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Ski-Lift Pricing, with an Application to the Labor Market

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Abstract

The market for ski runs or amusement rides often features admission tickets with no explicit price per ride. Therefore, the equilibrium involves queues, which are systematically longer during peak periods, such as weekends. Moreover, the prices of admission tickets are much less responsive than the length of queues to variations in demand, even when these variations are predictable. Despite the queues and sticky prices, we show that the outcomes are nearly efficient under plausible conditions. Then we show that similar results obtain for some familiar congestion problems and for profit-sharing schemes in the labor market.

During Christmas or Spring vacation, most ski areas have long lines. The same is true for Disneyland and other amusement parks in peak season. This type of crowding does not depend on surprises in demand, but instead is systematic. Most economists look at chronic queuing and conjecture that the suppliers would do better by raising prices. Further, most economists would argue that the failure to price properly leads to inefficient allocations of rides, as well as improper investment decisions. But the regular occurrence of lines in some markets suggests that it is economists, rather than suppliers (who have survived), who are missing something.

We argue in this paper that competitive suppliers of ski-lift services (amusement rides, etc.) may rationally set prices so that queues occur regularly and are longer at peak times. Under plausible assumptions, this method of pricing can support efficient allocative decisions. In equilibrium, owners of ski areas set prices for all-day lift tickets (or equivalently, for admission tickets to amusement parks) by maximizing profits subject to a downward-sloping demand curve. This appearance of monopoly power leads to no inefficiency. Moreover, the equilibrium price charged for a lift ticket may not rise with expansions of demand. Sticky prices may be consistent with optimization by suppliers and with efficient choices of quantities.

There are two distinct effects in operation under lift-ticket pricing, either of which is sufficient to imply that setting the marginal cost of a ride equal to zero does not lead to distortions. The first effect, which we call the package-deal effect, arises under any two-part pricing scheme with quantity constraints. Someone who buys 10 units of a good at $1 each from a local shopkeeper will be indifferent between standard pricing at $1 per unit and a two-part pricing scheme with a $10 entry fee, a per unit price equal to zero, and a limit of 10 units per customer.[1]

The second effect, which we label the homogeneity effect, describes conditions under which congestion leads to no efficiency losses. The usual argument, dating back to the two-roads problem of Knight (1924), is that free entry into one of two activities equates average rather than marginal returns, and thereby leads to welfare losses. This conclusion does not follow if the same quantity equates average and marginal returns. In the examples we consider, this coincidence arises under natural assumptions about how congestion affects individual utility. For example, suppose that total output in activity i takes the form $x_i f(n_i)$, where x_i is a measure of capacity or desirability and n_i is the number of people who participate. For activities 1 and 2, the values of n_1 and n_2 that equate the average product, $x_1 f(n_1)/n_1 = x_2 f(n_2)/n_2$, also equate the marginal product, $x_1 f'(n_1) = x_2 f'(n_2)$, if $f(n_i)$ is homogeneous of some degree. The same holds true if the dependence on x and n takes the form g(x,n) where g is homogeneous of degree 1.

In the case of ski area or amusement park pricing, we make an additional, subsidiary point. Queues may have an effect on the allocation of resources that has nothing to do with the cost of time. The package-deal effect applies only if there are quantity constraints. Ski areas and amusement parks place no explicit constraints on the quantities that each person can consume, but the queues impose an implicit constraint. If there are x total

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[1]This point is familiar in the context of the labor market. See Barro (1977) and Hall (1980).

rides available and n people show up to consume these rides, the queue may act purely as a symmetric allocation mechanism to provide x/n rides to each person. To emphasize this alternative view of the role played by queues, we make the extreme assumption that time spent at a ski area or amusement park is inherently valuable so that the cost of time spent in the queue is approximately zero. This assumption may be inappropriate for other settings, but it is inessential for the operation of either of the two basic effects. The homogeneity effect does not depend on any quantity constraints; the package-deal effect does require them, but these constraints can arise in ways other than queues.

Because ski-lift pricing illustrates both effects as well as the subtle fashion in which quantity constraints can arise, we start with this example and analyze it in detail. Section 2 considers the simplest case, where ski areas are identical and consumers differ only in the cost of going skiing. We compare the alternative modes of pricing, with emphasis on the assumption concerning the cost of time. Then we show why lift-ticket prices may not vary with changes in demand. Section 3 considers extensions where individuals differ in preferences, and ski areas differ in characteristics. Section 4 presents applications of the package-deal and homogeneity effects, including the two-roads problem, a common-property fishing problem, and profit-sharing arrangements for employees of a firm.

2. The Supply and Demand for Ski-Lift Services

To highlight the operation of the package-deal effect, we analyze first the equilibrium with conventional pricing for individual rides at a ski area, and then show how this equilibrium can be repackaged into one with an entry fee—the lift ticket—and a price per ride set equal to zero. Section 2.1 describes the market for rides on a ski lift and works out the equilibrium. Section 2.2 illustrates how the quantities and prices from this efficient solution can be replicated in an equilibrium with lift-ticket pricing and queues. Section 2.3 discusses how the results change when we allow for the opportunity cost of time spent in a queue, then considers the factors that might influence the choice between the alternative pricing arrangements. Section 2.4 illustrates why lift-ticket prices may not respond to changes in demand. The analysis in section 2 assumes that ski areas are identical and that individuals differ only in a limited sense. In this context only the package-deal effect is present. Section 3 discusses the homogeneity effect, which arises when we allow for differences among individuals and ski areas.

2.1 Equilibrium with Ride Tickets

Consider a group of identical, competitive ski-lift operators, each of whom sells ride tickets at a price P per ride. Each firm has a fixed capacity and therefore supplies inelastically the total quantity of rides x. Flexibility in this quantity at some positive marginal cost is more realistic, since suppliers can open more lift lines or perhaps operate the existing ones at greater speed. But these modifications do not change our results. In the present case the industry's total supply of rides is Jx, where J is the fixed number of firms.

Let q_i be the quantity of ski rides for the i^{th} person. This individual chooses q_i to maximize

$$U^i = U^i(q_i, z_i)$$

subject to $$Y_i = Pq_i + z_i + c_i,$$

where Y_i is real income, z_i is goods other than skiing (price normalized to one), and c_i is an individual specific, lump-sum cost of going skiing. This cost is quasi fixed because it depends only on the decision to go skiing, not on the number of ski runs consumed.

