Secrecy, Speculation and Policy (Revised)

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It is common for policymakers to be secretive about their actions.[1] Further, policymakers frequently rationalize their secretive behavior by contending that disclosure of information would somehow lead private individuals to take speculative actions that are socially inappropriate.

For those of us who are both democrats and believers in the efficacy of free markets, secrecy about policy actions is distasteful. Thus we frequently suggest that policymakers base secrecy on other than social objectives, such as a desire to cover up incompetence, etc. But, in fact, secrecy can be desirable when there are distortions present in the economic system.

This paper illustrates the potential costs and benefits of policy secrecy in a simple stochastic intertemporal framework which embodies rational expectations and competitive markets. The only exogenous random variable is government spending in the second of the model's two periods. The government can control the precision of private agents' forecasts of spending by the character of a signal it releases, which is usefully viewed as macroeconomic data. Better information--which produces more precise private forecasts--leads to welfare enhancing alterations in private investment ("speculation"), when government spending is financed by lump-sum taxation.

However, when government spending must be financed by proportionate taxation on investment, then it is no longer the case that it is always desirable to have better information. At the individual level--at given tax rates--there are private gains from information. But, at the social level,

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[1] Notably, Goodfriend (1986) documents the Federal Reserve's penchant for secrecy and its legal defense of secret open-market committee meetings.

tax rates depend on the aggregate of individual actions. Larger distortions can emerge if agents have better information. Thus, a tension emerges--overall it may or may not be desirable to have better information.[2]

The organization of the remainder of the paper is as follows. Section I lays out the structure of the model. Section II considers the effects of information when taxes are lump sum. Section III presents the effects of information when taxes are distortionary. Section IV discusses the relationship of the current study to recent analyses of the optimality of random taxation. Section V is a brief discussion of implications for public policy.

I. **Structure of the Model Economy**

Analysis of information and speculation requires a stochastic model with non-trivial intertemporal choices. This paper employs a basic member of this class--a two-period intertemporal equilibrium model in which the fundamental source of uncertainty is random government spending in the second period. The key elements involved are assumptions about preferences, production opportunities, uncertainties, and information.

Preferences

The representative agent's preferences are given by

$$(1) \quad U(c_1,c_2) = u(c_1) + u(c_2)$$

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[2]Hirshleifer (1971) considers the differential private and social value of information in an exchange economy that is free of the distortions that drive the present paper. Differential information access by private agents is a key part of Hirschleifer's analysis, but is not present here.

where the momentary utility function u( ) has positive and diminishing marginal utility. Under uncertainty, agents maximize expected utility.

Production

The intertemporal production structure of this model is very simple, in that each agent has an endowment of goods in period 1, y, that can be either consumed or invested. One unit of investment at date 1 yields one unit of consumption at date 2.

Uncertainty

The fundamental uncertainty in the model is government purchases of second-period goods in an exogenous, random amount ($\tilde{g}$). For simplicity, it is assumed that the government destroys the goods it purchases, but the qualitative implications are unchanged so long as the private sector regards public expenditure as a less than perfect utility substitute for private spending. Specifically, government spending takes on one of two possible values, either high or low, with $y > g_h > g_\ell \geq 0$.[3]

Information

Prior to period 1, each spending state is equally likely ($pr(\tilde{g}=g_\ell) = pr(\tilde{g}=g_h) = \frac{1}{2}$). But, at date 1, a random signal ($\tilde{s}$) occurs that shifts these probabilities, which it is useful to view as a release of a type of data that is valuable for forecasting second-period government spending.

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[3] In fact, if agents view public spending as a perfect substitute for private spending, then information has no value with lump-sum taxation. Thus, with distorting taxes, it is inevitable that complete secrecy is optimal.

The signal takes two values, which also occur with equal probability ($pr(\tilde{s}=s_h) = pr(\tilde{s}=s_\ell) = \frac{1}{2}$), i.e., the data can either indicate that high or low spending will occur in the future. The strength of the revision in probability beliefs about $\tilde{g}$ occasioned by $\tilde{s}$ is a parameter of the model (x), so that one can meaningfully discuss the marginal value of information. If the signal takes the value $\tilde{s} = s_h$, then there is a increased probability of a high spending state, $pr(\tilde{g}=g_h|s_h) = \frac{1}{2} + x$. If the signal takes the value $\tilde{s} = s_\ell$, then there is an increased probability of a low spending state, $pr(\tilde{g}=g_\ell|s_\ell) = \frac{1}{2} + x$.

The parameter x governs the extent of revision in probability beliefs occasioned by the signal, $\tilde{s}$. If x=0, then there is no information about future spending provided by the signal, a parametric case which we term complete secrecy. If $x = \frac{1}{2}$, then the signal provides accurate knowledge about the state of spending, which we term full information. With $0 < x < \frac{1}{2}$, there is partial secrecy. Equivalently, x governs the extent of remaining uncertainty after receipt of the signal as reflected by the conditional variance of $\tilde{g}$, which has the value $(\frac{1}{4} - x^2)(g_h - g_\ell)^2$.

Government

The government has two roles in this model, as spending is exogenous. First, it levies taxes to finance the exogenous stochastic spending level, in ways that are different in each of the sections below. Second, it chooses an information policy--a value of the parameter x--to maximize the expected utility of the representative individual. It is important to stress that the government does not select a value of the signal, but rather the characteristics of the probability structure, e.g., the size of a research and statistics staff.[4]

Timing of Economic Actions

At the beginning of the first period, the government chooses an information policy (i.e., a value of x) which is known by all economic agents. Then, a signal $\tilde{s}$ is generated according to the stochastic mechanism described previously. Lastly, first-period consumption and investment levels are chosen by private agents, who have rational expectations about period two circumstances.

