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Abstract

This paper analyzes the political economy of growth as an issue of inter-generational distribution. The first part of the paper develops a model of endogenous growth via accumulation of knowledge in a finite-horizon overlapping generations setting. Equilibrium growth is inefficient due to the presence of an intergenerational externality. We then analyze the outcome when the planner's objective mirrors those of the individuals in the economy. This results in a dynamic game between the current and future planners in which growth is again inefficiently low because future agents are unable to reward those currently alive to induce them to accumulate knowledge. Numerical examples suggest that the political equilibrium may be only marginally better than laissez-faire.

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In a world made up of individuals with finite horizons, any tradeoff between the well–being of those currently alive and economic growth will inevitably involve intergenerational conflict. Without the intergenerational markets needed to resove that conflict, economic growth enters the arena of political economy. While it is true almost by definition that a farsighted social planner can generally achieve efficiency, it is more realistic to consider the behavior of planners whose objectives reflect that of the population of which they are a part. Thus we analyze the decisions with regard to growth and intergenerational distribution of a planner with the same finite horizon as his constituents. The result is that in the absence of institutions that allow precommitment (or its equivalent in the form of "trigger strategy" equilibria), the sequence of finite–horizon planners will enact policies that while better than nothing exhibit inefficiently low growth.

The first part of the paper develops a simple endogenous growth model in which the engine of growth is the accumulation of knowledge. We assume that a higher level of knowledge attained by one generation reduces the cost of attaining that same level by the next, an externality which has the consequence that the laissez–faire equilibrium growth rate is inefficient. We then characterize the set of Pareto efficient accumulation paths and find that there is a continuum of efficient growth rate–interest rate combinations, the choice among which depends on the social discount rate. Competitive equilibrium with subsidized or mandated accumulation of knowledge may give rise to a Pareto efficient steady state, though for some parameters efficiency requires intergenerational redistribution.

The main contribution of the paper is to address the question of how a government whose decision–makers reflect the finite horizons of their constituents would choose policies that affect the accumulation of knowledge and of physical capital. Specifically we assume that each government maximizes a weighted sum of utilities of those currently alive. Policy decisions are modeled as the outcome of a non–cooperative dynamic Stackelberg game: Each period the government selects a policy that takes into account the effect (through state variables) on subsequent policy decisions (and hence on the welfare of the current young generation). Numerical methods are used to compute equilibria under specific parametric assumptions. The political equilibrium is generally inefficient, and only slightly superior to the *laissez–faire* equilibrium in terms of the growth rate.

This paper generalizes an earlier paper (Kahn (1996)) by including physical capital, a consequence of which is that finding the political equilibrium involves solving for equilibrium policy *functions* rather than just numbers. The findings confirm and indeed strengthen those in the earlier paper. A second contribution of this paper is that it provides a practical method for solving the problem of finite–horizon sequential decision–making in a fully dynamic infinite–horizon model. Other papers that have addressed related problems have typically resorted to shortcuts such as looking only at a two–period model, or making Nash rather than Stackelberg assumptions. The approach adopted here could have broad applicability for a number of political economy issues, including fiscal policy, capital taxation, and monetary policy.[1]

1. The Model

The model adapts the standard neoclassical overlapping generations model of capital accumulation to incorporate endogenous growth. In a sense it represents a cross between Diamond (1965) and Uzawa (1965).[2] Each generation (or "cohort") allocates time between labor and the accumulation of knowledge. Output depends on physical capital and effective labor, and exhibits constant

[1]See, for example, Persson and Svensson (1989), Persson, Persson, and Svenson (1987), Cukierman and Meltzer (1989).

[2]Azariadis and Drazen (1990) explore different issues with a similar extension of the Diamond model.

returns to scale. Knowledge is passed (at least to some degree) from one generation on to the next, along with physical capital. We assume only that a higher level of knowledge attained in one generation makes it less costly for the next generation to attain the same level. Thus the fact that the Wright brothers' generation discovered how to make airplanes fly did not mean that the next generation was born with this knowledge, only that it could attain that knowledge more easily, and without fully rewarding their predecessors (hence the externality).

We assume that within each period knowledge accumulated by an individual translates directly into his human capital, without any external spillovers. Hence in what follows we will speak of knowledge and human capital interchangeably. There is, however, an intergenerational externality, owing to the nonexcludability of knowledge across generations. That is, the older generation cannot sell its stock of knowledge to the young generation. In the model this is simply assumed, but even if it were technically possible to make the stock of knowledge excludable, the young have nothing to offer the old in exchange for it.[3]

Individuals live for two periods. All individuals within each cohort are identical. In their first period they allocate time between labor and accumulation of knowledge. We will refer to the time spent on human capital accumulation as "schooling", though a more apt interpretation is the share of flexible resources (in this case time) that productive individuals allocate to increasing their knowledge rather than producing. The wage they earn for labor depends on their accumulated human capital. They allocate their wage income in the first period between consumption when young and consumption when old. When old, individuals consume their savings plus interest.

[3]Of course in reality some knowledge is excludable. All that is required for the model is that some knowledge *not* be inter-generationally excludable. *Intra*-generational excludability is just a simplifying assumption.

Each individual solves the problem

$$\text{Max } u(c_{1t}) + \frac{1}{1+\alpha}u(c_{2t+1})$$

subject to

$$c_{1t} + c_{2t+1}/(1+r_{t+1}) = w_t H_t \ell_t \tag{1.1}$$

$$H_t = g(\ell_t)\bar{H}_{t-1} \tag{1.2}$$

where w_t is the wage per unit of human capital, H_t is the individual's human capital stock, $\bar{H}_{t-1}$ is the average human capital level of the previous generation, r_{t+1} is the interest rate, and $\ell_t \in [0,1]$ is the proportion of time allocated to labor. The remaining time $1-\ell_t$ is allocated to human capital accumulation. We assume that $g' \leq 0$, that $g(0) < \infty$, $g(1) \geq 0$, and that $u' > 0$, $u'' < 0$. Since all individuals within a cohort are assumed to be identical, we know that $H_t = \bar{H}_t$, so we will drop the distinction for the remainder of the paper.

The first order conditions for the individual's maximization problem are

$$u'(c_{1t}) = (1+\alpha)^{-1}(1+r_{t+1})u'(c_{2t+1}) \tag{1.3}$$

and

$$\ell_t g'(\ell_t) + g(\ell_t) = 0 \tag{1.4}$$

assuming interior solutions. Thus the individual simply chooses ℓ_t to maximize his earnings $w_t \ell_t H_t$, given 1.2. The solution to 1.4—and consequently the equilibrium growth rate—is independent of K_t and H_{t-1}.

Output is produced from a constant returns to scale production technology $F(K_t, N_t \ell_t H_t)$, where N_t is the number of individuals born in period t. We assume that $N_t = N_{t-1}(1+n)$. Competitive firms maximize profits, taking the wage and interest rate as given. Defining $k_t = K_t/(N_t \ell_t H_t)$, and $f(k_t) = F(k_t, 1)$, profit maximization implies

$$f'(k_t) = r_t, \tag{1.5}$$

and

$$f(k_t) - k_t f'(k_t) = w_t. \tag{1.6}$$

Thus the model is a straight generalization of Diamond's (1965) model. To reproduce that model we would set $g(\ell) = 1$. The equilibrium value of ℓ would be 1, the level of human capital would be fixed, and all of Diamond's results would follow.

In order to make the generalization interesting, we make one regularity assumption on $g(\ell)$. First define $\ell^* = \arg\max_{\ell} \ \ell g(\ell)$. Then we assume

A1: $\ell^* < 1$.

The assumption that $g(0) < \infty$ already rules out $\ell^* = 0$, so **A1** guarantees an interior solution for ℓ.

