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1. Introduction

The traditional macroeconomic view is that trends and cycles in economic activity are usefully investigated as distinct economic phenomena. The argument is that the trend elements of real per capita production, as well as relative prices, are explained by the permanent evolution of tastes and technology. For example, in the growth theory of Robert Solow [1970], a common deterministic trend in the per capita levels of consumption, investment, and output arises from technological change that augments general production opportunities. With growth explained in this manner, the traditional view attributes cyclical fluctuations in prices and production to other sources, including monetary events, fiscal policies, and temporary real shocks such as energy price movements.

Some key features of macroeconomic time series are widely viewed as supporting this traditional view. While measures of consumption, investment and output grow over time, the ratios of consumption or investment to national product have remained roughly constant in the United States. These observations are widely viewed as rationalizing the Solow growth dynamics and led Richard Kosobud and Lawrence Klein [1961] to include these in their list of "great ratios." On the other hand, since investment fluctuates much more than output, the investment ratio exhibits substantial variation over the business cycle; the consumption ratio fluctuates much less. To many, this has suggested that trend and cycle must be explained with different models or, at a minimum, with different impulses or sources of shocks.

In this paper, we depart from this traditional view by conducting an empirical investigation of the extent to which business cycle fluctuations can be understood as the result of one or more common unobserved *stochastic* trends. Shocks to these stochastic trends permanently shift the levels of consumption, investment and output but may leave the great ratios relatively unchanged in the long term. Working with post–war U.S. data, our results indicate that an important component of variation in output over the business cycle—between one half and four fifths—can be traced to such common stochastic trends. Thus, at this level, our results lend support to those model building efforts based on the real business cycle program—initiated by Finn Kydland and Edward Prescott [1982] and John Long and Charles Plosser [1983]—that seek to integrate the study of trends and cycles.[1] At the same time, our attempts to identify the source of these shocks turns up evidence that is not easily reconciled with real business cycle models or with any of the commonly discussed alternatives.

Our investigation draws together three lines of recent research activity. First, there is an accumulation of empirical evidence, stemming from the initial work of Charles Nelson and Charles Plosser [1982], that individual macroeconomic time series may contain important stochastic trend components. Second, theoretical research into real business cycle models—see, for example, Robert King, Charles Plosser and Sergio Rebelo [1988b, section 2]—shows that permanent productivity disturbances have important transitory effects on the fractions of national product that are invested and consumed. This suggests that it is important to design statistical procedures that permit shifts in stochastic trends to have different effects on various measures of economic activity at business cycles horizons. Third, the concept of cointegration as developed by Robert Engle and Clive W. J. Granger [1987] suggests a new set of econometric procedures that can be adapted to multivariate models with common stochastic trends. Specifically, methods based on cointegration enable us to move the study of stochastic trends beyond the univariate analyses mentioned above. In this regard, a key contribution of our analysis is to establish a natural, formal link between the theory of cointegrated econometric structures—where certain linear combinations of nonstationary variables are stationary—and models of common stochastic trends—where the nonstationary nature of a vector of variables is attributed to a smaller number of unobservable stochastic trends.

The organization of the paper is as follows. Section 2 provides a brief overview of the models of growth and fluctuations that motivate our research. Section 3 examines the trend and cointegrating properties of a variety of post-war U.S. economic time series. Section 4 develops a framework for extracting and identifying permanent components building on work by Stephen Beveridge and Charles Nelson [1981] and Engle and Granger [1987]. In section 5 these methods are applied in two stages. First, we study a three variable system consisting of consumption, investment and output. Second, we augment this system with additional variables, including real money balances, inflation and nominal interest rates. Analysis of this larger system allows us to investigate the sensitivity of the common stochastic trends model to the list of variables, the number of unobservable stochastic trends estimated and alternative procedures for their identification. Section 6 compares our estimate of the common stochastic trend with two variants of Solow's [1957] measure of factor productivity and Edward Denison's [1985] estimate of potential GNP. Section 7 summarizes our findings.

2. Growth and Fluctuations: A Perspective

The empirical hypothesis investigated in this paper is that macroeconomic time series contain common stochastic trends and that these stochastic growth components have a significant influence on the character of economic fluctuations. To motivate why we think it is important to investigate potential interactions between growth and fluctuations, we review some related empirical and theoretical research.

Econometric Background

Recent empirical research—stemming from the work of Nelson and Plosser [1982] and surveyed in James Stock and Mark Watson [1988a]—suggests that individual macroeconomic time series contain stochastic trends, including consumption, investment, output and measures of factor productivity. The stochastic trend in the logarithms of these macroeconomic quantities is modeled as a random walk, $\tau_t = \mu + \tau_{t-1} + \eta_t$, where τ_t is the value of the trend at date t, μ is the average change in τ and η_t

represents serially uncorrelated stochastic growth. In this stochastic difference equation with a "unit root," the influence of η_t on τ_t is permanent in the sense that it implies a parallel shift in the expected trend path at all future dates: $E_t\tau_{t+s} - E_{t-1}\tau_{t+s} = \eta_t$ for all $s > 0$. While many individual series appear to contain stochastic trends, univariate evidence such as that provided by Nelson and Plosser [1982] cannot determine whether these are common—affecting one or more series simulataneously—or specific to the individual series.

The recent literature on cointegration, stemming from Engle and Granger [1987], provides a natural framework for thinking about common stochastic trends. A column vector of n random variables X_t is said to be cointegrated if its elements are individually integrated of order one (i.e., the first difference of each of the elements of X_t is stationary) and if there is at least one linear combination of elements of X_t, say $\alpha' X_t$ that is stationary. The vector α is called the cointegrating vector. In recent years, there has been substantial work developing methods for estimation and hypothesis testing in systems with cointegrated variables. We use these methods since cointegration naturally arises when one or more common stochastic trends is the sole source of nonstationarity in a vector of variables.

Theoretical Background

In order to be more specific and to fix some notation for the analysis that follows, we outline a simple real business cycle model with permanent productivity shocks. (See King, Plosser and Rebelo [1988b] for a detailed discussion.) The standard neoclassical aggregate production function specifies that commodity output, Y_t, is produced according to a Cobb–Douglas technology with constant returns to scale,

$$Y_t = \lambda_t K_t^{1-\theta} N_t^{\theta} \tag{2.1}$$

where N_t is the number of units of labor effort employed and K_t is the capital stock at the beginning of date t. The capital stock evolves over time as the net result of gross investment, I_t, and depreciation at rate, δ, so $K_{t+1} = (1 - \delta)K_t + I_t$. The resource constraints are that consumption and investment exhaust output, $C_t + I_t = Y_t$, and time is allocated between work, N_t, and leisure, L_t.

Total factor productivity, λ_t, is assumed to follow a logarithmic random walk,

$$\log(\lambda_t) = \mu_\lambda + \log(\lambda_{t-1}) + \xi_t, \tag{2.2}$$

where the innovation ξ_t is taken to be independently and identically distributed with mean zero and variance σ^2. Thus the average growth of total factor productivity is μ_λ, although in any period the actual growth rate will deviate from μ_λ by some unpredictable amount ξ_t.

Restrictions on Trends. Within the basic neoclassical model with deterministic trends, it is familiar—from Solow [1970]—that consumption, investment and output all asymptotically grow at the rate μ_λ/θ. That is, the "great ratios"—of investment to output and consumption to output—are constant along a steady state path.

When uncertainty is added, realizations of ξ_t change the forecast of the trend path of productivity: $E_t\log(\lambda_{t+s}) = \mu_\lambda \bullet s + \log(\lambda_t)$. The net effect of $\xi_t > 0$ is to raise the expected long run growth path so that the levels of output, consumption, investment and the capital stock must be higher. This induces a common stochastic trend, τ, in consumption, investment, output and capital. The stochastic trend is related to total factor productivity by $\tau_t = \lambda_t/\theta$, which is the analogue of the restriction that the deterministic trend growth rates are μ_λ/θ. In the presence of common stochastic trends, the "great ratios" become stationary stochastic processes.

Cointegration and Common Trends: To interpret this economic result using the statistical concept of cointegration, suppose that the elements of the vector X_t are the logs of output, consumption and investment respectively. Each of these variables will be nonstationary because of the random walk character of productivity. The theory, however, predicts that these variables have a common stochastic trend. The the log of consumption minus the log of output will be stationary as will the log of investment minus the log of output. Thus, there are two independent cointegrating vectors; $\alpha_1' = [-1 \;\; 1 \;\; 0]$ and $\alpha_2' = [-1 \;\; 0 \;\; 1]$, which isolate the logs of the stationary "great ratios."[2]

Trends and Fluctuations. To study the dynamic adjustment process that results from a permanent increase in productivity, additional information on preferences is required (as in Lawrence Christiano [1988], Gary Hansen [1988], or King, Plosser and Rebelo [1988b]). But a general property of real business cycle models is that a positive shift in the stochastic trend sets in motion a protracted transition to a new steady state growth path. For example, in the model studied in King, Plosser and Rebelo [1988b, section 2] a one percent permanent increase in total factor productivity implies that the path of the capital stock, output, consumption and investment must all eventually be $1/\theta$ percent above their previous paths. In order to reach this higher growth path the investment ratio will temporarily rise, the consumption ratio will temporarily fall and work effort will increase in the short term relative to its constant steady state. During this transition the real interest rate will exceed the rate of time preference. Asymptotically, however, this divergence disappears so that the real interest rate is a stationary stochastic process, even though the level of total factor productivity follows a random walk.

Additional Sources of Real Stochastic Trends. It is possible to introduce a variety of additional sources of stochastic trends into macroeconomic models. For example, it is sometimes suggested that there are prolonged periods during which the rate of productivity growth is high or low (i.e. μ_λ is itself stochastic with most of its power at low frequencies). Such variation would induce long run changes in the fraction of output allocated to investment and in the rate of interest, since with more rapid growth individuals would effectively discount the future at a higher rate.

Additional stochastic trends might also arise from fiscal policies. For example, as noted by Andrew Abel and Olivier Blanchard [1983] and others, a change in the income tax rate—the proceeds of which are used to finance government purchases—works exactly like a productivity shock as long as one views private consumption and investment as constrained by private output. Thus, a change in the income tax rate in such an environment would induce equiproportionate variation in consumption, investment and output. Alternatively, with permanent changes in the relative taxation of different types of income, fiscal trends can induce changes in the composition of national product. For example, a permanent increase in the taxation of capital—with the proceeds used to finance transfer payments—will lead to a decline in the ratio of investment to national product.

Long Run Restrictions in Systems with Nominal Variables. In the real models discussed above, permanent productivity shocks have important implications for the cointegration properties of real variables. Additional cointegrating relations arise in systems that include nominal variables. In our empirical investigation below, we model the logarithm of real balances, $m_t - p_t$, as a linear function of the logarithm of output, y_t and the level of the nominal interest rate, R_t. That is, we specify a money demand relation:

$$m_t - p_t = \epsilon_y y_t + \epsilon_R R_t + v_t. \quad (2.3)$$

where v_t is the money demand disturbance. The conventional Fisher relation implies that

$$R_t = r_t + E_t \Delta p_{t+1}, \quad (2.4)$$

where r_t is the *ex-ante* real interest rate and the second term is the expected rate of inflation between t and t+1. These two specifications suggest further possible cointegrating relations. For example, if the real rate of interest is stationary, then the Fisher relation implies that nominal rates and inflation are cointegrated. That is, permanent variations in the inflation rate—as suggested by the empirical analyses of G. William Schwert [1987] and others—must be matched by permanent variations in nominal interest. Another possibility is that the disturbance term in the money demand relation (2.3) is stationary, so that real balances, output, and nominal interest rates are cointegrated. (See Robert Lucas [1988] and Dennis Hoffman and Robert Rasche [1989] for an alternative empirical investigation of the issue.) Such additional cointegrating relations imply the existence of additional common stochastic trends. In the empirical analysis we investigate the implications of these additional permanent components for economic fluctuations.

Implications for Empirical Research

Our empirical analysis is structured around three questions suggested by the preceding discussion. First, what is the nature of the cointegration properties in the post-war U.S. data, and are these properties broadly consistent with the cointegrating predictions of the dynamic economic models discussed above? Second, how important are the implied common stochastic trends over horizons that are typically associated with the business cycle? Third, is it possible to give a coherent economic interpretation to the common stochastic trends found in the data?

3. Long Run Properties of Real Flows, Money, Prices and Interest Rates

Our empirical analysis considers quarterly U.S. time series data on real aggregate national-income account flow variables, the money supply, inflation, and three interest rates. The three aggregate real flow variables are the logarithms of per capita real consumption expenditures (c), gross private domestic fixed investment (i), and "private" gross national product (y), defined as total gross national product less real total government purchases of goods and services. The measure of the money supply used is M2 (in logarithms, m). The price level is measured by the implicit price deflator for our measure of private GNP (in logarithms, p). We consider three interest rates; the rate on 3-month U.S. Treasury bills (R_{gs}), an average rate on 4 to 6 month commercial paper (R_{ps}), and the yield on a portfolio of high-grade longer term corporate bonds (rated AAA by Moody's; R_{pl}). The sample period used for statistical procedures that involve only real flows is 1949:1 through 1988:4. When money, interest rates, or prices are involved, the analysis uses data from 1954:1 to 1988:4 to avoid observations that occurred during periods of price controls, the Korean War, and the Treasury–Fed accord. Data prior to 1949:1 (respectively 1954:1) are used as initial observations in regressions that contain lags.[3]

Given that our national product measure is not the standard one we have graphed the logarithm of the key real variables (y, c, i and m–p) in Figure 1. These plots replicate the general features of the data familiar to students of economic growth and business cycles. Output, consumption and investment display strong upward trends. Investment is evidently the most volatile component, followed by output and then consumption. Real balances (m–p) also display an upward trend.

Figure 2 plots the logarithm of the consumption output ratio (c–y) and the logarithm of the investment ratio (i–y). Over the postwar period, these ratios display the stability reported by prior researchers; it is easy to view them as fluctuating around a constant mean. This suggests that the growth evident in Figure 1 occurs in a manner that is "balanced" between investment and consumption.

Univariate Unit Root Tests

Table 1 presents univariate unit root and trend test statistics for these series and their first differences. Following the work of David Dickey and Wayne Fuller [1979], we present two sets of unit root tests computed from univariate autoregressions with and without deterministic time trends. The inclusion of deterministic trends allows the regression to "partial out" deterministic growth in the series so that the regression coefficients on lagged dependent variables reflect the presence or absence of stochastic growth. The estimates $\hat{\rho}$ are of the sum of the autoregressive coefficients in a fifth–order difference equation. Focusing first on the real flow variables, the Dickey–Fuller statistics provide no evidence against the hypothesis the y and c individually contain a single unit root potentially with nonzero drift. For example, the

Dickey–Fuller (DF) "t" statistic for c is –2.09, well above the 10% critical value of –3.12. The unit root statistics for investment indicate that it may contain a deterministic trend but not a stochastic one. However, this finding may also reflect a more volatile temporary component that has adverse effects on the small properties of the DF testing procedure.

