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Abstract

This paper develops an analytically tractable model of a firm's production decisions in the face of demand uncertainty that takes explicit account of the non-negativity constraint on inventories. The model also allows a range of assumptions about the firm's ability to backlog its excess demand. Either the ability to backlog or positive serial correlation in demand causes the firm's production to exhibit more variance than its sales. Testable implications of the model are discussed, and modifications to the standard linear-quadratic specification are suggested.

> Manufacturers, setting industry in motion to provide for a scale of consumption which is expected to prevail some months later, are apt to make minor mistakes, generally in the direction of running a little ahead of the facts. When they discover their mistake they have to contract for a short time to a level below that of current consumption so as to allow for the absorption of the excess inventories; and the difference of pace between running a little ahead and dropping back again has proved sufficient in its effect on the current rate of investment to display itself quite clearly against the background of the excellently complete statistics now available in the United States.
>
> J. M. Keynes, The General Theory.

Economists have long recognized that inventories play an important role in business cycles, as the above quotation suggests. The importance of changes in inventory investment in cyclical fluctuations has been widely documented.[1] In eight post-war recessions, declines in inventory investment have on average accounted for more than 30 percent of the peak-to-trough declines (relative to trend) of real GNP, even though inventory investment itself represents a much smaller component of GNP. Blinder and Fischer's (1981) detailed accounting of the 1973-75 recession also suggests that most of the action in inventories occurs in the finished goods component, and that large changes in inventory investment occur one quarter after large changes in sales. Yet the common assumption that firms try to smooth production in the face of fluctuating demand, either because of diminishing returns or costs of adjusting production, suggests that inventories could be a stabilizing force.

A temporary decline in demand would, according to this view, result in some degree of accumulation of inventories (or, to be more precise, the inventory reduction that the firm undertakes should not be so great as to offset the unintentional accumulation that is the direct result of the demand shock), as the firm only partially adjusts its rate of production.

It is safe to say that few economists still believe that finished goods inventories are held only to smooth production in the face of fluctuating demand. The simple fact that the variance of production exceeds the variance of sales--even after controlling for obvious seasonal factors influencing production (see, e.g. Blanchard (1983))--strongly suggests that production-smoothing is of secondary importance. Two general approaches that have been considered in the literature are (1) to allow for a stockout-avoidance motive, taking account of the non-negativity constraint on inventories, and (2) to allow for cost or productivity shocks. In the spirit of the first approach is the inclusion in a linear-quadratic model of a (quadratic) cost of being away from some target level of inventory. If the target is proportional to expected next-period sales, for example, then the variance of production can exceed the variance of sales. The difficulties with this approach, apart from its obviously *ad hoc* nature, are, first, that it does not prevent inventories from being negative, and, second, that the more general variance inequality that it implies has also been rejected empirically (see West (1986)). Moreover, Abel's (1985) more rigorous treatment of the non-negativity constraint led him to the opposite conclusion, namely that production should be smoother than sales. The approach that allows for cost shocks, as exemplified by Eichenbaum (1984) and Blinder (1986), while able to account for production volatility, in practice still ignores the non-negativity constraint. Also, it suffers from the lack of better direct evidence of cost shocks.

This paper develops a model of production and inventory behavior that focuses exclusively on the stockout-avoidance motive. The model potentially explains many of the stylized facts about inventory behavior that have been documented in recent years, including the "excess" volatility of production (e.g. Blinder (1986) or West (1986)), and the positive correlation between inventory levels and sales (Rotemberg and Saloner (1985)). Inventories arise as a result of two plausible technological constraints: (1) Firms cannot costlessly short-sell their products; and (2) production decisions are made in advance, before the state of demand is fully known.[2] The model has linear costs, so that there is no conventional production-smoothing motive. With explicit specifications of uncertainty, the model yields analytical solutions to the firm's optimization problem. The solutions explicitly take into account the non-negativity constraint on inventories (i.e. the possibility of stockouts) and a range of assumptions about the firm's ability to backlog its excess demand.

The conditions under which the variance of production is greater than the variance of sales turn out to be quite broad. They include demand processes that are independently and identically distributed (i.i.d.), and processes that are positively serially correlated. Although the linear cost structure presumably overstates the potential for production counter-smoothing, the strength of the findings below suggests that production can be more variable than sales even if there is significant convexity of costs, and even if the only source of uncertainty is on the demand side. Hence the model is capable of rationalizing the violations of the production-smoothing models found by Blinder (1986) and West (1986). The model also suggests some ways in which the linear-quadratic framework employed in much empirical research (e.g. West (1986) and Blanchard (1983)) might be modified to make it a better approximation to more rigorously based models.

The plan of the paper is as follows: Section I presents the general version of the model, and solves for a representative firm's optimal price and production decisions. Section II contains the central results of the paper, namely conditions under which production will be more variable than sales. Section III illustrates the main points of the paper with an extended example. The final section summarizes the results and suggests areas for further research.

I. The Model

In this section I consider a monopolist's choice of price and production when it is uncertain about its demand. Each period the firm must decide how much it wishes to have available for sale in the coming period, and at what price, prior to knowing demand for that period.[3] If actual sales turn out to be less than the available stock, the firm may carry the remainder (at some cost) into the next period as inventory. If, on the other hand, the firm "stocks out", the result is lost and/or delayed sales.[4] The model of the firm is similar to that of Karlin and Carr (1962). The main generalization is that I allow the firm to backlog some portion of its excess demand in any period.

Given its current information, the firm chooses a price and a quantity of production for the succeeding period to maximize expected profits over an infinite horizon. After observing the realization of demand for that period and any other relevant news, the firm again chooses price and quantity for the next period, and so on. The stock of goods the firm has available for sale each period is equal to its leftover inventory plus that period's production. Actual sales equal either that period's total demand or the amount available for sale, whichever is smaller.

I make the extreme but useful assumption of a linear cost of production. Thus the firm will not explicitly use inventories to smooth production over time. There is nothing in this assumption that would bias the firm toward production counter-smoothing, though. A positive discount rate ensures that the firm will try to match production and sales, as the model implies that in the limit as the uncertainty about demand is eliminated, production becomes identical to sales.

