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Abstract

An explicit growth model with specialized inputs is used to show that a decentralized equilibirum can exist despite a form of aggregate increasing returns in production. In this setting, unceasing growth can arise engodenously. In contrast to models that focus on spillovers of knowledge, this model has no true externality; nonetheless, the equilibirum with differentiated products behaves as if it did. It is therefore possible to analyze specialization and growth in terms of Marshallian external economies despite the formal validity of the objections to this kind of analysis raised during the cost controversies of the 1920's.

*I have benefited from the comments of seminar participants at the University of Chicago, NYU, VPI, and SUNY Stonybrook. The usual disclaimer applies.

In his 1928 Presidential Address to the economics section of the British Association for the Advancement of Science, Allyn Young described a model of growth driven by increasing returns (Young, 1928). He argued that the increasing returns arose from specialization in production, and following Adam Smith, took the view that the degree of specialization at any point in time was limited by the extent of the market. He went on to suggest, somewhat vaguely, that the extent of the market is determined by purchasing power or the value of aggregate production. Thus, increases in income lead to increases in demand, which can in turn lead to increases in the extent of the market. These permit increases in specialization, which permit output to grow faster than inputs, so per capita income rises. Repeated in circular fashion, this argument appears to generate a process of unending economic growth. Moreover, Young argued, the increasing returns due to specialization could be viewed as being external to any individual firm in the sense proposed by Alfred Marshall, so this explanation of growth could be consistent with the existence of a decentralized, competitive equilibrium.

In this paper, I construct an explicit growth model along the lines proposed by Young and show that specialization alone can lead to unceasing growth. I also show that such a model can generate what appears to beexogenous labor augmenting technological change at the aggregate level. The growth model itself amounts to no more than an extended example that can be explicitly solved. Because it is the issue of external economies--rather than specialization per se--that has proven to be the most controversial part of models developed along these lines, most of the analysis is concerned with verifying the extent to which Young's claims about external and internal effects can be rigorously justified and making precise the sense in which interactions between firms can be modeled as externalities. I show that although the economy considered here does not have a true externality, it behaves as if it did. It does not possess a competitive equilibrium with externalities, but it does have an equilibrium that is formally indistinguishable from one. I also note that although the economy does not have true increasing returns, it behaves as if it did.

As a practical matter, the observation that the equilibrium is formally the same as one with increasing returns and true technological externalities is quite important, for it allows me to analyze the model here using existing tools for studying dynamic competitive economies with externalities. It also offers a justification for the persistent use of Marshall's notion of external economies in contexts where no true externality is apparent, and bears on the cost controversies of the 1920's and on Scitovsky's notion of a pecuniary externality.

The next section summarizes some of the historical origins of the present model. Section III develops a formal model of specialization in a static economy. Section IV describes the extent to which this model can be reinterpreted as a model with an externality in production. Section V analyzes a dynamic model of growth based on specialization.

II. Historical Origins of the Model

Young's thoughts on growth were heavily influenced by Alfred Marshall's discussion of increasing returns and external effects in Principles of Economics. To explain the ongoing process of growth that he observed, Marshall relied heavily on the presence of some form of increasing returns to offset the natural tendency towards diminishing returns. To avoid the conclusion that this necessarily leads to a collapse of competitive behavior, he introduced the notion of economies that are external to any firm because they depend on the scale of the industry or of the economy as a whole. He gives two examples of such effects. The first is an increase in "trade-knowledge" that cannot be kept secret (p. 237). In previous work, I developed a dynamic, equilibrium model of growth with external economies of this kind (Romer 1983, 1986). There, knowledge is the source of increasing returns, and spillovers of knowledge account for the external effect that one producer has on all other producers. The model can generate unceasing or even accelerating growth in a decentralized equilibrium, but does not allow any role for specialized goods in the growth process.[1]

Marshall's second example of an external economy is based on the growth of subsidiary trades that use "machinery of the most highly specialized character." (Marshall p. 225) This idea is at the heart of Young's model, so it is useful to quote Young at length concerning what he took to be the main features of specialization: "the growth of indirect or roundabout methods of production and the division of labour among industries." (Young 1928, p.529)

> It is generally agreed that Adam Smith, when he suggested that the division of labour leads to inventions because workmen engaged in specialized routine operations come to see better ways of accomplishing the same results, missed the main point. The important thing, of course, is that with the division of labour a group of complex processes is transformed into a succession of simpler processes, some of which, at least, lend themselves to the use of machinery. In the use of machinery, and the adoption of indirect processes there is a further division of labour, the economies of which are again limited by the extent of the market. It would be wasteful to make a hammer to drive a single nail. ... How far it pays to go in equipping factories with special appliances for making hammers ... depends again upon how many nails are to be driven. (p. 530)

[1]Other papers relying on external effects associated with knowledge include Arrow (1962), Lucas (1985), and Prescott and Boyd (1985). Arrow's model cannot generate unceasing growth for a stationary population. Lucas and Prescott and Boyd are concerned with knowledge as embodied as human capital.