The growth model suggests examining various ratios and growth rates. Results for c–y, i–y, and c–i suggest that these series are best viewed as stationary, containing neither a unit root (stochastic trend) or a deterministic trend, although there is some evidence of a small (0.1% per year) deterministic trend in c–y. There is no strong evidence of deterministic or stochastic trends in the growth rates (Δc, Δi and Δy) examined in Table 1. Overall, we interpret these results as being consistent with y, c, and i being individually nonstationary and jointly pair wise cointegrated with cointegrating vectors of (1, –1).[4]

The statistics for money and prices suggest that real balances (m–p) and inflation (Δp) contain stochastic trends, but that their first differences are stationary. This implies that both Δp and Δm are nonstationary, but contain the same stochastic trend, i.e., permanent changes in Δm are equal to permanent changes in Δp. Neither money growth nor inflation shows evidence of a deterministic trend. The statistics in Table 1 also suggest that velocity can be modeled as being stationary with zero drift (i.e., m–p and y are cointegrated).

While the results for the real flow variables, money and prices are easy to interpret, it is more difficult to interpret the results for nominal and real interest rates. If we rule out deterministic trends *a priori*, then nominal interest rates may be viewed as nonstationary. (That is, one cannot reject $\rho = 1$ given a DF "t" of –2.25 for R_{gs} and a 10% critical level of –2.57.) While this may not be surprising given the nonstationarity in inflation discussed earlier, Table 1 also provides evidence consistent with the hypothesis that all three ex post real interest rates are nonstationary. On the other hand, the public/private $(R_{gs}-R_{ps})$ and long/short $(R_{ps}-R_{pl})$ spreads appear stationary.

Cointegration Results

We present three types of statistics on unit roots in the multivariate systems, provided in Tables 2–5. First, we provide results based on an estimated fifth-order vector autoregression (VAR(5)). The long run behavior of linear stochastic difference equation systems is governed by the eigenvalues of the companion matrix (see, e.g., Gregory Chow [1986, Chapter 3]). Thus, the number of estimated eigenvalues with modulus close to unity gives us guidance as to how many stochastic trends are present. Second, we use multivariate unit root tests developed by Søren Johansen [1988] and Stock and Watson [1988b]. In these tests, we take as the null hypothesis that all series are integrated but that there is no cointegration (there is one unit root present for each series) and consider alternatives that there are a smaller number of unit roots (which implies existence of a particular number of cointegrating relations). Third, we provide estimates of cointegrating vectors, which can be compared to the null hypotheses specified by our theory.

Results for Consumption, Investment and Output: Table 2 examines 3-variable systems with y, c, and i. Panel A presents the six largest eigenvalues from estimated vector autoregressions (VAR's). An implication of y, c, and i being cointegrated with two cointegrating vectors is that the VAR companion matrix will have one unit root and the remaining roots will be less than one in modulus. Both specifications in Table 2 are consistent with this view.[5]

Panel B of Table 2 presents a battery of statistics designed to formally test the one unit root hypothesis in the multivariate model. The $J_T(r)$ statistic is Johansen's [1988] test of the null hypothesis of r cointegrating vectors (n–r unit roots) versus the alternative hypothesis of more than r cointegrating vectors (less than n–r unit roots).[6] The $q_T^f(m,k)$ statistic is Stock and Watson's [1988b] test of the null hypothesis of m unit roots in the n–variable system ($m \leq n$) against the alternative of k unit roots (i.e. n–k cointegrating vectors). The $J_T(0)$ and $q_T^f(3,2)$ statistics do not provide strong evidence against the null of three unit roots in favor of a two unit roots. The $q_T^f(3,1)$ test, however, strongly rejects three unit roots in favor of one unit root.

The final panel in Table 2 presents maximum likelihood estimates (MLE's) of the cointegrating vectors, conditional on the presence of one unit root in the VAR. The point estimates are close to (1, –1, 0) and (1, 0, –1), the values for balanced growth in output consumption and investment. Balanced growth imposes two constraints on the cointegrating vectors, which can be tested with a likelihood ratio (LR) test. This statistic fails to reject the balanced growth restrictions at the 10% level.[7]

Results for Inflation and Interest Rates: The univariate results in Table 1 suggest taking a closer look at the unit root properties of the ex–post real rate. Statistics summarizing the four–variable system of inflation and the three interest rates are presented in Table 3. The cointegration test statistics (panel B) provide evidence against the four unit root model and evidence in favor of the one unit root model, i.e. the model with three cointegrating vectors. The evidence presented by the estimated cointegrating vectors (panel C) is, however, mixed: although the point estimates appear consistent with the spreads and being stationary, they suggest that the real rate may be nonstartionary. Moreover, the joint hypothesis that the spreads and the real rate is stationary is rejected at the 5 percent level, while the hypothesis that the spreads are stationary is not rejected at the 10% level.

Table 4 explores two possibilities suggested by the apparent nonstationarity of the nominal and real interest rates over this period. First, although the results in Table 1 suggest that velocity can be modeled as being stationary, if the demand for money has a nonzero long–run interest elasticity then a nominal rate would logically enter the money—income cointegrating vector. This suggests estimating the long run money demand relation, $(m - p) = \epsilon_y\, y + \epsilon_R\, R_{gs}$. The estimate of the semi–elasticity ϵ_R (Table 4, row (1)) is negative, as predicted by money demand theory, and significantly different from zero; the joint hypothesis of stationary velocity is rejected against the alternative of non-unit income elasticity and nonzero interest semi-elasticity at the 5% level. Although the income elasticity is estimated rather precisely (its 95% confidence interval is (1.08, 1.28)), the interest rate semi-elasticity is estimated less precisely (its 95% confidence interval is (–7.22, –2.22)).[8]

The second issue examined in Table 4 is the possibility that the consumption/output and investment/output ratios might exhibit permanent shifts resulting from permanent shifts in real rates. Estimated bivariate cointegrating relations $(c - y) = \phi_1(R_{gs} - \Delta p)$ and $(i - y) = \phi_2(R_{gs} - \Delta p)$ are shown in Table 4, specifications (2) and (3). As predicted by the long-run theory of the growth model, for example, a higher real interest rate lowers the share of product going into investment and, symmetrically, raises the share of consumption. The long-run effects, however, are imprecisely estimated and small: a permanent increase in the annual real rate of one percentage point is associated with a decline in the investment/output ratio of 0.1 percentage points.

Results for Six-Variable Systems: The next set of results (Table 5) concerns two 6-variable systems, one with y, c, i, m – p, R_{gs}, and Δp (panel A) and one with y, c, i, R_{gs}, R_{ps}, and Δp (panel B). The foregoing analysis suggests that each of the variables entering these systems contains a unit root; Table 5 sheds light on the number and nature of cointegration relations.

In each panel, we present ML estimates of cointegrating vectors if there are three unit roots in the system. This unit root specification is suggested by the results of the prior analysis (Tables 1–4). Further, the Stock–Watson test for the number of unit roots—$q_\tau^f(6,3)$ statistic—supports the view that each system can be modeled as having three unit roots.[9]

Finally, each panel contains results of tests of some alternative hypotheses about cointegrating vectors. In panel A, the first hypothesis that we examine involves cointegration of output and consumption; output and investment; and real balances, real output and nominal interest rates. The χ^2 statistic indicates only weak evidence against this hypothesis (p–value = .528). In this specification, the real interest rate permitted to be nonstationary. Inducing stationarity via the requirement that R_{gs} ar Δp are cointegrated leads to the next hypothesis—we find stronger evidence against this set of restrictions. Our final two hypotheses permit a nonstationary real interest rate to be cointegrated with the "ratios"—under alternative money demand specifications—and we find no strong evidence against these hypotheses. In panel B, we similarly begin with a "balanced growth" specification without real interest rate stationarity and then investigate alternative specifications that permit cointegration of nominal interest and inflation (2) and real rates and ratios (3).

Taken together, the results of this section suggests that the money demand and interest rate spread cointegrating relations are consistent with the observed behavior of these time series. There is some evidence that the shares of consumption and investment change in tandem with permanent shifts in the ex–post real rate. However, this effect is economically small—at least in the long run—and the hypothesis of "balanced growth" also appears generally consistent with the data.

4. Reduced Form Analysis of Permanent Components: Methodology

This section describes a statistical methodology for analyzing the nature and quantitative importance of stochastic trend components of economic time series. In Section 5, we apply these statistical methods to some alternative reduced form econometric models containing the various series considered in the previous section. Throughout, our strategy is to develop empirical models with just enough assumptions to identify the summary statistics of interest.[10]

The empirical analysis of the previous section suggested that many macroeconomic variables are well described as integrated of order one i.e., as containing a unit root. Further, certain combinations of variables are well described as cointegrated. These considerations lead us to specify the following general model for the purpose of exploring the relationship between common trends and cointegration. As above, let X_t denote an n element column vector of two or more time series, each of which is assumed to be individually integrated. After differencing the time series are assumed to be stationary with the Wold moving average representation.

$$\Delta X_t = \gamma + C(L)\epsilon_t \tag{4.1}$$

where C(L) is an n × n matrix of polynomials in the lag operator L and the innovations ϵ_t are serially uncorrelated with mean zero (formally the ϵ's are a martingale difference sequence) and covariance matrix Σ_ϵ. In addition, we make the (weak) assumption that the moving average coefficients decay sufficiently rapidly so that $\Sigma_{j=0}^{\infty} j|C_j| < \infty$.

If X_t is cointegrated with $r \geq 1$ cointegrating vectors, represented as an n × r matrix α, then—by Engle and Granger's [1987] definition—it follows that $\alpha' X_t$ is integrated of order zero.[11] Given the representation (4.1), cointegration implies that $\alpha' C(1) = 0$, where $C(1) = \Sigma_{j=0}^{\infty} C_j$.

The Common Trends Model

On an intuitive level, if two time series are cointegrated, then they must share a common stochastic trend in the sense that they have a common integrated component. More generally, if ΔX_t has the representation (4.1) with $r \geq 1$, it is shown in the appendix that X_t then has the "common trends" representation,

$$X_t = X_0 + A\tau_t + D(L)\epsilon_t, \qquad \tau_t = \mu + \tau_{t-1} + \eta_t \tag{4.2}$$

τ_t is a k × 1 vector of random walks with drift μ and innovations η_t; and D(L) is a n × n matrix of lag polynomials. The number of trends (the dimension of τ_t) is equal to the number of variables minus the number of cointegrating vectors, i.e. $k = n-r$. To preserve the cointegration properties of X_t, the n × k matrix A has the property that $\alpha' A = 0$.

The common trends formulation (4.2) provides an explicit decomposition of X_t into permanent and transitory components. By construction, $A\tau_t$ is integrated of order one but $D(L)\epsilon_t$ is integrated of order zero. An implication of the assumption $\Sigma_{j=0}^{\infty} j|C_j| < \infty$ is that $\Sigma_{j=0}^{\infty}|D_j| < \infty$, so that $D(L)\epsilon_t$ has a finite, generally nonsingular covariance matrix. Thus $X_t^p \equiv A\tau_t$ can be thought of as the common trends present in X_t, while $X_t^s \equiv D(L)\epsilon_t$ can be thought of as a stationary or transient component. Thus (4.2) can be rewritten as a permanent–transitory decomposition,

$$X_t = X_0 + X_t^p + X_t^s. \tag{4.3}$$

which is a multivariate generalization of the decomposition proposed by Stephen Beveridge and Charles Nelson [1981].

Identification

The common trends model (4.2) is a special case of a dynamic factor model, where the k factors are the random walks τ_t and A is an $n \times k$ factor loading matrix.[12] Without additional restrictions on the relation between the innovations in the two components, this factor model is not generally identified. In our context, however, the fact that one of the components in (4.2) is nonstationary while the other is stationary means that certain implications can be investigated without imposing additional restrictions.

Four features of the model (4.2) are of particular empirical interest: (i) the number of stochastic trends (i.e., the dimension of τ_t, k); (ii) the permanent components X_t^p; (iii) the innovations in the permanent components, η_t; and (iv) various statistics describing the dynamic response of X_t to the permanent innovations (impulse responses and forecast error variance decompositions). Identification of these four properties require different assumptions, which we discuss in turn.

Identification of k. Because the number of common trends in (4.1) is n–r, the problem of identifying k is equivalent to identifying r, the number of cointegrating vectors. The number of linearly independent cointegrating vectors is by definition n minus the rank of C(1) (so rank (C(1)) = k). Thus k is identified directly from the Wold representation (4.1). (This observation was exploited by the cointegration tests reported in Section 3).

Identification of X_t^p. The decomposition of X_t into permanent and transitory components requires some additional restrictions. For example, it is insufficient to define the permanent component to be "the integrated component" of X_t, for this is not unique: both $A\tau_t$ and $A\tau_t + D(L)\epsilon_t$ are integrated, so by this definition either would do as a permanent component.[13]

In the common trends model (4.2), the vector of permanent components, $X_t^p = A\tau_t$ is identified by the assumption that τ_t is a vector random walk, so that X_t^p is also a random walk (with the singular covariance matrix $E\Delta X_t^p \Delta X_t^{p\prime} = A\Sigma_\eta A'$). As Beveridge and Nelson [1981], Andrew Harvey [1985], and Watson [1986] emphasize, the assumption that X_t^p follows a random walk provides a natural definition of trend as the long run forecast of X_t. For large m, the forecast of X_{t+m} made at time t, $(X_{t+m|t})$, is arbitrarily close (in mean square) to X_t^p; that is, $E[X_t^p - (X_{t+m|t} - \gamma m)]^2 -> 0$ as $m -> \infty$. Because $X_{t+m|t}$ can be formed given C(L), Σ_ϵ, and $\{\epsilon_t, \epsilon_{t-1}, ...\}$, it follows that the assumption that τ_t is a vector random walk suffices to identify X_t^p.

Identification of τ_t *and* η_t. Given k and X_t^p, additional assumptions are required to identify τ_t. Because $X_t^p = A\tau_t$, A and τ_t are only identified up to an arbitrary transformation by a nonsingular $k \times k$ matrix R (i.e. $A\tau_t = (AR)(R^{-1}\tau_t) = A^*\tau_t^*$). If k = 1, R is a scalar that normalizes the scale of η_t. However, if k > 1 additional considerations involving economic theory must be used to identify the k permanent shocks.

In our empirical analysis of section 5 below, we achieve the necessary identification in two stages. First, we use some *a priori* restrictions on the long run influence of changes in unobserved stochastic trends or observed variables. For example, we require that a long run change in the inflation rate has no long run effect on the level of economic activity. Second, we order the unobserved permanent shocks in a Wold causal chain, as in Christpoher Sims' [1980] work with VAR systems.[14]

Technically, this amounts to writing A as $A_0\Pi$, where the cointegration restriction is $\alpha' A_0 = 0$ and Π is a lower triangular $k \times k$ matrix with ones on the diagonal. The choice of A_0 reflects *a priori* considerations about which permanent shock impinges on which variable. The k permanent shocks are further assumed to be contemporaneously uncorrelated and Π is chosen to be the lower triangular Cholesky factorization of Σ_η. The assumptions embodied in the choice of A_0, combined with the assumption that the permanent shocks are uncorrelated, serve to identify A. Given A, τ_t is identified as $\tau_t = (A'A)^{-1}A'X_t^p$.

