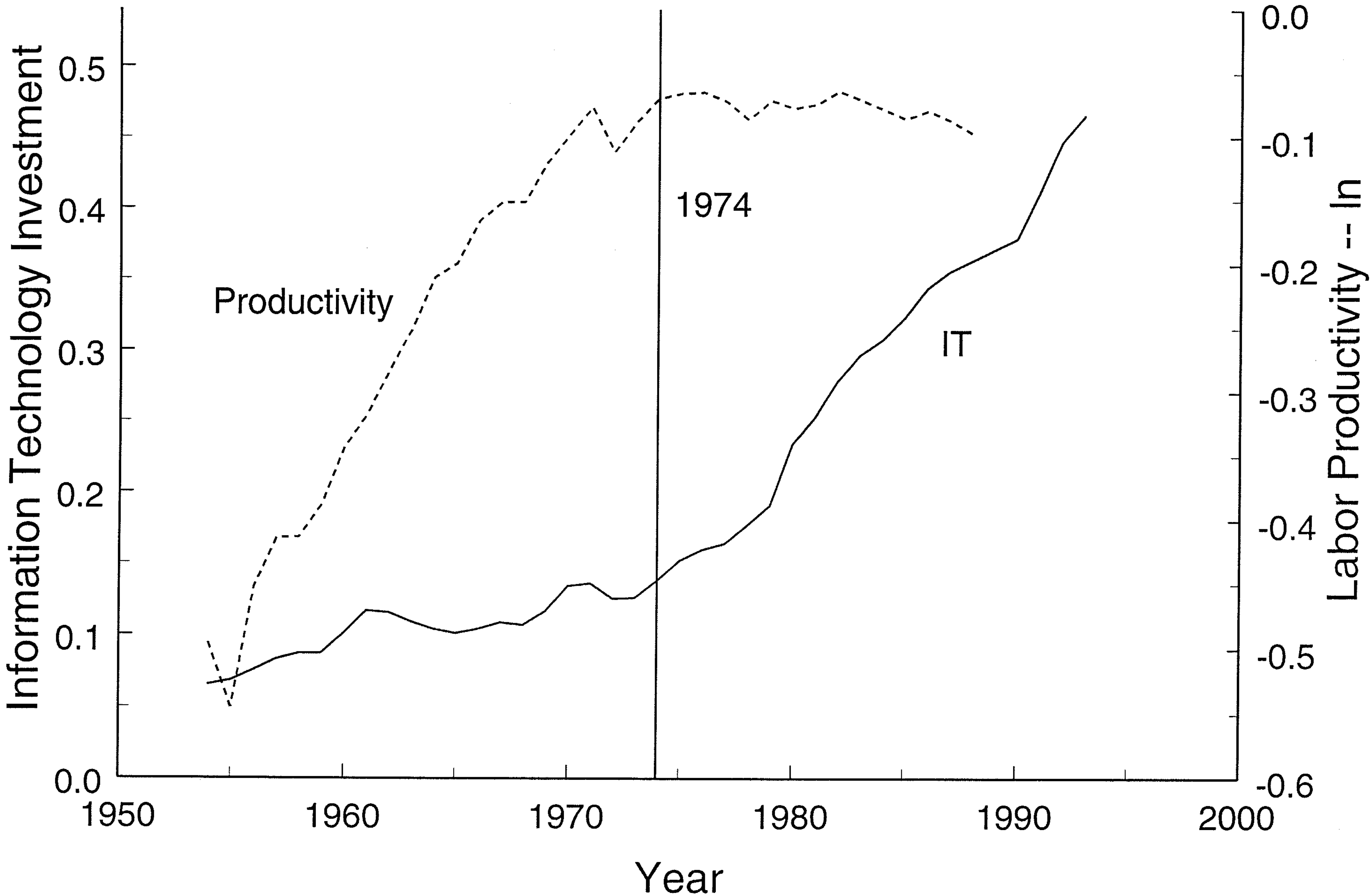


Figure 2: IT Investment and Productivity Slowdown



equipment can be taken as a measure of improved efficiency in equipment production, then the pace of technological change jumped up around 1974.[1] The rapid advance in technology since 1974 is undoubtedly linked to the development of information technologies. Figure 2 shows the phenomenal rise of IT investment (as a fraction of total equipment investment). Growth in labor productivity stalled with the rise in IT investment.

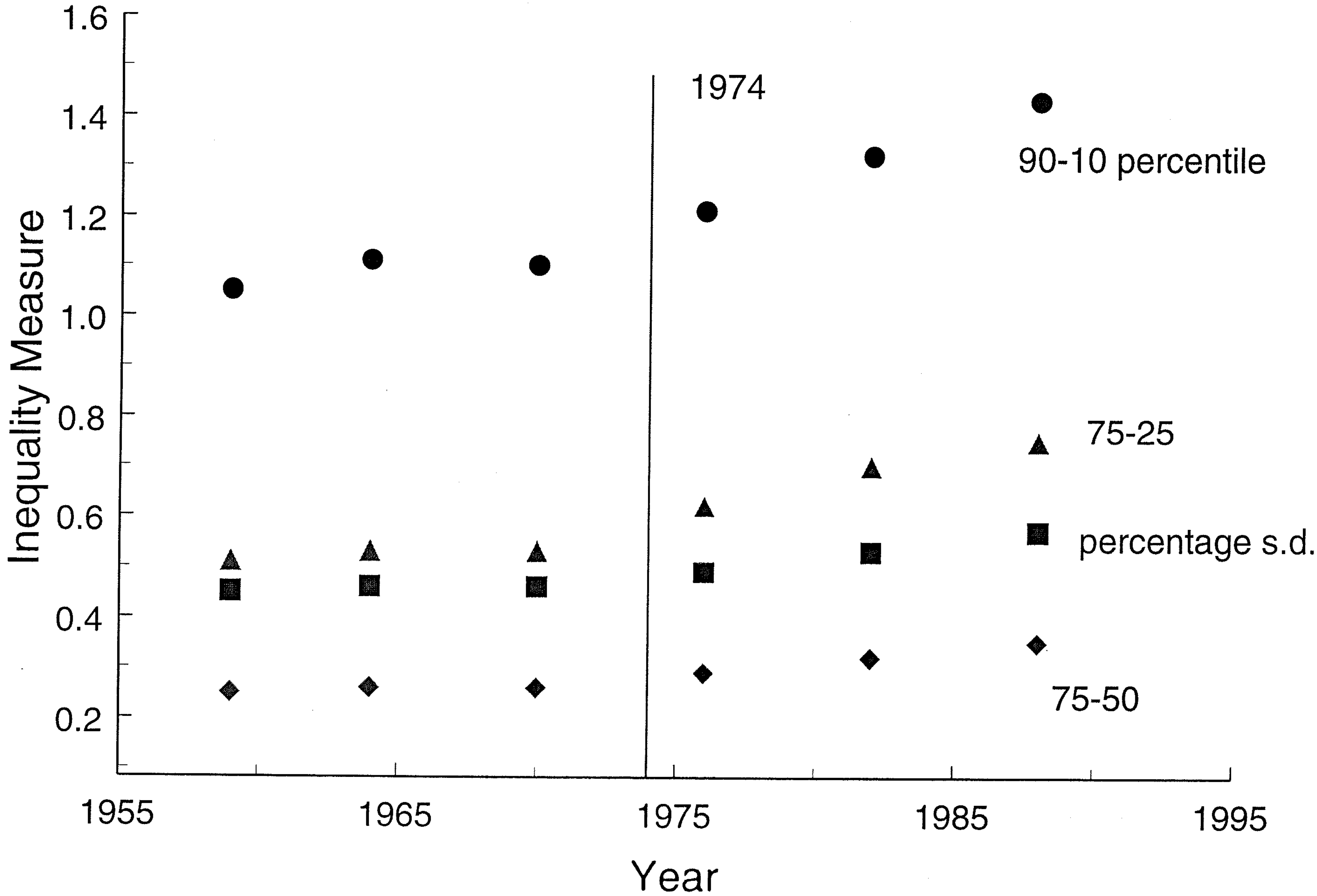
By most accounts wage inequality increased around 1974. Some postwar measures of income inequality taken from Juhn, Murphy and Pierce (1993, Table 1.B) are shown in Figure 3. As can be seen, the standard deviation of the logarithm of hourly wages for men remained constant between 1959 and 1970, while it rose 11 percentage points between 1970 and 1988. Likewise, the (logarithm of the) ratio of the wage earned by the upper quartile to the wage earned by the lower quartile remained roughly constant between 1959 and 1970. From 1970 to 1988 it rose by 22 percentage points.

1.2 *The Industrial Revolution*

The Industrial Revolution began in 1760. It witnessed the birth of several technological miracles, as chronicled by Mokyr (1994). For example, Crompton's mule revolutionized the spinning of cotton. Watt's energy efficient steam engine brought steam power to manufacturing.[2] When the mule was harnessed to steam power, the mechanization of manufacturing was inexorable. By 1841 the real price of spun cotton had fallen by two-thirds. In 1784 Cort introduced his puddling and rolling technique for making wrought iron, a product vital for the industrialization of Britain. Between 1788 and 1815 the production of wrought iron increased by 500 percent. The price of wrought-iron fell from £22 to £14 per ton from 1801 and 1815, despite the fact that between 1770 and 1815 the general level of prices rose by 50 percent. Last, the foundation of the modern machine-tool industry was constructed. A gun-barreling machine was designed by Wilkinson that could make cylinders for Watt's steam engines. Maudley introduced the heavy-duty lathe.

Skill undoubtedly played an important role in technological innovation and adoption

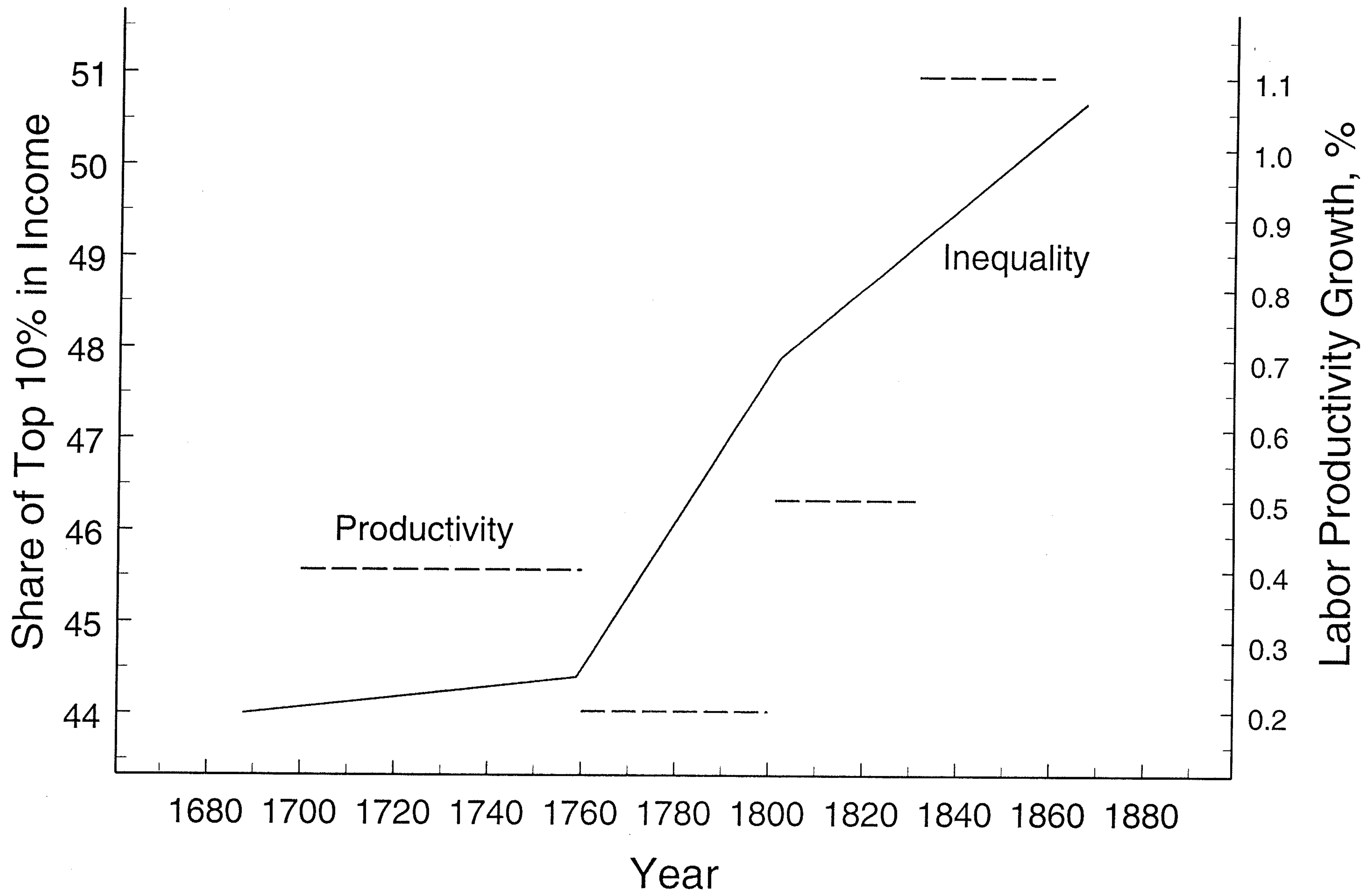
Figure 3: Measures of Wage Inequality



during the Industrial Revolution. While the Industrial Revolution was the age of handful of miracles, many historians view it also as an age of continuous and gradual smaller innovations — an age of learning. Implementing and operating brilliant inventions and effecting subsequent innovations is often demanding work requiring skill. For instance, von Tunzelmann (1994) reports that it took three months for someone brought up in a mill to learn how to operate either a hand mule or a self-acting mule. The former required three years to learn how to maintain while the latter demanded seven. Knowledge concerning improvements in the machinery continued throughout the worker's lifetime. It seems reasonable to conjecture that the demand for skill rose in the Industrial Revolution. As Mokyr (1994, p. 29) states "for the economy *as a whole* to switch from manual techniques to a mechanized production required hundreds of inventors, thousands of innovating entrepreneurs and tens of thousands of mechanics, technicians and dexterous rank and file workers".[3] In fact, income inequality rose throughout the Industrial Revolution as has been documented by Lindert and Williamson (1983, Table 3). Their data is plotted in Figure 4.[4]

The diffusion of new technologies is often slow because the initial incarnations of the underlying ideas are inefficient.[5] Getting new technologies close to their full potential may take a considerable period of time. According to von Tunzelmann (1994), Cort's famous puddling and rolling process went through a long incubation period and was commercially unsuccessful at first. Royalties had to be slashed to encourage adoption. Apparently, "both entrepreneurs and workers had to go through a learning period, making many mistakes that often resulted in low outputs of uneven quality."[6] It is interesting to note that Harley (1993, Table 3.5) calculates that productivity fell in the initial stages of Industrial Revolution. It took time for the fruits of the Industrial Revolution to ripen. This is also shown in Figure 4.