For those who ski ($q_i > 0$), the determination of q_i can be described in the usual way by a downward-sloping, income-compensated demand curve, $q_i = D^i(P)$. In this section we neglect variations across individuals in these demand functions (contingent on participation in skiing)—that is, we assume $q_i = D(P)$. A later section allows for heterogeneity of demands, which could reflect differences across the population of skiers in tastes for skiing or in net incomes, $Y_i - c_i$. Given the income-compensated demand, $D(P)$, we can calculate a monetary measure of the gain from skiing using the area under the compensated demand curve, $\phi(q) = \int_0^q D^{-1}(\tilde{q})d\tilde{q}$. This gain is the most that an individual would be willing to pay for the opportunity to ski q runs.

Individual i chooses to ski if the fixed cost, c_i, plus the explicit cost, $PD(P)$, is less than the gain from skiing, $\phi[D(P)]$. Our analysis allows the cost, c_i, to differ across persons and over time. On a given day, the cumulative distribution of the c_i's is described by F_s, so that the fraction of persons who choose to ski is $F_s\{\phi[D(P)] - PD(P)\}$. Since $\phi[D(P)] - PD(P)$ is decreasing in P, the number of skiers falls if P rises. The shift parameter s represents changes over time in the distribution of the c_i. For example, during weekends and vacation periods, the costs of going skiing for the typical person are relatively low, so the number who ski is relatively high. Overall, we can write the number of persons N who choose to ski as the function,

$$(1) \quad N = N(P,s),$$

where $\frac{\partial N}{\partial P} < 0$.

By specifying that ski areas are competitive, we mean that each is small enough that its actions have a negligible impact on aggregate quantities. In this model (though not in those that follow) competitive behavior implies that firms take prices as given. Equilibrium requires that the total capacity of rides, Jx, equal the total number demanded, qN—that is,

$$(2) \quad Jx = D(P)\bullet N(P,s).$$

For a given value of Jx, this condition determines the equilibrium price per ride P. As one would expect, the price P falls with an increase in total capacity, Jx, and rises with an increase in the level of demand such as that generated from a downward shift in the fixed costs, c_i.

Over the longer term the model also determines the size of the industry, Jx. This scale depends on the cost of building new capacity (either more firms J or more rides per firm x) and on the distribution of returns, as determined by equation (2) and the distribution function of the shift parameter s.

2.2 Equilibrium with Lift Tickets

We now show how the equilibrium described above can be implemented using an entry fee (i.e. an all-day lift ticket) and a price per ride set equal to zero. Let π_j denote the price of a lift ticket at area j, and let n_j be the number of skiers who ski there. Given the total capacity x, the maximum number of rides per skier will be $q_j = x/n_j$. In equilibrium each person will desire a greater number of rides than x/n_j at the zero marginal cost implied by lift-ticket pricing. Hence there is no problem in getting the customers to accept the quantity of rides available. In fact, people will queue up to receive the rides.

Each individual cares about the outlay on skiing, $c_i + \pi_j$, and the number of rides available, $q_j = x/n_j$. We assume that people do not care directly about the time spent waiting in lift lines, or about how the rides are distributed throughout the day. They would prefer shorter lift lines because they would prefer more rides; but given a fixed number of rides, they are indifferent between spending time outdoors in line or indoors in the lodge. The only function of the queue is to allocate the fixed number of rides x equally among the n_j skiers.

As noted in the introduction, this extreme assumption about the welfare cost of time spent in lift lines is a useful expository device because it shows that queues may play a role that has nothing to do with the usual arguments about the cost of time. We do not take the welfare implications of this assumption literally. In section 2.3, we discuss departures from this assumption.

Suppose that individual i considers the choice between areas j and k, which offer the respective quantity of rides, $q_j = x/n_j$ and $q_k = x/n_k$. Since the cost c_i of going skiing is the same for each area, the individual will be indifferent between areas j and k if the gain from skiing minus the cost of the lift ticket is the same. Therefore the equilibrium condition for people to be indifferent between areas is

$$(3) \qquad \phi(q_j) - \pi_j = \phi(q_k) - \pi_k.$$

A ski area that is small relative to the total market can choose its lift-ticket price, π_j, but the number of skiers, n_j, adjusts to keep the net surplus, $\phi(x/n_j) - \pi_j$, equal to that offered by other areas. Competitive behavior means that the area takes as given a reservation value for the net surplus, not the price of the lift ticket. This given level of the net surplus implies a downward-sloping number of customers, n_j, as a function of π_j. The nature of the relation between n_j and π_j can be determined by implicit differentiation of the terms on the left side of equation (3). Using $\phi'(x/n_j) = D^{-1}(x/n_j)$, the result can be expressed in terms of the elasticity,

$$(4) \qquad \frac{dn_j}{d\pi_j} \bullet \frac{\pi_j}{n_j} = -\frac{\pi_j}{D^{-1}(x/n_j)\bullet(x/n_j)}.$$

Since an area's costs are fixed and do not depend on n_j, each area seeks to maximize $\pi_j n_j$, taking as given the relation between the ticket price and the number of skiers implied by equation (4). As usual, maximization of revenue requires that the elasticity of n_j with respect to π_j equal -1, so that in equilibrium

$$(5) \qquad \frac{\pi_j}{q_j} = D^{-1}(q_j).$$

The left side of equation (5) is the amount paid per ride under lift-ticket pricing, which we define as the effective price per ride, $\hat{P}_j$:

$$(6) \qquad \hat{P}_j = \pi_j/q_j.$$

From equation (5) it follows that

$$(7) \quad q_j = D(\hat{P}_j).$$

Each person at area j ends up with the quantity of rides q_j that corresponds to the effective price per ride $\hat{P}_j$. Although people wait in line and face an explicit marginal cost for rides of zero, the results are as if each skier gets the quantity of rides that he or she would demand at an explicit market price per ride $\hat{P}_j$. These q_j rides have simply been combined into a package deal with a total cost of $\pi_j = \hat{P}_j q_j$.

Given the reservation value of net surplus, each area chooses its price, π_j, in accordance with equations (3) and (5) (taking account of the condition $q_j = x/n_j$). Since the areas have the same capacity x and are otherwise identical, they end up with the same values for the lift-ticket price, $\pi_j = \pi$, the number of customers, $n_j = N/J$, and the effective price per ride, $\hat{P}_j = \hat{P}$.