Impulse Responses and m–Quarter Ahead Forecast Errors. A major focus of our empirical investigation is on the dynamic properties of our cointegrated system. We examine these properties by estimating the responses of X_t to innovations in the permanent components and the fraction of the forecast error variance of X_t attributable to the permanent components. There are two ways to proceed at this point.

First, to separate the influence of the permanent innovations from the influence of the transitory innovations, we must make an assumption about the correlation between these sets of innovations. The precise assumption that we use is stated algebraically in the Appendix. There we show that the k permanent innovations η_t can be written as linear combinations of the n one–step ahead forecast errors ϵ_t in equation (4.1). The influence of these permanent innovations on X_t can be identified by assuming that they are uncorrelated with the other n–k linear combinations of the ϵ_t that make up the purely transitory innovations. This assumption allows us to separate the joint effects of the permanent shocks from the joint effects of the transitory shocks.

Second, we can proceed further and identify the separate dynamic response of X_t to each of the permanent innovations by making the necessary assumptions specified previously to identify A and τ_t.

In the empirical work that follows we pursue both of these approaches. The first has the advantage of being invariant to the ordering of the permanent innovations and the other assumptions necessary to identify A and τ_t from X_t^p. The second procedure is potentially interesting because one can investigate the character and implications of the independent stochastic trends. In so doing, one may be able to provide some interpretation of the underlying unobservable factors.

Estimation

A common procedure for estimating the moving average representation of a stationary multivariate time series model is to invert the estimated finite order VAR. Granger's Representation Theorem (Engle and Granger [1987]) implies that a similar procedure can be used with the cointegrated system (4.1), or equivalently (4.2), except that the cointegrating conditions are imposed by estimating a vector error correction model (VECM) rather than a VAR. In the empirical implementation, the VECM is assumed to be

$$(4.4) \qquad \Delta X_t = g + B(L)\Delta X_{t-1} - d(\alpha' X_{t-1}) + \epsilon_t$$

where ϵ_t are the same innovations as in (4.1), g is n × 1, d is n × (n–k), and B(L) is an n × n matrix lag polynomial of order p. The n–k stationary variables $\alpha' X_t$ are the "error–correction" terms. Thus C(L) and ϵ_t are computed by estimating (4.4), given α, and inverting the resulting VECM.

As discussed above, if k=1 then $\alpha' A=0$ identifies A up to scale. For k>1, A is written as $A=A_0\Pi$, where A_0 is known and Π is a lower triangular matrix with ones on the diagonal and unknown parameters below the diagonal. Given A_0, Π and A are estimated from α, $\hat{C}(1)$, and $\hat{\Sigma}_\epsilon$.[15]

The innovations statistics are computed from the estimated moving average decomposition $\hat{C}(L)$ and the estimated transformation matrix $\hat{F}^*$ which transforms the underlying errors into the permanent innovation (i.e., $F^*\epsilon_t = \eta_t^*$ where η_t^* is the vector of orthogenized permanent innovations). The first k rows of $\hat{F}^*$ are given by $\hat{F} = (A_0'A_0)A_0'\hat{C}(1)$, and the additional assumptions that the permanent and transitory shocks are uncorrelated and that the permanent shock precedes the transitory shock in a Wold causal sense permits computing moving average representation of the vector X_t to the permanent innovations. The details underlying the construction of $\hat{F}^*$ are given in the Appendix.

In summary, the common trends model (4.2) has three desirable features. First, the model involves no overidentifying restrictions beyond the testable restrictions imposed by cointegration, although additional structure is needed to interpret its dynamic properties. Second, it permits rather general correlations between specific permanent and transitory shocks. Third, the model is easily estimated using a finite order VECM.

5. Permanent Shocks and Economic Fluctuations: Empirical Results

In this section we investigate the quantitative importance of permanent components in the fluctuations of output, consumption and investment at business cycle horizons. We begin by considering the simple three variable system introduced in section 3 which contains one common stochastic trend. Next we expand the system to include real balances, a nominal interest rate and inflation. This expanded system enables us to explore the implications of additional common stochastic trends and to investigate the sensitivity of the results to a variety of identifying assumptions.

A Three Variable System of Real Flow Variables

Our results are based on an estimated VECM using five lags of the first differences of y, c and i with an intercept and the two theoretical error correction terms, y–c and y–i. Imposing the two balanced growth cointegrating vectors implies that the factor loading matrix A is a 3x1 matrix with equal elements. We adopt the normalization

A=[1 1 1]', so that a one percent increase in the permanent component increases y, c and i by one percent in the long run.

Since the system has only one stochastic trend, the analysis of the dynamics only requires knowledge of the cointegrating vectors to estimate the VECM and an assumption that the permanent shock is uncorrelated with and causally prior to the transitory shock. Given these assumptions we have constructed the impulse response of y, c and i to a one unit innovation in τ_t. These estimated impulse responses are plotted in Figure 3, where we also plot the responses plus and minus one standard deviation.[16] In response to a shock that leads to a one percent increase in the permanent level, the values of output and investment increase by more than one percent in the near term (one to two years), while consumption moves only slightly. These results are consistent with the simple theoretical model discussed in section 2 where the capital stock rapidly increases at the short run cost of consumption. Virtually all of the adjustment is completed within four years.

Are these responses large enough to explain a substantial fraction of the short run variation in the data? This question is addressed in Table 6 and Figure 4. Table 6 shows the fraction of the forecast error variance that is attributed to an innovation in the common stochastic trend, at horizons of 1 to 24 quarters. These variance decompositions indicate that innovations in the permanent component appear to play a dominant role in the variation in GNP and consumption. At the one to four–quarter horizon, the point estimates suggest that 50% to 60% of the fluctuations in private GNP can be attributed to the permanent component. This increases to 70% at the two year horizon and to 80% at the six–year horizon. The results for consumption are similar. Interestingly, the permanent component explains a much smaller fraction of the movements in investment. The point estimate is that the permanent component explains only 25% of the one year ahead variance in unexpected investment. This increases to 40% at the six year horizon.

These results are illustrated in Figure 4, which presents plots of the three year ahead forecast errors for y, c and i and the portion of that error that is attributable to the common stochastic trend. As can be seen, the variation due to the permanent component tracks the major movements in the forecast errors of y and c very closely. On the other hand, the permanent component offers less of an explanation for variations in investment.

This evidence suggests the existence of a strongly persistent—potentially permanent—component that shifts the composition of real output between consumption and investment. (If there were temporary components with negligible effect on forecast errors after three or more years, then we would expect the variance ratios in Table 6 to increase more sharply at the longer horizons.) Thus, the results motivate us to investigate the possibility of a permanent component in our six-variable system that might account for such compositional effects.

<u>Six-Variable Systems with Nominal Variables</u>

This section examines the sensitivity of the main result of the three variable system—that permanent shocks can account for much of the short run movements in output and consumption—to the incorporation of additional variables into the system. This is done by estimating a six-variable system that includes the three real flow variables plus real balances, interest rates and inflation, thereby expanding the potential range of shocks. Our strategy is to analyze in detail a "benchmark" specification and then briefly summarize a range of alternative formulations.

Cointegration Constraints and Estimates of the Permanent Component. The results of section 3 suggest that a resonable specification incorporates three cointegrating relations (and thus three common trends) among the six variables. For our benchmark model we have chosen the following cointegrating relations: $c - y = \phi_1(R_{gs} - \Delta p)$, $i - y = \phi_2(R_{gs} - \Delta p)$ and $m - p = \epsilon_y\, y - \epsilon_R\, R_{gs}$. The first two permit variation in the real ratios arising from permanent shifts in the "real interest rates" and the third says that "money demand" disturbances are stationary. The values in Table 4 are used as estimates of ϕ_1, ϕ_2, ϵ_y and ϵ_R.

Given these cointegrating relations we are able to identify the total permanent component X_t^p, and assuming that the permanent shocks are uncorrelated with and causally prior to the temporary shocks, we are able to identify its short run effects on each of the six variables. In order to individually identify the three stochastic trends we must make some further assumptions. As discussed in the previous section, we do this by specifying the 6x3 factor loading matrix A and an ordering of the stochastic trends. The parameterization adopted reflects the requirement that $\alpha'A = 0$ and is designed to offer a potential interpretation of the three trends. The specific formulation is:

$$(5.1)\quad \begin{bmatrix} y \\ c \\ i \\ m-p \\ R_{gs} \\ \Delta p \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & \phi_1 \\ 1 & 0 & \phi_2 \\ \epsilon_y & \epsilon_R & \epsilon_R \\ 0 & 1 & 1 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ \pi_{21} & 1 & 0 \\ \pi_{22} & \pi_{32} & 1 \end{bmatrix} \tau_t + D(L)\epsilon_t,$$

with the column vector of stochastic trends evolving according to $\tau_t = \mu + \tau_{t-1} + \eta_t$.

To interpret this specification, temporarily suppose that $\pi_{21} = \pi_{31} = \pi_{32} = 0$. The trend τ_1 has a balanced effect of y, c, and i, plus an effect on real balances determined by ϵ_y; by construction, it has no long-run effect on inflation and the interest rate. In this 6-variable system, then, we interpret τ_1 as the counterpart to the "balanced growth" component in the previous 3-variable system. A change in τ_2 has a unit positive effect on nominal interest and inflation, no effect on real quantities and an effect on real balances governed by the interest elasticity of real money balances. Hence, we interpret τ_2 as a neutral shift in the level of the inflation rate. A change in τ_3 has a unit effect on R_{gs} but no effect on Δp, so it is interpreted as a permanent change in the real interest rate. Its effect on real balances is given by the interest rate elasticity. Nonzero ϕ_1 and ϕ_2 admit the possibility that this shock will have a compositional effect on real activity.

In general, π_{12}, π_{13}, and π_{23} are nonzero. As discussed in Section 3, there is no unique choice of the factorization of the covariance matrix of the permanent when $k>1$. In the benchmark model, we adopt a factorization analogous to that of Sims' [1980], but vary this as part of the sensitivity analysis below. For a discussion of these issues in the context of conventional VAR's, see Blanchard and Watson [1986].

It is important to recall that our specification of the factor loading matrix is just one possibility. After all, the τ's are fundamentally unobservable factors. Our choice of parameterization is an attempt to interpret these factors as being associated with the sorts of economic mechanisms we have described above. Nevertheless, nothing we have done makes our interpretation unique or guarantees that the "true factors" have any such interpretation.

Our benchmark model (5.1) is estimated over the period 1954:1 to 1988:4 using a VECM with eight lags and the three error correction terms implied by the cointegrating relations. The first aspect of the model we investigate is the behavior of the total permanent component, X_t^p . The summary statistics for this feature of the model do not depend on our parameterization of the factor loading matrix or the ordering of the individual stochastic trends. They do depend on the cointegrating relations and the assumption that the permanent shocks are causally prior to the temporary shocks.

Table 7 presents information on the explanatory power of the total permanent component. These variance decompositions indicate that the three common stochastic trends together account for over 60% of the one quarter ahead forecast error of output and over 80% at the 12 quarter horizon. Interestingly the fraction of consumption's forecast error explained by the permanent component is actually smaller at short horizons in the six variable system than the three variable system, although the ratio is roughly the same (60%) at the three year horizon in both setups. The permanent component in this six variable system, however, explains a much larger fraction of investment than in the three variable system, almost 80% at the three year horizon. The short run variation in the remaining variables in the system, m–p, r_{gs} and Δp, are also explained to a substantial degree by permanent components.

By parameterizing the factor loading matrix, we can be more specific about the implications and character of the three common trends. Table 8 presents the variance decompositions of the forecast errors from the benchmark model. Four aspects of this table are of particular interest. First, the importance of the real or "balanced growth" shock (the first shock) in explaining the movements in output and consumption is reduced, especially at the one to four quarter horizon. At the three to five year horizon, however, it remains true that this shock has important explanatory power: roughly one half of the variation in these forecast errors is attributable to the first permanent component. Second, including the additional shocks in this expanded model does not change the inability of the first permanent component to explain the short run variations in investment. Third, the second component (associated with permanent movements to inflation) explains a considerable amount of the variation in inflation at medium to long horizons, but little else. Fourth, the third component (associated with permanent movements in the real interest rate) explains most of the forecast errors for the nominal rate and substantial amounts of the output and investment forecast errors.

Figure 5 visually illustrates the roles played by the different shocks by plotting the forecast error at the three year horizon and the portion attributable to each stochastic trend as well as the total, X_t^p , for y c and i. These plots highlight the negligible role of the second or "inflation" shock and the substantial role played by the first or "balanced growth" shock and the third or "real interest rate" shock. Looking at specific episodes in this figure, one finds that stochastic trend #1 has particular explanatory power for the sustained growth of the 1960s. On the other hand, stochastic trend #3 seems particularly important in the contraction of 1974 and the the 1981–82 recession.

The last part of the dynamic analysis is to investigate the impulse response functions of the key real variables, y,c and i, to the three permanent shocks. In Figure 6 we have plotted these responses to unit impulses in the "balanced growth" shock (trend #1), the "inflation" shock (trend #2) and the "real interest rate" shock (trend #3). The estimated standard deviations of these underlying shocks are 0.7%, 0.08% and 0.12% per quarter. The response of output to the "balanced growth" shock (shock #1) is negligible over the first few quarters, while consumption increases slightly and investment declines. By a year, however, major increases in output, consumption and investment are present. While these responses are smaller that those observed in the three variable model, they seem to conform to how one might think a system would respond to "news" about technological developments.

The "inflation" shock is associated with some curious responses. Both output and consumption respond negatively to this shock and then tend to oscillate between positive and negative. Recall, however, that the model constrains this shock to have zero long run impact on output. Investment, on the other hand, shows a positive response to this shock for the first three quarters. Although not shown here, real balances also respond positively to this shock in the short run. While these responses do not correspond to how most economists would describe the effects of such a nominal disturbance, it must be remembered that this shock plays very little role in explaining any of these variables.

We have already shown that the shock we have labeled a "real interest rate" shock (shock #3), plays an important role in explaining the short run behavior of output and investment. The impulse response functions make interpreting this shock as a permanent change in the real rate of interest somewhat difficult. All three of the real flow variables have an initial response to a permanent increase in this "real interest rate" that is strongly positive, before turning negative two to three quarters out. While there may be economic models that predict such responses to a permanent change in the real rate, standard ones do not: our findings lead us to question our interpretation of this third and important common trend, as a real rate disburbance.

We draw four main conclusions from our analysis of the benchmark model. First, permanent components arising from common stochastic trends are empirically important factors influencing economic fluctuations. Second, the behavior of the "balanced growth" factor, while having less explanatory power in the six variable system than in the three variable system, retains a significant role in explaining movements at horizons greater than 2 years. Third, a large fraction of the short–run (0–2 year) variability in output and investment is explained by a factor that is related to persistent movements in the real rate of interest. Fourth, the impulse response functions appear consistent with our interpretation of the first shock as a real or "balanced growth" shock, but lead us to question our interpretation of the third as a "real rate" shock, at least within the context of standard models of the macroeconomy.