Figure 4: Industrial Revolution



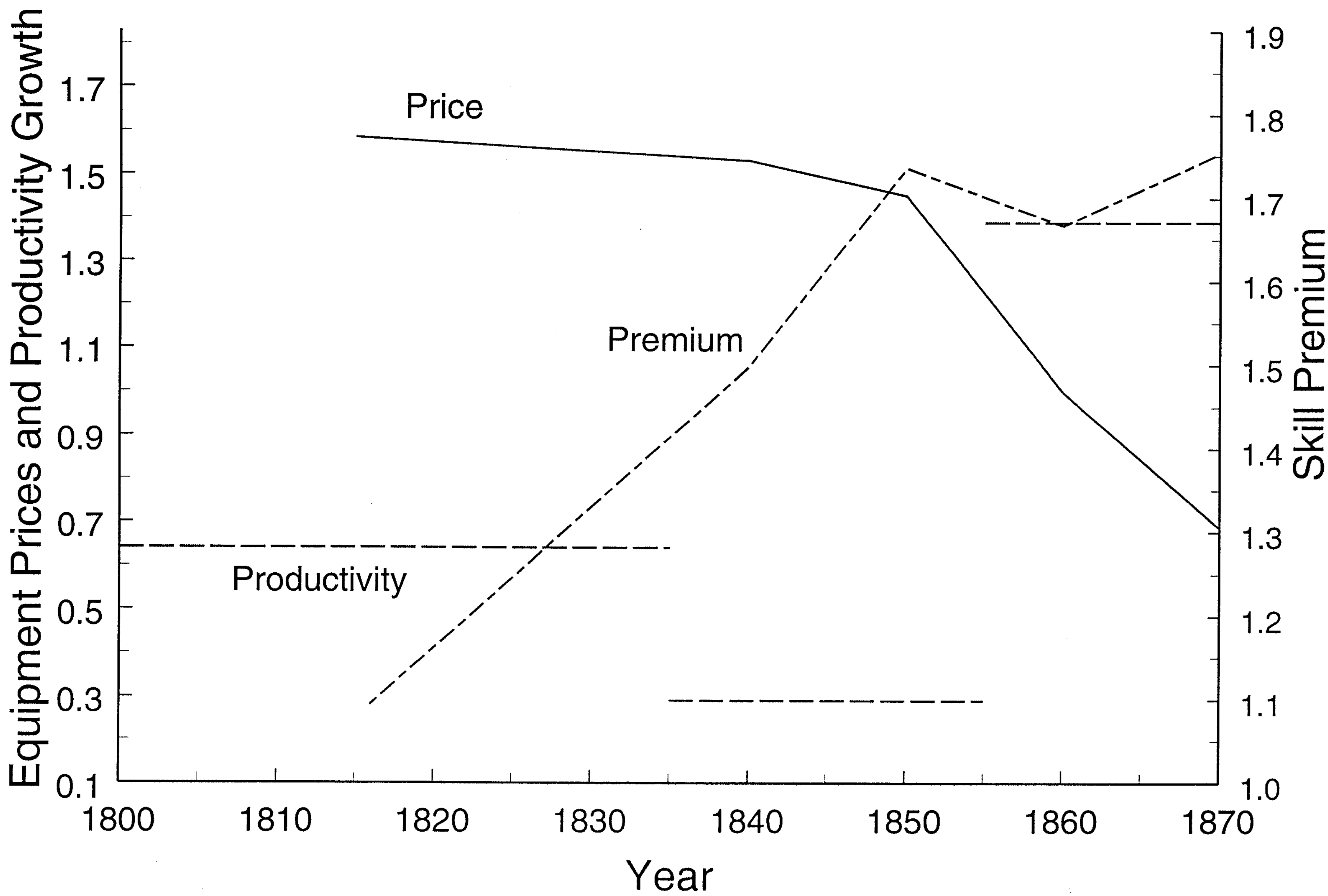
1.3 *The American Antebellum Period*

The Industrial Revolution spread to the U.S. in the nineteenth century. The nation industrialized at a rapid clip over this period. This was an era of tremendous investment-specific technological change. Figure 5 shows the dramatic decline in the relative price of equipment that occurred. This series is based upon some calculations using data presented in Gallman (1992). For the period 1774 and 1815 the real stock of equipment per capita grew at roughly 0.7% per year. Between 1815 and 1860, however, the average annual growth was a very robust 2.8%. This jumped up to a whopping 4.5% over the interval from 1860 to 1900. Two examples might help to illustrate this incredible pace of industrialization. In 1830 there were just 30 miles of railroad tracks in the U.S.. By 1840 this had risen to 2,808 miles, while in 1860 the number was 30,000.[7] Likewise the aggregate capacity of U.S. steam engines more than quadrupled between 1840 to 1860 from 760,000 to 3,470,000 horsepower. It rose another one and half times by 1870 to 5,590,000. The antebellum period saw a dramatic surge in the skill premium as Figure 5 illustrates, using data reported in Williamson and Lindert (1980, Appendix D). Not surprisingly skilled workers, such as engineers, machinists, boilermakers, carpenters and joiners, all saw their wages rise relative to the common laborer.[8] Last, it is interesting to note that the figures in Abramovitz and David (1973, Table 2) imply a slowdown in both labor and total factor productivity growth for the 1840's just as the American Industrial Revolution was gaining steam; the numbers for labor productivity are plotted in Figure 5.[9]

1.4 *More on The Hypothesis*

The idea to be entertained here is that the adoption of new technologies involves a significant cost in terms of learning and that skill facilitates this learning process. That is, skill is important for adapting to change. There is considerable evidence for learning effects. For example, using a data set from 1973 to 1986 consisting of 2,000 firms from 41 industries, Bahk and Gort (1993) find that a plant's productivity increases by 15 percent over the first

Figure 5: U.S. Antebellum Period



fourteen years of its life due to learning effects. A variety of learning curves from angioplasty surgery to steel finishing are documented in Jovanovic and Nyarko (1995). Last, Yorukoglu (1995) reports a steep learning curve associated with investment in information technologies.

There is also evidence that skill plays an important role in facilitating the adoption of new technologies. Findings reported in Bartel and Lichtenberg (1987) support the joint hypothesis that (i) educated workers have a comparative advantage in implementing new technologies because they are better at assimilating new ideas and (ii), the demand for educated versus less-educated workers declines as experience is gained with a technology. Flug and Hercowitz (1995) find, using a cross-country panel data set, that a rise in equipment investment leads to an increase in the skill premium, and higher relative employment for skilled labor. It is important to note that the hypothesis to be developed here is different from the capital-skill complementarity hypothesis, as advanced by Griliches (1969) of which a modern reincarnation can be found in Krusell et al (1996). This hypothesis states that skilled labor is more complementary with capital in production than is unskilled labor. Krusell et al (1996) argue that the recent rise in the skill premium is consistent with capital-skill complementarity and an increase in the rate of investment-specific technological change. The idea in the current paper is that a successful implementation of a new technology requires skilled labor. Moreover, as a technology becomes established the production process substitutes away from expensive skilled labor toward more economical unskilled labor. Therefore, in times of heightened technological progress the demand for skill should rise, since this type of labor has a comparative advantage in speeding up and easing the process of technological adoption. Such times should therefore be associated with a rise in the skill premium. If this notion is correct, once the recent burst of investment-specific technological change subsides, as IT matures, the skill premium should decline.[10]

All work stands on the shoulders of others, and this paper is no exception. Nelson and Phelps (1966) developed an early model where skill speeds up the technological diffusion process. Furthermore, in their setting the benefits from skill are greater, the faster is the pace of innovation. The analysis here has a similar flavor. Rather than modeling skill as important for allowing a *given* technology to catch up with the state of art in the economy as Nelson and Phelps (1966) do, though, here skill is taken to be instrumental in facilitating the adoption of *new* technologies. Unlike Nelson and Phelps (1966), the focus of the current analysis is on the effect that technological change has on the skill premium and labor productivity. This involves some new considerations. First, the analysis must explicitly incorporate into it both skilled and unskilled labor. Second, the modelling of the adoption of new technologies, and the shutting down of old ones, is undertaken in a general equilibrium setting so that a connection between an economy's growth rate and its skill premium can be made. Jovanovic (1995) presents some back-of-the-envelope calculations suggesting that the costs of adopting new technologies exceed inventions cost by a factor of 20 to 1. He suggests that adoption costs may amount to 10% of GDP. Surely, adoptions costs must be large. How else can the long diffusion lags for new technologies be explained, as well as the continual investment in dominated technologies at the level of households, firms and countries. And surely a large part of these adoption costs must be in acquiring or developing the skills needed to implement the new technologies.

2 The Economic Environment

Imagine an economy consisting of households, and a firm. The firm lives forever and produces output at a variety of plants using capital and two types of labor, viz skilled and unskilled. Each household lives m years. When young a household must make an irreversible decision about whether to become skilled or not. Households earn income by supplying labor and lending funds to firms.

2.1 *Firm*

The firm in the economy produces at a number of plants.[11] A plant is indexed by the age or vintage of its capital stock denoted by j. This capital is purchased the period before the plant is opened. The price of a unit of capital declines over time due to investment-specific technological change. Specifically, imagine that one unit of consumption can purchase γ^{t+1} units of capital in period t. Assume that the blue prints for a new plant under construction in period t call for $(\gamma^{t+1})^{\frac{1}{1-\alpha}}$ units of capital. Therefore, the consumption cost of a new plant in period t is $(\gamma^{t+1})^{\frac{\alpha}{1-\alpha}}$. Suppose that capital depreciates at the rate δ per period. Thus, an age-j plant in period t will have $k_{j,t} = (1-\delta)^{j-1}(\gamma^{t+1-j})^{\frac{1}{1-\alpha}}$ units of capital in it. There is a nonconvexity associated with operating a plant. In particular, operating the plant requires a minimum of $\bar{l}$ units of unskilled labor. The firm is free to open or close plants as desired. The number of vintage-j plants owned in the current period by the firm is represented by p_j.

The production technology for a plant of vintage j is given by,

$$F(k_j, l_j, h_j, \mu_j) = \begin{cases} \mu_j k_j^{\alpha}(l_j - \bar{l})^{\beta} h_j^{\zeta}, & \text{if } l_j \geq \bar{l}, \\ 0, \text{otherwise}, \end{cases} \quad \text{for } 0 < \alpha, \beta, \zeta, \alpha + \beta + \zeta < 1, \tag{2}$$

where k_j is the capital stock, l_j and h_j are the inputs of unskilled and skilled labor, and μ_j is the total factor productivity of the plant.

Learning. Total factor productivity evolves over time due to investment in learning. Skilled labor is essential to this learning process. The law of motion for total factor productivity has the form

$$\mu_{j+1} = G(\mu_j, e_j) = (1-\kappa)\mu_j + \vartheta(1-\mu_j)e_j^{\phi}\ , \text{ for } j \geq 0, \tag{3}$$

where e_j is the amount of skilled labor hired by the plant to facilitate the adoption of a new technology. Let a plant's initial level of productivity be represented by μ_0. This form resembles Nelson and Phelps (1966, eq 8). The improvement in a vintage-j plant's practice, or $\mu'_{j+1} - \mu_j$, depends upon the amount of skilled labor hired, e_j. As the amount of unrealized potential, or $1 - \mu_j$, shrinks it becomes increasingly difficult for skilled labor to effect an improvement. Note that in order to prevent a regress in productivity some skilled labor must always be employed. Following Yorukoglu (1995) the starting value for the learning process, or μ_0, is taken to be inversely related to the current rate of investment-specific technological change. That is, as the rate of technological progress increases the more costly it becomes to adopt the new technology since agents will be less familiar with it. Specifically, let

$$\mu_0 = \chi\gamma^{-\tau}. \tag{4}$$

The above learning process is undoubtedly mechanistic. The process of adopting and implementing new technologies is often uncertain by nature, a bit like sailing on uncharted waters. The trial and error process of adjusting one's actions based on successes and failures may be better modelled from a Bayesian perspective, as in Jovanovic and Nyarko (1995, 1996). But doing this in a general equilibrium framework, such as the one adopted here, looks like a daunting task.

Again there is considerable evidence for learning effects at both the plant and firm level. As a case in point consider the Lawrence #2 mill, a cotton mill in the antebellum period studied by David (1975). This mill was built in 1834 in Lowell, Massachusetts. Detailed inventories of the equipment at this plant show that no new machinery was added between 1836 and 1856. Thus, it seems reasonable to infer that any increases in productivity over this period arose purely due to learning effects. In fact, output-per-manhour in this plant grew on average at 2.3% per year over this period. Figure 6 shows the learning curve materializing from David's (1975) analysis. The four observations pertain to years when it is known that the plant was operating at full capacity.

The Plant's Problem. The plant's optimization problem is summarized by

$$V(k_j, \mu_j; \cdot) \quad = \quad \max_{l_j, h_j} \{F(k_j, l_j, h_j, \mu_j) - wl_j - vh_j \qquad \text{P(1)}$$

$$+\max\{\max_{e_j}[\frac{V(k'_{j+1},\mu'_{j+1};\cdot')}{1+r}-ve_j],0\}\},$$

subject to

$$\mu'_{j+1}=G(\mu_j,e_j), \tag{5}$$

where v and w represent the skilled and unskilled wage rates, respectively, and r is the interest rate. Note that equation P(1) defines the value of an age-j plant.[12]

The efficiency conditions for unskilled and skilled labor used in direct production are

$$\beta\mu_j k_j^{\alpha}(l_j-\bar{l})^{\beta-1}h_j^{\zeta}=w, \tag{6}$$

and

$$\zeta\mu_j k_j^{\alpha}(l_j-\bar{l})^{\beta}h_j^{\zeta-1}=v. \tag{7}$$

The one for skilled labor used in adoption (assuming that the plant is still in operation next period) reads

$$\frac{V_2(k'_{j+1},\mu'_{j+1};\cdot')}{1+r}G_2(\mu_j,e_j)=v, \tag{8}$$

which can be rewritten as[13]