> The successors of the early printers, it has often been observed, are not only the printers of today, with their own specialized establishments, but also the producers of wood pulp, of various kinds of paper, of inks and their different ingredients, of type-metal and of type, the group of industries concerned with the technical parts of the producing of illustrations, and the manufacturers of specialized tools and machines for the use in printing and in these various auxiliary trades. The list could be extended, both by enumerating other industries which are directly ancillary to the present printing trades and by going back to industries which, while supplying the industries which supply the printing trades, also supply other industries, concerned with preliminary stages in the making of final products other than printed books and books and newspapers. ... It is sufficiently obvious ... that over a large part of the field of industry an increasingly intricate nexus of specialized undertakings has inserted itself between the producer of raw materials and the consumer of the final product. (p. 537-8)

Following Young, the model presented below removes from consideration increasing returns from investments in knowledge and external effects due to knowledge spillovers. It focuses entirely on the role of specialization. A more realistic and more ambitious model would examine both effects simultaneously.

The problem with the suggestive discussion of specialization offered by Marshall and Young is that it contains nothing resembling a true externality in production. Rather, it seems to be based on an underlying model of differentiated commodities that are used as inputs in the production of final goods. As part of the Cost controversies of the nineteen twenties, authors as diverse as Frank Knight (1925) and Piero Srafa (1926) objected to this kind of attempt to force a square observation through a round theoretical hole. Knight went so far as to claim that the notion of an economy that was external to the firm but internal to the industry was an "empty economic box." (Knight 1925, p. 333) Economists in areas like trade or urban economics have nonetheless clung to Marshall's notion of external economies, presumably because the familiar framework of competitive equilibrium (with externalities) could be used and because the results of this kind of analysis seemed suggestive. See the survey by Helpman (1984) for a discussion of the use of this framework in international trade, or Papaeorgiou and Smith (1983) for an application of "agglomeration externalities" in a model of the formation of cities.

Once the idea of an externality was formalized by Meade (1952) and the existence of a competitive equilibrium with externalities demonstrated by Chipman (1970), it was clear that Marshall's concept could be rigorously justified. It is now widely accepted that the inclusion of knowledge as an input to production can lead to increasing returns and that spillovers of knowledge between firms can be treated as externalities because patent protection is incomplete. Yet beyond this, Knight seems to have been largely correct. No convincing examples of external economies have been offered. In particular, the notion emphasized by Young--that there could be improvements in the organization of the industry or the economy as a whole as production is divided between firms producing more specialized outputs--cannot be captured as an externality in the modern sense.

In a paper that builds on the analysis of externalities by Meade, Scitovsky (1954) confronts the problem with the conventional use of external economies. To preserve the basic character of the kind of analysis used by Marshall in settings beyond that involving knowledge, he introduces the term "pecuniary externality" to describe a broader notion of externality that he believes economists have in mind in discussions of this kind. He concedes that he can find no precise definition of this concept, but he makes clear that as opposed to a purely technological external effect, a pecuniary external effect is one that operates through the market mechanism; yet it still leads to deviations from optimality. He recognizes that this kind of externality can only arise in a setting in which the equilibrium is not Pareto optimal, yet fails to make clear the sense in which the presence of a pecuniary externality is logically distinct from the mere fact of suboptimality. Despite these shortcomings, his term seems to capture a concept of interest to economists, for it continues to be used. (See for example Hart, 1980)

The interpretation suggested here is that a pecuniary externality may be said to be present if the economy behaves as if a true externality were present; that is, a pecuniary externality is present if there is an isomorphism between the formal model of the economy under consideration and the model for an economy with a true externality. The analysis of the model with specialized goods offered below illustrates this kind of isomorphism in detail.

III. A Static Model of Specialization

The first step in the construction of a model where specialization leads to some form of increasing return has already been taken by Ethier (1985). He suggests that we reinterpret as a production function the utility function used by Dixit and Stiglitz (1977) to capture a preference for variety. In this reinterpretation, the output of final consumption goods is an increasing function of the total number of specialized intermediate inputs used by a final goods producer. In a continuum version of this model where the range of potential intermediate goods is the positive half-line, the list of intermediate inputs used in final good production is a function $x:\mathbb{R}_+ \longrightarrow \mathbb{R}$, where $x(i)$ denotes the amount of intermediate good i used. The production function (really functional) $Y(x)$ analagous to the Dixit-Stiglitz utility function then takes the form

$$(1) \qquad Y(x) = \int_{\mathbb{R}_+} x(i)^{\alpha} di, \text{ where } \quad 0 < \alpha \le 1.$$

Let $\{N,M\}$ denote the list of inputs $x(i)$ which takes on the constant value $x(i) = N/M$ on the range $i \in [0,M]$. Thus, M measures the range or total number of inputs used, and N measures the total amount of inputs; the graph of $x(i)$ is a rectangle of width M lying on the i axis and having a total area equal to N. Since

$$(2) \qquad Y(\{N,M\}) = M^{1-\alpha}N^{\alpha},$$

output of the final good increases without bound with the range of inputs M when the total amount of inputs N is held constant. Stated this way, Y appears to be a constant returns production function, but N and M are not the relevant inputs. As a function of the lists of intermediate inputs $x(i)$, Y is a strictly concave production function that is homogeneous of degree α.