Sensitivity Analysis

We now turn our attention to the sensitivity of the main conclusions of the above empirical analysis to various modifications to the benchmark six variable model. Specifically, we consider changes in cointegrating vectors (and thus A matrices), changes in the ordering of the permanent components, different dimension of the system, and choice of variables. To save space, we focus on a key measure, the fraction of the variance of the 3–year ahead forecast error in each of the variables explained by the real or "balanced growth" component. The results of the specification analysis are summarized in Table 9.[17]

Two robust conclusions emerge from the results in Table 9. First, looking across specifications a substantial fraction of the forecast errors in output and consumption are explained by movements in the "balanced growth" component; the point estimates fall in the range of one–third to two–thirds. Second, the fraction of the forecast error variance of investment explained by the real permanent component is never large (at most 41% in model B.6. These are consistent with the findings for the benchmark model B.1 in Table 9.

In addition to these main conclusions, our work with this battery of models has led to a number of additional results of interest. First, a comparison of the models in panels B with those in panels C indicates that the real component has less explanatory power for output when real balances are included in the system. A mechanical explanation of this finding is that, in these systems, the real shock by construction must explain part of the long-run movements in real balances, so that the effect on output is attenuated. Second, whether ϕ_1 and ϕ_2 are set to zero makes little difference for the forecast error variance decompositions. Third, changing the ordering of the shocks (for example, putting the permanent real shock last in the Cholesky factorization, as in model B.7) is quantitatively noticeable but does does not change the overall qualitative features of the results. Fourth, the variance decompositions for output and investment exhibit substantial stability when the model is estimated over various subsamples. Fifth, the forecast error variance decompositions for the *total* permanent component X_t^p is almost always greater than 80% at the three-year horizon as long as the cointegrating vectors permit a stochastic trend in the "real interest rate." In other models it ranges between 55% and 70%.

6. Analysis of Trend Components of Private GNP

Our method can be used to provide a decomposition of each element of X_t into a permanent (or trend) and a stationary (or cyclical) component. In this section, we examine the trend component of private gross national product by comparing it to standard estimates of productivity growth and trend output.

Trend Private GNP. The trend component of output, y_t^p, is plotted in Figure 7 along with Denison's [1985] estimate of real potential GNP per capita.[18] Despite the very different approaches used to construct the two trend estimates they are broadly similar. The three major differences between the two series are the prolonged growth of the 1960s (where the benchmark model ascribes more of this growth to a shift in the trend), the 1974 contraction, and the slowdown of the late 1970s.

Solow Residuals and the Balanced Growth Stochastic Trend. In the neoclassical growth framework of Section 2, the common long run movements in aggregate variables arise from changes in productivity. Is there any evidence that productivity movements are related to innovations in the trend component of GNP or, more generally, to η_t? We investigate this by comparing these estimated innovations to a popular estimate of the change in total factor productivity in the economy, the residual of Robert Solow [1957]. If the economy can be characterized by a Cobb–Douglas production function—as in the theoretical model of Section 2—the Solow residual has the convenient interpretation of being exactly ξ_t in (2.1). Two measures of this productivity residual are used: Robert Hall's [1986, Table 1] for total manufacturing and that produced by Prescott [1986]. Hall's series is reported annually, and Prescott's quarterly series was aggregated to the annual level for comparability.[19]

The time path of the Solow residual and the change in the permanent component of private GNP from the benchmark 6–variable model are plotted in Figure 8a for Hall's measure and in Figure 8b for Prescott's measure. Visual inspection suggests a modest relation between Hall's Solow residual and extracted permanent components (the correlation is .56) and a stronger relationship between Prescott's Solow residual and the permanent component (the correlation is .70).

The Solow residual is well understood to be an imperfect measure of technical change. For example, Prescott [1986] points to errors in measuring the variables used in its construction and Hall [1986] has suggested that this measure of productivity will misrepresent true technological progress in noncompetitive environments where price exceeds marginal cost. These caveats suggest carrying out a more extensive investigation into the relationship between our estimated permanent innovations and various productivity measures. Nonetheless, these results suggest a fairly close link between real permanent shocks from the benchmark model and the longer swings in the two measures of the Solow residual. These comparisons thus lend some credence to the interpretation in Section 5 of the permanent real shocks as measuring economy–wide shifts in productivity.

7. Conclusion

This empirical investigation suggests four general conclusions. First, real per capita private output, consumption, and investment, money balances, and interest rates appear to be well–characterized as containing common stochastic trends. Cointegrating relations appear present among the real flow variables; among money, output and interest rates (as a long run money demand relation); and among nominal interest rates (as stationarity of the spreads).

Second, when these common stochastic trends are modeled as random walks, innovations in these trends appear to account for a substantial fraction of the movements in the real and nominal variables, even over short horizons. In particular, in the case of real per–capita GNP, approximately one–third to two–thirds of its forecast errors at the 3–year horizon can be explained by movements in the permanent real (balanced growth) shock.

Third, comparison of changes in the real permanent components with independent estimates of Solow's (1957) measure of total factor productivity lends some support to the interpretation of this change in the real permanent component as in innovation to economy–wide productive opportunities.

Fourth, thus interpreted, our results lend some support to the empirical relevance of the class of neoclassical growth models discussed in Section 2, in which productivity shocks play a key role in generating business cycle fluctuations. While shocks to the real permanent "balanced growth" component explain a sizable fraction of the movements in output at business cycle horizons, this explanatory power mainly arises from some specific periods, notably the sustained growth of the 1960's.

Yet, shocks to the "balanced growth" permanent component shed little light on the fluctuations during other important episodes, such as the 1974 contraction and the 1981–82 recession. Moreover, these permanent real shock accounts for less than two–fifths of the movements in investment, even at the 6–year horizon.

Our investigation also indicates the explanatory power of an additional permanent component, associated with interest rates, that is closely associated with swings in investment. Further, in the 1974 contraction and the early 1980's, this component is associated with the forecast errors in output as well. These observations suggest that models in which productivity shocks are the sole source of cyclical fluctuations fail to capture empirically important features of the postwar U.S. experience.

Notes

[1]For some recent discussions, see Prescott [1986], King, Plosser and Rebelo [1988a,b] and Plosser [1989]).

[2]The log of consumption minus the log of investment is also stationary, but this is not an independent cointegrating vector since it is just a linear combination of the first two $(\alpha_1 - \alpha_2)$.

[3]All data were obtained from Citibase. Using the Citibase mnemonics for the series, the precise definitions of the variables are GC82/P16 (consumption), GIF82/P16 (investment), and (GNP82–GGE82)/P16 (real private output). The Citibase M2 series (FM2) was used for 1959:1–1985:4; the earlier M2 data was formed by splicing the M2 series reported in Banking and Monetary Statistics, 1941–1970, Board of Governors of the Federal Reserve System to the Citibase data in January 1959. (We thank Dennis Kraft for his advice on this matter.) The monthly data were averaged to obtain the quarterly observations. The price deflator was obtained as the ratio of nominal private GNP (the difference between Citibase series GNP and GGE) and real private GNP (the difference between Citibase series GNP82 and GGE82). The interest rates are FYGM3, FYCP, and FYAAAC.

[4]These univariate results are not robust to certain changes in the definition of the variables. We highlight three examples. First, while the ratio of consumption to private GNP appears to be stationary over the 1949–1988 period, there is a clear upward trend in the ratio of consumption to total GNP over the 1953–1988 period. Second, the ratio of real consumption of durables to total GNP (or private GNP) shows a strong upward trend over the sample period. The inclusion of consumer durable purchases in investment rather than consumption reduces the trend in the consumption to total GNP ratio, but increases in trend in the investment to total GNP ratio. Third, the investment/output ratio is more volatile when inventory changes are included in the measure of investment.

[5]There are three difficulties with the interpretation of these estimated roots. First, just as in the univariate analyses, when the true root is unity its estimate is biased towards zero. Second, when the true root is unity the distribution, including the mean, of its estimator changes when a time trend is added to the system. Third, the distribution of the roots depends on the additional parameters describing the short–run dynamics in the VAR, and thus will vary from one n–variable system to the next. Thus the point estimates are only suggestive; the formal unit root/cointegration hypothesis tests provide more precise information regarding the number of unit roots.

[6]The τ subscript on the statistics presented in Panel B of Table 2 indicate that they were constructed using linearly detrended data. The asymptotic critical values for J_τ differ from those tabulated in Johansen [1988], which are appropriate only for tests constructed using data that have not been demeaned or detrended. It is straightforward to use the results in Sims, Stock, and Watson [1990] to derive the asymptotic null distribution of the J_τ statistics. We have done this, and the p–values shown in the table are based on this asymptotic distribution.

[7]Formally, the LR statistic tests the hypothesis that the estimated cointegrating vectors fall in the subspace spanned by the hypothesized cointegrating vectors, against the alternative that they do not, under the maintained hypothesis that that the number of cointegrating vectors is correctly specified. Johansen (1988) shows that the likelihood ratio test statistics is distributed as a χ^2 random variable under the null hypothesis if the first differences of the data have a mean of zero; this is extended to the case with nonzero mean in Johansen [1989].

[8]The imprecision in $\hat{\epsilon}_R$ highlights a point made by Stock [1988] about estimated cointegrating vectors. While these estimators are "super-consistent" (Stock [1987]), they may imprecisely estimated in a fixed sample; although the discrepancy between $\hat{\epsilon}_R$ and ϵ_R converges to zero quickly as the sample size grows, it can still be large for a given fixed sample size.

[9]The point estimates of coefficients on nominal interest rates and inflation in the consumption and investment equations make little sense and differ sharply from those in Table 4. Since the real interest rate appears to be borderline nonstationary, the nominal interest rate and inflation are very highly correlated in the long run. This leads to a problem similar to multicolinearity in classic regression models. In this framework, small sample bias in the individual coefficients as well as imprecision is likely to result from the problem. The variance decompositions and impulse response functions reported below were calculated using the point estimates of the cointegrating vectors from Table 4.

[10]An alternative approach to identifying these long run properties would be to take a structural model explicitly derived from economic theory and to impose the implied restrictions on the reduced form. The main difficulty with this approach is the lack of a set of readily agreed upon restrictions that could be imposed and used to interpret the data. Moreover, this would defeat our objective of summarizing long-run empirical regularities while imposing a minimal number of questionable theoretical restrictions.

[11]In this section we use I(1) to denote stationary processes and I(0) to denote covariance stationary processes with invertible Wold representations. In general I(q) processes need not be covariance stationary.

[12]General discussions of dynamic factor models are provided by John Geweke and Kenneth Singleton [1981] and Mark Watson and Robert Engle [1983].

[13]The only long-run property of X_t that is identified directly from the Wold decomposition (4.1) is the spectral density of ΔX_t at frequency zero, which has rank k. This is an alternative way of describing the fact that the number of common trends can be determined without additional identifying assumptions.

[14]Additional assumptions would be necessary to compute impulse responses and variance decompositions with respect to the transitory innovations.

[15]From (4.1), (4.2) and (4.3) it follows that, conditional on $\epsilon_s = 0$ for $s \leq 0$, $X_t^p = C(1)\Sigma_{s=1}^t \epsilon_s = A\tau_t$. Combining this with the assumption that $E_{\eta t}\eta_t' = I_k$ (the k × k identity matrix), if follows that $C(1)\Sigma_\epsilon C(1)' = A_0\Pi\Pi'A_0$. Given estimates of Σ_ϵ and C(1), Π is therefore estimated as the lower triangular Cholesky factor of $(A_0'A_0)^{-1}A_0'\hat{C}(1)\hat{\Sigma}_\epsilon\hat{C}(1)'A_0(A_0'A_0)^{-1}$.

[16]The reported impulse responses are deviations from a (common) deterministic time trend. Thus a permanent response of 1 percent actually represents a shift upwards of the long-run growth path by 1 percent. The standard errors for the impulse response functions and variance decompositions were computed using 500 simulations as discussed by Thomas Doan and Robert Litterman [1986], page 19–4.

[17]The models in panel A and C were estimated using 5 lags. The results from these models were robust to changing the lag length to 8 or 10 lags. The results in panel B, based on models with 8 lags, were sensitive to decreases but not increases in the lag lengths. Likelihood ratio test statistics suggested that lags 6–8 of the nominal variables entered some of the equations, particularly the equations for money and inflation. For panel B, this suggested using the specifications with the longer lags to limit bias.

[18]Denison's measure of potential output is computed by adjusting actual output using an Okun's law relationship, by adjusting for capacity utilization, and by making other adjustments such as for labor disputes, the weather, and the size of the armed forces. Source: Denison (1985), Tables 2–4.

[19]We are grateful to Gary Hansen for providing us with Prescott's Solow residual series.

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Appendix

Common Trends and Cointegration

This appendix presents the derivation of the "common trends" model (4.2) from Engle and Granger's (1987) cointegrated model. Suppose that the nx1 vector X_t has a moving average representation in first differences, perhaps with a nonzero nx1 drift γ.

$$\text{(A.1)} \qquad \Delta X_t = \gamma + C(L)\epsilon_t$$

where $E(\epsilon_t | \epsilon_s, s < t) = 0$, $E(\epsilon_t \epsilon_r' | \{\epsilon_s\}), s < \min(t, r)) = 0$, $t \neq r$, and $= \Sigma$, $t = r$. Engle and Granger (1987) define X_t to be cointegrated if there exists a nxr matrix α such that $\alpha' X_t$ is integrated of order zero, which implies that $\alpha' C(1) = 0$. Cointegration can in principle be defined so that $\alpha' X_t$ might have a deterministic (but not stochastic) trend. Here, however, we use the stronger definition that this deterministic trend is zero, i.e., that $\alpha'\gamma = 0$. Assume that there are $r = n-k$ cointegrating vectors.

To obtain the common trends representation (4.2), first rewrite (A.1) as

$$\text{(A.2)} \qquad X_t = X_0 + \gamma t + C(1)\xi_t + D(L)\epsilon_t$$

where $D_j = -\Sigma_{i=j+1}^{\infty} C_i$ and $\xi_t = \Sigma_{s=1}^{t} \epsilon_s$. Note that ξ_t is a n×1 random walk. Define A_0 to be a fixed kxn matrix of full column rank such that $\alpha' A_0 = 0$, and let H be the n×n matrix,

$$H = \begin{bmatrix} (A_0' A_0)^{-1/2} A_0' \\ (\alpha'\alpha)^{-1/2}\alpha' \end{bmatrix} = \begin{bmatrix} H_1 \\ H_2 \end{bmatrix}$$

where $M^{1/2}$ denotes the matrix square root of M. Note that $HH' = I_n$ (the n×n identity matrix), so that $H^{-1} = H'$. Also note that

$$HC(1) = \begin{bmatrix} H_1C(1) \\ H_2C(1) \end{bmatrix} = \begin{bmatrix} (A_o'A_o)^{-1/2}A_o'C(1) \\ 0 \end{bmatrix} .$$

Thus, $C(1) = H^{-1}HC(1) = [H_1' \ H_2']HC(1) = H_1'H_1C(1) = A_o(A_o'A_o)^{-1}A_o'C(1) = A_oF$, where $F = (A_o'A_o)^{-1}A_o'C(1)$ is k×n. Similarly, $\alpha'\delta = 0$ implies that $\delta = A_o\bar{\mu}$, where $\bar{\mu} = (A_o'A_o)^{-1}A_o'\delta$ is k×1. Thus,

$$\text{(A.3)} \qquad X_t = X_o + A_o\bar{\mu}t + A_oF\xi_t + D(L)\epsilon_t = X_o + A_o\bar{\tau}_t + D(L)\epsilon_t$$

where $\bar{\tau}_t = \bar{\mu}t + F\xi_t = \bar{\tau}_{t-1} + \bar{\eta}_t$, where $\bar{\eta}_t = F\epsilon_t$.

