Job Mobility and the Information Content of Equilibrium Wages: Part 1

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Abstract

In this part a simple complete contingent claims general equilibrium model is presented. The economy is constructed so as to yield agent-specific uncertainty—in particular, concerning the productive attributes of workers—which generates intertemporal job mobility in equilibrium. It is first shown that although the model is quite general in many respects, it has one very strong testable implication: all workers earning a given wage at some point in time have the same probability of job mobility; and this probability is not influenced by age, tenure or present job, etc. Since the model does not even impose restrictions sufficient to imply standard mobility "facts"—for example, the simple correlation between labor market experience and job mobility—there is a sense in which this prediction is the job matching approach's most fundamental.

PART I. A FINITE STATE SPACE ECONOMY

1. Introduction

In this part a simple complete contingent claims general equilibrium model is presented. The economy is constructed so as to yield agent-specific uncertainty--in particular, concerning the productive attributes of workers--which generates intertemporal job mobility in equilibrium. It is first shown that although the model is quite general in many respects, it has one very strong testable implication: all workers earning a given wage at some point in time have the same probability of job mobility; and this probability is not influenced by age, tenure or present job, etc. Since the model does not even impose restrictions sufficient to imply standard mobility "facts"--for example, the simple correlation between labor market experience and job mobility--there is a sense in which this prediction is the job matching approach's most fundamental.

Next, attention is directed towards assessing the testability of the prediction. As the argument proceeds, it will become apparent that though the hypothesis may be very simple, the data actually required to subject it to test may be very difficult to obtain; some kinds of measurement error may render the procedure vacuous. Two points are made. One is that the prediction holds under a surprisingly general measurement error setup. The second is that if the data generation process is assumed to be such as to prevent empirical confrontation of the basic proposition, other types of results--indeed some of which are familiar--might be obtained under suitable additional restrictions. However, to do so an economy with a much larger state space will likely prove a much more tractable setting; see Part II.

The reader will note that a nontrivial amount of notation is required to construct the model, and perhaps will suspect that a simpler environment might be preferable. In this regard there are two points to be made. First, as indicated, operating in a general setting identifies the key prediction of job matching theory. If this result can be shown to fail empirically a whole collection of models can be eliminated from further consideration. Second, a variety of fairly specific settings (although not strictly specializations of what is to follow) have already been explored; for example Jovanovic (1979, 1984), MacDonald (1982), Miller (1984) and Flinn (1986). A notable deficiency of this entire program is that it has not been shown that the ideas central to these models can in fact be embedded in a fully articulated economy.

The remainder of Part I proceeds as follows. The contingent claims economy is described and its equilibrium displayed. Subsequently, the main proposition is stated (formal demonstration appearing in the Appendix) and the testability issues addressed.

2. Time

Time is taken to be discrete. If desired, this assumption can be thought of as continuous time with minimum intervals within which instantaneous rates of production and consumption cannot be adjusted. In either case time periods are indexed by "dates" t; $t \in \mathcal{D} \equiv \{1,2,\ldots,T\}$, $T < \infty$.

3. Agents

The traditional dichotomy between "consumers" and "producers" is maintained. Here, owing to the focus on job mobility, consumers will be re-labelled "workers". Workers are indexed by i, $i \in \mathcal{W}$, and firms by $f \in \mathcal{F}$, where $\mathcal{W}$ and $\mathcal{F}$ are finite sets.

4. Commodities

Commodities are differentiated according to physical characteristics, including date of production and place of delivery. The list of different possible physical characteristics is indexed by j; $j \in \mathcal{J}$, where $\mathcal{J}$ is a finite set.

Less conventional, though certainly consistent with the standard formulation, is that the working time with which each worker i will be endowed is treated as a separate commodity. Typically, this explicit treatment of individual workers is not required because work time is assumed to generate a service flow of known constant rate for all i, and these service flows are perfect substitutes in production; i.e. total labor services purchased are known, and are the relevant input. In what follows, the rate at which i's time converts to productive services will not be understood with certainty, but will exhibit intertemporal covariation. In this case, the price at which a unit of i's time trades at any t may depend on the events which have been observed to date, and "wage rates" might be worker-specific in the sense of depending on i's observed history, or indeed other elements of the economy's history.

As indicated, a variety of random influences (such as the rates at which i's time converts to service flow) impact on the economy at each date, and claims to acquire or supply commodities may be made contingent on those occurrences observed at or before t. For the moment, the description of the random events may be quite general. Let H^t be the (finite) set of events which could be observed at date t, and h^t a particular element: $h^t \in H^t$. The vector $\lambda^t \equiv (h^1, \ldots, h^t)$ comprises a particular history of observations made up to and including date t, and $\Lambda^t = \overset{t}{\underset{\tau=1}{X}} H^\tau$ is the set of possible histories; note that Λ^t is a finite set $\forall\ t \in \mathcal{D}$. Contingent commodities for date t are thus made contingent on histories $\lambda^t \in \Lambda^t$.

Bringing physical commodities together with the notion of histories in the usual way, the collection of entities traded in the model is a list of claims to receive or deliver goods at date t if and only if some particular history λ^t occurs. The price system associated with these "contingent claims" is $(p_j(\lambda^t),\ w_i(\lambda^t))$ where $j \in \mathcal{J}$, $i \in \mathcal{W}$, $\lambda^t \in \Lambda^t$ and $t \in \mathcal{D}$. $p_j(\lambda^t)$ is the unit price, as of t=1, of a claim to one unit of good j delivered at t if and only if λ^t occurs. Similarly, $w_i(\lambda^t)$ is the price of a claim to one unit of i's time at t if and only if λ^t occurs.

5. Endowments

At every date and for all histories, all workers $i \in \mathcal{W}$ are endowed with some of each commodity apart from working time.[1] For simplicity, i's endowment of commodity j, $\omega_{ij} \geq 0$, will be assumed constant over both time and histories. Similarly, i has a claim to a fraction v_{if} of producer f's net revenue: $v_{if} > 0\ \forall\ i \in \mathcal{W}$, $f \in \mathcal{F}$, and

$$\sum_{i \in \mathcal{W}} v_{if} = 1, \quad f \in \mathcal{F}.$$

Denoting producer j's net revenue from trades in claims by R_f, i's share is thus $v_{if}\ R_f$.

Not all workers have the same working life. Formally, partition $\mathcal{F}$ into

$\mathcal{W}_\tau$, $\tau \in \mathcal{D}$; that is, for $\tau \neq \tau'$ $\mathcal{W}_\tau \cap \mathcal{W}_{\tau'} = \phi$, $\cup_\tau \mathcal{W}_\tau = \mathcal{W}$, and $\mathcal{W}_\tau \neq \phi$. Worker i, $i \in \mathcal{W}_\tau$ has an endowment of L_i units of working time at t if and only if $t \in \mathcal{D}_i \equiv \{\tau, \tau+1, \ldots, \min(T, \tau+A-1)\}$. That is, worker $i \in \mathcal{W}_\tau$ may work beginning at date τ, and for a total of $A \geq 1$ periods provided the economy has not come to a halt. $\mathcal{D}_i$ is i's potential working life and comprises periods during which an exogenous and multidimensional flow of productive services (which may have many alternative uses, including household production) emerges from the worker for a total of L_i units of time. For $t \notin \mathcal{D}_i$, i's time generates a service flow which is useful only in the home. Essentially, earliest possible labor force entry and latest possible exit are treated as exogenous. If preferred, given the preferences and technology specified below, the model can be recast in "overlapping generations" form with all agents in a given generation able to work at any point in their lifetime.[2]

6. Worker's Preferences

The economy has a large number of contingent commodities, over which preference must be defined. Let $c_{ij}(\lambda^t)$ denote i's consumption of good j at t if λ^t occurs, and $\ell_i(\lambda^t)$ the quantity of i's working time used in home production (or the excess over that which is exogenously allocated to home production each period) under the same contingency. The set of consumption/time pairs which are feasible for i taken to be:

$$\mathcal{C}_i \equiv \{c_{ij}(\lambda^t) \geq 0, \quad \ell_i(\lambda^t) \geq 0; \quad j \in \mathcal{J}, \quad \lambda^t \in \Lambda^t, \ t \in \mathcal{D}\}.$$

What are i's preferences over elements of $\mathcal{C}_i$? For simplicity it will be assumed that all individuals i) have identical preferences; ii) maximize expected utility where expectations are taken with respect to objective probabilities, and preferences are state independent; and iii) are risk neutral. These restrictions permit the exact form of the equilibrium presented below to be computed easily but they are not essential to the basic result that workers having a given wage are equally likely to change jobs. Consequently, they are adopted in the interest of simplifying the presentation.

Consider a sequence of histories beginning with some h^1, and computing λ^t according to $\lambda^t = (h^t, \lambda^{t-1})$ for some $h^t \in H^t$; $t \in \mathcal{D}$. With each such sequence of histories, there is an associated $c_{ij}(\lambda^t)$ and $\ell_i(\lambda^t)$ sequence. Under the assumptions listed above, this sequence is evaluated according to the subutility

$$\sum_{t\in\mathcal{D}} \{ \sum_{j\in\mathcal{J}} \beta_j(t)\, c_{ij}(\lambda^t) + \gamma(t)\, \ell_i(\lambda^t)\},$$

where $\beta_j(t) \geq 0$ and $\gamma(t) \geq 0$ are the marginal utilities of the jth consumption good and time respectively at date t. A good deal of clutter is eliminated by assuming $\beta_j(t) = \rho^{t-1}\beta_j$ and $\gamma(t) = \rho^{t-1}\gamma$ for some constants ρ, β_j and γ.

