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Living with Risk

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LIVING WITH RISK*

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Abstract

Living with risk can lead to anticipatory feelings such as anxiety or hopefulness. Such feelings can affect the choice between lotteries that will be played out in the future - choice may be motivated not only by the (static) risks involved but also by the desire to reduce anxiety or to promote savoring. This paper provides a model of preference in a three-period setting that is axiomatic and includes a role for anticipatory feelings. It is shown that the model of preference can accommodate intuitive patterns of demand for information such as information seeking when a favorable outcome is very likely and information aversion when it is more likely that the outcome will be unfavorable. Behavioral meaning is given to statements such as "individual 1 is anxious" and "2 is more anxious than 1". Finally, the model is differentiated sharply from the classic model due to Kreps and Porteus.

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1. INTRODUCTION

Living with risk can lead to anticipatory feelings such as anxiety that the eventual outcome will be bad, or hopefulness that it will be good.[1] Such feelings can affect the choice between lotteries that will be played out in the future - choice may be motivated not only by the (static) risks involved but also by the desire to reduce anxiety or to promote savoring. This paper provides a model of preference in a three-period setting that is axiomatic and includes a role for anticipatory feelings. It is intuitive that the latter affect the demand for information. It is shown that the model of preference can accommodate intuitive patterns of demand for information such as information seeking when a favorable outcome is very likely and information aversion when it is more likely that the outcome will be unfavorable. Behavioral meaning is given to statements such as "individual 1 is anxious" and "2 is more anxious than 1". Finally, the model is differentiated sharply from the classic model due to Kreps and Porteus [9].

Consider risky prospects that pay off at a fixed time in the future. Standard expected utility maximizers care about the riskiness of prospects but they are indifferent to when risk is resolved. Thus it is not possible within the standard framework to distinguish between individuals who prefer early resolution, perhaps because they are anxious and cannot bear to live with risk, and those who prefer to delay resolution, perhaps because they wish to savor the prospect (or illusion) of a favorable outcome. Kreps and Porteus, henceforth KP, permit such a distinction. A key to their model is expansion of the domain of objects of choice from the set of lotteries (or lottery streams) to the domain of dynamic choice problems (p. 187). The latter includes in particular, the set of multi-stage or temporal lotteries that distinguish between risks according to their temporal resolution.

We adopt the KP domain, though specialized to our setting of three periods and terminal payoffs.[2] The common domain permits a sharp comparison of the two models. The key difference is that the KP model violates our central axiom, called Expected Stationarity, the essence of which is the assumption that for lotteries that resolve next period, the agent expects her future preference over such lotteries to be the same as her current one, that is, the ranking of such "one-step-ahead lotteries" is expected to be independent of the calendar date. Given such an expectation of stationarity, we show that if the agent cares about

[1] See Caplin and Leahy [2] for references to psychological research on anticipatory emotions.

[2] Henceforth, when referring to the Kreps-Porteus model, the intention is to this specialization of their model.

the temporal resolution of risk, then she will be led to value commitment. In contrast, commitment is never valuable within the KP model. This is due to their implicit assumption, made explicit here, that risk preferences are expected to be nonstationary. Since such nonstationarity is arguably unnatural in many settings, our analysis raises questions about the suitability of the KP model as a model of living with risk.[3]

Consider a concrete example - we focus on savoring, though examples highlighting anxiety can also be constructed (see Section 5.1). There are three time periods, $0, 1, 2$. Let p and q be two lotteries, where p represents a lotto ticket that is resolved at time 2 and q is riskless. In choosing between them at time 0, the agent is influenced not only by the riskiness of p, but also by the long time interval during which she can savor the possibility of winning a large prize. The latter may dominate and lead to the choice of p over q. (In the formal model, the choice of p amounts to the commitment to receiving p at time 1. Thus, while she may not physically hold the ticket at 0, she is certain then that the ticket will be her's at time 1, and thus it is intuitive that she may already begin to savor the possibility of a good outcome.) The influence of savoring on choice is demonstrated by considering also the choice, still at time 0, between q and the hypothetical lottery $\widehat{p}$, where $\widehat{p}$ has the same probability distribution over outcomes as does p, but differs from p in that it is completely resolved at time 1. Since savoring with regard to $\widehat{p}$ is limited to a shorter period, risk aversion may dominate and lead to the time 0 strict preference for q over $\widehat{p}$. Consider finally the choice at 0 between $\{p\}$ and the menu $\{p, q\}$, where $\{p\}$ represents commitment to p as above, and $\{p, q\}$ represents the option of deferring to period 1 the decision between p and q. We argue that the preceding, including the strict preference for q over the hypothetical lottery $\widehat{p}$, plus "stationarity", imply the value of commitment, specifically the strict preference for $\{p\}$ over $\{p, q\}$: the latter ranking hinges on what choice the agent expects to make out of $\{p, q\}$ at time 1 should she choose that menu at 0. But the comparison at 1 between p and q is completely analogous to that between $\widehat{p}$ and q at time 0 - in particular, in both cases, the agent compares the deterministic prize q with a lotto ticket that is realized in the next period. Thus, if there is no reason for the difference in calendar dates to matter,[4] the agent should

[3] There are other reasons for caring about the temporal resolution of risk - one is that early resolution may facilitate planning - and KP are noncommital about which story they have in mind. Our objection is only to the anxiety, or living-with-risk, story.

[4] Calendar dates may matter if savoring applies also to deterministic outcomes. Then the choice between p and q at time 1 is different from the hypothetical choice problem at 0, because

expect her time 1 ranking to agree with her time 0 preference for q over $\widehat{p}$ (this is the expectation of stationarity referred to above). Therefore, she would expect the menu $\{p, q\}$ to lead ultimately to the choice of q. But, as described above, p is the "correct" choice from the time 0 perspective which attaches a large weight to savoring. Foreseeing all this, she would prefer to commit herself and choose $\{p\}$ over $\{p, q\}$. Note that *commitment is valuable only* because of a difference in the evaluations of p versus $\widehat{p}$, that is, *because the temporal resolution of a given risk matters*. Finally, commitment is not valuable in the KP model because they implicitly assume that the agent expects her time 1 ranking of p versus q to coincide with her ranking at time 0, even though the importance of savoring is presumably different in those two situations.

Caplin and Leahy [2] emphasize that dynamic inconsistency arises naturally in the presence of anticipation. They propose a model where preferences are defined not only over temporal lotteries (of the sort considered by KP) but also over "psychological lotteries." Elsewhere [3], they acknowledge that such an expanded domain poses challenges for gathering evidence, and they suggest that surveys, physiological measures and brain scans might serve as sources of evidence. Here we follow the more traditional revealed preference approach in which economic choices alone constitute the relevant evidence.

Given "changing tastes" as above, behavior could be determined as in Strotz [16], by assuming that the agent chooses the plan that is optimal amongst those that will be implemented. We adapt instead the alternative approach put forth by Gul and Pesendorfer in a series of papers, whereby a single preference, albeit over choice problems rather than over lotteries, determines the choice of plans.[5] A choice problem, which limits options for actions ex post, is selected ex ante. The individual's expectation is that later, when she decides on an action, she may be tempted to deviate from the choice that would be optimal ex ante were commitment possible. In the general model of [6], henceforth GP, self-control might be exerted and the temptation resisted. A special case assumes no self-control and this model is closest to ours. The GP analysis does not apply directly, however, because they assume the Independence axiom and, for reasons that will be evident later, we do not. Neither is the later paper [7] directly applicable. Here

in the former consumption occurs in the next period while in the latter consumption lies two periods into the future. We focus on modeling "living with risk" and thus assume that savoring and anxiety are limited to risky prospects.

[5] Gul and Pesendorfer [7] describe advantages of their approach, which apply also to our setting.

the authors deal exclusively with temptation in the absence of self-control, but they restrict themselves to finite choice problems (and thus Independence is not meaningful). We deal with continuous choice problems without Independence. However, we do not claim any technical novelty. In particular, the axiomatics are simple in our setting, because the domain (including *two-stage* lotteries) and our specific story of why temptation arises facilitate a straightforward axiomatic characterization; this is because, under suitable assumptions, they allow us to express the "temptation ranking" explicitly in terms of the given preference over choice problems.

In spite of our results being simple from a technical point of view, we feel they are useful for four (related) reasons. First, it has not been noted previously that a GP-style temptation model can be applied to capture anticipatory feelings. Second, the results demonstrate that anticipatory emotions can be accommodated within a revealed preference framework. Third, they cast new light on the seminal model of KP. Finally, the model suggests a positive answer to the question posed by Eliaz and Spiegler [5] "can anticipatory feelings explain anomalous choices of information sources?" In particular, the intuitive pattern of information demand described in the opening paragraph above can be accommodated (see Section 5).

2. THE MODEL

2.1. Random Menus

There are three periods, $t = 0, 1, 2$. Let Z denote a space of outcomes. We assume that Z is compact metric and connected. Elements of $\mathcal{K}(\Delta(Z))$ are called *menus*.[6] Objects of choice at time 0 are lotteries over menus, or *random menus*. Thus preference $\succeq$ is defined on $\Delta(\mathcal{K}(\Delta(Z)))$.