The innovations $\bar{\eta}_t$ have nondiagonal covariance matrix $\Sigma_\eta = F\Sigma_\epsilon F'$. Define η_t to be the transformed innovation such that $\bar{\eta}_t = \Pi\eta_t$, where Π is some nonsingular k×k matrix such that $E\eta_t\eta_t'$ is diagonal. Then (A.3) can be written in the common trends form,

$$\text{(A.4)} \qquad X_t = \gamma + A\tau_t + D(L)\epsilon_t$$

where $A = A_o\Pi$ and $E(\Delta Y \Delta Y')$ is diagonal. Note that Π can always be chosen to be lower triangular with ones on the diagonal, which is normalization adopted in the text.

As discussed in Section 4, if $k > 1$ then further identifying assumptions are needed to compute the innovation statistics. With the additional assumptions that (i) the permanent and transitory shocks are uncorrelated and (ii) the permanent shocks appear first in a Wold causal ordering, the combined vector of (correlated) permanent innovations $\bar{\eta}_t$ and transitory innovations $(\bar{\nu}_t)$ can be written,

$$(A.5) \qquad \eta_t^+ = F^+\epsilon_t = \begin{bmatrix} F\epsilon_t \\ G\epsilon_t \end{bmatrix} = \begin{bmatrix} \bar{\eta}_t \\ \bar{\nu}_t \end{bmatrix}$$

where G is such that F^+ is nonsingular. Let Π^* be a lower triangular matrix such that Π^* has ones on the diagonal and that $F^*\Sigma_\epsilon F^{*\prime} = \Pi^* D^* \Pi^{*\prime}$, where D^* is diagonal. Then, the desired moving average representation of the orthogonal permanent innovations is $C^*(L) = C(L)(F^+)^{-1}\Pi^*$. Note that the upper left-hand (k,k) block of Π^* is Π. Because only the first k column of $C^*(L)$ are of interest, it is inconsequential how G is computed as long as $F^+\Pi^*$ is nonsingular. A computationally convenient procedure (used in the empirical work) is to let the rows of G be the right eigenvectors corresponding to the (n–K) nonzero eigenvalues of $I_n - F'(FF')^{-1}F$.

In the univariate case, F = –1 and the common trends representation (A.4) reduces to the stationary/nonstationary decomposition proposed by Beveridge and Nelson [1981].

Table 1
Tests for Stochastic and Deterministic Trends

Series[a]	Estimate of Largest Root In Univariate Autoregression[b]				Estimate of Deterministic Trend Slope Coefficient[a]			
	w/o Deterministic Trend		w/Deterministic Trend		Levels Regession		Mean Change	
	$\hat{\rho}$	DF "t"	$\hat{\rho}$	DF "t"	$\hat{\mu}_1$	t_1	$\hat{\mu}_2$	t_2
A. Output, Consumption, and Investment, 1949:1 - 1988:4								
y	1.00	-.90	.94	-2.45	1.63	2.50	1.72	3.10
c	1.00	-1.22	.96	-2.09	1.67	2.01	1.89	6.12
i	.99	-.98	.89	-3.44	1.66	3.39	1.70	1.41
Δy	.12	-5.92	.12	-5.93	-.00	-.45	.00	.01
Δc	.14	-5.41	.12	-5.49	-.01	-.89	-.00	-.03
Δi	.27	-5.49	.27	-5.48	.00	.01	.02	.04
c-y	.85	-3.46	.77	-4.21	.10	2.26	.17	.46
i-y	.81	-4.02	.81	-3.99	.00	.02	-.01	-.02
c-i	.84	-4.10	.82	-4.26	.10	1.18	.18	.18
B. Money, Prices, and Interest Rates, 1954:1 - 1988:4								
m-p	1.00	-.83	.96	-2.78	1.57	2.34	1.54	2.70
$\Delta m-\Delta p$	.61	-4.01	.61	-4.00	-.00	-.08	-.00	-.00
m-p-y	.88	-3.67	.89	-3.53	.02	.26	-.26	-.44
Δp	.87	-2.19	.85	-2.10	.02	.54	-.05	-.51
$\Delta^2 p$	-1.44	-6.52	-1.42	-6.48	-.00	-.79	.03	.23
R_{gs}	.95	-2.25	.86	-3.73	.05	2.02	.04	.59
R_{ps}	.94	-2.33	.84	-3.85	.06	2.36	.04	.52
R_{pl}	.99	-1.34	.94	-2.13	.07	2.15	.05	.99
ΔR_{gs}	.11	-4.90	.10	-4.90	-.00	-.24	.01	.20
ΔR_{ps}	.08	-4.95	.08	-4.94	-.00	-.17	.00	.06
ΔR_{pl}	.27	-4.85	.26	-4.87	-.00	-.59	-.00	-.11
C. Spreads and Real Interest, 1954:1-1988:4								
$R_{gs}-R_{ps}$	.51	-4.78	.49	-4.89	-.00	-.98	.00	.01
$R_{ps}-R_{pl}$	.78	-4.45	.74	-4.87	.01	1.49	.00	.02
$R_{gs}-\Delta p$	.86	-1.82	.80	-2.30	.03	1.49	-.00	-.04
$R_{ps}-\Delta p$	.84	-2.01	.76	-2.61	.03	1.75	-.00	-.14
$R_{pl}-\Delta p$	.90	-1.69	.81	-2.52	.05	1.94	-.00	-.03

Notes to Table 1.

[a]The variables y, c, i, m, and p are in logarithms. The interest rates are on a quarterly decimal basis.

$\bar{\mu}_1$ is an estimate of trend growth (at annual rates) formed from a regression of the level of the series onto a constant, a time trend, and 5 lags of the series, and t_1 is the t-test for $\mu_1 = 0$ from this regression. μ_2 is an alternative estimate of the trend growth (at an annual rates) in the series; it is the sample mean of the first difference of the series, and t_2 is the t-test that the true mean is zero constructed using μ_2, with the asymptotic variance (the spectral density of the first difference at frequency zero) estimated by a time-domain average of the first 5 autocovariances.

[b]Unit Root Tests: Following Dickey and Fuller [1979], the largest root can be estimated with or without a simultaneous estimation of a deterministic trend. In the columns marked "w/o deterministic trend," we report the former. In these columns, $\hat{\rho}$ is the sum of autoregressive coefficients from a regression of level of the series on a constant and 5 lags of the series. DF "t" is the appropriate Dickey-Fuller t-test for $\rho = 1$. We report the alternative estimate in the columns marked "with deterministic trend." In these columns, $\hat{\rho}$ is the sum of autoregressive coefficients from a regression of level of the series on a constant, a time trend, and 5 lags of the series. DF "t" is the appropriate t-test for $\rho = 1$.

Critical values for the Dickey-Fuller "t tests" differ across the two sets of tests. Without the deterministic trend, the critical values are: 10%, -2.57, 5%, -2.86. With estimation of a deterministic trend, the critical values are: 10%, -3.12, 5%, -3.41.

Table 2

Cointegration Statistics:
Income, Consumption, Investment (y, c, i), 1949:1 – 1988:4

A. Results from Unrestricted Levels Vector Autoregressions

Largest Eigenvalues of Estimated Companion Matrix

VAR(5) with Constant			VAR(5) with Constant and Trend		
Real	Imaginary	Modulus	Real	Imaginary	Modulus
1.00	.00	1.00	.96	.00	.96
.77	.20	.79	.76	.20	.79
.77	–.20	.79	.76	–.20	.79
.77	.00	.77	.77	.00	.77
–.47	–.44	.65	–.47	–.44	.65
–.47	.44	.65	–.47	.44	.65

B. Multivariate Unit Root Tests

Statistic	Value	P-Value	(Null/Alternative)
$J_T(0)$	35.0	.14	(3 unit roots/at most 2 unit roots)
$q_T^f(3,2)$	–25.9	.31	(3 unit roots/at most 2 unit roots)
$q_T^f(3,1)$	–28.3	.01	(3 unit roots/at most 1 unit root)

C. Estimated Cointegrating Vectors

Variable	Null Hypothesis α_1	Null Hypothesis α_2	MLE α_1	MLE α_2
y	1	1	–1.06 (.02)	–1.00 (.03)
c	–1	0	1.00[a]	0.00[a]
i	0	–1	0.00[a]	1.00[a]

Likelihood Ratio test:
LR statistic = 3.42, df = 2, p-value = 18%

Notes: All statistics were computed using 5 lags in the relevant vector autoregressions. The MLE's of the cointegrating regressions (panel C) were computed using the algorithm given in Stock and Watson (1989). t-statistics constructed using the indicated values in parentheses below the point estimates have a large-sample normal distribution. The likelihood ratio statistic is computed using the algorithm given in Johansen (1988).

[a]Normalized

Table 3
Cointegration Statistics:
Inflation and Interest Rates (Δp, r_{gs}, r_{ps}, r_{pl}), 1954:1 – 1988:4

A. Results from Unrestricted Vector Autoregression
VAR(5) with Constant
Largest Eigenvalues of Companion Matrix:

Real	Imaginary	Modulus
.96	00	.96
.92	.00	.92
.90	.00	.90
–.03	.81	.81
–.03	–.81	.81
.72	–.32	.79
.72	.32	.79
.19	.72	.75

B. Multivariate Unit Root (Cointegration) Tests

Statistic	Value	P–Value	(Null/Alternative)
$J_T(0)$	61.4	<.01	(4 unit roots/at most 3 unit roots)
$q_T^f(4,3)$	–65.5	<.01	(4 unit roots/at most 3 unit roots)
$q_T^f(4,2)$	–65.5	<.01	(4 unit root/at most 2 unit roots)
$q_T^f(4,1)$	–15.1	.01	(4 unit roots/at most 1 unit root)

C. Estimated Cointegrating Vectors: Interest Rates and Inflation

Variable	Null Hypotheses			MLE		
	α_1	α_2	α_3	α_1	α_2	α_3
Δp	1	0	0	1.00[a]	0.00[a]	0.00[a]
R_{gs}	–1	1	0	–.60 (.08)	–1.02 (.01)	–1.08 (.04)
r_{ps}	0	–1	1	0.00[a]	1.00[a]	0.00[a]
r_{pl}	0	0	–1	0.00[a]	0.00[a]	1.00[a]

LR statistic $(\alpha_1, \alpha_2, \alpha_3)$ = 10.6, df = 3, p–value = 1.4%
LR statistic (α_2, α_3) = 4.42, df = 2, p–value = 11%

Notes: See the notes to Table 2. No deterministic trend is included in panel A. $LR(\alpha_1, \alpha_2, \alpha_3)$ tests the joint hypothesis that α_1, α_2, and α_3 equal their value. $LR(\alpha_1, \alpha_2)$ tests the hypothesis that α_2 and α_3 equal their null value.
[a]Normalized

Table 4

Further Estimated Cointegrating Vectors[a]

Estimated Relation	Cointegration Tests	
A. Money Demand, 1954:1–1988:4		
1. $m-p = \underset{(.05)}{1.18}\, y - \underset{(1.25)}{4.72}\, R_{gs}$	$q_{\tau}^{f}(3,2) = \underset{(.46)}{-22.3}$	$J_{\tau}(0) = \underset{(.02)}{43.8}$
Test of Velocity Restriction[b]: $\chi^2 = \underset{(.005)}{7.61}$		
B. Real Ratios and Real Interest, 1954:1–1988:4		
1. $c-y = \underset{(.54)}{1.16}\, (R_{gs}-\Delta p)$	$q_{\mu}^{f}(2,1) = \underset{(<.01)}{-63.1}$	$J_{\tau}(0) = \underset{(.03)}{19.6}$
2. $i-y = \underset{(.35)}{-.55}\, (R_{gs}-\Delta p)$	$q_{\mu}^{f}(2,1) = \underset{(<.01)}{-62.0}$	$J_{\tau}(0) = \underset{(.01)}{22.1}$

Notes: [a]See notes to Table 2. p-values for the test statistics are reported in parentheses. [b]The χ^2 statistic tests the joint hypothesis that the coefficients on y and R_{gs} are respectively 1 and 0.

Table 5
Cointegration Statistics:
Two Six-Variable Mixed Real and Nominal Systems, 1954:1 – 1988:4

A. Real Quantities, Real Balances, Treasury Bill Rate and Inflation

Cointegrating Vector MLE's
Coefficient Estimate
(Standard Errors)

Variable	α_1	α_2	α_3
y	-1.12 (.02)	-1.12 (.05)	-1.15 (.03)
c	1.00[a]	0.00[a]	0.00[a]
i	0.00[a]	1.00[a]	0.00[a]
m-p	0.00[a]	.08	1.00[a]
R_{gs}	1.52 (.38)	0.98 (1.13)	3.61 (.54)
Δp	1.58 (.41)	2.52 (1.42)	0.62 (.64)

Multivariate Unit Root Test:
$q_\tau^f(6,3) = -20.4$ (p-value = .068)

Wald Tests of Cointegrating Vectors

Null Hypotheses	Statistic (ρ value)
$(y-c)$, $(i-c)$, $(m-p)-\epsilon_y y-\epsilon_R R_{gs}$:	$\chi^2_7 = 6.1$ (.528)
$(y-c)$, $(i-c)$, $(m-p)-\epsilon_y y-\epsilon_R R_{gs}$, $R_{gs}-\Delta p$:	χ^2_6 7.4 (.286)
$(y-c)-\phi_1(R_{gs}(-\Delta p)$, $(i-c)-\phi_2(R_{gs}-\Delta p)$, $(m-p)-\epsilon_y y-\epsilon_R R_{gs}$	$\chi^2_5 = 4.5$ (.480)
$(y-c)-\phi_1(R_{gs}-\Delta p)$, $(i-c)-\phi_2(R_{gs}-\Delta p)$, $(m-p)-y$	$\chi^2_7 = 14.4$ (.045)

B. Real Quantities, Public and Private Interest Rates, and Inflation

Cointegrating Vector MLE's
Coefficient Estimate
(Standard Errors)

Variable	α_1	α_2	α_3
y	-1.12 (.02)	-1.12 (.05)	0.000 (.002)
c	1.00[a]	0.00[a]	0.00[a]
i	0.00[a]	1.00[a]	0.00[a]
R_{gs}	1.52 (.38)	0.98 (1.13)	-1.01 (.04)
Δp	.01 (.00)	- .04 (.00)	.00 (.00)

Multivariate Unit Root Test:
$q_\tau^f(6,3) = -20.2$ (p-value = .075)

Wald Tests of Cointegrating Vectors

Null Hypotheses	Statistic (ρ value)
$(y-c)-\phi_1(R_{gs}-\Delta p)$, $(i-c)-\phi_2(R_{gs}-\Delta p)$, $R_{gs}-R_{ps}$	$\chi^2_7 = 11.8$ (.130)
$(y-c)$, $(i-c)$, $R_{gs}-R_{ps}$	$\chi^2_9 = 11.8$ (.224)
$(y-c)$, $(i-c)$, $R_{gs}-R_{ps}$, $R_{gs}-\Delta p$	$\chi^2_8 = 13.7$ (.090)

Notes to Table 5.: See notes to Table 2. The Wald statistics, testing whether the true cointegrating subspace is spanned by the hypothesized cointegrating vectors, have a chi-squared distribution with the indicated degrees of freedom. (The reduced degrees of freedom for some statistics allows for the estimation of elasticities under the null.) When the subspace hypothesis concerns r cointegrating vectors, it is maintained that there are n-r unit roots in the system. For all test statistics, p-values follow in parentheses. Estimates of ϵ_y, ϵ_R, ϕ_1, and ϕ_2 (used in specifying the constrained cointegrating vectors) were taken from Table 4.