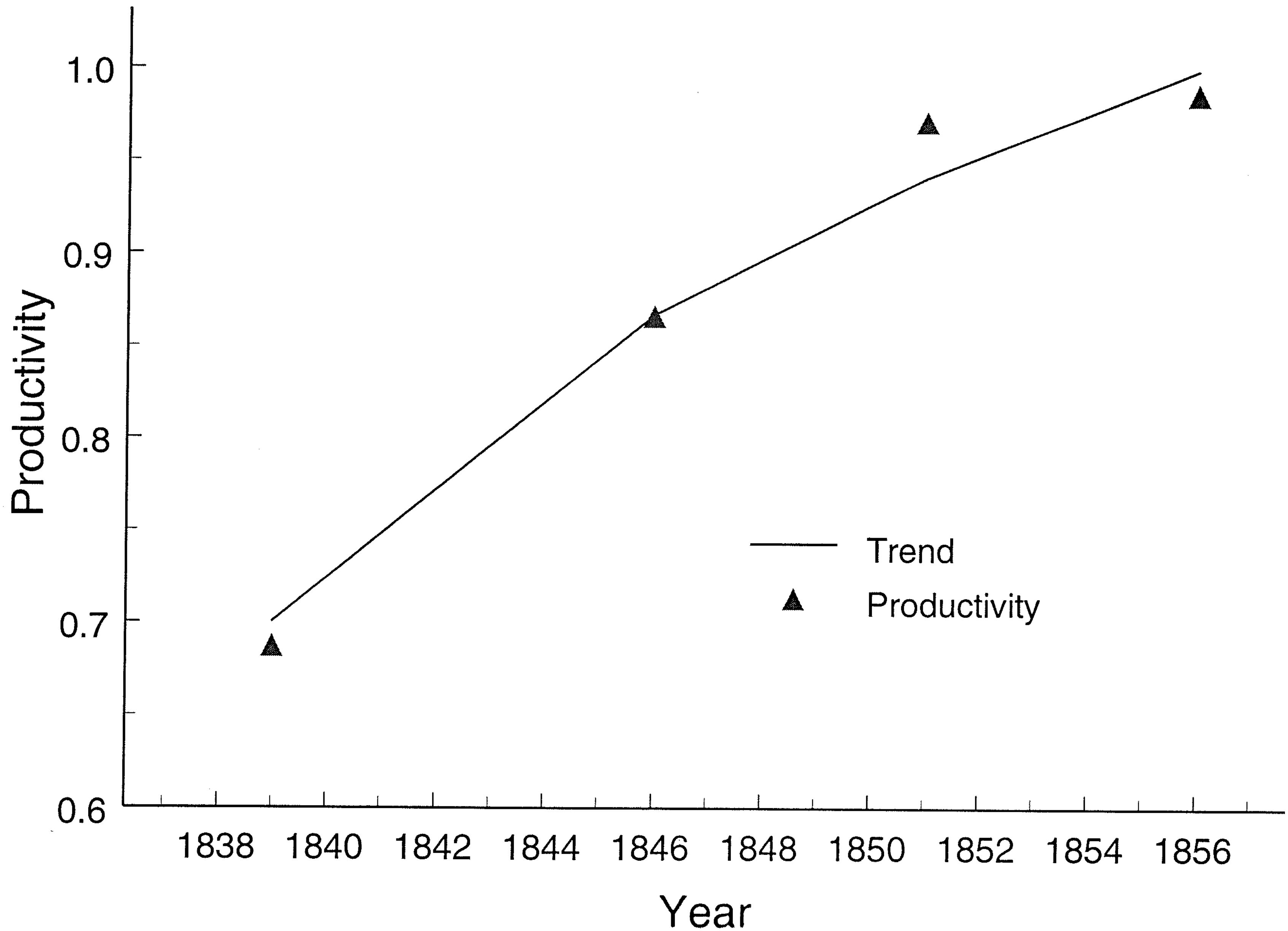
$$\frac{k_j'^{\alpha}(l_j'-\bar{l})^{\beta}h_j'^{\zeta}+v'G_1(\mu'_{j+1},e'_{j+1})/G_2(\mu'_{j+1},e'_{j+1})}{1+r}G_2(\mu_j,e_j)=v. \tag{9}$$

Exit and Entry. The firm must decide when to open and close a plant. Clearly, an age-j plant should be shut down whenever the present discounted value of its profits become negative. Otherwise, it should remain open. Thus, if

$$\max_{e_j}\{V(k'_{j+1},G(\mu_j,e_j);\cdot')/(1+r)-ve_j\}\begin{cases}<0, & \text{then } p'_{j+1}=0,\\ =0, & \text{then } 0\le p'_{j+1}\le p_j,\\ >0, & \text{then } p'_{j+1}=p_j \text{ (for } j\ge 1).\end{cases} \tag{10}$$

Likewise, a plant will only be created whenever the present-value of creating one is nonnegative. Since anyone can start a firm, in equilibrium there must be zero rents from doing so.

Figure 6: Lawrence Number 2 Cotton Textile Mill



Therefore, if

$$\max_{e_0} \{V(k_1', G(\mu_0, e_0); \cdot')/(1+r) - (k_1')^\alpha - ve_0\} \begin{cases} = 0, & \text{then } p_1' \geq 0, \\ < 0, & \text{then } p_1' = 0. \end{cases} \tag{11}$$

Observe that the plant chooses the level of productivity that it opens with, or $\mu_1' = G(\mu_0, e_0)$; the higher this level, the higher is the start-up cost in terms of skill labor, ve_0.

2.2 *Asset Pricing Structure*

Suppose in each period that the firm pays out its profits less investment spending in dividends. Current dividends, d, will then be given by $d = \sum_{j=1}^{n} p_j[F(k_j, l_j, h_j, \mu_j) - wl_j$ - $v(h_j + e_j)] - p_1'[(k_1')^\alpha + ve_0]$. The value of the firm in the current period after paying dividends is

$$q = \frac{\sum_{j=1}^{n'} p_j' V(\cdot_j')}{(1+r)}, \tag{12}$$

where p_j' denotes the number of age-j plants that the firm will own next period. Observe that $p_1'' V(\cdot_1'')/(1+r') = p_1''[(k_1'')^\alpha + v'e_0']$, from the free entry condition. Using P(1) and (12) it is easy to see that the following arbitrage condition must hold:

$$1 + r = \frac{q' + d'}{q}. \tag{13}$$

2.3 *Households*

At any point in time there will be m generations of households coexisting. In each cohort, some agents will be skilled, others will be unskilled. Consider an unskilled agent who is currently i years old. Suppose that he has assets in the amount a_i. These assets will earn the amount $a_i d$ in dividends in the current period, after which they could be sold for $a_i q$. The agent will also earn the amount w in wage income. The agent must decide how to divide his current income, $a_i(d + q) + w$, between consumption, c_i, and asset holdings for next period, a'_{i+1}. Let the agent's momentary utility function be given by $U(c) = \ln c$ and assume that he discounts the future at rate ρ. The optimization problem for an agent in the i-th generation will take the form

$$W_i(a_i;\cdot) = \max_{a'_{i+1}} \{U(c_i) + \rho W_{i+1}(a'_{i+1};\cdot')\}, \tag{P(2)}$$

subject to

$$qa'_{i+1} = a_i(d+q) + w - c_i. \tag{14}$$

The agent's first-order condition is

$$U'(c_i) = \rho \underbrace{[(q'+d')/q]}_{1+r} U'(c'_{i+1}). \tag{15}$$

Each agent is indexed by an ability variable $\lambda \in [1, \infty)$, where λ is distributed over the population in line with the cumulative distribution function $\Lambda(\lambda)$. The ability variable gives the number of efficiency units of skilled labor that the agent is capable of providing. Consider an age-i skilled agent of ability level λ, where $i > 1$. This individual will earn the amount λv in labor income each period. Denote his current asset holdings by $b_i(\lambda)$. The agent must decide how to divide his current income, $b_i(\lambda)[d+q] + \lambda v$, between consumption, $z_i(\lambda)$, and holdings of assets for next period, $b'_{i+1}(\lambda)$. The optimization problem for an age$-i$ skilled agent of ability λ is

$$S_i(b_i(\lambda), \lambda; \cdot) = \max_{b'_{i+1}(\lambda)} \{U(z_i(\lambda)) + \rho S_{i+1}(b'_{i+1}(\lambda), \lambda; \cdot')\}, \tag{P(3)}$$

subject to

$$qb'_{i+1}(\lambda) = b_i(\lambda)[d+q] + \lambda v \ - z_i(\lambda), \tag{16}$$

where $i > 1$ and $U(z) = \ln z$. His first-order condition is given by

$$U'(z_i(\lambda)) = \rho[(q'+d')/q]U'(z'_{i+1}(\lambda)). \tag{17}$$

Observe that the decision for $b'_{i+1}(\lambda)$ is homogenous of degree one in λ and $b_i(\lambda)$. Thus, taking age as given, a skilled agent's consumption and asset holdings will be proportional to his skill index.

In the first period of his life an agent must decide, once and for all, whether to become skilled or not. The potential benefit of becoming skilled is clear; it may allow an agent to earn more in labor income. The costs of becoming skilled are twofold. First, there is an opportunity cost of o units of time to become skilled. Second, there is a utility cost of $\Theta\lambda^{-\theta}$, where $\Theta \geq 0$ and $\theta \geq 1$. Note that this utility cost is decreasing in the ability index λ. The less your ability, the harder it is to become skilled. Clearly, an agent will become skilled if $S_1(0, \lambda) - \Theta\lambda^{-\theta} > W_1(0)$, and will remain unskilled if $S_1(0, \lambda) - \Theta\lambda^{-\theta} < W_1(0)$.[14] The lower the value of λ, the less likely it is that an agent will choose to become skilled. Define f_i to be the fraction of the age-i generation who are unskilled. The fraction of the current generation who choose to remain unskilled will be given by

$$f_1 = \Lambda(\lambda_1), \tag{18}$$

where λ_1 solves

$$S_1(0, \lambda_1; \cdot) - \Theta\lambda_1^{-\theta} = W_1(0; \cdot). \tag{19}$$

Clearly, the upper end of the income distribution will be made up by skilled agents. Empirically, the tail of the income distribution can be well approximated by a Pareto distribution, which is also easy to work with. Therefore, let Λ be represented by the Pareto distribution so that $\Lambda(\lambda) = 1 - \lambda^{-\iota}$, for $\lambda \geq 1$.

2.4 *Competitive Equilibrium*

The competitive equilibrium under study will now be spelled out. The aggregate state of the world for the economy under study is described by the lengthy vector $s = (p_1, ..., p_n, k_1, ..., k_n, \mu_0, ..., \mu_n, a_2, ..., a_m, b_2, ..., b_m, f_2, ..., f_m)$. This gives the number of plants of each age, their capital stocks, their stocks of experience, the economy's wealth distribution over generations and skills, and the distribution of skills across generations. The equilibrium wage rates, interest rate, dividend payments, and the share price of the firm can all be expressed as a function of this aggregate state of the world.

Definition: A competitive equilibrium is a set of allocation rules for $l_j = L_j(s)$, $h_j = H_j(s)$, $e_j = E_j(s)$, $p'_j = P_j(s)$, $a'_i = A_i(s)$, $b'_i = B_i(s)$, and $f_1 = F(s)$, together with a set of pricing functions, $w = W(s)$, $v = V(s)$, $d = D(s)$, $q = Q(s)$, and $r = R(s)$ such that:

1. Plants hire unskilled and skilled labor in line with problem P(1), with the equilibrium solution to these problems satisfying $l_j = L_j(s)$, $h_j = H_j(s)$, $e_j = E_j(s)$.

2. The age distribution of plants, as given by $p'_j = P_j(s)$, is determined in accordance with the entry and exit criteria (11) and (10).

3. Unskilled households solve problem P(2), with the equilibrium solution to this problem satisfying $a'_i = A_i(s)$.

4. Skilled households solve problem P(3), with the equilibrium solution satisfying $b'_i \equiv \int_{\lambda_i}^{\infty} b'_i(\lambda)\Lambda(d\lambda) = B(s)$, where $\lambda_1 = \Lambda^{-1}(F(s))$.

5. The fraction of the new generation choosing to remain unskilled, as given by $f_1 = F(s)$, is determined in line with (18) and (19).

6. All markets clear implying

$$\sum_{i=1}^{m}[f_i a'_{i+1} + b'_{i+1}] = 1, \tag{20}$$

$$\sum_{j=1}^{n} p_j l_j = \sum_{i=1}^{m} f_i, \tag{21}$$

and

$$\sum_{j=1}^{n} p_j h_j + \sum_{j=1}^{n} p'_j e_{j-1} = \sum_{i=1}^{m} \int_{\lambda_i}^{\infty} \lambda\Lambda(d\lambda) - o\int_{\lambda_1}^{\infty} \lambda\Lambda(d\lambda). \tag{22}$$

3 Balanced Growth

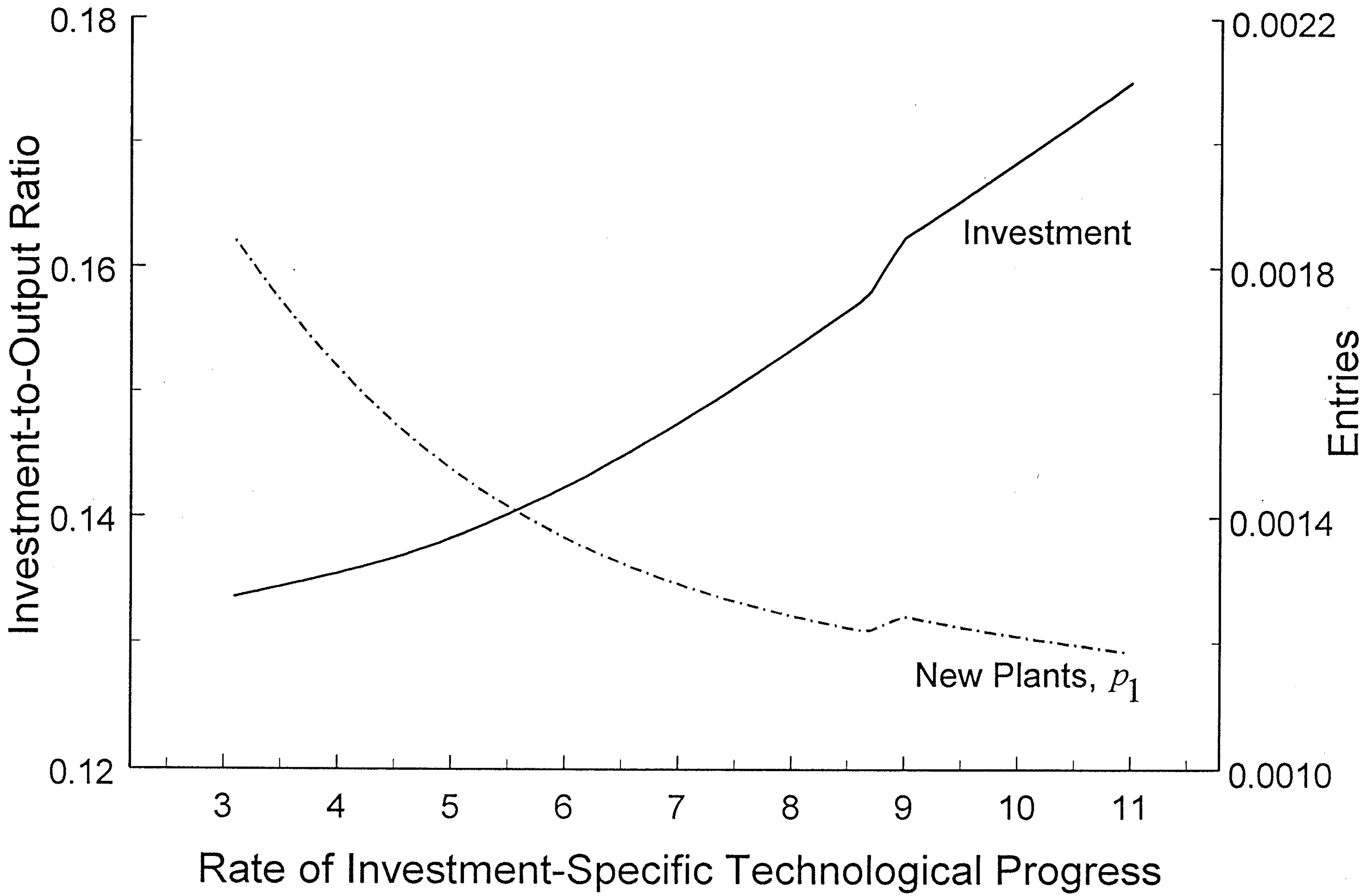
The speed of investment-specific technological progress in the model is given by γ, the rate at which the consumption price of a unit of capital declines. Let the age distribution of plants at a point in time be represented by $(p_1, p_2, ..., p_n)$. Along a balanced growth path this age distribution will be constant. So will the amounts of unskilled and skilled labor, l_j, h_j, and e_j, used by an age-j plant. This implies that the distribution of productivity $(\mu_1, ..., \mu_n)$ is also stationary. The distribution of capital across plants $(k_1, k_2, ..., k_n)$ is growing by construction at rate $\gamma^{\frac{1}{1-\alpha}}$. Output, or $y = \sum_{j=1}^{n} p_j \mu_j k_j^{\alpha}(l_j - \bar{l})^{\beta} h_j^{\zeta}$, therefore grows at rate $\gamma_y = \gamma^{\frac{\alpha}{1-\alpha}}$. It is easy to check that the marginal products of skilled and unskilled labor, or v and w, must grow at the same rate as output, as must dividends, d, and the value of the firm, q (providing that the interest rate is constant).