Let $\pi(\lambda^T)$ be the probability with which λ^T is to occur;

$$\sum_{\lambda^T \in \Lambda^T} \pi(\lambda^T) = 1,$$

and $\pi(\lambda^T) > 0 \quad \forall \quad \lambda^T \in \Lambda^T$. Then expected utility is given by

$$\mathcal{U} = \sum_{\lambda^T \in \Lambda^T} \pi(\lambda^T) \sum_{t\in} \rho^{t-1} \sum_{j\in} \beta_j c_{ij}(\lambda^t) + \gamma\, \ell_i(\lambda^t)\ .$$

Nothing that $\pi(\lambda^t) = \sum_{h^{t+1}\in H^{t+1}} \cdots \sum_{h^T\in H^T} \pi(h^T,\ldots,h^{t+1},\lambda^t)$,

minor manipulation gives[3]

$$\mathcal{U} = \sum_{t\in\mathcal{D}} \sum_{\lambda^t\in\Lambda^t} w(\lambda^t)\,\rho^{t-1}\Big\{\sum_{j\in\mathcal{J}} \beta_j c_{ij}(\lambda^t) + \gamma\,\ell_i(\lambda^t)\Big\}. \qquad (1)$$

7. **Worker i's Budget**

Given the price system and endowments, the set of $c_{ij}(\lambda^t)$ and $\ell_i(\lambda^t)$ which are affordable for worker i is given by those which satisfy

$$\sum_{t\in\mathcal{D}} \sum_{\lambda^t\in\Lambda^t} \sum_{j\in\mathcal{J}} p_j(\lambda^t)[c_{ij}(\lambda^t) - \omega_{ij}]$$

$$+ \sum_{t\in\mathcal{D}_i} \sum_{\lambda^t\in\Lambda^t} w_i(\lambda^t)[\ell_i(\lambda^t) - L_i] - \sum_{f\in\mathcal{F}} v_{if}R_f \le 0. \qquad (2)$$

8. **Worker i's choice of $c_{ij}(\lambda^\tau)$ and $\ell_i(\lambda^t)$**

Worker i's problem is to choose, for all $t \in \mathcal{D}$ and $\lambda^t \in \Lambda^t$, $c_{ij}(\lambda^t)$ and $\ell_i(\lambda^t)$. Let ξ_i, $\phi_{ij}(\lambda^t)$, $\bar{\phi}_i(\lambda^t)$, and $\phi_{-i}(\lambda^t)$ be nonnegative multipliers associated with (2), $c_{ij}(\lambda^t) \ge 0$ and $L_i \ge \ell_i(\lambda^t) \ge 0$ respectively. Since $\mathcal{U}$ is a concave function, and feasibility and affordability define a nonempty convex set (assuming R_f is not "too small", as will be shown to hold in equilibrium) necessary and sufficient conditions characterizing the solution to worker i's problem are

$$\pi(\lambda^t)\rho^{t-1}\beta_j - \xi_i p_j(\lambda^t) + \phi_{ij}(\lambda^t) = 0 \quad \forall\ t \in \mathcal{D},\ j \in \mathcal{J} \text{ and } \lambda^t \in \Lambda^t; \quad (3)$$

$$\pi(\lambda^t)\rho^{t-1}\gamma - \xi_i w_i(\lambda^t) + \phi_{-i}(\lambda^t) - \bar{\phi}_i(\lambda^t) = 0 \ \forall\ t \in \mathcal{D}_i \text{ and } \lambda^t \in \Lambda^t; \quad (4)$$

$$\xi_i \{ \sum_{t\in \mathcal{D}_i} \sum_{\lambda^t \in \Lambda^t} \sum_{j \in \mathcal{J}} p_j(\lambda^t)\,[\omega_{ij} - c_{ij}(\lambda^t)]$$

$$+ \sum_{t\in \mathcal{D}_i} \sum_{\lambda^t \in \Lambda^t} w_i(\lambda^t)[L_i - \ell_i(\lambda^t)] + \sum_{f\in} v_{if} R_f \} = 0; \quad (5)$$

$$\phi_{ij}(\lambda^t)c_{ij}(\lambda^t) = 0 \quad \forall\ t \in \mathcal{D},\ j \in \mathcal{J} \text{ and } \lambda^t \in \Lambda^t; \quad (6)$$

and

$$\left.\begin{array}{l} \phi_{-i}(\lambda^t)\ell_i(\lambda^t) \\ \\ \bar{\phi}_i(\lambda^t)[L_i - \ell_i(\lambda^t)] \end{array}\right\} = 0 \quad \forall\ t \in {}_i \text{ and } \lambda^t \in \Lambda^t. \quad (7)$$

9. <u>Evolution of History</u>

Before proceeding to technology and producer optimization, more details should be given regarding the stochastic structure through which λ^t is generated. Producer f's problem, like worker i's, can be developed without doing so. But following that route generates a certain amount of backtracking later. Thus, from an expositional standpoint, a more explicit discussion of λ^t is called for at this point.

At each date t, an observation associated with worker i becomes available. This observation is intended to be relevant (in a manner described below) for determining i's productive traits. So for $t \notin \mathcal{D}_i$, the observation is taken to be a trivial one, labelled $\hat{q}$. For $t \in \mathcal{D}_i$, observations are generated as follows. Let Q_s be a random variable. There are a variety of such random variables, and s indexes them; $s \in \mathcal{S}$, where $\mathcal{S}$ is a finite set.[4]

Q_s may take on values q_m, $m \in \mathcal{M}$, where $\mathcal{M}$ is a finite set.

$-\infty < \inf_{m \in \mathcal{M}} \{q_m\} < \sup_{m \in \mathcal{M}} \{q_m\} < \infty$. For $s \in \mathcal{S}$, let

$$\delta_{ms} \equiv \Pr[Q_s = q_m].$$

The q_m should be thought of as comprising a complete list of all possible values of a "service flow" (more on this interpretation later) so that it is appropriate to imagine each random variable Q_s to take on the same set of q_m, $m \in \mathcal{M}$, as opposed to a distinct set of values, say q_{ms} for each s.

Data relevant to worker i are observations of either $\hat{q}$ or Q_s(i.e. depending on whether $t \in \mathcal{D}_i$). When Q_s is observed, observations are independent across time and workers. The main assumption is as follows: direct information on which Q_s is being observed at any point is not available to any agent. That is, for $t \in \mathcal{D}_i$, some value q_m is observed, and the δ_{ms} satisfy

$$\forall\ m \in \mathcal{M} \text{ and } s \in \mathcal{S}, \quad \delta_{ms} > 0.$$

Under this assumption, no observed q_m may rule out any $s \in \mathcal{S}$, and some residual uncertainty as to which s is generating the data always remains. At the cost of introducing a good deal of notation, this assumption can be weakened significantly.

How is the Q_s relevant to any i determined? Nature assigns each $i \in \mathcal{W}$ an element of $\mathcal{S}$ at the outset, and does so in the following fashion. Let S be a random variable taking on values $s \in \mathcal{S}$, with

$$\Pr[S = s] = \zeta_s, \qquad \sum_{s \in \mathcal{S}} \zeta_s = 1,$$

and $\zeta_s > 0 \ \forall\, s \in \mathcal{S}$.

For each i , nature produces a single independent realization of S, S = s, then assigns Q_s to worker i. The agents do not observe this process, but know of its existence and parameters.

Given the setup, for each $i \in \mathcal{W}$ and $t \in \mathcal{D}$, an observation Q_i is available, where

$$Q_i = \begin{cases} \hat{q} & t \notin \mathcal{D}_i \\ q_m & \text{some } m \in \mathcal{M},\ t \in \mathcal{D}_i. \end{cases}$$

When it is necessary to indicate the date at which Q_i is observed, Q_i^t will be used. It then follows that $h^t = (Q_i^t;\ i \in \mathcal{W})$, and λ^t is constructed as before. $\pi(\lambda^T)$ is generated in the obvious fashion. First the probability with which λ^T occurs given any assignment of s to i, $i \in \mathcal{W}$, is calculated. Next, the conditioning on the assignment is removed. Specific expressions will be given as needed.

Three points should be noted. First, this information-generation cannot be avoided. If, for example, i chosen not to work at some $t \in \mathcal{D}_i$, home production reveals Q_i. This restriction is needed because its absence would permit the economy's state space (essentially Λ^T) to depend on choices made in the economy. While economic activity might be organized so that which state occurs does not affect the outcome, that the state occurs and is observed anyway is central to the operation of the contingent claims structure. From a practical standpoint, for the issues at hand, this restriction is not of great importance because economies where all agents spend some time at work when possible will be the focus of the analysis. And it is reasonable to suppose that all work activities generate some information regarding productive traits.

Second, Q_i is observed by all agents; no private information. As usual, this is an assumption about individual ability to conceal information, as well as incentives to do so. In any economy with a great deal of heterogeneity, as the one under consideration will turn out to be, incentives to conceal are much reduced (contrast MacDonald (1980) with Riley (1976), for example). Also, Q_i will (below) be inferrable, in a straightforward way, from entities which can plausibly be imagined to be cheaply observed by all interested parties. Thus, for the problem at hand, absence of a major influence due to private information would seem to be the leading case.