[a]Normalized.

Table 6
Forecast Error Variance Decompositions:
3-variable real model (y, c, i), 49:1 - 88:4

Fraction of the forecast error variance attributable to the real permanent shock

Horizon	y	c	i
1.	.54	.75	.09
	(.24)	(.26)	(.13)
4.	.61	.78	.26
	(.22)	(.23)	(.18)
8.	.68	.69	.33
	(.18)	(.22)	(.17)
12.	.70	.69	.33
	(.16)	(.21)	(.17)
16.	.74	.73	.35
	(.15)	(.19)	(.16)
20.	.76	.78	.37
	(.14)	(.16)	(.16)
24.	.79	.81	.39
	(.13)	(.14)	(.16)
∞	1.00	1.00	1.00

Note: Approximate standard errors are shown in parentheses.

Table 7
Forecast Error Variance Decomposition:
6 - Variable Model 54:1 - 88:4

	Fraction of Forecast Error Variance Attributable to Permanent Component X_t^p					
Horizon	y	c	i	$m-p$	R_{gs}	Δp
1	.63	.33	.68	.84	.80	.70
	(.16)	(.25)	(.17)	(.18)	(.17)	(.20)
4	.80	.34	.82	.86	.88	.66
	(.14)	(.23)	(.17)	(.15)	(.14)	(.14)
8	.79	.42	.73	.91	.92	.71
	(.12)	(.21)	(.13)	(.12)	(.12)	(.12)
12	.83	.60	.77	.94	.92	.76
	(.11)	(.17)	(.12)	(.10)	(.11)	(.13)
16	.87	.69	.80	.95	.93	.78
	(.11)	(.14)	(.11)	(.09)	(.10)	(.13)
20	.88	.75	.78	.94	.94	.80
	(.10)	(.11)	(.11)	(.07)	(.09)	(.14)
24	.88	.79	.77	.94	.94	.81
	(.09)	(.09)	(.11)	(.07)	(.09)	(.14)
∞	1.00	1.00	1.00	1.00	1.00	1.00

Note: Approximate standard errors in parentheses.

Table 8
Forecast Error Variance Decompositions:
6 - Variable model (5.1), 54:1 - 88:4

A. Fraction of the forecast error variance attributable to permanent shock #1

Horizon	y	c	i	m-p	R_{gs}	Δp
1.	.00	.02	.14	.75	.14	.31
	(.16)	(.09)	(.18)	(.32)	(.20)	(.15)
4.	.04	.14	.05	.76	.13	.23
	(.16)	(.13)	(.13)	(.29)	(.21)	(.08)
8.	.21	.31	.13	.72	.12	.21
	(.12)	(.18)	(.11)	(.28)	(.23)	(.07)
12.	.44	.49	.27	.74	111	.17
	(.14)	(.23)	(.15)	(.28)	(.23)	(.06)
16.	.55	.59	.32	.76	.11	.16
	(.17)	(.23)	(.17)	(.27)	(.24)	(.07)
20.	.59	.64	.32	.76	.12	.15
	(.17)	(.22)	(.16)	(.25)	(.25)	(.08)
24.	.63	.67	.32	.78	.13	.14
	(.17)	(.20)	(.15)	(.25)	(.26)	(.09)
∞	1.00	.94	.99	.80	.21	.04

B. Fraction of the forecast error variance attributable to permanent shock #2

Horizon	y	c	i	m-p	R_{gs}	Δp
1.	.01	.03	.07	.02	.03	.38
	(.15)	(.12)	(.18)	(.15)	(.16)	(.20)
4.	.03	.01	.21	.06	.04	.34
	(.17)	(.12)	(.19)	(.14)	(.18)	(.14)
8.	.03	.01	.20	.02	.02	.34
	(.15)	(.14)	(.15)	(.16)	(.20)	(.12)
12.	.02	.00	.12	.01	.02	.44
	(.14)	(.15)	(.14)	(.16)	(.19)	(.15)
16.	.02	.01	.10	.01	.03	.49
	(.14)	(.15)	(.14)	(.16)	(.19)	(.17)
20.	.02	.02	.11	.01	.03	.53
	(.13)	(.14)	(.13)	(.15)	(.19)	(.19)
24.	.02	.03	.10	.01	.02	.55
	(.13)	(.14)	(.13)	(.15)	(.18)	(.20)
∞	.00	.02	.00	.00	.01	.96

C. Fraction of the forecast error variance attributable to permanent shock #3

Horizon	y	c	i	m-p	R_{gs}	Δp
1.	.62	.27	.47	.07	.63	.01
	(.29)	(.21)	(.25)	(.16)	(.31)	(.13)
4.	.72	.18	.55	.04	.72	.10
	(.32)	(.18)	(.28)	(.15)	(.34)	(.08)
8.	.54	.11	.40	.17	.77	.16
	(.23)	(.10)	(.20)	(.17)	(.36)	(.08)
12.	.37	.11	.38	.19	.79	.14
	(.15)	(.10)	(.17)	(.18)	(.37)	(.08)
16.	.30	.08	.37	.18	.79	.13
	(.12)	(.10)	(.18)	(.17)	(.38)	(.07)
20.	.26	.08	.35	.16	.79	.12
	(.11)	(.09)	(.16)	(.16)	(.38)	(.07)
24.	.24	.09	.34	.15	.79	.11
	(.10)	(.08)	(.16)	(.15)	(.38)	(.07)
∞	.00	.05	.01	.20	.79	.00

Note: Approximate standard errors are given in parentheses.

Notes to table 9:

Models with Real Balances

Model B.2. is identical to the main model in the text, except that the coefficients ϕ_1 and ϕ_2 are set to zero, i.e., cointegration of shares and the real interest rate is dropped.

Model B.3. is a two stochastic trend model, obtained by assuming that the real interest rate is stationary, i.e., dropping the third column of the factor loading (A) matrix reported in the main text.

Model B.4. is a two stochastic trend model, obtained by assuming that the inflation rate is stationary, i.e., dropping the second column of the factor loading (A) matrix reported in the main text.

Model B.5. is a two stochastic trend model for a five variable system (y,c,i m-p and R), i.e., dropping the second column and sixth row of the factor loading (A) matrix reported in the main text. Hence, this model contains a balanced growth and nominal interest rate stochastic trends.

Model B.6 is identical to model B.5. except that the ordering of stochastic trend innovations is reversed.

Model B. 7 is identical to model B.1. except that the stochastic trend innovations are reordered to place the inflation shock first, the real interest rate shock second and the balanced growth trend third.

Model B. 8 is identical to model B.1. except that stationary velocity is imposed, i.e., we impose the parameter restrictions $\epsilon_y = 1$ and $\epsilon_R = 0$.

Models with Two Interest Rates

Model C.1 is a six variable system with variables ordered y, c, i, r_{gs}, r_{ps} and δ_p. We require that the spread $R_{ps} - R_{gs}$ is stationary. This delivers the following factor loading matrix analogous to A in the main text. The first column is [1 1 1 0 0 0]', which derives from the balanced growth restriction on the effects of the first stochastic trend. The second column is [0 0 0 1 1 1]', which derives from the uniform effect of trend inflation on the two nominal interest rates. The third column is [0 ϕ_1 ϕ_2 1 1 0]', which derives from (i) cointegration of shares and the real interest rate; and (ii) stationarity of the interest rate spread.

Model C.2 is a six variable, two system obtained by imposing stationary of the real interest rate, i.e., dropping the third column of the factor loading matrix used in model C.1.

Model C. 3. is identical to model C.1. without cointegration of shares and the real interest rate, i.e., imposing $\phi_1 = \phi_2 = 0$.

Model C.4. is identical to model C.1. with the balanced growth trend innovations ordered last.

Table 9

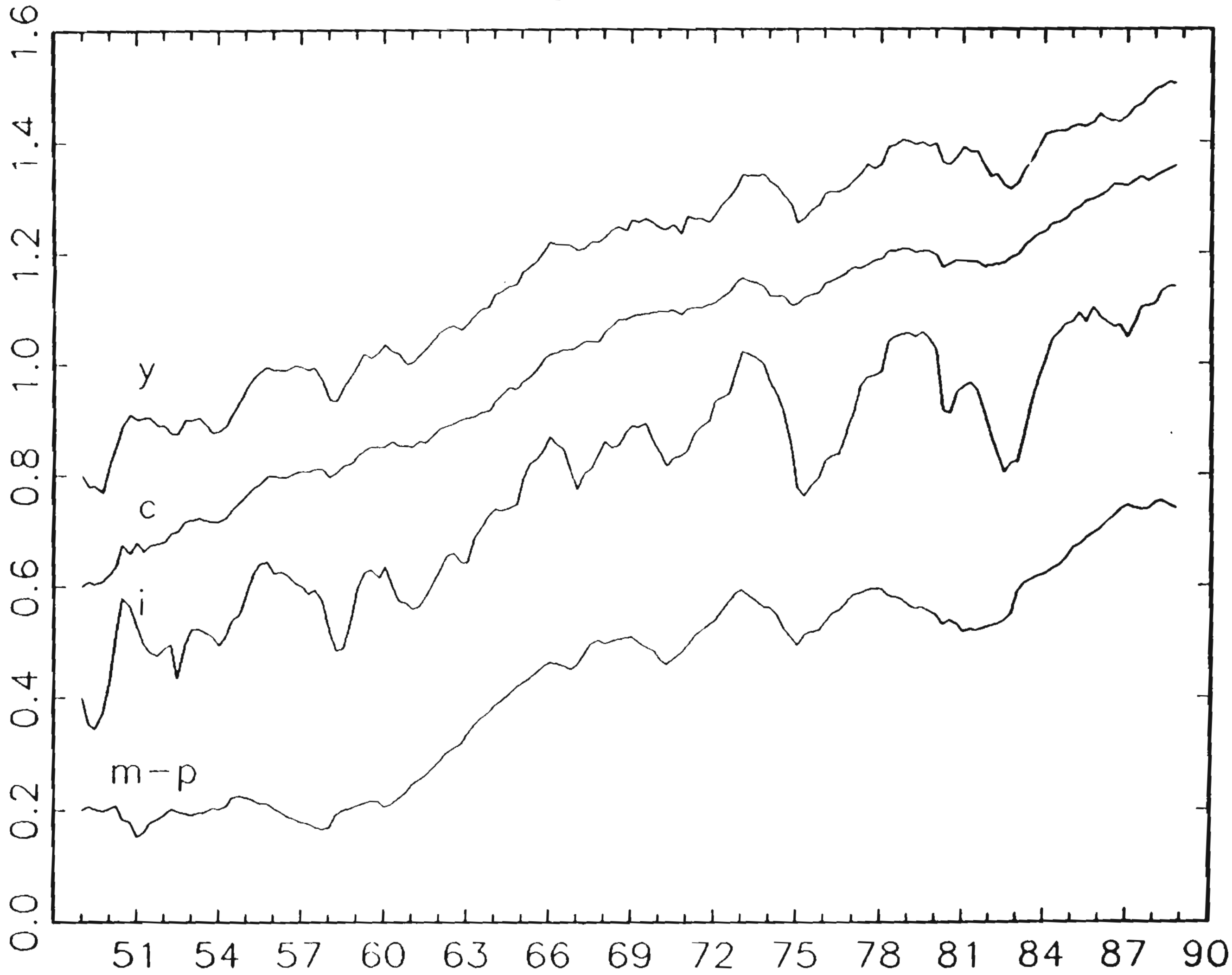
3-Year Ahead Forecast Error Variance Decompositions:
Summary of Results for Various Models

		LR test of cointegrating Vectors (df, p-value)	Fraction of forecast error variance attributed to the permanent real shock						
			y	c	i	m-p	R_{gs}	Δp	R_{ps}
A. Real Model 1949:1 - 88:4		4.1 (2, .120)	.70	.69	.33	–	–	–	–
B. Monetary Models	B.1	4.5 (5, .480)	.44	.49	.27	.74	.11	.17	–
	B.2	6.1 (7, .528)	.43	.54	.26	.72	.10	.17	–
1954:1 - 88:4	B.3	7.4 (6, .285)	.35	.39	.14	.54	.01	.19	–
	B.4	7.4 (6, .285)	.37	.40	.15	.56	.01	.18	–
	B.5	2.8 (4, .600)	.43	.49	.25	.68	.08	–	–
	B.6	8.2 (4, .083)	.58	.75	.41	.80	–	.16	–
	B.7	*	.35	.29	.12	.30	.02	.18	–
	B.8	14.4 (7, .045)	.55	.58	.41	.87	.29	.21	–
C. Two Interest Rate Models	C.1	11.2 (7, .130)	.66	.43	.32	–	.04	.06	.04
	C.2	11.8 (9, .224)	.61	.54	.26	–	.01	.05	.01
1954:1 - 88:4	C.3	13.7 (8, .090)	.61	.62	.27	–	.01	.05	.02
	C.4	**	.53	.53	.21	–	.01	.03	.02

*The hypothesized cointegrating vectors are the same as in model (B.1).