If backlogging is feasible, the firm's total demand may include its excess demand from previous periods. A backlog, for the purposes of this paper, occurs when a firm and a customer agree to execute a transaction at some period in the future, as a consequence of the firm having stocked out in the current period. I assume that the firm can backlog some known portion of its excess demand in any period, an assumption which includes as special cases Abel's (1985) assumption of no backlogs, and West's (1983, Ch. 3) assumption of 100 percent backlogs. The cost to the firm of backlogging is the interest lost on the price of the goods, since money is assumed to change hands only at the time of the sale. The price for the backlogged sale is the price as of the original demand, not the price at the time of the sale.[5]

The assumptions regarding the timing and availability of production are worth emphasizing, because they differ somewhat from those of other authors (e.g. West (1983), Abel (1985)). I assume that the production resulting from decisions based on period t-1 information is available for sale in period t. There are several possible interpretations of this assumption. One is that the production decision based on period t-1 information is implemented in period t, but quickly enough that the production is available for sale in period t as well. A second interpretation is that the decision is implemented in period t-1, with the production available for period t. Still a third is that production and sales activity take place in alternate sub-periods, night and day, for example, with each night's production based on information including the day's sales. The convention for this paper will be the first, so the production indexed by t will always be based on information as of t-1, but available for sale in period t (see Figure 1). The substantive difference between this assumption and that of the above-mentioned authors is that they assume a longer lag between the receipt of information and the availability of production for sale, namely that the production based on period t-1 information is not available for sale until period t+1. Which assumption is better is an empirical question, and may differ from one industry to another. I would argue that this paper's assumption is probably closer to the truth for production-to-stock industries. Industries with lengthy production periods (e.g. airplane manufacturing, construction) tend to produce "to order" rather than to stock, and therefore do not hold finished goods inventories. Figure 2 depicts one possible realization of inventories, sales, and production, under the assumption that the latter two activities take place at constant rates within a period (except in the event of a stockout). During period t-2, sales are occurring at a faster rate than production, so inventories are drawn down. The opposite is the case in period t-1. In period t, sales occur at such a fast rate that the firm stocks out during the period, at which point the sales are constrained by production. The next period's production is then increased, with the goal of building inventories back up to a more desirable level. Again, there are other possible timing conventions that would lead to different within-period behavior of the endogenous variables that are entirely consistent with the results presented below.

I assume that the firm's demand in period t has a stochastic component representing new demand for that period, and a certain component representing backlogs from earlier periods. Prices enter the stochastic component multiplicatively, so that the firm's total demand in period t can be expressed as:

$$x_t = h(p_t)S_t + b_{t-1}, \tag{1.1}$$

where b_{t-1} is the stock of backlogs, S_t is an exogenous stationary stochastic process, and the function h() is non-stochastic.

If the firm does stock out in period t, then its excess demand is just $x_t - (n_{t-1} + y_t) = x_t - z_t$. I assume that the firm can backlog a portion q of its excess demand, where q is a known constant. We have, therefore

$$(1.2) \qquad b_t = q(x_t - z_t).$$

The assumption that q is a known constant is restrictive, and in many cases might not be very realistic. It does, though, represent a generalization of the assumptions made by West (1983) (who assumed q=1), and Abel (1985) (q=0).

I assume that the only direct costs facing the firm are a linear cost of production, cy_t, and its opportunity cost of funds, which implies a discount factor $\beta < 1$.[6] Both β and c are constant over time and across firms. The firm's realized profit in period t is

$$(1.3) \qquad \pi_t = p_t z_t - cy_t.$$

Inventory carried into the next period satisfies the identity

$$(1.4) \qquad n_t = n_{t-1} + y_t - z_t,$$

and sales can be no greater than the available stock, nor greater than demand at a given price:

$$(1.5) \qquad z_t \leq \min[n_{t-1} + y_t, x_t].$$

In what follows, I will assume that (1.5) holds with equality, i.e. that the firm never turns down a sale. This will be justified below in the case of multiplicative demand shocks by Propositions 1 and 2, which show that the firm earns a constant positive markup in each period.

At the beginning of each period, the firm chooses price and production to maximize the present discounted value of expected profits. Thus it solves

$$E_{t-1}[\sum_{s=t}^{\infty} \beta^{s-t}(p_s z_s - cy_s)]$$

subject to the inventory-production-sales identity (1.4), the non-negativity constraint on inventories (1.5), the demand function (1.1), and the backlog equation (1.2). The multiplicative nature of the stochastic component of demand turns out, not surprisingly, to be a helpful restriction. I will take it one step further and assume that innovations in the demand process enter multiplicatively, i.e.

$$S_t = E_{t-1}(S_t)u_t,$$

where the u_t are independently and identically distributed. I choose this particular specification of uncertainty because of its simplicity (it will turn out that price is independent of the state variables) and because of its realism as compared to other simple specifications. The additive uncertainty assumed in Section III, for example, would imply that uncertainty diminishes in relative importance with the scale of operations.

To summarize: Each period the firm's demand consists of its backlogged sales from previous periods, about which it is certain, plus new demand, which it does not know at the time it makes its production decision. In choosing its level of production, the firm must weigh the possibility that it will stock out against the possibility that it will end up with inventory to be carried over into the next period. The key assumption is that the firm cannot literally sell a good that it does not have in stock.

I now solve the firm's profit maximization problem using standard dynamic programming techniques. The fundamental dynamic programming equation is given by

$$(1.6) \qquad J(n_{t-1}, b_{t-1}, \Gamma_{t-1}) =$$

$$\max_{p,y} E_{t-1}(p_t z_t - cy_t + \beta J(n_t, b_t, \Gamma_t))$$

subject to the constraints (1.1), (1.2), (1.4), and (1.5), where Γ_t denotes the set of state variables relevant to the firm's forecasting problem. The linear cost structure assumed in this problem greatly facilitates its solution. The primary effect of the assumption of multiplicative uncertainty is to make the solution time-invariant, as we shall see.

Let G() denote the cumulative distribution function (assumed to be continuous and strictly monotonic) for the forecast error u_t. Also, let $Q_t \equiv [n_{t-1} + y_t - b_{t-1}]/[h(p_t)E_{t-1}(S_t)]$, the ratio of the amount the firm makes available for sale (in excess of its backlogs) to its expected demand (again, not counting its backlogs). The probability that the firm stocks out in period t is $1 - G(Q_t)$, which is the probability that the forecast error is greater than Q_t. Using the fact that the derivative of expected sales with respect to production is equal to the stockout probability, one can show that the first-order condition for the production decision is:

$$(1.7) \qquad 0 = (p_t - c)(1-\beta q)[1 - G(Q_t)] - c(1-\beta)G(Q_t).$$

This condition can be interpreted as follows: At a given level of production, the firm will stock out with probability 1–G, in which case it loses p–c in the current period, while backlogging a portion q of the excess demand. Thus the net gain to the firm from producing another unit this period, conditional on stocking out, is $(p-c)(1-\beta q)$. With probability G, however, the firm will not stock out, in which case an increase in production this period is just a shift from the next period. Hence, conditional on not stocking out, the firm loses $c(1-\beta)$ as a result of producing an extra unit this period. At the optimum, the net expected gain from producing another unit should equal zero. It is easy to verify that at the optimum, G will lie strictly between 0 and 1, provided $p > c$ and $0 \leq \beta < 1$.