For interpretation, see the time line below. At $t = 0$, the agent chooses a random menu X. At $t = 1^-$, X delivers a menu $x \in \mathcal{K}(\Delta(Z))$. At $t = 1$, she chooses $p \in x$, and finally, all risk is resolved and an outcome is realized at $t = 2$. Note that the time line is intended as a description of the agent's perception when she evaluates random menus at time 0. Thus the time 1 choice of p from x is her time 0 expectation of what she will do if facing the menu x.

[6] For any compact metric space Y, $\Delta(Y)$ denotes the space of Borel probability measures endowed with the weak convergence topology, and $\mathcal{K}(Y)$ denotes the space of compact subsets of Y endowed with the Hausdorff metric. Both $\Delta(Y)$ and $\mathcal{K}(Y)$ are compact metric. Finally, δ_y is the probability measure on Y that assigns probability 1 to y.

$$-_{choose\quad X\in\Delta(\mathcal{K}(\Delta(Z)))}\bullet - - - - - - - - - -_{choose\ p\in x}\bullet - - - - - - - - - -_{payoff}\bullet - -$$

Think of any random menu X as modeling a physical action undertaken at time 0. Such an action determines, along with stochastic factors described by the probabilities prescribed by X, a set of options for further action at time 1; the latter are modeled by lotteries over Z, the set of outcomes or payoffs. A random menu X that has support on singleton menus leaves room for only trivial choices at time 1 and commits the agent to a two-stage lottery. In fact, the set $\Delta(\Delta(Z))$ of two-stage lotteries, can be identified with the set of elements in $\Delta(\mathcal{K}(\Delta(Z)))$ that provide commitment.

The choice of the set of random menus as our domain can be "rationalized" as follows:[7] the proper domain should include both the set $\Delta(\Delta(Z))$ of two-stage lotteries, in order to address the attitude towards the temporal resolution of risk, and also $\mathcal{K}(\Delta(Z))$, in order to model the demand for commitment. Indeed, our central axioms and the principal content of our model concern only preference restricted to their union $\mathcal{D}$,

$$\mathcal{D} = \Delta(\Delta(Z)) \cup \mathcal{K}(\Delta(Z)). \tag{2.1}$$

We could have adopted $\mathcal{D}$ as the domain of preference. We chose instead to adopt the larger domain $\Delta(\mathcal{K}(\Delta(Z)))$ because: (i) it is unifying and more elegant; (ii) preference can be extended uniquely from $\mathcal{D}$ to $\Delta(\mathcal{K}(\Delta(Z)))$ under relatively mild assumptions (as explained below); (iii) KP also use $\Delta(\mathcal{K}(\Delta(Z)))$ as their domain (specializing their model to our simpler setting of three periods and terminal payoffs), and having a common domain facilitates comparison of the two models, which is a principal objective; and (iv) the larger domain broadens the range of applicability of the model to include the (arguably typical) case where the set of options available at time 1 is not entirely within the agent's control, but depends also on stochastic factors.

The adoption of a three-period horizon is not innocuous. It is well-known that Strotz-like representations may not be well-defined given longer horizons (see Peleg and Yaari [11], for example). To accommodate an arbitrary finite horizon, Gul and Pesendorfer [7] adopt two alternative strategies. In one, they limit the agent to finite choice problems; in the second, they show in a setting with infinitely

[7] A different domain is called for if one wishes to admit subjective beliefs. For that purpose, one could take as domain $\Delta(\mathcal{K}(\mathcal{H}))$, where $\mathcal{H}$ is the set of Anscombe-Aumann acts over a state space S. The analysis to follow can be adapted to this domain.

many choices that Strotz-like behavior can be approximated arbitrarily well by a well-defined representation. We suspect that both strategies could be adapted to our setting, but we have chosen instead to focus on the simplest framework that permits modeling the notion of living with risk.

2.2. Axioms

We adopt several axioms for the binary relation $\succeq$ on $\Delta(\mathcal{K}(\Delta(Z)))$.

Axiom 1 (Order). *$\succeq$ is complete and transitive.*

Identify $\mathcal{K}(\Delta(Z))$ with a subset of $\Delta(\mathcal{K}(\Delta(Z)))$, where x is identified with δ_x. Then $\succeq$ induces a ranking of menus, also denoted by $\succeq$. Thus we often write $x' \succeq x$ rather than $\delta_{x'} \succeq \delta_x$.

In the standard model, a menu is as good as the best alternative that it contains, a property that is captured by the following axiom:

Strategic Rationality (SR): For all menus x and y, $x \succeq y \Longrightarrow x \sim x \cup y$.

Strategic Rationality is not intuitive in our setting, as illustrated by the example in the introduction. To see the intuition in slightly more general terms, consider the agent at time 0 evaluating the menu $x \subset \Delta(Z)$ from which a choice p is to be made at time 1. Her ex ante view of these lotteries includes not only the risk associated with each p, but also the fact that she will have to live with this risk for two periods - anticipatory feelings or anxiety affect her evaluation of each lottery and therefore also of x. But these are less relevant at time 1 and thus she may view lotteries differently then. Being forward-looking, she foresees this consequence of the passage of time when evaluating x, or when choosing between any two menus. As a result, she may value commitment and thus violate SR.

For example, suppose that $\{p\} \succeq \{q\}$, reflecting the fact that at 0, when she must live with risk for 2 periods, she would prefer to commit to p rather than to q. SR would require that $\{p, q\} \sim \{p\}$. This is possible here if p is preferable also at 1. But suppose that at 1, when savoring and anxiety are less important, that q is more attractive. Then she will choose q if it is feasible, that is, if $\{p, q\}$ is chosen at 0. Thus both $\{p, q\}$ and $\{q\}$ lead to the ex post choice of q. All this is foreseen. Therefore, $\{p, q\} \sim \{q\}$. This is the intuition for the following weakening of SR:

Axiom 2 (No Self-Control (NSC)). *For all x and y,*

$$x \cup y \sim x \text{ or } x \cup y \sim y. \tag{2.2}$$

The axiom strengthens GP's central axiom Set-Betweenness so as to express a lack of self-control. In our case, if at time 1, when anticipatory emotions are not as important as they were ex ante, the agent is tempted to choose a lottery that was not optimal ex ante under commitment, there is no reason for her to resist. That is, she does not exert self-control in the face of such temptations.

According to NSC, for every pair of menus with $x \succ y$, either there is no temptation ($x \sim x \cup y$), or ex post choice is from the tempting menu ($x \cup y \sim y$). We now go further and specify circumstances when each case obtains. Consider lotteries over Z that are resolved at time 2. The noted circumstances center on how such lotteries are evaluated from the perspectives of times 0 and 1. Usually it is assumed that the ranking of lotteries that are resolved and paid at a fixed time T is the same regardless of when this ranking is done. This is decidedly not the case here - anticipatory feelings depend on the temporal distance from T; in the present three-period setting, they are presumably more important at time 0. Thus we consider both perspectives $t = 0$ and $t = 1$ explicitly.

Consider the ranking of lotteries $\Delta(Z)$ at time 0 when the agent can commit. Such rankings take the form $\delta_{\{p'\}} \succeq \delta_{\{p\}}$, or given the notational convention introduced above, $\{p'\} \succeq \{p\}$. Since the above lotteries are not resolved until time 2, they constitute *delayed risks* from the perspective of time 0. To emphasize this, we introduce special notation, and we define the order $\succeq_{del}$ on $\Delta(Z)$ by

$$p' \succeq_{del} p \text{ iff } \{p'\} \succeq \{p\}.$$

Given any lottery over Z, we can imagine it alternatively playing out earlier, at time 1. Thus, for any lottery p in $\Delta(Z)$ having finite support, $p = \Sigma_z p(z)\,\delta_z$, define the random menu $X_p \in \Delta(\Delta(Z))$ by

$$X_p = \Sigma_z p(z)\,\delta_{\{\delta_z\}}.$$

Thus X_p yields the terminal payoff z with probability $p(z)$, just as does p, but for X_p the outcome will be known at time 1.[8] Therefore, from the perspective of time 0, any such X_p constitutes an *immediate risk*. More generally, for any $p \in \Delta(Z)$, the immediate risk corresponding to p is the two-stage lottery $X_p \in \Delta(\Delta(Z))$ defined by:

$$X_p(B) = p\left(e^{-1}(B)\right),$$

for any measurable $B \subset \Delta(Z)$, where $e : Z \to \Delta(Z)$ is the natural embedding, $e(z) = \delta_z$. We introduce notation also for the ranking of immediate risks: let

[8] In the introductory example, X_p was denoted by $\widehat{p}$.

$\succeq_{imm}$ be the ranking on $\Delta(Z)$ be defined by

$$p' \succeq_{imm} p \text{ if } X_{p'} \succeq X_p.$$

Then $p' \succeq_{imm} p$ indicates the time 0 preference for the risk or lottery p' over p when these are to be resolved next period, while $p' \succeq_{del} p$ indicates the same ranking when the two lotteries are to be resolved only at time 2.