In the second period, taxes are levied to finance realized $\tilde{g}$, then government and private consumption take place.

II. **Investment, Information, and Welfare with Lump-Sum Taxes**

When government spending is financed with lump-sum taxes in the second of the model's two periods, more information is always desirable, as it aids agents in smoothing consumption against the adverse affects of government spending.

Equilibrium Investment and Welfare

Our analysis begins by assuming that there is a given probability of the low spending state, p, calculating equilibrium investment and expected

[4]Permitting the government to make announcements that differed from actual signal outcomes would introduce a number of interesting additional considerations not explored here which amount to complicating the private inference problem

utility. With a given level of investment, I, expected utility is given by

$$(2) \quad EU = u(y-I) + \{pu(I-g_{\ell}) + (1-p)u(I-g_h)\}$$

The representative individual picks an efficient level of I by equating private costs and benefits, i.e.,

$$(3) \quad Du(y-I) = \{pDu(I-g_{\ell}) + (1-p)Du(I-g_h)\}$$

as discussed by Sandmo (1970) in the context of saving under uncertainty.

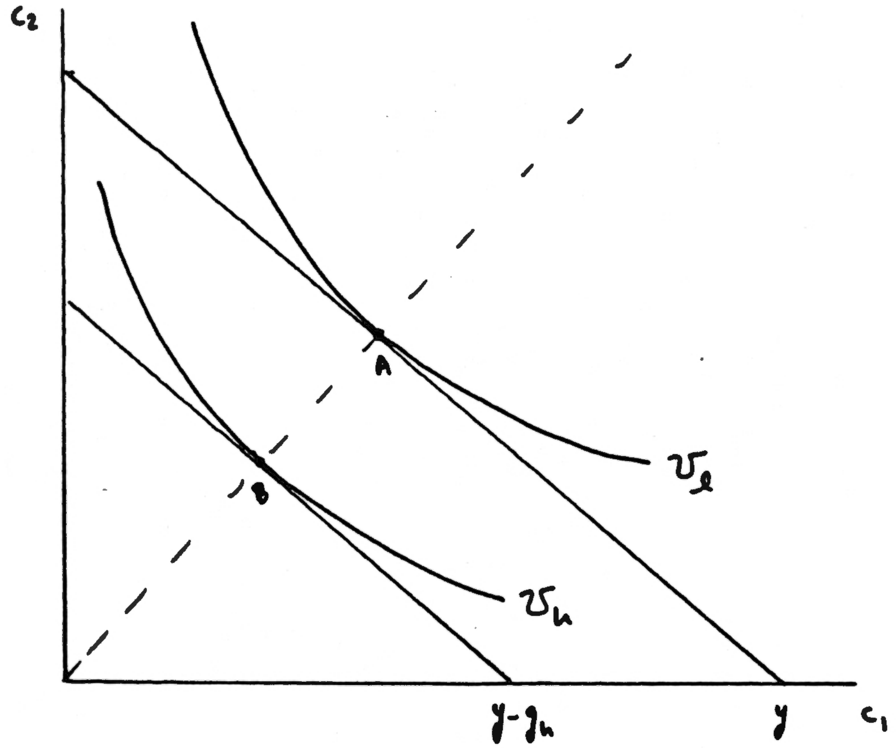
Let I(p) denote the efficient level of investment for given probability and V(p) denote the expected utility obtained by pursuing that policy. It is easy to determine that the efficient level of investment implied by (p) depends negatively on p (see Appendix A), which reflects the fact that lower government spending entails a smaller transfer of resources to the future to smooth consumption against government spending. Similarly, expected utility is increasing in p because the low spending is more likely.

Secrecy versus Full Information

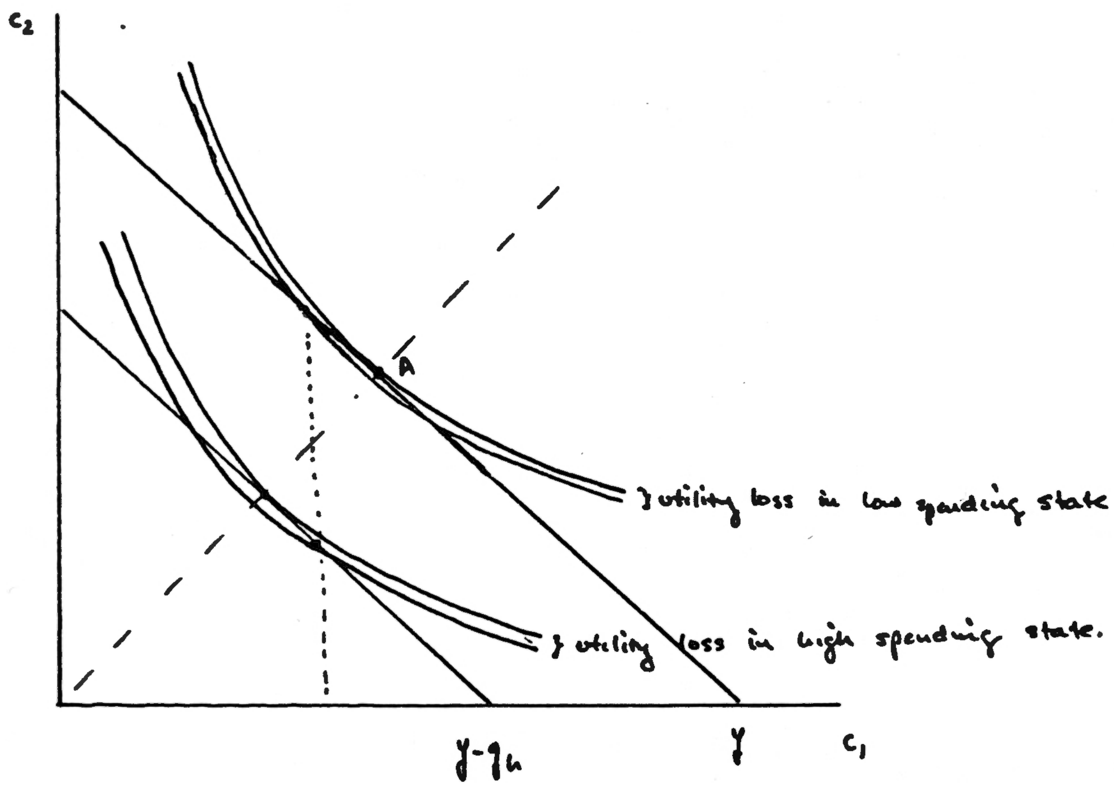
It is useful to illustrate the costs of complete secrecy, relative to full information, in a diagram in anticipation of developments in Section III below. Under full information, individuals choose to consume an equal amount in each period, $c = \frac{1}{2}(y-\tilde{g})$, given that the pure rate of time preference and physical net return to storage are zero, as illustrated in Figure 1-A, which is drawn for the special case in which $g_{\ell} = 0$. In each case, the level of