One immediate implication of equation (1.7), because it is expressed in terms of Q_t, is that the firm always produces enough to satisfy the certain component of its demand, which in this model is the stock of backlogged sales. This makes sense, because if $p > c$, the firm profits on each sale; hence it would never wish to delay or miss out on a certain sale.

Equation (1.7), or something very close to it, holds much more generally than just under the assumptions in this paper. The only differences would be that with a more general specification of uncertainty, G would be replaced by the distribution function for demand, and the parameters assumed to be constant would be time-varying, and possibly functions of the control variables y and p. But for given values of p, c, β, q, and for a given distribution of demand, (1.7) must hold at the optimum. It essentially expresses the tradeoff between stocking out and carrying over inventory as described earlier. Note, for example, that a larger markup implies a lower stockout probability (that is, _ceteris paribus_, more production), while a greater ability to backlog increases the firm's optimal stockout probability.

The first-order condition with respect to price is a little more complicated. Letting $\underline{u}(Q_t) \equiv E(u_t | u_t < Q_t)$, and $e_t \equiv - \frac{\partial h}{\partial p_t} \frac{p_t}{h}$ (the price elasticity of expected demand), we have

$$(1.8) \qquad 0 = \underline{u}G + Q_t(1-G) + \beta q[1 - (\underline{u}G + Q_t(1-G))] - e_t \left\{ \frac{p_t - \beta c}{p_t} \underline{u}G + \beta q \frac{p_t - c}{p_t} [1-\underline{u}G] \right\}.$$

This condition is perfectly analogous to the standard monopolist's first-order condition for price. An increase in price raises revenue on existing sales, but drives some customers away. The positive terms in (1.8) equal expected sales (expressed as a proportion of expected demand), including those sales that are backlogged, i.e. not executed until the following period. The negative term takes account of the fact that some customers will, in expectation, be lost as a result of the price increase, and is proportional to the elasticity of expected demand. The primary difference between (1.8) and the corresponding condition under certainty ($(p-c)/p = 1/e$) is the possibility of no reduction in sales in response to a price increase, which arises because of the positive stockout probability. In other words, the elasticity of expected sales will generally be less than the elasticity of expected demand, because with probability 1-G the firm will stock out, in which case there will be no change in sales, on the margin, in response to a price increase. Intuition should therefore suggest that, because of this, the markup will be greater than in the certainty case, and indeed this is confirmed by the following result.

Proposition 1: The firm's markup is greater than it would be under certainty.

(Proof: Using (1.7) to simplify (1.8) yields the following expression for the markup (note, however, that (1.9) below is not a closed form expression for $(p_t - c)/p_t$):

$$(1.9) \qquad \frac{p_t - c}{p_t} = \frac{1}{e_t}\left[\frac{[\underline{u}G + Q(1-G)](1-\beta q) + \beta q}{\underline{u}(1-\beta q) + \beta q}\right]$$

The factor multiplying 1/e is greater than one, because $Q_t > \underline{u}$.)

There is a long tradition in the economics literature, going back at least to Mills (1959), of concern with the question of how uncertainty affects the pricing decision of a monopolist.[7] Proposition 1 essentially extends the result of Karlin and Carr (1962) to the case where the monopolist can backlog its excess demand.

Equation (1.9) makes clear that, as with the textbook monopolist, some restrictions on $h(p_t)$ will generally be necessary for any equilibrium to exist. For the present purposes, it will suffice to assume that the elasticity of expected demand is bounded from above and below. This will ensure that the markup is positive but finite, and that the firm makes positive expected profits. The next result then follows immediately from equation (1.9):

Proposition 2: The profit-maximizing markup, and hence the price, is constant over time.

(Proof: (1.7) implies that Q_t depends only on p_t, β, c, q, and on the distribution of u_t. Also, the elasticity depends only on p_t. Therefore, no state variables enter the right side of (1.9), so price is constant over time.)

This result is helpful from an analytical standpoint because it enables us to focus on production and sales behavior without regard to price changes. It also greatly simplifies the solution to the problem of optimal production, as we shall see. The counter-smoothing results derived in the next section are not at all sensitive to assumptions about pricing behavior, though. For example, the result carries over to the case of a competitive equilibrium in which price adjusts ex post to clear the market (see Kahn (1986, Ch. 2)).

The intuition for the constant price result is simply that the markup depends on the elasticity of expected sales, which is constant, not on the expected level of sales. A firm with, for example, an unexpectedly large stock of inventories does better by saving on current production costs and selling entirely out of inventory than by cutting its price and maintaining its level of production. Although with non-multiplicative uncertainty, firms would generally respond to changes in expected demand by changing their prices, the variance of production and the variance of sales would, to a first approximation, be affected the same way. Procyclical price changes, for example, would smooth both production and sales. Hence a more general specification of uncertainty would not be likely to overturn the results presented in the next section.

In what follows, I will define ${}_{t-1}\underline{x}_t$ to equal the expectation of $h(p)S_t$, the firm's new demand (demand exclusive of backlogs) in period t, conditional on information as of time t-1. The constancy of the profit-maximizing price leads immediately to the following result about the production decision:

Proposition 3: The firm chooses production so that the probability of stocking out is the same every period.

(Proof: Equation (1.7) expresses Q_t as an implicit function of p, c, q, β, and the distribution function G, all of which are constant. Therefore the stockout probability $1-G(Q_t)$ is constant.)

Proposition 3 is a consequence of the result that all of the parameters that affect the tradeoff between stockouts and inventory carrying costs are constant over time. If, for example, the markup were procyclical, the stockout probability would be countercyclical, since the costs of stocking out would be greater in periods of high demand.