Last, in balanced growth the distribution of share holdings across unskilled and skilled agents, or $(a_2, ..., a_m)$ and $[b_2(\lambda), ..., b_m(\lambda)]$, remains time-invariant. There will be a constant interest rate, r, that ensures the asset market always clears. The distributions of consumptions across unskilled and skilled agents $(c_1, c_2, ..., c_m)$ and $[z_1(\lambda), z_2(\lambda), ..., z_m(\lambda)]$ grow at the same rate as output, a fact evident from (14) and (16). Observe that an individual's consumption does *not* grow at the rate γ_y over his lifetime. From the Euler equations (15) and (17) it is apparent that an individual's consumption will grow at rate $\rho(1+r)$. Given the overlapping generations structure of the model it transpires that $(1+r) > \gamma_y/\rho$ so an agent's consumption grows at a faster rate than the economy. The fraction of agents choosing to become skilled, or f_1, will remain fixed, since $S_1(0, \Lambda^{-1}(f_1)) - W_1(0)$ is a constant in balanced growth given the logarithmic form of momentary utility — see the Appendix for more detail.

What happens to the model's balanced growth path as the rate of investment-specific technological change increases? To answer this question, the solution to model is computed numerically. The model was not tuned, by choice of parameter values, to be in harmony with any particular features of the U.S. data. A list of the parameter values used for the analysis is presented in the Appendix. Four years is the unit of time.

Aggregate investment (as a fraction of output) increases with a rise in the rate of investment-specific technological change. This is shown in Figure 7. But also observe that, for the most part, the number of new plants decreases with the pace of technological change.

Figure 7: Investment



The size of new plants, measured in term of capital or employment, increases with growth — Figure 8. Along a balanced growth path the number of exits must equal the number of entrants, or else the number of firms would be changing. Plants exit in the model because eventually they become unprofitable to operate in face of the relentless increase in real wages. The oldest plants in the economy have less capital than do newer plants and are therefore less profitable to operate, other things equal. This disparity increases with the rate of technological progress. The increase in the rate of growth of real wages, caused by a faster rate of technological advance, results in less of the oldest plants surviving.

Steady-state entries and exits, p_1, are not a smooth function, however, of the rate of investment-specific technological change (Figure 7). This transpires because the model exhibits two modes of behavior.[15] In the first mode all plants of the oldest age are scrapped; the rest remain standing. In this mode $p_1 = ... = p_n$ so that trivially the number of entries, p_1, equals the number of exits, p_n. This mode corresponds to the situation where equation (10) is always slack. Here cutting down on the oldest of plants, in face of an increase in the growth of wages, implies cutting down on the number of entries. Thus, the number of entries decreases with the rate of progress. In the second mode some of the next-to-oldest plants are scrapped as well. Here $p_1 = ... = p_{n-1} > p_n$, with entries, p_1, equalling exits, $p_n + (p_{n-1} - p_n)$. In this situation equation (10) holds with strict equality (for p_n). Here, entries increase with the rate of technological progress. Cutting back on the number of age-n plants no longer implies that new entries must be reduced (since now $p_n \neq p_1$).

As the pace of investment-specific technological change picks up income inequality worsens, as can be seen from Figure 9. Skilled labor is at a premium since it facilitates plant-level learning. Not surprisingly, as the skill premium increases, the fraction of young agents choosing to become skilled increases as well. Flug and Hercowitz (1995, Table 3, eq. 1) found that a 1 percentage point increase in the equipment investment-to-output ratio lead to a 1.90 percentage point increase in the skilled-to-unskilled employment ratio. The number computed here is 2.53.

Figure 8: Plant Size

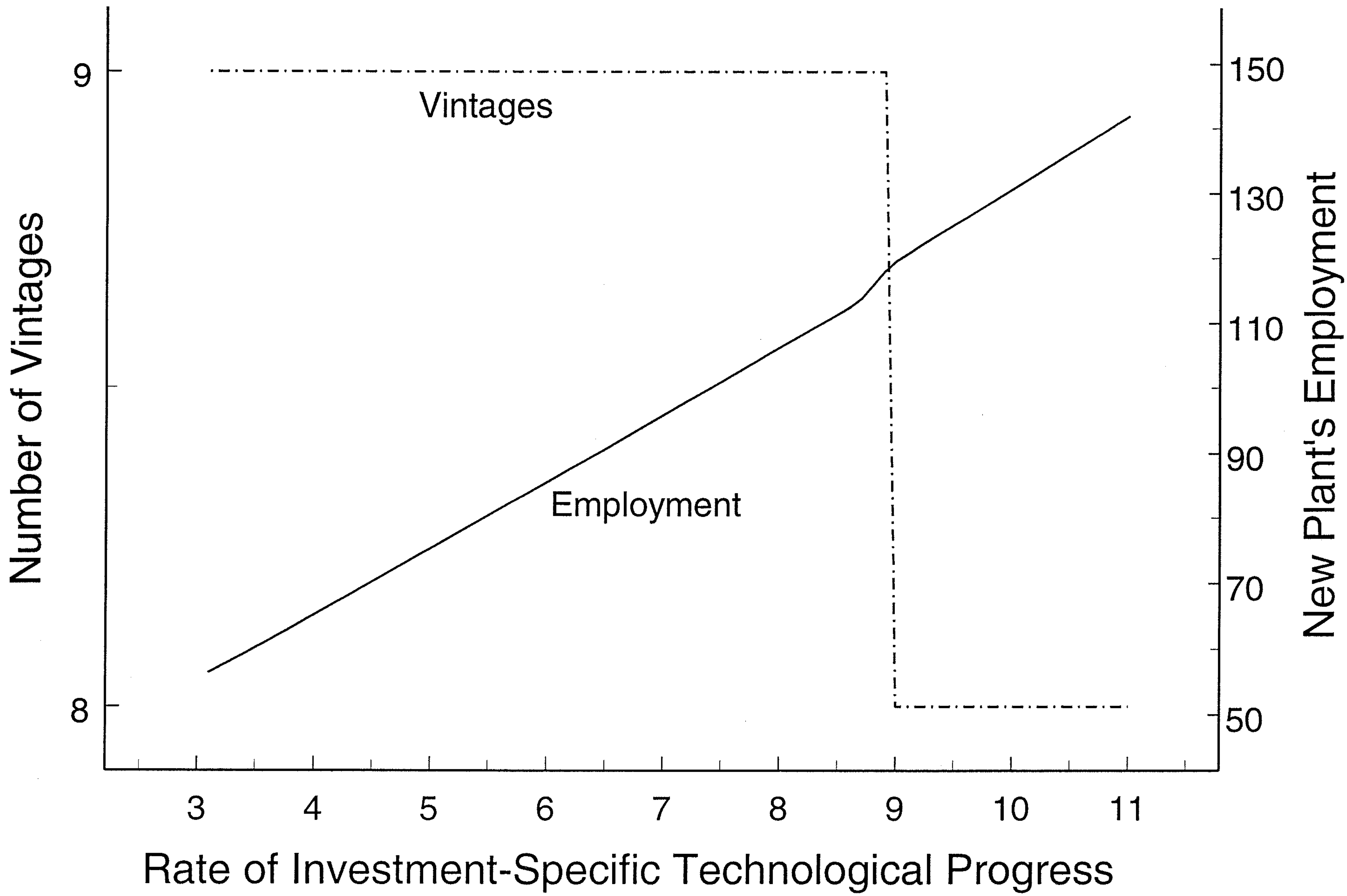


Figure 9: Income Inequality

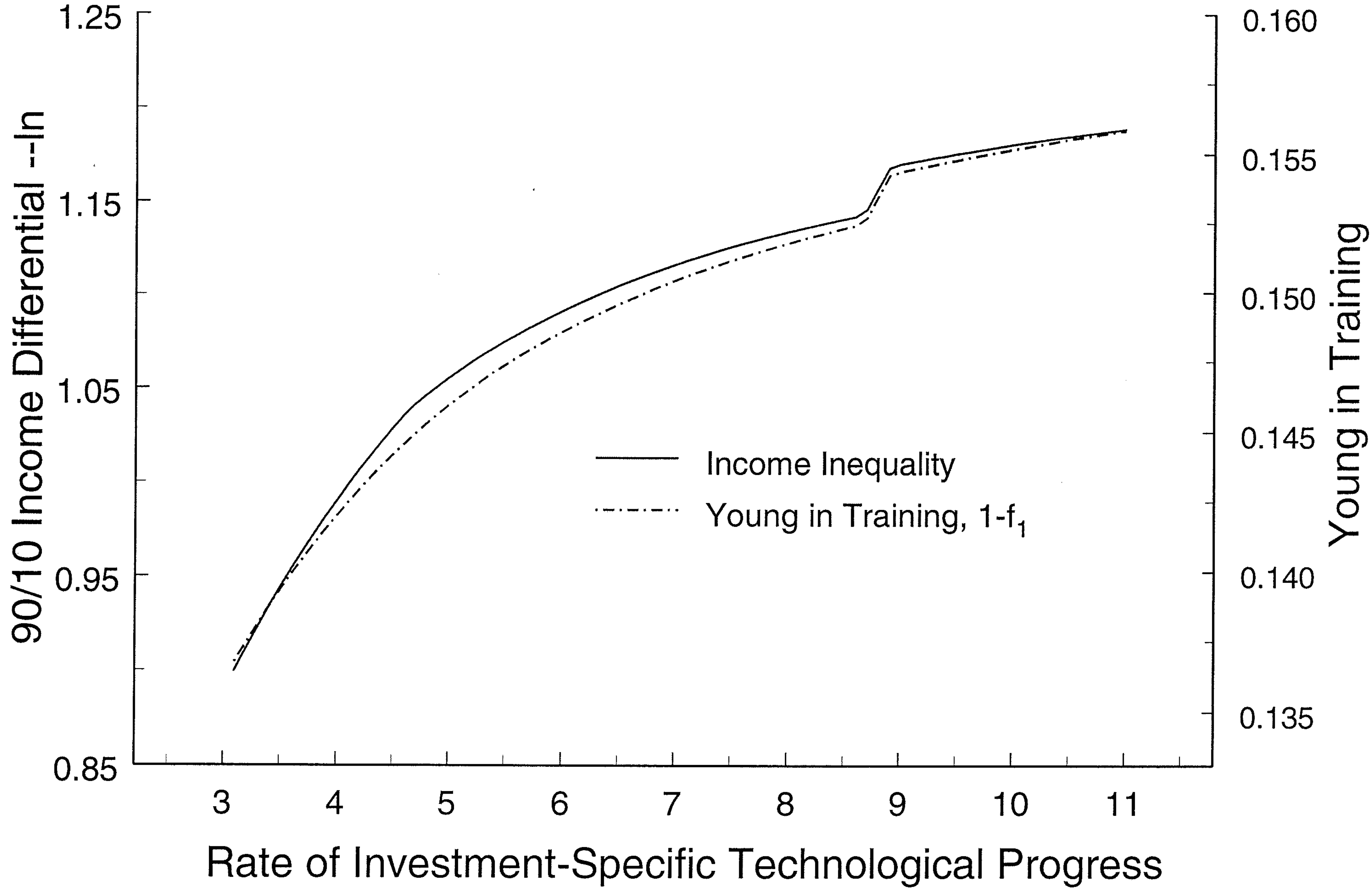


Figure 10 shows the learning curve for a plant. A plant's total factor productivity increases by approximately 15 percent over its first 16 years in operation, a number not too far off from Bahk and Gort's (1993) estimate of a 15 percent increase over the first 15 years. The learning curve is $\cap$-shaped. Most of the learning comes early on, over 80% within the first years. This is consistent with the case studies reported in Jovanovic and Nyarko (1995). The speed of learning picks up considerably with the rate of technological advance. Also, as a plant ages it is no longer profitable to hire the skilled labor necessary to keep productivity up. The share of skilled labor in total labor cost decreases monotonically with plant age, as Figure 10 illustrates. Bartel and Lichtenberg (1987, eq. 4) report that a one year increase in the average age of capital is associated with a drop of 0.78 percentage points in the share of skilled labor in the wage bill. The number here is 0.57.

3.1 *Investment in Learning*

Investing in learning today increases output tomorrow, just as investing in physical capital does. Each period $\sum_{j=1}^{n} p_j' v e_{j-1}$ is spent by plants to hire skilled labor to improve future productivity in the economy. GDP, as conventionally defined, is given by $\sum_{j=1}^{n} p_j \mu_j k_j^{\alpha} (l_j - \bar{l})^{\beta} h_j^{\zeta}$. Investment in plant-level learning as fraction of GDP is computed to be

$$\frac{\sum_{j=1}^{n} p_j' v e_{j-1}}{\sum_{j=1}^{n} p_j \mu_j k_j^{\alpha} (l_j - \bar{l})^{\beta} h_j^{\zeta}}. \tag{23}$$

Figure 11 gives these learning costs as a fraction of GDP. Not surprisingly, they increase with the rate of investment-specific technological change. They are not out of line with Jovanovic's (1995) calculation of the costs of technological adoption.

