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On the Axiomatic Method Part II: Its Scope and Recent Applications to Game Theory and Resource Allocation

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Abstract

This is Part II of a study of the axiomatic method and its recent applications to game theory and resource allocation. Part I is a user's guide. Part II discusses alternatives to the axiomatic method and answers criticisms often addressed at the axiomatic method. It delimits the scope of the method and illustrates its relevance to the study of resource allocation and the study of strategic interaction. Part II also provides extensive illustrations of the considerable recent success that the method has met in the study of a number of new models.

Contents

1 Introduction

This is Part II of a study of the axiomatic method and its recent applications to game theory and resource allocation. Part I is a user's guide. This second part discusses alternatives to the axiomatic method and answers criticisms often addressed at the axiomatic method. It delimits the scope of the method and illustrates its relevance to the study of resource allocation and the study of strategic interaction. Finally, it provides extensive illustrations of the considerable recent success that the method has met in the study of a number of new models.

Each section opens with a compact statement of the point I am attempting to make. This statement is usually italicized. I continue with examples illustrating the point, in one or several indented paragraphs. In addition to the theory of resource allocation, these examples are most often taken from the theory of cooperative games.

2 Alternatives to the axiomatic method

In this section, I describe alternatives to the axiomatic method, and show that not only they are compatible with it, but that in fact, they often naturally lead to it; at the very least, they are very usefully complemented by it.

2.1 Basing solutions on the "intuitive" appeal of their definitions

According to some authors, a solution may be so intuitive that it does not require an axiomatic justification. *The intuitive appeal of a definition is indeed seen by many as a substitute for an axiomatic justification.*

> For instance, Peleg (1985) opens his study of *consistency* for coalitional form games, in which he provides the first characterization of the core, by stating that this solution is so natural that there may be no need to characterize it.

I have of course no objection to relying on intuition since intuition also underlies the formulation of the axioms. Moreover, the view just expressed is in complete agreement with the position developed in these pages, provided terms are properly defined. Indeed, we have seen a number of axioms that pertain to only one problem at a time in the domain of definition. Let us refer to them as *one-problem axioms.* When such an axiom actually applies to *every* problem in the domain — I will say that it has *full coverage* — it automatically defines a solution.

> For most classes of problems, the concept of *Pareto-optimality* can be used either to define an axiom imposed on solutions, or to define a solution, simply the solution that selects for each problem its set of Pareto-optimal outcomes.[1] Similarly, conditions such as *individual rationality* and *no-envy* can be used either as axioms or solutions. By contrast, *symmetry* (two agents with identical characteristics should be treated in the same way) is a one-problem axiom that does not have full coverage, since there are problems in which not all agents have identical characteristics. In fact, most problems are of this kind, so that we cannot define a solution on the basis of considerations of symmetry alone, except perhaps in the following trivial way: for each economy to which *symmetry* applies, only select allocations recommended by the axiom; for each other problem, select the whole feasible set.

To the extent that a solution is intended to provide as precise a prediction or recommendation as possible, it may be natural to focus on the axiom interpretation of a test if many alternatives pass it, and on the solution interpretation if the opposite holds.

> If this language is adopted, and pursuing our earlier examples, *Pareto-optimality* and *individual rationality* should be called axioms since for most economies many allocations pass either test, whereas the Walrasian solution should be referred to as a solution since there are typically few Walrasian allocations. The core is somewhere in between; indeed, depending upon the model and the number of agents, there may be few core allocations (think of a large exchange economy), or a large set of them (convex games are an example).

[1]This is under the proviso that Pareto-optimal outcomes always exist, since I have required solutions always to be non-empty valued. For most classes of problems, the existence of Pareto-optimal outcomes is guaranteed. This is the case for all of the models discussed in this paper.

Alternatively, we could think of the solution that associates with each problem its set of feasible outcomes satisfying some basic set of properties as a "presolution". The term suggests that further restrictions need to be imposed on outcomes.

In the theory of resource allocation, the correspondence that selects for each economy its set of Pareto-optimal allocations, or the correspondence that selects for each problem its set of individually rational allocations, are examples of such presolutions.

In the theory of coalitional form games, the notion of an imputation, an efficient payoff vector meeting the individual rationality constraints, can also be understood in this way, as providing a first reduction of the set of payoff vectors worth considering.

2.2 Justifying solutions on the basis of the recommendations they make for test problems

Another approach consists in simply "producing" solutions, and evaluating them by verifying that they give appropriate answers in situations in which we feel that intuition is a reliable guide. This approach is the most frequently taken.[2] Here, the merits of solutions are assessed by applying them to examples. Solutions are promoted when they provide intuitively correct recommendations or predictions for the examples, and criticized when they do not.

This is a very valuable way to proceed but in my view, the lessons to be learned by examining examples are often not drawn with sufficient care.

Consider the extension of the Shapley value known as the λ-transfer value from coalitional form games with transferable utility to coalitional form games without transferable utility, and to resource allocation problems. (i) It had of course been known for a long time that on the subclass of coalitional form games with transferable utility whose core is

[2]This is illustrated by the following list of examples of solutions that were introduced in this way: for bargaining problems, the Raiffa solution (1953); for coalitional form games with transferable utility, the core (Gillies, 1959); for normal form games, the Nash equilibrium solution (1951); for extensive form games, the subgame perfect equilibrium solution (Selten, 1975); for exchange economies, the Walrasian solution; and for economies with single-peaked preferences, the uniform rule (Bennassy, 1982).

non-empty, the Shapley value may select payoff vectors outside of the core. Given the compelling definition of the core, this had been seen as a problem. (ii) Examples of games without transferable utility illustrating additional difficulties with the λ-transfer value were also developed by Roth (1980). (iii) Shafer (1980) constructed an exchange economy in which the λ-transfer value assigns a positive part of society's resources to an agent whose endowment is zero. The Shafer and Roth examples have been the object of an extensive literature. (See Aumann, 1985b; Roth, 1986; Scafuri and Yannelis, 1984; Yannelis, 1982).

In exchange economies, much has been made of certain "paradoxical" behaviors of the Walrasian solution. For instance, (i) economies exist in which it allocates all of the gains from trade to only one of the agents; or (ii) economies in which an agent's welfare decreases when his endowment increases; or (iii) economies where an agent's welfare increases when he transfers some of his endowment to another agent whereas the recipient's welfare decreases (this is the well-known "transfer problem"). (iv) It is also manipulable by misrepresentation of preferences.

First, I will note that it should not come as a surprise that any given solution would on occasion not make the right recommendation. But I would mainly like to argue that instead of serving as an indictment of the solutions in the study of which they were developed, the examples should instead be used in a constructive way to establish a new vista from which to consider the field. The axiomatic method suggests that the following protocol be set in motion.

1. *First, we should identify the class of situations that the examples illustrate.* The examples will be informative only to the extent that they are representative of sufficiently wide classes of situations. *Each class should then inspire the formulation of a general property that can be incorporated as an axiom in the analysis*: the axiom simply places restrictions on how the solution should behave on the class. This process is not a substitute for intuition but it articulates it into operationally useful conditions. The questions can then be asked: How restrictive is the axiom? Which ones of the standard solutions satisfy it? Which other properties is it compatible with? Which combinations of properties is it compatible with? Which maximal combinations of properties is it compatible with?

Possible requirements on a solution suggested by the examples presented above are listed next. (i) In the context of coalitional form games, a solution should be a selection from the core. (ii) In the context of resource allocation, a solution should not attribute to an agent more of every good that he owned initially; (iii) it should assign to an agent a welfare level that is monotonic with respect to his endowment; (iv) it should be immune to the "transfer problem". (v) In the context of the problem of fair division, a solution should assign to each gent a welfare level that is monotonic with respect to the social endowment. (vi) In the context of a wide variety of resource allocation problems, a solution should be immune to manipulation by misrepresentation of preferences ...