Equilibrium requires 1.3–1.6 and

$$N_t c_{1t} + N_{t-1} c_{2t} + K_{t+1} = F(K_t, H_t N_t \ell_t) + K_t \tag{1.7}$$

or

$$c_{1t} + c_{2t}/(1+n) = H_{t-1} g(\ell_t) \ell_t [f(k_t) + k_t - (1+n) g(\ell_{t+1}) k_{t+1} \ell_{t+1}/\ell_t] \tag{1.8}$$

where H_{t-1} and K_t are predetermined state variables for period t. Since ℓ^* is independent of the state variables, we can fix $g(\ell)$ and ℓ $\forall t$. The equilibrium conditions imply that

$$c_{2t} = (1+n)\ell^* H_t k_t (1 + f'(k_t)) \tag{1.9}$$

$$c_{1t} = \ell^* H_t[f(k_t) - (1+n)g(\ell^*)k_{t+1} - k_t f'(k_t)] \tag{1.10}$$

$$u'(c_{1t}) = (1+\alpha)^{-1}(1+f'(k_{t+1}))u'((1+n)\ell^* H_{t+1}k_{t+1}(1+f'(k_{t+1}))) \tag{1.11}$$

Given H_{t-1} and k_t, we have $H_t = g(\ell^*)H_{t-1}$, and equations 1.5–1.6, 1.9–1.11 determine c_{1t}, c_{2t}, k_{t+1}, w_t, and r_t.

We will focus on balanced growth steady states in which k is constant, under the assumption

A2: $u(c) = \begin{cases} c^{1-1/\sigma}/(1-1/\sigma), & \text{if } \sigma \neq 1 \\ \log(c) \text{ otherwise} \end{cases}$

In such a steady state, K/N, H, c_1, and c_2 all grow at the rate $g(\ell) - 1$. Conditional on ℓ^*, analysis of competitive equilibrium proceeds entirely as in Diamond (1965), albeit with a fixed growth rate $g(\ell^*) - 1$. In particular, the equilibrium may or may not be dynamically efficient. We shall see shortly, however, that the competitive outcome is *always* Pareto inefficient. We first analyze the problem of a planner with a fixed social discount rate.

1.1. A Social Planner's Problem

We first consider the solution of an infinitely lived social planner who discounts the utility of generations at rate ρ. At time 1 he chooses a path $\{c_{1t}, c_{2t}, \ell_t\}$ from $t = 1$ to ∞ to solve the problem

$$\text{Max} \sum_{t=1}^{\infty}(1+\rho)^{-t+1}N_t[u(c_{1t}) + \frac{1}{1+\alpha}u(c_{2t+1})] \tag{P2}$$

subject to

$$N_t c_{1t} + N_{t-1}c_{2t} + K_{t+1} = F(K_t, H_t N_t \ell_t) + K_t, \tag{1.12}$$

$$H_t = H_{t-1}g(\ell_t) \tag{1.13}$$

given K_1, H_0, and c_{21}. N_t enters the objective for convenience, but does not affect the analysis, since it just implies an effective discount factor of

$(1+n)/(1+\rho)$. Thus we will need to assume

A3: $\rho > n$

to assure a well-defined problem.

We can set up the following Lagrangian:

$$\begin{aligned}\mathcal{L} = \ & \sum_{t=1}^{\infty}(1+\rho)^{-t+1}\Big(N_t[u(c_{1t})+\tfrac{1}{1+\alpha}u(c_{2t+1})]+ \\ & \lambda_t[F(K_t, H_t N_t \ell_t)+K_t-N_t c_{1t}-N_{t-1}c_{2t}-K_{t+1}]- \\ & \mu_t[H_t-H_{t-1}g(\ell_t)]\Big)\end{aligned} \tag{1.14}$$

where λ_t and μ_t are multipliers associated with the two transition equations. The first order conditions for the solution of the optimization problem in $\{K_{t+1}, H_t, c_{1t}, c_{2t}, \ell_t, \lambda_t, \mu_t\}$ are

$$u'(c_{1t}) = \lambda_t \tag{1.15}$$

$$u'(c_{2t}) = \lambda_t(1+\alpha)/(1+\rho) \tag{1.16}$$

$$\lambda_t N_t H_t F_2(K_t, N_t H_t \ell_t) = -\mu_t g'(\ell_t)H_{t-1} \tag{1.17}$$

$$\lambda_t N_t \ell_t F_2(K_t, N_t H_t \ell_t) = \mu_t - \mu_{t+1}g(\ell_{t+1})/(1+\rho) \tag{1.18}$$

$$\lambda_t[1+F_1(K_t, N_t H_t \ell_t)] = \lambda_{t-1}(1+\rho) \tag{1.19}$$

along with the two constraints 1.12 and 1.13.

Although the adjustment to a steady state is of interest, we will focus only on the optimal balanced growth steady state in which k and ℓ are constant. First, 1.15 and 1.16 imply that the growth rates of c_{1t} and c_{2t} are the same in the steady state, as one would expect. Also, the homogeneity of F implies that the rate of growth of *per capita* consumption is equal to the rate of growth of human capital. With the CES utility function assumed above, and with

$F_1(K, N\ell H) = f'(k)$, $F_2(K, N\ell H) = f(k) - kf'(k)$, we have

$$g(\ell)^{1/\sigma} = \lambda_t/\lambda_{t+1}. \tag{1.20}$$

Equation 1.19 implies that

$$\lambda_t/\lambda_{t+1} = [1 + f'(k)]/(1+\rho). \tag{1.21}$$

Hence from equation 1.17 and 1.20 we have $\mu_{t+1}/\mu_t = (1+n)g(\ell)^{-1/\sigma}$, which, after some straightforward substitutions, yields:

$$1 + g'(\ell)\ell/g(\ell) = (1+n)g(\ell)^{1-1/\sigma}/(1+\rho). \tag{1.22}$$

Finally, 1.20 and 1.21 imply

$$1 + f'(k) = (1+\rho)g(\ell)^{1/\sigma}. \tag{1.23}$$

Equations 1.22 and 1.23 determine the planner's choice of ℓ, denoted ℓ_p, which in turn determines the optimal growth rate $g(\ell)$. While 1.23 is a standard $MRS = MRT$ condition, equation 1.22 equates the marginal foregone output from additional work to the discounted value of the resulting increased output the following period, in utility terms.

We can compare 1.22 with the equilibrium condition implied by (1.4), $1 + g'(\ell)\ell/g(\ell) = 0$. The two conditions coincide when $\rho = \infty$, as one might expect, because then $f'(k) = \infty$ by 1.23. The optimal and equilibrium growth rates also coincide when σ, the intertemporal elasticity of substitution, is zero. As σ increases the optimal growth rate increases as well, although it is necessary for ρ to increase with σ to keep the maximization problem well-defined. Except for the extreme cases, the planner's optimal ℓ is lower than the equilibrium ℓ, which means that the optimal growth rate generally exceeds the equilibrium growth rate for any $\rho < \infty$.

1.2. Efficient Knowledge Accumulation

The social planner's optimum yields a particular set of Pareto efficient allocations associated with different social discount rates, but as is well known from the work of Diamond (1965), Cass (1972), and others, the fundamental theorems of welfare economics do not apply to these economies. The competitive equilibrium need not be Pareto efficient, and the Pareto optima given by the planner's problem may not be achievable by decentralized equilibrium. It is also unclear whether equations 1.22 and 1.23 fully characterize the set of Pareto optimal steady state allocations, given that they come from a particular intergenerational weighting scheme.

This section analyzes the relationship between efficiency and equilibrium. We already know that ℓ^* is too large in equilibrium, so our default assumption is that a planner can impose a choice of ℓ directly (presumably the efficient choice) while allowing competitive equilibrium to determine the other endogenous variables. We will provide an alternative derivation of efficiency conditions for ℓ that are independent of the social discount rate. First, however, we will note necessary conditions for efficiency of k given a choice of ℓ.

The work of Cass (1972) and others suggests that a sufficient condition for dynamic efficiency of the path $\{K_t\}$, conditional on $\{\ell_t\}$, is that

$$\lim_{t\to\infty} \prod_{s=0}^{t} [1 + f'(k_t)]/[(1+n)g(\ell_t)] > 0 \tag{1.24}$$

In a steady state this condition translates into

$$1 + f'(k) \geq (1+n)g(\ell) \tag{1.25}$$

which, as we have seen, is satisfied by the planner's optimum. Also in a steady state the resource constraint 1.12 becomes

$$c_t + c_{2t}/(1+n) = H_t\ell[f(k) + k - (1+n)g(\ell)k]. \tag{1.26}$$

If $1 + f'(k) < (1+n)g(\ell)$, then reducing k would increase steady state consumption, a contradiction of efficiency.