To make these statements about the function Y rigorous, I need to be more precise about its domain. In what follows, it is sufficient to think of Y as being defined on the set of piecewise continuous functions with compact support. Dixit and Stiglitz actually use the analogue of the function Y raised to the power $1/\alpha$ so that their function is homogeneous of degree 1. The form used here is more convenient for the subsequent discussion because it allows me to calculate derived demands directly from profit maximization. It has the disadvantage that firms in this industry appear to earn pure profits. If so, the set of output firms needs to be exogenously specified, but this in not the only interpretation that is available. Any strictly concave production function can be interpreted as a homogeneous of degree 1 production function with a fixed factor of production. Thus, suppose that the true production function is of the form $\mathcal{Y}(L,x) = L^{1-\alpha}Y(x)$, where L is the amount of labor used in final goods production. Under this interpretation, the apparent profits are payments to labor, and firms earn zero profits. If labor can be freely allocated among firms, the number of firms and the scale of operation of each firm is indeterminate. If labor is available in a fixed, inelastically supplied quantity normalized to be equal to one, I am free to work with an industry-wide aggregate production function $Y(x) = \mathcal{Y}(1,x)$. Because it simplifies the analysis, I suppress the argument L in the arguments that follow, and reintroduce it only at the very end.

Some notion of fixed cost appears to be central to the idea emphasized by Smith and Young that the degree of specialization is limited by the extent of the market. Thus, I assume that the intermediate inputs $x(i)$ are produced from a primary input Z according to a cost function that has a U-shaped average cost curve. Preserving the symmetry in the model, assume that an amount $x(i)$ of any good i can be produced at a cost $\kappa(x(i))$. Inaction at zero cost is feasible, so $\kappa(0)$ equals zero; but at any positive level of production, $\kappa(x)$ must be greater than some quasi-fixed cost $\bar{\kappa}$. For simplicity, I assume that this cost is measured purely in terms of the primary input and ignore labor inputs in the production of intermediate inputs. Since this cost is measured in units of the primary good per unit of infinitesimal length di, the resource constraint faced by the economy as a whole is

$$\int_{\mathbb{R}_+} \kappa(x(i))di \leq Z. \tag{4}$$

With this specification for costs, the feasible range of intermediate inputs is finite, and final goods output must also be also finite.

Together, a production function like Y and a cost function like κ offer an extremely crude representation of Young's intricate "nexus of specialized undertakings" that intervene between raw materials and consumption goods, but one that is sufficient for the purposes at hand.[2] It is intended as a kind of reduced form, and clearly does not capture the multiple levels of intermediate inputs and of specialization illustrated in Young's description of the printing trade or his example of the production of special machinery for the production of hammers, which are themselves specialized tools for driving nails. Modeling the output of a firm in the consumption goods sector as a deterministic function of the entire set of specialized inputs that are available is a convenient simplification that cannot be taken literally. Besides allowing for multiple stages of intermediate inputs, a more realistic approach would extend this model in precisely the way that Sattinger (1984), Perloff and Salop (1985), and Hart (1985) extend the Dixit-Stiglitz model of consumer preferences, allowing for many producers of final goods, each of whom has a technology that is most productive with a specific, small subset of all potential intermediate inputs. If the particular inputs that are most productive are distributed symmetrically across a large number of final goods producing firms, the aggregate effect should be similar to that achieved in the model here. If one allows for the possibility of household production, the model can accommodate an apparent preference for variety on the part of

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[2]See Vassilakis (1986) for an alternative model of specialization that explicitly captures the idea that production can take place in an unlimited number of stages, with specialized outputs from one stage being used to produce the specialized inputs used in the next stage.

consumers as well. Radial tires and food processors have as much claim as hammers or wood pulp to the label of intermediate inputs in production.[3]

A decentralized equilibrium for this economy consists of an undetermined number of final output producing firms that can be described in terms of the aggregate production function Y and a continuum of firms in the intermediate goods sector, each of which is the single producer of a particular intermediate input. As will be clear, the existence of the fixed costs and the unlimited number of potential inputs means that there will never be more than a single firm producing any particular intermediate input, so I can index all potential intermediate inputs and the firms that can potentially produce them by $i \in \mathbb{R}_+$. Each of these firms (both potential and actual) is assumed to be a price taker in the market for the primary resource Z. (Implicitly, I also assume that the market for the suppressed labor input is competitive.) Using the primary good as numeraire, let q denote the price of the final output good. Assuming for simplicity that the primary input has no alternative use in consumption or production, preferences can be any increasing function of final good consumption. For now, all that I need to specify about the demand side of the economy is that the individual consumers are price takers and are endowed with the supply of labor and the primary resource.