Third, the observation on Q_i at t will be inferred from work activities undertaken at t-1. Thus, by assuming some $Q_i = q_m$, rather than $\hat{q}$, is observed during the first period of working life, strictly it is being supposed that pre-working life activities generate information too. Doing so is purely a convenience which allows i to choose a first job with some person-specific information. This assumption can be dropped with minor inconvenience and no substantive effect.

10. Technology

Each producer $f \in \mathcal{F}$, has access to a one output technology. Recalling that goods $j \in \mathcal{J}$ are consumed, and assuming there are no other outputs (intermediate goods, capital, etc.) $\mathcal{F}$ can be partitioned into subsets $\mathcal{F}_j$, $j \in \mathcal{J}$ (with $\mathcal{F}_j \cap \mathcal{F}_{j'} = \phi$ and $\bigcup_j \mathcal{F}_j = \mathcal{F}$) and $f \in \mathcal{F}_j$ interpreted as meaning that f may produce good j. A producer $f \in \mathcal{F}_j$ will be called a "type j" producer.

If $f \in \mathcal{F}_j$ purchases $x_i(\lambda^t)$ units of i's working time for use at date t, when history is λ^t, output emerges at date t+1. f's output, called y_f, depends on the value of Q_i relevant to i for period t, Q_i^{t+1}. (Recall that Q_i is associated with activities at date t if observed at t+1). Thus f's output at t+1 depends on h^{t+1}. It is temporarily convenient to express this dependence in terms of λ^{t+1}.

Producer f's technology is as follows. If λ^{t+1} occurs, output is given by [5]

$$y_f(\lambda^{t+1}) = \sum_{i \in \mathcal{W}} (a_j + b_j Q_i^{t+1}) x_i(\lambda^t) \tag{8}$$

where $f \in \mathcal{F}_j$, and $a_j > 0$ and $b_j \neq 0$ are finite constants. Under (8), i has a well- defined personal output if employed by $f \in \mathcal{F}_j$: $y_{if}(\lambda^{t+1}) \equiv (a_j + b_j Q_i^{t+1}) x_i(\lambda^t)$, from which Q_i^{t+1} may be inferred from $y_{if}(\lambda^{t+1})$ as $Q_i^{t+1} = [y_{if}(\lambda^{t+1})/x_i(\lambda^t) - a_j]/b_j$. The interpretation of (8) is straightforward. a_j is the instantaneous non "match" (in the sense of i-j, not specifically i-f) specific rate of output. The match specific flow of output is given by $b_j Q_i^{t+1}$. That is, $x_i(\lambda^t)$ converts to match specific productive effort at rate Q_i^{t+1}, so all q_m are in efficiency units of skill per unit of time.

Note that b_j need not be positive, although it is supposed that

$\forall j \in \mathcal{J}$, $\inf_{m \in \mathcal{M}} \{a_j + b_j q_m\} > 0$. The idea here is that skill is multi-

dimensional and that any Q_i is merely a summary measure, say the brains/

brawn ratio, in which case production of some goods may find a relatively more brawn-intensive service flow useful. Interpretation of Q_i as a summary measure is only required because Q_i is one dimensional. The case where Q_i is a vector is fairly straightforward and makes interpretation of Q_i easier, but adds a good of notation with no extra results.

Note that a single value of Q_i applies to all instants worked within the period, irrespective of match type. This is not an additional restriction, since a period is defined as the shortest time over which flows in the model may not be changed. From that viewpoint, the restriction is that Q_i is permitted to vary on a period by period basis; i.e. that it may change as frequently as any other entity in the model. It appears that this latter assumption can be relaxed in two ways. It is easy to permit Q_i to remain fixed for some integer number of periods; and the value of the new results seems accurately to reflect the marginal resource cost. More interesting, and difficult, is to allow the random variables Q_s to be described by a general stochastic process on $\{q_m, m \in \ \}$. Such can be done by specifying Q_s's $(s \in \ )$ stochastic structure by a one-step transition matrix with elements that depend on all previous Q_i. This setup provides, even relative to the current setup, a plethera of mobility behaviours, but the central result stated below does not seem sensitive to this extension.

Note also that even though Q_s is chosen independently for each s, Q_i will exhibit intertemporal covariation. The information $Q_i^t = q_m$ is useful for determining which Q_s is generating the data on i, and thus affects the probability with which $Q_i^{t+1} = q_{m'}$ when direct conditioning on s is not permitted. That is, $Pr(Q_i^{t+1} = q_{m'}|s) = \delta_{m's} =$ $Pr(Q_i^{t+1} = q_{m'}|Q_i^t = q_m,s)$, but in general $Pr(Q_i^{t+1} = q_{m'}) \neq Pr(Q_i^{t+1} = q_{m'}|Q_i^t = q_m)$.

11. <u>Producer f's choice of $x_i(\lambda^t)$</u>

Producer $f \in \mathcal{F}_j$ seeks to maximize the net revenue earned from the sale of promises to deliver output and purchase working time conditional on the various possible contingencies.

Recalling that $h^t = (Q_i^t;\ i \in \mathcal{W})$ and $\lambda^t = (h^t, \lambda^{t-1})$, $t > 1$, use of (8) generates f's net revenue as

$$R = \sum_{t \in \mathcal{D}-\{T\}} \sum_{\lambda^t \in \Lambda^t} \sum_{i \in \mathcal{W}} \Big[\sum_{h^{t+1} \in H^{t+1}} p_j(\lambda^{t+1})(a_j + b_j Q_i^{t+1}) - w_i(\lambda^t) \Big] x_{if}(\lambda^t). \tag{9}$$

Note that no output can be produced at t=1, and that (9) takes into account that there is no point in purchasing x_i at t=T.

Necessary and sufficient conditions characterizing a (finite) maximum of R_f ($f \in \mathcal{F}_j$, $j \in \mathcal{J}$) subject to $x_{if}(\lambda^t) \geq 0$ are: $\forall\ i \in \mathcal{W}$, $f \in \mathcal{F}_j$, $t \in \mathcal{D}-\{T\}$, $\lambda^t \in \Lambda^t$,

$$\sum_{h^{t+1} \in H^{t+1}} p_j(\lambda^{t+1})(a_j + b_j Q_i^{t+1}) - w_i(\lambda^t) + \psi_{if}(\lambda^t) = 0 \tag{10}$$

where $\psi_{if}(\lambda^t) x_{if}(\lambda^t) = 0$ and $\psi_{if}(\lambda^t) \geq 0$. If

$$\sum_{h^{t+1} \in H^{t+1}} p_j(\lambda^{t+1})(a_j + b_j Q_i^{t+1}) - w_i(\lambda^t) > 0,$$

the problem has no finite maximum.

When (10) holds with $\psi_{if}(\lambda^t) = 0$, the interpretation is as follows. Hiring an additional instant of i's time at t when λ^t has occurred costs $w_i(\lambda^t)$. That unit will generate output at rate $a_j + b_j Q_i^{t+1}$, for some value of Q_i^{t+1}, which would permit any of a variety of (slightly larger) claims to be filled. The value of such marginal additions to the set of claims is what is compared to the wage when deciding how much of i's time to purchase.

12. Equilibrium

The model is a special case of the economy studied by Debreu (1959). Accordingly, it has at least one equilibrium. The point of introducing the additional structure imposed here is that equilibrium takes on a particular form; one in which job mobility arises as a result of worker-specific information.

What form does the equilibrium take? For all possible histories, each worker i has on endowment $\omega_{ij} > 0$ of good j. Further it was assumed that no worker-firm pairing actually causes negative output (i.e. $\inf_{m\in\mathcal{M}} \{a_j+b_j q_m\} > 0$ $\forall\ j \in \mathcal{J}$). It follows that under all contingencies, a positive but finite quantity of each good j is available, and hence that workers must be willing to demand this quantity in equilibrium. From (3) and (6), it then follows that

$$\forall\ t \in \mathcal{D}\ ,\ j \in \mathcal{J} \text{ and } \lambda^t \in \Lambda^t,$$
$$\pi(\lambda^t)\ \rho^{t-1}\beta_j - \xi_i p_j(\lambda^t) = 0 \tag{11}$$

Since all workers have the same nonsatiated preferences $\mathcal{U}$, and $\mathcal{U}$ is linear in $c_{ij}(\lambda^t)$ and $\ell_i(\lambda^t)$, the normalization $\xi_i = 1$ may be imposed. Then (11) gives commodity prices

$$p_i(\lambda^t) = \pi(\lambda^t)\ \rho^{t-1}\beta_j. \tag{12}$$

Substitute (12) into the equation characterizing producer f's decision rule, (10):

$$\sum_{h^{t+1}\in H^{t+1}} \pi(\lambda^{t+1})\,\rho^{t}\beta_j(a_j+b_jQ_i^{t+1}) - w_i(\lambda^t) + \psi_{if}(\lambda^t) = 0.$$

Suppose $\psi_{if}(\lambda^t) > 0$. Then $w_i(\lambda^t)$ is such that f prefers not to purchase any of i's time at t if λ^t occurs. If, looking back at (4), for this same history λ^t (and assuming $t \in \mathcal{D}_i$), $\pi(\lambda^t)\rho^{t-1}\gamma - w_i(\lambda^t) - \bar{\phi}_i(\lambda^t) = 0$, with $\bar{\phi}_i(\lambda^t) > 0$, i would wish to set $\ell_i(\lambda^t) = L_i$; i.e. not sell any working time. Indeed, if $\forall\ j \in \mathcal{J}$

$$\pi(\lambda^t)\rho^{t-1}\gamma > \sum_{h^{t+1}\in H^{t+1}} \pi(\lambda^{t+1})\rho^{t}\beta_j(a_j+b_jQ_i^{t+1}),$$

the equilibrium involves no trade of i's time given λ^t; that is, the least i would accept in return for giving up some working time exceeds the most any firm would be willing to pay.