Howitt (1995) argues that GDP should be adjusted upward to incorporate investment in learning. That is, the national income identity should be rewritten as

$$\sum_{j=1}^{n} [f_i c_i + \int_{\lambda_i}^{\infty} z_i(\lambda) \Lambda(d\lambda)] + p_1'(k_1')^{\alpha} + \sum_{j=1}^{n} p_j' v e_{j-1} = \sum_{j=1}^{n} p_j \mu_j k_j^{\alpha} (l_j - \bar{l})^{\beta} h_j^{\zeta} + \sum_{j=1}^{n} p_j' v e_{j-1}. \tag{24}$$

Observe that investment in learning has been added to both sides of the equation. So, by this accounting, GDP would be 9% higher — at a 3% rate of investment-specific technological

Figure 10: Learning Curve

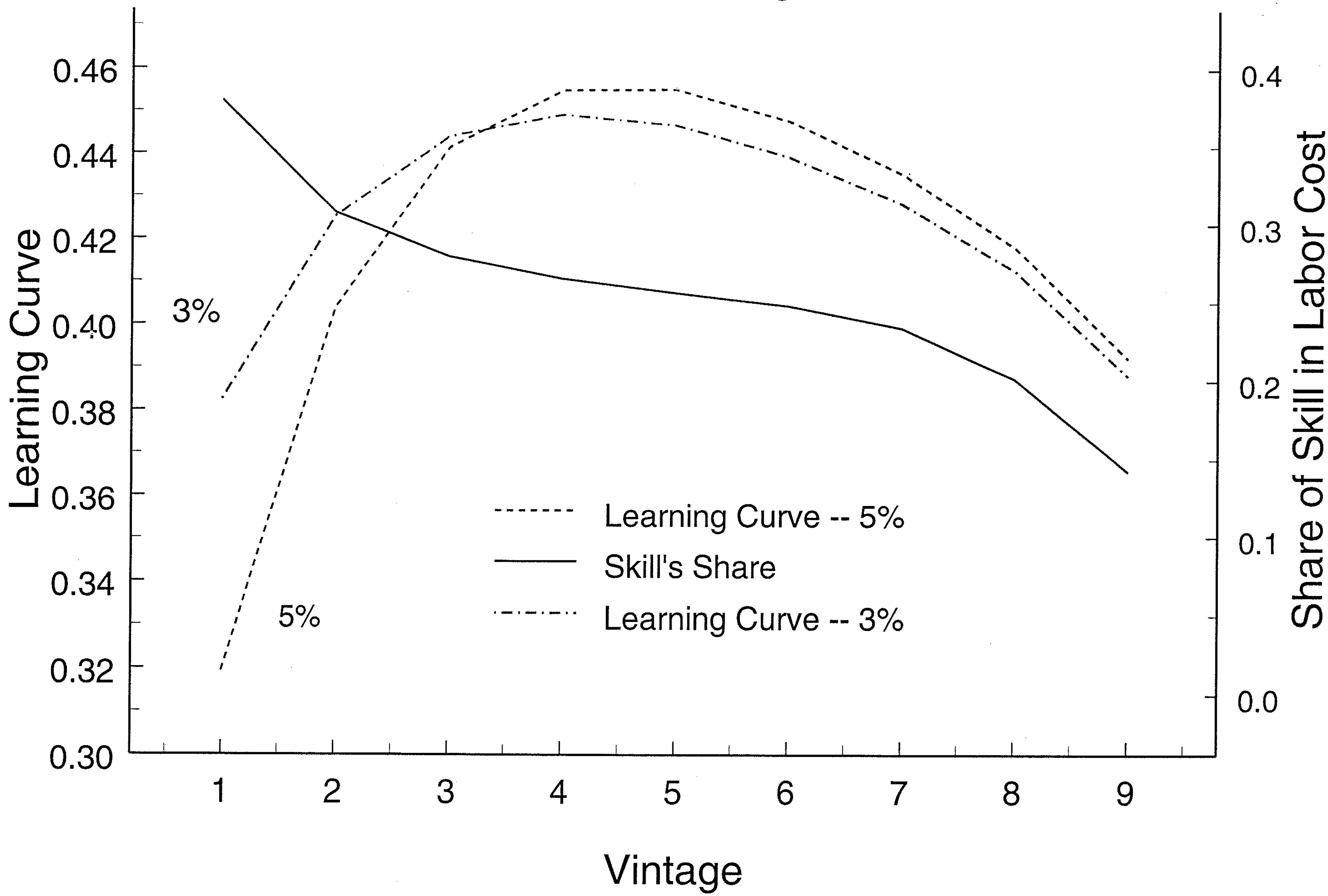
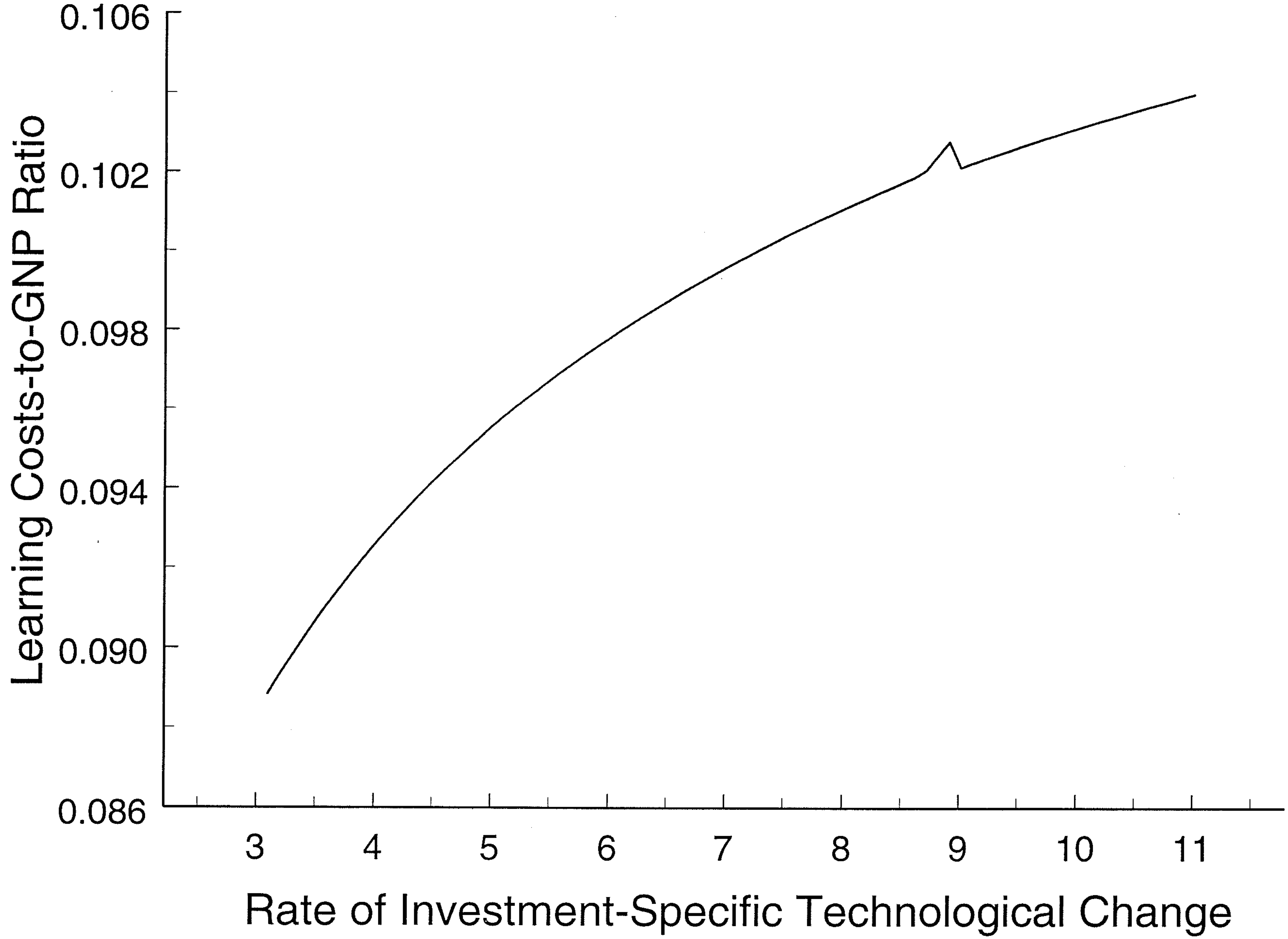


Figure 11: Learning Costs



change. In conventional GDP accounting expenditures on learning are taken out of profits (or expensed) so that profits, $\sum_{j=1}^{n} p_j[\mu_j k_j^{\alpha}(l_j - \bar{l})^{\beta} h_j^{\zeta} - vh_j - wl_j] - \sum_{j=1}^{n} p_j' v e_{j-1}$, plus payments to labor, $\sum_{j=1}^{n} p_j(vh_j + wl_j) + \sum_{j=1}^{n} p_j' v e_{j-1}$, add up to GDP, $\sum_{j=1}^{n} p_j \mu_j k_j^{\alpha}(l_j - \bar{l})^{\beta} h_j^{\zeta}$. In the adjusted accounts this wouldn't be the case. Here profits would be $\sum_{j=1}^{n} p_j[\mu_j k_j^{\alpha}(l_j - \bar{l})^{\beta} h_j^{\zeta} - vh_j - wl_j]$, which when added to labor income, $\sum_{j=1}^{n} p_j(vh_j + wl_j) + \sum_{j=1}^{n} p_j' v e_{j-1}$, give adjusted GDP, $\sum_{j=1}^{n} p_j \mu_j k_j^{\alpha}(l_j - \bar{l})^{\beta} h_j^{\zeta} + \sum_{j=1}^{n} p_j' v e_{j-1}$. It is clear the value of the economy's capital stock — both physical and informational — is given by q. Breaking this down into stocks of physical and informational capital would be a messy task, especially in the real world. Computing the rates of obsolescence for these stocks is yet another problem.

4 Transitional Dynamics

Imagine an economy riding along its balanced growth path. Now, suppose that suddenly the rate of investment-specific technological progress jumps up to towards a new higher level as a new technology comes on line. The impact of this technological change on income and productivity is likely to be regulated by two related factors: the speed of learning and the speed of diffusion. The more costly it is for economic agents to learn about a new technology the slower will be its speed of diffusion. But the faster a new technology diffuses through an economy the easier it may be to learn about it. If a new technology represents a radical or discrete departure from past technologies, society's knowledge about it may be quite limited at first. As use of the technology becomes widespread, society's stock of experience with it increases, and the technology's productivity rises. To capture such effects, let the baseline level of expertise associated with the adoption of the new technology, μ_0^*, be expressed as

$$\mu_0^* = \chi \gamma^{-\tau} [\frac{\sum p_j^*}{\sum p_j + \sum p_j^*}]^{\sigma}, \tag{25}$$

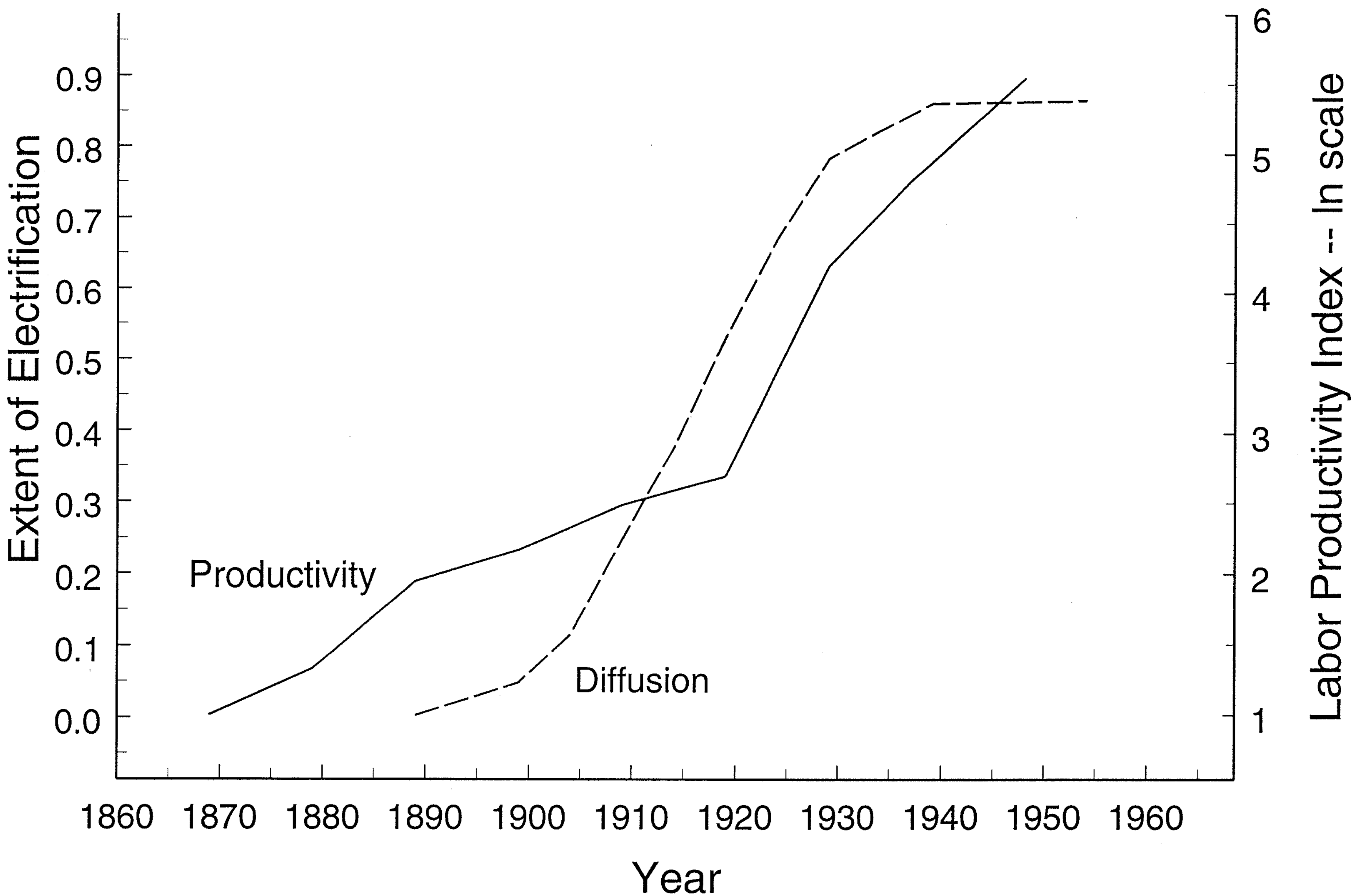
where p_j^* is the number of age-j plants using the new technology. Thus, the new technology's productivity increases with the fraction of plants using it. This formulation is rooted in David (1991, Technical Appendix A).