Figure 1

A. Competitive Consumption with Full Information



B. Utility Losses under Complete Secrecy



investment is the distance $I = y-c_1 = \frac{1}{2}(y+\tilde{g})$, so investment <u>rises</u> with government spending.

Under complete secrecy, individuals choose a level of investment between the full information values above, with the exact position depending on the extent to which they wish to hedge against adverse spending realizations. But, whichever state occurs, individuals suffer a lower realized level of utility than under full information (see Figure 1-B). Thus, complete secrecy is less desirable than full information according to the expected utility criterion.

Optimal Information Policy

The government selects a degree of information provision (x) to maximize the expected utility of the representative agent prior to the receipt of the signal, i.e., $EV = (\frac{1}{2})V(\frac{1}{2} + x) + (\frac{1}{2})V(\frac{1}{2} - x)$, where $V(\frac{1}{2} + x)$ is expected utility conditional on $\tilde{s} = s_\ell$ and $V(\frac{1}{2} - x)$ is expected utility conditional on $\tilde{s}=s_h$.

The effects of increasing the degree of information provision on welfare are straightforward, as derived in Appendix A. Increasing information is always desirable, so it is optimal for the government to provide full information ($x = \frac{1}{2}$), in the absence of information costs. That is, expected utility is strictly increasing in x, so that it is optimal to set ($x = \frac{1}{2}$), which is the full information solution.

Formally, as demonstrated in Appendix A, the effect of a change in the precision of information (x) on expected utility is

$$\frac{\partial EV}{\partial x} = \frac{1}{2}\{DV(\frac{1}{2} + x)\} - \frac{1}{2}\{DV(\frac{1}{2} - x)\}$$

$$= \frac{1}{2}\{u(I(\frac{1}{2} + x) - g_\ell) - u(I(\frac{1}{2} + x) - g_h)\}$$

$$- \frac{1}{2}\{u(I(\frac{1}{2} - x) - g_\ell) - u(I(\frac{1}{2} - x) - g_h)\}.$$

The expression $\{u(I - g_\ell) - u(I - g_h)\}$ reflect the utility gain from having the good state occur, i.e., the realization of low government spending. We know that investment is negatively influenced by a higher probability (p) of the low spending state, i.e., $I(\frac{1}{2} - x) > I(\frac{1}{2} + x)$. Thus, from strict concavity of u( ), it follows that $\{u(I(\frac{1}{2} + x) - g_\ell) - u(I(\frac{1}{2} + x) - g_h) > \{u(I(\frac{1}{2} - x) - g_\ell) - u(I(\frac{1}{2} - x) - g_h)\}$. That is, there is a smaller utility cost from having the high spending state if one had previously invested a higher amount. The value of more precise information (higher x) arises from the ability to eliminate these sorts of costs, so it follows that $\partial EV/\partial x > 0$, i.e., information has positive value.

III. **Investment, Information, and Welfare**

When government spending is financed with a tax on investment, then it is no longer necessarily the case that more information is always desirable. Fundamentally, this reflects the fact that private and social returns to investment are no longer coincident.

Taxes, Spending and Investment

The government taxes investment at rate $\tilde{\tau}$, which is stochastic, with the rate set so as to balance the government's budget $I\tilde{\tau} = g$. Thus, there is an inequivalence of private and social returns, which reflects the standard assumption that an individual agent treats the distribution of $\tilde{\tau}$ as given in picking his investment level, but that the simultaneous actions of all (identical) agents determine the tax rate.

Analysis of Investment with Given Tax Rates

With this tax structure in place, an individual agent thus faces a private rate of transformation of $(1-\tilde{\tau})$ between consumption in period one and period two, which is an example of the rate of return uncertainty studied by Sandmo (1970). For an individual agent, with a given distribution of tax rates $\tilde{\tau}$, efficient investment satisfies the necessary condition.

$$(4)\quad Du(y-I) = E\{(1-\tilde{\tau})\ Du((1-\tilde{\tau})I)\}$$

As previously, we can ask how an increase in the probability of the good (low government spending) state (p) will affect investment. Our intuition suggests that high private expected returns to investment/saving will lead to a higher level of investment/saving. As Sandmo (1970) explains, however, this requires a condition on preferences, which insures that the relevant substitution effect dominates the relevant income effect. (That is, as explained in Appendix B, we require that the elasticity of marginal utility be less than unity, which implies a high degree of intertemporal substitution). Under this preference condition, which we impose throughout our discussion, the direction of investment's response to information is reversed when taxes are distorting. That is, investment rises with a higher probability of low spending under distorting taxes while it falls with lump-sum taxes in Section II above.