In what follows, attention is confined to economies for which the equilibrium involves all workers selling working time whenever they have any to sell. A necessary and sufficient parameter restriction implying this outcome is

$$\forall\ t \in \mathcal{D}_i \text{ and } \lambda^t \in \Lambda^t,\ \pi(\lambda^t)\gamma \le \sup_{j\in\mathcal{J}}\Big\{\sum_{h^{t+1}\in H^{t+1}} \pi(\lambda^{t+1})\rho\beta_j(a_j+b_jQ_i^{t+1})\Big\},$$

in which case the equilibrium wage involves $\psi_{if}(\lambda^t) = 0$ in (10) for some j' and all $f \in \mathcal{F}_j$ with $\psi_{if} > 0$ otherwise:

$$w_i(\lambda^t) = \sup_{j\in\mathcal{J}} \{ \sum_{h^{t+1}\in H^{t+1}} \pi(\lambda^{t+1})\rho^t \beta_j (a_j + b_j Q_i^{t+1}) \}. \qquad (13)$$

Thus the equilibrium price system is given by (12) and (13). There are many equilibrium commodity vectors associated with these prices. However, all share the feature that $\ell_i(\lambda^t) = 0$, and the worker sells all available working time to the firm type j' for which $\psi_{if} = 0$, $f \in \mathcal{F}_j'$. Given this labor supply behavior, and noting that (13) implies $R_f = 0$ for all f, any feasible $c_{ij}(\lambda^t)$ which satisfies the budget constraint (2) with equality at the stated prices, and for which aggregate consumption of good j equals aggregate production for each λ^t, is an equilibrium allocation. (Absence of storage possibilities is implicit in (8)).

13. Job Mobility

As time passes and successive histories unfold, workers may change jobs in the sense of selling their working time to a type j producer in one period and a type j' the next. The model does not determine the assignment of workers to firms within a collection of producers of the same type. Thus no interproducer-intratype pattern of mobility is inconsistent with the model. The model's predictive contract is in terms of inter-type mobility. More on this point below.

This kind of reallocation occurs at the beginning of period t+1 t if and only if

$$\operatorname*{argmax}_j \{ \sum_{h^{t+1}\in H^{t+1}} \pi(\lambda^{t+1})\rho^t \beta_j (a_j + b_j Q_i^{t+1}) \}$$

$$\neq \operatorname*{argmax}_j \{ \sum_{h^{t+2}\in H^{t+2}} \pi(\lambda^{t+2})\rho^{t+1} \beta_j (a_j + b_j Q_i^{t+2}) \}, \qquad (14)$$

in which case, at the equilibrium prices, a type j producer would be willing to hire i given λ^t but not given λ^{t+1}, and conversely for a type j' firm; $j \neq j'$.

It is useful to provide a characterization, which is simpler than (14), of the circumstances under which mobility occurs. Consider the most a type j firm would be willing to pay to hire i given λ^t:

$$\sum_{h^{t+1} \in H^{t+1}} \pi(\lambda^{t+1}) \rho^t \beta_j (a_j + b_j Q_i^{t+1})$$

$$= \sum_{\lambda^{t+1} \in H^{t+1}} \frac{\pi(\lambda^{t+1})}{\pi(\lambda^t)} \rho^t \pi(\lambda^t) \beta_j (a_j + b_j Q_i^{t+1})$$

$$= \rho p_j(\lambda^t) \sum_{h^{t+1} \in H^{t+1}} \pi(h^{t+1} | \lambda^t)(a_j + b_j Q_i^{t+1}) \quad \text{from (11)}$$

$$= \rho p_j(\lambda^t)[a_j + b_j E(Q_i^{t+1} | \lambda^t)]. \tag{15}$$

Moreover, since the random variables Q_s, $s \in \mathcal{J}$, are independent across workers, $E(Q_i^{t+1}|\lambda^t) = E(Q_i^{t+1}|\lambda_i^t)$, where $\lambda_i^t \equiv (Q_i^1, \ldots, Q_i^t)$. (15) becomes[6]

$$\rho p_j(\lambda^t)[a_j + b_j E(Q_i^{t+1}|\lambda_i^t)] \tag{16}$$

Now, compare the values of (16) across j:

$$\sup_{j \in \mathcal{J}} \{\rho p_j(\lambda^t)[a_j + b_j E(Q_i^{t+1} | \lambda_i^t)]\}$$

$$= \rho^t \pi(\lambda^t) \sup_{j \in \mathcal{J}} \{a_j \beta_j + b_j \beta_j E(Q_i^{t+1} | \lambda_i^t)\} \tag{17}$$

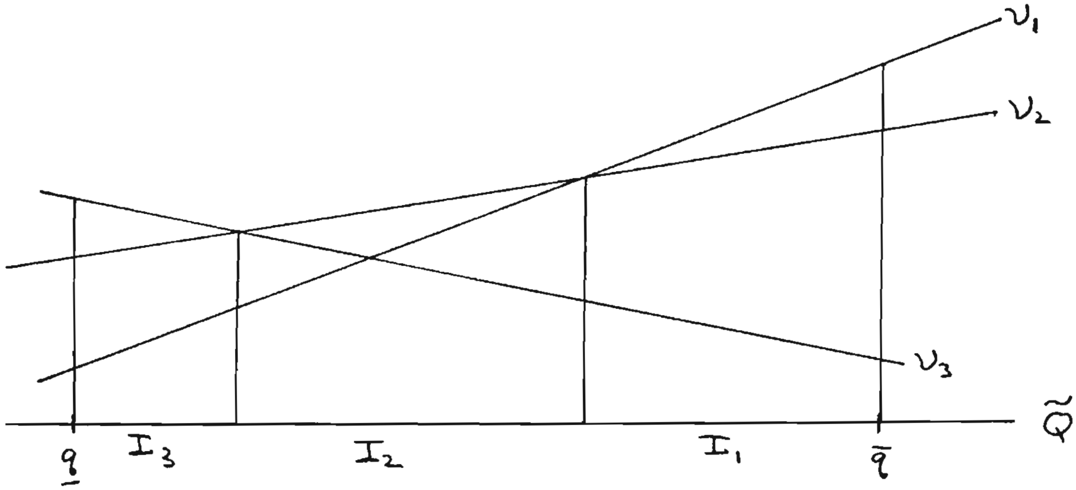
Since a_j, b_j and β_j do not vary over time, it is immediate that equilibrium job mobility behavior depends solely on $E(Q_i^{t+1} | \lambda_i^t)$.

Now, for any history λ_i^t,

$$\underline{q} \equiv \inf_{s\in\mathcal{S}} \left\{ \sum_{m\in\mathcal{M}} q_m \delta_{ms} \right\} \le E(Q_i^{t+1} | \lambda_i^t) \le \sup_{s\in\mathcal{S}} \left\{ \sum_{m\in\mathcal{M}} q_m \delta_{ms} \right\} \equiv \bar{q}.$$

Now consider the $\#(\mathcal{J})$ functions $\nu_j(\tilde{Q}) = a_j\beta_j + a_j\beta_j\tilde{Q}$, where $\tilde{Q}\in[\underline{q}, \bar{q}]$.[7] Without loss of generality it may be assumed that for every $j \in \mathcal{J}$, there is some subset of $[\underline{q}, \bar{q}]$ such that $j = \underset{j'\in\mathcal{J}}{\text{argmax}}\ \nu_{j'}(\tilde{Q})$.[8] Call this set I_j. Clearly $\bigcup_{j\in\mathcal{J}} I_j = [\underline{q}, \bar{q}]$. Moreover, owing to the linearity of ν_j, I_j is a closed interval and $\text{Int}\ I_j \cap \text{Int}\ I_{j'} = \phi \quad \forall\ j, j' \in \mathcal{J}$ See Figure 1.

Figure 1



It thus follows that at date t, worker i is hired by some producer $f \in \mathcal{F}_j$ if and only if $E(Q_i^{t+1}|\lambda_i) \in I_j$. The intervals I_j depend on neither time nor history, which follows from risk neutrality, the linearity of the production processes, and the absence of type-specific shocks (say to a_j). Thus mobility can be given a very simple characterization: i moves from a type j producer to a type j' at the beginning period t+1 if and only if

$$E(Q_i^{t+1} | \lambda_i^t) \in I_j \text{ and } E(Q_i^{t+2} | \lambda_i^{t+1}) \in I_{j'}.$$

An immediate and obvious implication is that a sufficient condition for i and i' to have the same probability of job mobility, given observation of λ_i^t and $\lambda_{i'}^t$, is that $\lambda_i^t = \lambda_{i'}^t$, up to order. In that case,

$$\Pr(Q_i^{t+1} = q_m | \lambda_i^t) = \Pr(Q_{i'}^{t+1} = q_m | \lambda_{i'}^t) \quad (18)$$

for all $m \in \mathcal{M}$. This implies $E(Q_i^{t+1} | \lambda_i^t) = E(Q_{i'}^t | \lambda_{i'}^t)$, so that i and i' are both employed by the same firm type j. Also, define $H(i, j, j', \lambda_i^t) \equiv \{h^{t+1} |\ E(Q_i^{t+2} | \lambda_i^{t+1}) \in I_{j'}$ given $E(Q_i^{t+1} | \lambda_i^t) \in I_j\}$. $H(\cdot)$ is the set of period t+1 events which cause i, given employment by j in period t, and history λ^t, to move to j' at t+1. Then (18) immediately gives equal "destination-specific" mobility probabilities:

$$\sigma_{jj'}(\lambda_i^t) \equiv \Pr[(Q_i^{t+1}, h_{-i}^{t+1}) \in H(i, j, j', \lambda_i^t)]$$
$$= \Pr[(Q_{i'}^{t+1}, h_{-i'}^{t+1}) \in H(i', j, j', \lambda_{i'}^t)] \equiv \sigma_{jj'}(\lambda_{i'}^t)$$

Summing over j' gives equal unconditional odds of i (or i') leaving j: $\sigma_j(\lambda_i^t)$.