To complete the description of the equilibrium, it remains to determine the value of the common lift-ticket price, π, or equivalently of the effective price per ride $\hat{P}$. We can analyze individuals' decisions to incur the fixed cost to go skiing just as in the first model, except that the effective price per ride $\hat{P}$ now replaces the explicit price P. (Recall from equation (7) that people end up with the quantity of rides that they would demand at the effective price $\hat{P}$.) The analogue to equation (2) is now

$$(8) \quad Jx = D(\hat{P}) \bullet N(\hat{P}, s)$$

Because this condition is the same as the one that determined the price per ride P in the ride ticket equilibrium, the effective price $\hat{P}$ takes on the same value. Finally, equations (6) and (7) imply that the common lift-ticket price is determined by the effective price per ride,

$$(9) \quad \pi = \hat{P}q = \hat{P}\bullet D(\hat{P}).$$

Since the equilibrium with lift-ticket pricing yields an effective price per ride $\hat{P}$ equal to the explicit price per ride P in the first equilibrium, skiers receive the same number of rides at the same cost in each case. The same people end up participating, and each ski area receives the same revenue.

The equality of π_j and n_j across areas arises here because all ski areas and individuals are identical. Section 3 shows that π_j and n_j can vary across areas if there are differences in skiers' preferences or in the characteristics of ski areas. What generalizes is the result that lift-ticket pricing can replicate the quantities and marginal valuations generated under ride-ticket pricing. The lift-ticket equilibrium bundles the number of rides per person from the ride-ticket equilibrium into a single package that is sold at a price equal to the number of rides times the price per ride.

In the longer run context where the capacity Jx is variable, suppliers have the same incentives to invest under the two systems of pricing because the revenue generated by an additional ride corresponds in each case to the skiers' marginal valuation of rides, $D^{-1}(q)$. Thus—given our assumption that people care about the number of rides but not directly about the time spent in line—there are no inefficiencies implied by the existence of queues, which reflect the explicit marginal cost of zero for rides. Allocative decisions are still based on the proper shadow price, $\hat{P} = D^{-1}(q)$.

Although the lift-ticket equilibrium is only a repackaged form of the original competitive equilibrium, the superficial appearances are strikingly different. The lift-ticket solution features quantity rationing by means of queues, as well as ticket prices that seem to be set by firms with market power. Although individual ski areas have no true market power, the demand for lift tickets at each area is the downward-sloping curve $n_j(\pi_j)$ characterized by equation (4), and each area maximizes revenue subject to this curve.

2.3 Ride Tickets versus Lift Tickets

Given the assumptions so far, there is no basis for predicting which of the two forms of pricing will prevail. They lead to identical allocations and effective prices. Ski areas charging on a per ride basis could coexist with others charging on a lift-ticket basis. One can readily verify that an area could also use a combination of a lift ticket (i.e. an entry fee) and a charge per ride.

The description of the world implicit in this model misses important features of reality. For some aspects, such as the determination of the price per ride P or $\hat{P}$, these features may not be important. However, in the choice between two otherwise equivalent pricing schemes, these features may be decisive. The most obvious elements neglected so far are

a) the costs that must be incurred by an area to enforce contracts—for example, to avoid the theft of rides;

b) the differences in rides—they are heterogeneous goods indexed by the time of day and by contingencies such as breakdowns of equipment and arrivals of skiers;

c) the time spent waiting in line, which is likely to have a positive opportunity cost.

We can conjecture what the inclusion of these features would imply. Given the allocation of rides common to the two kinds of equilibria (and to any mixture of these two), the form of pricing that minimizes the neglected costs will be selected. Ride-ticket pricing will generally have higher monitoring and set-up costs than lift-ticket pricing. Since it would be extremely expensive to set up a complete set of markets in time and contingency specific rides and to enforce contracts written in this form, some amount of queuing would be expected even under pure ride pricing. On the other hand, lift-ticket pricing imposes costs in terms of time. Relative to a system with an extensive system of reservations, each individual must spend more time at a ski area to achieve a given allocation of rides. However, if the typical skier's fixed cost, c, for getting to the ski area is large, and if waiting in line is preferred to spending time on other available activities (aside from skiing), then this last element would be relatively unimportant.

Queues also have the advantage that they permit an automatic form of ex post settling-up to operate. Hence transactions can take place before all the relevant contingencies have been realized, without the need for any ex post payments or recontracting. Consider the operation of ride-ticket pricing under the plausible assumption that there is uncertainty about the number of skiers who come to a ski area on a given day. To avoid the costs of repeated purchases of tickets throughout the day, skiers would presumably want to purchase all of their ride tickets for the day when they arrive. But if individuals arrive at a ski area and purchase tickets sequentially, the price for ride tickets offered to the first purchasers of the day will inevitably turn out to be incorrect ex post. For example, if more skiers than expected appear, too few ride tickets will remain for the late arrivals. If the ski area increases the explicit price per ride for late arrivals, early purchasers will have bought at a price that is too low; the marginal value of the last ticket held by an early purchaser will be less than what it could be sold for. Full efficiency would require trades between the early and late arrivals. Under lift-ticket pricing, the price per ride adjusts automatically. When more people show up—i.e. n_j is larger—the effective price per ride, $\hat{P}_j = \pi_j n_j / x$, increases even if π_j is held constant. Section 2.4 shows that in some cases this automatic price change is of exactly the right size.

As far as we know, ski areas use only the lift-ticket form of pricing. Oi (1971) describes how Disneyland once followed a combination form of pricing with an entry fee and a charge per ride. In contrast to the explanation offered here, Oi interprets this scheme as evidence of market power. Disneyland has since shifted to a pure entry fee. We take these observations as evidence that the costs of allocating rides using ride tickets are higher than those using entry fees. Presumably the cost of implementing reservations and collecting ride tickets outweigh the value of the savings in the time required to acquire a given number of rides. This outcome is likely if the lump-sum costs of participating are large, and if time spent at a ski area or amusement park is valued for its own sake.

2.4 Shifts in Demand

The foregoing arguments demonstrate that there may be little or no deadweight loss associated with the use of lift-ticket pricing, rather than ride-ticket pricing. But the results do not yet explain why ticket prices would be "sticky." Over the course of a season, variations in the shift parameter s—such as those reflecting weekends and vacation periods—cause predictable changes in demand. Lift lines vary markedly, as do prices for accommodations, but lift-ticket prices apparently change relatively little. The main variations seem to be discounts during periods of very low demand, such as non-vacation weekdays or the final days of the ski season.