Incidentally, general theorems describing the limited extent to which such requirements are compatible with other appealing ones have now been established, largely exonerating the λ-transfer value and the Walrasian solution from the limitations that the examples mentioned above had illustrated. These difficulties are now understood to be widely shared, and largely unavoidable on classical domains, although as we will see, quite a few interesting situations have also been identified where they do not occur.

2. *Once the axioms have been formulated, and when the goal is to understand the merits of a particular solution, we can turn to the identification of the subdomain of problems for which the solution does provide the right answer.* If it is relatively large, we might be willing to accept undesirable behavior of the solution on the complementary class.[3]

In bargaining theory and in the theory of coalitional form games without transferable utility, a number of conditions are satisfied by some of the central solutions under the assumption of strict comprehensiveness of problems[4] but violated if that assumption is not made. Violations only occur on the "boundary" of the domain.

In economic models of resource allocation, strengthening monotonicity assumptions on preferences has similar consequences: when we go from the domain of weakly monotonic preferences to the domain of strictly

[3]When probabilistic information is available, this information can be used to quantify the severity of the problem.

[4]This is the assumption that the undominated boundary contain no non-degenerate subset parallel to a coordinate subspace.

monotonic preferences, we find that a number of properties hold that cannot be satisfied otherwise.

3. *If the subdomain over which the violations of an axiom by a particular solution occur is large enough, we may need to restrict the domain of definition of the solution to the complementary subdomain.*[5]

The Shapley value, when applied on the domain of convex coalitional form games with transferable utility, and when used as a solution to resource allocation problems, enjoys properties (core selection, various monotonicities), that it does not satisfy in general (Moulin, 1992).

In exchange economies, and under the assumption of gross substitutability of preferences, the Walrasian solution satisfies many properties (stability, uniqueness, various monotonicities) that it violates on general domains (Polterovich and Spivak, 1983; Moulin and Thomson, 1988). Other restrictions on preferences, such as homotheticity, normality, and quasi-linearity imply better behavior of the Walrasian solution (and others) than on standard domains.

4. *Alternatively, we may keep the same domain of definition for the solution but restrict the range of application of the axiom to a subdomain.*

In formulating the properties of *feasible set monotonicity* and *population-monotonicity* of bargaining solutions, we can restrict attention to strictly comprehensive problems. It is quite useful to know that on this large subdomain, the lexicographic extension of the egalitarian solution satisfies the properties (this is because it coincides there with the egalitarian solution, a solution that enjoys them in general).

5. *Another option is to weaken the conclusion of the axiom, provided we do not lose too much of the essential idea of its initial formulation.*

The egalitarian bargaining solution is not *consistent* but it so happens that the solution outcome of a reduced problem always Pareto-dominates

[5] Of course, restricting the domain is not always an option. The pathological examples may be ones for which it is particularly important that we be able to make recommendations.

the restriction of the original solution outcome to the subspace pertaining to the agents involved in the reduction, (instead of coinciding with that restriction as required by *consistency*; Thomson, 1984). For most problems however, *consistency* and this property are equivalent.

In bargaining theory, applying the axioms only when certain smoothness conditions are satisfied, when corner situations do not occur, or when the feasible set is strictly comprehensive, are other typical ways in which useful reformulations can be obtained. In exchange economies, smoothness of preferences and interiority of allocations often play a role too.

6. *Finally, we may redefine the solution altogether.* Of course, the price of using such redefinitions may be that some previously satisfied property will now fail to be met.

In bargaining theory, in order to obtain *Pareto-optimality*, the egalitarian solution which only satisfies *weak Pareto-optimality* can be replaced by its lexicographic extension (Imai, 1983).[6] In the process however *continuity* is lost as well as a number of monotonicity properties.

3 Common criticisms addressed at the axiomatic method

The criticism is sometimes levelled against the axiomatic method that the studies that have made use of it too often consist in the formulation of a large number of axioms and in the analysis of their logical relations, only to end in some impossibility result. Other criticisms are that when these studies do not end in impossibilities, the recommendations they make often conflict one with the other. Also, characterizations are obtained on "too large" a domain. Finally, the axioms are often criticized for not being descriptive of behavior. I will take up each of these criticisms in turn and draw on the theory of cooperative games and on the theory of resource allocation to show that they are unfounded.

[6] On the domain of strategic games, either in normal form or in sequential form, Nash equilibrium can be replaced by undominated Nash equilibrium, or subgame perfection respectively. See below for a further discussion.

3.1 Too many axioms

Considering first the claimed multitude of axioms, I will actually assert the opposite. *In spite of the great variety of models that have now been the object of axiomatic analysis, and the apparently large number of axioms that have been used in these analyses, all of these axioms are expressions for each model of just a handful of elementary principles with wide appeal and relevance.* They are the following:

1. **Efficiency.** The principle of *efficiency*, or *Pareto-optimality* (and weaker versions such as *weak Pareto-optimality* and *unanimity*), is of course the most prominent one.

2. **Symmetry.** Many studies also involve some form of symmetry. An example is *equal treatment of equals*, which requires that identical agents be treated identically (at each chosen alternative, or globally). A related condition is *anonymity*, which states that the solution should be invariant under "permutations" of agents.

3. **Invariance.** *Invariance principles* with respect to certain choices of utility functions play an important role in models where utility information is used (d'Aspremont and Gevers, 1977; Sen, 1977).

The general principles described next have underlied a great number of recent developments.

4. **Consistency and its converse.** The *consistency principle* states the independence of a solution with respect to the departure of some of the agents with their assigned payoffs. It allows us to deduce, from the desirability of an outcome for some problem faced by some group, the desirability of each restriction of the outcome to each subgroup, for the problem obtained by imagining the members of the complementary subgroup to leave with their assigned payoffs; these are the associated "reduced problems". The *converse* of this principle permits us to infer the desirability of an outcome for the problem faced by any group on the basis of the desirability of the restrictions of the outcome to all two-person subgroups in the associated reduced problems (see Driessen, 1991, and Thomson, 1996a, for surveys.)

5. **Monotonicity.** Consider now problems that can be described in terms of a parameter that belongs to a set endowed with an economically meaningful order structure (feasible set in utility space, technological opportunities in commodity space, population size). The *monotonicity principle* requires the welfares of all or some selected subset of the agents to be affected in a particular direction by changes in parameters that can be evaluated accord-

ing to that order (see Thomson, 1995b, for a survey of the applications of the principle to variations in populations).

6. **Replacement.** The *replacement principle* asserts that any change in any parameter entering the description of the class of problems under consideration, whether or not it can be evaluated in some order, should affect the welfares of all relevant agents in the same direction (Thomson, 1990a). A primary example of such a parameter is preferences.

Both the *monotonicity* and *replacement* principles are formalizations of the central idea of solidarity, with the latter expressing the strongest demands.[7]

7. **Informational simplicity.** Principles of *informational simplicity* have also been considered. They express in various ways the idea that solutions should only depend on the essential features of each problem, either to facilitate the calculation of the desired outcomes, or to help guarantee that the agents will have a good understanding of it (examples are *contraction independence* of Nash, 1950; *local independence* of Nagahisa, 1991, 1994, and Nagahisa and Suh, 1995 ; see also Diamantaras, 1992). These conditions turn out to have considerable relevance to strategic issues, discussed next.