By itself, this proposition is both trivial and, from a practical standpoint, empirically vacuous. It is trivial because the working times of workers who share a common personal history λ_i^t are, virtually as a matter of definition, the same input. Since, for any history, work time has a unique (up to rearranging among $f \in \mathcal{F}_j$) best use in this model, identical inputs will have identical mobility patterns; that is, identical sequences of best uses. At first blush, the proposition looks very promising from an empirical standpoint. One way to look at it is that it requires that no variable (job tenure etc.) has any influence on the probability of changing jobs except λ_i^t--a strong set of non parametric "zero restrictions". However, to test these restrictions directly λ_i^t must be observed in its entirety, from a practical standpoint ruining the hypothesis.

A more useful result can be obtained by exploiting the model's structure more fully. Let $\hat{w}_i = w_i(\lambda_i^t)$; i.e. $\hat{w}_i$ is the image of $\lambda_i^t(\cdot)$ under $w_i(\cdot)$.

<u>Proposition</u>: <u>For almost all economies</u>, $\hat{w}_i = \hat{w}_{i'} \Leftrightarrow \lambda_i^t = \lambda_{i'}^t$, up to order.

Informally, the proposition states that workers i and i' earn the same wage if and only if when the wage was observed (they need not be observed contemporaneously) they had identical (up to order) histories; in particular they are at the same stage in their working life. And this holds except for economies with "knife-edge" type parameter restrictions. Put differently, as was mentioned earlier, if i and i' have the same history, then their working times are perfect substitutes. As a consequence they will earn the same wage. The new part is necessity of identical histories for earning identical wages.

While, as will be discussed below, this proposition also has its practical limitations, it is clear that they are vastly less severe than the requirement that λ_i^t be observed in detail.

The proof of this proposition requires an elementary but fairly long mathematical argument, and is therefore left to the Appendix. But the basic logic of the result can be understood easily from the following example. The example assumes that workers begin their working life with no history other than $\hat{q}$, contrary to the analysis in the rest of the text. The modification merely eliminates some algebra in this example. Some other restrictions are added --for example, regarding the q_m -- whose relaxation only serves to strengthen the argument.

Suppose the length of the working life is A=3, so that workers may sell working time on the basis of histories of length zero through 2. Let there be just two goods, $\#(\mathcal{J}) = 2$, the marginal utilities of which are $\beta_1 = \beta_2 = 1$. Further, assume technologies are such that $a_1 = a_2 = 1$. The maximum producers of type j would be willing to pay at t to hire a worker with $E(q_i^{t+1} | \lambda^t) = \tilde{Q}$

is $$\nu_j = 1 + b_j \tilde{Q}, \; j = 1,2.$$

The random variables generating $\tilde{Q}$ are assumed to be two in number (#( )=2), each possibly taking on values

$$q_m = \begin{cases} 0 & m=1 \\ 1 & m=2 \end{cases}$$

with probabilities[9]

$$\Pr(Q_1 = 1) \equiv \delta_1 \equiv 1 - \Pr(Q_1 = 0),$$

$$\Pr(Q_2 = 1) = \delta_2 = 1 - \Pr(Q_2 = 0); \quad \Pr(S=1) = \zeta \in (0,1).$$

The possible histories are then (starting at t=0).

$H_i^0 = \{\phi\}$, $H_i^1 = \{o, 1\}$, $H_i^2 = \{(0,0), (1,0), (0,1), (1,1)\}$. The usual computations yield the six possible values for $\tilde{Q}$:

$$E(Q_i^0 | \phi) = \delta_1 \zeta + \delta_2 (1 - \zeta)$$

$$E[Q_i^1 | (1)] = \frac{\delta_1^2 \zeta + \delta_2^2 (1 - \zeta)}{\delta_1 \zeta + \delta_2 (1 - \zeta)}$$

$$E[Q_i^1 | (0)] = \frac{\delta_1 (1 - \delta_1)\zeta + \delta_2 (1 - \delta_2)(1 - \zeta)}{(1 - \delta_1)\zeta + (1 - \delta_2)(1 - \zeta)}$$

$$E[Q_i^2 | (1,1)] = \frac{\delta_1^3 \zeta + \delta_2^3 (1 - \zeta)}{\delta_1^2 \zeta + \delta_2^2 (1 - \zeta)}$$

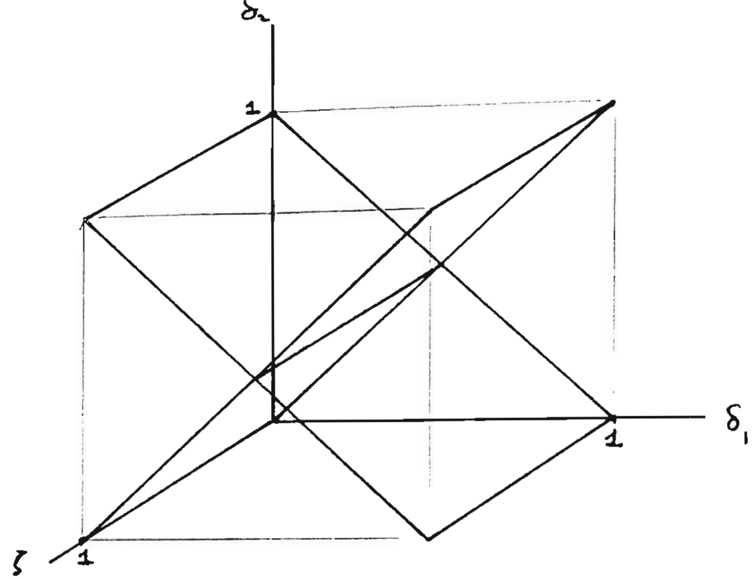
$$E[Q_i^2 | (0,1)] = E[Q_i^2 | (1,0)]$$

$$= \frac{\delta_1^2 (1 - \delta_1)\zeta + \delta_2^2 (1 - \delta_2)(1 - \zeta)}{\delta_1 (1 - \delta_1)\zeta + \delta_2 (1 - \delta_2)(1 - \zeta)}$$

$$E[Q_i^2 | (0,0)] = \frac{\delta_1 (1 - \delta_1)^2 \zeta + \delta_2 (1 - \delta_2)^2 \zeta}{(1 - \delta_1)^2 \zeta + (1 - \delta_2)^2 \zeta}$$

First, what restrictions on δ_1 and δ_2 are required for two or more of the possible values of $\tilde{Q}$ to equal one another? Set $E(Q_i^0 | \phi) = E[Q_i^1 | (1)]$ for example. Minor manipulation gives $\delta_1 = \delta_2$. That is, the information $Q_i^0 = 1$ must be equally likely for both $s \in \mathcal{S}$. Proceeding in the same fashion for all the other pairs gives either $\delta_1 = \delta_2$ or $\delta_1 = 1 - \delta_2$. Thus, in the space if possible $(\zeta, \delta_1, \delta_2)$ triples, the unit cube in Figure 2, the set which generates nondistinct $E(Q_i^{t+1} | \lambda^t)$ is the three dimensional "x".

Figure 2



Formally, the "x" has measure 0 in the unit cube. Roughly speaking, if parameters (ζ, δ_1, δ_2) were chosen at random from any continuous distribution function on the unit cube, the probability that the choice would lie on the "x" is zero. Thus, for almost all stochastic structures (ζ, δ_1, δ_2) possible the various values of $\tilde{Q}=E(Q_i^{t+1}|\lambda^t)$ are distinct.

Do these distinct values generate distinct wages? The answer is again yes, except for a measure 0 set of b_j. To see why, let the six distinct values of $\tilde{Q}$ be $\tilde{Q}_1$, ..., $\tilde{Q}_6$. For distinct wages <u>not</u> to emerge from this collection, at least one of the following equalities must hold

$$1 + b_j\tilde{Q}_k = 1 + b_{j'}\tilde{Q}_{k'}$$

where $j, j' = 1,2$;

$k, k' = 1,\ldots,6$

and, $j = j'$ and $k = k'$ do not hold simultaneously.

Each of these equalities defines a line in the set of possible b_j, here equal to $\{(b_1, b_2) | \infty > b_1 > -1, \infty > b_2 > -1\}$. As above, this set of lines has measure 0 in the relevant parameter set.

In summary, unless a particular set of coincidences are imposed on the economy, all individuals earning a given wage have the same value of $E(Q_i^{t+1} | \lambda_i^t)$, and the same history λ_i^t, and therefore the same probability of mobility. In other words, the price at which i's time trades in the market summarize all the economically relevant information about worker i.