As one would expect, equation (8) implies that the effective price per ride $\hat{P}$ varies in the same direction as the level of demand, with the sensitivity depending inversely on the magnitude of the price elasticity of the overall demand for rides (that is, of $D(\hat{P})\cdot N(\hat{P},s)$). Thus, the effective price per ride is high when the level of demand is high, and vice versa. However, the price π for a lift ticket does not necessarily vary in the same direction as the level of demand. From equation (9), the effective price per ride is $\hat{P} = \pi/q = \pi n/x$. Even with π (and x) fixed, the extra crowding associated with the increase in n (which equals N/J) itself generates a higher effective price per ride. The lift-ticket price π increases when $\hat{P}$ increases only if the associated fall in rides per person, $x/n = D(\hat{P})$, is less than equiproportional. Using equation (9), the effect of a change in $\hat{P}$ on π is

$$(10) \qquad d\pi/d\hat{P} = D(\hat{P})(1 + \eta_{D,\hat{P}}) \gtreqless 0,$$

where $\eta_{D,\hat{P}} < 0$ is the elasticity of rides demanded per person with respect to the effective price per ride. If this elasticity is less than 1 in absolute value, π rises along with $\hat{P}$ and, hence, with the level of demand. But if the elasticity is greater than 1 in absolute value, π falls when $\hat{P}$

increases. Finally, if the elasticity is close to -1, π shows little sensitivity to fluctuations in demand.[2] In this case competitive forces are consistent with nearly constant lift-ticket prices, even though the times of peak demand exhibit lines that are much longer than those during non-peak times.

This result suggests an additional advantage to lift-ticket pricing. If the elasticity of demand for rides per person is close to -1, it is unnecessary to incur the "menu costs" of changing the stated price at a ski area in response to changes in demand. The effective price per ride changes in nearly the right way if the price of lift tickets is held constant. As suggested in the last section, this may be important even over the course of a day. If ticket sales take place sequentially as customers arrive, the cost of changing the ticket price as information on the size of demand accumulates includes not just a menu cost of changing signs, but also the costs of recontracting with previous purchasers.

If demand falls to very low levels, the condition $|\eta_{D,\hat{P}}| < 1$ almost surely applies. Since the consumption of a ski run requires a minimum amount of a skier's time, the demand curve for rides as a function of the price has a finite intercept equal to the maximum number of ski runs that can be taken in a given day. As the effective price of a ski run approaches zero, the elasticity of demand must also approach zero. In this region equation (10) implies $d\pi/d\hat{P} > 0$—that is, the model predicts discounts on lift tickets during the times of greatest slack. However, the model also suggests the

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[2]The same mechanism works for shifts in supply, Jx, although these seem less important in the short run in the present context.

possibility of a substantial interval—such as the comparison between a normal weekend and the peaks during vacation periods—where lift-ticket prices would show little or no variation with demand.

The same mechanism may explain why the explicit prices for goods such as airline tickets and restaurants often do not vary betweeen peak and off-peak periods. At busy times the effective amount of service diminishes because planes and restaurants are more crowded. Thus, the price per effective unit of service rises automatically if the explicit price is held fixed. Under such circumstances, the results with fixed explicit prices may roughly replicate the equilibrium where the price per effective unit of service fluctuates and where customers are free to choose how much service to purchase. In these examples the package-deal effect operates with quantity constraints that are implicit rather than explicit, and the results do not depend on queues per se.

Constant lift-ticket prices work exactly only if the elasticity of the demand for rides per person equals -1. But if the menu costs or the costs of recontracting due to sequential service are large enough to play a decisive role in the choice of the pricing format, a two-part pricing scheme with an entry fee and a price per ride can be implemented to avoid price changes even when the elasticity differs from -1. This consideration does not appear to be relevant for ski areas, where charges per ride do not seem to be used, but may be a factor in the choice of such a scheme by some amusement parks.

Consider an amusement park with capacity x, which charges an entry fee π and a price per ride r. If n people visit the park and the value of r is small (so that π is positive in equilibrium), the quantity of rides per person is $q = x/n$. As before, each park takes as given the net surplus attained by participants, as given now by $\phi(x/n_j) - rx/n_j - \pi$. Setting the total differential of the net surplus to zero and equating the elasticity of n with respect to π to -1 leads to

$$(11) \qquad r + \frac{\pi}{q} = D^{-1}(q),$$

which extends equation (5). The effective price per ride is now[3]

$$(12) \qquad \hat{P} = r + \frac{\pi}{q} = r + \frac{\pi}{D(\hat{P})}.$$

Solving for π gives

$$(13) \qquad \pi = (\hat{P} - r)D(\hat{P}).$$

As before (equation (8)), the equation of total supply to total demand determines $\hat{P}$. Then the reaction of π to changes in $\hat{P}$ follows from equation (13) as

$$(14) \qquad \frac{d\pi}{d\hat{P}} = D(\hat{P})[1 + \frac{\hat{P}-r}{\hat{P}} \eta_{D,\hat{P}}].$$

Consider small fluctuations in demand or supply that induce fluctuations in the effective price $\hat{P}$ around some level $\hat{P}_0$. Let $\eta_{D,\hat{P}}$ be the elasticity of the demand for rides with respect to the effective price. For a given value of $\eta_{D,\hat{P}}$, the price per ride r can be chosen so that $\frac{\hat{P}-r}{\hat{P}} \eta_{D,\hat{P}}$ is equal to -1 when evaluated at $\hat{P} = \hat{P}_0$.[4] Then $\frac{d\pi}{d\hat{P}}$ will be zero when evaluated at $\hat{P}_0$ and

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[3]If $\hat{P} > r$, which we assume, the quantity demanded at the explicit price r exceeds the amount available, $q = x/n$. Therefore, although the explicit price is now positive, the demanders still queue up for the available rides. These queues typically applied at Disneyland even when ride tickets were used.

[4]However, the value of r would be negative if $\eta_{D,\hat{P}}$ were less than 1 in absolute value.

it will be small in a neighborhood of $\hat{P}_0$. For small fluctuations in demand or supply, an equilibrium with constant prices r and π will be approximately equivalent to the conventional equilibrium with no entry fee and a fluctuating price P per ride.

3. Elaborations of the Model

In Section 2, ski areas were identical and individuals differed only in terms of the fixed cost c_i; conditional on participation, they too were identical. In this section we illustrate how the previous results change when there are differences in characteristics of ski areas and in individuals' preferences for ski runs. Differences among ski areas lead to results that complement those above about sticky prices. The conditions that cause lift-ticket prices to be invariant with demand also cause these prices to be the same at areas with different characteristics. By considering two distinct kinds of differences across areas, we are also able to illustrate more clearly the separate roles of the homogeneity effect and the package-deal effect.