4.1 *The Electrification of America*

The electrification of America, as masterfully chronicled and analyzed by David (1991), illustrates the delays in the successful exploitation of new technologies. The era of electricity dawned around 1900. Electricity was obviously useful as a source of lighting in homes and businesses, but it had to supplant water and steam as source of power in manufacturing.[16] This was made difficult by the fact that there were large stocks of equipment and structures already in place geared to these sources of power. Thus, in the early stages, electricity tended to be overlaid onto existing systems already in place. In particular, the mechanics of steam and water power favored one power unit driving a group of machines. Hence, early electric motors were also used to drive a group of machines. The benefits of electricity derived from the savings in power requirements and the greater control over machine speed. The group drive system of belts and shafting used by steam and water power were retained. Not surprisingly, electric power tended to be used mostly in those industries that were rapidly expanding, since new plants could be designed to better accommodate this power source.

By around 1910 it was apparent that machines could be driven with individual electric motors. This had a large impact on productivity in the workplace. The belt-drive apparatus used in the group drive system could now be abandoned. Factory construction no longer needed to allow for the heavy shafting and belt-housing required for the group drive power transmission. Additionally, the labor needed to maintain this system was eliminated. Furthermore, flexibility in the production process rose for several reasons. The entire power system no longer needed to be shut down for maintenance or replacement purposes. Also, since each machine could be more accurately controlled, increases in the quantity and quality of output obtained. Machines could now be located, and moved more freely, to accommodate better the production process. Last, the workplace was made considerably safer. Figure 12 shows the diffusion of electric motors in manufacturing, where the data source is David (1991, Table 3). Electric motor horsepower, as a fraction of the horsepower of the total mechanical drive in manufacturing establishments, follows a typical S-shaped diffusion pattern.

Figure 12: The Electrification of America



It is interesting to note that labor productivity growth in manufacturing slows down at the time of electricity's introduction, where again the data is based on David (1991, Table 2).

4.2 *The Computational Experiment*

Returning to the issue at hand, suppose that the rate of investment-specific technological change jumps up from $\gamma = 3\%$ to $\gamma^* = 5\%$ with the *unanticipated* arrival of a new technology (numbers roughly in line with the pre- and post-1974 experience).[17] Assume that there is no further development of the old technology upon the announcement of the new one. David (1991) suggests that the opportunities for furthering the mass production technologies developed in the first half of this century may have been exhausted by 1974. In any event, note that in the absence of learning effects this experiment would correspond, more or less, to an increase in the rate of exogenous (investment-specific) technological change in a standard growth model. Thus, any interesting dynamics derive from the modelling of learning.

At the time of impact, the firm has a portfolio plants $(p_1, ..., p_n)$ using the old technology. In each period it must decide whether to use the new or old technology for the plants that are under construction. Clearly, it will pick the most profitable technology. Therefore, if

$$\{\max_{e_0^*} [\frac{V(k_1^{*\prime}, G(\mu_0^{*\prime}, e_0^*); \cdot^\prime)}{1+r} - (k_1^{*\prime})^\alpha - ve_0^*] - \max_{e_0} [\frac{V(k_1^\prime, G(\mu_0^\prime, e_0); \cdot^\prime)}{1+r} - (k_1^\prime)^\alpha - ve_0]\}$$

$$\begin{cases} < 0, & \text{then } p_1^{*\prime} = 0, \text{ and } p_1^\prime \geq 0, \\ = 0, & \text{then } p_1^{*\prime} \geq 0, \text{ and } p_1^\prime \geq 0, \\ > 0, & \text{then } p_1^{*\prime} \geq 0, \text{ and } p_1^\prime = 0. \end{cases} \tag{26}$$

Note that while new technology is technically superior to the old one, in the sense that $k_1^{*\prime} > k_1^\prime$, the baseline level of knowledge associated with operating it is initially less, or $\mu_0^{*\prime} < \mu_0^\prime$. At first, no one may adopt the new technology. Further technical improvement in the new technology will eventually be great enough, however, to entice some agents to use it. When this happens the economy's baseline level of knowledge associated with operating the new technology will begin to rise, in accordance with (25). In the long run, the economy will converge to a stationary portfolio of plants, $(p_1^*, ..., p_n^*)$, where just the new technology is used.

Figure 13 illustrates the impact that an unanticipated increase in rate of investment-specific technological change has on the economy. As can be seen, 2 periods (or 8 years) elapse before the new technology gets adopted and it is roughly 6 periods (or 24 years) before half of the plants are using the new technology. The new technology's diffusion is mildly S-shaped. Recall that condition (26) implies that firms only adopt the new technology when it is in their own best interest to do so. Investment in the old technology is skewed toward the beginning of the interim period before the adoption of the new one. Investing in the old technology clearly becomes less attractive relative to waiting to invest in the new technology as the adoption date approaches. In fact, the announcement of the new technology may cause investment in the old technology to rise initially.

Along the transition path income inequality rises and then falls to a new higher steady-state level. The upward jump in the rate of investment-specific technological progress increases the demand for skilled labor, since skill facilitates the adoption of new technologies. This drives up the skill premium, as Figure 14 illustrates. The rise in the skill premium entices more people to become skilled. Figure 14 also shows this. It is interesting to note that both the skilled and unskilled wage rates *fall* for a while along the transition path — Figure 15. The drop is quite significant for the unskilled wage rate. The announcement of the new technology is met by a boom on the stock market as is shown in Figure 16.

Who gains and loses from the introduction of the new technology? At the announcement date, the first four generations of skilled and unskilled agents realize a drop in their welfare. The last six find their lot in life improving. This transpires since older agents have more capital income and are affected less by the declining wage path. Thus, at the announcement date the majority of the population are made better off by the introduction of the new technology. It takes one generation (4 years) before the newly borne skilled become better off, and three generations (12 years) for the unskilled entering the world to see their lives

Figure 13: Diffusion

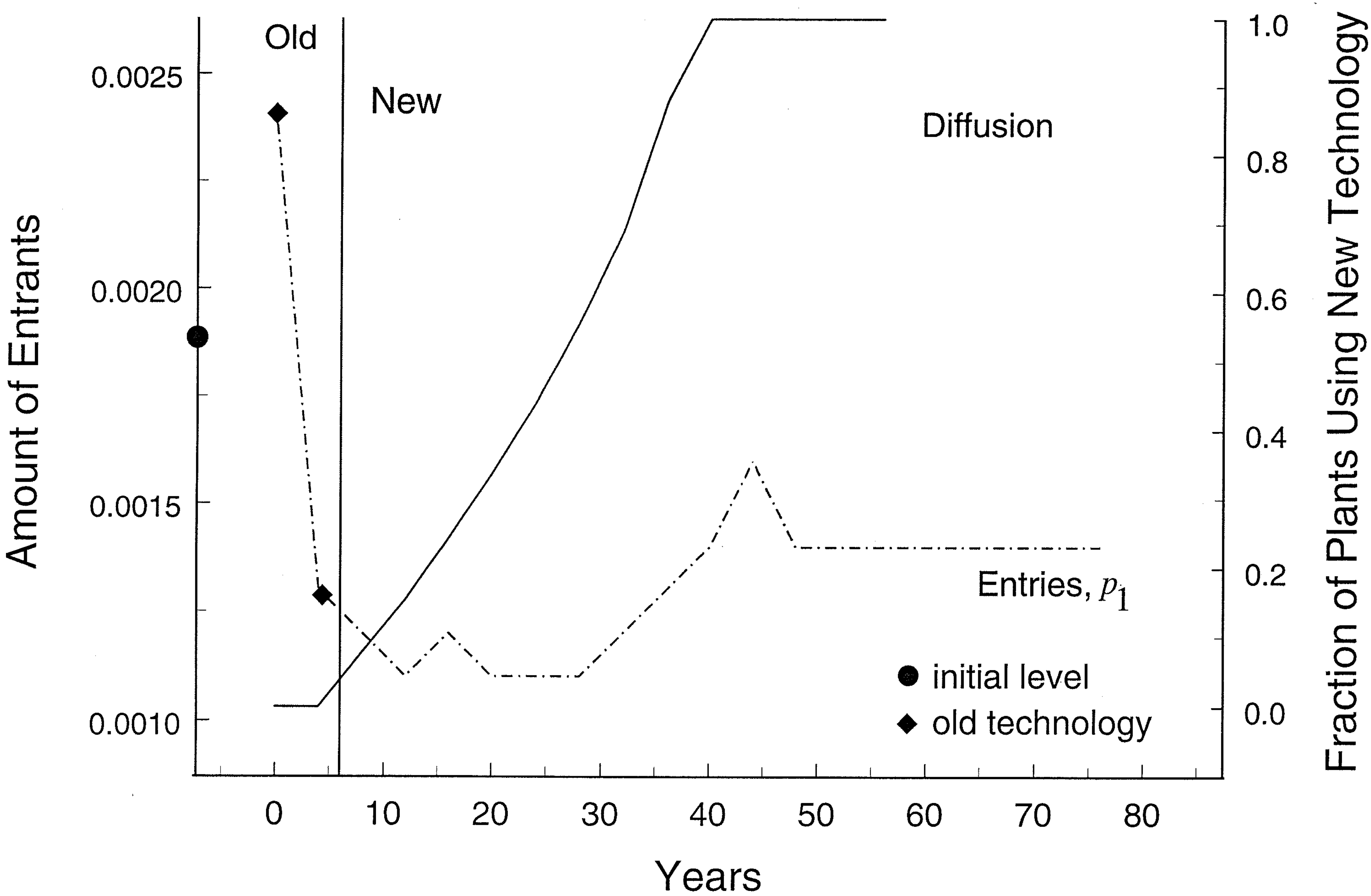


Figure 14: Income Inequality

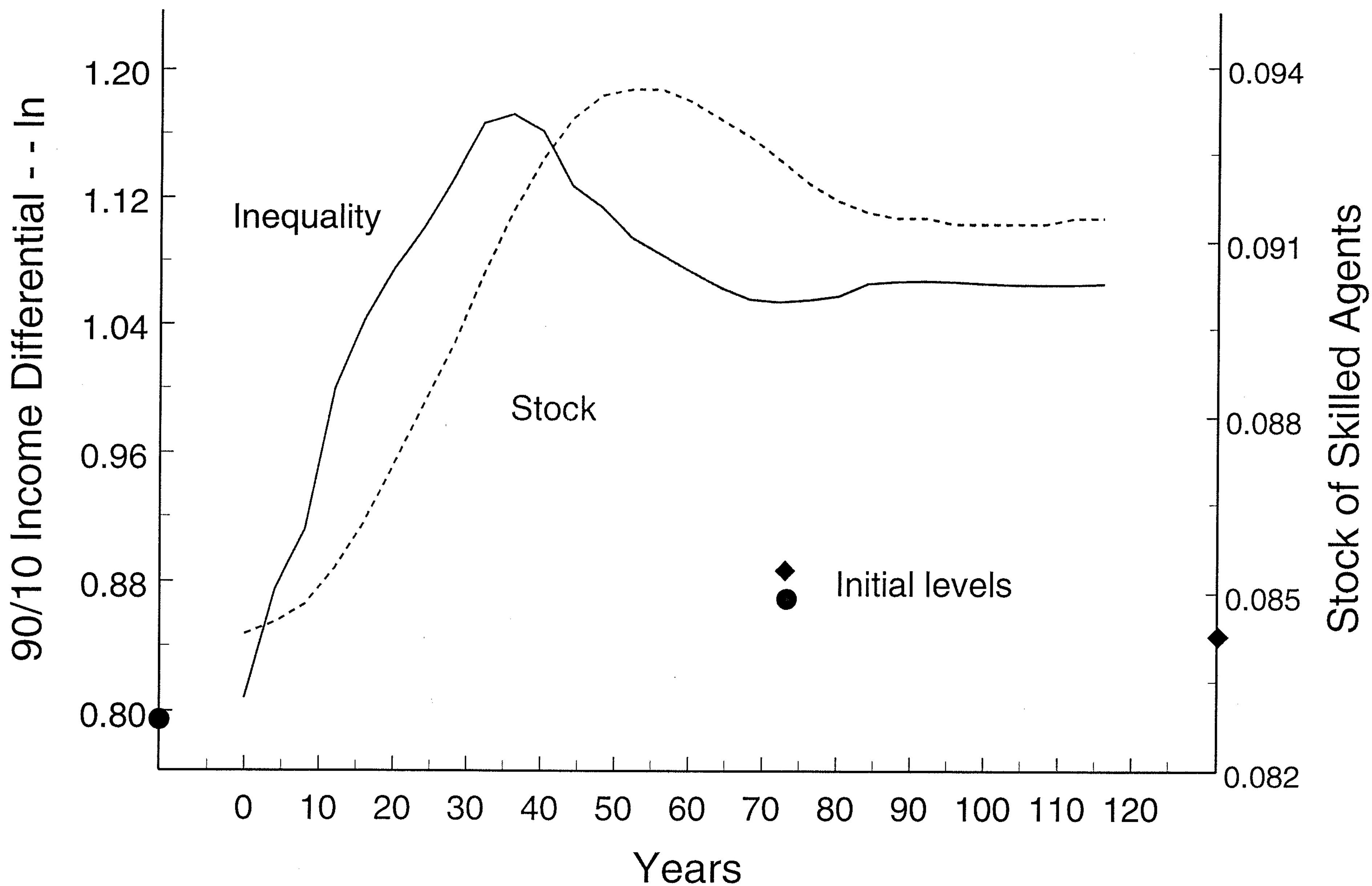


Figure 15: Wage Rates

Figure 16: Stock Price

improve.

What happens to the evolution of labor productivity when the pace of investment-specific technological change picks up? Figure 17 provides an answer to this question. Labor productivity slows down with the dawning of the new technology. The slowdown lasts for about 5 periods or 20 years. It occurs for two reasons. First, it takes time for the new technology to get adopted. Second, once adopted, it takes time for plants to get the new technology operating at, or near, its full potential. This is reflected in the high costs of adoption along the transition path — Figure 18. Note that while productivity rebounds quickly after 20 years it still about takes about 50 years to get up to its old trend path — Figure 17. What would happened if the old line of technology is alternatively allowed to continue to improve at an annual rate of 3%? In the short run there will be less incentive to adopt the new technology. The gap in productivity between the new and old technologies grows over time, however, so that eventually the new technology will get adopted. The evolution of labor productivity under this alternative scenario is also shown in Figure 17. It now takes 12 years, as opposed to 8 years, for the new technology to be adopted. Thus, it takes longer for the slowdown to emerge but when it does the results are similar to those found before. The time paths for the skilled and unskilled wage rates, and income inequality, mirror those plotted above for the original experiment.