Next consider the perspective of time 1. Though we are given only the time 0 preference $\succeq$, it suggests a time 1 perspective as we now show. At the intermediate time, objects of choice are lotteries that are resolved one period later, that is, risks that are immediate from the time 1 perspective. At time 0, immediate risks are ranked via $\succeq_{imm}$. It follows that in a 'stationary' environment, where the calendar date alone is not important, the time 1 ranking of lotteries should also be given by $\succeq_{imm}$. Suppose that the agent foresees these time 1 preferences. Then she foresees choosing lotteries out of menus so as to maximize $\succeq_{imm}$. But at time 0, those lotteries constitute delayed risks and thus are ranked according to $\succeq_{del}$. Thus where these orders disagree, she will prefer to limit options for future choice. Specifically, we adopt:

Axiom 3 (Expected Stationarity). *For all lotteries p' and p in $\Delta(Z)$, $\{p'\} \succ \{p', p\}$ if and only if $p' \succ_{del} p$ and $p' \prec_{imm} p$.*

Above we gave intuition for the 'if' part of the axiom. The converse ensures that the difference between $\succ_{del}$ and $\succ_{imm}$, and hence the effects of differing temporal distance from the time of resolution, are the *only* reason for commitment. (There is an implicit tie-breaking rule: when $p' \sim_{imm} p$, she assumes that she will make the choice that is best according to $\succeq_{del}$.) In its absence, the remainder of the axiom ('if') could, for example, be satisfied vacuously if $\succ_{del}$ and $\succ_{imm}$ are identical, and commitment could be valuable for reasons having nothing to do with savoring or anxiety. (See the example in Section 4 labeled 'Time-varying risk aversion'.)

We offer three more remarks on the axiom. First, though its interpretation refers to expectations about future behavior, the axiom is exclusively an assumption about the time 0 ranking of random menus. Second, note that the demand for commitment expressed in Expected Stationarity could be due either to anxiety or to savoring. Finally, if our model is truly about living with risk, then there should not be any demand for commitment when the prospects involved are deterministic; in other words, it should be that *$\succeq_{del}$ and $\succeq_{imm}$ agree on Z*. But that is easily seen to be true:

$$\delta_{z'} \succeq_{imm} \delta_z \Longleftrightarrow \delta_{\{\delta_{z'}\}} \succeq \delta_{\{\delta_z\}} \Longleftrightarrow \delta_{z'} \succeq_{del} \delta_z. \tag{2.3}$$

Some form of continuity is needed. GP observe that continuity of preference (closed weakly better-than and weakly worse-than sets) is not to be expected in a model of temptation without self-control, and they use a weaker form of continuity (Axioms 2a-2c). The following adapts their axioms to our setting.

Axiom 4 (Limited Continuity). *(a) Upper Semi-Continuity: The sets* $\{y \in \mathcal{K}(\Delta(Z)) : y \succeq x\}$ *are closed.*

(b) Lower Singleton Continuity: The sets $\{p' \in \Delta(Z) : p' \preceq_{del} p\}$ *and* $\{p' \in \Delta(Z) : p' \preceq_{imm} p\}$ *are closed.*

(c) For every $x \in \mathcal{K}(\Delta(Z))$, *there exists* $p \in \Delta(Z)$ *such that* $x \sim \{p\}$.

Parts (a) and (b) imply that both $\succeq_{del}$ and $\succeq_{imm}$ are continuous. We use condition (c) to prove that $\succeq$ has a utility function representing it on $\Delta(\mathcal{K}(\Delta(Z)))$, which is otherwise not guaranteed in the absence of continuity. (GP exploit instead the Independence axiom to prove existence of a representation.)

Let Y be any (compact metric) space and $\gtrapprox$ an order on $\Delta(Y)$. Say that $\gtrapprox$ is *FSD-increasing* if, for all lotteries p' and p in $\Delta(Y)$, $p' \gtrapprox p$ whenever

$$p'(\{y : \delta_y \gtrapprox \delta_{y^*}\}) \geq p(\{y : \delta_y \gtrapprox \delta_{y^*}\}) \text{ for every } y^* \text{ in } Y, \tag{2.4}$$

that is, if, for every y^*, the set of outcomes better than y^* according to $\gtrapprox$ has larger probability under p' than under p; refer also to any representing utility function as FSD-increasing. If (2.4) is satisfied, write $p' \gtrapprox^{FSD} p$, which is to be read as "p' first-order stochastically dominates p with respect to the order on Y induced by $\gtrapprox$". The preceding applies in particular to $\succeq$, an order on $\Delta(\mathcal{K}(\Delta(Z)))$, and to $\succeq_{imm}$ and $\succeq_{del}$, both of which are orders on $\Delta(Z)$. In all of these cases, the indicated sets $\{y : \delta_y \gtrapprox \delta_{y^*}\}$ are closed, hence measurable, by Limited Continuity.

The assumption that preference on a space of lotteries is FSD-increasing is common and is not especially problematic for our setting. Therefore, we assume:

Axiom 5 (Monotonicity). *Both* $\succeq$ *and* $\succeq_{del}$ *are FSD-increasing.*

Given the other axioms, if $\succeq$ is FSD-increasing, then so is $\succeq_{imm}$.[9] However, that $\succeq_{del}$ is FSD-increasing is not implied, and the axiom imposes this requirement separately.

Recall the discussion of domain surrounding (2.1). Apart from Order, the preceding axiom is the first to restrict preference outside of $\mathcal{D} = \Delta(\Delta(Z)) \cup \mathcal{K}(\Delta(Z))$. In fact, as the proof of our representation result makes clear, a characterization of utility on $\mathcal{D}$ alone, rather than on all of $\Delta(\mathcal{K}(\Delta(Z)))$, can be obtained under the weaker assumption that $\succeq$ is FSD-increasing when restricted to $\Delta(\Delta(Z))$. Thus the 'cost' paid for employing the entire domain of random menus rather than the subdomain $\mathcal{D}$ is the difference between assuming that $\succeq$ is FSD-increasing on $\Delta(\mathcal{K}(\Delta(Z)))$ rather than only on $\Delta(\Delta(Z))$.

To understand the role of the stronger FSD assumption, note that it implies: two random menus X' and X must be indifferent if they induce the same distribution, that is, if there is equality in the appropriate form of (2.4) for every menu y^* in $\mathcal{K}(\Delta(Z))$. If we denote the latter condition by $X' \approx^{FSD} X$, then this implication can be written in the form

$$X' \approx^{FSD} X \implies X' \sim X. \tag{2.5}$$

Now suppose that $\succeq^0$ is defined only on $\mathcal{D} = \Delta(\Delta(Z)) \cup \mathcal{K}(\Delta(Z))$, and that it is complete and transitive and satisfies Limited Continuity. Then $\succeq^0$ can be *extended uniquely* to a binary relation $\succeq$ on all of $\Delta(\mathcal{K}(\Delta(Z)))$ that is complete and transitive and that satisfies (2.5):[10] let $X = \Sigma_x X(x)\,\delta_x$ and suppose that $x \sim^0 \{p_x\}$. Observe that $\Sigma_x X(x)\,\delta_{\{p_x\}}$ is a two-stage lottery, hence an element of $\mathcal{D}$, and that $\Sigma_x X(x)\,\delta_{\{p_x\}} \approx^{FSD} X$. Then define $\succeq$ by: for any two random menus X' and X,

$$X' \succeq X \text{ if } \Sigma_x X'(x)\,\delta_{\{p_x\}} \succeq^0 \ \Sigma_x X(x)\,\delta_{\{p_x\}}. \tag{2.6}$$

It follows from (2.5) that $\succeq$ is well defined (and the other claims above are readily proven). Since $\succeq$ is determined uniquely by $\succeq^0$, the model's content lies primarily in the latter, that is, in the nature of preference on $\mathcal{D}$.

[9] Given a menu x^*, Limited Continuity and connectedness of Z imply that there exists z^* in Z such that $\{\delta_{z^*}\} \sim x^*$. Then $\{z : \{\delta_z\} \succeq x^*\} = \{z : \{\delta_z\} \succeq \{\delta_{z^*}\}\}$. It follows, using also (2.3), that $p' \succeq_{imm}^{FSD} p \Longrightarrow X_{p'} \succeq^{FSD} X_p \implies X_{p'} \succeq X_p \Longrightarrow p' \succeq_{imm} p$.

[10] We give a proof only for finite-support lotteries. That is, denote by $\Delta_s(\cdot) \subset \Delta(\cdot)$ the subset of finite-support, or *simple*, lotteries. Then $\succeq^0$ is assumed to be defined on $\Delta_s(\Delta(Z)) \cup \mathcal{K}(\Delta(Z))$ and it is extended to a binary relation $\succeq$ on $\Delta_s(\mathcal{K}(\Delta(Z)))$.

3. UTILITY

The two primitive components of the functional form are utility functions $U, V : \Delta(Z) \to \mathbb{R}$; they will represent $\succeq_{del}$ and $\succeq_{imm}$ respectively. We assume that they are continuous, FSD-increasing and that they are ordinally equivalent on Z. Refer to any pair of utility functions (or corresponding orders) satisfying these properties as *compatible*. Then it is wlog, by taking a monotonic transformation of U or V, to assume that

$$V(\delta_z) = U(\delta_z) \text{ for every } z \text{ in } Z. \tag{3.1}$$

(To see why, ordinal equivalence on Z implies that $V(\delta_z) = \phi(U(\delta_z))$ for some strictly increasing and continuous

$$\phi : U(Z) \equiv \{U(\delta_z) : z \in Z\} \to \mathbb{R}.$$

Since Z is connected, $U(Z) = U(\Delta(Z))$, that is, for every p there exists z such that $U(p) = U(\delta_z)$. Therefore, ϕ is strictly increasing and continuous on $U(\Delta(Z))$, and $\phi(U)$ is ordinally equivalent to U on $\Delta(Z)$. Then (3.1) is satisfied if we use $\phi(U)$ in place of U.)