A simple argument by contradiction shows that this economy cannot have a competitive equilibrium in the usual sense. Given any list of prices $p:\mathbb{R}_+ \longrightarrow \mathbb{R}$ for the intermediate inputs, each price taking firm in the final

[3] Judd (1985), Stokey (1986), and Schmitz (1986) are examples of dynamic models that use a preference functional similar to the production functional used here. Judd is concerned with patent rights on new goods. Stokey asks why some goods disappear as others appear. Schmitz's analysis emphasizes the importance of new goods in sustaining growth and is closest to the analysis of the dynamic model offered in Section V below.

goods sector will demand a positive amount of every input; but positive production of all intermediate inputs is not feasible. Note that this result depends on the non-convexity introduced by the fixed costs, not on the unbounded range of potential inputs. The same argument would go through if potential inputs were constrained to lie in a compact interval $[0,\mathcal{M}]$ for large enough $\mathcal{M}$.

The kind of equilibrium that will obtain is a monopolistically competitive equilibrium as described by Dixit and Stiglitz. Assuming that the number of firms producing the final output good is large, each of these firms will take prices as given for all the intermediate input goods that are produced. Formally, I can allow them to take prices for all goods as given if I set the price for any intermediate good that is not produced equal to $+\infty$. Given a list of prices $p:\mathbb{R}_+ \longrightarrow \mathbb{R} \cup \{+\infty\}$ and a price q for final output, it is straightforward to derive demands for all inputs. For the functional form suggested here, this can be done explicitly. The demand for any particular input is proportional to (the inverse of) the derivative of the power function x^{α} that appears as the integrand in Y:

$$
\begin{aligned}
p(i) &= \alpha q x(i)^{\alpha-1}, \\
x(i) &= \left[\frac{\alpha q}{p(i)}\right]^{\frac{1}{1-\alpha}}.
\end{aligned}
\tag{4}
$$

Individual firms producing capable of producing intermediate goods do not take prices as given. For example, a potential producer of an intermediate input that is not produced cannot be assumed to choose quantities taking a price $p(i) = \infty$ as given. Rather, both potential and actual producers take these demand curves and the price q as given.[4]

Given the cost function κ that describes the technology for converting primary inputs into intermediate inputs, all intermediate input firms (both potential and actual) maximize profits subject to the given demand curves. Since many firms do not produce in equilibrium and since any firm that is not producing can duplicate the inputs and outputs of any firm that is producing, the equilibrium is a zero profit, monopolistically competitive equilibrium. If I specify a particular functional form for the cost function κ, it is possible to calculate this equilibrium explicitly. Given the derived demands, profit maximization on the part of intermediate goods producers leads to values of $x(i)$ that depend on the output price q. The price q is determined by the requirement that profits for the intermediate goods producers must be zero.

For example, let κ take the form $\kappa(x) = (1+x^2)/2$. (This is convenient because it has achieves the minimum average cost of 1 at an output of 1 and has linear marginal cost.) In this case, the equilibrium has

$$x(i) = \left[\frac{\alpha}{2-\alpha}\right]^{1/2}, \qquad (5)$$

[4]Strictly speaking, it is not meaningful to talk of the response of the final output firms to a change in the price of a single intermediate input or in the prices for any other measure zero set of inputs. The limiting argument I have in mind to justify this equilibrium is one in which input producing firms are partitioned into collusive coalitions of finite measure. As the size of all such coalitions is driven to zero, the influence of any coalition over the price q for final output (or over the price for the entrepreneurial factor) will go to zero, and the limit equilibrium will be the one described.

on a set of inputs i of length

$$M = Z(2-\alpha), \tag{6}$$

with $x(i) = 0$ otherwise. It is also straightforward to calculate the quantities that would be chosen by a social planner who maximizes output subject to the constraints imposed by the technology. A curious feature of the iso-elastic production function used here is that the quantities from the social optimum coincide with those in the decentralized equilibrium.

This appears to be a convenient feature of this form of production. Given the level of Z, all I need to do to calculate a static equilibrium is to solve a social optimization problem. In fact, this is much less helpful than it appears, because it relies crucially on the fact that the stock of Z is given. In any extension of this model that allows an alternative use for Z, the decentralized equilibrium will differ from the social optimum. (These results are all clear from the discussion in Dixit and Stiglitz 1977.) Direct calculation shows that the marginal rate of transformation between the resource Z and consumption goods differs from the price q that obtains for consumption goods; it is too large by a factor of α^{-1}. Consequently, any model that explains growth by allowing individuals to forego current consumption and accumulate additional units of the resource Z will necessarily have an equilibrium that differs from the social optimum.

For given Z, the optimal values of $x(i)$ and M obtain in equilibrium because of two offsetting influences. The first is due to the presence of a downward sloping demand curve faced by actual producers of intermediate goods. This leads each individual producer of intermediate inputs to produce at a point where the marginal cost of an additional unit is less than its marginal productivity. This reduces the equilibrium level of $x(i)$ for goods that are produced and therefore raises the equilibrium level of M that can be supported.

The opposing effect arises from the fact that the payments to labor (or under the alternative interpretation, the profits in the final goods producing sector) are increasing in the range of goods produced; but in the absence of economy wide vertical and horizontal integration, there is no way for workers or firms in the output sector to increase the range of goods produced. This effect by itself leads to a value of $x(i)$ for inputs that are produced that is too big and to a range of inputs M that is too small. The iso-elastic functional form happens to be a case where these effects on quantities exactly cancel. But both effects lead to an undervaluation of additional units of the primary resource.