We can now completely characterize the decision rule of the firm in terms of the state variables of the problem. Substituting for Q_t in (1.7), and defining the parameter

$$k \equiv G^{-1}\left[\frac{p-c}{p - \dfrac{\beta c(1-q)}{1-\beta q}}\right]$$

(the optimal value of Q_t), we have

$$n_{t-1} + y_t = b_{t-1} + k_{t-1}\underline{x}_t \tag{1.10}$$

Equation (1.10) shows that the firm makes available for sale in period t an amount equal to its backlogged sales b_{t-1}, plus a multiple k of its expected demand. Strictly speaking, k could be any positive number, depending on the distribution of u, the markup, and the other parameters. For most plausible parameter values, though, k will be greater than 1, as the following example suggests.

Example: Suppose new demand $\underline{x}_t$ is of the constant elasticity form

$\Gamma_{t-1} = p_t^{-e}u_t$, where Γ_{t-1} is a scalar, and let u be distributed uniformly over the interval [0,2], so that G(u) = (1/2)u. It turns out that the markup satisfies a quadratic equation involving the discount rate β, the elasticity of demand e, and the backlog factor q. Tables 1A and 1B give values for the markup (defined as p/c) and for k implied by various values for e, with q equal to zero or one. Table 1A assumes $\beta = 0.995$, and Table 1B assumes $\beta = 0.99$.

A number of interesting features of the model emerge from these calculations. First, the ability to backlog has, perhaps not surprisingly, a huge effect on inventory levels. With q equal to zero, k is substantially greater than unity even for very small markups, whereas with q equal to one, k is small at even fairly substantial markups. At moderate markups (e.g. less than 33 percent), firms that are able backlog their excess demand have inventory levels that are, on average, less than half of those of firms that cannot backlog.

Second, inventory holdings are not very sensitive to the discount rate. The rate assumed in Table 1B is roughly twice that in Table 1A, and yet for the most part the optimal values of k hardly differ. Since empirical researchers have rarely found inventory investment to be very sensitive to interest rates, it is encouraging that, at least in this example, the model implies that inventory levels are not substantially affected by changes in the discount factor.[8] The last column of Table 1B gives the percentage difference in expected inventories between the two cases (with Table 1A as the base case), for q equal to zero. (This difference is negligible in the case of q equal to 1.)

Finally, note that the although the markups are larger than the static monopolist's markup p/c = e/(e-1), the difference is very small. This is because the discount factor β is very close to one in either case. It is easy to verify that if $\beta=1$ exactly, the markup is exactly that of the static monopolist. This is because it is costless to hold inventories if the discount rate is zero, and therefore the firm will never allow itself to stock out. Hence the elasticity of expected sales is equal to the elasticity of expected demand, so the reason for the higher markup as described earlier goes away.

It should be no surprise that the model is not very well-behaved as the elasticity of expected demand goes to infinity. In the example above, as the elasticity approaches infinity, price approaches cost, and the firm ceases to produce to stock. This reflects the fact that a constant p=c could not be a competitive equilibrium, because inventory carrying costs (the positive discount rate) imply that firms' expected profits would be negative. With monopolistic price-setting, profits may be negative if the elasticity of demand is too large. Kahn (1986, Ch. 2) provides a model of a competitive equilibrium analogous to the monopoly model considered here. The behavior of production and sales in that setting is the essentially the same; only the interpretation differs, since by assumption individual consumers and producers behave as if they can buy and sell as much as they want.

Another interesting feature of this model is that the derivative of production with respect to expected sales need not equal one, and is likely to be greater than one. The technical reason for this "multiplier" effect is that firms choose production with regard to stockout probabilities, not first moments. In general, the difference between the means of two probabi-lity distributions will not equal the difference between the quantiles (i.e. the points at which the distribution function is equal to some given value). Therefore, unless the optimal stockout probability happens to be such that k equals one, a change in expected demand should lead to something other than a one-for-one change in expected production.[9] The results of the next section concerning production counter-smoothing, however, hold for <u>any</u> value of $k > 0$.

We can, by a series of substitutions, solve for production and sales in terms of demand expectations and innovations. Equation (1.10) and the identity $y_t = z_{t-1} + a_t - a_{t-1}$ together imply

$$(1.11) \qquad y_t = z_{t-1} + k({}_{t-1}\underline{x}_t - {}_{t-2}\underline{x}_{t-1}) + b_{t-1} - b_{t-2}.$$

Thus production is equal to the most recent period's sales, plus the change in the stock of backlogged sales, plus a factor proportional to the change in expected new demand. We can immediately see that if no backlogging were possible (q=0), <u>and</u> if demand were i.i.d., then production would just equal lagged sales, and the variance of production would, of course, equal the variance of sales.[9] The interest of this model, then, depends on the failure of at least one of those restrictions.

Equations (1.5) and (1.10) together imply the following expression for sales:

$$(1.12) \qquad z_t = b_{t-1} + {}_{t-1}\underline{x}_t v_t$$

where $v_t \equiv \min[k, u_t]$. Equation (1.12) says that actual sales equal the stock of backlogs, plus new demand or the remaining stock, whichever is smaller. A stockout occurs if $u_t > k$.

Combining the above expressions for production and sales with the specification of backlogs (1.2), we get the following equations for production, sales, backlogs, and inventories:

$$(1.13) \qquad y_t = k({}_{t-1}\underline{x}_t - {}_{t-2}\underline{x}_{t-1}) + {}_{t-2}\underline{x}_{t-1} v_{t-1} + q\,{}_{t-2}\underline{x}_{t-1}(u_{t-1} - v_{t-1})$$

$$(1.14) \qquad z_t = {}_{t-1}\underline{x}_t v_t + q\,{}_{t-2}\underline{x}_{t-1}(u_{t-1} - v_{t-1})$$

$$(1.15) \qquad b_t = q_{t-1}\underline{x}_t(u_t - v_t)$$

$$(1.16) \qquad n_t = {}_{t-1}\underline{x}_t(k - v_t).$$

These expressions can be readily understood if one bears in mind the relationships between u, v, and k. If $u < k$, then $v = u$; otherwise $v = k$. In the first case, backlogs are zero, leftover inventories are positive, and sales are equal to total demand. In the second, leftover inventories are zero, backlogs positive, and sales are constrained by the available stock.[10]

The results from this section suggest a straight-forward test of the model. If $q = 0$, for example, the model implies that the quantity $n_{t-1} + y_t$ is an efficient forecast of period t demand, and that actual period t sales are equal to the forecast plus an error term that should be white noise uncorrelated with anything in the period t-1 information set. One could regress $\log(z_t)$ on $\log(n_{t-1}+y_t)$, and the nothing else in the period t-1 information set should enter significantly except for factors that might affect k, namely interest rates, factor prices, and so forth. (See Kahn (1986, Ch. 3) for a more detailed derivation of this and related tests of the model.)