To describe how U and V determine a utility function on the entire domain $\Delta(\mathcal{K}(\Delta(Z)))$, we proceed in stages. Since the formulae for finite support lotteries are more transparent, we define utility first for $\Delta_s(\mathcal{K}(\Delta(Z)))$, the set of *simple* random menus. We do this by describing first how utility is defined on menus $(\mathcal{K}(\Delta(Z)))$, then how it is defined on the set $\Delta_s(\Delta(Z))$ of (simple) two-stage lotteries, and finally, how it is extended to all simple random menus. Finally, we extend the definition of utility to all random menus.

Consider the agent at time 0 evaluating a menu $x \subset \Delta(Z)$ from which a choice p is to be made at time 1. Think of U as describing the time 0 valuation of lotteries to be played out beginning at time 1 (delayed risks), and suppose that she expects V to describe risk preferences at time 1. Suppose further that the agent anticipates that she will not exert self-control at time 1. Therefore, she expects the time 1 choice out of any menu x to maximize V; maximization of U enters only when there is indifference according to V. This leads to the Strotz-like utility for any menu given by

$$\mathcal{U}(x) = \max\left\{U(p) : p \in \arg\max_{p' \in x} V(p')\right\}. \tag{3.2}$$

In particular,

$$\mathcal{U}(\{p\}) = U(p), \tag{3.3}$$

so that $U(\cdot)$ ranks delayed risks, and hence represents $\succeq_{del}$. Note also that strategic rationality is satisfied if and only if U and V describe the same risk preferences (that is, they are ordinally equivalent on $\Delta(Z)$).

Next define utility on $\Delta_s(\Delta(Z))$. Let $X = \Sigma_p X(\{p\})\,\delta_{\{p\}}$ be a two-stage lottery. Since X provides perfect commitment, its evaluation is based on the time 0 perspective alone - there is no conflict with later preferences and thus no reason to violate recursivity. Therefore, utility is computed by backward induction: for each p that is realized at the first stage, replace it by a certainty equivalent $z_p \in Z$. In this way, X is transformed into the single-stage lottery $\widehat{X} = \Sigma_p X(\{p\})\,\delta_{\{\delta_{z_p}\}}$, which is assigned a suitable utility level. The question is how to compute certainty equivalents at the second stage and utility levels at the first stage. The function U is used to compute certainty equivalents, that is, z_p is defined as any outcome in Z satisfying

$$U(p) = U\left(\delta_{z_p}\right). \tag{3.4}$$

(There exists such a z_p because U is continuous and Z is connected.) We use U because each p is a delayed risk (it is resolved only at time 2) and because, as just shown, U gives the utility of delayed risks. On the other hand, the single stage lottery $\widehat{X}$ constructed above has all risk resolved by time 1 - at that point, the agent will receive some δ_{z_p} and thus she will be certain that z_p will be forthcoming at time 2. The utility function V is used to evaluate immediate risks. Putting the two steps together yields the following expression for the utility of X:

$$\mathcal{U}\left(\Sigma_p X(\{p\})\,\delta_{\{p\}}\right) = V\left(\Sigma_p X(\{p\})\,\delta_{z_p}\right), \tag{3.5}$$

where z_p is *any* solution to (3.4).[11]

The preceding expression applies in particular to a two-stage lottery $X = \Sigma_p X(\{p\})\,\delta_{\{p\}}$ that is an immediate risk. Then each p is degenerate, $X = \Sigma_z X(z)\,\delta_{\{\delta_z\}}$, and, taking z to be a certainty equivalent for δ_z,

$$\mathcal{U}(X) = \mathcal{U}\left(\Sigma_z X(z)\,\delta_{\{\delta_z\}}\right) = V\left(\Sigma_z X(z)\,\delta_z\right), \tag{3.6}$$

that is, V represents $\succeq_{imm}$.

[11] We show in the theorem below, using (3.1), that utility is well-defined on $\Delta_s(\Delta(Z))$, that is, (i) the right side of (3.5) is invariant to the choice of z_p's, and (ii) the utility values defined by (3.5) and (3.2) agree on the intersection $\Delta_s(\Delta(Z)) \cap \mathcal{K}(\Delta(Z))$.

On the surface, there may appear to be a contradiction between the way we arrived at (3.2) versus the way in which we implemented the recursive calculation (3.5). In the context of the former, we referred to V as describing expected risk preferences at time 1, while in the latter, V was used to evaluate an immediate risk from the perspective of time 0. These dual roles for V are perfectly consistent and reflect our axiom Expected Stationarity - the expectation that 'one-step-ahead risks" will be evaluated in the same way regardless of the calendar date.

Finally, with regard to (simple) two-stage lotteries, there is indifference to the temporal resolution of risk if and only if U and V are ordinally equivalent on $\Delta(Z)$, which, in turn, is equivalent to strategic rationality.

Thus far we have defined utility $\mathcal{U}$ on $\Delta_s(\Delta(Z)) \cup \mathcal{K}(\Delta(Z))$. But it follows from the discussion at the end of the previous section that $\mathcal{U}$ can be extended uniquely to $\Delta_s(\mathcal{K}(\Delta(Z)))$ in such a way as to be FSD-increasing. Translating the argument there, specifically (2.6), into utility terms, we can write the utility of any finite-support random menu $X = \Sigma_x X(x)\,\delta_x$ in the form

$$\mathcal{U}(\Sigma_x X(x)\,\delta_x) = \mathcal{U}\left(\Sigma_x X(x)\,\delta_{\{p_x\}}\right), \tag{3.7}$$

for any $p_x \in \Delta(Z)$ such that $\mathcal{U}(x) = \mathcal{U}(\{p_x\})$ for all x.[12]

Example (Linear model): Let U and V be continuous and linear. In order that they be ordinally equivalent on Z (and satisfy the normalization (3.1)), let[13]

$$V(p) = \Theta^{-1}(E_p\Theta(u)),$$

for some $\Theta : U(\Delta(Z)) \to \mathbb{R}$ strictly increasing and continuous. Then U and V are compatible (any linear utility function is FSD-increasing).

Denote by u the vNM index of U. The utility of a (nonrandom) menu x is

$$\mathcal{U}^{lin}(x) = \max\left\{E_p u : p \in \arg\max_{p' \in x} E_{p'}\Theta(u)\right\}.$$

[12] The utilities of $\Sigma_x X(x)\,\delta_{\{p_x\}}$, a two-stage lottery, of every menu x and of every delayed risk p_x have already been defined. In addition, any lottery p_x that is maximizing in (3.2) satisfies $\mathcal{U}(x) = U(p_x) = \mathcal{U}(\{p_x\})$.

[13] E_p denotes expectation with respect to p.

Evidently, preference is strategically rational if and only if Θ is linear (U and V represent the same risk preferences). The utility of any two-stage lottery $X \in \Delta_s(\Delta(Z))$ is

$$\mathcal{U}^{lin}(X) = \Theta^{-1}\left(\Sigma_x X(\{p\})\,\Theta(E_p u)\right).$$

There is indifference to the temporal resolution of risk if and only if Θ is linear, thus tying together violations of strategic rationality and the nonreduction of two-stage lotteries.

The unifying expression that describes the utility of any random menu $X \in \Delta_s(\mathcal{K}(\Delta(Z)))$ is

$$\Theta\left(\mathcal{U}^{lin}(X)\right) = \Sigma_x X(x)\,\Theta\left(\max\left\{E_p u : p \in \arg\max_{p' \in x} E_{p'}\Theta(u)\right\}\right).$$

This completes the example.

The preceding example has an obvious extension to nonsimple random menus. The same is true of our general model as we now describe. Specifically, we extend the utility specification (3.5) and (3.7); the specification (3.2) is unaltered and defines utility $\mathcal{U}$ on $\mathcal{K}(\Delta(Z))$.

Let $\theta : \Delta(Z) \to Z$ be any measurable map such that $U(p) = U\left(\delta_{\theta(p)}\right)$ for all p.[14] Then any $X \in \Delta(\Delta(Z))$ induces the measure $X \circ \theta^{-1}$ on Z defined in the usual way by[15]

$$\left(X \circ \theta^{-1}\right)(B) = X\left(\theta^{-1}(B)\right), \;\; B \subset Z \text{ measurable.}$$

Define $\mathcal{U}$ on $\Delta(\Delta(Z))$ by

$$\mathcal{U}(X) = V\left(X \circ \theta^{-1}\right). \tag{3.8}$$

For the generalization of (3.7), let $\phi : \mathcal{K}(\Delta(Z)) \to \Delta(Z)$ be any measurable map such that $\mathcal{U}(x) = U\left(\delta_{\phi(x)}\right)$. Then $X \in \Delta(\mathcal{K}(\Delta(Z)))$ implies that $X \circ \phi^{-1} \in \Delta(\Delta(Z))$, where utility is defined above. Define $\mathcal{U}$ on $\Delta(\mathcal{K}(\Delta(Z)))$ by:

$$\mathcal{U}(X) = \mathcal{U}\left(X \circ \phi^{-1}\right). \tag{3.9}$$

[14] Existence of such a measurable map follows from Filipov's Implicit Function Lemma [1, p. 507]. Similarly, for ϕ below.