3.1. Differences in Qualities of Ski Areas

The lift-ticket equilibrium derived above does not require separate tickets at each area. Suppose that the ski operators set a single entry fee, which equals the common lift-ticket price. Then skiers will sort themselves so that each area has the same number of skiers. This sorting sets the average return to attendance at each area to a single value, and simultaneously sets the marginal value of an additional ski run at each area to a different common value. Since the areas are identical, there is no conflict between equating marginal and average quantities. In terms of the analogy with the classical two-roads problem, if the roads are identical and individuals are free to choose between them, the number of vehicles on each will be the same and there will be no efficiency loss from misallocation of cars between the roads. Identical roads or ski areas is a special case, but this section shows that this result generalizes to allow for at least one kind of difference across areas.

For simplicity we now suppress the participation decision and consider a pool of N identical skiers who have decided to go skiing. Skiers choose among J areas, which now have different effective capacities x_j. These differences could reflect variations in lift capacities or in lengths of ski runs.

As in section 2, we can derive the equilibrium for this extended model under lift-ticket pricing. All areas charge the same lift-ticket price, but the number of skiers n_j varies one for one with the capacity x_j. Each skier receives the same amount of skiing, x_j/n_j, and this amount coincides with the quantity that each would receive if the operators charged an explicit price per unit of skiing. Moreover, the results would be the same if the operators levied a single entry fee for skiing and allowed the skiers to allocate themselves among the areas.

To reconcile this result with intuition about congestion costs, consider the analogues to average and marginal costs. A single lift ticket combined with free movement among areas means that the surplus per person, $\phi(x_j/n_j)$, and hence the amount of skiing per person, x_j/n_j, are the same at all areas. On the other hand, a social planner who allocates skiers across areas would seek to maximize the total gain from skiing, $\sum n_j\phi(x_j/n_j)$, subject to $\sum n_j = N$. The first-order condition for this problem is that the expression, $\phi(x_j/n_j) - (x_j/n_j)\phi'(x_j/n_j)$, be the same at all areas. This condition also implies that the amount of skiing per person is the same at all areas. Hence the allocation of skiers coincides with the one chosen privately. Since $n\phi(x/n)$ is homogeneous of degree 1 in x and n, the finding is a special case of the result noted in the introduction; if g(x,n) is homogeneous of degree 1, then equating the average product, g/n, leads to the same answer as equating the marginal product, $\frac{\partial g}{\partial n}$.

This argument applies also within a given ski area or amusement park. Ski areas may not have to charge different prices for runs of different lengths or qualities, and amusement parks may not have to charge different prices for rides with different durations or levels of excitement. If a ride on a roller coaster is x times more satisfying than one on a bumper car, lines at the two activities will adjust so that each person can consume x times as many rides per day on a bumper car as compared to a roller coaster.

3.2 Differences in Transportation Costs

Now suppose that ski areas can differ by their costs of access. To simplify matters, assume again that the areas have the same capacity x. Let b_j denote the cost for any skier to travel to area j; for example, b_j could depend on the distance of the area from a major urban center. As before, π_j is the lift-ticket price, $q_j = x/n_j$ is the amount of skiing received by each person, and $\hat{P}_j = \pi_j/q_j$ is the effective price per ski run. By extension of equation (3), an individual is indifferent between areas j and k, if

$$(15) \quad \phi(q_j) - \pi_j - b_j = \phi(q_k) - \pi_k - b_k.$$

As before, a change in the lift-ticket price, π_j, causes the number of skiers, n_j, to adjust so that the net surplus on the left side of equation (15) remains constant. Hence, as in equation (5), revenue maximization by the firm implies $\pi_j = q_j D^{-1}(q_j)$. Inserting this result into equation (15) gives

$$(16) \quad \phi(q_j) - D^{-1}(q_j)q_j - b_j = \phi(q_k) - D^{-1}(q_k)q_k - b_k.$$

The term, $\phi(q_j) - D^{-1}(q_j)q_j$, in equation (16) is the standard measure of consumer surplus—that is, the maximum amount that a consumer would pay for the privilege of buying q_j rides at price $D^{-1}(q_j)$. The equation says that this measure of consumer surplus at the two areas must differ by the difference in the transportation costs, $b_j - b_k$. For example, suppose that area j is closer than area k, so that $b_j < b_k$. Then, since consumer surplus is increasing in q, equation (16) implies $q_j < q_k$, and hence $\hat{P}_j = D^{-1}(q_j) > \hat{P}_k = D^{-1}(q_k)$. Thus, closer areas have higher effective prices per ski run and are more crowded in the sense of offering fewer rides per person.

Given the determination of $\hat{P}_j$, the solution for π_j follows from $\pi_j = D^{-1}(q_j)q_j = \hat{P}_j D(\hat{P}_j)$. Thus, the relation between lift-ticket prices, π_j, and the cost of access, b_j, depends again on the elasticity of the demand for rides per person. A lower value of b_j implies a higher value of $\hat{P}_j$ and hence a higher value of π_j if the elasticity of $D(\hat{P})$ with respect to $\hat{P}$ is less than one in magnitude (in the relevant range of demand). But a low b_j implies a low π_j (along with a high $\hat{P}_j$) if the elasticity exceeds one in magnitude.

Finally, if the elasticity equals -1 in some range, then lift-ticket prices do not vary in this range across areas that differ in their costs of access.

Except when the elasticity is equal to -1, different areas must charge different amounts for lift tickets. A single entry fee for skiing with free choice among areas will not achieve the social optimum because the homogeneity effect does not operate. To see why, note that the total output from area j, net of transportation cost, is $h(b_j,n_j) = n_j\phi(x/n_j) - b_jn_j$. This function, $h(b_j,n_j)$, is not homogeneous of degree one in b_j and n_j, and cannot be written in the form, $h(n_j,b_j) = b_jf(n_j)$ for some homogenous function $f(n_j)$.

The next section shows that differences in individual preferences cause lift-ticket prices to differ among areas. But to the extent that these differences are small, the present results permit a kind of cross-sectional check on the explanation proposed above for the stickiness of lift-ticket prices. Many explanations can be offered for price stickiness over time, but it is harder to explain cross-sectional stickiness. If the demand curve for ski runs per person is close to unit elastic, then there should be less variation in lift-ticket prices than in the number of skiers or the length of lift lines, both in comparisons over time and among areas at a point in time. In both dimensions, it will appear that quantities respond more than prices.

3.3. Differences in Preferences

Suppose now that the demand curves for lift rides, $D_i(P)$, differ across individuals. These differences could reflect variations in preferences or incomes. At a given effective price, $\hat{P}$, the quantity of rides demanded per person differs from one person to another. All of the previous equilibria with lift tickets used a queuing mechanism to deliver the same number of rides to all skiers. Since this mechanism does not discriminate among people with different preferences, it cannot allocate different numbers of rides per day, $D_i(\hat{P})$, to them. To achieve an allocation that does discriminate, different areas (or different classes of tickets at a single area) will have to cater to different types of individuals.