It is important to stress that there is private value to information in this economy, i.e., if a single individual could obtain $\tilde{s}$ at zero cost, he would surely want to adjust investment. (See Appendix B for a proof). But, because the signal is public information, it also alters the investment behavior of other agents and, hence, tax rates in the equilibrium to be developed below. Consequently, the social value of information may be negative.

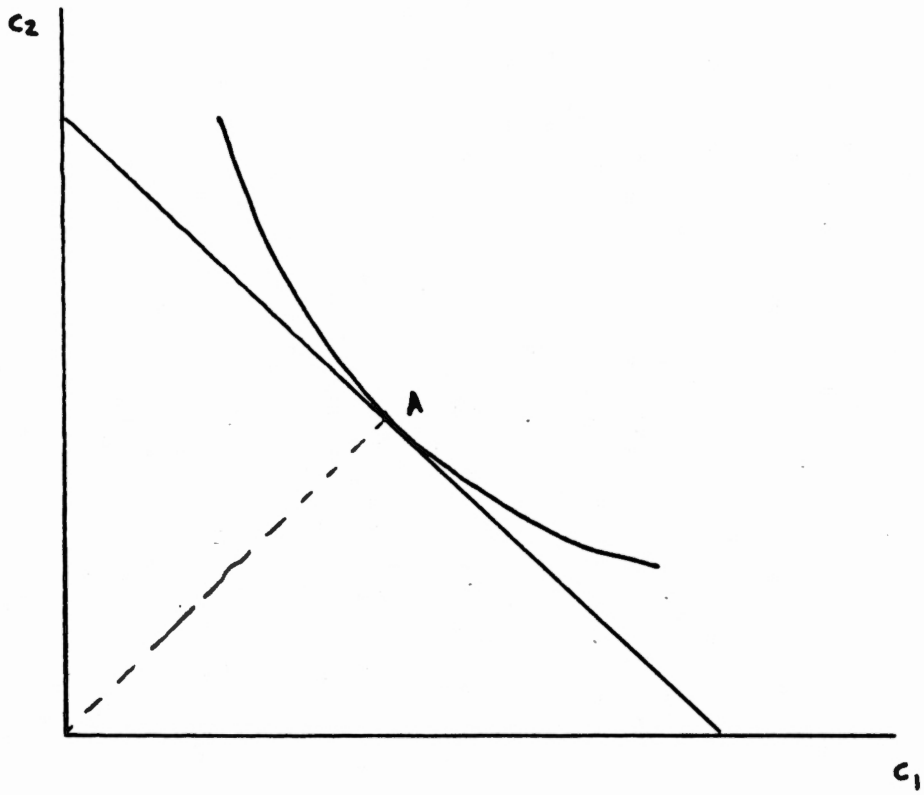
Equilibrium Investment

Competitive equilibrium investment involves fulfillment of the private efficiency condition (4) plus satisfaction of the government budget constraint. As shown in Appendix B, if one is on the desirable side of the Laffer curve, then a rise in p raises equilibrium investment by a greater amount than that considered with fixed tax rates, because tax rates can fall as the tax base expands. Thus, equilibrium investment is an increasing function of p, $I_e(p)$, under the preference restriction considered above.

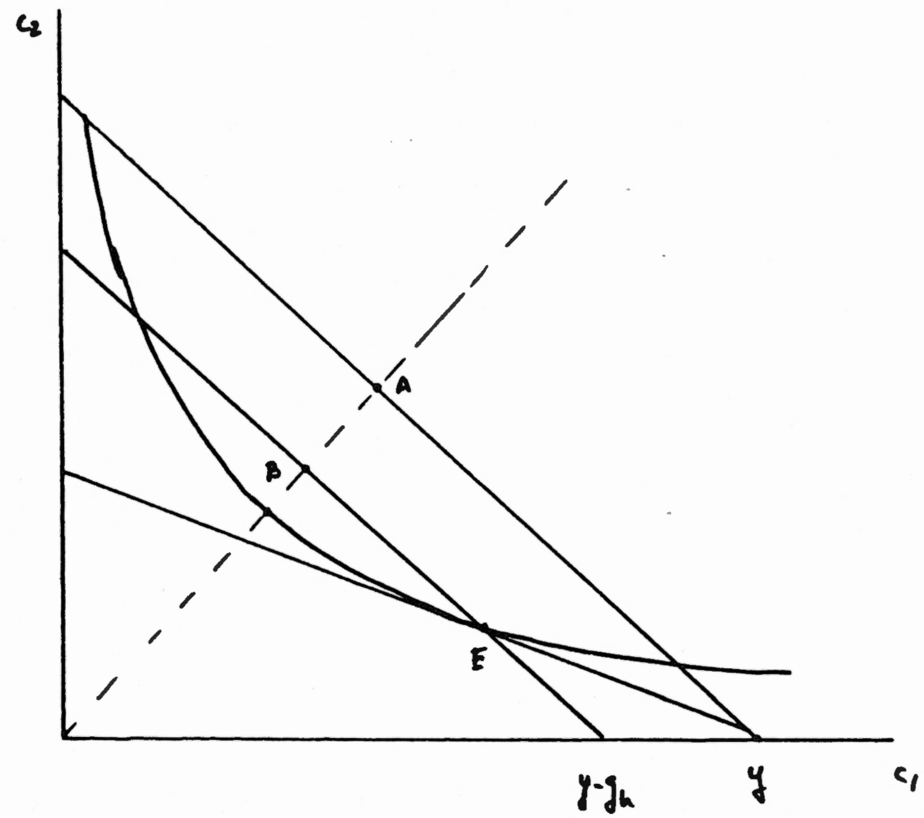
Figure 2 illustrates the model's competitive quilibrium when there is full information. If the low spending state occurs, then there is no distortion because $g_\ell = 0$ and no taxes must be levied, as illustrated in 2-A. With a distorting tax in the high spending state, a competitive equilibrium is the point E in Figure 2-B, where there is private utility maximization (indifference curve tangent to private opportunities, $c_2 = (1-\tau_h)(y-c_1)$) and government budget balance (consumer equilibrium occurs on the social