The first effect turns out not to be central to the results in this paper. It is the second effect that seems to have some connection with the external effects that Marshall and Young perceived. Without taking a stand on how important market power for actual intermediate goods producers is as a practical matter, it is useful for expository purposes to assume it away and focus exclusively on the second issue, the divergence between the private and social gains from the introduction of new goods.

With the functional form used so far, it is impossible to separate these two effects. Both depend on the the curvature paramenter α for the power function x^{α} that appears as the integrand in the definition of Y. For a given value of q, the demand schedule faced by a firm producing an intermediate input is proportional to the derivative of this power function and is downward sloping if that function is strictly concave. As is clear from equation (2), the extent to which output is increasing in the range of inputs also depends on α and goes to zero as α goes to 1.

To disentangle the effects, consider an alternative form of production Y. Suppose now that Y takes the form

$$(7) \qquad Y(x) = \int g(x(i))di,$$

where the function $g: \mathbb{R}_+ \longrightarrow \mathbb{R}$ replaces the power function x^α. Provided that g is concave and nondecreasing, this will still be a well defined, concave production function. To preserve the result that final output depends nontrivially on the range of inputs used, g must have some degree of curvature; but to avoid a difference between the marginal cost and marginal productivity of goods that are produced, individual producers of intermediate outputs must face demand curves that are horizontal. Since these demand curves are proportional to the derivative of g, these requirements cannot both be satisfied globally. However, for an intermediate goods producer taking this demand curve as given, all that is required to ensure that price equals marginal revenue is that the demand curve be flat around the point where it intersects marginal cost.

Consequently, suppose that the function g is at least twice continuously differentiable with the following properties. On the interval $[0,x_0]$, g is strictly concave, with $g(0) = 0$, $g'(x_0) = 1$. On the interval $[x_0,\infty)$, let g have a constant slope equal to 1. In the graph of g, let Γ denote the intercept that is defined by tracing the constant slope of 1 back to the vertical axis. Thus, for $x > x_0$, $g(x) = \Gamma + x$. The curvature in the interval $[0,x_0]$ is needed simply to satisfy the requirement that $g(0) = 0$ without violating continuity. The derived inverse demand curve

$p(i) = q\, g'(x(i))$ is a differentiable curve that may or may not have a finite intercept. It is downward sloping on the interval $[0,x_0]$, and takes on the constant value of q on $[x_0,\infty)$.

Now, consider the output from $Y(x)$ with this function g. As before, let $\{N,M\}$ denote the rectangular list of inputs with a range of M different specialized inputs each supplied at the level $x(i) = N/M$ so that the total amount of inputs used in N. If N/M is greater than x_0 (and by choice of a small enough x_0, these will turn out to be true for all relevant lists of inputs), the expression for output as a function of N and M is

$$(8) \qquad Y(\{N,M\}) = \Gamma M + N.$$

As before, this is increasing in the range of inputs M when total inputs N are held constant. With this function, I can repeat the analysis given above. The geometrical construction leading to the calculation of the monopolistically competitive equilibrium is as follows. The derived inverse demand curve that the producer of good i faces depends on the level of q, $p(i) = q\, g'(x(i))$. For given q, $x(i)$ and $p(i)$ are chosen by the intersection of the marginal revenue schedule with the marginal cost curve $\kappa'(x)$. The equilibirum value of q is determined by the requirement that the resulting price-quantity pair lie on the average cost curve.

Suppose that $q = 1$. Given that marginal cost $\kappa'(x) = x$, the assumption that x_0 is small relative to 1 implies that marginal cost intersects the marginal revenue schedule at the point $(p,x) = (1,1)$ where it coincides with the demand curve. This is also a point on the average cost curve--in fact the point of minimum average cost--so this corresponds to a potential equilibrium. Given that x_0 is small and provided that the demand price $g'(x)$ does not go to ∞ too rapidly as x goes to zero, the U-shaped average cost curve will lie above the demand curve for all other values of x, tangent only at the point (1,1). Then this is the unique monopolistically competitive equilibrium.

For the functions used here, the equilibrium price for output will equal 1, the ultimate slope of the function g. The equilibrium list of inputs $x(i)$ will take on the value 1 for a set of inputs i of measure $M = Z$ and will be zero elsewhere. It is also a simple matter to calculate the solutions to the social planning problem for this economy,

$$x^*(i) = \left[\frac{1-\gamma}{1+\gamma}\right]^{1/2} \text{ and } M^* = Z(1+\gamma), \tag{9}$$

where

$$\gamma = \left[\frac{\Gamma^2}{1+\Gamma^2}\right]^{1/2}. \tag{10}$$

For this form of production, the decentralized equilibrium leads to a range of output goods that is too small relative to that achieved in the social optimum. Here, all firms that are producing intermediate goods do so up to the point at which the marginal cost equals the marginal product, so there is no force to offset the tendency for the equilibrium to provide too small a range of inputs.