To illustrate the results, suppose that there are two types of customers. Avid skiers have the demand $q^A = D^A(P)$, while less avid skiers have the demand $q^B = D^B(P)$, where $q^A > q^B$ for any value of P. The J ski areas will end up dividing themselves into two types; a number J^A that cater to type A customers, and a number $J^B = J - J^A$ that serve the type B customers. Given the numbers J^A and J^B, the determination of effective prices per ski run, $\hat{P}^A$ and $\hat{P}^B$, and lift-ticket prices, π^A and π^B, proceeds as before. In particular, assuming that A-type skiers go to A-type areas, and similarly for B-types, the conditions are

$$N^A(\hat{P}^A, s) \bullet D(\hat{P}^A) = J^A x$$

$$\pi^A = \hat{P}^A \bullet D(\hat{P}^A)$$

and analogously for the B's.

Ski areas can choose between proclaiming themselves as type A, with revenue $\hat{P}^A x$, or type B, with revenue $\hat{P}^B x$. Hence, the numbers J^A and J^B adjust in an equilibrium to attain $\hat{P}^A = \hat{P}^B = \hat{P}$.[5] In that case we find

[5] We neglect integer constraints on the solution. If the number of areas (that serve a given locality) is large, then this problem would be unimportant. Also, differences in capacities, x_j, make this issue less serious.

$$(17) \qquad q^A = D^A(\hat{P}) > q^B = D^B(\hat{P})$$

and

$$(18) \qquad \pi^A = \hat{P} \bullet D^A(\hat{P}) > \pi^B = \hat{P} \bullet D^B(\hat{P})$$

That is, more avid skiers receive more rides and pay higher lift-ticket prices.[6]

Recall that $D^A(\hat{P}) = x/n^A$ and $D^B(\hat{P}) = x/n^B$, where n^A and n^B are the number of skiers at each type of area. Hence, equation (18) implies

$$(19) \qquad \frac{n^A}{n^B} = \frac{\pi^B}{\pi^A}$$

In other words, in deciding whether to charge π^A or π^B—that is, whether to be a type A or type B ski area—each area faces a demand in terms of numbers of skiers, n, that has an elasticity of precisely -1 with respect to the lift-ticket price. Correspondingly, the area's revenue, πn, is invariant with the choice among the π's. The areas are indifferent between charging a high lift-ticket price and catering with short lines to the skiers who demand lots of rides per person, or charging a low price and servicing with long lines those who demand few rides. Note also that, as an area changes its lift-ticket price, it also changes the entire class of skiers that choose to patronize it. That is, a shift from π^A to π^B means that the n^A previous customers all leave, while n^B new customers show up.

[6] Note that type A skiers prefer q^A rides at price π^A to q^B rides at price π^B (since $q^A = D^A(\hat{P})$ is the quantity demanded at the effective price $\hat{P}$). Similarly, type B skiers prefer q^B rides at price π^B to q^A rides at price π^A. Therefore the separating equilibrium that we propose is viable.

The results generalize to multiple skier types, which lead to multiple values of q^k and π^k, but still a single value of $\hat{P}$.[7] A ski area's revenues must still be invariant to its choice of type—that is, of π^k. Therefore, over the set of values for π^k that prevail in equilibrium, it agan follows that each area faces a demand in terms of number of skiers, n^k, that has an elasticity of -1 with respect to π^k. This equiproportional change in the number of skiers in response to a change in the lift-ticket price does not depend on the elasticity of the aggregate demand for lift rides or of individuals' demands for rides per person. The result obtains whenever a range of lift-ticket prices π^k prevails in equilibrium.

Except for the restriction to a finite number of individual types, and hence a finite number of observed prices π^k, the lift-ticket equilibrium in the presence of different tastes resembles the equilibrium with differentiated products and hedonic prices as described in Rosen (1974). Each type of ski area offers a different type of skiing experience, indexed by q^k, the number of ski runs available per skier. With identical competitive producers, profit is invariant to the type of good offered, and the price function $\pi(q)$ traces out the structure of the demand side of the market. In Rosen's model, producers with no market power choose the type of good offered and the price charged from the locus described by $\pi(q)$. Here, the departure from the standard model of competitive price taking is even

[7]However, the integer problem mentioned in n. 6 becomes more serious when there are multiple types. If the number of types is much greater than the number of ski areas, then each area will have to cater to a range of skier types. In this case the use of lift tickets will involve an additional welfare loss relative to an equilibrium with ride tickets.

sharper. Firms simply choose a price π; quality—that is, the number of skiers—adjusts endogenously. It is interesting to note that, until recently, the Metro in Paris used a similar scheme to sort people by tastes. Purchasers of first-class tickets rode in separate cars, which were not physically different from second-class cars, except that the first-class cars were less crowded (in equilibrium).[8] Roughly speaking, individuals with a stronger preference for ski rides or with a greater distaste for congestion are willing to pay more for the opportunity to pay more.

4. Applications to Other Markets

In this section we apply the paradigm of ski-lift pricing to two classic problems of congestion, the two-roads problem noted above and an open-access fishing problem. We conclude with an application to employment contracts with profit sharing. Our objective is to illustrate the applicability of the approach to a variety of problems, and to use some well-known examples to clarify the distinction between the package-deal effect and the homogeneity effect.

4.1 Classical Congestion

Suppose that there are two roads that connect a pair of cities. Let $v(x,n)$ describe the speed of cars traveling on each road as a function of the

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[8] We are told that the abolition of this vestige of the class system was one of the promises made in the presidential campaign of Francois Mitterand. After his election, a compromise was reached whereby this system was not allowed to operate during the morning and evening periods of peak demand (where it presumably would be most useful), but remained in effect during the middle of the day.

capacity x of each road and the number of cars n. Thus, $h(x,n) = nv(x,n)$ is the rate of flow of cars along each road. As Knight (1924) pointed out, if no toll is charged for the use of either road, individuals will sort themselves so that the average output, $h(x,n)/n$, is the same on the two roads; that is, the speeds and travel times will be the same. However, social optimality requires that the aggregate travel time summed over all individuals be minimized, which is equivalent to maximizing the total flow, $h(x_1,n_1) + h(x_2,n_2)$. As noted in the introduction, the private and social choices coincide if h is homogeneous of degree 1, or if it can be written in the form, $h(x,n) = xf(n)$, where f is homogeneous of some degree. The first specification implies that speed depends on the relationship of capacity to the number of cars, but is homogeneous of degree 0. Thus, as seems reasonable, doubling the capacity of the road and the number of vehicles leaves the speed unchanged.