Figure 2:
Competitive Equilibrium with Full Information and Distortions

A. Low Government Spending State (g_1=0)



B. High Government Spending State



opportunity frontier, $c_2 = y-g_h-c_1$)). The cost of distorting taxes, in utility terms, reflects the fact that point E and point B do not coincide. The assumption that preferences reflect sufficient intertemporal substitutability so that investment rises with a higher probability of lower tax rates locates point E to the right of point A.

Effects of Better Information

As a result of the distortions present in the current system, the ex ante expected utility effects of better information are ambiguous. First, a given increase in the probability of the low government spending state raises equilibrium expected utility, denoted $V_e(p)$, both because the good state becomes more likely and because the distortion arising from non-lump-sum taxation becomes smaller, i.e.,

$$(5)\quad DV_e(p) = \{u(I_e(p) - g_\ell) - u(I_e(p) - g_h)\}$$

$$+ E\left\{\frac{\tilde{g}}{I_e(p)} DU(I_e(p) - \tilde{g})\right\} DI_e(p).$$

The latter term reflects the fact that a rising tax base enables the distribution of tax rates to decline and, hence, the distortion to fall.

The effect on ex ante expected utility of an increase in the precision of information involves, as previously, offsetting effects, i.e.,

$$(6)\quad \frac{\partial EV_e}{\partial x} = \frac{1}{2} DV_e(\frac{1}{2} + x) - \frac{1}{2} DV_e(\frac{1}{2} - x).$$

From (5), there are two components to the effect of a change in x. The first is the utility consequence of a mistake in determining which state will occur, analyzed previously in the no distortions case.

$$(7) \quad \frac{1}{2}\left\{u(I_e(\tfrac{1}{2} + x) - g_\ell) - u(I_2(\tfrac{1}{2} + x) - g_h)\right\}$$

$$- \left\{u(I_e(\tfrac{1}{2} - x) - g_\ell) - u(I_e(\tfrac{1}{2} - x) - g_h)\right\}$$

Recall that with our preference assumption, investment decreases with the probability of the high spending state, i.e., $I_e(\frac{1}{2} + x) > I_e(\frac{1}{2} - x)$, because of the substitution effect of taxation. But this implies that the preceeding expression is negative, for there is a bigger utility cost if the high spending is lower. Fundamentally, because investment moves in the "wrong direction" in response to information, expected utility declines with informaton on this account. The second component of the expected utility is the effect on marginal distortion occasioned by the precision of information

$$(8) \quad E\left\{\frac{\tilde{g}}{I_e(\frac{1}{2} + x)} Du(I_e(\tfrac{1}{2} + x) - \tilde{g})\right\} DI_e(\tfrac{1}{2} + x)$$

$$- E\left\{\frac{\tilde{g}}{I_e(\frac{1}{2} - x)} Du(I_e(\tfrac{1}{2} - x) - \tilde{g})\right\} DI_e(\tfrac{1}{2} - x)$$

So far, it has not been proved possible (for me) to make much progress with this component, in general. But there is an important special case. If there is no distortion in the low spending state ($g_\ell = 0$) and one starts from an initial position of complete information ($x = \frac{1}{2}$), then it follows that this expression is unambiguously negative, as the first term is zero. Thus, these conditions are <u>sufficient</u> for a small amount of secrecy to be desirable.