**The hypothesized cointegrating vectors are the same as in model (C.1).

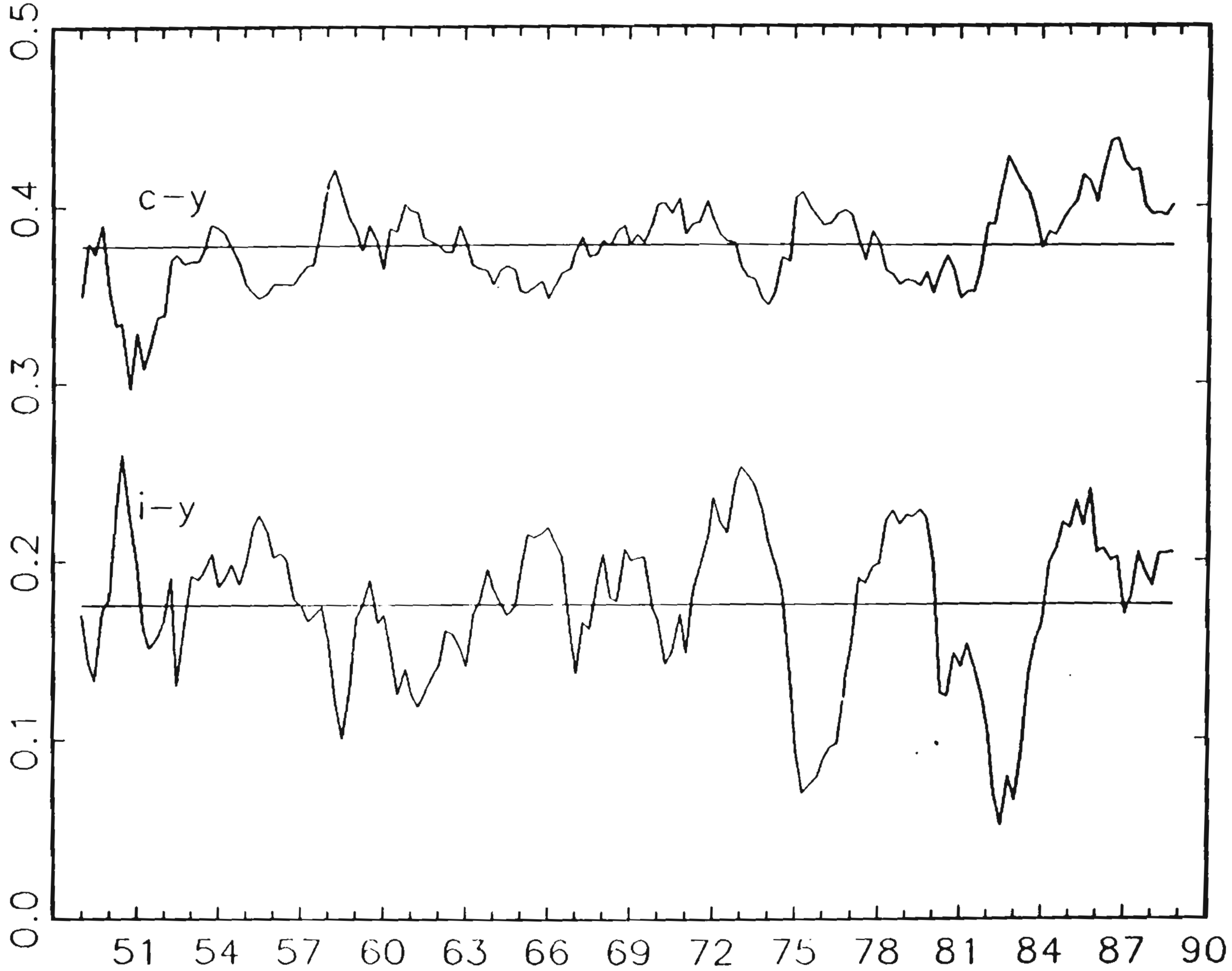
Figure 1



Logarithms of Private Output (y), Consumption (c), Investment (i)

and Real Money Balances (m−p)

Figure 2



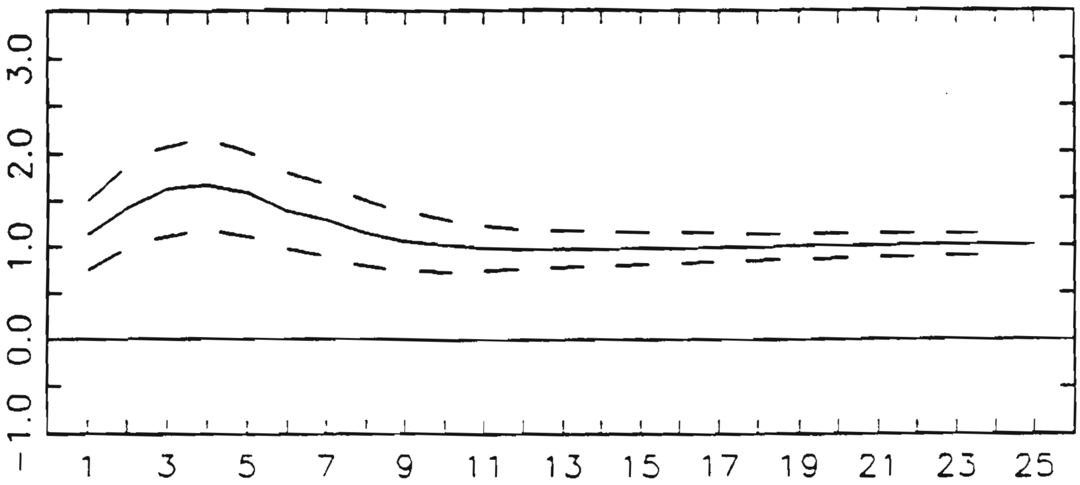
Logarithms of the Consumption−Output (c−y)
and Investment−Output (i−y) Ratios

Notes to Table 9:

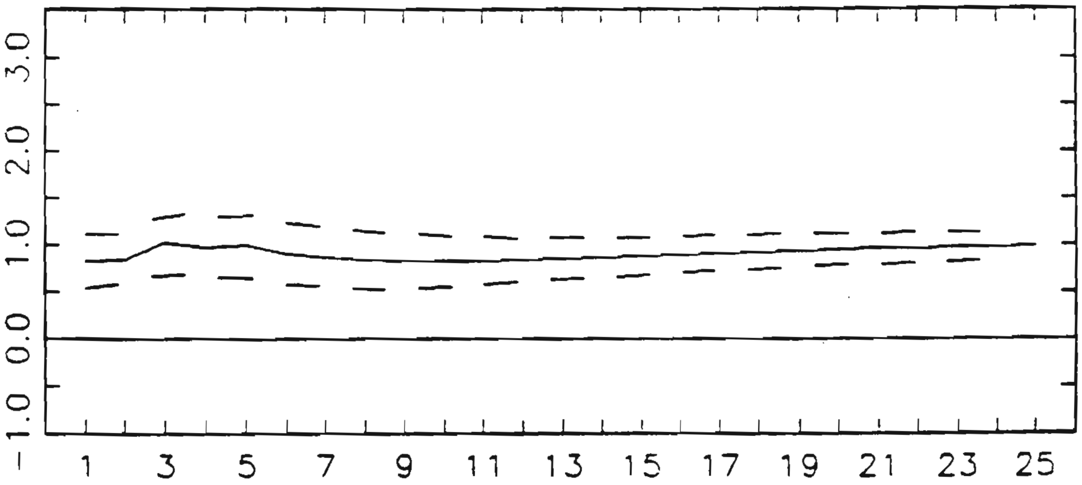
The estimation period denotes the sample used to estimate the VECM. The LR statistic and p-values test the hypothesis that the true cointegrating subspace is orthogonal to the columns of the A_0 matrix. The VECM's in panel B were estimated using 8 lags, the VECM's in panels A and C with 5 lags.

Figure 3

Responses in 3-variable Model to a Unit Shock
in the Real Permanent Component



(a) y



(b) c

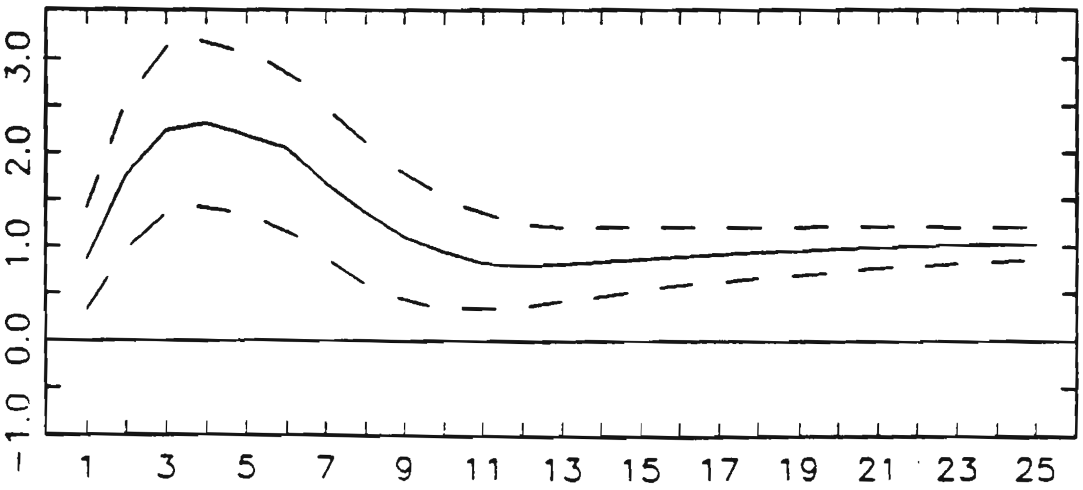


Figure 4

Historical Forecast Decomposition
3-variable Model

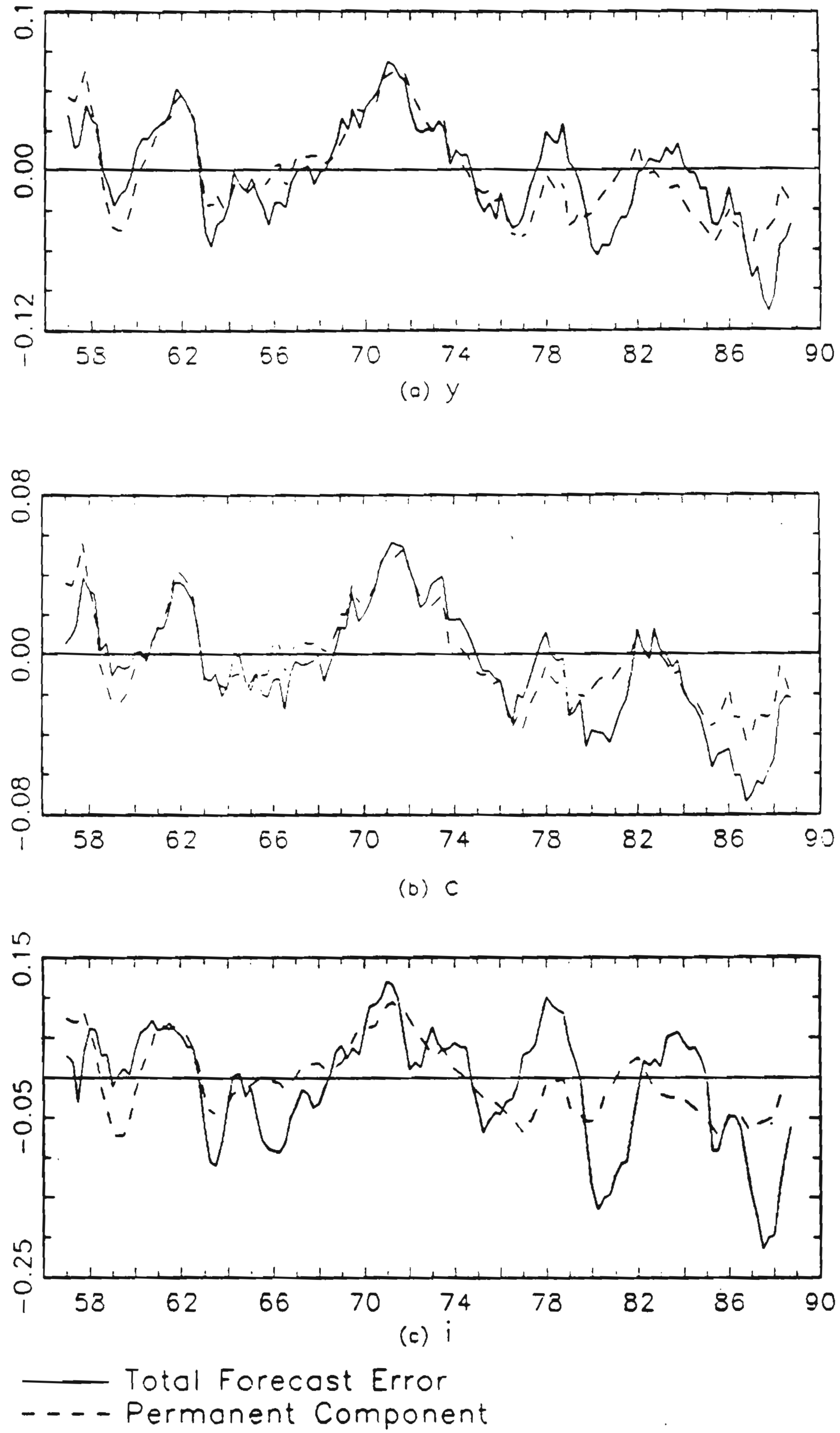


Figure 5

Historical Forecast Decomposition
6 variable Benchmark Model

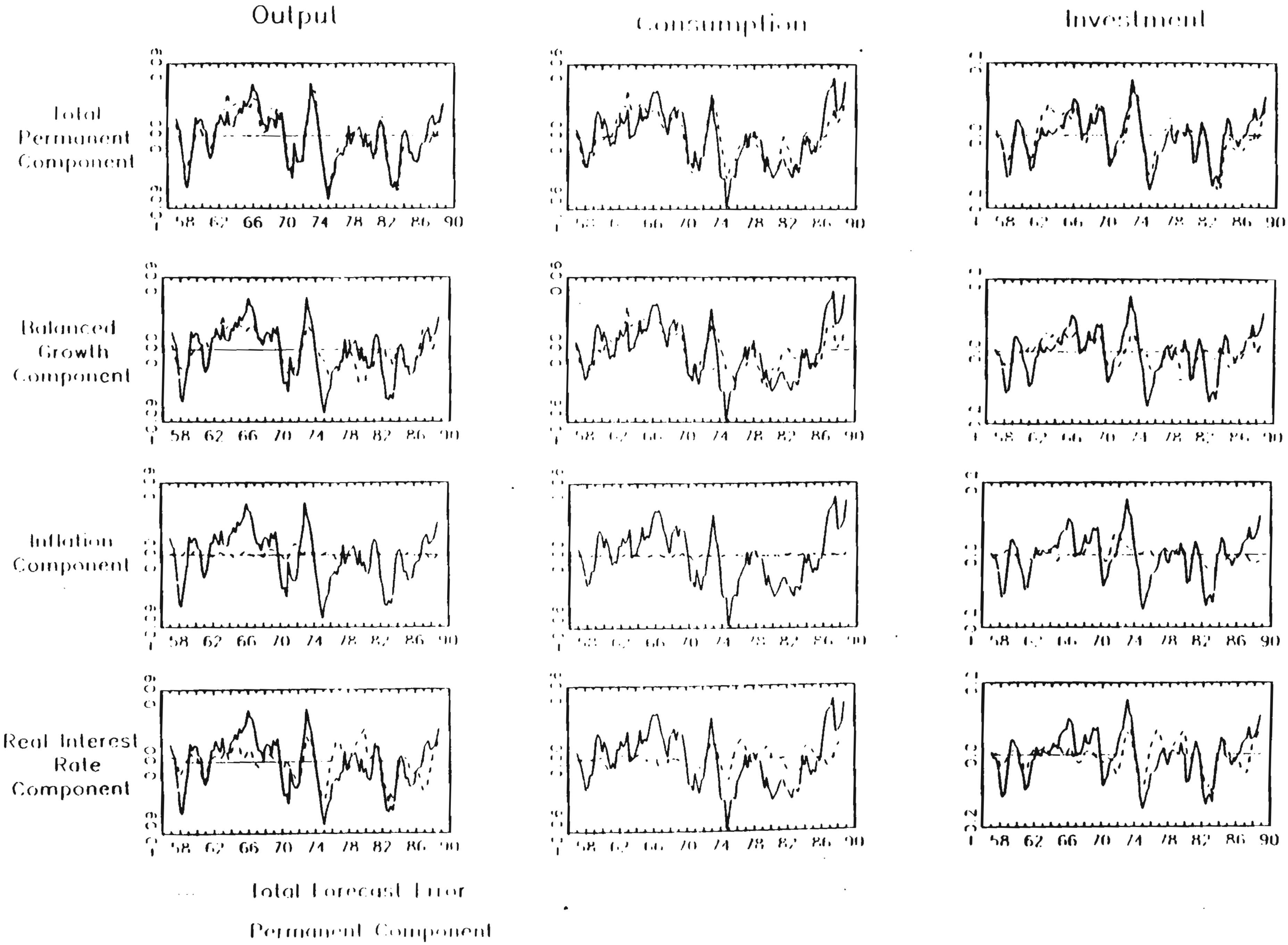


Figure 6

Selected Impulse Responses in 6-variable Model (5.1)