The next section considers the question of whether inventory behavior as derived above is likely to lead to production counter-smooothing.

II. The Variance of Production

A number of authors have suggested that a target inventory motive might account for the fact that the variance of production typically exceeds the variance of sales. The purpose of this section is to determine sufficient conditions for production counter-smoothing to occur. To understand why some sort of target motive is a logical candidate to explain the "excess" variance of production, we can rearrange the identity (1.4) as follows. Let

$a_t \equiv n_{t-1} + y_t$, the total stock available for sale by the end of period t. Then we have another identity

(2.1) $$a_t \equiv a_{t-1} + y_t - z_{t-1}$$

or, with production on the left side:

(2.2) $$y_t \equiv z_{t-1} + a_t - a_{t-1}.$$

Thinking of a_t as the target, and y_t as the instrument by which the firm achieves the target, equation (2.2) has the behavioral interpretation that production must make up for the previous period's sales (that is, replenish the inventory stock), <u>and</u> respond to changes in the desired stock available for sale. The corresponding variance relationship is

(2.3) $$\mathrm{var}(y) = \mathrm{var}(z) + \mathrm{var}(\Delta a) + 2\mathrm{cov}(z_{-1}, \Delta a)$$

The firm's desired level of a_t presumably depends on expected future sales, so changes in a_t would depend on changes in expected future sales. Under this interpretation, then, the variance of production would exceed the variance of sales unless changes in expected future sales were negatively correlated with the most recent period's sales. Even so, it is still not clear under what circumstances production counter-smoothing would occur. For example, Abel (1985) shows, in a model very similar to West's (1983, Ch. 3), that profit-maximizing firms may smooth production even if their costs are linear. Abel considers only i.i.d. demand processes, however, and does not allow his firms to backlog their excess demand. One of the implications of what follows is that Abel's results depend very much on both of those restrictions.[11]

We have seen already that with i.i.d. demand and no backlogging, the two variances are identical, since production equals lagged sales. The first result of this section shows that backlogs alone induce production counter-smoothing.

Proposition 4: If demand is i.i.d. and $q > 0$, then the variance of production exceeds the variance of sales.

(Proof: With i.i.d. demand, equations (1.13) and (1.14) simplify to

$$y_t = [v_{t-1} + q(u_{t-1} - v_{t-1})]\underline{x},$$

and

$$z_t = [v_t + q(u_{t-1} - v_{t-1})]\underline{x},$$

where $\underline{x}$ is the (constant) expectation of new demand. Since u and v are serially uncorrelated, it follows that

$$\mathrm{var}(y) = \mathrm{var}(z) + 2q\underline{x}^2\mathrm{cov}(v,u-v).$$

But v and u-v are nondecreasing functions of u, so the covariance term is positive.)

The intuition for Proposition 4 is that backlogs shift sales away from periods of large unexpected demand, while production responds to the previous period's excess demand. This is easiest to see in the case q=1. Production each period equals lagged sales plus the change in the stock of backlogs. Recalling that each period's backlogs are always filled, we then have that production equals last period's sales to new customers, plus the current stock of backlogs (since $z_{t-1} + (b_{t-1} - b_{t-2}) = (z_{t-1} - b_{t-2}) + b_{t-1}$). Backlogs, though, are the difference between the last period's demand and sales, if positive; hence production equals lagged new demand, and the variance of production equals the variance of new demand.[12] The time series of sales, on the other hand, will clearly be smoother than that of new demand, since sales will be shifted away from periods with large positive demand shocks.

Now suppose that q=0, but that demand is not i.i.d. The next result shows that the production counter-smoothing result found in Section I holds in this more general setting in which the firm chooses price optimally.

Proposition 5: If demand is not i.i.d. (in the sense that ${}_{t-1}\underline{x}_t$ is not a constant), and if demand expectations are positively related to current sales, then the variance of production exceeds the variance of sales.

(Proof: With q=0, and using the notation $\Delta_s w_t \equiv {}_s w_t - {}_{s-1}w_{t-1}$ for any variable w, we have

$$y_t = z_{t-1} + k\Delta_{t-1}\underline{x}_t$$

so $\mathrm{Var}(y) = \mathrm{Var}(z) + k^2\mathrm{var}(\Delta_{t-1}\underline{x}_t) + 2k\ \mathrm{cov}(z_{t-1},\Delta_{t-1}\underline{x}_t)$. If ${}_{t-1}\underline{x}_t$ depends positively on z_{t-1}, then $\mathrm{cov}(z_{t-1},\Delta_{t-1}\underline{x}_t)$ will be positive, which also implies $\mathrm{var}(y) > \mathrm{var}(z)$.)

Thus changes in expectations must be negatively correlated with sales for the variance of production to be less than the variance of sales, even without backlogging. Note that this result holds for any $k > 0$. So even if production responds less than one-for-one with changes in expectations, counter-smoothing may still occur. It is the serial correlation properties of the demand process that in this case determine the extent to which production will be more variable than sales. If expected demand depends only on lagged demand (or lagged sales), and if there is no backlogging, then production counter-smoothing will occur if and only if demand is positively serially correlated.

The intuition for Proposition 5 is the same as in the case considered in Section I. A demand shock has two effects: it changes inventories, and it changes expectations about future demand. If demand is positively serially correlated, the two effects work in the same direction. A positive shock, for example, lowers inventories (thereby increasing optimal production one-for-one), and increases expected demand for the following period. The latter effect also increases optimal production, so the total innovation in production is larger than the innovation in sales. Thus both backlogs and non-i.i.d. demand have the effect of increasing the variance of production relative to the variance of sales.[13] That these effects can be quantitatively significant is established in Kahn (1986, Ch. 1), in which simulations with plausible stochastic processes for demand yield variances of production two to three times the variances of sales.

III. Stockouts and Backlogs: An Extended Example

This section presents two simple cases that highlight the results from the previous section, i.e. that firms' ability to backlog excess demand and positive serial correlation in demand both lead to production counter-smoothing. To keep matters as simple as possible, in this section I consider only the production decision of the firm, and assume that the output price is constant. I also let uncertainty enter additively rather than multiplicatively, so as to be able to calculate variances directly.