Secrecy versus Full Information

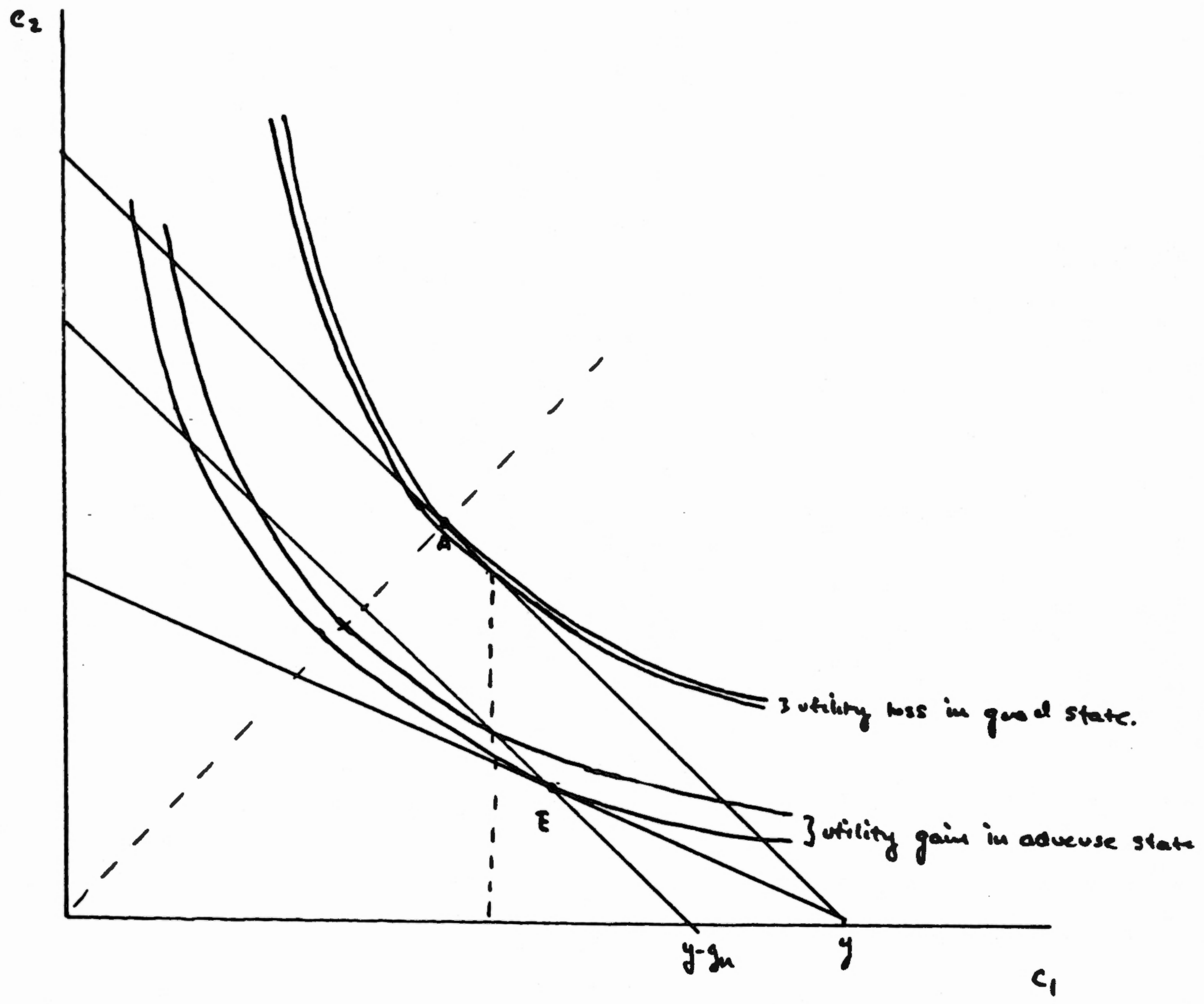
Imposition of complete secrecy implies that the level of investment again occurs between the values implied by full information, as depicted in Figure 3. If the low spending state occurs, individuals will have invested too much under complete secrecy, so that there is a utility loss in that state. But, in the high spending state, investment will be higher under secrecy than it would be under full information. Since competitive equilibrium investment is too low in the presence of distortions, secrecy results in a utility gain in the high spending state.

Notice that if the competitive equilibrium lies directly below point A (which occurs if $u(c) = \log c$), then it is irrelevant whether secrecy occurs or not, i.e., information has no implications for unconditional expected utility. Further, if the competitive equilibrium lies to the left of A--investment rises with spending because individuals have a small degree of intertemporal substitution--then secrecy cannot be desirable, because losses occur in both states with imposition of secrecy.

In addition to displaying these general conclusions, Figure 3 is drawn under the assumption that investment under complete secrecy is equal to its expected value under complete information, i.e., $I_e(p)$ is approximately linear in p. In this case, it is clear that because there is a one-for-one

Figure 3:

Utility Consequences of Policy Secrecy and Distortions



exchange between states (and the utility value is higher in the adverse state), expected utility rises unambiguously.

IV. **Relationship to Analyses of Random Taxation**

The current analysis of secrecy concerning future taxation is a cousin to recent work on the potential desirability of random taxation. (See Weiss (1976) and Stiglitz (1982)). The shared family tie is that increased tax rate uncertainty or policy secrecy may motivate actions that reduce other distortions. But there are important differences in the conditions that rationalize secrecy and random taxation.

In the present savings/investment context, an analysis along the lines of Stiglitz (1982)--with appropriate translation--considers an economy with a constant level of second-period spending that must be financed with a tax on investment, i.e., $\tau I = g$. One can potentially introduce randomization in two ways. (For simplicity, in each case we consider taxing half the population at rate $\tau+z$ and half at the rate $\tau-z$.) With random <u>ex ante</u> taxation, agents know the tax rate they face prior to making the investment decision--tax rates are announced at date 1. With random <u>ex post</u> taxation, agents do not learn the tax rate until date 2, but take tax induced rate of return randomness in account in their date 1 investment decision. Stiglitz (1982) discusses conditions under which random <u>ex ante</u> and random <u>ex post</u> taxation dominate deterministic taxation. (See Appendix C for a translation of Stiglitz' analysis to the two-period saving/investment context).

A necessary condition for random taxation to be optimal is that investment increase with tax uncertainty, as stressed by Stiglitz (1982).

That is, it is central that individual uncertainty about taxes raise investment, so that the average tax rate can be lowered. Sandmo (1970) discusses the conditions under which uncertainty about the rate of return raises saving, which involve the relevant income effect dominating the relevant substitution effect. Thus, exactly the opposite preference restrictions are needed to insure that random taxation is desirable and that secrecy is desirable in the present context.

V. **Implications for Public Policy**

What lessons can be drawn from this paper for the conduct of policy? It is probably best to stress that the example provided in the paper does not consititute a rational for secrecy in policymaking in any arbitrary context. Rather, in the model economy studied, the desirability of secrecy can be established only through policy analysis within the established traditions of macroeconomics and public finance. That is, one must isolate the distortion (or externality) present in the system and analyze how its expected value is altered by the use of secrecy as a policy instrument. That policy evaluation would typically involve detailed knowledge of the structure of the economic system, including parameters of production technology and preferences.

It is widely held that central banks are among the most secretive of policy authorities. Potentially, a positive analysis based on the present paper's key elements--the interaction of information and distortion--could rationalize this penchant for secrecy. But, a first necessary step is to isolate the external, distortionary elements in the monetary system, a goal which has so far proved elusive.