The characterization of efficiency or inefficiency in $\{\ell_s\}$ is a problem of a different nature, because it is no longer just a matter of aggregate consumption efficiency. We can see from 1.26 that given H_{t-1} and K_t, maximizing $N_t\ell_tH_t$ yields the most resources to divide between c_{1t}, c_{2t}, and K_{t+1}. Given any choice of K_{t+1}, consumption efficiency would appear to require just such a maximization, and that is what occurs in the competitive equilibrium. It is easy to show, however, that this cannot generally be efficient. Suppose we fix c_{2t} and consider the effects of reducing ℓ_t below ℓ^*. Intuitively, this has zero first order effect on $N_t\ell_tH_t$, since we are starting from an interior maximum. Consequently we can leave c_{1t} and K_{t+1} unaffected on the margin. But it has a first–order effect on H_t, which carries over into $t+1$. Hence we can make the individual born at time t strictly better off, at least insofar as he has some positive elasticity of substitution between c_1 and c_2.

Starting from some path in which $\ell_t = \ell^*\ \forall t$, consider a perturbation of ℓ_t, holding fixed everything but the path of H_t and the consumption of cohort t. The effect on c_{1t} is

$$dc_{1t}/d\ell_t = F_2(K_t/N_t, \ell_tH_t)H_{t-1}[\ell_tg'(\ell_t) + g(\ell_t)] \tag{1.27}$$

which is zero at ℓ^*. The effect on H_{t+1} from the change is

$$dH_{t+1}/d\ell_t = H_{t-1}g(\ell^*)g'(\ell^*) \tag{1.28}$$

which is positive (for a marginal *decrease* in ℓ_t). Now consider the possibilities for c_{2t+1}. Even if we have $\ell_{t+s} = \ell^*$, $s = 1, 2, \ldots$, which means that H is on a permanently higher path as a result of the change in ℓ_t, the effect on c_{2t+1} is

$$dc_{2t+1}/d\ell_t = (N_{t+1}/N_t)F_2(K_{t+1}/N_{t+1}\ell^* H_{t+1})H_{t-1}\ell^* g(\ell^*)g'(\ell^*) \qquad (1.29)$$

which again is positive for a marginal decrease in ℓ_t. Thus we can make the generation born at t better off without making anyone else worse off.

To characterize efficient growth we can proceed as above, except that we need to take account of the fact that subsequent generations are made better off. We need to maximize the increase in c_{2t+1} as the consequence of lowering ℓ_t, which means leaving cohort $t+1$ no better off. In other words, a path for ℓ is efficient if it cannot be altered to increase some cohort's lifetime utility without reducing some other cohort's lifetime utility. So now in considering the effect of changing ℓ_t on c_{2t+1}, we will allow for the possibility of lowering ℓ_{t+1} so as to leave all future cohorts unaffected.

Now consider a path $\{c_{1s}, c_{2s}, K_s, H_s, \ell_s\}_{s=t}^{\infty}$. If it is efficient, then we should not be able to make cohort t better off by changing ℓ_t, while leaving subsequent cohorts no worse off. This will require

$$u'(c_{1t})dc_{1t}/d\ell_t + (1+\alpha)^{-1}u'(c_{2t+1})dc_{2t+1}/d\ell_t = 0 \qquad (1.30)$$

where $dc_{1t}/d\ell_t$ and $dc_{2t+1}/d\ell_t$ are constructed so as to leave all other cohorts' consumptions unchanged. Since there is no presumption that $\ell_t = \ell^*$ (in fact we know that $\ell_t < \ell^*$), we have to take account of the effect on c_{1t}. We also want to increase ℓ_{t+1} to the point that H_{t+1} is left unaffected, i.e. so that

$$dH_{t+1} = H_{t-1}[g(\ell_{t+1})g'(\ell_t)d\ell_t + g'(\ell_{t+1})g(\ell_t)d\ell_{t+1}] = 0 \qquad (1.31)$$

This implies

$$d\ell_{t+1}/d\ell_t = -g(\ell_{t+1})g'(\ell_t)/[g'(\ell_{t+1})g(\ell_t)] \tag{1.32}$$

Also, holding c_{2t} fixed, we have

$$dc_{1t}/d\ell_t = F_2(K_t/N_t, \ell_t H_t)H_{t-1}[\ell_t g'(\ell_t) + g(\ell_t)] \tag{1.33}$$

which is positive.

Next we have the effect on c_{2t+1}, which is

$$\begin{aligned} dc_{2t+1}/d\ell_t &= (N_{t+1}/N_t)F_2(K_{t+1}/N_{t+1}, \ell_{t+1}H_{t+1})H_{t-1} \\ &\quad \times [g(\ell_t)[\ell_{t+1}g'(\ell_{t+1}) + g(\ell_{t+1})]d\ell_{t+1}/d\ell_t + g'(\ell_t)\ell_{t+1}g(\ell_{t+1})] \end{aligned} \tag{1.34}$$

Substituting for $d\ell_{t+1}/d\ell_t$ using 1.32, and noting that $F_2(K_t/N_t, \ell_t H_t) = (1-\beta_t)f(k_t)$, where β_t is capital's share, we have

$$dc_{2t+1}/d\ell_t = -(1+n)(1-\beta_{t+1})f(k_{t+1})H_{t-1}g(\ell_{t+1})^2 g'(\ell_t)/g'(\ell_{t+1}) \tag{1.35}$$

Consequently a necessary condition for efficient growth is (from substituting equations 1.33 and 1.35 into 1.30):

$$u'(c_{1t})f(k_t)(1-\beta_t)[\ell_t g'(\ell_t) + g(\ell_t)] = \tag{1.36}$$

$$(1+\alpha)^{-1}u'(c_{2t+1})(1+n)(1-\beta_{t+1})f(k_{t+1})g(\ell_{t+1})^2 g'(\ell_t)/g'(\ell_{t+1})$$

Equation 1.36 is a necessary condition for the path $\{\ell_t\}$ to be Pareto efficient, provided $\ell_t \in (0,1)$. A similar perturbational argument for K_{t+1} yields another more familiar efficiency condition:

$$u'(c_{1t}) = (1+\alpha)^{-1}[1 + f'(k_{t+1})]u'(c_{2t+1}) \tag{1.37}$$

Combining 1.36 and 1.37, we have

$$f(k_t)(1-\beta_t)[\ell_t g'(\ell_t)+g(\ell_t)] = \tag{1.38}$$

$$(1+n)(1-\beta_{t+1})f(k_{t+1})g(\ell_{t+1})^2 g'(\ell_t)/g'(\ell_{t+1})/[1+f'(k_{t+1})]$$

The left side of 1.38 is proportional to the change in earnings from a change in ℓ_t, and the right side is the corresponding discounted change in earnings from the offsetting change in ℓ_{t+1}.

1.3. Steady State Analysis

Now consider a steady state in which ℓ and k are constant. From 1.38 we have

$$1+\ell g'(\ell)/g(\ell) = (1+n)g(\ell)/[1+f'(k)] \tag{1.39}$$

a condition that depends only on the economy's technology. In fact this condition is implied by the conditions 1.22 and 1.23, as can be seen by substituting one into the other. But equilibrium conditions will determine k, and these will generally depend on preferences, population growth, and government policies.

Note that 1.39 implies

$$1+f'(k) > (1+n)g(\ell) \tag{1.40}$$

for any steady state that has positive production. That is, in any efficient steady state with positive production, k must be strictly smaller than that which maximizes consumption per worker. This is because $1+f'(k)=(1+n)g(\ell)$ and (4.2) together would imply $\ell g'(\ell)/g(\ell)=0$, or $\ell=0$. Consequently if ℓ is chosen efficiently, steady state dynamic efficiency in k is assured

How could a Pareto efficient outcome be implemented? Essentially all that would be necessary is some mechanism to control ℓ, e.g. "mandatory schooling", plus in some instances the ability to make intergenerational transfers. Together with competitive labor and goods markets, these suffice to bring about a Pareto efficient steady state. Note, however, that the equilibrium k is normally increasing in ℓ (i.e. decreasing in the growth rate). This is because a higher growth rate causes reduced savings.

We can let $\psi(\ell)$ denote the competitive equilibrium steady state value of k as a function of an exogenously imposed ℓ. Let $\zeta(\ell)$ denote the steady state value of k as a function of ℓ that satisfies the efficiency condition 1.39. With $\psi(\ell)$ upward–sloping, and $\zeta(\ell)$ downward sloping, the intersection yields the unique efficient steady state (ℓ, k) under the assumption that a planner chooses the optimal ℓ while k is determined competitively.