8. **Implementability.** Finally, we have principles pertaining to the strategic behavior of the agents. *Strategy-proofness* states that it should always be in an agent's best interest to tell the truth about his characteristics, typically his preferences, but also the resources he controls (endowments of physical goods, knowledge of technologies, of likelihood of uncertain events ...) (see Barberà, 1996, for a perspective and Sprumont, 1995a, for a survey). *Implementability* says that there should be a game form such that for each economy, its set of equilibrium outcomes of the induced game coincides with the set of outcomes that the solution would have selected on the basis of truthful information (see Maskin, 1985, Postlewaite, 1985; Moore, 1992, for surveys, and Corchón, 1996, for comprehensive treatments).

It occasionally takes time to discover that a single principle underlies developments in several distinct areas. But once the principle has been recognized and given a general formulation, it can serve as a very useful link across models, providing conceptual unity and common elements of proof techniques.

A striking example illuminating this phenomenon is the *consistency*

[7] In some models, the *monotonicity* and *consistency* principles can actually be seen as "conditional" forms of the *replacement* principle (Thomson, 1995b).

principle just mentioned. The principle, which likely underlies a method of adjudicating conflicting claims suggested in the Talmud, a body of Jewish laws and commentaries that is more than 2,000 years old (O'Neill, 1982; Aumann and Maschler, 1985), made a first explicit appearance in early studies of the bargaining problem (Harsanyi, 1959) and in the theory of coalitional form games with transferable utility (Davis and Maschler, 1965). After a twenty-year lull, researchers returned to it, and its implications have now been very fully explored in a wide variety of areas: apportionment (Balinski and Young, 1982), coalitional form games with transferable utility (Sobolev, 1975; Peleg, 1986; Tadenuma, 1992), bargaining (Lensberg, 1985, 1988), various models of fair allocation (Tadenuma and Thomson, 1991, 1993; Thomson, 1988, 1994b), coalitional form games without transferable utility (Peleg, 1985), quasi-linear cost allocation (Moulin, 1985; Chun, 1986), and bankruptcy and taxation (Young, 1987, 1988; Dagan and Volij, 1994), each time under a different name[8]. In the late 80's, it was recognized as a general principle, and the terminology settled on *consistency*.

It is true that some minimal adaptation of a general principle to each specific domain is usually necessary, so that the principle may give rise to a constellation of specific properties.

> Pursuing the theme of *consistency*, a variety of formulations have been considered depending upon whether (i) the model is discrete, (ii) the decision to be made pertains to utility levels or to physical goods, (iii) all subgroups or only selected ones are allowed to leave (small groups, groups belonging to a family endowed with a particular structure) ...

However, in most cases, this adaptation is a fairly straightforward operation.[9] What is important is to understand the essential logic of, and motivation for, the principle behind its various avatars.

[8]The following names have been used: ("uniformity", "stability", "stability under arbitrary formations of subgroups", the "reduced game property", "bilateral equilibrium", "separability").

[9]For instance, a property such as *strategy-proofness* always takes the same form independently of which model is being considered.

3.2 Too many impossibilities

Turning now to the claim that axiomatic analysis has too often resulted in impossibilities, I find that it too has little merit. First of all, *impossibilities do not invalidate axiomatic analysis: they simply reflect mathematical truths that cannot and should not be ignored. Moreover, an impossibility is often a characterization with one axiom too many, and it is a matter of presentation whether the focus is on the characterization or the impossibility.* If we have the expectation or the hope that a certain list of desirable properties are compatible, but in fact they are not, our conclusion will take the form of an impossibility theorem and of course the tone will be disappointment.

In abstract social choice, this is undoubtedly the message conveyed by Arrow's work and much of the literature that followed it. However, it is now well-understood that the impossibility theorems of Arrovian social choice are mainly due to the analysis being conducted on unstructured domains of alternatives, and to the search being for general methods that satisfy a restrictive independence condition. By focusing on concretely specified models and not insisting on the independence condition, a large number of meaningful positive results have now been uncovered, as we will see in the remaining pages of this essay.

3.3 Too many conflicting recommendations

Concerning the claim that when axiomatic analysis has not led to impossibilities, it has too often produced conflicting recommendations, I will first point out that whenever this has been the case, once again nobody should be blamed for results that may not fulfill our hopes. To the contrary, the axiomatic method should be credited for having led to their discovery, and to have permitted a clarification of the relative merits of *a priori* reasonable solutions. Moreover, *for several important domains, just a few solutions have in fact been identified as being clearly more deserving of our attention than other candidates*, as now illustrated.

1. **Bargaining problems.** I have already noted that in spite of the multiplicity of the solutions that had been proposed for bargaining problems, only three (and natural variants), have come up again and again in the literature. They are Nash's (1950) original solution, the Kalai-Smorodinsky solution, and the egalitarian- equivalence solution.

The Kalai-Smorodinsky solution was introduced and characterized by Kalai and Smorodinsky (1975), and the egalitarian solution was first characterized by Kalai (1977). The Nash solution has usually come up in connection with some independence property, and the Kalai-Smorodinsky and egalitarian solutions when some monotonicity property is required. The other solutions have played a role on rare occasions, or never. Since the egalitarian solution requires interpersonal comparisons of utility, it follows that in contexts where for conceptual or practical reasons such comparisons are deemed unacceptable, we are left with just two principal contenders! (see Roth, 1979, Peters, 1992, Thomson and Lensberg, 1989, Thomson, 1996c, for surveys of this literature).

2. **Coalition form games with transferable utility.** Similarly, a great many solutions have been proposed in the theory of coalitional form games with transferable utility, but one has been derived repeatedly in axiomatic analysis, namely the Shapley value (see Aumann, 1985b, who emphasizes this point). Together with the core and the nucleolus — the latter has been important in recent developments — we only have three solutions on which to focus.

Further relevant criteria to rank them may be existence — recall that non-emptiness of the core is far from being always guaranteed — and *singlevaluedness* — when non-empty, the core often selects mutiple allocations.

3. **Resource allocation.** In the study of allocation of private goods, it is also true that no single solution has always been shown superior to the others, but we can with a large degree of confidence eliminate from contention all but a few. The Walrasian solution has come out of axiomatic analyses on numerous occasions, and the egalitarian-equivalence solution and various selections from it have played an important role in recent literature.

The Walrasian solution has been derived primarily when *informational efficiency* (Hurwicz, 1977; Jordan, 1982), *implementability* (Hurwicz, 1979; Gevers, 1986), or *consistency* (Thomson, 1988; Thomson and Zhou, 1993) properties are imposed. Selections from the egalitarian-equivalence solution (Pazner and Schmeidler, 1978) have emerged from considerations of *monotonicity*, with respect to endowments or technology (Thomson, 1987b; Moulin, 1987), or considerations of *welfare domination* pertaining to simultaneous changes in preferences and populations (Sprumont and Zhou, 1995; Sprumont, 1995c).

The final examples pertain to somewhat narrower domains but for them, an even sharper focus on a small number of solutions and sometimes a single solution, has been obtained.

4. **Auctioning a single indivisible good.** For the allocation of a single indivisible good when monetary transfers are possible, the solution that selects for each economy the allocation at which the winner of the indivisible good is indifferent between his assigned bundle and the common bundle of the losers has come up on several occasions.

Considerations of *consistency* and *population-monotonicity* (Tadenuma and Thomson, 1993, 1995), and of *welfare-domination under preference-replacement* (Thomson 1994c), have led to that selection.[10]

5. **Allocation of a private good when preferences are single-peaked.** For the allocation of a single infinitely divisible good when preferences are single-peaked, the same solution, known as the uniform rule, has come up in virtually all cases.