IV. A Related Model with Externalities

In the example just described, it is not difficult to calculate a decentralized equilibrium directly. It requires explicit consideration of demand functions and zero profit conditions with an infinite number of goods, but because of the symmetry in this example, this is essentially a one dimensional problem. Once the model is extended to a growth model with goods at an infinite number of dates, explicit consideration of demands for dated goods as functions of prices over time becomes much more difficult. Rather than pursue this direct approach, I want to use an analogue of the usual approach in growth models where computation of the equilibrium reduces to the solution of a dynamic optimization problem. Calculation of the social optimum for a dynamic version of this model is straightforward, but for the reasons noted above, this cannot be supported as a decentralized equilibrium. Because each intermediate goods producer sells to a large number of final goods producers, vertical integration alone will not be sufficient.

One indication that some kind of maximization approach to the computation of the equilibrium should be feasible is given in Theorem 2 in Hart (1980). It states that in an equilibrium where agents take prices as given for all goods that are actually produced, the outcome satisfies a restricted form of optimality. Loosely speaking, the equilibrium solves the optimization problem that would be faced by a planner who takes as given the set of goods that can be produced and maximizes welfare subject to this constraint. This observation closely resembles the idea exploited in my earlier work on a dynamic model with externalities (Romer 1983, 1986); in a competitive equilibrium with externalities, the equilibrium quantities solve the social planning problem that would be faced by a planner who takes as given the level of the aggregate external effect and maximizes welfare.

This section makes precise the formal similarity between the model with differentiated products given above and a model with true externalities. To do this, I construct an artificial economy that has a true externality and show how all the quantities from the model with differentiated intermediate inputs can be inferred from the competitive equilibrium with externalities for the artificial economy. This correspondence between the variables in the artificial economy and the original economy is emphasized by the use of the same symbols N, M, Z,... in both cases. However, in the artificial economy I assume that there are a continuum of firms indexed on the unit interval, and use lower case letters to distinguish per firm quantities from aggregate quantities. (Since the mass of firms is assumed to be one, the aggregate value will actually be the same number as the per firm value, but with different units.)

Let each firm in the new economy be endowed with z units of a primary resource and assume that the following technology for converting the primary resource into an intermediate good is present. Let a function $n(m,z)$, $n:\mathbb{R}_+^2 \longrightarrow \mathbb{R}$, describe the amount of an artificial good n that is produced from z when some intermediate control variable is set equal to m. Define this function implicitly by the requirement that

$$m\kappa\left(\frac{n(m,z)}{m}\right) = z, \tag{11}$$

where κ is the same cost function as before. (The expression n/m appears as the argument of κ because N/M is the value of x for any rectangular input list in the original economy.) This means in effect, that the artificial economy has the same technology for converting the primary resource into intermediate outputs as did the original economy.

Recall that $Y(\{N,M\})$ was the notation used to indicate the value of the production function in the original economy when it was evaluated at a list of inputs $x(i)$ that took on the constant value N/M on a set of measure M. Because of the symmetry in the functional forms used for Y, these input lists are the only relevant ones, so it is useful to define a function $\hat{Y}:\mathbb{R}_+^2 \longrightarrow \mathbb{R}$ by

$$(12) \qquad \hat{Y}(N,M) = Y(\{N,M\}).$$

For a symmetric functional Y, much of the information about the technology can be summarized in terms of $\hat{Y}$.

Now, for each firm, define a production function for an individual firm $j \in [0,1]$ as

$$(13) \qquad f(m_j,M,z_j) = \hat{Y}(n(z_j,m_j),M),$$

where M is the economy wide level of the individual choices of m, $M = \int_0^1 m_j dj$. This function f explicitly allows for an externality in production. Each firm chooses its own level of the control variable m, taking as given the economy wide level M. In terms of the analogy that I am developing between the two models, it is as if each firm takes as given the "range" of inputs available when it makes its own production decisions.

The calculation of the competitive equilibrium with externalities for the artificial economy is now trivial. Like a Pareto optimal equilibrium with identical agents, it can be calculated directly in terms of quantities, without explicit reference to prices or demands. The social optimization problem for a planner who can simultaneously set the aggregate and the individual firm levels of m is characterized by the first order condition for maximizing output

$$(14) \qquad D_1f(m^*,m^*,z) + D_2f(m^*,m^*,z) = 0.$$

The solution to the maximization problem of an individual firm in the competitive equilibrium with externalities is characterized by the first order condition

$$D_1 f(\tilde{m},M,z) = 0. \tag{15}$$

After substituting in the equilibrium condition that $M = \tilde{m}$, this becomes

$$D_1 f(\tilde{m},\tilde{m},z) = 0. \tag{16}$$

Provided that the underlying functional $Y(x)$ is concave as a function of the list of arguments x, the related function $\hat{Y}(N,M)$ will be concave in N for any fixed M. If the cost function κ is convex except for the discontinuity at 0, then $f(m,M,z)$ is a concave function of m and z for any fixed M, and a standard application of the Kuhn-Tucker theorem can be used to generate prices that support the quantities from this equation in a competitive equilibrium with externalities for the artificial economy.