4.3 *Some Other Ideas*

Technological advance is generally characterized by a continual flow of minor innovations, but once in a while a truly major innovation such as the steam engine or the microchip comes along. On both theoretical and empirical grounds one might expect that it may take a long time for the fruits of a truly major innovation to bear.[18] The above story presented a simple model of learning to explain this. This story could be augmented in several ways. First, in Andolfatto and MacDonald (1993) each major invention ushers in a wave of imitators. The odds of imitating a new invention depends on the number of firms who have already

Figure 17: Labor Productivity

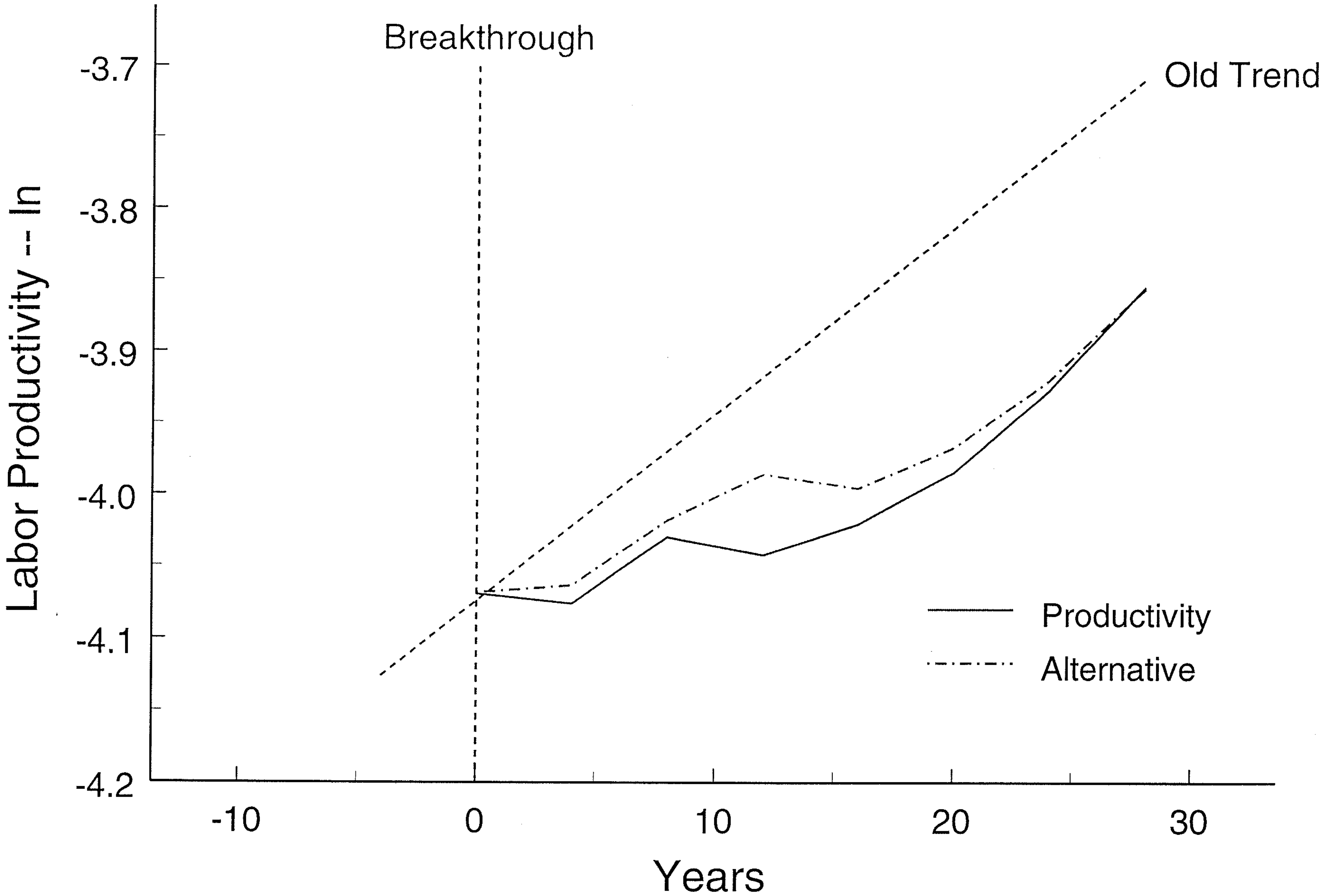
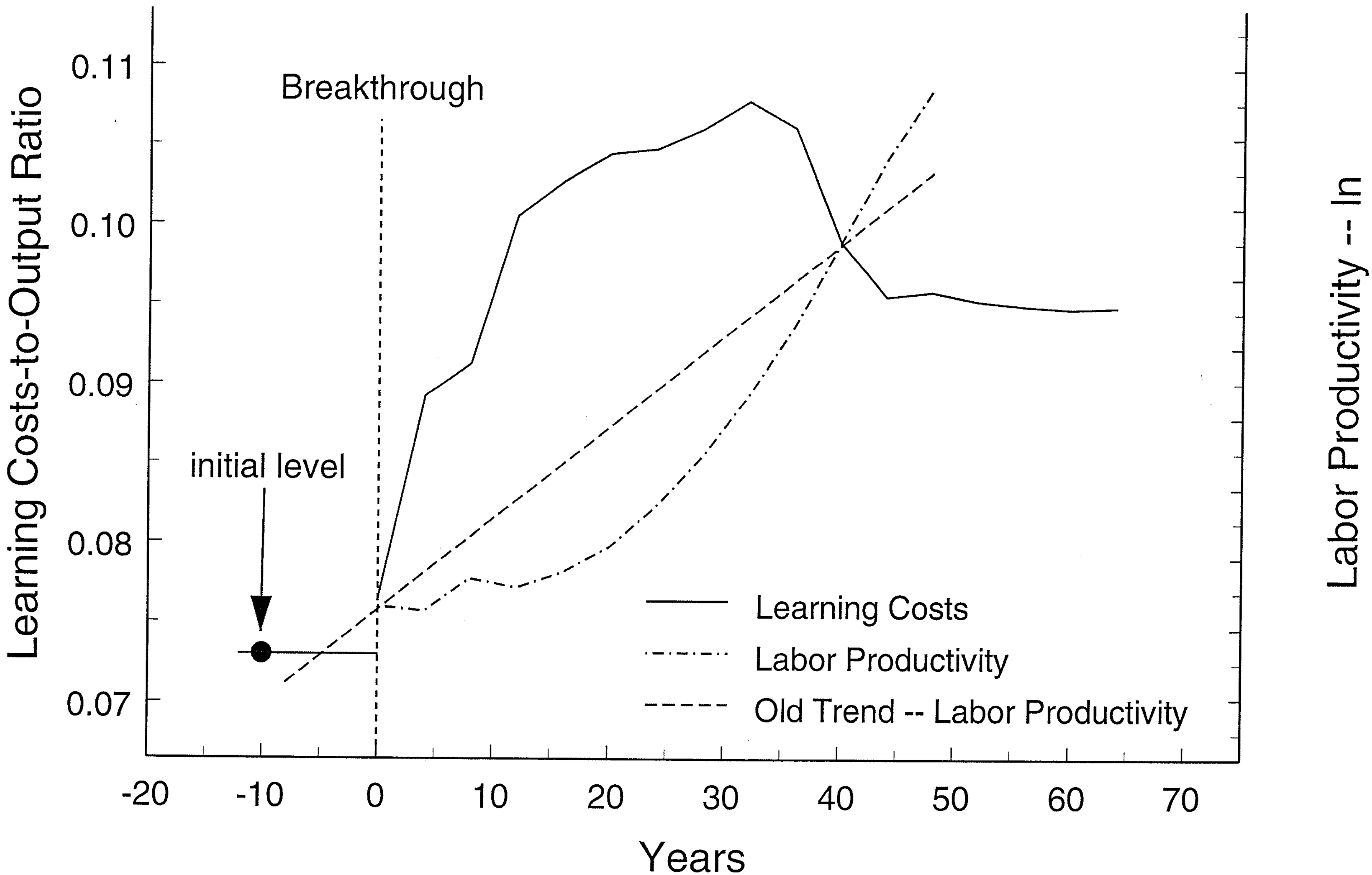


Figure 18: Learning Costs



successfully adopted the new invention — in a manner similar to (25). They view the time path for GDP as being made up by a pasted-together staircase of S-shaped curves, reflecting the invention, imitation, and invention cycle. Second, Helpman and Trajtenberg (1995) dub those technologies, which have universal and far reaching applications throughout the economy, general purpose technologies. They suggest that the advent of a general purpose technology may spawn a flood of innovation in complementary technologies such as software and communication devices. A lot of time and resources may have to be spend developing these complementary technologies. This phase may be associated with a productivity slowdown. Third, as highlighted by Jovanovic and Nyarko (1995), learning about multifaceted technologies will be slower than for single dimensional ones. Fourth, some agents may choose never to adopt a radically different technology even if it is better than an old one. This may happen when an agent is knowledgable about an old technology and knowledge is not very transferable across the old and new technologies — see Jovanovic and Nyarko (1996).

There are of course other explanations of the productivity slowdown. For example, Hornstein and Krusell (1996) and Howitt (1995) argue that technological progress is characterized by an improvement in the quality of goods and services produced and that a large part of this improvement in quality goes unmeasured.[19] If this is so, then productivity growth will be understated. But why the slowdown since 1974? Their answer is that the bounty of the recent burst of investment-specific technological progress may lie in those goods and services where quality improvements are hardest to measure. Hornstein and Krusell (1996, Table 5) present evidence suggesting that the share of these types of goods and services in national income has been steadily growing and that they now make up more than 60% of U.S. GDP. This story is complementary to the one told here.[20]

5 Conclusion

Plunging prices for new technologies, a surge in wage inequality, and a slump in the advance of labor productivity — could this be the hallmark of the dawn of an industrial revolution?

Just as the steam engine shook 18th century England, and electricity rattled 19th century America, are information technologies now rocking the 20th century economy?

The story told here is simple. Technological innovation is embodied in the form of new producer durables or services. The prices of these goods decline rapidly in periods of high innovation. Adopting new technologies is costly. Setting up, and operating, new technologies often involves acquiring and processing new information. Skill facilitates this adoption process. Therefore, times of rapid technological advancement should be associated with a rise in the return to skill. At the dawn of an industrial revolution, the long-run advance in labor productivity temporarily pauses as economic agents undertake the (unmeasured) investment in information required to get new technologies operating closer to their full potential.

A Appendix

A.1 *Parameter Values*

In order to simulate the model values must be assigned to its parameters. An inventory of the model's parameters and their assigned values is taken here.

1. Tastes: $\rho = 0.97^4$.

2. Production: $\alpha = 0.3$, $\beta = 0.3$, $\zeta = 0.05$, $\delta = 1 - (1 - .05)^4$.

3. Learning Curve: $\phi = 0.3$, $\kappa = 0.2$, $\vartheta = 1.0$, $\tau = 7.0$, $\chi = 1.0$, $\sigma = 2.0$.

4. Skill Distribution: $\iota = 8.0$.

5. Schooling Costs: $o = 0.5$, $\theta = 15.0$, $\Theta = 350.0$.

A.2 *Simulation Method*

A key step in solving the model is to deflate all nonstationary variables by a transformation of γ^t to render them stationary. To this end, the following definitions are made: $\widehat{c}_{i,t} = c_{i,t}/(\gamma_y)^t$, $\widehat{z}_{i,t} = z_{i,t}/(\gamma_y)^t$, $\widehat{k}_{j,t} = k_{j,t}/(\gamma_y)^{t/\alpha}$, $\widehat{w}_t = w_t/(\gamma_y)^t$, $\widehat{v}_t = v_t/(\gamma_y)^t$, $\widehat{q}_t = q_t/(\gamma_y)^t$, and $\widehat{d}_t = d_t/(\gamma_y)^t$. It is then easy to deduce that $\widehat{V}(\cdot_{j,t}) = V(\cdot_{j,t})/(\gamma_y)^t$, $\widehat{W}_i(\cdot_t) = W_i(\cdot_t)$ - $(\ln \gamma_y) \sum_{j=i}^{m} \rho^{j-i}(t + j - i)$, and $\widehat{S}_i(\cdot_t) = S_i(\cdot_t)$ - $(\ln \gamma_y) \sum_{j=i}^{m} \rho^{j-i}(t + j - i)$.[21] Also, note that $\widehat{S}_i(b_{i,t}(\lambda), \lambda; \cdot) = \widehat{S}_i(b_{i,t}(1), 1; \cdot) + \ln \lambda \sum_{j=i}^{m} \rho^{j-i}$, if $b_{i,t}(\lambda) = \lambda b_{i,t}(1)$.

Labor Allocations

$$(l_j - \bar{l}) = \left(\frac{\mu_j \widehat{k}_j^\alpha}{\mu_1 \widehat{k}_1^\alpha}\right)^{1/(1-\beta-\zeta)} (l_1 - \bar{l}), \text{ for } j = 2, ..., n, \tag{A.1}$$

$$h_j = \left(\frac{\mu_j \widehat{k}_j^\alpha}{\mu_1 \widehat{k}_1^\alpha}\right)^{1/(1-\beta-\zeta)} h_1, \text{ for } j = 2, ..., n,$$

Assuming that a plant of age j for $0 \leq j \leq n$ lives on for at least two more years then

$$\frac{\gamma_y[\widehat{k}'^{\alpha}_{j+1}(l'_{j+1}-\overline{l})^{\beta}h'^{\zeta}_{j+1}+\widehat{v}'G_1(\mu'_{j+1},e'_{j+1})/G_2(\mu'_{j+1},e'_{j+1})]}{1+r}G_2(\mu_j,e_j)=\widehat{v}.$$

Otherwise

$$e_j=0,\ \text{(assuming no more life)},$$

or

$$\frac{\gamma_y\widehat{k}'^{\alpha}_{j+1}(l'_{j+1}-\overline{l})^{\beta}h'^{\zeta}_{j+1}}{1+r}G_2(\mu_j,e_j)=\widehat{v},\ \text{(assuming one more year of life)}.$$

Entry and Exit Conditions

$$\max_{e_0}\{\widehat{V}(\cdot'_1)/(1+r)-\gamma_y-\widehat{v}e_0\}=0, \tag{A.2}$$

$$\max_{e_j}\{\widehat{V}(\cdot'_{j+1})/(1+r)-\widehat{v}e_j\}\begin{cases}<0, & \text{then } p'_{j+1}=0,\\ =0, & \text{then } 0\le p'_{j+1}\le p_j,\\ >0, & \text{then } p'_{j+1}=p_j \text{ (for } j\ge 1),\end{cases}$$

where the $\widehat{V}(\cdot_j)$'s are determined from the recursion

$$\widehat{V}(\cdot_j)=\ \max\{\widehat{F}(\cdot_j)-\widehat{w}l_j-\widehat{v}h_j+\gamma_y\max\{\max_{e_j}[\frac{\widehat{V}(\cdot'_{j+1})}{1+r}-\widehat{v}e_j],0\}\}.$$

Consumptions

$$\widehat{c}_i=\frac{1-\rho}{1-\rho^{m-i+1}}[a_i(\widehat{q}+\widehat{d})+\widehat{I}_i], \tag{A.3}$$

with

$$\widehat{I}_i=\widehat{w}+\frac{\gamma_y\widehat{I}'_{i+1}}{1+r}\ \text{and}\ \widehat{w}=\beta\mu_1\widehat{k}_1^{\alpha}(l_1-\overline{l})^{\beta-1}h_1^{\zeta},$$

and

$$\widehat{z}_i=\frac{1-\rho}{1-\rho^{m-i+1}}[b_i(\widehat{q}+\widehat{d})+\widehat{P}_i],$$

with[22]

$$\widehat{P}_i=\widehat{v}\int_{\lambda_i}^{\infty}\lambda\Lambda(d\lambda)+\frac{\gamma_y\widehat{P}'_{i+1}}{1+r}\ \text{and}\ \widehat{v}=\zeta\mu_1\widehat{k}_1^{\alpha}(l_1-\overline{l})^{\beta}h_1^{\zeta-1}.$$