[15] More generally, adopt the following notation: Let (S_i, Σ_i) be measurable spaces for $i = 1, 2$, m_1 a measure on Σ_1, and $\phi : (S_1, \Sigma_1) \to (S_2, \Sigma_2)$ a measurable map (S_2-valued random variable). Then $m_1 \circ \phi^{-1}$ denotes the measure on Σ_2 induced by m_1 and the random variable ϕ, that is, $\left(m_1 \circ \phi^{-1}\right)(B_2) = m_1\left(\phi^{-1}(B_2)\right)$ for all B_2 in Σ_2.

This completes the utility specification.

We can now state our main result.

Theorem 3.1. *(a) Utility is well-defined on* $\Delta(\mathcal{K}(\Delta(Z)))$ *by (3.2), (3.8) and (3.9). The corresponding preference* $\succeq$ *satisfies Order, No Self-Control, Expected Stationarity, Limited Continuity and Monotonicity. Finally,* U *and* V *represent* $\succeq_{del}$ *and* $\succeq_{imm}$ *respectively.*

(b) Let $\succeq$ *be a binary relation on* $\Delta(\mathcal{K}(\Delta(Z)))$ *satisfying the axioms in (a). Then there exists a compatible pair of utility functions* $U, V : \Delta(Z) \longrightarrow \mathbb{R}$ *such that* $\succeq$ *admits a representation of the form (3.2), (3.8) and (3.9). Moreover, preference is represented in this way also by* $U', V' : \Delta(Z) \longrightarrow \mathbb{R}$ *if and only if they are ordinally equivalent to* U *and* V *respectively.*

The bulk of the proof is provided in the appendix, but the uniqueness property asserted in (b) is easily understood. First, if U' and V' also represent the preference $\succeq$ via (3.2), (3.8) and (3.9), then, by (a), they represent $\succeq_{del}$ and $\succeq_{imm}$, as do U and V - hence the asserted ordinal equivalences. Conversely, it is easily seen that the definition of (ordinal) utility $\mathcal{U}$ on $\Delta(\mathcal{K}(\Delta(Z)))$ uses only the ordinal properties of U and V.

It is noteworthy, at both conceptual and practical levels, that the model described in the theorem is completely specified by a compatible pair of (ordinal) utility functions on $\Delta(Z)$, or equivalently, by the corresponding orders. Modeling anticipatory feelings does not require consideration of "psychological lotteries" - it is sufficient to specify two preferences over ordinary lotteries, interpreted as the rankings of delayed and immediate risks. Moreover, the model is rich in that *any* compatible pair of orders can be taken as primitives. The large literature, both theoretical and empirical, concerning the ranking of lotteries, makes this starting point convenient. In particular, any specific model of risk preference (satisfying suitable FSD-monotonicity) can be integrated into our model of anxiety axiomatically - one need only assume, in addition to compatibility, that each order satisfies the axioms that characterize the specific model of risk preference that is of interest. Here we presume that both induced orders conform to the same model of risk preference, which we view as the natural specification. In the example above (linear model), the relevant model of risk preference is expected utility; below we provide an example where risk preferences are nonlinear.

4. EXAMPLES

Example: (Kreps-Porteus): This is the classic model of preference where the temporal resolution of risk matters. As we will see, it violates our axioms.

Preference on $\Delta_s(\mathcal{K}(\Delta(Z)))$ is represented by[16]

$$\Theta\left(\mathcal{U}^{KP}(X)\right) = \Sigma_x X(x)\,\Theta\left(\max_{p\in x} E_p u\right),$$

where u and Θ are as in the previous example ($u : Z \to \mathbb{R}$ is continuous and $\Theta : u(Z) \to \mathbb{R}$ is strictly increasing and continuous). Kreps and Porteus [9] formulate utility not only for two-stage lotteries, which is how their model is often described, but also for all random menus. They model agents who care about how risk resolves over time, but who are also dynamically consistent in the usual sense that commitment is never valuable. In particular, and in contrast with our general model,

$$\mathcal{U}^{KP}(x) \leq \mathcal{U}^{KP}(x \cup y)\text{, for all menus } x \text{ and } y.$$

More formally, the order $\succeq_{del}$ is represented by U, where U is linear with vNM utility index u, and $\succeq_{imm}$ is represented by V, $V(p) = \Theta^{-1}(E_p\Theta(u))$. Apart from the extreme case where Θ is linear, $\succeq_{del}$ and $\succeq_{imm}$ are distinct and thus Expected Stationarity is violated: even though the agent's time 0 ranking of delayed risks differs from her ranking of immediate risks, she does not value commitment. The reason is that when evaluating a menu at time 0, the agent expects her choice out of the menu at time 1 to be guided by $\succeq_{del}$ which also describes her time 0 ranking of delayed risks. She holds this expectation even though (i) the time 1 choice is between immediate risks, and (ii) her current ranking of immediate risks is given by by $\succeq_{imm}$.

It is readily verified that all other axioms are satisfied; indeed, Kreps-Porteus preference satisfies Strategic Rationality, which is stronger than NSC.

Note finally that $\mathcal{U}^{KP}$ is distinguishable from $\mathcal{U}^{lin}$ *only if* we can observe rankings of menus. In particular, in both cases the utility of two-stage lotteries is given by

$$\mathcal{U}(X) = \Theta^{-1}\left(\Sigma_p X(\{p\}\Theta(E_p u))\right).$$

This merits emphasis: a recursive structure for utility on the domain of two-stage lotteries does *not* imply that commitment has no value.

[16]Throughout this section, we describe utility only for finite-support lotteries.

Example (RDEU): Here we describe a special case of our general model where the induced orders $\succeq_{del}$ and $\succeq_{imm}$ both conform to rank-dependent-expected utility (RDEU), a model of risk preferences that has played a large role in attempts to accommodate evidence, such as the Allais Paradox, contradicting the vNM model. See the survey by Starmer [15], for example.

Let $g, h : [0,1] \rightarrow [0,1]$ be increasing and surjective; since they are used to transform (or distort) probabilities, they are sometimes referred to as *distortion functions*. Let $u : Z \longrightarrow \mathbb{R}$ be continuous and define U, for $p = \Sigma_i p_i \delta_{z_i}$, by

$$U(p) = \Sigma_i \left[g(\Sigma_{j \geq i} p_j) - g(\Sigma_{j \geq i+1} p_j) \right] u(z_i), \tag{4.1}$$

where outcomes are ordered so that $u(z_i) \leq u(z_{i+1})$ for all i. Define V similarly using the distortion function h and the vNM index $v = \Theta(u)$, where $\Theta : u(Z) \rightarrow \mathbb{R}$ is strictly increasing and continuous. In order to satisfy the normalization (3.1), let

$$V(p) = \Theta^{-1} \left(\Sigma_i \left[h(\Sigma_{j \geq i} p_j) - h(\Sigma_{j \geq i+1} p_j) \right] \Theta(u(z_i)) \right).$$

Then U and V constitute a compatible pair and thus they determine a utility function, denoted $\mathcal{U}^{rdeu}$, consistent with our axioms.

This example generalizes the linear model (our first example), to which it reduces if both g and h are identity functions. We show below that nonlinear distortion functions are useful for modeling intuitive patterns of the demand for information.

Axiomatic foundations for RDEU preferences over lotteries can be found in [12, 14, 15]. As described in the last section, axiomatic foundations for $\mathcal{U}^{rdeu}$ follow by adding these RDEU axioms, applied to $\succeq_{del}$ and $\succeq_{imm}$, to those in Theorem 3.1. The cited axiomatic studies and the survey by Starmer [15] can be brought to bear on the plausibility or appeal of $\mathcal{U}^{rdeu}$. The only difference here from the literature on risk preferences is that the orders $\succeq_{del}$ and $\succeq_{imm}$ deal with lotteries that are resolved only with the delay of at least "one period," and where the length of a period should be significant (on the scale of days or weeks rather than minutes) in order that anticipatory feelings be relevant. However, the axioms characterizing RDEU seem as appealing intuitively (or no more problematic) in our setting; and while we are not aware of any experimental evidence on the descriptive validity of RDEU when resolution is delayed significantly, there is no reason to expect the axioms to perform less well for such risks. Thus we view $\mathcal{U}^{rdeu}$, which we will use in the next section to model the attitude towards information, as being well-founded.

Segal [13] models preference over two-stage lotteries where preference at each stage conforms to RDEU. His model, extended to the domain of random menus by assuming Strategic Rationality, is related to $\mathcal{U}^{rdeu}$ in the same way that $\mathcal{U}^{KP}$ is related to $\mathcal{U}^{lin}$.