Direct calculation shows that the equilibrium values of $M = \tilde{m}$, $N = n(Z,\tilde{m})$, and $x = N/M$ calculated for this artificial economy with externalites are identical to those derived for the original economy with differentiated products. The quantities for the social optimum are the same as well. Any theoretical intervention in the original economy, such as a change in the endowment of Z or the introduction of a tax-subsidy scheme, can be mapped into an intervention in the artificial economy with externalities and studied in this simpler context. In this sense, Marshall and his followers may not have been too far off the mark in referring to this as a case of external economies. This economy might also be said to possess a pecuniary externality, that is, to behave as if it had a true externality. Whether this will prove to be a useful interpretation of Scitovski's term in the other contexts in which it has been used remains to be seen.

V. A Dynamic Model

The obvious way to make the previous static model into a growth model is to allow for the accumulation of the primary resource Z. Most models of growth place a great deal of emphasis on the distinction between human capital and physical capital and on the different technologies for accumulating the two types of capital. Typically, they make the very strong assumption that human capital cannot be accumulated at all and that output can be converted one-for-one into new capital. For simplicity, I will follow this convention and treat Z as accumulated physical capital and treat labor L as being in fixed supply. However, this is surely as important a limitation on the present model as the exclusion of knolwedge as a distinct input into production. For simplicity, I will also treat the supply of labor as being exogenous and neglect both a labor-leisure trade-off and population growth, but the inclusion of these elements into the model should be straight forward. Recall also that labor is a factor of production only in the final goods sector; I assumed that intermediate goods were produced at a cost only in terms of the primary resource, which I am now calling capital. In a more detailed treatment of this model, this would have to be fixed up as well.

The specification of intertemporal preferences is conventional,

$$\int_0^\infty U(c(t))e^{-\rho t}dt. \tag{17}$$

For convenience, let there be a continuum of identical consumers indexed on the interval $[0,1]$, each endowed with an amount $z(0)$ of the total initial stock of the primary capital. So that I can work interchangeably with per capita and per firm quantities, let there also be a continuum of firms in the final goods producing sector also indexed on $[0,1]$, all producing at the same level. Consumers will rent their capital to intermediate goods producing firms, who use it to produce intermediate inputs $x(i,t)$ according to the technology defined by the cost function κ. As before, the feasible set of intermediate inputs is constrained by the equation

$$\int_{\mathbb{R}_+} \kappa(x(i,t))di \le Z(t). \tag{18}$$

Note that the quasi-fixed cost in the production of any intermediate good is a flow cost not a one time cost.

Each individual in this economy receives per capita output (equal to per firm output) of $Y(x)$. This must be allocated between consumption $c(t)$ and investment in additional capital. The simplest investment technology is one that permits foregone output to be converted one-for-one into new capital, but that also allows for exponential depreciation at the constant rate δ. Thus,

$$\dot{z} = Y(x) - c - \delta z. \tag{19}$$

Direct calculation of a dynamic equilibrium for this model would go something like this. As in the static model, let $q(t)$ denote the spot price of the output good at time t measured in units of current capital goods. Given a value of $Z(t)$, $q(t)$ would be determined as above, using the derived demand curve for intermediate inputs and the zero profit condition. Accumulation of Z by private agents would be determined by an intertemporal utility maximization problem where agents take the path for $q(t)$ and the interest rate as given. The interest rate is determined by the discount rate ρ and the equilibrium rate of change of marginal utility $U'(c(t))$.

Rather than attack this problem directly, I exploit the analogy between the true model with specialization and the artificial model with externalities. Per capita and per firm output at time t can be described as a function $f(m,M(t),z(t))$. In this artificial economy with externalities, each firm will take as given the entire path for $M(t)$ when it makes its decisions concerning $m(t)$. Just as in the static example described above, the equilibrium can be calculated by taking the first order conditions for a social optimization problem where the path of $M(t)$ is taken as given, then substituting in the equilibrium condition $M(t) = m(t)$. Thus, consider a family of intertemporal optimization problems that take an arbitrary path for $M(t)$ as given:

$$(20) \qquad P(M) \qquad \max_{m,c} \int_0^\infty U(c(t))e^{-\rho t}dt$$

$$\text{subject to} \quad \dot{z}(t) = f(m(t),M(t),z(t)) - c(t) - \delta z(t)$$

$$z(t) \geq 0,$$

$$z(0) \text{ given.}$$

The first order conditions for this problem are a system of differential equations for the state variable $z(t)$, a co-state variable or multiplier, say $\lambda(t)$, and the control variables $c(t)$ and $m(t)$. All these equations will depend on the path for $M(t)$. Once $m(t)$ is substituted in for $M(t)$, they form an autonomous system of differential equations that can be studied in the usual fashion. The shadow price $\lambda(t)$ can be interpreted as a market price for units of the capital stock Z at time t and can be used to derive the interest rates and spot prices for the equilibrium.