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Appendix A:

Effects of Information without Distortions

With a given probability of the low spending state (p), investment is chosen to maximize expected utility,

$$\text{(A1)} \quad EU = u(y-I) + pu(I-g_\ell) + (1-p)u(I-g_h).$$

This outcome of this maximization process is a decision rule for investment I(p) and a maximal level of expected utility or value function V(p). Our initial objective is to characterize I(p) and V(p). The first-order condition for investment is

$$\text{(A2)} \quad -Du(y-I) + pDu(I-g_\ell) + (1-p)Du(I-g_h) = 0,$$

where $D_i f$ is the partial derivative of f with respect to its ith argument. (The subscript is omitted when there is a single argument.) It is clear from inspection that the full information values are $I(0) = (y+g_h)/2$ and that $I(1) = (y+g_\ell)/2$. From the implicit function theorem's application to (A2), we find that

$$\text{(A3)} \quad DI(p) = \frac{Du(I-g_\ell) - Du(I-g_h)}{-\{D^2u(y-I) + E\ D^2u(I-\tilde{g})\}} < 0,$$

where the numerator is negative due to the fact that $g_h > g_\ell$ and the denominator is positive to due concavity of u( ).

Indirect utility is increasing in p as this implies that the probability of the economy's good state is larger,

$$(4) \quad DV(p) = u(I-g_\ell) + [-Du(y-I) + E\{Du(I-\tilde{g})\}]DI(p)$$
$$= u(I-g_\ell) - u(I-g_h),$$

where the second line of equality follows from application of the first-order condition, further,

$$(5) \quad D^2V(p) = \{Du(I-g_\ell) - Du(I-g_h)\}DI(p)$$

$$= \frac{\{Du(I-g_\ell) - Du(I-g_h)\}^2}{-\{D^2u(y-I) + E(D^2u(y-\tilde{g}))\}} > 0.$$

Unconditional expected utility--prior to the receipt of the signal--is thus given by

$$(A6) \quad EV(x) = \frac{1}{2} V(\frac{1}{2} + x) + \frac{1}{2} V(\frac{1}{2} - x)$$

Thus, the effect of a marginal change in x is given by

$$(A7) \quad DEV(x) = V(\frac{1}{2} + x) + \frac{1}{2} V(\frac{1}{2} - x)$$
$$= \frac{1}{2} \{u(I(\frac{1}{2} + x) - g_\ell) - u(I(\frac{1}{2} + x) - g_h)\}$$
$$= \frac{1}{2} \{u(I(\frac{1}{2} - x) - g_\ell) - u(I(\frac{1}{2} - x) - g_h)\}$$

with

$$\text{(A8)} \quad D^2EV(x) = \frac{1}{2} D^2V(\frac{1}{2} + x) + \frac{1}{2} D^2V(\frac{1}{2} - x) > 0.$$

Further, at $x = 0$, it is direct that $DEV(x) = 0$ so that the marginal utility of information is positive and increasing over $0 < x \leq \frac{1}{2}$.

Thus, it is straignt forward that optimal information policy in the absence of distortions involves $x = \frac{1}{2}$.

Appendix B

Interaction of Information and Distortions

With a given probability of the low spending taxation state (p), individuals choose investment so as to maximize

$$\text{(B1)}\quad EU = u(y-I) + pu((1-\tau_\ell)I) + (1-p)u((1-\tau_h)I),$$

treating tax rates as invariant to their actions. But, taken together, agent's actions determine the tax rate.

Information, Distortion and Individual Investment

We start by analyzing investment, welfare and information with exogenous tax rates. The efficiency condition for private investment is

$$\text{(B2)}\quad -Du(y-I) + E\{(1-\tilde{\tau})Du((1-\tilde{\tau})I\} = 0.$$

To explore the implications of a change in p for investment given the levels of tax rates, we apply the implicit function theorem to (B2),

$$\text{(B3)}\quad D_1 I(p;\tau_h, \tau_\ell) = \frac{\{(1-\tau_\ell)Du((1-\tau_\ell)I) - (1-\tau_h)Du((1-\tau_h)I)\}}{-\{D^2u(y-I) + E((1-\tilde{\tau})^2D^2u((1-\tilde{\tau})I)\}} ,$$

where the denominator is positive due to the concavity of u( ). The sign of the numerator depends on the form of u( ), in ways that are familiar from the theory of saving (e.g., Sandmo (1970)). Fundamentally, the indeterminacy reflects the offseting income and substitution effects of the change in the interest rate distribution implied by p. An increase in p implies that there is a higher probability of the higher return, so if $p_a > p_b$ then the distribution of returns at p_a stochastically dominates that at p_b.

A sufficient condition for $D_1I(p;\tau_h, \tau_\ell)$ to be everywhere positive is that $\sigma(c) = -cD^2u(c)/Du(c) < 1$. To see this, write the numerator of (B3) as $\{c_\ell Du(c_\ell) - c_h Du(c_h)\}/I$, where $c_\ell = (1-\tau_\ell)I$ is second-period consumption in the low spending state, i.e., $c_\ell > c_h$. Then, it follows that

$$\text{(B4)}\quad c_\ell Du(c_\ell = \int_{c_h}^{c_\ell} \frac{d}{dz}[zDu(z)]\, dz + c_h Du(c_h)$$

$$= \int_{c_h}^{c_\ell} Du(z)[1-\sigma(z)]dz + c_h Du(c_h),$$

which implies $c_\ell Du(c_\ell) > c_h Du(c_h)$ when $\sigma(z) < 1$, as Du(z) is positive. This restriction is identical to the sufficient condition for the substitution effect of an interest rate change to globally dominate the income effect in the saving problem under certainty, where the efficiency condition $Du(y-I) = RDu(RI)$ yields $DI(R) = Du(RI)[1 - \sigma(RI)]/-\{D^2u(y-I) + R^2D^2u(RI)\}$.