Part A considers a firm whose demand is serially correlated. In Part B the firm's demand is i.i.d., but its excess demand can be backlogged.

A. Serially Correlated Demand

Suppose that the firm's demand, denoted x, is an AR(1) process:

$$x_t = d + \rho x_{t-1} + u_t, \tag{3.1}$$

where $-1 < \rho < 1$. The forecast error u_t is an i.i.d. normally distributed random variable with mean zero and variance σ^2, and is uncorrelated with anything known at time t-1. The maximum amount the firm can sell in period t is its inventory as of the end of the previous period, plus whatever it produces during period t:

$$z_t = \min[n_{t-1} + y_t, x_t], \tag{3.2}$$

where z_t denotes actual sales in period t. The cost of producing y_t is again cy_t, so the firm's problem is of the form considered in Section I, except with a constant price p.

The first-order condition implied by equation (1.7) in this case is:

$$0 = (p-c)(1 - \varphi(A_t/\sigma)) - c(1-\beta)\varphi(A_t/\sigma), \tag{3.3}$$

where

$$A_t \equiv n_{t-1} + y_t - (d + \rho x_{t-1})$$

(the difference between the amount the firm makes available for sale and its expected demand), $\varphi()$ is the standard normal c.d.f., and β is the discount factor. Interpretation of this condition is reasonably straight-forward. First, note that the firm will stock out if and only if $u_t > A_t$; thus $1 - \varphi$ is the stockout probability. In the event of a stockout, the gain from having produced an additional unit is p-c; in the event that there is stock left over at the end of the period, the firm is worse off as a result of having produced the additional unit by an amount $c(1-\beta)$, since it could have delayed the expenditure by one period. At the optimum, the expected marginal gain must equal zero. Note that equation (3.3) is a special case of (1.7) with $q = 0$.

Letting $k = \sigma\varphi^{-1}((p-c)/(p-\beta c))$, we get the decision rule

$$a_t = E_{t-1}x_t + k \tag{3.4}$$

where $a_t \equiv n_{t-1} + y_t$ (the total stock available for sale in period t), so that we have

$$n_{t-1} + y_t = d + \rho x_{t-1} + k. \tag{3.5}$$

That is, the optimal a_t equals expected demand plus a factor that depends on the markup, the discount factor, and the variance of the forecast error. These are the parameters that determine the cost of stocking out relative to carrying over inventories. We can use this rule together with (3.2) to solve for the behavior of production, sales, and inventories in terms of the characteristics of the demand process:

$$y_t = d + \rho x_{t-1} + \min[u_{t-1},k]. \tag{3.6}$$

$$z_t = d + \rho x_{t-1} + \min[u_t,k]. \tag{3.7}$$

(3.8) $$n_t = k - \min[k,u_t].$$

Thus production and sales follow very much the same sort of process. However, the important difference is that x and u are contemporaneously correlated, while u is uncorrelated with previous realizations of x. Letting $v \equiv \min[u,k]$, we have

(3.9) $$y_t = d + \rho x_{t-1} + v_{t-1}$$

so that

(3.10) $$\mathrm{var}(y) = \rho^2\sigma^2/(1-\rho^2) + \mathrm{var}(v) + 2\rho\mathrm{cov}(u,v) \;.$$

The same calculations for the sales process yield

(3.11) $$\mathrm{var}(z) = \sigma^2\rho^2/(1-\rho^2) + \mathrm{var}(v) \;.$$

Since the covariance of v and u is positive, it is clear that production is more variable than sales in this example if and only if $\rho > 0$.

The intuition for this result is that if demand exhibits positive serial correlation, production will respond to an innovation in sales by more than one-for-one. A positive demand shock, for example, reduces inventories <u>and</u> increases the optimal level of a_t. The first effect alone would make production exactly as variable as sales; the changes in optimal a_t that result from changes in expected demand cause production to be more variable than sales, the more so the more positively serially correlated is demand.

B. Backlogs

In this section I set ρ from equation (3.1) equal to zero, so that demand is i.i.d., but allow the firm to backlog its excess demand. The total demand facing the firm in each period now includes any excess demand that may be left over from previous periods. The conditional expectation of "new" demand (that is, demand exclusive of backlogs) is now assumed to be constant. We have

$$x_t = d + (x_{t-1} - z_{t-1}) + u_t. \tag{3.12}$$

The second term is, of course, non-negative, and equals zero in the event that the firm has inventory leftover from the previous period. The firm knows $x_{t-1} - z_{t-1}$ when it chooses y_t.

In this case the first-order condition for optimal production turns out to be

$$0 = (p-c)(1-\beta)[1-\varphi(A_t/\sigma)] - c(1-\beta)\varphi(A_t/\sigma). \tag{3.13}$$

which is a special case of (1.7) (q = 1). The difference between this condition and (3.3) in Part A is that now the gain to the firm from having produced an additional unit, conditional on stocking out, is the difference between selling this period and selling next period, i.e. $(p-c)(1-\beta)$. In the previous case, excess demand was assumed to be lost forever, whereas with backlogging, the sales are only delayed by one period. Note that the loss from having produced an additional unit in the event of a shortfall in demand is also proportional to $1-\beta$, so that the discount factor a actually does not affect the production decision in this case.

As before, the firm's decision rule takes the following form:

(3.14) $$a_t = E_{t-1}x_t + \kappa$$

or

(3.15) $$n_{t-1} + y_t = d + (x_{t-1} - z_{t-1}) + \kappa,$$

where $\kappa = \sigma\varphi^{-1}((p-c)/p)$. Ceteris paribus, κ is less than k as defined in the previous example, again because the cost of stocking out is smaller in this case.

Given the equation for sales (3.2), and defining $\upsilon_t \equiv \min[u_t,\kappa]$ as before, we have

(3.16) $$z_t = d + (x_{t-1} - z_{t-1}) + \upsilon_t.$$

End-of-period inventories are then

(3.17) $$n_t = \kappa - \upsilon_t,$$

and excess demand in period t can be expressed as

(3.18) $$x_t - z_t = u_t - \upsilon_t,$$

which is simply the difference between the forecast error and the truncated forecast error for period t. For u_t small, excess demand is zero, but for $u_t > \kappa$, it is positive.

