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ON

INCOME TAXATION AND THE CORE

by

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Since the important work of Mirrlees(1971), economists have been concerned with refining the necessary conditions for a utilitarian-optimal income tax schedule given resource constraints and incentive compatibility constraints. The incentive constraints are based on the notion that individuals' wage levels or characteristics are unknown to the government, so the optimal income tax schedule must separate individuals and is therefore generally second best.

The necessary conditions for optimization generally include a zero marginal tax rate for both the highest and lowest wage individuals. Intuitive and algebraic derivations of this result can be found in Seade (1977), whose framework we employ below.

The purpose of this note is to examine the core of this model in order to find further necessary conditions on optimal (or core) tax schedules. The utilitarian framework translates directly into a transferable utility framework in the context of cooperative game theory. We retain the incentive constraints for coalitions that may form; coalitions also must separate individuals.

Following Seade (1977), we describe consumers' characteristics by a single variable $h \in [\underline{h},\bar{h}]$. The distribution of consumer characteristics or wage levels has density $f(h)$. $x = x(h)$ is the vector of N commodities consumed by a person of type h after taxes. $U^h = u(x(h),h)$ is a cardinal utility function for consumers of type h. Let θ be the (closed) choice set for all consumers $(x(h) \in \theta)$. The utilitarian problem is to maximize

$$W \equiv \int_{\underline{h}}^{\bar{h}} u(x(h),h)f(h)dh.$$

The feasibility constraints for consumption and production are given by

$$\int_{\underline{h}}^{\bar{h}} x(h)f(h)dh \leqq X$$

and

$$\phi(X) \leqq 0$$

respectively.

The individual behavior constraint is obtained by imposing the condition that $\operatorname{argmax}_h u(x(h),h') = h'$, so that consumers do not misrepresent themselves. Thus,

$$u_x x' = 0$$

where u_x is the partial derivative of u with respect to x and x' is the gradient of x (with respect to h). The assumptions and a theorem of Seade (1977) are stated next.

<u>Assumption 1:</u> $u(\cdot,h)$ is continuous, monotonic, strictly concave and everywhere differentiable in x. Besides, u_{ih} is bounded for $0 \ll x \ll \infty$.

<u>Assumption 2:</u> The density f(h) does not oscillate infinitely often as h changes.

<u>Assumption 3:</u> The welfare function(al) is utilitarian and its integral over a zero-measure set of people is zero.

<u>Assumption 4:</u> The allocation vector function x(h) and the resulting tax schedule are continuous and smooth.

<u>Theorem (Seade):</u> Consider an optimal tax regime and assume that assumptions 1-4 hold for $\underline{h}$. Then if $0 \ll x(\underline{h}) \ll \infty$, all marginal taxes must be zero at $\underline{h}$. Similarly for $\bar{h}$.

This holds for any desired revenue collection, since we can embed such a collection in ϕ. Next, we proceed to define the core. A <u>coalition</u> is a density c on $[\underline{h},\bar{h}]$ with $c(h) \leq f(h)$ a.s.

To each coalition c we associate a production possibility set ϕ^c. This set can capture various revenue collections required from coalitions as well as coalition endowments and technologies. We do not require any continuity or concavity of ϕ^c as c varies. As will become obvious, these properties are not relevant to the problem at hand. For a fixed coalition c, an after tax consumption vector x(h) is <u>c-feasible</u> if

$$\int_{\underline{h}}^{\bar{h}} x(h)c(h)dh \leqq X, \tag{1}$$

$$\phi^c(X) \leqq 0 \tag{2}$$

and

$$\underset{h}{\operatorname{argmax}}\ u(x(h),h') = h' \ \forall h' \text{ with } c(h') > 0.$$

Since we will deal only with coalitions that have support on an interval, the last condition is equivalent to

$$u_x x' = 0 \quad \forall h' \text{ with } c(h') > 0 \tag{3}$$

for such coalitions.

For a given tax system and resulting allocation x(h), a coalition c is said to <u>block</u> x(h) if there is a c-feasible function $\bar{x}(h)$ with

$$\int_{\underline{h}}^{\bar{h}} u(\bar{x}(h),h)c(h)dh > \int_{\underline{h}}^{\bar{h}} u(x(h),h)c(h)dh.$$

Just as for the optimal income tax problem, this represents a transferable utility framework. The <u>core</u> of an income tax model is the set of tax systems or post-tax allocation vectors that are not blocked by any coalition.

<u>Theorem:</u> Suppose that assumptions 1-4 hold for all $h \in [\underline{h},\bar{h}]$ and $0 \ll x(\underline{h}) \ll \infty$. Then a necessary condition for a tax system or its associated after-tax

allocation to be in the core is that marginal tax rates on $[\underline{h},\bar{h}]$ must be zero. That is, any tax system in the core is a pure head tax.

Proof: Fix $\underline{a},\bar{a}$ with $\underline{h} \leq \underline{a} < \bar{a} \leq \bar{h}$. Consider a coalition c defined by

$$c(h) = \begin{cases} f(h) & \text{for } h \in [\underline{a},\bar{a}] \\ 0 & \text{otherwise.} \end{cases}$$

If x(h) is in the core, it must be that x(h) maximizes $\int_{\underline{h}}^{\bar{h}} u(x(h),h)c(h)dh$ $= \int_{\underline{a}}^{\bar{a}} u(x(h),h)f(h)dh$ subject to (1), (2), and (3). Using the theorem of Seade cited earlier, it must be that the marginal tax rates at $\underline{a}$ and $\bar{a}$ are zero. Since this is true for all $\underline{a}$ and $\bar{a}$, it must be that marginal tax rates are zero everywhere.

Q.E.D.

Corollary: Suppose that h represents a wage rate and that $\underline{h} = 0$. Further, suppose that assumptions 1-4 hold for all $h \in [\underline{h},\bar{h}]$ and $0 \ll x(\underline{h}) \ll \infty$.

A) If desired total revenue collection from the income tax is zero, then the core contains only the no tax (first best) regime.

B) If desired total revenue collection from the income tax is positive, then the core is empty.

When this project was begun, it was hoped that a more restrictive notion of optimality, such as core, would yield further qualitative conditions that must be satisfied by an optimal tax regime. This does not appear to be the case.

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