Example (Time-varying risk aversion): Time-varying risk aversion is another possible reason for commitment. Consider the utility function

$$\mathcal{U}^{tvra}(X) = \Sigma_x X(x) \max \left\{ U(p) : p \in \arg\max_{p' \in x} U'(p') \right\}, \quad X \in \Delta(\mathcal{K}(\Delta(Z))),$$

where U and U' are (ordinally distinct) continuous linear functions on $\Delta(Z)$. Then $\succeq_{del}$ and $\succeq_{imm}$ are *both* represented by U, yet commitment is valuable, thus violating Expected Stationarity.[17] The reason for commitment differs here. In particular, an individual with utility function $\mathcal{U}^{tvra}$ does not care when risk is resolved: any two-stage lottery $X = \Sigma_p X(\{p\}) \delta_{\{p\}}$ has utility

$$\mathcal{U}^{tvra}\left(\Sigma_p X(\{p\}) \delta_{\{p\}}\right) = \Sigma_p X(\{p\}) U(p) = U(\Sigma_p X(\{p\}) p),$$

which depends only on the induced distribution over outcomes $\Sigma_p X(\{p\}) p$. Yet she values commitment because she expects her risk preferences to change, and therefore, to choose out of menus at time 1 according to U', while her time 0 utility function over lotteries is U.

Example (Self-control): Define $\mathcal{U}^{sc}$ as in the linear example, except that (3.2) is replaced by

$$w(x) = \max_{p \in x} \left[U(p) + \Theta^{-1}(E_p \Theta \circ u)\right] - \max_{p' \in x} \Theta^{-1}(E_p \Theta \circ u).$$

Then preference violates NSC, though it satisfies GP's weaker axiom Set-Betweenness ($x \succeq y \implies x \succeq x \cup y \succeq y$). Our other axioms are satisfied. In particular, Expected Stationarity is readily verified because $\succeq_{imm}$ has utility function V, where $V(p) = \Theta^{-1}(E_p \Theta \circ u)$.

[17] All the other axioms of our model are satisfied.

5. DEMAND FOR INFORMATION

5.1. Information, Anxiety/Savoring and Commitment

Consider the attitude towards information when it has only psychic, as opposed to planning, consequences. This is reflected in the ranking of random menus that provide commitment, that is, in the ranking of two-stage lotteries.

For simplicity, restrict attention to two-stage lotteries with finite support. Given $X = \Sigma_p X(\{p\})\,\delta_{\{p\}}$, define $EX \in \Delta(Z)$ by

$$EX = \Sigma_p X(\{p\})\,p.$$

Then EX describes the probability distribution over outcomes induced by X, where the temporal resolution of this risk has been removed. Since EX describes the prior risk, it is the counterpart of the Bayesian prior in a model with states of the world and subjective uncertainty. We can modify the temporal resolution prescribed by X and consider two extremes. *No information* (at the first stage) corresponds to $\delta_{\{EX\}}$. The other extreme, all risk being resolved at time 1, corresponds to the two-stage lottery $\Sigma_z(EX)(z)\,\delta_{\{\delta_z\}}$; we refer to this as *perfect information*. For brevity, we examine these extremes only, though intermediate cases could be considered.

Say that the agent is *information seeking at EX* if she prefers perfect information to no information, that is, if

$$\Sigma_z(EX)(z)\,\delta_{\{\delta_z\}} \succeq \delta_{\{EX\}};$$

if the reverse ranking holds, refer to her as *information averse at EX*. These notions are weak - an agent can satisfy both, in which case we refer to her as *information neutral* at EX.

Theorem 5.1. *The agent with preference $\succeq$ satisfying our axioms is information seeking (averse) at EX if and only if, for every z in Z,*

$$\delta_z \succeq_{imm} EX \implies (\Longleftarrow)\ \delta_z \succeq_{del} EX.$$

Proof. By the representation, information seeking at EX is equivalent to

$$V(EX) = V(\Sigma_z(EX)(z)\,\delta_z) = \mathcal{U}\left(\Sigma_z(EX)(z)\,\delta_{\{\delta_z\}}\right) \geq \mathcal{U}\left(\delta_{\{EX\}}\right) = U(EX),$$

or

$$V(EX) \geq U(EX). \tag{5.1}$$

Since U and V represent $\succeq_{del}$ and $\succeq_{imm}$ and since they are equal on Z, the asserted condition follows. Similarly for information aversion. ■

The condition in the theorem corresponding to information seeking asserts that whenever $\succeq_{imm}$ would reject the lottery EX in favor of a certain outcome, then so would $\succeq_{del}$. In that standard sense, $\succeq_{del}$ is more risk averse than $\succeq_{imm}$ at EX. Thus the agent is *information seeking at EX if and only if $\succeq_{del}$ is more risk averse than $\succeq_{imm}$ at EX*. Information aversion corresponds to $\succeq_{imm}$ being more risk averse.[18]

In the linear model, the condition (5.1) becomes (writing $p = EX$)

$$\Theta^{-1}\left(E_p \Theta\left(u\right)\right) \geq E_p u, \tag{5.2}$$

which is true if Θ is "convex at $E_p u$". This condition is familiar from the KP model as the condition describing a preference for early resolution of risk. The similarity with what we know from KP is not surprising since we have already noted that their model coincides with our linear model (the example in Section 3) on the subdomain of two-stage lotteries; similarly, our general model coincides there with a nonlinear version of KP. The distinctive feature of our model is the connection it implies between the demand for information and the value of commitment. We noted earlier that indifference to the temporal resolution of risk (or a zero demand for information) is equivalent in our model to commitment having no value. Now we go further and relate information seeking (or aversion) to the sort of commitments that are or are not valuable.

Theorem 5.2. *The agent with preference $\succeq$ satisfying our axioms is information seeking at $p \in \Delta(Z)$ if and only if, for every z in Z,*

$$\{p\} \succ \{\delta_z\} \Longrightarrow \{p\} \sim \{p, \delta_z\}; \tag{5.3}$$

and she is information averse at p if and only if, for every z in Z,

$$\{\delta_z\} \succ \{p\} \Longrightarrow \{\delta_z\} \sim \{p, \delta_z\}. \tag{5.4}$$

Proof. Assume information seeking at p. Then, by (5.1), $V(p) \geq U(p)$. If also $\{p\} \succ \{\delta_z\}$, then $U(p) > U(\delta_z) = V(\delta_z) \Longrightarrow V(p) > V(\delta_z)$. Therefore,

[18] We are not aware of much experimental evidence on how delayed resolution affects risk aversion. Some relevant experiments are reported in Liberman, Sagristano and Trope [10]. But the stakes involved are too small to plausibly generate anxiety or savoring.

p is better than δ_z according to both U and V. Conclude that $\{p\} \sim \{p, \delta_z\}$. Conversely, assume (5.3), which is equivalent to: for every z,

$$U(p) > U(\delta_z) \Longrightarrow V(p) \geq V(\delta_z).$$

But then $V(p) \geq U(p)$, which implies information seeking at p. If not, then $V(p) < U(p)$, and (because U and V are continuous and Z is connected), there exists z such that $V(p) < V(\delta_z) = U(\delta_z) < U(p)$, a contradiction.

The proof for information aversion is similar. ■

We interpret information seeking (aversion) as the behavioral manifestation of anxiety (savoring). Therefore, both theorems above describe the revealed preference implications of anxiety and of savoring. In fact, to highlight this identification, below we use the terms anxiety, savoring and neutrality (at a lottery p) *interchangeably* with information seeking, aversion and neutrality (at p).

The above characterizations are intuitive. Consider the characterization for information seeking at p, and, for concreteness, interpret p as representing the risk of a large loss due to a house fire or car accident. Living with this risk entails anxiety, and thus leads to a preference for early resolution. Complete insurance is available at a price that would leave the agent with the certain outcome z. Suppose, however, that in spite of the anxiety, she strictly prefers at time 0 to remain uninsured ($\{p\} \succ \{\delta_z\}$). Then having the option to postpone the insurance decision to time 1 is a matter of indifference ($\{p\} \sim \{p, \delta_z\}$) - the agent is certain that insurance will be declined because at time 1 the anxiety argument for insurance is weaker. Conversely, suppose that for *any* price, if insurance is declined at time 0, then it would be declined also at time 1. Then the psychic cost of the risk p is smaller at the later time, presumably because it can cause less anxiety at that point. But if p is a source of anxiety, then its early resolution would be preferred. The bottom line is that, given our axioms, *an anxious individual is one who would never strictly prefer to commit to not insuring.*

It might appear surprising that anxiety and savoring are characterized by conditions that express a limited form of strategic rationality. However, the qualification 'limited' is crucial - other commitments may be strictly valuable. For example, for an anxious individual, the ranking

$$\{\delta_z\} \succ \{p, \delta_z\} \sim \{p\} \tag{5.5}$$

is intuitive: insurance could be chosen at time 0 to provide peace-of-mind, while if the decision is left for a later time, when anxiety is less important, the individual might decide not to insure. Thus she may *strictly prefer to commit to insurance.*

The indicated strict preference to commit reflects a strict form of anxiety. More generally, we use the terms information seeking (anxiety) and information aversion (savoring) in the weak sense. Therefore, since every preference is either information seeking or information averse at the given p, the way to express (weak) information aversion (hence savoring) is to exclude strict information seeking, that is, to exclude (5.5). This explains the characterization (5.4) of information aversion.

The characterizations in the theorem are not valid in the KP model. Since commitment is never valuable in their model, both conditions (5.3) and (5.4) are satisfied globally, without implications for the nature of information demand.