Whether *strategy-proofness* (Sprumont, 1991, Ching, 1992, 1994a; Barberà and Jackson, 1994), *implementability*, *monotonicity* with respect to *resources* or with respect to *population*, *welfare-domination under preference-replacement*, or *consistency*, are imposed, (Thomson, 1990b, 1994a, 1994b, 1995a, 1997; Dagan, 1996; Moreno, 1995; Klaus, Peters, and Storcken, 1995, 1996), the uniform rule has emerged as the most important solution.

6. **Public choice when preferences are single-peaked.** Finally, for the problem of choosing the level of a public good from an interval when preferences are single-peaked, a family of solutions, the generalized Condorcet solutions, and various subfamilies, have been characterized in several ways.

[10]This is the primary solution for this domain. Virtually all other solutions coincide with it.

Characterizations of these families have been obtained from considerations of *strategy-proofness* (Moulin, 1980, 1984; Ching, 1994b), *consistency* (Moulin, 1984), *population-monotonicity* (Ching and Thomson, 1992), and *welfare-domination under preference-replacement* (Thomson, 1993; Vohra, 1997).

These examples certainly do not guarantee that the same phenomenon will always occur but they do show that for several models, some very useful priorities among solutions are obtained by applying the axiomatic method.

Incidentally, note that if the objective of an axiomatic study were taken to be the characterization of a particular solution, the fact that the solution has been characterized in some earlier work might diminish the interest of the result. On the other hand, if we do not lose sight of the objective of the axiomatic *program*, which I have argued should be to identify as completely as possible which combinations of desirable properties are compatible, and how, then the fact that a certain solution comes up once again in a characterization should be celebrated: this may give us the hope that the class of problems under study has only one reasonable solution, or at least only a few such solutions. When it comes to actually making a choice, a consensus will then be much more likely.

3.4 Too large a domain

A concern that is sometimes expressed is that for axioms to be effective in proofs, the domain of problems under consideration has to be "large", perhaps "too large". Characterizations depend too much on the solutions being defined for a wide range of problems, including problems that are not likely to occur frequently, or even are at the limit of what is plausible. Moreover, crucial steps in proofs are made possible by precisely drawing on these problems that lie at the "boundary of the domain".

This criticism is unjustified. An axiomatic study properly conducted begins with the mathematical specification of the relevant domain for the range of economic situations we have in mind. If the domain has not been specified correctly, then our conclusions will not be useful. It is true that in practice, there is often some flexibility in specifying the domain, and this is why I argued earlier that studying the sensitivity of our conclusions to the choice of domains should be part of our analysis. If we find that particular problems carry much of the burden of the proofs, then it is critical to make sure that they should be included.

> For instance, in the study of resource allocation, we often include economies with an arbitrarily large degree of substitutability between goods or an arbitrarily large degree of complementarity (linear preferences and Leontieff preferences). Moreover, these preferences are often used in proofs. If in the particular class of situations that we have in mind, natural (upper or lower) bounds on degrees of substitutability between goods do exist, then of course these bounds should be imposed. There are however interesting situations where no such bounds exist, where for instance certain goods may truly be undistinguishable, so that allowing for perfect substitutability is indeed quite legitimate. Then the domain should include these preferences, and there is nothing wrong if they appear in proofs.

We often start working with a standard domain, not knowing how much of a role its size will play, but as results accumulate, we typically gain insights into the issue. For certain properties, we now have a very good understanding of it, an understanding that should be part of our program. The development of the literature on *strategy-proofness* illustrates well how concerns about largeness of domains can be completely alleviated as a field evolves.

> The following history of the literature concerning *strategy-proofness* should illustrate the point. (i) The first studies pertained to abstract domains of Arrovian social choice, in which the set of alternatives is unstructured and preferences are unrestricted. The central result of that literature, the Gibbard (1973)-Satterthwaite (1975) theorem, essentially states that on such a domain, a solution can be *strategy-proof* only if it dictatorial. The question was then whether this theorem, proved on such a large domain, had any relevance to concretely specified economic models, models in which the set of alternatives is equipped with certain mathematical structures and preferences are correspondingly restricted.
>
> (ii) Major progress on this issue was achieved in the early 90's by Barberá and Peleg (1990) who derived the dictatorship conclusion for a model in which the space of alternatives is endowed with a topological structure and preferences are required to be continuous. However, they imposed no convexity assumption on preferences. Moreover, in their proofs, they

used preferences having several local maxima. This kind of preferences are usually excluded from our economic models.

(iii) Zhou (1991) imposed all of the classical assumptions and still derived the dictatorship conclusion: on domains of preferences of the kind typically considered in our microeconomic textbooks, dictatorship cannot be escaped.

(iv) Schummer (1997) imposed additional qualitative restrictions on preferences, such as homotheticity and even linearity, and he showed that the dictatorship conclusion still holds for such narrow domains.

(v) Moreover, and this must be close to the end of this journey, in the case of linear preferences, Schummer (1997) was able to exactly calculate how large the number of possible preferences had to be to force dictatorship. Remarkably, *only four* preferences suffice.

For economies with public goods and economies with indivisible goods, Schummer (1996a, 1996b) has similarly shown that extremely narrow classes of problems lead to dictatorship.

After the initial results of Gibbard and Satterthwaite, we could entertain doubts about the relevance of their conclusion to concretely specified models of resource allocation. Thanks to these recent developments, we now know that dictatorship is essentially inescapable.

The attention that has been lavished on *strategy-proofness* is unequalled however. For other properties, and other classes of problems, we often do not know how sensitive to largeness of domains our conclusions are. Such analysis will have to be part of the axiomatic program as it develops further.

There is of course no reason why progress should only be in the direction of successive narrowing of domains. Sometimes, starting from a characterization, we may be curious about how much and in what direction the domain can be widened without losing existence and uniqueness.

For *strategy-proofness*, Alcade and Barberá (1994) have explored this issue in the context of matching theory. So have Barberá, Sonnenschein, and Zhou (1991) in the context of the election of a committee, Ching and Serizawa (1994) in the context of allocation when preferences are single-peaked, and finally Berga and Serizawa (1996) in the context of public decision, again when preferences are single-peaked. In each of these studies, the authors have been able precisely to answer the question whether a characterization obtained on a certain domain would persist when the domain is extended at all. When no extension is possible, the domain is "maximal" for the list of properties that are being investigated.

3.5 Axioms are not descriptive of behavior

An additional criticism often addressed at the axiomatic method is that "people do not behave according to the axioms". Here the issue has to do with the scope of the axiomatic method discussed at length in the next section. Axiomatic studies are not necessarily concerned with behavior, but nothing prevents them from being so concerned. I will in particular discuss the usefulness of the axiomatic method to the study of equilibrium in games. There, the axioms are meant to capture "components" of behavior. For instance, is it reasonable to think that players discard dominated strategies? If yes, we may consider writing this down as one of the axioms that will compose the behavioral portrait of the players.

On the other hand, in the normative analysis of allocation problems, the axioms are not intended to reflect behavior but rather values. In formulating the rules according to which goods will be produced or exchanged, should we care about efficiency? Should we care about how gains made possible by future improvements in technologies should be distributed? These are essentially normative and not descriptive issues.

4 The scope of the axiomatic program

In this section, I discuss the scope of the axiomatic method. I feel that many researchers are unfortunately not aware of its wide relevance, and in fact, think of its being limited to the study of abstract models and of cooperative situations.