14. Testability of the Basic Prediction

Consideration is first given to how the basic hypothesis could be subjected to test. Subsequently, problems with the required methodology are analysed.

Suppose observations on physical reality are measurements from one of the sample paths of the equilibrium of the economy set out above. What might be observed? It will be assumed that exogenous entities -- the parameters of tastes (ρ, β_j, γ), endowments (L_i, ω_{ij}, v_{it}), technologies (a_j, b_j), and information (ζ_s, δ_{ms}, q_m), as well as the outcomes of the underlying forcing variables λ^t -- are not directly observable. On the other hand, endogenous variables, ℓ_i, c_{ij}, w_i, p_j, x_{if}, might be observed. Of course, some of these data might reveal some of the underlying exogenous variables; L_i for example. But this fact is not relevant for the present discussion.

The underlying random variables, S and Q_s, induce a joint probability distribution for all the endogenous variables at all dates. In general, all of recorded history represents just a single observation from one marginal distribution (obtained by integrating out the future endogenous variables) and virtually no hypothesis could be rejected. However, given the independence of assignment of Q_s to workers, the marginal distribution of worker i - specific variables summarising labor market activities (the sequence of wages and type of producer at which i works at each t) is the same across workers, and i's experience is independent of i''s . This outcome occurs because the I_j are nonstochastic. Thus cross-section observations on workers can be used to learn about this distribution. If some variables (say past wages) are not observed, then the marginal distribution of the observed variables can be obtained in the usual fashion provided the observation process does not depend on the realized values of the unobserved variables.

Suppose that observations are taken at some dates t and t+1 for a random sample of workers i; i $\in \hat{\mathcal{W}}$ where $\hat{\mathcal{W}} \subset \mathcal{W}$ is the sample. Minimal required data would appear to be i) the type of firm at which each i was employed at each date; ii) the wage at t; (iii) any other nonempty set of worker i - specific variables at t. Examples of (iii) would be previous wages and other aspects of the work history. Let

$$\theta_i = \begin{cases} 1 & \text{if i changed jobs between t and t+1} \\ 0 & \text{otherwise,} \end{cases}$$

w_i be i's observed wage, and Z_i the vector of other observed characteristics.

The random sample $\hat{\mathcal{W}}$ can be used to construct the joint empirical distribution function $\hat{F}(\theta, w, z)$ given the independence assumptions. The basic prediction is then $F(\theta|w, z) = F(\theta|w)\ \forall\ z$, which can be tested given $\hat{F}$.

Several points are noteworthy. One is that observation of nontrivial z is necessary to confront the hypothesis; an identification condition.

Second, at this level of generality, the theory does not assert that $F(\theta|w^0)$ bears any particular relation to $F(\theta|w^1)$ for some $w^1 \neq w^0$. However, it can be shown that $F(\theta|w^0) = F(\theta|w^1)$ for some $w^0 \neq w^1$ fails for almost all economies. That aside, the point is that some parametric procedure, say of the probit type, not only requires a variety of purely statistical restrictions, but also alters the basic hypothesis unless w is permitted to enter in a very flexible manner.

There are various difficulties which might be encountered in implementing this testing procedure.

One type of problem is generated by some of the arbitrary elements in the model. How long is a period? How can the investigator tell when a job change is across firm types as opposed to within? A resolution of the first issue is not too difficult. For the phenomenon at hand, a period is the duration of time which is required for new information to emerge. From any given initial condition, not only is it the case that individuals having the same wage also have the same one-period probability of switching jobs, they also have the same k-period probability; $k > 1$. Consequently the investigator need only choose a spacing of observations which can reasonably be assumed to correspond to at least one of the model's periods. The second problem is more difficult. The most reasonable approach would be to suppose that no mobility occurs without some reason, and to interpret all mobility as inter-type, seeing as the model offers no other reason for mobility. (It should be noted here that it is possible to extend the model to allow for temporal variation of a real business cycle variety. For example, a_j can be allowed to vary.

While mobility patterns are altered by doing so, the basic result still applies.) This approach essentially blurs the distinction between firms and types. The economy can be seen as containing one large producer producing many goods with independent production technologies, or as many producers with differentiated products, or (as presented above), somewhere in between. The model predicts when reallocation of working time to different activities will occur, and it is such reallocations which are therefore properly labelled "job mobility". From the empirical standpoint this issue is less difficult than it might first seem. Suppose that certain events lead to within firm (comprising several "types") reallocation, which goes unobserved by the investigator, and that other events lead to observed inter-firm mobility. While the latter is not all the mobility which occurs, the model still implies that the wage contains all relevant information, and hence the probability of observed mobility depends only on the wage.

A second general type of complication involves some of the simplifications in the model; no specific human capital, no desire for variety, no family behavior, no costs of moving, etc. Any of the above can plausibly be argued to influence job mobility. One response is that with varying degrees of difficulty, these elements can be included. But in a sense this misses the point. The whole idea of the preceding exercise was to determine what is implied for mobility behavior by assuming that the sole reason for changing jobs (or not doing so) is new information. Allowing for other factors, while likely to be useful later in the study of mobility, merely muddies waters which are already quite murky.

The most serious problem involves measurement error. If it is not possible to determine what the wage in fact is, conditioning on the wage is not possible.

In one sense this issue is not problemmatic. Suppose the investigator observes the true wage, perturbed by some random measurement error whose probability structure does not depend on the wage and which attaches probability to a countable set (actually any measure 0 subset of the real numbers will do) of values; a rational-valued random variable is an example. Then, the necessity part of the main result continues to hold. While workers who have the same wage will have distinct observed wages, only workers who have the same true wage can have the same observed wage (for almost all economies). Consequently, equal observed wages again implies equal true wages and hence identical histories.

If, on the other hand the wage is only observable in intervals, for example, the situation may be more serious. Proceeding along the lines of the proof of the main proposition, it can be shown that for all economies apart from a set of small (as opposed to zero) measure, the set of wages consists of isolated points. That is, no two wages are arbitrarily close together. Thus if the intervals are small, workers whose wage is reported to lie within any given interval will have the same true wage, and the result again follows. Think of wage observations being in clumps. Just how wide the intervals can be depends on how many types of producers there are, how often new information emerges, etc; that is, how much information the wage must convey.

If the nature of the observation process is such that should two workers have the same observed wage, it cannot be supposed that they have the same true wage, the observed wage no longer summarizes all information relevant to job mobility, and the testability of the proposition breaks down.

What can be said about this kind of environment? At the level of generality utilized so far, the answer seems to be "not much". The reason is simply that if the observed wage does not imply suitable restrictions on the true wage, the observed wage contains too little information and there are many ways in which the other pieces of information might make themselves apparent. The probability of mobility, given the observed wage, may depend on many other aspects of the worker's history (say tenure on present job) simply because such observation are correlated with λ^t. However, no positive results are yet available for such an environment.

In part II an infinite state space model is examined. While it obviously does not necessarily generate the same results as a finite state space model with suitably confounding measurement error, it does generate the same type of results in that nonwage variables influence job mobility. The infinite state space model appears to be a more tractable environment for analyzing such interrelations.

FOOTNOTES

[1]That every agent is endowed with some of each commodity is purely a convenience.

[2]This modification is straightforward in this model because in equilibrium, no agents will desire intertemporal trades.

$$\sum_{\lambda^T \in \Lambda^T} \pi(\lambda^T) \sum_{t \in \mathcal{D}} \rho^{t-1} \sum_{j \in \mathcal{J}} \{\beta_j c_{ij}(\lambda^t) + \gamma \ell_i(\lambda^t)\}$$

$$= \sum_{\lambda^T \in \Lambda^T} \pi(h^1)\, \pi(h^2, \dots, h^T | h^1) \sum_{j \in \mathcal{J}} \{\beta_j c_{ij}(h^1) + \gamma \ell_i(h^1)\}$$

$$+ \dots + \sum_{\lambda^T \in \Lambda^T} \pi(\lambda^T) \sum_{t \in \mathcal{D}} \rho^{t-1} \sum_{j \in \mathcal{J}} \{\beta_j c_{ij}(\lambda^T) + \gamma \ell_i(\lambda^T)\}$$

$$= \sum_{h^1 \in H^1} \pi(h^1) \sum_{j \in \mathcal{J}} \{\beta_j c_{ij}(h^1) + \gamma \ell_i(h^1)\}$$

$$+ \dots + \sum_{\lambda^T \in \Lambda^T} \pi(\lambda^T)\, \rho^{T-1} \sum_{j \in \mathcal{J}} \{\beta_j c_{ij}(\lambda^T) + \gamma \ell_i(\lambda^T)\}.$$

[4]Q_s may be a allowed to be vector, which is more in the spirit of the interpretation given below.

[5]Other factors of production are ignored.

[6](16) should be interpreted with reference to the contingent claims structure. Partition λ^{t+1} into $(Q_i^{t+1}, h_{-i}^{t-1}, \lambda^t)$, where $h_{-i}^{t+1} = (Q_{i'}^{t+1}, i' \in \mathcal{W}, i' \neq i)$.