Example: Suppose $f(k) = Ak^{\beta}$, and again assume $u(c) = \log(c)$, $g(\ell) = G(1 - \ell^{\nu})^{\xi}$, where $\nu > 1$, $\xi < 1$. Figure 1 displays the equilibrium for the parameters $A = 20$, $\beta = .5$, $G = 2$, $\nu = 2$, and $\xi = .5$ (so $g(\ell) = 2\sqrt{1 - \ell^2}$). The efficient ℓ, denoted ℓ_e, is approximately 0.57, which corresponds to $g = 1.65$. With a 25 year time period this would be approximately 2 percent annual growth. Equilibrium ℓ^*, on the other hand is 0.71, $g(\ell^*) = 1.41$, so growth would be less than 1.5 percent. Shifts in policy correspond to shifts in $\varphi(\ell)$, which would correspondingly shift the efficient ℓ and growth rate.

To summarize, the endogeneity of growth in this model implies that equilibrium is always inefficient, but that the way to efficiency *must* involve increasing knowledge accumulation, and need not involve reductions in physical capital.

1.4. Efficient Growth and Policy

Any government policies that affect k will shift the $\psi(\ell)$ schedule, implying that if ℓ is shifted accordingly to maintain efficiency, that the policies alter the growth rate of the economy. Differences in preference parameters or in population growth will also alter the efficient growth rate. For example, consider a pay–as–you–go social security system. This would be associated with a smaller steady state value of k, and consequentially the efficient growth rate is smaller as well. This represents a movement along the Pareto frontier, favoring the current old at the expense of the young and of future generations.

The same is true of any other policy that affects the equilibrium level of k, though the model in its present form is not rich enough to permit a variety of government policies. But, for example, if the government cannot set ℓ but instead has to achieve a desired value via taxes and subsidies, the ℓ it wishes to achieve (and consequently the growth rate) will depend, for example, on whether wages or interest earnings are taxed, and on whether deficit or surplus financing is used.

The remainder of the paper will drop the assumption that governments necessarily implement efficiency, and replace it with an assumption that governments have the same time horizon as their constituents, and act sequentially and in an uncoordinated fashion to maximize their welfare.

2. Political Economy

The normative implications of the model for government policy are straightforward, as we have seen. In particular, with the ability to make lump–sum transfers between individuals, government policy can in principle attain any point on the Pareto frontier. As a positive matter as well it would seem that a rational government ought to be interested in efficiency, regardless of how it chooses to split the rents. When distortions arise from the fact that individuals have finite horizons, however, it is less obvious that governments composed of such individuals will necessarily opt for efficiency. First, it might be necessary that those currently alive collectively appropriate the full gains from increased efficiency, or else they will lack the incentive to pursue it. Second, the gains must be distributed among those alive in accordance with the government's preferences. Otherwise the government could face a tradeoff between efficiency and the distribution of wealth.

In this part of the paper the political system is assumed each period to maximize a weighted sum of the utilities of those currently alive, taking into account the fact that the same decision process will take place in the next period, and that the choice today will influence next period's choice through its influence on the state variables of the economy. Thus political choice is depicted as a dynamic Stackelberg game between governments at different time periods. The decision problem within each period is treated like a bargaining problem, with the government selecting some point on the contract curve.[4] A solution technique is developed to solve for the equilibrium of this game as applied to the model from the first part of the paper. We assume that the political system chooses ℓ and the size and direction of intergenerational transfers.

In general the inability to coordinate with subsequent governments gives rise to inefficiency in the steady state. It turns out that the government improves upon the competitive equilibrium, but does not achieve Pareto efficiency. There exists a steady state policy that would make everyone better off by increasing growth (at the expense of current output) and increasing transfers to the old. That policy is not selected, however, because each government cannot coordinate with subsequent governments to carry out the transfer that results in the Pareto improvement. In equilibrium some of the gains from growth spill over to those not yet alive. Consequently governments opt for inefficiently low growth.

The model economy is the same as in Section 1 except that it will now incorporate an explicit policy of lump–sum intergenerational transfers. The consumption and savings decisions of individuals are determined in a

[4]Majority voting would not be very interesting in this context with only two types of agents.

competitive equilibrium in which each individual takes the political decision as given. The political decision, however, takes into account its effect on individual decision–making, and hence on the political decision of the next period. We introduce at this time a minor refinement in notation: $\bar{k}_t$ denotes the aggregate per capita quantity (which individuals view as exogenous), while k_t denotes the value that a representative individual chooses. Of course in equilibrium the two quantities are identical. Hence the individual's budget constraint is

$$c_{1t} + c_{2t+1}/(1 + r_{t+1}) = w_t H_t \ell_t - \tau_t H_t + \tau_{t+1} H_{t+1}(1 + n)/(1 + r_{t+1}) \qquad (2.1)$$

where w_t is the wage, and τ_t is the politically–determined lump–sum transfer (scaled by the level of the economy so that τ will be constant in a balanced–growth steady state) from cohort t to cohort $t-1$ at date t.

Market equilibrium requires $r_t = f'(\bar{k}_t)$ and $w_t = f(\bar{k}_t) - \bar{k}_t f'(\bar{k}_t)$. The first order conditions for the individual's maximization problem are as before:

$$u'(c_{1t}) = (1 + \alpha)^{-1}(1 + f'(\bar{k}_{t+1}))u'(c_{2t+1}) \qquad (2.2)$$

and the budget constraint 2.1. Equilibrium still requires 1.12 and 1.13, the equations that give the evolution of K_t and H_t. Consequently we have

$$c_{1t}/H_{t-1} = g(\ell_t)\left(\ell_t[f(\bar{k}_t) - \bar{k}_t f'(\bar{k}_t)] - g(\ell_{t+1})(1 + n)k_{t+1}\ell_{t+1} - \tau_t\right) \qquad (2.3)$$

$$c_{2t}/H_{t-1} = (1 + n)g(\ell_t)[\ell_t k_t(1 + f'(\bar{k}_t)) + \tau_t] \qquad (2.4)$$

For a given path of the policy variables τ_t and ℓ_t, the model can be solved for the equilibrium path of k_t, c_{1t}, c_{2t}, w_t, and r_t.

The political system at time t is assumed to choose τ_t and ℓ_t to solve

$$\operatorname*{Max}_{\ell_t, \tau_t} \frac{\theta}{1 + \alpha} u(c_{2t}) + (1 - \theta)[u(c_{1t}) + \frac{1}{1 + \alpha} u(c_{2t+1})] \qquad (\text{P2})$$

given $\bar{k}_t$ and $\bar{H}_{t-1}$, given 2.2–2.4 and knowing that at $t+1$ the same decision process will determine ℓ_{t+1} and τ_{t+1}.[5] Thus it follows that the political decision at t takes into account its effect on all future political decisions, since the decision at $t+1$ takes into account its effect on $t+2$, and so forth.

The result is a decision for (τ_t, ℓ_t) that should only depend directly on $\bar{k}_t$, H_{t-1} and next period's decision *rule* $(\tau_{t+1}, \ell_{t+1}) \equiv \Gamma_{t+1}(\bar{k}_{t+1}, H_t; ...)$. Consequently we have $\Gamma_t(\bar{k}_t, H_{t-1}; \Gamma_{t+1}(\bar{k}_{t+1}, H_t; \Gamma_{t+2}(\bar{k}_{t+2}, H_{t+1}; ...), ...))$. But in a symmetric equilibrium the state of the system at entering time t is fully described by $\bar{k}_t$ and H_{t-1}, so the equilibrium strategy can be described simply as $\Gamma(\bar{k}, H_{-1})$.

Even so, actually finding an equilibrium policy function remains a difficult task. It is possible in general only to characterize equilibrium sufficiently so that numerical techniques can find a solution under specific parametric assumptions. The results are suggestive of more general conclusions, and in any case can be compared to the "cooperative" solution of a longer– or infinitely–lived social planner. We do not address the questions of existence and uniqueness of equilibrium.

2.1. Solution Technique

The technique for solving the model consists of starting at an arbitrary time t with an arbitrary policy rule $\Gamma_{t+1}(\bar{k}_{t+1}, H_t)$ specified for the next period. This generates first–order conditions that characterize a policy rule $\Gamma_t(\bar{k}_t, H_{t-1}|\Gamma_{t+1}(\bar{k}_{t+1}, H_t))$. This process can be repeated until the function so generated converges to a rule $\Gamma(\bar{k}, H_{-1})$. The iteration process should not be thought of as dynamic convergence to a "steady state" $\Gamma(\cdot)$ function; it is just an expositional method for characterizing the equilibrium. The function so

[5] Although some types of intergenerational altruism in which agents effectively have an infinite horizon—such as in Barro (1974)—would make this problem completely uninteresting, the results that follow are not sensitive to the inclusion of a conventional bequest motive.

computed is valid globally, not just in steady state.