5.2. Comparative Anxiety

Above we provided behavioral characterizations of anxiety. Here we go further and give behavioral meaning to statements about comparative anxiety across agents. The obvious modifications corresponding to comparative savoring are left to the reader.

For concreteness, think of the insurance example. We know from Theorem 5.2 and the ensuing discussion that (in the absence of neutrality) anxiety about the possible loss is reflected via the desire to commit to insurance, that is, through rankings of the sort

$$\{\delta_z\} \succ \{p, \delta_z\} \sim \{p\}.$$

Let both $\succeq^1$ and $\succeq^2$ be anxious at p. Say that 2 is *more anxious at p than* 1 if whenever 1 strictly prefers to commit to insurance, then so does 2; that is, if for every $z \in Z$,

$$\{\delta_z\} \succ^1 \{p, \delta_z\} \sim^1 \{p\} \Longrightarrow \{\delta_z\} \succ^2 \{p, \delta_z\} \sim^2 \{p\}. \tag{5.6}$$

Theorem 5.3. *Let $\succeq^1$ and $\succeq^2$ satisfy our axioms and let both be anxious at p. Suppose in addition that $\succeq^1$ and $\succeq^2$ agree on Z. Then 2 is more anxious at p than 1 iff either (i) 1 is neutral at p , or (ii) for every z,*

$$\delta_z \succeq^2_{imm} p \Longrightarrow \delta_z \succeq^1_{imm} p \quad (\succeq^2_{imm} \text{ is less risk averse than } \succeq^1_{imm} \text{ at } p) \tag{5.7}$$

and

$$\delta_z \succeq^1_{del} p \Longrightarrow \delta_z \succeq^2_{del} p \quad (\succeq^2_{del} \text{ is more risk averse than } \succeq^2_{del} \text{ at } p). \tag{5.8}$$

The proof is not particularly revealing and is relegated to the appendix.

To interpret the theorem, consider first the role of neutrality (condition (i)). If 1 is neutral at p, then 1 is both anxious and savoring (and is neither in the strict sense). Therefore, the antecedent in (5.6) is not satisfied for any z, and the defining condition is satisfied vacuously. Since 2 is anxious by assumption, it makes sense to refer to her as being weakly more anxious than 1.

Theorem 5.1 characterizes anxiety in terms of $\succeq_{imm}$ being more risk averse than $\succeq_{del}$. This makes conditions (5.7)-(5.8) intuitive: an agent becomes more anxious if she becomes less averse to immediate risks and more averse to delayed risks. Conversely, this is necessary for increased anxiety at p unless (5.6) is satisfied vacuously, that is, 1 is neutral at p.

These characterizing conditions are readily expressed in terms of the representations. Let (U_i, V_i) represent $\succeq^i$, $i = 1, 2$, as in Theorem 3.1. By construction, $U_i = V_i$ on Z. By the hypothesis that $\succeq^1$ and $\succeq^2$ agree on Z, it follows that U_1 and U_2 are ordinally equivalent on Z. Therefore, by applying a common monotonic transformation to U_1 and V_1, we can assume wlog that

$$U_1 = U_2 = V_1 = V_2 \text{ on } Z. \tag{5.9}$$

Using these utility functions, it is easily shown (see the proof) that 2 is more anxious at p than 1 if and only if

$$V_2(p) \geq V_1(p) \text{ and } U_2(p) \leq U_1(p).$$

Finally, consider the assumption that the two agents have the same ranking on Z. This restriction seems natural; for example, for preferences over lotteries with vector outcomes, Kihlstrom and Mirman [8] have pointed out that 'more risk averse than' can be meaningfully defined only when the two agents agree on the ranking of outcomes. However, we have not succeeded in finding a similar conceptual justification for that restriction here - indeed, our definition of "more anxious than" does not require it.

5.3. 'Anomalous' Demand for Information

In the linear special case of our model, condition (5.2) shows that the attitude towards information depends on properties of Θ at $E_p u$ rather than on the prior p separately. Thus the linear model cannot accommodate information attitudes that vary with the prior. Eliaz and Spiegler [5] emphasize that such dependence on the prior is anomalous in an 'expected-utility based' model.[19]

[19] Their model differs from ours in details, but their point is still valid here.

As an example of such an anomalous information attitude, they consider the intuitive hypothesis that the agent is *information seeking (averse) when the favorable (unfavorable) event is very likely.* Though inconsistent with the linear model, the hypothesis is not anomalous relative to our general model. For example, consider binary lotteries with outcomes z_1 and z_2, where $u(z_1) < u(z_2)$, and take the RDEU model (with Θ linear and hence dispensable). Then, by (5.1), the hypothesis is satisfied if and only if

$$(1 - h(p_2))\,u(z_1) + h(p_2)\,u(z_2) > (1 - g(p_2))\,u(z_1) + g(p_2)\,u(z_2)\,, \text{ if } p_2 \text{ is near } 1,$$

$$(1 - h(p_2))\,u(z_1) + h(p_2)\,u(z_2) < (1 - g(p_2))\,u(z_1) + g(p_2)\,u(z_2)\,, \text{ if } p_2 \text{ is near } 0.$$

These conditions are satisfied if (and only if) h lies above g for probabilities near 1 and below g for probabilities near 0. Refer to this pattern as "h is s-shaped relative to g".

As emphasized in the last section, the RDEU special case of our model is axiomatically well-founded. Thus the preceding reflects on Eliaz and Spiegler's [5, p.16] skepticism about the usefulness of non-expected utility theories for addressing anomalous attitudes towards information. Admittedly, they describe other anomalies in addition to the one we have been considering. However, these seem intuitively to be due to something other than anticipation or anxiety, (cognitive dissonance or confirmatory bias, for example) and thus are most naturally addressed by other models.

One might wonder also whether the hypothesis that h is s-shaped relative to g is consistent with evidence. Note first that it is consistent with risk aversion for both $\succeq_{del}$ and $\succeq_{imm}$, for which it suffices that u and $\Theta(u)$ be concave and that both g and h be convex (see Chew, Karni and Safra [4]).[20] Second, there exists evidence about the shapes of distortion functions needed in order for the RDEU risk preference model to accommodate Allais-type behavior; subject to the qualification described in the discussion of the RDEU example, this evidence is relevant here.[21] However, even if both g and h have these shapes, their *relative* shapes are not pinned down by available evidence. What is needed to determine relative shapes is evidence on how individuals rank *both* immediate risks (risks

[20] Assume $Z \subset \mathbb{R}^n$ so that risk aversion can be defined.

[21] Starmer [15, p. 348] describes some support for an *inverted* s-shape (relative to the 45° line) for distortions - concave and lying above the 45° line for small probabilities - those smaller than some p^* - and convex and above the 45° line for probabilities greater than p^*; Quiggin [12] proposed this form with $p^* = 1/2$. See Tversky and Wakker [18] for a discussion of the connection between the shape of the distortion function and theoretical properties of preference.

that resolve within one period), *and* delayed risks (those that resolve within two periods), where payment in both cases is received in period 2.

Other patterns of information attitudes can also be accommodated within the RDEU model. For example, suppose that $h(\cdot) - g(\cdot)$ is single-peaked with peak at $p_1 = p_2 = \frac{1}{2}$. This specification models an agent who is information seeking when facing any risk, but 'particularly so' when she is less certain about the outcome. Alternatively, if $h - g$ is positive except near $p_1 = p_2 = \frac{1}{2}$, then there is information seeking only when the agent is nearly certain ex ante about which outcome will be realized (resembling Eliaz and Spiegler's Example 3).

Finally, we emphasize that the RDEU model is but one example of the general framework characterized in Theorem 3.1, and that adopting other models of risk preference will lead to alternative implications for the attitude to information. As one illustration, consider the generalization of RDEU called cumulative prospect theory (Tversky and Kahneman [17]), in which (real-valued, for example) outcomes are measured relative to a reference point and there is risk aversion in gains and risk loving in losses. By assuming that both $\succeq_{del}$ and $\succeq_{imm}$ conform to cumulative prospect theory, one can model anxiety for lotteries that involve only losses and savoring for those that involve only gains.

A. APPENDIX

Proof of Theorem 3.1. (a) Utility is well-defined: Show that utility on $\Delta(\Delta(Z))$ that is defined in (3.8) is invariant to the choice of θ. If θ_1 is another such map, then $U(p) = U\left(\delta_{\theta_1(p)}\right)$

$$\begin{aligned}
&\Longrightarrow \quad U\left(\delta_{\theta_1(p)}\right) = U\left(\delta_{\theta(p)}\right) \\
&\Longrightarrow \quad V\left(\delta_{\theta_1(p)}\right) = V\left(\delta_{\theta(p)}\right) \quad (U \text{ and } V \text{ ordinally equivalent on } Z) \\
&\Longrightarrow \quad X \circ \theta_1^{-1} \approx^{FSD} X \circ \theta^{-1}, \ \text{ for any } X \in \Delta(\Delta(Z)), \\
&\Longrightarrow \quad V\left(X \circ \theta_1^{-1}\right) = V\left(X \circ \theta^{-1}\right),
\end{aligned}$$

where: $\approx^{FSD}$ is defined as in (2.5), and the last equality follows from the assumption that V is FSD-increasing. The second step in showing that utility is well-defined requires showing that the utility values defined by (3.8) and (3.2) agree on the intersection $\Delta(\Delta(Z)) \cap \mathcal{K}(\Delta(Z))$. An element in the intersection must have the form $\{p\}$, for some delayed risk $p \in \Delta(Z)$ that resolves at time 2. Then p can also be viewed as the two-stage lottery that produces p with certainty at the first stage. According to (3.8), the latter has utility $V\left(\delta_{z_p}\right)$ where z_p solves (3.4), and according to (3.2), (see also (3.3)), the singleton menu $\{p\}$ has utility $U(p) = U(z_p)$. But $V\left(\delta_{z_p}\right) = U\left(\delta_{z_p}\right)$ by (3.1).