We can use (3.14)-(3.18) to solve for production and sales:

(3.19) $$y_t = d + u_{t-1}$$

$$z_t = d + (u_{t-1} - \nu_{t-1}) + \nu_t. \tag{3.20}$$

But the quantity $d + u_{t-1}$ is exactly equal to lagged new demand, and therefore production is exactly as variable as new demand. Sales will clearly be less variable than new demand, though, since when demand is unexpectedly high, sales are shifted to subsequent periods, and indeed one may show rigorously that the variance of (3.19) is greater than the variance of (3.20).

These special cases highlight the two sources of production counter-smoothing considered in this paper: serial correlation in demand, and the backlogging of excess demand. The two are clearly related, since backlogging imparts some serial correlation to the <u>total</u> demand faced by the firm. A large positive shock to demand in one period, for example, is carried over into the next period's total demand by means of the backlog. The two sources are conceptually different, though, so it is useful to consider them separately. Backlogs are negligible or non-existent in many industries that have sigificant serial correlation in demand.

IV. Conclusions

This paper has demonstrated the importance of taking explicit account of both stockouts and backlogs in inventory models, as both considerations were shown to increase the variance of production relative to the variance of sales in the presence of serially correlated demand. With strictly convex costs, of course, it is likely that production would be smoothed relative to the predictions of the model, but a simple continuity argrument implies that strict convexity of the cost function is consistent with production counter-smoothing. Moreover, there are a number of other factors left out of the formulation that could work in the direction of further increasing the volatility of production. For example, changes in interest rates will generally affect both the demand for durable goods and the cost of holding inventories. An increase in the interest rate, for example, would lower both expected demand and desired inventory accumulation. Thus to the extent that interest rates affect demand even greater volatility of production might occur.

This last observation suggests one area for further research. It would be useful to model the demand side of the model in greater detail. The general equilibrium results alluded to earlier (Kahn (1986), Ch. 2) suggest that the results of this paper are robust to more rigorous treatments of demand. That model assumes time-separable utility, though, which is not very realistic in the case of consumer durables. A more complete treatment of intertemporal substitution on the part of consumers would be a useful extension, especially in the model with backlogs.

Another question that arises is the relationship between this model and the linear-quadratic specification of as in Blanchard (1983) and West (1986). First, the "target" inventory behavior that comes out of this model is similar to the *ad hoc* specification employed in those models, except that <u>current period</u> production is added to the end-of-period inventory stock. Thus for the linear-quadratic target specification to have more of a claim to being an approximation to an optimal policy its target inventory component should be modified accordingly. The second difference is the convex costs of production and of changing production in the linear-quadratic model. Both of the above-mentioned authors imposed a strict quadratic specification of production cost (i.e. $c(y) = hy^2$). If those models were estimated with a linear and a quadratic cost term (which would be useful anyway, given West's excess volatility result), along with the modified target inventory specification, then there might be some basis for the claim that the specification approximates the objective of a firm with general convex costs and a non-negativity constraint on inventories, especially if the quadratic coefficients turn out to be small or insignificant. Conversely, a test for the significance of the quadratic coefficients could be a valid test of the constant marginal cost assumption employed in this paper.

In terms of policy implications, one clear message of this paper is that the degree of uncertainty surrounding a particular policy option, both in terms of what its effect on aggregate demand will be, and whether or not it will be implemented, may play as important a role in determining the policy's impact as its direct effect. A policy that increases short-term uncertainty about demand without affecting its expectation, for example, may have substantial real affects, since changes in the dispersion of the demand distribution have first-order effects on optimal inventory accumulation. Similarly, the multiplier effects of counter-cyclical policies will depend very much on how they affect firms' uncertainty about demand. On the other hand, such policies could end up exacerbating the fluctuations, given that firms may respond to changes in expectations with more than one-for-one changes in production.

Notes

1. See, for example, Blinder and Fischer (1981), and Dornbusch and Fischer (1984, Ch. 7).

2. In Kahn (1986, Ch. 3) the second assumption is weakened to allow for revisions to planned production (with increasing marginal cost) when demand information becomes available. The qualitative features of the model remain the same, except that actual production ends up deviating from planned production by some amount that is correlated with the demand forecast error.

3. Thus the firm is in a so-called "production to stock" industry (see, for example, Abramowitz (1950), or West (1983)).

4. One might argue that stockouts cannot be very important because we never see them (i.e. zero inventories) in the data. First, the optimal stockout probability derived in the paper can be arbitrarily close to zero. Second, we only observe inventories aggregated across goods and at points in time, so even if the stockout probability is high we would not expect to see zero inventories. Moreover, extension of the model to a firm that produces multiple products is reasonably straight-forward, and the qualitative features of the one-good model remain. For example, a firm that produces M different goods will behave the same as in the one-good case (with respect to each good), provided its customers will not accept different goods as substitutes for their first choices in the event of a stockout. Otherwise there would be "economies of scope", since a firm that provided many related goods could economize on inventories without losing sales. In the limiting case in which the M goods are perfect substitutes the firm behaves exactly as if it only produces one good.

5. A better assumption in many instances might be that the price the backlogged customer pays is the minimum of the current and past price. In this model, though, the two prices turn out to be the same.

6. Inclusion of other linear costs, such as a cost of carrying over inventory, or a cost of stocking out, is a straight-forward extension of the model, and does not affect the qualitative behavior of the firm.

7. See the related work of Zabel (1970,1972), who also finds that multiplicative uncertainty leads to a higher price than under certainty. Blinder (1982) considers the pricing behavior of inventory-carrying firms facing various kinds of additive uncertainty.

8. Tables 1A and 1B compare the steady-state values of k for different discount factors. This is, of course, different from calculating the effect on inventory <u>investment</u> (changes in inventories) of a change in the interest rate. Nonetheless, the last column of Table 1B should give a pretty fair indication of how interest rates affect inventory investment in that particular example.

9. Abel (1985) derives this result in his discussion of the importance of the longer production lag in his model. His results suggest that assuming a longer production lag might weaken, but not reverse, the results in this paper.

10. It is possible for (1.13) to yield a negative solution for production, since the first term (which reflects changes in expectations) can in theory be arbitrarily negative for a given period. In practice, though, expectations are not likely to be so volatile as to produce this result, so little would be gained by imposing an additional non-negativity constraint on the firm's problem. Technically one can rule out such an outcome by bounding the support of the forecast error distribution.

11. Abel shows that production will vary <u>less</u> than sales in a model with

a longer production lagged (as described earlier) and i.i.d. demand. Abel points out that his model with the shorter production lag as in this paper and constant marginal cost yields equal variances of production and sales. Taken together, Abel's results and those in this paper suggest that adding serial correlation to demand makes production more variable, while lengthening the production lag tends to smooth production relative to sales.