Although both $\bar{k}_{t-1}$ and H_t are state variables, in fact the model has been formulated in such a way that the two policy instruments ℓ_t and τ_t will only depend on $\bar{k}_t$. This is because of the homotheticity built into both preferences and technology. Substituting 2.3 and 2.4 into P2, we can express the political decision problem as

$$\operatorname*{Max}_{\ell_t,\tau_t} \frac{\theta}{1+\alpha}u((1+n)g(\ell_t)[\ell_t k_t(1+f'(\bar{k}_t))+\tau_t])+ \tag{P2$'$}$$

$$(1-\theta)\{u(g(\ell_t)[\ell_t[f(\bar{k}_t)-\bar{k}_t f'(\bar{k}_t)]-g(\ell_{t+1})(1+n)k_{t+1}\ell_{t+1}-\tau_t])+$$

$$\frac{1}{1+\alpha}u((1+n)g(\ell_t)g(\ell_{t+1})[\ell_{t+1}k_{t+1}(1+f'(\bar{k}_{t+1}))+\tau_{t+1}])\}$$

subject to 2.2–2.4, given $\bar{k}_t = k_t$ and H_{t-1}, and given $\tau_{t+1}(\bar{k}_{t+1})$, $\ell_{t+1}(\bar{k}_{t+1})$.

Note that 2.2–2.4 determine a function $k_t(\tau_t, \tau_{t+1}(\bar{k}_{t+1}), \ell_t, \ell_{t+1}(\bar{k}_{t+1}))$. That is, individuals choose savings taking policy variables as given. But they know that $\bar{k}_{t+1} = k_{t+1}$; hence if τ_t or ℓ_t change, with perfect foresight consumers take account of the effect on τ_{t+1} through the effect on $\bar{k}_{t+1}$. So to get, for example, the total effect of a change in τ_t on k_{t+1} (and hence on $\bar{k}_{t+1}$), we have

$$\frac{dk_{t+1}}{d\tau_t} = \frac{\partial k_{t+1}}{\partial \tau_t} + \left[\frac{\partial k_{t+1}}{\partial \tau_{t+1}}\frac{d\tau_{t+1}}{d\bar{k}_{t+1}} + \frac{\partial k_{t+1}}{\partial \ell_{t+1}}\frac{d\ell_{t+1}}{d\bar{k}_{t+1}} + \frac{\partial k_{t+1}}{\partial \bar{k}_{t+1}}\right]\frac{d\bar{k}_{t+1}}{d\tau_t}, \tag{2.5}$$

and since $\frac{d\bar{k}_{t+1}}{d\tau_t} = \frac{dk_{t+1}}{d\tau_t}$, we have

$$\frac{dk_{t+1}}{d\tau_t} = \frac{\partial k_{t+1}}{\partial \tau_t}\left[1 - \frac{\partial k_{t+1}}{\partial \tau_{t+1}}\frac{d\tau_{t+1}}{d\bar{k}_{t+1}} - \frac{\partial k_{t+1}}{\partial \ell_{t+1}}\frac{d\ell_{t+1}}{d\bar{k}_{t+1}} - \frac{\partial k_{t+1}}{\partial \bar{k}_{t+1}}\right]^{-1} \tag{2.6}$$

We similarly have

$$\frac{dk_{t+1}}{d\ell_t} = \frac{\partial k_{t+1}}{\partial \ell_t}\left[1 - \frac{\partial k_{t+1}}{\partial \tau_{t+1}}\frac{d\tau_{t+1}}{d\bar{k}_{t+1}} - \frac{\partial k_{t+1}}{\partial \ell_{t+1}}\frac{d\ell_{t+1}}{d\bar{k}_{t+1}} - \frac{\partial k_{t+1}}{\partial \bar{k}_{t+1}}\right]^{-1} \tag{2.7}$$

for the total effect of ℓ_t on k_{t+1}.

The effects given by 2.6 and 2.7 will enter the political decision process for τ_t and ℓ_t. They can be found by differentiating 2.2, and are detailed in the Appendix. As one would expect, the direct effect of a transfer from young to old is normally to decrease the saving of the young (i.e. $dk_{t+1}/d\tau_t < 0$), while the effect of increased time working relative to accumulating knowledge is to increase saving (i.e. $dk_{t+1}/d\ell_t > 0$), assuming that the marginal effect on current earnings is positive, which it always will be at the optimum.

Let $1 + \gamma_{t+1} \equiv (1+n)g(\ell_{t+1})$ and $q_t \equiv g'(\ell_t)/g(\ell_t)$. After some tedious but straightforward manipulations, the first-order conditions for (P2$'$) turn out to be

$$\theta(1+n)u'(c_{2t}) = (1-\theta)u'(c_{2t+1}) \times \tag{2.8}$$

$$\left\{(1+f'(k_{t+1})) - (1+\gamma_{t+1})\frac{dk_{t+1}}{d\tau_t}\left(\tau_{t+1}q_{t+1}\frac{d\ell_{t+1}}{dk_{t+1}} + k_{t+1}\ell_{t+1}f''(k_{t+1}) + \frac{d\tau_{t+1}}{dk_{t+1}}\right)\right\}$$

and

$$\begin{aligned} &\theta(1+n)u'(c_{2t})[(1+q_t\ell_t)k_t(1+f'(k_t)) + \tau_t q_t] = \\ &(1-\theta)u'(c_{2t+1})\{(1+f'(k_{t+1}))[-(1+q_t\ell_t)(f_t - k_tf'(k_t)) + \tau_t q_t] - \\ &(1+\gamma_{t+1})\left[q_t\tau_{t+1} + \tfrac{dk_{t+1}}{d\ell_t}\left(\tau_{t+1}q_{t+1}\tfrac{d\ell_{t+1}}{dk_{t+1}} + \ell_{t+1}k_{t+1}f''(k_{t+1}) + \tfrac{d\tau_{t+1}}{dk_{t+1}}\right)\right]\} \end{aligned} \tag{2.9}$$

Given k_t and sufficiently well-behaved functions $\tau_{t+1}(k_{t+1})$ and $\ell_{t+1}(k_{t+1})$, equations 2.2–2.4 and 2.8–2.9 can (in principle) be solved for τ_t and ℓ_t as function of k_t. An **equilibrium** is a pair of policy functions $\tau(k)$, $\ell(k)$ such that if $\tau_{t+1} = \tau(k_{t+1})$ and $\ell_{t+1} = \ell(k_{t+1})$, then the τ_t and ℓ_t values that satisfy 2.8 and 2.9, given that k_{t+1} comes from 2.2–2.4, are $\tau(k_t)$ and $\ell(k_t)$.

If we combine 2.8 and 2.9 to eliminate the marginal utility terms we get

(after some simplification)

$$1+q_t\ell_t = \frac{1+\gamma_{t+1}}{1+f'(k_{t+1})} \times \tag{2.10}$$

$$\left\{-q_t\tau_{t+1} + \left[[(1+q_t\ell_t)k_t(1+f'(k_t))+\tau_t q_t]\frac{dk_{t+1}}{d\tau_t} - \frac{dk_{t+1}}{d\ell_t}\right] \times \right.$$

$$\left.\left(\tau_{t+1}q_{t+1}\frac{d\ell_{t+1}}{d\bar{k}_{t+1}} + k_{t+1}\ell_{t+1}f''(k_{t+1}) + \frac{d\tau_{t+1}}{d\bar{k}_{t+1}}\right)\right\} \times \frac{1}{(f(k_t)+k_t)}$$

Using the relationship (A3) from the Appendix to eliminate $dk_{t+1}/d\ell_t$ we get

$$1+q_t\ell_t = \frac{1+\gamma_{t+1}}{1+f'(k_{t+1})} \times \tag{2.11}$$

$$\left\{-q_t\tau_{t+1} + [q_t\ell_t k_t(1+f'(k_t))+\tau_t q_t + f(k_t)+k_t]\frac{dk_{t+1}}{d\tau_t} \times \right.$$

$$\left.\left(\tau_{t+1}q_{t+1}\frac{d\ell_{t+1}}{d\bar{k}_{t+1}} + k_{t+1}\ell_{t+1}f''(k_{t+1}) + \frac{d\tau_{t+1}}{d\bar{k}_{t+1}}\right)\right\} \times \frac{1}{(f(k_t)+k_t)}$$

Recall again that the *laissez-faire* equilibrium has $1+q_t\ell_t = 0$, while the optimal steady state has $1+q\ell = (1+\gamma)/(1+f'(k))$. The above condition clearly differs from either of these cases, but it is difficult to say much more than that without either simplifying the model or looking at numerical examples. We will take the latter route. See Kahn (1996) for a model without physical capital in which analytical solutions are possible.