Skill Decision

$$f_1=\Lambda(\lambda_1), \tag{A.4}$$

where λ_1 solves

$$\widehat{S}_1(0,\lambda_1;\cdot) - \Theta\lambda_1^{-\theta} = \widehat{W}_1(0;\cdot),$$

and $\widehat{S}_i(\cdot)$ and $\widehat{W}_i(\cdot)$ are determined by the recursions

$$\widehat{W}_i(\cdot) = \max\{\widehat{U}(\cdot_i) + \rho\widehat{W}_{i+1}(\cdot')\},$$

$$\widehat{S}_i(\cdot,1;\cdot) = \max\{\widehat{U}(\cdot_i) + \rho\widehat{S}_{i+1}(\cdot',1;\cdot)\}.$$

Market Clearing Conditions

$$\sum_{i=1}^{m}[f_i a'_{i+1} + b'_{i+1}] = 1, \tag{A.5}$$

$$\sum_{j=1}^{n} p_j l_j = \sum_{i=1}^{m} f_i,$$

and

$$\sum_{j=1}^{n} p_j h_j + \sum_{j=1}^{n} p'_j e_{j-1} = \sum_{i=1}^{m}\int_{\lambda_i}^{\infty}\lambda\Lambda(d\lambda) - o\int_{\lambda_1}^{\infty}\lambda\Lambda(d\lambda).$$

Share Price

$$\widehat{q} = \frac{\gamma_y \sum_{j=1}^{n'} p'_j \widehat{V}(\cdot'_j)}{(1+r)}.$$

Wealth Dynamics

Substituting the equations for c_i and z_i, given by (A.3), into (14) and (16), it is easy to see that distribution of wealth evolves according to

$$\widehat{q}a'_{i+1} = a_i(\widehat{d}+\widehat{q}) + \widehat{w} - \frac{1-\rho}{1-\rho^{m-i+1}}[a_i(\widehat{q}+\widehat{d}) + \widehat{I}_i], \tag{A.6}$$

and

$$\widehat{q}b'_{i+1} = b_i(\widehat{d}+\widehat{q}) + \widehat{v}\int_{\lambda_i}^{\infty}\lambda\Lambda(d\lambda) - \frac{1-\rho}{1-\rho^{m-i+1}}[b_i(\widehat{q}+\widehat{d}) + \widehat{P}_i].$$

In the current period the state of the system is given by $\widehat{s} = (p_1, ..., p_n, \widehat{k}_1, ..., \widehat{k}_n, \mu_0, ..., \mu_n, a_2, ..., a_m, b_2, ..., b_m, f_2, ..., f_m)$. The endogenous variables determined in this time period are the allocations l_j, h_j, e_j, p'_j, $\widehat{c}_i$, $\widehat{z}_i$, $\widehat{a}'_i$, $\widehat{b}'_i$, λ_1, and f_1, together with the prices w, v, r and q, and the auxiliary variables $\widehat{V}(\cdot_j)$, $\widehat{S}_i$, $\widehat{W}_i$, $\widehat{I}_i$, and $\widehat{P}_i$. Given values for the future variables l'_j, h'_j, e'_j, $\widehat{V}'(\cdot_j)$, $\widehat{S}'_i$, $\widehat{W}'_i$, $\widehat{I}'_i$, $\widehat{P}'_i$, $\widehat{c}'_i$, $\widehat{z}'_i$, the blocks (A.1) - (A.6) describe a system of $4n + n' + 8m + 7$ equations in $4n + n' + 8m + 7$ unknowns.

This system of equations can be solved, using the *extended-path* technique, in *roughly* the following way. To see how this can be done, express the above system of difference equations more compactly as:

$$\Delta(s', x; s, \overrightarrow{s}'', \overrightarrow{x}') = 0. \tag{A.7}$$

Here s' and x are the vectors of state variables and other endogenous variables that are to be determined in the current period. For each period, the system of equations represented by (A.7) determines a solution for s' and x as a function of the current state of the world, s, and the time paths for the state and other variables from next period on denoted by $\overrightarrow{s}''$and $\overrightarrow{x}'$. In writing (A.7) the variables $\widehat{V}'(\cdot_j)$, $\widehat{S}'_i$, $\widehat{W}'_i$, $\widehat{I}'_i$, and $\widehat{P}'_i$ have been solved out for in terms of the time paths for the other variables, so that there is no need to carry these variables around in the system.[23] The algorithm used to solve the difference equation system is now described.

1. A steady-state solution, s^* and x^*, for the system is computed. This steady state solution satisfies the condition $\Delta(s^*, x^*; s^*, \overrightarrow{s}^*, \overrightarrow{x}^*) = 0$. It is assumed that the system reaches this steady state by time $T + 1$.

2. An initial guess is made for $\{s_t\}_{t=1}^{T}$and $\{x_t\}_{t=1}^{T}$; i.e., for the time path of the system from period one on. Denote this guess by $\overrightarrow{s_1}^0 \equiv \{s_t^0\}_{t=1}^{T}$ and $\overrightarrow{x_1}^0 \equiv \{x_t^0\}_{t=1}^{T}$.

3. (Iteration j). The guess from the previous iteration, or $\overrightarrow{s_1}^{j-1}$and $\overrightarrow{x_1}^{j-1}$, is used to solve out recursively for a new time path for the s_t's and x_t's. Specifically, using (A.7), the period-one solution for s' and x, or s_2 and x_1, is found. This is done given a starting point for the system, $s = s_1$, and the guess $\overrightarrow{s}'' = \overrightarrow{s_3}^{j-1}$and $\overrightarrow{x}' = \overrightarrow{x_2}^{j-1}$. Let this solution be denoted by s_2^j and x_1^j. Then, using (A.7), the period-two solution for s'

and x, or s_3 and x_2, is obtained, given the current state of the system $s = s_2^j$ and the guess $\overrightarrow{s}'' = \overrightarrow{s_4}^{j-1}$and $\overrightarrow{x}' = \overrightarrow{x_3}^{j-1}$. This solution is represented by s_3^j and x_2^j. This procedure is continued until the terminal period T is reached.

4. A revised guess path $\overrightarrow{s_1}^j$ and $\overrightarrow{x_1}^j$ is created. This is done by letting $\overrightarrow{s_1}^j = (s_1^j, s_2^j, ...)$ and $\overrightarrow{x_1}^j = (x_1^j, x_2^j, ...)$. Step 3 is now repeated using these new guesses in (A.7) to obtain new sequences for the s's and x's.

5. Steps 3 and 4 are repeated until $(\overrightarrow{s_1}^j, \overrightarrow{x_1}^j) \to (\overrightarrow{s_1}^{j+1}, \overrightarrow{x_1}^{j+1})$. Then the time path's, $s_1, s_1, ...,$ and $x_1, x_1, ...$, solving (A.7) have been found.

Footnotes

1. The data is from Greenwood, Hercowitz and Krusell (1996), who use this series to calculate that 60% of postwar U.S. growth may be attributed to investment-specific technological change.

2. A Watts steam engine cost somewhere between £500 and £800, while a 40 spindle Jenny cost £5 or £6 (McPherson, 1994 p. 16). Operating a steam engine, though, was enormously expensive. They were hungry beasts. Landes (1969, p. 99-101) quotes a writer in 1778 as saying " the vast consumption of fuel in these engines is an immense drawback on the profits of our mines, for every fire-engine of magnitude consumes £3,000 of coals per annum. This heavy tax amounts almost to a prohibition." By comparison, it cost only £900 to feed 500 horses, which apparently could produce the same amount of work. Thus, the pursuit of an energy efficient steam engine was on. The Newcomen steam engine of 1769 needed 30 pounds of coal per horsepower hour while a Watts engine of 1776 required 7.5 pounds. By 1850 or so, this number had been reduced to 2.5 (Landes, 1969, p. 103).

3. Interestingly Mokyr (1994) emphatically states the notion that Britain's Industrial Revolution was due to its more advanced science is false. Rather ideas flowed from the Continent to Britain and then working technologies flowed back from Britain to the Continent. He cites (p. 38) an engineer of the day as stating "the prevailing talent of English and Scottish people was to apply new ideas to use and to bring such applications to perfection, but they do not imagine as much as foreigners." Mokyr (1994, p. 39) concludes that "Britain's technological strength during the industrial revolution depended above all on the abundance and quality of its skilled mechanics and practical technicians who could turn great insights into productive applications."

4. The number for 1867 was supplied by Peter Lindert. It reflects an adjustment to convert the data from individual income to household income to place this year more on par with the earlier years. This adjustment lessens the rise in income inequality [as compared with Lindert and Williamson (1983)]. The magnitude of the rise of income inequality over

the industrial revolution is a controversial topic in economic history. Given the limited availability of data for this period, it is probably best to view any estimate as lying within a large confidence interval.

5. It's ironic that one of the least productive inventions of the Industrial Revolution is the foundation of the current Information Age. In the period 1823-32 Charles Babbage created his "Difference Engine", which was a mechanical computer. Part of the insight for this invention came from a binary coded loom invented in 1801 by Jean-Marie Jacquard that used punchcards to control fabric patterns.

6. The quote is by C.K. Hyde (1977), *Technological Change and the British Iron Industry*, as cited by von Tunzelmann (1994, p. 277).

7. In 1840 roughly 30 percent of pig iron production was devoted to producing railway tracks, and the railway was using 30 percent of the country's steampower capacity (McPherson, 1994, Chap 3).

8. The labor force grew rapidly between 1820-1860, in large part due to immigration. Immigrants tended to take unskilled jobs so this should have exerted an upward pressure on wage premium. In fact, Williamson and Lindert (1980, Figure 9.1, p. 205) do document a positive correlation between labor force growth and inequality between 1820 and 1973. Thus, the simple story told here neglects an important aspect of U.S. economic history.

9. Additionally, numbers presented in Gallman (1992, Table 2.10.C) suggest that total factor productivity growth for the 1840-1860 subperiod was below the average for the longer 1840-1900 period.

10. By contrast this is not an implication of the capital-skill complementarity hypothesis. Suppose that skilled labor is more complementary with equipment than is unskilled labor. Then, other things equal, the skill premium should rise so long as the stock of equipment increases. That is, there should be a *secular* rise in the skill premium. See Krusell et al (1996) for more detail.

11. The vintage capital model developed here derives from the framework presented in

Cooley, Greenwood, and Yorukoglu (1995).

12. By running the recursion forward it is easy to see that

$$V(\cdot_{j,t}) = \Pi(\cdot_{j,t}) + \sum_{h=1}^{N-j}[\prod_{s=0}^{h-1} \frac{1}{1+r_{t+s}}]\Pi(\cdot_{j+h,t+h}),$$

where $\Pi(\cdot_{j,t}) \equiv F(\cdot_{j,t}) - w_t l_{j,t} - v_t(h_{j,t} + e_{j,t})$ and N is the maximum age of the plant.

13. Assume that all age-j plants, still operating next period, hire in the current period the same amount of skilled labor for adoption purposes. There is no intrinsic reason for this to be true, because some age-j plants may exit earlier than others and therefore may want to invest less in learning.

14. Suppose $o < 1$. Then when $i = 1$ problem P(3) has the same form with one exception: the budget constraint now reads $qb'_{i+1}(\lambda) = b_i(\lambda)[d + q] + (1 - o)\lambda v - z_i(\lambda)$. For the case where $o > 1$ problem P(3) would have to be altered for some $i > 1$ as well.

15. Similar behavior is also exhibited in the Cooley, Greenwood, and Yorukoglu (1995) vintage capital model.

16. While only 3% of households used electric lighting in 1899, almost 70% did by 1929 (David 1991, Table 3).

17. How much of the information age was, and will be, anticipated is an open question. Just after the World War II *Popular Mechanics* (March 1949) wrote: "Where a calculator on the ENIAC is equipped with 18,000 vacuum tubes and weighs 30 tons, computers in the future may only have 1,000 vacuum tubes and weigh only 1 1/2 tons."

18. The diffusion of new innovations is slow. Gort and Klepper (1982) study 46 product innovations, beginning with phonograph records in 1887 and ending with lasers in 1960. They trace diffusion by examining the number of firms that are producing the new product over time. On average there were only 2 or 3 firms producing each new product for the first 14 years after its commercial development, upon which there was a sharp increase in the number of firms (on average 6 firms per year over the next 10 years). It is interesting to note that prices fell rapidly following the inception of a new product (13% a year for the first 24 years). Using a 21 product subset of the Gort and Klepper data, Jovanovic and Lach

(1996) report that it took approximately 15 years for the output of a new product to rise from the 10 to 90% diffusion level. They also cite evidence from a study of 265 innovations that found that it took a new innovation on average 41 years to move from the 10 to 90% diffusion level. For instance, in the U.S. it took the steam locomotive 54 years to move from the 10 to 90 percent diffusion level and the diesel (a smaller innovation) 12 years.

19. Improvement in quality is not a new thing. According to McPherson (1994 p. 1), "In 1770, the average European farmed from sunrise to sunset six days a week. This individual ate mostly bread and owned one outfit of clothing. If this person was British he was slightly richer: He probably owned a pair of shoes. Travel to the next village was an occasion to remember for a lifetime. People went to bed when the sun went down because oil lamps were expensive and homemade candles and fat lamps were not bright enough to allow much activity at night." One can only surmise, then, that the advent of gas lighting (one of the first networks), which found its way into homes in the early 1800's, had a big impact on the quality of life. David (1991) claims that the development of electric trams cut the average urban worker's transportation time by somewhere between 30 and 45 minutes.

20. Hornstein and Krusell (1996) also engage the notion that learning effects associated with introduction of new capital goods might temporarily slowdown advances in productivity.

21. The notation $\widehat{F}(\cdot)$ is used to signify that the arguments in the function F are being evaluated at their transformed values.

22. Alternatively,

$$\widehat{I}_{i,t} = \widehat{w}_t + \sum_{h=1}^{m-i} (\gamma_y)^h [\prod_{s=0}^{h-1} \frac{1}{1+r_{t+s}}] \widehat{w}_{t+h}$$

and

$$\begin{aligned} \widehat{P}_{i,t} &= \widehat{v}_t \int_{\lambda_{i,t}}^{\infty} \lambda \Lambda(d\lambda) \\ &\quad + \sum_{h=1}^{m-i} (\gamma_y)^h [\prod_{s=0}^{h-1} \frac{1}{1+r_{t+s}}] \widehat{v}_{t+h} \int_{\lambda_{i,t}}^{\infty} \lambda \Lambda(d\lambda). \end{aligned}$$

23. For instance, $\widehat{I}_i'$ can be written in terms of w',w'', ... , and r', r'', ... , etc.

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