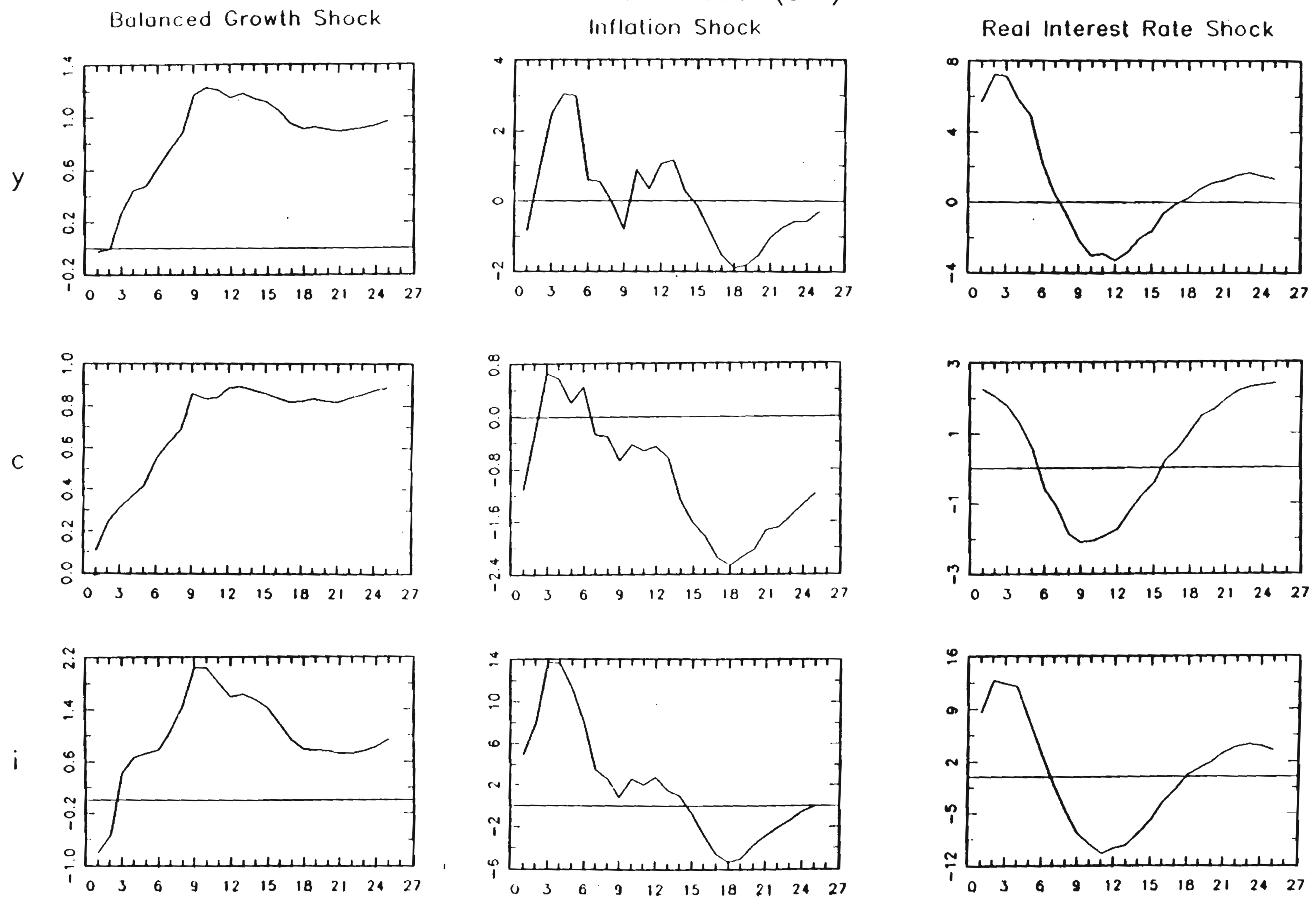


Figure 7

Estimates of Annual Trend Output

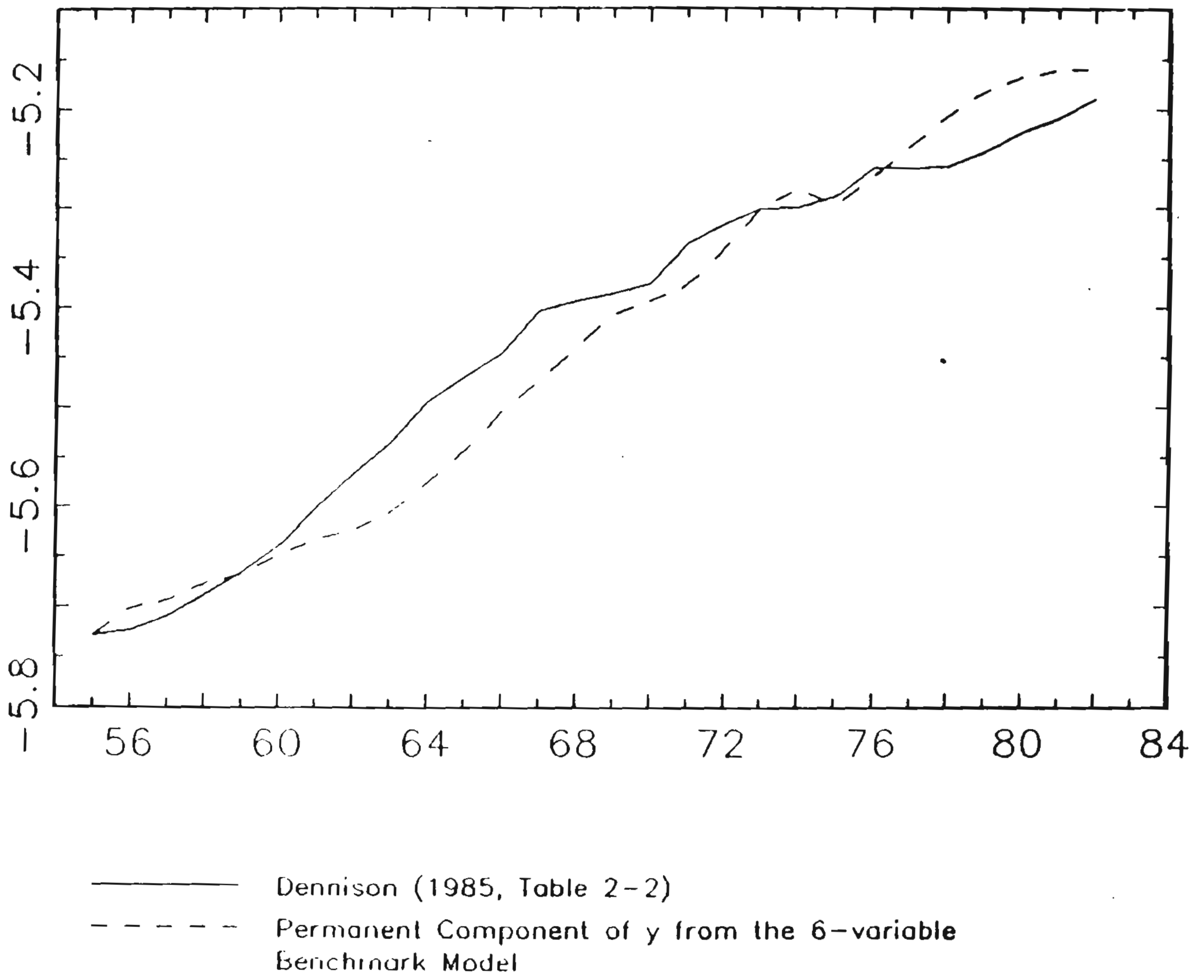


Figure 8a

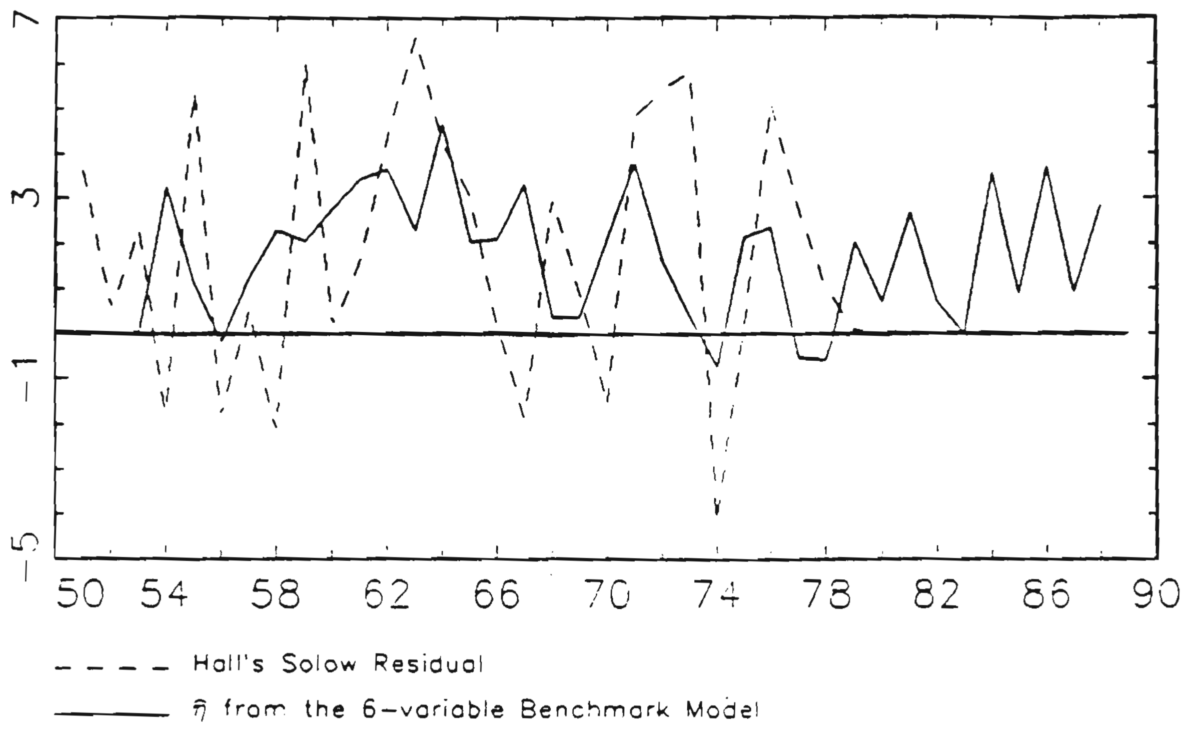
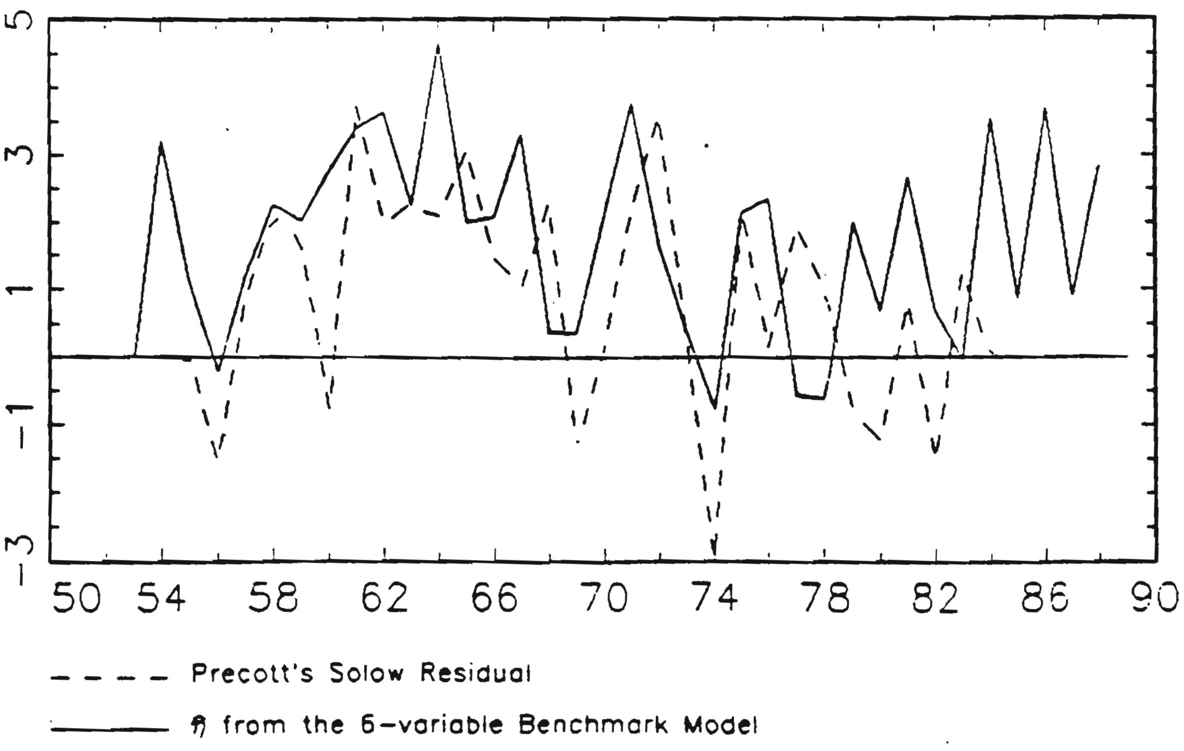


Figure 8b



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