Now consider a type j producer considering hiring worker i (i.e. demanding $x_{if}(\lambda^t) = L_i$) having observed λ^t. Claims to the subsequent output may be sold for all λ^{t+1}, given λ^t, where the quantity of output does not vary with h_{-i}^{t+1}, but only with Q_i^{t+1}. So <u>for given</u> Q_i^{t+1}, the total value of claims sold is (from (12))

$$(a_j + b_j Q_i^{t+1}) L_i \sum_{h_{-i}^{t+1} \in {}_{-i}^{t+1}} \pi(Q_i^{t+1}, h_{-i}^{t+1}, \lambda^t) \rho^t \beta_j$$

$$= (a_j + b_j Q_i^{t+1}) L_i \, \pi(\lambda^t) \rho^t \beta_j \sum_{h_{-i}^{t+1} \in {}_{-i}^{t+1}} \frac{\pi(Q_i^{t+1}, h_{-i}^{t+1}, \lambda^t)}{\pi(\lambda^t)}$$

$$= (a_j + b_j Q_i^{t+1}) L_i \rho p_j(\lambda^t) \, \pi(Q_i^{t+1} | \lambda^t)$$

Summing this expression over the possible values Q_i^{t+1} might take on yields the total value of claims that might be sold, or (16) multiplied by L_i ((16) is expressed per unit of i's time).

Another way to look at (16) is to note that it equals expected period t+1 output, valued at the period t price of filled claims, and discounted, the latter operation to reflect the fact that output does not arrive until t+1. In equilibrium, the firm behaves as if maximizing profits from the sale only of output, not claims (i.e. incomplete markets) assuming current date prices of delivered goods will remain constant over time.

[7]For any finite set A, #(A) is the number of elements in A.

[8]Otherwise, type j producers will not hire at any date and can be eliminated from the model.

[9]To eliminate notation $\delta_{12} = 1 - \delta_{11}$ and $\delta_{22} = 1 - \delta_{21}$ is utilized here.

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APPENDIX

1. Proof of Proposition 1

The proof makes use of the following fact. Lebesque measure is denoted by μ, gradient by ∇, and zero vector by $\underset{\sim}{0}$.

Lemma: Let X be a measurable subject of $\mathbb{R}^n$, f: X→$\mathbb{R}$ be a once continuously differentiable function, and $k \in f(X)$ a constant. Then

$$\mu\{x \in X | f(x) = k\} \leq \mu\{x \in X | \nabla f(x) = \underset{\sim}{0}\}$$

It was established in the text that $w_i(\lambda^t$ depends nontrivially only on λ_i^t. Thus in what follows, the subscript i is suppressed. Also, since realizations of Q_s are independent, it is immediate that the order of events h^t in any given history λ^t is irrelevant.

The set of all possible histories is $\tilde{\Lambda} = \bigcup_{t \in \mathcal{D}} \Lambda^t$, with typical element $\tilde{\lambda}$. Define the binary relation "~" on $\tilde{\Lambda} \times \tilde{\Lambda}$ by $(\tilde{\lambda}^0 \sim \tilde{\lambda}^1) \Leftrightarrow (\tilde{\lambda}^1$ is a permutation of $\tilde{\lambda}^0$). Let the collection $\{\tilde{\Lambda}_1, \ldots, \tilde{\Lambda}_K\}$ be the equivalence classes in $\tilde{\Lambda}$ induced by ~; $K < \infty$. For each k=1, ..., K, choose one arbitrary element $\tilde{\lambda}_k$. Let $\Lambda = \{\tilde{\lambda}_1, \ldots, \tilde{\lambda}_K\}$, and let λ represent a typical element. Λ is the set of histories such that i) every possible history is either equal to an element of Λ, or differs by a permutation, and ii) no two histories are the same.

The wage mapping w: $\Lambda \to \mathbb{R}$ depends on the parameters of the economy. The parameter space, constructed in detail below, is denoted Ω, with typical element ω; $\mu\, \Omega < \infty$. Let $\Omega^w = \{\omega \in \Omega |$ w: $\Lambda \to$ is one to one}. That $\mu\, \Omega^w = \mu\, \Omega$ is to be shown.

Ω is the Cartesian product of the following sets (M is a large finite real number):

i) $\overset{\#(\mathcal{M})}{\underset{i=1}{\times}} [-M, M]$ -- the $\#(\mathcal{M})$ - dimensional space of which the vectors $(q_m; m \in \mathcal{M})$ are members;

ii) $\overset{\#(\mathcal{S})}{\underset{i=1}{\times}} \{(x_m; m \in \mathcal{M}) \mid x_m > 0, \sum_{m \in \mathcal{M}} x_m = 1\}$ -- the $[\#(\mathcal{M})-1] \times \#(\mathcal{S})$ - dimensional space from which the δ_{ms} are taken;

iii) $\{(x_s; s \in \mathcal{S}) \mid x_s > 0, \sum_{s \in \mathcal{S}} x_s = 1\}$ -- the $[\#(\mathcal{S}) - 1]$ - dimensional space from which the ζ_s are taken;

iv) $\overset{\#(\mathcal{J})}{\underset{i=1}{\times}} (0, M]$ -- the $\#(\mathcal{J})$ - dimensional space from which the vector of taste parameters $(\beta_j; \in \mathcal{J})$ are drawn.

v) $\{\overset{\#(\mathcal{J})}{\underset{i=1}{\times}} (0, M]\} \times \{\overset{\#(\mathcal{J})}{\underset{i=1}{\times}} [-M, M]\}$ -- the $2 \cdot \#(\mathcal{J})$ - dimensional space from which the vector of technology parameters $(a_j; j \in \mathcal{J}; b_j; j \in \mathcal{J})$ are taken.

Note two points here:

a) Ω comprises only those parameters upon which the mapping w depends nontrivially. The full parameter space of the economy is the Cartesian product of Ω with another finite dimensional space, say Δ. However, $\mu\, \Omega = \mu\, \Omega^w$ implies $\mu(\Omega \times \Delta) = \mu(\Omega^w \times \Delta)$. Thus $\mu\, \Omega = \mu\, \Omega^w$ is what is demonstrated.

b) In the text several inequality restrictions involving Ω are introduced -- $\inf(\{a_j + b_j q_m\} > 0$ -- as well as one involving

$$\Omega \times (0,M) \times (0,M) \text{ --- } \pi(\lambda_i^t)\gamma \leq \sup_j \{ \sum_{h^{t+1} \in H^{t+1}} \pi(\lambda^t)\rho\beta_j(a_j + b_j Q)\}.$$

Thus the "relevant" parameter space is that part of Ω for which the inequality restrictions are satisfied for some γ and ρ, say Ω^r. It is simple to show that $0 < \mu\, \Omega^r \leq \mu\, \Omega$. Also, since $\Omega^r \subset \Omega$ and $\Omega^w \subset \Omega$, $\mu(\Omega \cap \Omega^r) = \mu[(\Omega - \Omega^w) \cap \Omega^r] + \mu(\Omega^w \cap \Omega^r)$. But $\mu[(\Omega - \Omega^w) \cap \Omega^r] = 0$ if $\mu\, \Omega = \mu\, \Omega^w$, and $\Omega \cap \Omega^r = \Omega^r$. Thus, if $\mu\, \Omega = \mu\, \Omega^w$, $\mu\, \Omega^r = \mu(\Omega^w \cap \Omega^r)$, in which case w is one to one for almost all parameter values satisfying the inequality restrictions. Thus demonstrating $\mu\, \Omega = \mu\, \Omega^w$ is again sufficient.

w is the composition of two mappings: E: $\Lambda \to \mathbb{R}$ yields $E(Q|\lambda)$ for any $\lambda \in \Lambda$. W: $\mathbb{R} \to \mathbb{R}$ provides $\sup_j\{a_j\beta_j + b_j E(Q|\lambda)\}$ for any $E(Q|\lambda)$. Hence $w = W \circ E$. A sufficient condition for w to be one to one is that W and E are one to one. It therefore suffices to demonstrate that and E are one to one off a set of measure 0 in Ω; $\mu(\Omega - \Omega^w) = 0$.

Let Ω^0 be the Cartesian product of the sets in (i) - (iii) above, with typical element ω^0, and Ω^1 the Cartesian product of the sets in (iv) - (v), with typical element ω^1; $\Omega = \Omega^0 \times \Omega^1$. The parameters of E are elements $\omega^0 \in \Omega^0$, and the parameters of W are elements $\omega^1 \in \Omega^1$. If E fails to be one to one only for $\omega^0 \in S^0 \subset \Omega^0$, and $\mu\, S^0 = 0$, then $S^0 \times \Omega^1 \subset \Omega$ and $\mu(S^0 \times \Omega^1) = 0$. Similarily, if W fails to be one to one for $\omega^1 \in S^1 \subset \Omega^1$ and $\mu\, S^1 = 0$, then $\Omega^0 \times S^1 = \subset \Omega$ and $\mu(\Omega^0 \times S^1) = 0$. Since $\Omega^w = \Omega - \{(S^0 \times \Omega^1) \cup (\Omega^0 \times S^1\}$, $\mu\, S^0 = 0 = \mu\, S^1$ imply $\mu\, \Omega^w = \mu\, \Omega$. It thus suffices to show that E( ) is one to one for parameters off a set of measure 0 in Ω^0 (Ω^1).