4.1 Is the axiomatic method mainly suited to the analysis of abstract models?

Axiomatic studies of the abstract models of social choice, bargaining, and coalitional form games are quite numerous, whereas until recently the number of axiomatic studies of concretely specified classes of resource allocation problems had been rather limited. This may suggest that the axiomatic method is mainly suited to the study of abstract domains. I do not believe so, for the following reasons:

1. *First, enough evidence has accumulated in the last ten years to make a convincing case that the axiomatic method is not only conceptually compatible with concrete formulations but also operationally useful*; it does offer a workable and productive way of analyzing concretely specified economic models. The conceptual apparatus that has been elaborated, the proof techniques that have been developed, and the body of results that have been obtained, together provide what I consider to be compelling evidence in support of this position.

> In addition to the examples used throughout this paper, see Young, 1994, Moulin, 1995, Thomson, 1996b, or Moulin and Thomson, 1997, for surveys of the literature on resource allocation; also see the various references of Section 11.3 concerning strategic analysis.

2. Conversely, and with the possible exception of Arrovian social choice, the impression that *the theory of abstract models had progressed only, or principally, in the axiomatic mode, is greatly mistaken.*

> The historical record is clear: in the theory of bargaining, between Nash's publication of his classic article (1950) and the middle seventies, when the literature underwent a significant revival thanks to Kalai and Smorodinsky (1975) and Kalai (1977), only a handful of axiomatic studies of the bargaining problem appeared.
>
> Similarly, in the theory of coalitional form games with transferable utility, no axiomatization of solutions other than the Shapley value and variants of it was developed in almost thirty years following Shapley's classic 1953 paper. Apart from Sobolev's work (1975) on the prenucleolus (Schmeidler, 1969), work that did not become known in the West for several years,[11] it is only in the early eighties that axiomatic analysis took a preeminent position in that branch of the literature. Then, axiomatic derivations of the central solutions were finally obtained, for the core (Gillies, 1959) and the prekernel (Davis and Maschler, 1965), by Peleg (1986). At that time, characterizations of the Shapley value from new

[11] To this date, there is no published English translation of Sobolev's fundamental characterization of the prenucleolus, although several have been circulated.

perspectives were also discovered (Young, 1985; Hart and Mas-Colell, 1989).

Nash's and Shapley's founding papers did give an axiomatic "tone" to the theory of bargaining and to the theory of coalitional form games with transferable utility respectively,[12] but as the above references indicate, these authors were essentially not followed in their methodology until relatively recently, and in fact quite recently as far as the latter is concerned.

An even more striking example is the theory of coalitional form games without transferable utility. Until the late 1980's, that literature had been entirely non-axiomatic: none of the central solutions, the core, the λ-transfer value (Shapley, 1969), the Harsanyi value (Harsanyi, 1959, 1963), were given axiomatic justifications until twenty or thirty years after they were introduced. These characterizations are due to Peleg (1985) for the core, Aumann (1985a) for the λ-transfer value, and Hart (1985) for the Harsanyi value. Then, other solutions were also discovered in the course of axiomatic analysis — an example here is Kalai and Samet's (1985) egalitarian solution.

4.2 Is the axiomatic method mainly suited to the analysis of cooperative situations?

A common perception is that the axiomatic method is mainly suited to the study of cooperative models. I argue below that this view is mistaken and I devote Section 12 to a discussion of the relevance of the axiomatic method to the study of strategic interaction.

5 On the relevance of the axiomatic method to the study of resource allocation

Here, I discuss the relevance of axiomatic studies of abstract models to the understanding of concrete resource allocation problems.

[12]This may explain the mistaken view about the role played by the axiomatic method in the development of the theory of cooperative games described above, since no game theory textbook goes much beyond these two papers, and most students of the field obtain a flavor of the methodology through the abbreviated treatment that they find there.

Instead of directly analyzing a class $\mathcal{C}$ of resource allocation problems specified with all of their physical details, a standard way of proceeding is to "reduce" them first so as to obtain abstract problems in a class $\mathcal{A}$ that we understand, and then to apply the conclusions derived in the analysis of $\mathcal{A}$.

1. *A first issue in evaluating the legitimacy of this approach is whether each concrete problem in $\mathcal{C}$ is mapped into one of the abstract problems in the class $\mathcal{A}$.* The answer is yes for several important classes.

> Consider the problem of allocating private goods: under standard assumptions on preferences, endowments, and technologies, by taking the image in utility space of the set of feasible allocations (this is the reduction alluded to above), we obtain a problem satisfying the assumptions typically made in the theory of bargaining (non-degeneracy, convexity, compactness, and comprehensiveness).
>
> If coalitions can form and preferences are quasi-linear, we can associate with each economy a coalitional form game with transferable utility (by defining the worth of a coalition to be the maximal aggregate utility the coalition can achieve by redistributing among its members the resources under its control), and in fact this game satisfies the balancedness condition that has been central to the theory of these games (Shapley and Shubik, 1969).
>
> If general preferences are permitted, we end up with problems belonging to one of the classes that are standard in the theory of coalitional form games without transferable utility.

2. *However, whether each resource allocation problem in $\mathcal{C}$ maps to some problem in $\mathcal{A}$ is not sufficient to justify applying the results obtained in the study of $\mathcal{A}$. Since these results pertain to solutions defined on the whole of $\mathcal{A}$, we need to know whether conversely, each of the problems in $\mathcal{A}$ can be derived from some problem in $\mathcal{C}$.* We do have fairly general, and positive, answers to this kind of questions, at least when the class of concrete problems are exchange economies. Unfortunately, for other domains, not much is known.

> Billera (1974) and Billera and Bixby (1973a, 1973b) have shown that if a bargaining problem satisfies the standard conditions mentioned in item 1, then indeed it is the image in utility space of some problem of distribution of private goods satisfying standard assumptions.
>
> Similarly, Shapley and Shubik (1969) have shown that each totally balanced coalitional form game with transferable utility can be derived

from some economy satisfying commonly imposed assumptions. The main restriction in each of these studies has to do with the number of goods, which should be sufficiently large.

Sprumont (1995b) has initiated the investigation of the conditions that a coalitional form game with transferable utility has to satisfy in order to arise from some economy with public goods.

3. *Further, consider a requirement $P_{\mathcal{A}}$ involving pairs of abstract problems, and a requirement $P_{\mathcal{C}}$ involving pairs of concrete problems, such that the images in utility space of two concrete problems satisfying the hypotheses of $P_{\mathcal{C}}$ are two abstract problems satisfying the hypotheses of $P_{\mathcal{A}}$. Suppose that we have been able to determine the implications of $P_{\mathcal{A}}$. We would like to know whether we can deduce from this knowledge the implications of $P_{\mathcal{C}}$. To answer this, we need to know whether the inverse operation to that described in the previous paragraph is possible, namely whether for each pair of problems satisfying the hypotheses of $P_{\mathcal{A}}$, there is a pair of concrete problems satisfying the hypotheses of $P_{\mathcal{C}}$ and whose images in utility space are the two abstract problems.*

This point is somewhat more subtle and the following example might be more illuminating that the general statement. Suppose that the analysis of $\mathcal{A}$ has involved axioms pertaining to pairs of problems. In bargaining theory, an example is when two problems are related by inclusion, a situation to which the requirement of *strong monotonicity* pertains: it says that all agents should weakly gain as a result of an expansion of the feasible set. It is often motivated by reference to an economic situation in which physical resources increase, and the desire to make all agents benefit from such increases. The implications of this requirement are well understood: in the presence of efficiency, only the so-called monotone path solutions are acceptable (Kalai, 1977; Thomson and Myerson, 1980). The issue in applying this result to economies is whether, given two bargaining problems related by inclusion, there exist two economies that differ only in their endowments of resources — the endowment of one should dominate the endowment of the other — and such that their images in utility space coincide with the two bargaining problems.