12. Another way to understand this case is that with $q > 0$, the firm can act more like a "production-to-order" firm (see West (1983)), so that its production will be more backward-looking. The model with $q = 1$ is not, however, a model of a production-to-order industry, since some output is produced to stock. It is best thought of as an industry in which firms can backlog, but competitive pressures force some degree of production to stock.

13. Backlogs and serial correlation together do not interact perversely to overturn the results of Propositions 4 and 5. For example, it is straightforward to show that if $q=1$, then the variance of production exceeds the variance of <u>demand</u> if demand exhibits positive serial correlation, and that the variance of demand in turn exceeds the variance of sales, the latter being smoothed to some extent by the backorder technology.

Figure 1: Timing Conventions

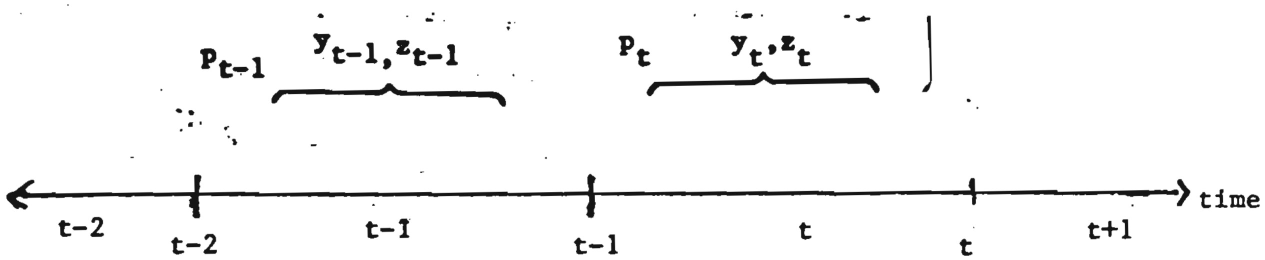

Figure 2: Inventories, Sales, and Production

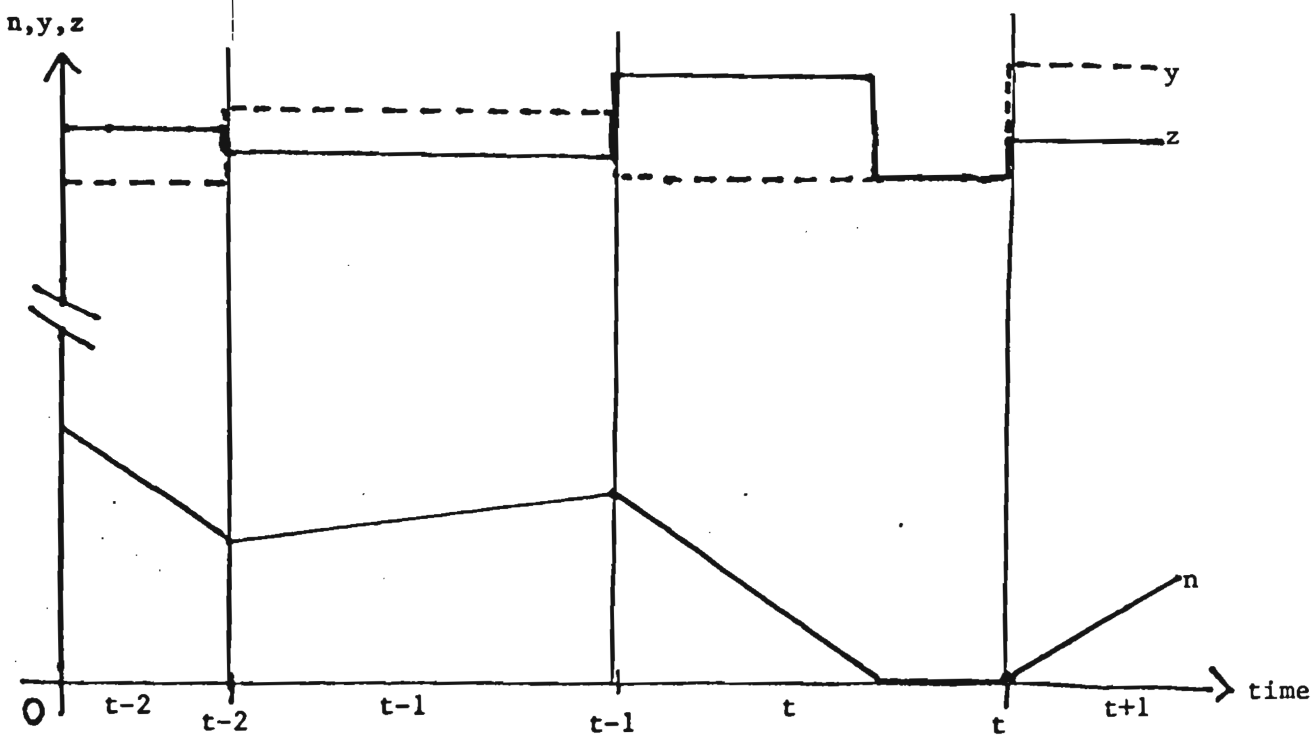

Table 1A: Markups and Inventories, β = 0.995

	q = 0		q = 1	
e	p/c	k	p/c	k
1.1	11.06	1.999	11.05	1.819
1.5	3.02	1.995	3.01	1.335
2	2.01	1.990	2.01	1.001
4	1.34	1.971	1.33	0.500
10	1.12	1.918	1.11	0.200
50	1.02	1.655	1.02	0.040
100	1.01	1.443	1.01	0.020

Table 1B: Markups and Inventories, β = 0.99

	q = 0		q = 1		
e	p/c	k	p/c	k	$\%\Delta n^{*}$
1.1	11.11	1.998	11.09	1.820	-0.0%
1.5	3.03	1.990	3.01	1.336	-0.3
2	2.02	1.980	2.01	1.002	-0.6
4	1.35	1.944	1.33	0.501	-1.3
10	1.12	1.847	1.11	0.200	-3.8
50	1.03	1.447	1.02	0.040	-12.6
100	1.01	1.177	1.01	0.020	-18.3

*This column gives the percentage change in expected inventories that results from setting a equal to 0.99, as compared to a=0.995, for the case q=0. This difference is negligible in the case q=1.

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