2.2. Numerical Methods and Results

We now turn to solving for the equilibrium policy functions $\tau(k)$ and $\ell(k)$ numerically. The method to be used here will be to assume that they can be approximated by a polynomial. Specifically we will assume that

$$\tau(k) = \sum_{i=0}^{m} \upsilon_i p_i(k) \tag{2.12}$$

$$\ell(k) = \sum_{i=0}^{m} \omega_i p_i(k) \tag{2.13}$$

where p_i is the ith–order Chebyshev polynomial in k (with the appropriate domain adjustment). The Chebyshev polynomials are a family of orthogonal polynomials defined by $p_0(x) = 1$, $p_1(x) = x$, $p_i(x) = 2xp_{i-1}(x) - p_{i-2}(x)$, on the interval $[-1, 1]$.

If $\tau(k)$ and $\ell(k)$ satisfy the above, then $\tau'(k)$ and $\ell'(k)$ are defined accordingly. The solution procedure involves selecting a value of m and finding values of w and z that approximately satisfy the system (2.8)–(2.9) . Of course unless the true solution is a polynomial of order less than or equal to m, there will not be a solution at each stage that holds for all values of k_t. A variety of methods can be used to find solutions that are good approximations. One convenient method advocated by practitioners of numerical techniques (e.g. Judd (1991)) is to solve the system exactly at $m+1$ points, specifically the roots of p_{m+1}. The accuracy of the fit can than be checked at intermediate points, and in particular at the steady state value of k.

Results were computed for a the case of Cobb–Douglas production $f(k_t) = Ak_t^{\beta}$ and CES utility $u(c) = c^{1-1/\sigma}/(1-1/\sigma)$ under a variety of parametric assumptions. It turns out that relatively low order polynomials (e.g. $m = 4$, meaning a cubic equation) provide a good approximation to the true equilibrium policy functions, at least for k not too small. Figure 2 plots a representative graph of the steady state equilibrium interest rate $1 + f'(k)$ and aggregate equilibrium growth rate $g(\ell_e) - 1$ against θ. Also plotted are the steady state efficient growth rate $(g(\ell_p) - 1)$ given the same steady–state k and the laissez–faire growth rate $g(\ell^*) - 1$. (The rates are annualized percentage rates based on a 30 year period). The specific parametric and functional form assumptions are $\beta = 0.3$, $\sigma = 1$, $A = 6$, $n = 0.3$, and $g(\ell) = 2\sqrt{1-\ell^2}$. Note that for these parameters the efficient planner's problem is only well–defined for $\theta > 0.51$, because the implicit social discount rate α associated with lower values of θ would fall below n, and the maximization problem (P2) would have no solution.

Figure 3 plots the equilibrium and efficient total growth rates $(1+n)g$ against the interest rate, again in annualized percentage rates. The diagonal dotted line represents the 45° line, so anything to the left of it is dynamically inefficient. Again the efficient planner's problem is not defined in that region. The equilibrium growth rate here is seen only to kick up in the dynamically inefficient region.

Finally, Figure 4 plots the two equilibrium policy functions $\tau(k)$ and $\ell(k)$ for the case of $\theta = 0.6$. Note that these policy functions are valid for any value of k, not just in steady state. At this value of θ the equilibrium per capita growth rate is 1.34 percent, while the efficient growth rate is 1.75 percent.

The main finding is that for moderate values of θ (say between 0.5 and 0.7) the equilibrium growth rate falls substantially short of the efficient growth rate. This is because the equilibrium growth rate is essentially flat with respect to the interest rate, hence there is no k for which the equilibrium growth rate would be efficient. For the case plotted in the figures the *per capita* equilibrium growth rate hovers at about 1.3 percent all but extreme values of θ, while the efficent rate varies between 1.5 and 2.3 percent. By comparison, the *laissez–faire* equilibrium growth rate is just under 1.2 percent. Similar results were obtained for a variety of parameters.

The intuition for the qualitative result is that the benefits of growth largely spill over onto subsequent generations. There is no mechanism available by which a subseqent generation can commit to reward the previous generation for its sacrifices. To some extent each generation can extract some reward for growth via its influence on subsequent policy decisions through the state variables of the economy. The government is assumed to exploit this to the extent possible in choosing a point along a pseudo–Pareto frontier. In the examples computed this effect is rather meager, and leads to only a slight improvement over *laissez–faire.*

The other notable feature of the numerical results is that the equilibrium $\ell(k)$ function is virtually flat, and that steady state ℓ also does not vary much with θ or with k, in or out of steady state. This would appear to rule out explaining differences in growth rates by differences in social policy preferences (as represented by θ), in contrast to the infinite horizon case where the social discount rate matters a lot.

2.3. Trigger–Strategy Equilibria

As in Kahn (1996), the analysis would not be complete without some discussion of trigger–strategy equilibria. It is clearly possible to sustain an equilibrium that is superior to the Markovian outcome if appropriate out–of–equilibrium beliefs are specified. In particular, the efficient solution is a trigger–strategy subgame perfect equilibrium under the following beliefs: If the planner at date t deviates in any way, subsequent planners revert to the Markovian equilibrium.Given those beliefs, the best deviation a planner could make is the Markovian allocation itself, which is demonstrably inferior to the efficient solution. So no planner would chose to deviate, and the efficient alllocation is sustained.

Thus such a trigger–strategy equilibrium is effectively equivalent to having a planner with an infinite horizon, or one who can commit future planners to a particular path of policies. The problem is obviously that it is only one of many equilibria, and whether one wants to contrast the Markovian equilibrium with the trigger–strategy efficient outcome or with the infinite–horizon planner, the basic point is the same: More than one outcome is possible, and some are distinctly inferior to others. Moreover, the trigger–strategy equilibrium suffers from the conceptual drawback that it relies on the belief that a deviation will give rise to a punishment that—if the deviation were to occur—would be undesirable to carry out.

3. Discussion and Conclusions

This paper has developed a model of sequential government decision–making in a finite–horizon setting, and applied it to a simple endogenous growth model. The approach yields explicit policy outcomes in equilibrium, and we suspect that it could be useful for a variety of policy questions beyond those addressed here. Each government's objective mirrors the objectives of the individuals currently alive. Each rationally takes its effect on subsequent governments' actions into account when making its policy decision. We have limited our attention to Markovian solutions, i.e. those in which policies only depend on the state of the economy.[6] While this ignores potential history–dependent equilibria that involve (for example) trigger strategies, the Markovian solutions have the virtue of being renegotiation–proof. Nonetheless we expect that analyses of history–dependent equilibria may prove useful and would be interesting topics for future research.

The lack of coordination in this model has symptoms that are similar to those from more familiar models. In monetary models (e.g. Samuelson (1958)) each young generation's willingness to accept money for goods is dependent on their belief that the subsequent generation will accept it from them. In the capital accumulation model each young generation's willingness to transfer wealth to the old is dependent on their belief that the same thing will happen in the subsequent time period. The ability to bind subsequent generations does *not* in itself induce the socially desirable outcome in the current period. Indeed the fact that subsequent governments are bound to their policies (or that subsequent beliefs are independent of whatever happens in the current period) makes it all the more tempting for the current young to exploit the situation. In the

[6]See, for example, Kotlikoff, Persson, and Svensson (1988).

monetary model they could consume all of their endowment, and then reintroduce money in the following period. In the capital accumulation model the young could refuse to transfer to the old, and then still obtain transfers the next period by virtue of the government's being bound. In other words, the incentive to deviate is present with or without precommitment. By themselves (i.e. without some kind of mechanism to resolve the intergenerational conflict) these models are not equipped to deal with the types of positive policy questions addressed in this paper.