Slightly more formally, given a pair of problems in $\mathcal{A}$ related by inclusion (a situation to which we would like to apply the axiom of *strong monotonicity*), when are they the images of the two versions of a given problem in $\mathcal{C}$ resulting from two choices of the social endowment, one of which dominates the other, (a situation to which the axiom of *resource-monotonicity* applies)? That this operation be possible is important, but I am not aware of any general study of it. Certainly, we know from our previous discussion that a general positive answer should not be expected.

4. The operation may not always be critical however, for the following reason. In a characterization proof, not all possible problems or pairs of problems are used. Then, *the more limited question that needs to be asked is whether the pairs used in the proof of the characterization can be obtained from pairs of concrete problems satisfying the hypotheses of the axiom.*

In our example, not all pairs related by inclusion are used in deriving a characterization of the class of *strongly monotonic* solutions to the bargaining problem; in fact, much more restricted classes of such pairs are needed.

5. A limitation of the abstract model is that changes in the parameters as described in the hypotheses of an axiom unfortunately may occur not only in the concrete circumstances motivating the condition but also in circumstances that are unrelated to them. The description of the model not being rich enough for the investigator to verify when the motivating situation applies, other situations may be "smuggled in" that were not intended, widening the scope of the condition too much. *To avoid this pitfall, it is important to directly study how a given solution defined on $\mathcal{A}$ and in which one may be interested behaves, when applied to the images of pairs of problems in $\mathcal{C}$.*

For such studies, see Roemer (1986a,b, 1988, 1990, 1996) and Chun and Thomson (1988), who considered which monotonicity and consistency conditions are satisfied by solutions to the bargaining problem when they are used to define solutions to resource allocation problems.

In this regard, it is useful to note however that for a number of properties, as the number of commodities increases, what can be achieved enlarges considerably. In fact, as soon as the number of commodities is equal to two, the behavior of bargaining solutions when applied to economic problems is essentially what it is on abstract domains (Chun and Thomson, 1988). These results show that the one-commodity case is quite special, invalidating many studies that have taken it as the canonical example.

The advantage of working within a concretely specified model is that we can exactly identify the circumstances under which the possibility of an enlargment of the feasible set occurs, and decide case by case how the solution should respond. Altogether, and in the absence of complete answers to some of the questions just raised, it may be safer to work directly with concretely specified resource allocation models rather than abstract problems. The numerous references that I have given to recent studies of such models were intended to show that this position is not only methodologically sound but also operationally productive.

6 On the relevance of the axiomatic method to the study of strategic interaction

In this section, I discuss the possible applications of the axiomatic method to strategic models.

6.1 The conceptually flawed opposition between axiomatic game theory and non-cooperative game theory

As a preface to this discussion, I will clarify what I perceive to be a frequent misunderstanding pertaining to the traditional division of game theory into its "cooperative" and "non-cooperative" branches. The former is thought of by many as the natural domain of application of the axiomatic method, and it is often referred to as "axiomatic game theory", non-cooperative games being the domain of "strategic" analysis. For instance, the axiomatic theory of bargaining is commonly opposed to its non-cooperative counterpart: axiomatic game theory is understood to be normative, that is, its objective is to recommend normatively appealing compromises; by contrast, non-cooperative game theory is supposed to be descriptive of the way a group of agents, each of them intent on promoting his own interest, would solve conflicts without outside interference.

My first observation is that *this opposition between the axiomatic approach and the non-cooperative approach is conceptually flawed.* Indeed the term "axiomatic" refers to the *methodology* of the investigator, who is outside of the game, and the term "non-cooperative" to the *behavior* of the agents involved in the game. Moreover, as I will discuss later, nothing prevents the axiomatic method to be applied to the study of non-cooperative games, and in fact I will close this essay by urging that more efforts be made in this direction.

Here, I will suggest instead that it is more useful to distinguish between modes of analysis on the basis of the degree of concreteness with which we define the problems that we consider. It is this distinction that motivates the following sections.

6.2 Are abstract models of game theory more general, or less general, than concrete models?

Abstract models have been criticized for not providing adequate representations of the richness of actual conflicts. But they have also been praised for allowing a wider coverage: by discarding information about the concrete details of actual problems, we can handle within a single theory a much broader class of situations. Which viewpoint is the correct one?

1. In support of the first position, note that a game tree can be "collapsed" into a normal form game by ignoring all information about the tree structure and retaining only strategies and their associated payoffs, and a normal form game can in turn be collapsed into an abstract problem by ignoring all strategic information and retaining only the set of feasible payoffs. Therefore any solution defined on a class of abstract problems specified in utility space, can be mapped into a solution on a class of normal form games, and this solution can in turn be mapped into a solution on a class of extensive form games. *The conclusion is therefore mathematically unescapable that a possibly greater class of solutions is available for concretely specified models.*

In support of the second position, I simply note that natural procedures can often be defined for associating with each normal form game an extensive form game, and for associating with each abstract problem a normal form game. Then, a solution to extensive form games can be mapped into a solution to normal form games. Similarly, a solution to normal form games can be mapped into a solution to abstract bargaining problems.

An operation of this latter kind was performed by Nash (1950) who suggested associating with each bargaining problem a certain strategic "game of demands". Another such procedure, a "game of solutions", was developed by van Damme (1986). Starting from a game specified in concrete terms, Stahl (1972) and Rubinstein (1982) have also proposed ways of associating with it a certain strategic game in extensive form, a "game of alternating offers". Gül (1989) and Hart and Mas-Colell (1996) have considered coalitional form games and associated with each such game a sequential bargaining process.

2. *In actual conflicts, agents' actions are constrained in a variety of ways, due to tradition, laws, or historical accidents. It is often argued that it is these constraints that give each problem its specific character, and that without a realistic description of them, there is no hope of understanding how it will be solved.* Although the existence of such constraints cannot be denied, it is also true that a considerable flexibility remains.

Bargaining does not take place according to the rigid scenarios spelled out in our formal studies. The order in which agents move is quite variable; so is the time interval that separates an offer from a counter-offer; and the nature of these offers and counter-offers varies considerably.[13]

Of course, no mathematical model can possibly take into account all of this detail, and a focus on the central aspects of the negotiations is required. This is where the judgment of the modeler comes in, a judgment that only robustness analysis can test. If it is true that alternative modelings of a given bargaining situation essentially all lead to the same outcome, then a justification for the model has been obtained. A model of bargaining should be formulated so as to capture the essential elements of a class of relevant situations. The only way to become convinced of whether modeling has been successful is to perform this robustness analysis.

3. *A counter-argument is that situations where some flexibility seems to exist have been mis-specified.*

If the time at which bargaining has to be concluded is flexible, and is actually under the control of the players, then this flexibility should be incorporated into the analysis. If medical benefits may be part of the

[13] See Perry and Reny (1994), for an analysis where some flexibility is modeled.

negotiations, the choice of the players to bring up this issue should also be put into the model. The possibilities of throwing away utility, being represented by a third party, extending the scope of negotiation to new issues, calling in an arbitrator, setting the agenda, ..., can all in principle be incorporated in the game form or the tree, strengthening the argument that there is never any need to consider anything more than the actual game form or the tree.

This argument is formally correct, but it actually begs the issue: until we understand well how these various changes in the game form or the tree affects the outcome, it is sterile to claim that only exactly specified game forms or trees should be analyzed. A successful negotiator is not one who only understands whatever explicit rules are given but rather one who knows how to manipulate the rules, that is, understands what "the implicit game".

Political scientists, who have had to be concerned with procedures more than economists, have contributed importantly to the understanding of how they affect the outcome of games.