To illustrate this procedure, suppose that the utility function $U(c)$ takes the iso-elastic form

$$U(c) = \frac{c^{1-\sigma}-1}{1-\sigma}, \tag{21}$$

where $\sigma \in [0,\infty)$. Let the cost function $\kappa(x)$ take the form used above, $\kappa(0) = 0$, $\kappa(x) = (1+x^2)/2$ for $x > 0$, and let Y take form used with the function g as the integrand. For relevant input lists, $\hat{Y}(N,M) = \Gamma M + N$. The function $n(m,z)$ then takes the form

$$n(m,z) = (2zm-m^2)^{1/2}, \tag{22}$$

and $f(m,M,z)$ takes the form

$$f(n,M,z) = \Gamma M + (2zm-m^2)^{1/2}. \tag{23}$$

The necessary conditions for the problem $P(M)$ are most easily derived by defining a present valued Hamiltonian

$$H(z,\lambda,M) = \max_{m,c} U(c) + \lambda(f(m,M,z)-c-\delta z). \tag{24}$$

For the functional forms used here, the first order conditions for m and c in the definition of H are

$$m = z, \tag{25}$$

$$c = \lambda^{-1/\sigma}. \tag{26}$$

The equation for the evolution of λ is

$$\dot{\lambda} = \lambda\rho - \frac{\partial H}{\partial z} = \lambda\rho - \lambda(1-\delta), \tag{27}$$

so $\lambda(t)$ takes the form

$$\lambda(t) = \lambda(0)e^{\rho+\delta-1}. \tag{28}$$

Substituting this into the expression for $c(t)$ and substituting the result plus the equilibrium condition $M(t) = m(t)$ into the evolution equation for $z(t)$ gives a linear differential equation in $z(t)$ that can be solved explicitly in terms of $z(0)$ and $\lambda(0)$. Then the transversality condition $\lim_{t\to\infty} \lambda(t)z(t)e^{-\rho t} = 0$ can be used to solve for $\lambda(0)$. The result is an equilibrium in which $\tilde{c}(t)$ and $\tilde{z}(t)$ both grow at the exponential rate $(1-\rho-\delta)/\sigma$. The spot price $q(t)$ for output in terms of the capital good Z

is identically equal to 1. The instantaneous interest rate takes on the constant value $1-\delta$.

The calculation of the social optimum is essentially the same except that the substitution of $m(t)$ for $M(t)$ is done prior to the differentiation of the Hamiltonian. Recall the definition of

$$(29) \qquad \gamma = \left[\frac{\Gamma^2}{1+\Gamma^2}\right]^{1/2}.$$

As in the static model, the optimal level of m is larger than the equilibrium level for any fixed z:

$$(30) \qquad m^*(t) = z^*(t)(1+\gamma).$$

The optimal paths $z^*(t)$ and $c^*(t)$ both grow at the faster rate

$$(31) \qquad \frac{\dot{z}^*}{z^*} = \frac{\dot{c}^*}{c^*} = \frac{1}{\sigma}\left[\left[\frac{1+\gamma}{1-\gamma}\right]^{1/2} - \rho - \delta\right].$$

The optimal paths differ from the equilibrium paths for two reasons. First, for any given level of z, the output in the optimum is higher because the level of m that is chosen is different in the two cases. This is exactly the effect derived from the static model. The second effect, unique to the dynamic model, is that the fraction of output that is devoted to consumption is different in the two cases because the decentralized equilibrium does not offer the correct incentives for the accumulation of the capital good. In the decentralized equilibrium, output has a price in units of the capital good that differs from the socially feasible rate of transformation between capital goods and output.

Ex post it is easy to see that the equilibrium calculated for the artificial economy corresponds to an equilibrium for the economy with specialization. In fact, given the constancy of $q(t)$, it might have been possible to skip the step of constructing the model with the externality and to quess the solution for the dynamic model with specialization directly. In more complicated problems, for example ones that cannot be explicitly solved and can only be characterized by the geometry of the phase plane, I suspect that this computational device will nonetheless prove useful.

The final feature of this model worth noting is the sense in which it exhibits increasing returns. To do this, it is useful to reintroduce labor as an explicit argument in the production function for final output. Recall that true production is assumed to be a homogeneous of degree one function of the form $\mathcal{Y}(L,x)$. The explict form for $\mathcal{Y}$ corresponding to the function Y that is used in this section can be written as

$$\mathcal{Y}(L,x) = L\int_{\mathbb{R}_+} g\left[\frac{x(i)}{L}\right]di. \tag{32}$$

Despite the fact that this is a well behaved concave, constant returns to scale production function, it appears to exhibit increasing returns. Evaluated along a the usual rectangular list of inputs $\{N,M\}$ (and assuming that N/LM is greater than the critical level x_0 below which g is strictly concave)

$$\mathcal{Y}(L,\{N,M\}) = \Gamma LM + N. \tag{33}$$

In equilibrium, the list of inputs $x(\cdot,t)$ at any time t is $x(i) = 1$ on a set of measure $M(t) = Z(t) = N(t)$ and zero elsewhere. Output at time t is $Z(t)(\Gamma L+1)$, which is split into total payments to the labor equal to $Z(t)\Gamma L$ and total payments to owners of the resource equal to $Z(t)$. The apparent surplus accrues to labor. As $Z(t)$ grows, the shares of income accruing to capital and labor will be constant and the equilibrium will look like one with exogenous labor-augmenting technological change. Because of the casual treatement of labor in this model and the many simplifying assumptions, these observations are at most suggestive, but they do seem to offer an alternative interpretation of the conventional stylized facts about long run growth.

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