The extension of utility to $\Delta(\mathcal{K}(\Delta(Z)))$ is well-defined by (3.7) because $\mathcal{U}$ is FSD-increasing on $\Delta(\Delta(Z))$.

Necessity of axioms: Order is obvious. Monotonicity and Expected Stationarity are readily verified.

NSC: Let $U(x) = \max_{p \in x} U(p)$, $U^{-1}(x) = \arg\max_{p \in x} U(p)$, and similarly for V. In this notation,

$$\mathcal{U}(y) = \max\{U(p) : p \in V^{-1}(y)\}.$$

(i) $V(x') > V(x) \Longrightarrow x' \sim x' \cup x$: Hypothesis implies that $V^{-1}(x' \cup x) = V^{-1}(x')$.
(ii) $V(x') < V(x) \Longrightarrow x \sim x' \cup x$: Hypothesis implies that $V^{-1}(x' \cup x) = V^{-1}(x)$.
(iii) $V(x') = V(x) \Longrightarrow x' \sim x' \cup x$ if $x' \succeq x$: Hypothesis implies that $V^{-1}(x' \cup x) = V^{-1}(x') \cup V^{-1}(x)$, and hence $\mathcal{U}(x' \cup x) = \max\{\mathcal{U}(x'), \mathcal{U}(x)\}$.

Limited Continuity: (a) follows because $\mathcal{U}$ defined in (3.2) is usc on $\mathcal{K}(\Delta(Z))$ by a form of the Maximum Theorem. Parts (b) and (c) are obvious.

(b) *Sufficiency of the axioms*:

Step 1. There exists a representation w of $\succeq$ on $\mathcal{K}(\Delta(Z))$: $\Delta(Z)$ is separable (because it is compact metric) and connected. Hence $\succeq_{del}$ has a continuous (and FSD-increasing) utility function $U : \Delta(Z) \longrightarrow \mathbb{R}$. For any menu x, define

$$w(x) = U(p)$$

for any p such that $x \sim \{p\}$. Existence of p is ensured by Limited Continuity. If p and p' are two such measures, then $p' \sim_{del} p$ and thus $U(p') = U(p)$; hence, w is well-defined. Moreover,

$$w(\{p\}) = U(p) \text{ for every } p.$$

Step 2. Let V be a continuous utility function for $\succeq_{imm}$. It exists since $\succeq_{imm}$ is continuous by Limited Continuity and it is FSD-increasing by Monotonicity.

As observed in (2.3), $\succeq_{imm}$ agrees with $\succeq_{del}$ on Z. Therefore, it is wlog to assume (3.1) - U and V are identical on Z.

Step 3. $\{p'\} \sim \{p', p\}$ if $p' \succeq_{imm} p$ and $p' \succeq_{del} p$: Let $p' \succeq_{imm} p$ and $p' \succeq_{del} p$. By definition, the latter implies $\{p'\} \succeq \{p\}$. By NSC, there are 2 cases. Case 1: $\{p'\} \sim \{p', p\} \succeq \{p\}$. This is consistent with the desired conclusion. Case 2: $\{p'\} \succ \{p', p\} \sim \{p\}$. Then Expected Stationarity implies $p' \prec_{imm} p$, contradicting our hypothesis.

Step 4. $\{p'\} \sim \{p', p\}$ if $p' \succ_{imm} p$: Suppose $p' \succ_{imm} p$ and $\{p'\} \not\sim \{p', p\}$. Then Step 3 implies that $p' \not\succeq_{del} p$, that is, $p' \prec_{del} p$. Thus $\{p\} \succ \{p', p\}$ by Expected Stationarity. But this contradicts NSC.

Step 5. Prove the representation for finite nonrandom menus: Argue as in GP (p. 1429). Let x be finite and let $p^* \in x$ satisfy

$$w(\{p^*\}) = \max\{w(\{p\}) : p \in \arg\max_{p' \in x} V(p')\}.$$

Note that $x = \cup_{p' \in x}\{p^*, p'\}$. Since w represents $\succeq$ and the latter satisfies NSC, then

$$w(x) = w(\{p^*, p'\}) \text{ for some } p' \in x.$$

Since V represents $\succeq_{imm}$, Steps 3-4 imply that

$$w(\{p^*, p'\}) = w(\{p^*\}) = U(p^*).$$

This yields the desired result

$$w(x) = w(\{p^*\}) = \max\{U(p) : p \in \arg\max_{p' \in x} V(p')\}.$$

Step 6. Extend the representation to $\mathcal{K}(\Delta(Z))$: GP's Lemma 8 and the ensuing paragraph (p. 1430) deliver the extension. Limited Continuity provides the continuity properties needed by their argument. It follows from the Maximum Theorem that w is usc.

Step 7. Prove the desired representation on $\Delta(\Delta(Z))$: Define utility via (3.8), that is, for any $X \in \Delta(\Delta(Z))$,

$$\mathcal{U}(X) = V\left(X \circ \theta^{-1}\right),$$

where $\theta : \Delta(Z) \to Z$ satisfies $U(p) = U\left(\delta_{\theta(p)}\right)$. Argue as in the proof of (a) to show that utility is well-defined.

It follows from Monotonicity, specifically from $\succeq$ being FSD-increasing, that[22]

$$X \sim \left((X \circ \theta^{-1}) \circ e^{-1}\right).$$

Informally: If X has finite support, then it assigns probability $X(p)$ to each p in its support, while $(X \circ \theta^{-1}) \circ e^{-1}$ assigns probability $X(p)$ to $\delta_{\theta(p)}$, the certainty equivalent of p according to U. 'Therefore, they are indifferent by backward induction.'

Now, for any $X', X \in \Delta(\Delta(Z))$, $X' \succeq X$

$$\begin{aligned}
&\iff \left((X' \circ \theta^{-1}) \circ e^{-1}\right) \succeq \left((X \circ \theta^{-1}) \circ e^{-1}\right) \\
&\iff (X' \circ \theta^{-1}) \succeq_{imm} (X \circ \theta^{-1}) \\
&\iff V\left(X' \circ \theta^{-1}\right) \geq V\left(X \circ \theta^{-1}\right) \quad \text{(by Step 2)} \\
&\iff \mathcal{U}(X') \geq \mathcal{U}(X).
\end{aligned}$$

Step 8: *Extend the representation to all random menus:* Define utility on $\Delta(\mathcal{K}(\Delta(Z)))$ by (3.9), that is, for any $X \in \Delta(\mathcal{K}(\Delta(Z)))$,

$$\mathcal{U}(X) = \mathcal{U}\left(X \circ \phi^{-1}\right),$$

where $\phi : \mathcal{K}(\Delta(Z)) \to \Delta(Z)$ satisfies $U(x) = U\left(\delta_{\phi(x)}\right)$; note that $X \circ \phi^{-1}$ is a two-stage lottery and thus its utility was defined in the previous step. Argue as in the proof of (a) to show that utility is well-defined.

It follows from $\succeq$ being FSD-increasing, that

$$X \sim X \circ \phi^{-1}.$$

[22] Recall the definition of $\succeq_{imm}$ and that $e : Z \longrightarrow \Delta(Z)$ is the natural embedding.

Since $\mathcal{U}$ represents preference on $\Delta(\Delta(Z))$, the extension defined here represents preference on $\Delta(\mathcal{K}(\Delta(Z)))$. ■

Proof of Theorem 5.3: Let (U_i, V_i) represent $\succeq^i$, $i = 1, 2$, and satisfy (5.9).

If 1 is neutral at p, then (5.6) is satisfied vacuously. Assume (5.7)-(5.8). Then, by the representations,

$$V_2(\delta_z) \geq V_2(p) \Longrightarrow V_1(\delta_z) \geq V_1(p) \text{ and} \tag{A.1}$$

$$U_1(\delta_z) \geq U_1(p) \Longrightarrow U_2(\delta_z) \geq U_2(p). \tag{A.2}$$

It follows from continuity of the representing functions, connectedness of Z and from (5.9) that

$$V_2(p) \geq V_1(p) \text{ and } U_2(p) \leq U_1(p). \tag{A.3}$$

This, in turn, implies, given (5.9), that

$$[U_1(\delta_z) > U_1(p), V_1(\delta_z) < V_1(p)] \Longrightarrow [U_2(\delta_z) > U_2(p), V_2(\delta_z) < V_2(p)], \tag{A.4}$$

which is equivalent to (5.6).

Conversely, suppose that 1 is not neutral at p. Since 1 is anxious, $U_1(p) < V_1(p)$. Then (5.6) $\Longrightarrow$ (A.4) $\Longrightarrow$ (A.1)-(A.2) $\Longrightarrow$ (5.7)-(5.8). ■

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