The suboptimality studied by Knight relied on the assumption that one of the roads had a capacity that was so large that the speed was independent of the number of cars, $v_1(n) = a$. The second road was assumed to be subject to congestion; for example, $v_2(x,n) = x_2f(n)$ for some decreasing function f. The free-access equilibrium is then suboptimal, but no justification was given for the different functional dependencies on capacity.

Consider now the case of J fishing ponds, with n_j fishermen at pond j. For the moment, we treat the total number of fisherman, $N = \Sigma n_j$, as fixed. We assume that the output of fish at pond j takes the form

$$(20) \qquad y_j = x_j(n_j)^{\alpha}$$

where x_j is the intrinsic quality of the pond and $0 \leq \alpha < 1$. The condition $\alpha = 0$ corresponds to the case of a ski lift with fixed capacity, x_j. The case $\alpha > 0$ means that an additional fisherman raises the total catch, but if $\alpha < 1$, the marginal and average product diminish with n_j. Thus the pond is subject to congestion; adding an additional fisherman reduces the catch of the previous fishermen. Now suppose that each pond has the same value of α—that is, although the x_j's can vary, crowding sets in at the same proportionate rate at each pond.

If there are no admission fees each fisherman goes to the pond that promises the highest average product, so that in equilibrium, the average products, $x_j n_j^{\alpha-1}$, are equal at each pond. For fixed N, a social planner would seek (in this static problem) to maximize the total current output of fish, $Y = \Sigma y_j$. This maximization requires the marginal product, $\alpha x_j n_j^{\alpha-1}$, to be equal at each pond. But this condition generates the same number, n_j, as the private choices. In other words, despite the congestion problem, the decentralized solution with no expicit prices achieves a Pareto optimal allocation of fishermen.

The result depends on the assumption that crowding sets in at the same proportionate rate at each pond, as implied by the form of equation (20). To see this, relax the assumption that N is fixed, and assume instead that fishermen have a distribution of costs for going fishing, c_i. This means that each person has available an alternative activity—such as staying home—where the output does not involve the same sort of crowding as prevails at each fishing pond. In the decentralized solution a person chooses to fish if c_i < APL, where APL is the common value of the average product of labor.

(This condition applies to commercial fishing and assumes no utility from fishing, per se.) On the other hand, the social planner would assign a person to fishing if c_i < MPL, where MPL is the common value of the marginal product. Since MPL < APL, we get the standard result that too many people choose to fish under the decentralized solution. However, to attain a Pareto optimum, it is necessary to charge only a single price π to fish—i.e., a fishing license.[9] It is unnecessary to have different prices at the various ponds, even though they differ by their intrinsic qualities, x_j. In equilibrium the better ponds are more crowded—but to exactly the extent required to attain the optimal allocation of fishermen.

The fishing problem has an analogue to the assignment of people to rooms for sessions at a professional meeting. Sessions differ by their intrinsic quality, x_j. However, as more people crowd in, it becomes more difficult to see or hear, so that the "quantity" received per person declines with n_j. For example, if crowding sets in at the same proportionate rate at each session, then equation (20) describes the total "output" of session j. If the total number of participants, N, is fixed, then the decentralized choices achieve a Pareto optimum without having explicit prices for each session. If the number N is variable—in particular, if people have access to some alternative activity that is not subject to congestion—then too many people

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[9] The price is $\pi = APL - MPL = (1-\alpha)x_j(n_j)^{\alpha-1}$, where j is any of the ponds, and the total number of fishermen N is the number for whom $c_i + \pi < APL = x_j(n_j)^{\alpha-1}$ (or $c_i < MPL = \alpha x_j(n_j)^{\alpha-1}$). If there is a downward shift in the c_i's, then N, and hence n_j, rise, which implies a decline in π. Hence with shifts in labor supply, fishing licenses should be cheaper when the ponds are more crowded!

attend sessions in a decentralized equilibrium. However, the attainment of a Pareto optimum requires only a single fee (a registration charge), and not prices that vary across sessions in accordance with their intrinsic qualities.

4.2 Profit Sharing

In the labor market a fully flexible wage rate corresponds to a flexible price per lift ride. The case of a lift ticket relates to alternative methods of labor compensation, such as profit-sharing schemes. From the standpoint of an individual worker, the firm's total profit looks like a ski operator's total capacity. In particular, the amount that each person gets (share of profits or number of lift rides) varies inversely with the number of other people who show up. (Profit per worker falls with more workers as long as the average product of labor exceeds the marginal product.) But, as in the ski example, competition among firms causes the parameters of the profit-sharing rule to adjust so as to reproduce the outcomes that would emerge under flexible wages. Further, under some conditions, it would be satisfactory to have fixed wages and fixed parameters for the profit-sharing formula.

Suppose that each of J identical, competitive firms has the production function,

$$(21)\quad Y = A \bullet F(n),$$

where Y is output, A is a technological shift parameter, and n is the number of workers. We assume that each worker has the same productivity and works a fixed number of hours per day. We deal initially with a standard setting where the real wage rate, w, is flexible. Given this wage, profit maximization for each firm entails

$$(22) \quad AF'(n) = w.$$

Equation (22) determines each firm's labor demand. Aggregate labor demand is the multiple J of the demand per firm.

The economy has a population of M potential workers who have a distribution of reservation wages, c_i; those with $c_i < w$ choose to work. Hence the aggregate labor supply function is

$$(23) \quad N = N(w, \theta)$$

where $N \leq M$, $\frac{\partial N}{\partial w} \geq 0$, and θ represents factors (including wealth effects) that influence the position of the distribution of reservation wages.

The equation of aggregate labor supply to aggregate labor demand determines the market-clearing values of the wage rate, w^*, and employment, N^*. Then each firm's employment is $n^* = N^*/J$. We assume that variations over time in wages rates and employment reflect shifts in the technological parameter, A, or in the reservation-wage parameter, θ. (A shift in the parameter A, to the extent that it changes wealth, could imply a simultaneous shift in θ.)

As in the ski-lift example, the competitive equilibrium in the labor market can be supported by pricing mechanisms other than the obvious one of freely flexible wage rates per worker. For example, assume that the wage rate is fixed at some value $w' < w^*$. The wage w' parallels the explicit price per ride, r, in the ski-lift case. Therefore, $w' = 0$ corresponds to pure lift-ticket pricing, where $r = 0$.