In some contexts, it has been shown that an appropriate choice of agenda could lead to any point in policy space (McKelvey, 1976).

4. In order to be effective, the axiomatic method typically requires that the domain be "large enough," whereas players engaged in a particular conflict situation need not be concerned about other conflict situations. And indeed, why should they be? The answer to this very legitimate question is twofold: first, it is hard to imagine a player selecting a strategy in the particular game that he is facing today without drawing on his experience in previous situations of the same kind, and attempting to formulate general rules as to how he should play similar games in the future. Minimally, he has to speculate about what his opponent(s) will do, so that his thinking should cover at least two game situations, not just one. Altogether, rationality on the part of a player requires that he develops some theory of how to play games that extends beyond the particular game that he is currently playing.

Second, as analysts, and *even if the players are assumed to play only one game, we will feel confident about our conclusions only when we have understood how the solution that we are proposing behaves on more than one game.* Our theory can only gain strength by being tested on a whole class of games.

When it comes to the recommendations that a judge or arbitrator should make, the need for a general procedure is also quite clear. Consider for instance the problem of dividing the liquidation value of a firm, say 12, between two claimants with claims 8 and 10. Without a general procedure for solving such bankruptcy problems, what should one think of the awards of 5 to claimant 1 and 7 to claimant 2? It is virtually impossible to evaluate such a recommendation in isolation, but by bringing within the scope of the exercise other situations of the same kind, one can begin to form an opinion. For instance, it is easier to evaluate both the above recommendation *and* the awards of 5 to the first claimant and 8 to the second claimant when the liquidation value is 13, when these two situations are considered together. More generally, by extending the class of problems to be solved, we will be better able to decide what to do for each of them.

I also believe that the parties involved are much more likely to accept the decision of the bankruptcy judge if he provides reasons for his decision. Such reasons are most likely to refer to other similar situations.

6.3 Early achievements of the axiomatic method applied to strategic models

It is obvious that there is no intrinsic reason why abstract models should be analyzed only axiomatically, and conversely, as I have attempted to show, the axiomatic method has been profitably applied to concrete classes of resource allocation problems. I will now argue that *there is also no reason why strategic interaction should not be studied axiomatically. A number of axiomatic studies of strategic models have recently been conducted, and they amply demonstrate the relevance and the usefulness of the approach.* Given the proliferation of solutions for strategic models that has occurred recently (van Damme, 1991), the axiomatic method might in fact be quite welcome in sorting them out. I now give a list of contributions that are particularly significant in this regard.

1. Harsanyi and Selten (1988) is a primary illustration. They consider normal form games and formulate a variety of conditions on solutions, such as the basic *invariance with respect to isomorphisms*, which says that two games that are the same up to a linear transformation of utilities and renaming of agents should be solved in the same way up to that transformation; the self-explanatory *invariance with respect to payoff transformations that*

preserve the best reply structure; *payoff monotonicity*, which says that if a pure strategy combination is chosen for some game and the payoff function is changed by increasing the payoffs at that strategy combination, then it should still be chosen for the new game; *cell consistency*, which says that the solution outcome of a game should agree with the solution outcomes of its cells; *truncation consistency*, which says that the solution outcomes of a truncated game should agree with the solution outcomes of the non-truncated game. Other axioms are *invariance with respect to sequential agent splitting*, *partial invariance with respect to inferior choices*, *partial invariance with respect to duplicates*. Harsanyi and Selten establish a large number of compatibility and incompatibility theorems. A related contribution is by Selten (1995).

2. Abreu and Pierce (1984) consider extensive form games and investigate the existence of solutions satisfying the following three axioms. (i) *Normal form dependence*: two games having the same normal form are solved in the same way. (ii) *Dominance*: no dominated strategy is part of any solution outcome, and if $\hat{T}$ is obtained from T by eliminating a dominated choice, then the solution outcomes of $\hat{T}$ are the projections of the solution outcomes of T on $\hat{T}$. (iii) *Subgame replacement*: replacing a subgame which has a unique equilibrium outcome in pure strategies by the corresponding payoffs, gives a game whose solution outcomes are the restriction of the solution outcomes of the initial game on the new game. They show that no solution satisfies both *normal form dependence* and *subgame replacement*, and that no solution satisfies *dominance*.

3. Kohlberg and Mertens (1986) consider sequential games and formulate several requirements on a solution for such games: (i) *existence*, (ii) *connectedness*, (iii) *backwards induction*, (iv) *invariance*, the requirement that two games with the same reduced normal form should be solved in the same way, (v) *admissibility*, and (vi) *iterated dominance*. See also Mertens (1989, 1991, 1992).

4. Bernheim (1988) considers normal form games and formulates a number of axioms pertaining to each player's choice of an action to maximize his payoff subject to probability beliefs about his opponent's choices, under the assumption that players do not assign positive probability to choices of the other players that are judged "irrational". Under these assumptions, there remain the issues (i) *whether priors are common or not*, and (ii) *whether the choices of the other players are perceived as independent random events or not*. The four combinations of the two axioms and their two negations characterize four equilibrium concepts, iterated dominance, correlated equilibrium,

rationalizationability, and Nash equilibrium. Also, see Brandenburger and Dekel (1987), and de Wolf and Forges (1995, 1996).

5. Peleg and Tijs (1996) derive most of the familiar equilibrium notions for games in strategic forms from considerations of *consistency* and various notions of *converse consistency*.[14,15] Additional axiomatic derivations of Nash equilibrium along these lines are obtained by Peleg, Potters, and Tijs (1994), Peleg and Südholter (1994), Norde, Potters, Reijnierse, Vermeulen (1993), and Shinotsuka (1994).

6. Jackson and Srivastava (1996) identify a general property of solutions (a property they call "direct breaking") that guarantees a certain kind of implementability.

7. Kaneko (1994) provides an axiomatic characterization of Nash equilibrium on the basis of epistemic considerations.

8. Peters and Vrieze (1994) derive a selection from the subset of the convex hull of the set of Nash equilibrium payoffs by translating the axioms used by Nash in deriving his solution to the bargaining problem in terms of the data entering the definition of normal form games.

9. Samet (1996) gives an axiomatization of operators describing the way agents formulate hypotheses about the way a game will be played.

10. Tan and Werlang (1988), Basu (1990), Salonen (1992), Ben-Porath and Dekel (1992), Börgers and Samuelson (1992), Tedeschi (1995), and Kaneko and Mao (1996) are other studies in which the axiomatic method is used, explicitly or implicitly.

6.4 On the interplay between the axiomatic and non-axiomatic modes of analysis

Instead of pitting the axiomatic approach to the study of conflict situations against non-axiomatic approaches, or abstract models against concrete models, a multifaceted approach seems the most promising. The merits of such an

[14] In this context, *consistency* says that if a strategy profile is selected by a solution for a game G, then in the "reduced game" obtained from G by imagining some of the agents playing their assigned components of the profile, and appropriately redefining the payoff function, the solution would still select the restriction of the original profile to the remaining agents. *Converse consistency* pertains to the opposite operation.

[15] When a strategy profile is such that its restrictions to subgroups of players are chosen by the solution for the associated reduced games, then it is selected by the solution for the large game.

approach were certainly recognized by the founders of game theory. Nowadays, it is true however that game theorists have often fallen victims to the need for specialization that in the last two decades may have been a necessary accompaniement of the considerable expansion of the field. I will therefore conclude with further illustrations of the useful role that the axiomatic method can play in the study of strategic interaction.