The inability to coordinate is the crucial factor that leads to inefficiency in equilibrium. Even if each government ignored its effect on subsequent governments' decisions the outcome would be inefficient. Indeed in numerical solutions it appeared that the likelihood of inefficiency was actually greater under the naive behavior than under the more sophisticated. The naive behavior is analogous to the Cournot assumption in models of imperfect competition, where each producer takes the others' quantities as given in its own quantity decision. The sophisticated behavior corresponds to the Stackelberg assumption that one producer can act first and take the others' responses into account. As in the imperfect competition models, in which neither Cournot nor Stackelberg maximizes joint profits, here neither the naive nor sophisticated behavior necessarily guarantees efficiency. Only full cooperation accomplishes that. But surely between the sophisticated, Stackelberg–like behavior and the the naive Cournot–like behavior, the former is *a priori* the preferred assumption.

The more fundamental question is whether this model has anything to say about differences in growth rates across countries. Clearly if the model is simply applied to all countries, then all should have the same low growth rate, since differences in policy preference parameters appeared to have little effect on the equilibrium growth rate. What the analysis suggests, rather, is the possibility that the model might apply more to some countries than others, perhaps because of differences in political stability. A country that can set up a stable intergeneral redistribution institution that rewards human capital accumulation appropriately can clearly do better than one that cannot. Endogenizing the ability to create such an institution is beyond the scope of the present paper, but certainly a stable political system would be one ingredient.

In terms of empirical support, we know that less developed countries have a significantly higher return to human capital accumulation (e.g. the return to schooling) than developed countries (see Psacharopolous (1973)). Kahn (1996) provides additional evidence that measures of schooling and human capital investment are negatively related to political instability, even after controlling for income level.

If developed countries have overcome this obstacle, it must be either the result of institutions that allow those who accumulate human capital to recoup more of the benefits, or the result of a longer horizon. But it is doubtful that redistributional mechanisms such as Social Security serve the purpose of inducing human capital accumulation. Moreover, rapid growth in developed countries preceded the development of such institutions. So it is probably hard to sustain the case that the redistributive mechanism itself is crucial. Institutions that enhance property rights to knowledge may be more important. This research shows, though, that a finite horizon has potentially catastrophic effects on growth, and that the ability to set up institutions that overcome this by appropriately rewarding human capital accumulation may be important.

Appendix

Each of the components of 2.6 and 2.7 can be found by total differentiation of 2.2. After converting second derivatives of utility into relative risk aversion, and substituting 2.2 in various places, we get

$$(1+\gamma_{t+1})\frac{dk_{t+1}}{d\tau_t} = -\varphi_{t+1}/$$

$$\Big\{(1+f'(k_{t+1})+\varphi_{t+1})\left(\ell_{t+1}+(1+q_{t+1}\ell_{t+1})k_{t+1}\frac{d\ell_{t+1}}{d\bar{k}_{t+1}}\right)+ \tag{A1}$$

$$\tau_{t+1}q_{t+1}\frac{d\ell_{t+1}}{d\bar{k}_{t+1}}+\frac{d\tau_{t+1}}{d\bar{k}_{t+1}}+f''(k_{t+1})[\ell_{t+1}k_{t+1}-\sigma(\ell_{t+1}k_{t+1}+\tau_{t+1}/[1+f'(k_{t+1})])]\Big\}$$

and

$$(1+\gamma_{t+1})\frac{dk_{t+1}}{d\ell_t} =$$

$$\varphi_{t+1}\left((1+q_t\ell_t)(f(k_t)-k_tf'(k_t))-q_t[g(\ell_{t+1})\ell_{t+1}k_{t+1}(1+n)+\tau_t]\right)-$$

$$(1+\gamma_{t+1})q_t[\ell_{t+1}k_{t+1}(1+f'(k_{t+1}))+\tau_{t+1}]/ \tag{A2}$$

$$\Big\{(1+f'(k_{t+1})+\varphi_{t+1})\left(\ell_{t+1}+(1+q_{t+1}\ell_{t+1})k_{t+1}\frac{d\ell_{t+1}}{d\bar{k}_{t+1}}\right)+$$

$$\tau_{t+1}q_{t+1}\frac{d\ell_{t+1}}{d\bar{k}_{t+1}}+\frac{d\tau_{t+1}}{d\bar{k}_{t+1}}+f''(k_{t+1})[\ell_{t+1}k_{t+1}-\sigma(\ell_{t+1}k_{t+1}+\tau_{t+1}/[1+f'(k_{t+1})])]\Big\}$$

where $1+\gamma_{t+1}=g(\ell_{t+1})(1+n)$, $q_t=g'(\ell_t)/g(\ell_t)$, and $\varphi_{t+1}=[(1+f'(k_{t+1}))/(1+\alpha)]^{\sigma}$. Further manipulations of the numerator of (A2) (using the fact that φ_{t+1} also equals c_{2t+1}/c_{1t}) lead directly to the result that

$$\frac{dk_{t+1}}{d\ell_t} = -[f(k_t)-k_tf'(k_t)]\frac{dk_{t+1}}{d\tau_t} \tag{A3}$$

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<u>Figure 1</u>

Efficient Growth and Competitive Equilibrium

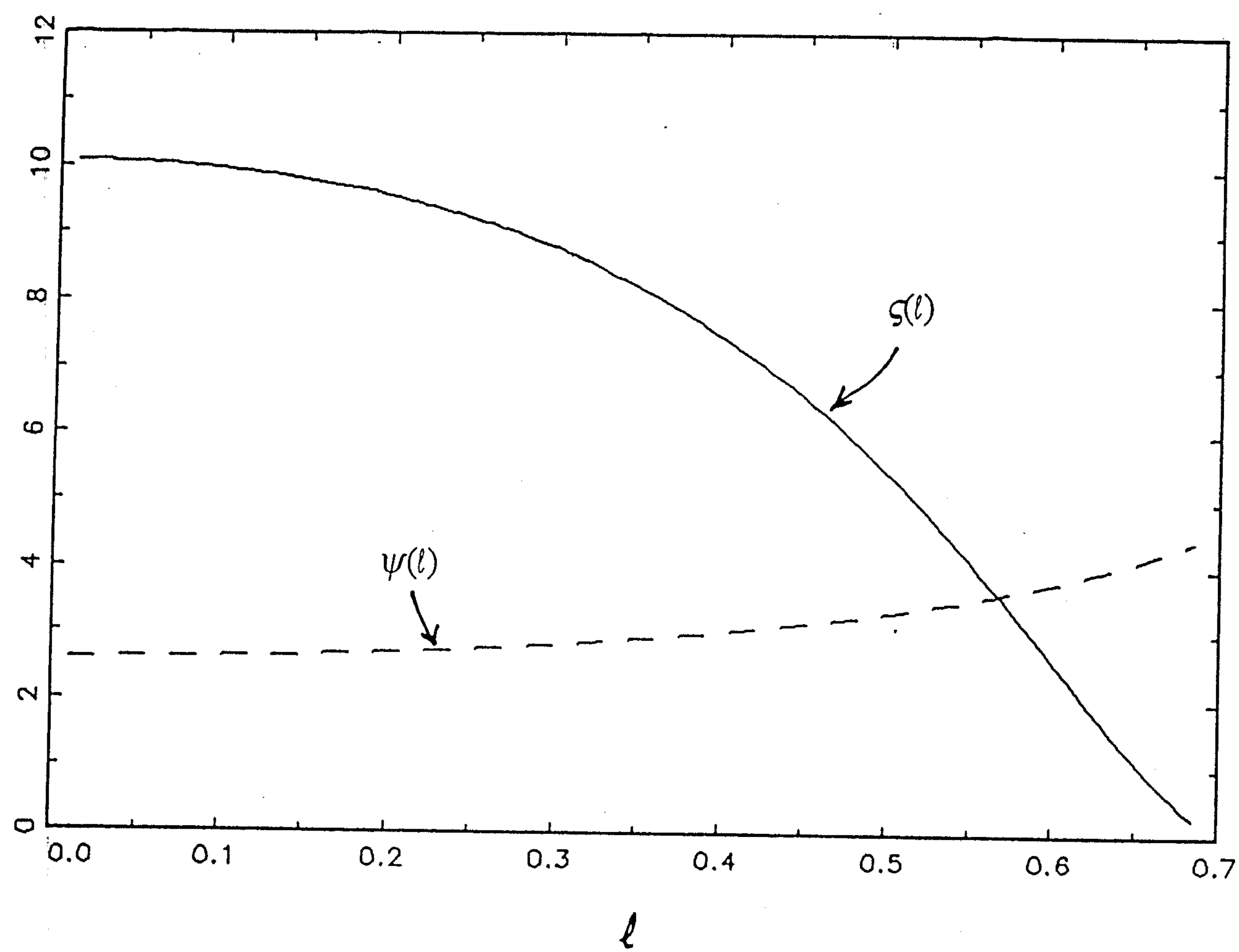

Figure 2: Equilibrium versus Efficient Growth Rates.