Take E first. Let $N_n : \Lambda \to \{0, 1, \ldots, T\}$ be the number of occurences of q_m in λ. Since $\forall\ \lambda^0, \lambda^1 \in \Lambda$, $\lambda^0 \neq \lambda^1$, it follows that for some m $N_m(\lambda^0) \neq N_m(\lambda^1)$. Calculation gives

$$E(Q|\lambda) = \sum_{m\in\mathcal{M}} q_m \left\{ \sum_{s\in\mathcal{S}} \delta_{ms}\, \zeta_s \left[\frac{\prod_{m'\in\mathcal{M}} \delta_{m's}^{N_{m'}(\lambda)}}{\sum_{s'\in\mathcal{S}} \zeta_{s'} \prod_{m'\in\mathcal{M}} \delta_{m's'}^{N_{m'}(\lambda)}} \right] \right\}.$$

It is immediate that E is single-valued, in which case E is one to one if each point in $E(\Lambda)$ is the image of exactly one point in λ. Suppose not, then for some $\lambda^0, \lambda^1 \in \Lambda$,

$$\sum_{m\in\mathcal{M}} q_m \sum_{s\in\mathcal{S}} \delta ms\, \zeta_s \left[\frac{\prod_{m'\in\mathcal{M}} \delta_{m's}^{N_{m'}(\lambda^0)}}{\sum_{s\in\mathcal{S}} \zeta_{s'} \prod_{m'\in\mathcal{M}} \delta^{N_{m'}(\lambda^0)}} - \frac{\prod_{m'\in\mathcal{M}} \delta^{N_{m'}(\lambda^1)}}{\sum_{s\in\mathcal{S}} \zeta_{s'} \prod_{m'\in\mathcal{M}} \delta_{m's'}^{N_{m'}(\lambda^1)}} \right] = 0. \quad \text{(A-1)}$$

Write (A-1) as $F(\omega^0; \lambda^0, \lambda^1) = 0$ where $F: \Omega^0 \times \Lambda \times \Lambda \to \mathbb{R}$, and let $S^0(\lambda^0, \lambda^1) = \{\omega^0 \in \Omega^0 | F(\omega^0, \lambda^0, \lambda^1) = 0\}$. Note that F is continuously differentiable in ω^0 everywhere. Now, using the lemma, $\mu\, S^0(\lambda^0, \lambda^1) \leq \mu\, \{\omega^0 \in \Omega^0 | \nabla_{\omega^0} F = \underset{\sim}{0}\}$ where $\nabla_{\omega^0} F$ is the gradient of F with respect to elements of ω^0. A necessary condition for $\nabla_{\omega^0} F = 0$ is $\partial F/\partial q_m = 0$ for each m.

Thus $\mu\{\omega \in \Omega^0 | \nabla_{\omega^0} F = \underset{\sim}{0}\} \leq \mu\{\omega^0 \in \Omega | \partial F/\partial q_m = 0\ \forall m \in \mathcal{M}\} \leq \mu\{\omega^0 \in \Omega^0 |$

$\partial F/\partial q_m = 0$, some m}. Fix m. A necessary condition for $\partial F/\partial q_m = 0$ is that the expression in braces in (A-1) equal zero. Call this expression $G(\omega^0, \lambda^0, \lambda^1)$, where $G: \Omega^0 \times \Lambda \times \Lambda \to \mathbb{R}$. Then $\mu\{\omega^0 \in \Omega^0 | \partial F/\partial q_m = 0\} \leq \mu\{\omega^0 \in \Omega^0 | G = 0\}$. Applying the lemma again, and proceeding as above, $\mu\{\omega^0 \in \Omega^0 | G = 0\} \leq \mu\{\omega^0 \in \Omega^0 | \nabla_{\omega^0} G = \underset{\sim}{0}\} \leq \mu\{\omega^0 \in \Omega^0 | \partial G/\partial \zeta_s = 0 \ \forall \ s \in \mathcal{S}\}$

$\leq \mu\{\omega^0 \in \Omega^0 | \partial G/\partial \zeta_s = 0$, some s}. After rearrangement,

$$\frac{\partial G}{\partial \zeta_s} \propto \frac{\prod_{m \in \mathcal{M}} \delta_{ms}^{N_m(\lambda^0)}}{\sum_{s' \in \mathcal{S}} \zeta_{s'} \prod_{m \in \mathcal{M}} \delta_{ms'}^{N_M(\lambda^0)}} - \frac{\prod_{m \in \mathcal{M}} \delta_{ms}^{N_m(\lambda^1)}}{\sum_{s' \in \mathcal{S}} \zeta_{s'} \prod_{m \in \mathcal{M}} \delta_{ms'}^{N_m(\lambda^1)}},$$

in which case $\partial G/\partial \zeta_s = 0$ requires the expression in square brackets to equal zero. Call this expression $J(\omega^0; \lambda^0, \lambda^1)$, where $J: \Omega^0 \times \Lambda \times \Lambda \to \mathbb{R}$. Then $\mu\{\omega^0 \in \Omega^0 | \partial G/\partial \zeta_s = 0$, some s$\} \leq \mu\{\omega^0 \in \Omega^0 | J = 0\}$.

J = 0 can be rewritten

$$\sum_{s' \in \mathcal{S}} \zeta_{s'} \left[\prod_{m \in \mathcal{M}} \left(\frac{\delta_{ms'}}{\delta_{ms}} \right)^{N_m(\lambda^0)} - \prod_{m \in} \left(\frac{\delta_{ms'}}{\delta_{ms}} \right)^{N_m(\lambda^1)} \right] = 0.$$

Applying the lemma again, $\mu\{\omega^0 \in \Omega^0 | J = 0\} \leq \mu\{\omega^0 \in \Omega^0 | \nabla_{\omega^0} J = \underset{\sim}{0}\} \leq$

$\mu\{\omega^0 \in \Omega^0 | \partial J/\partial \zeta_{s'} = 0 \ \forall s' \in \mathcal{S}\} \leq \mu\{\omega^0 \in \Omega^0 | \partial J/\partial \zeta_{s'} = 0$, some s'}.

Fixing s', $$\frac{\partial J}{\partial \zeta_{s'}} \propto \prod_{m \in \mathcal{M}'} \left(\frac{\delta_{ms'}}{\delta_{ms}} \right)^{N_m(\lambda^0)} - \prod_{m \in \mathcal{M}'} \left(\frac{\delta_{ms'}}{\delta_{ms}} \right)^{N_m(\lambda^1)}$$

where $\mathcal{M}' = \{m \in \mathcal{M} | N_m(\lambda^0) \neq N_m(\lambda^1)\}$. $\partial J/\partial\zeta_{s'} = 0$ can be solved uniquely for any $\delta_{ms'}$ such that $m \in \mathcal{M}'$. That is, the set of $\omega^0 \in \Omega^0$ for which $\partial J/\partial\zeta_{s'} = 0$ is the graph of a single valued function. Such graphs have measure 0 in the cross product of the domain and range of the function, hence a subset of Ω^0.

Thus $\mu S^0(\lambda^0, \lambda^1) = 0$. Since S^0 is the finite union

$$S^0 = \bigcup_{\substack{(\lambda^0, \lambda^1) \in \Lambda x \Lambda \\ \lambda^0 \neq \lambda^1}} S^0(\lambda^0, \lambda^1) ,$$

$\mu(S^0) = 0$. Thus E is one to one for parameters off a set of measure 0 in Ω^0.

Now consider the mapping $W: \mathbb{R} \to \mathbb{R}$ defined by

$$W(\tilde{Q}) = \sup_j \{a_j\beta_j + b_j\beta_j\tilde{Q}\}$$

where $\tilde{Q} \in \{E(Q|\lambda) | \lambda \in \Lambda\}$. It has already been shown that off a set of measure 0 in Ω^0, hence Ω, $\#\{E(Q|\lambda)|\lambda \in \Lambda\} = \#(\Lambda)$. Thus $\{E(Q|\lambda)|\lambda \in \Lambda\}$ can be expressed more conveniently as the finite set $e = \{e_1,...,e_K\}$ for some $K < M$.

W is single valued, so verifying that it is one to one involves showing that each $w \in W(e)$, is the image of exactly one $e_k \in e$.

Suppose not, then for some $e_{k^0} \neq e_{k^1}$, and $j \neq j'$ (since neither $b_j = 0$ nor $\beta_j = 0$ are permitted):

$$(a_j\beta_j + b_j\beta_j e_{k^0}) - (a_{j'}\beta_{j'} + b_{j'}\beta_{j'} e_{k^1}) = 0. \qquad \text{(A-2)}$$

Viewing (A-2) as $L(\omega^1, e_{k^0}, e_{k^1}) = 0$ for some $L\colon \Omega^1 \times e \times e \to \mathbb{R}$, and defining $S^1(e_{k^0}, e_{k^1}) \equiv \{\omega^1 \in \Omega | L(\omega^1, e_{k^0}, e_{k^1}) = 0\}$, the lemma gives $\mu\, S^1(e_{k^0}, e_{k^1}) \leq \mu\{\omega^1 \in \Omega^1 | \nabla_{\omega^1} L = \underset{\sim}{0}\}$, where $\nabla_{\omega^1} L$ is the vector of partial derivatives of L with respect to $\omega^1 \in \Omega^1$. But $\partial L/\partial a_j = 0$ only if

$\beta_j = 0$,

which is already excluded. Thus $\mu\, S'(e_{k^0}, e_{k^1}) = 0$.

Since

$$S^1 = \bigcup_{\substack{(e_{k^0}, e_{k^1}) \in e \times e \\ e_{k^0} \neq e_{k^1},}} S^1(e_{k^0}, e_{k^1}),$$

it follows that

$$\mu\, S^1 = 0. \qquad \|$$

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