6.4.1 The axiomatic and non-axiomatic approaches applied to game theoretic models have sometimes met in surprising and illuminating ways

In several interesting situations, axiomatic and non-axiomatic approaches have led to the same, or closely related conclusions. In such cases, each approach lends support to the other. I will give three illustrations, already mentioned earlier, taken from the theory of bargaining.

1. The first illustration is of course Nash's own work. Nash (1950) gives an axiomatic characterization of the Nash bargaining solution. In (1953), he also shows that the equilibria of a certain strategic game superimposed on his abstract model — in this game, strategies are utility levels — produce the very same outcomes.

2. Van Damme (1986) formulates a different game, in which players' demands have to be justified as resulting from the application of well-behaved bargaining solutions to the problem at hand, but its equilibria also lead to the Nash outcomes.

3. Finally, Stahl (1972) and Rubinstein (1982) reformulate the process of bargaining by incorporating temporal elements in the negotiations. Their strategic game of alternating offers generates equilibrium outcomes that also coincide with the Nash outcome under an appropriate limit argument.

6.4.2 Axiomatic analysis provides the basis for understanding why different approaches may lead to the same outcomes

Next I would like to suggest that axiomatic analysis can go further and sometimes offer general results providing the basis for the understanding of why different approaches lead to the same conclusions.

Consider the following theorem, which is a variant of a result due to Hurwicz (1979): if a correspondence defined on some standard class of exchange economies (i) always selects Pareto-optimal and individually rational allocations, and (ii) in cases where the initial allocation is Pareto-optimal, selects all individually rational allocations, and finally, (iii) is *Maskin-monotonic*,[16] then it contains the Walrasian solution.

This result teaches us a very general lesson about games. Indeed, since the (Nash) equilibrium correspondence of any game is necessarily *Maskin-monotonic*, and often satisfies the first two conditions of the theorem, then for a large class of games, (games defined on classes of exchange economies), their equilibrium correspondences always include the Walrasian solution, a rather remarkable fact.

Recently, a number of authors have explicitly searched for principles underlying general results pertaining to strategic interaction. The potential of this approach is beautifully illustrated by the success that it has met in connection with *consistency*, a condition already discussed here on a number of occasions.

1. Krishna and Serrano (1996) demonstrate how a strategic interpretation of the *consistency* condition shown by Lensberg (1988) to characterize the Nash bargaining solution in the context of a model with a variable population, would lead to the Nash solution. In his studies of non-cooperative models of bargaining and bankruptcy, Sonn (1994) finds the *monotonicity* and *consistency* conditions developed in the axiomatic theory of bargaining to be central to the derivation of the equilibrium equations. In a series of contributions, Serrano (1993, 1995a, 1995b) uses similar arguments to derive the nucleolus, the core, and the kernel.

2. Hart and Mas-Colell (1996) consider a non-cooperative bargaining process for coalitional form games without transferable utility and identify a particular solution which is also the one that comes out of axiomatic considerations. Here too, *consistency* plays an important role.

3. I have already discussed the characterizations of solutions to games in strategic form obtained by Peleg and Tijs (1996). These results are based on the application of notions of *consistency* and *converse consistency*, which until then had been exclusively seen from the normative

[16] This says that if an allocation is chosen for some profile of preferences and preferences change in such a way that the allocation does not fall in anybody's preferences, then it is still chosen for the new profile.

angle. For other contributions on the subject, see Peleg, Potters, and Tijs (1994), Peleg and Südholter (1994), and Shinotsuka (1994).

4. Dagan, Serrano, and Volij (1993) consider a strategic game for bankruptcy problems and exploit *consistency* in order to characterize its equilibria.

5. Moldovanu (1990) similarly identify the equibria of a game of offers in a model of assignment by drawing on the *consistency* of a certain solution.

6.4.3 The axiomatic method sometimes usefully complements strategic analysis

One of the central results in the theory of repeated games is the so-called "folk theorem," which states that any outcome Pareto-dominating the maximin point can be obtained at equilibrium. Therefore, the predictive power of strategic analysis is sometimes very low. In situations where an equilibrium results from preplay communication, the question arises how players will ever agree on any one equilibrium. Selection of an equilibrium on the basis of normative considerations examined in the axiomatic mode may provide an answer.

6.5 Implementation theory as the domain par excellence of axiomatic analysis

Most importantly perhaps, and if one of our goals as social scientists is not only to understand the way conflicts are solved in the world, but also to discover and promote methods of conflict resolution that are more likely to result in good outcomes, the rules of the game should be an object of choice. Implementation theory is concerned with identifying which social objectives are realistically achievable in the face of strategic behavior of the agents. This field is among those that have benefitted the most from axiomatic analysis.

Indeed, the axiomatic method has assisted at all levels, in the determination of (i) which normatively appealing social objectives are compatible, (ii) which equilibrium concepts are appropriate in the analysis of the games to which agents will be confronted (Jackson and Srivastava, 1996), and (iii) which social objectives can be implemented with respect to each chosen equilibrium concept. (iv) More recently, much attention has been devoted to the characterization of which solutions can be implemented by means of games satisfying additional properties of interest, mainly intended to permit simplicity of the procedure; here too, the approach has been mainly axiomatic, with the axioms capturing notions of computational simplicity (Dutta, Sen, and Vohra, 1995; Saijo, Tatamitami, and Yamato, 1993; Sjöström, 1996).

7 Conclusion

In this essay, I have described the axiomatic method and attempted to refute arguments against it. I have also presented recent accomplishments, focusing on resource allocation in concretely specified economic models. I hope that these recent successes will motivate applications to yet other areas.

8 Appendix

This appendix contains short descriptions of the various models most often used as illustrations in the main body of the paper.

(1) A **bargaining problem** is a pair (B, d) of a non-empty, convex and compact subset of $\mathbb{R}^n_+$ and a point d in B. The set B is interpreted as a set of utility vectors attainable by the n agents if they reach a consensus on it, and d is interpreted as the alternative that will occur if they fail to reach any compromise. Let $\mathcal{B}^n$ denote the class of all such problems.

(2) A **transferable utility game in coalitional form** is a vector v in $\mathbb{R}^{2^n-1}$. The coordinates of v are indexed by the non-empty subsets of the set of players. A coordinate is interpreted as the amount of "collective utility" that the members of the corresponding coalition can obtain. Let $\mathcal{U}^n$ denote the class of these problems.

(3) A **normal form game** is a pair (S, h) where $S = S_1 \times \ldots \times S_n$ and $h: S \to \mathbb{R}^n$. For each player i, S_i is a set of actions that he may take, and the function h gives the payoffs received by all the players for each profile of actions. Let $\mathcal{G}^n$ denote the class of all such games.

(4) An **extensive form game** is a tree T, where each non-terminal node bears as label an element of $\{1, \ldots, n\}$, and each terminal node bears as label a point in $\mathbb{R}^n$. As compared to the previous class of games, a sequential structure is added to the set of actions, and the nodes indicate times at which agents choose actions. Let $\mathcal{E}^n$ denote the class of all such trees.

(5) An **exchange economy** is a list $(R_1, \ldots, R_n, \omega_1, \ldots, \omega_n)$ where each R_i is a continuous and monotonic preference relation defined on $\mathbb{R}^\ell_+$, and $\omega_i \in \mathbb{R}^\ell_+$ is agent i's endowment. The integer ℓ is the number of commodities. Let $\mathcal{H}^n$ denote the class of all such economies.

(6) An **economy with single-peaked preferences** is a list $(R_1, \ldots, R_n, \Omega)$ where R_i is a single-peaked preference relation defined over the non-negative reals. The number Ω gives the amount of a non-disposable good to be divided among the n agents. Let $\mathcal{S}^n$ denote the class of all such economies.

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