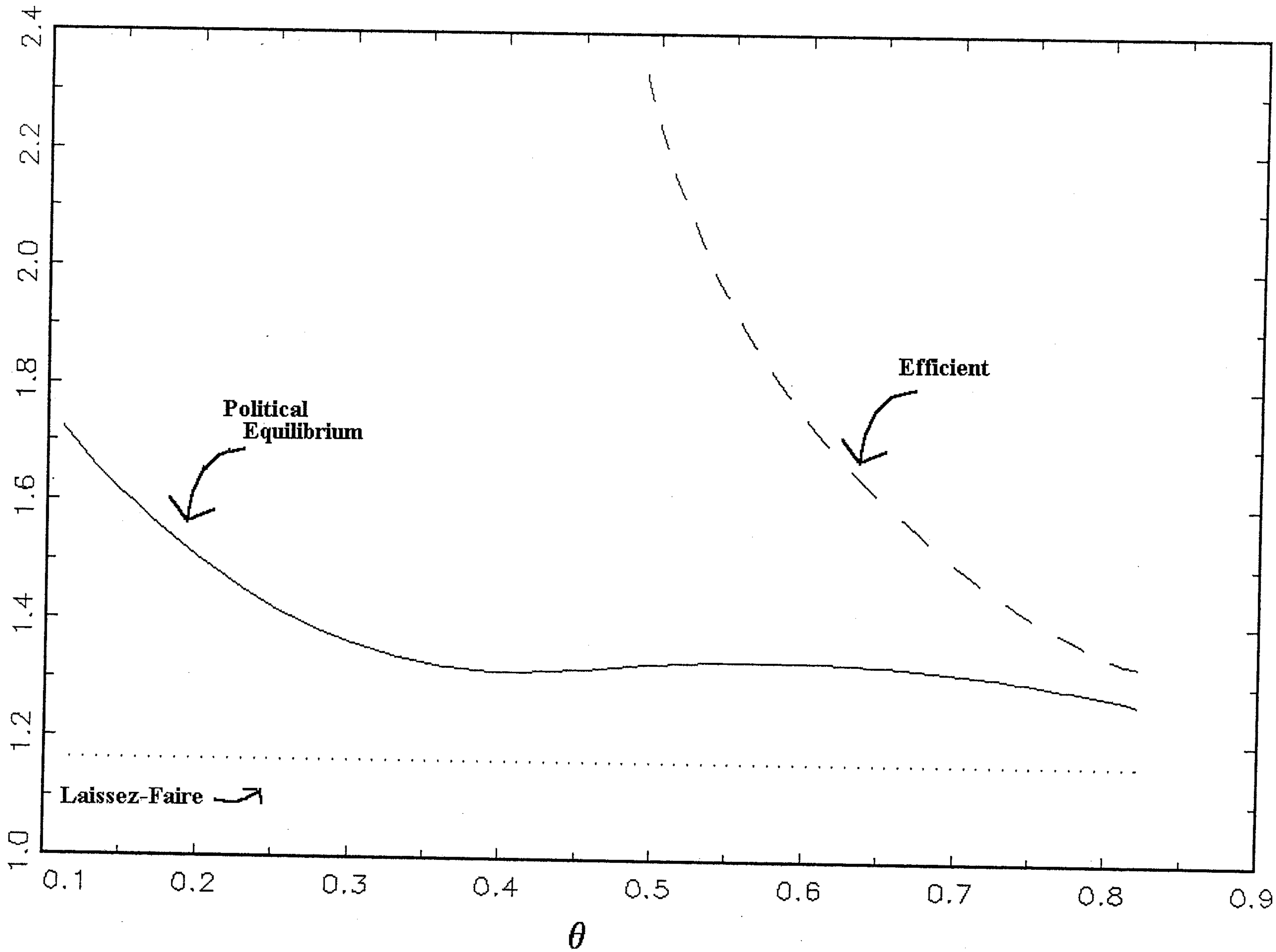

Figure 3: Equilibrium and Efficient Growth vs. Real Interest Rates

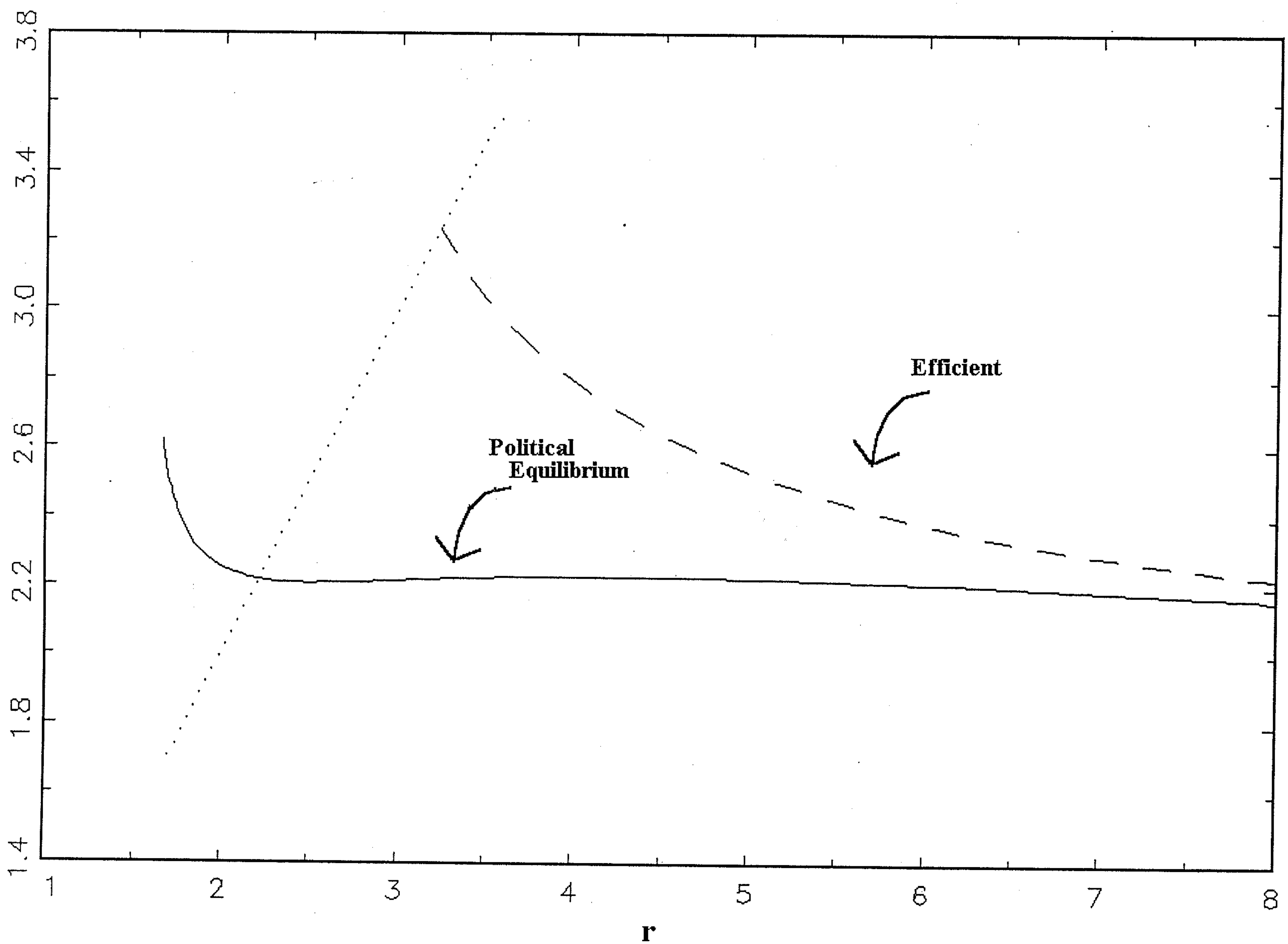

Figure 4: Equilibrium Policy Functions