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Working Paper No. 82
May 1987

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February 1987
Revised, May 1987

*This research is supported by the National Science Foundation. I have benefited from comments by Marvin Goodfriend, Bob King, Ben McCallum, and Bill Schwert.

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Abstract

The paper develops a model in which targeting of the nominal interest rate is a reasonable guide for monetary policy. Expected real interest rates and output are exogenous with respect to monetary variables, and the central bank ends up influencing nominal interest rates by altering expected inflation. In this model the monetary authority can come arbitrarily close in each period to its (time-varying) target for the nominal interest rate, even while holding down the forecast variance of the price level. The latter objective pins down the extent of monetary accommodation to shifts in the demand for money and other shocks, and thereby makes determinate the levels of money and prices at each date. Empirical evidence for the United States in the post-WWII period suggests that the model's predictions accord reasonably well with observed behavior for nominal interest rates, growth rates of the monetary base, and rates of inflation. Earlier periods, especially before WWI, provide an interesting contrast because interest-rate smoothing did not apply. The behavior of the monetary base and the price level at these times differed from the post-WWII experience in ways predicted by the theory.

Central bankers, including those at the Federal Reserve, seem to talk mainly in terms of controlling or targeting interest rates. Given the pervasiveness of this outlook, it would probably be useful for economists to assign interest rates a major role in a positive theory of monetary policy. Nevertheless, many monetary theorists—especially those of an "equilibrium" persuasion (and sometimes called "monetarists")—have viewed monetary policy mainly in terms of the behavior of monetary aggregates. In this view the targeting of interest rates is either impossible or undesirable (see, for example, Friedman, 1968, and Brunner, 1968). One aspect of modern versions of this skepticism concerns price-level determinism under an interest-rate rule (see Sargent and Wallace, 1975, and McCallum, 1981). A major result here is that an interest-rate target requires some additional mechanism to pin down the levels of nominal variables. However, this observation does not distinguish an interest-rate rule from rules related to monetary growth or inflation, which may also be incomplete with respect to the levels of money and prices. In any event, since any of these rules can be extended to achieve price-level determinism, this criticism does not constitute a serious attack on the logic or desirability of this class of policies.

Part of the difficulty in thinking of monetary policy in terms of interest rates concerns the familiar distinction between real and nominal rates. It may be that systematically and significantly influencing expected real interest rates—which is what many macroeconomists imagine when they view monetary policy in terms of interest rates—is beyond the power of monetary authorities over periods of interesting length. In fact, my assumption throughout this paper is that expected real interest rates are exogenous with respect to monetary policy. But even with this extreme assumption about real rates, the nominal interest rate is a perfectly fine nominal variable that the monetary authority should be able to control—at least if it does not try to regulate independently some other nominal rate of change, such as the inflation rate, the growth rate of a monetary aggregate, or the rate of change of the exchange rate. Moreover, since interest rates can be observed rapidly and with great accuracy, they are good candidates for variables that the monetary authority could monitor and react to in a feedback manner. In this respect, feedback from nominal interest rates to monetary instruments seems more attractive than some alternative suggestions that involve the inflation rate or the growth rate of nominal GNP.

In this paper I explore the behavior of monetary policy—in particular, the behavior of the monetary base—that is consistent with an objective of interest-rate smoothing. I argue that such an objective appears reasonable, and leads in a theoretical model to well-defined behavior for money and prices. Furthermore, this behavior for money and prices provides testable hypotheses about these variables under a regime where the monetary authority targets nominal interest rates. The empirical results suggest that this regime is a good approximation to reality in the United States in the post-World War II period, and perhaps also in the interwar period. The sample before World War I reveals very different behavior for the nominal interest rate, and therefore provides an interesting contrast to the recent experience.

Part I sets out the theoretical model. Part II considers optimal monetary policy within this model. Part III views this optimal policy as a positive theory to derive hypotheses about the behavior of the nominal interest rate, the growth rate of the monetary base, and the inflation rate. Part IV extends the analysis to incorporate seasonal elements. Then Part V relates the theory to empirical evidence for the United States since 1890.