Expected utility is increasing in p as previously, i.e.,

$$\text{(B5)}\quad D_1V(p;\ \tau_\ell,\ \tau_h) = u((1-\tau_\ell)I) - u((1-\tau_h)I) > 0$$

and

$$D_{11}V(p;\ \tau_\ell,\ \tau_h) = \{(1-\tau_\ell)Du((1-\tau_\ell)I) - (1-\tau_h)Du((1-\tau_h)I)\}DI(p) > 0.$$

Consequently, there is <u>private value</u> to information with tax distortions in the system. That is, if a single individual could obtain access to the signal--without any equilibrium adjustment in tax rates--it would prove valuable in adjusting investment. Formally, expected utility prior to receipt of information is

$$\text{(B6)}\quad EV(x;\ \tau_\ell,\ \tau_h) = \frac{1}{2}\, V(\frac{1}{2} + x;\ \tau_\ell,\ \tau_h) + V(\frac{1}{2} - x;\ \tau_\ell,\ \tau_h)$$

and thus

$$\text{(B7)}\quad D_1EV = \frac{1}{2}\, D_1V(\frac{1}{2} + x;\ \tau_\ell,\ \tau_h) - \frac{1}{2}\, D_1V(\frac{1}{2} - x;\ \tau_\ell,\ \tau_h).$$

As previously, this expression is zero at $x = 0$ and positive for $0 < x \leq \frac{1}{2}$, as

$$\text{(B8)}\quad D_{11}EV = \frac{1}{2}\, D_{11}V(\frac{1}{2} + x;\ \tau_\ell,\ \tau_h) + \frac{1}{2}\, D_{11}V(\frac{1}{2} - x;\ \tau_\ell,\ \tau_h) > 0.$$

Thus, there is private value to information.

Increases in tax rates which occur proportionnately are of some interest in the equilibrium analysis to be conducted below. Thus, we explore the effect of changing τ_ℓ, τ_h to $(1+d\phi)\tau_\ell$ and $(1+d\phi)\tau_h$.

$$\text{(B9)}\quad \left.\frac{\partial I(x;\ \phi\tau_\ell,\ \phi\tau_h)}{\partial\phi}\right|_{\phi = 1} = \frac{-E\{\tilde{\tau}(1-\sigma((1-\tilde{\tau})I))Du((1-\tilde{\tau})I)\}}{-\{D^2u(y-I) + E[(1-\tilde{\tau})^2D^2u((1-\tilde{\tau})I)]\}}$$

With $\sigma(\tilde{c})$ less than one, a proportionate increase in taxes unambiguously lowers investment.

Equilibrium Investment and Welfare

If aggregate investment rises exogenously, the the distribution of tax rate falls proportionately, $\partial\tilde{\tau}/\partial I = -\tilde{\tau}/I < 0$. Thus, the equilibrium response of investment to p is greater than the simple individual effect calculated above, i.e., if $I_e(p)$ is equilibrium investment,

$$\text{(B10)}\quad DI_e(p) = \frac{\partial I(p,\ \tilde{\tau})/\partial p}{\{1 + \frac{\partial I(p,\phi\tilde{\tau})}{\partial\phi}\Big|_{\phi=1}(\frac{I}{\tau})\}}\ .$$

The "multiplier" reflects the fact that increases in investment raises the tax bae and permits a lowering of tax rates. Notice that this involves an assumption that one is on the desirable side of the economy's "Laffer curve".

Analogously to the discussion above, we can define $V_e(p)$ as equilibrium expected utility with probability beliefs p, i.e.,

$$\text{(B11)}\quad V_e(p) = u(y-I_e(p)) + pu(I_e(p)-g_\ell) + (1-p)u(I_e(p) - g_h)$$

The effects of a change in p are

$$\begin{aligned}\text{(B12)}\quad DV_e(p) &= u(I_e(p) - g_\ell) - u(I_e(p) - g_h) \\ &\quad + \{-Du(y-I_e(p)) + EDu(I_e(p) - \tilde{g})\}DI_e(p) \\ &= u(I_e(p) - g_\ell) - u(I_e(p) - g_h) \\ &\quad + E\{\frac{\tilde{g}}{I_e(p)} Du(I_e(p) - \tilde{g})\}DI_e(p),\end{aligned}$$

where the final equality comes from an application of the individual's first-order condition.

Appendix C:

Random *Ex Post* Taxation of Investment

This appendix adapts Stiglitz's (1982, pp. 15-17) analysis of random *ex post* taxation to the savings/investment context so as to demonstrate that analyses of secrecy and random taxation are conceptually distinct.

Private agents face a random tax rate, i.e., $\tau \pm z$ with probability one half. They choose I so as to maximize

$$\text{(C1)} \quad EU = \frac{1}{2} U(y-I), \ (1-\tau-z)I) + \frac{1}{2} U(y-I, \ (1-\tau+z)I).$$

Let us denote the outcomes of this maximization routine as $I(z, \tau)$ and $EV(z, \tau)$. The revenue constraint of the government is

$$\text{(C2)} \quad g = \frac{1}{2} (\tau+z)I + \frac{1}{2} (\tau-z)I = \tau I.$$

Stiglitz (1982), pp. 15-17) shows that $D_1 EV(z, \tau) = 0$ for $z = 0$, which is the idea that an arbitrarily small amount of a fair bet has no utility consequence. Further, he shows that

$$\text{(C3)} \quad D_1^2 EV(z, \tau)\Big|_{z=0} = \frac{Du(y-I)I}{(1-\tau)} \left\{\sigma + \frac{d^2\tau}{(dz)^2} (1-\tau)\right\}$$

where $\sigma > 0$ is the elasticity of first-period marginal utility. Thus, following Stiglitz, small amounts of randomness are desirable only if