Assume now that each worker's compensation consists of the wage w' plus a bonus B. We consider a profit-sharing scheme, similar to Weitzman (1985), where the bonus to each worker is the fraction β of profit per worker; that is,

$$(24) \qquad B = \beta \left[\frac{AF(n)}{n} - w'\right],$$

where $0 \leq \beta \leq 1$. Therefore, a worker's total compensation, Ω, is given by

$$(25) \qquad \Omega = B + w' = \beta AF(n)/n + (1 - \beta)w'.$$

Since potential workers care only about Ω, and not on its division between B and w', each competitive firm takes as given the value of Ω that it must pay. Hence, for fixed w', equation (25) determines how the quantity of labor supplied to the firm, n, varies with the profit-sharing fraction, β. That is, each firm can call out a value of β (along with an arbitrary w'), and the number n adjusts to make the overall compensation, Ω, equal to its competitively determined value. This adjustment of n to a change in β parallels the response of the number of skiers to a shift in the lift-ticket price.

Setting the differential of Ω in equation (25) to zero leads to

$$(26) \qquad \frac{dn}{d\beta} = \frac{F(n) - nw'/A}{\beta\left[\frac{F(n)}{n} - F'(n)\right]} > 0.$$

The denominator is positive from the usual assumptions about the production function—that is, the average product, AF(n)/n, exceeds the marginal product, $AF'(n)$. The numerator is positive if, as we assume, w' is less than the average product, $AF(n)/n$.[10]

Each firm chooses β to maximize profit, as given by

$$(1 - \beta)[AF(n) - nw'],$$

subject to the relation between n and β from equation (26). Setting to zero the derivative of profit with respect to β (taking account of the response of n from equation (26)) leads to the condition

$$(27) \qquad AF'(n) = \Omega$$

That is, labor's marginal product equals the total compensation, Ω, not the explicit wage w'. Hence, with the substitution of Ω for w, the result parallels that with a flexible wage in equation (22). In particular, labor demand, n, depends on Ω exactly as it depended before on w.

Potential workers participate in the market if their reservation wage, c_i, is below the total compensation Ω. Therefore, the aggregate labor supply function is now

$$(28) \qquad N = N(\Omega, \theta),$$

which parallels the flexible-wage case in equation (23), except for the substitution of Ω for w.

Equations (27) and (28) imply that aggregate labor demand and supply depend on Ω exactly as they depended on w in the flexible-wage case. Therefore the market-clearing value of total compensation, Ω^*, equals the market-clearing flexible wage rate, w^*. It follows that all allocations—including employment per firm, n^*, and labor-force participation, N^*—coincide with those in the flexible-wage case.

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[10]The assumption $w' < w^*$ turns out to guarantee this condition at the equilibrium value of n.

Note that, although the allocations are the same, the appearances are again quite different. Under the bonus arrangement, the explicit wage, $w' < w^*$, is rigid, but each worker also gets a share of the profits. In deciding whether to work, each person looks only at the total compensation, $\Omega = w' + B$, and neglects the negative effect of his participation on the profit distributed to the other workers (which occurs because the marginal product of labor is below the average product). This interaction parallels the negative effect of an additional skier's participation on the rides available for others. Nevertheless, as in our previous example for skiing, the profit-sharing scheme reproduces the results for employment and total compensation per worker that would arise under flexible wages.

It also follows that firms would eagerly hire more workers than are available at the going wage w' ($< w^*$) and the prescribed terms for sharing profits. (This result parallels the eagerness of skiers to queue up for lift rides.) But more workers than n^* do not present themselves because the total compensation, $\Omega = w'+B$, would then fall below the value w^*, which is the reservation wage of the marginal worker (when total employment is $N^* = Jn^*$). As with flexible wages, employment is determined so that labor's marginal product equals the competitive wage w^*. In other words, profits are maximized subject to the constraint that firms pay each worker a total compensation that equals the marginal worker's reservation wage. Thus, the outcomes are Pareto optimal despite rigid wages and the apparent common-property problem associated with the sharing of profits. Even though the marginal cost to the firm of an additional worker is less under the profit-sharing arrangement, the equilibrium level of employment is the same as that with flexible wages. Correspondingly, the firms in each regime face the appropriate shadow price of labor (w^*), and thereby make correct decisions with respect to investment in capital, entry and exit, etc.

All of the results so far parallel those from section 2. In particular, they depend only on the package-deal effect. The package offered here—in this case by a buyer of labor services—is w' of wage dollars plus $B = w^* - w'$ of bonus dollars per unit of labor.

The discussion of amusement parks noted that, for any given elasticity of demand, there would exist a constant entry fee and a constant price per ride such that small shocks generated outcomes that approximated those from a flexible price equilibrium. A similar local result holds here. There will exist fixed values of w' and β such that disturbances generate outcomes that approximate those supported by flexible prices.

There is also an interesting special case where the parameters, β and w', can be constant in the face of global shocks to supply or demand. (This parallels the ski-lift example where the lift-ticket price does not vary with shocks if the elasticity of demand for rides per person equals -1.) By substituting for Ω from equation (25), the equilibrium condition from equation (27) is

$$(29) \qquad AF'(n) = \beta AF(n)/n + (1 - \beta)w'.$$

A change in the reservation-wage parameter, θ, leads to a change in n, which leads generally to a shift in β for a given w'. Total differentiation of equation (29) with respect to n and β (for fixed A and w') leads to

$$\frac{d\beta}{dn} = H[\beta F(n) - \beta nF'(n) + n^2F''(n)],$$

where H is a positive expression. This derivative will equal zero for all values of β and n if the production function has the Cobb-Douglas form, $F(n) = n^{\alpha}$, and if β is set equal to α.[11] It can be verified from equation (29) that, for this production function, the share parameter β also does not change with a shift in the technological parameter, A. Substitution of $F(n) = n^{\beta}$ into equation (29) shows that the level of total compensation is correct here only if $w' = 0$. Hence, if production is Cobb-Douglas with labor's share β, then the firms can pay workers a zero explicit wage, w', and a fraction β of the profits. Under this scheme, the values of β and w' do not have to change with variations in labor supply or proportional shifts to the production function in order to support the competitive allocations.

The present results do not imply that profit sharing is superior to other schemes that allow the bonus (and thereby total labor compensation) to move along with w*. Also, the analysis does not suggest that a framework with fixed wages and a flexible bonus (related, say, to profits) would be superior to a setup with flexible wages. As was the case for ski areas, the choice of compensation scheme must be based on elements of reality that are excluded from this model.

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[11] For $\beta \neq 0$, the general solution for $d\beta/dn = 0$ is $F(n) = c_1 n^{\beta} + c_2 n$, where c_1 and c_2 are arbitrary constraints. However, β varies with shifts in the parameter A unless $c_2 = 0$.

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