I. The Basic Theoretical Model

I use a simple stochastic model of money supply and demand, which builds on models of Goodfriend (1986), McCallum (1986), and Hetzel (1987). The private economy is described by two equations, the first pertaining to interest-rate determination, and the second to the real demand for money:

$$(1) \quad R_t = E_t p_{t+1} - p_t + r_t + v_t$$

$$(2) \quad m_t - p_t = \alpha_t - \beta R_t + \eta_t$$

where the variables are

R_t: nominal interest rate,

p_t: log of price level,

$E_t p_{t+1}$: expectation of next period's log of price level, based on information available at date t,

m_t: log of quantity of money (measured empirically as the monetary base),

r_t: "permanent" part of the expected real interest rate,

v_t: temporary shock to the expected real interest rate, distributed independently as white noise, (mean 0, variance σ_v^2),

α_t: permanent part of level of real demand for money,

η_t: temporary shock to real demand for money, distributed independently as white noise, $(0, \sigma_\eta^2)$,

$\beta > 0$: coefficient of the nominal interest rate in the money-demand function,

The permanent components of the expected real interest rate and money demand follow random walks,

$$
(3) \qquad \begin{aligned} r_t &- r_{t-1} + w_t \\ \alpha_t &= \alpha_{t-1} + a_t \end{aligned}
$$

where w_t and a_t are distributed independently as white noise, (mean 0, variances σ_w^2 and σ_a^2, respectively). If the expected real interest rate is stationary, then $\sigma_w^2 = 0$. The shifts to money demand, a_t and η_t, include the effects from changes in output (permanent and temporary, respectively), which are treated as exogenous. Also, with m_t interpreted as the monetary base, the money-demand shocks would include effects from changes in reserve requirements. In the model the important assumptions are that the shocks to money demand (a_t and η_t) and to the expected real interest rate (w_t and v_t) are exogenous with respect to monetary movements.

There are, of course, equilibrium models (such as Lucas, 1972, 1973, and Barro, 1976, 1980) in which monetary shocks can affect the expected real interest rate and output (and hence the quantity of real money demanded for a given nominal interest rate). The real effects of money in these models depend on incomplete information about monetary aggregates and price levels.

Since the gaps in information may be small and short lived, the quantitative significance of these effects has often been questioned on a priori grounds. Even when the information lags are important, the direction of effect of money on the real variables is ambiguous (Barro and King, 1984). In any case, the empirical evidence (Barro, 1981a) suggests that the impact of monetary shocks on expected real interest rates is small.

Other models where money has real effects involve the influence of expected inflation on transaction costs and the quantity of real cash balances. However, these channels are usually viewed as quantitatively unimportant.

Finally, money may influence the expected real interest rate and output in models with sticky prices, although convincing theoretical accounts of stickiness that matters for real allocations are still absent. In any event, the flexibility of prices can be viewed as one of the underlying assumptions that the model uses to generate testable hypotheses.

Overall, I treat the expected real interest rate as exogenous with respect to monetary variables because I lack an alternative specification that I regard as theoretically or empirically superior. However, even if this assumption is wrong, it may still be satisfactory in the present context if the connection between money and the expected real interest rate is less important than that between money and expected inflation, and hence the nominal interest rate.

The monetary authority has the target, $\bar{R}_t$, for the nominal interest rate at date t. It turns out in this model that the authority has the ability and incentive to keep the actual rate, R_t, close to $\bar{R}_t$ in each period.

Therefore, if R_t were constant, the model would predict little variation in nominal interest rates. But it is well known that, especially in recent years, nominal interest rates move around a good deal and in a largely unpredictable manner. In fact, even for short-term rates, a random walk turns out to be a pretty good description of the data. In order to accord with this observation, the model incorporates a time-varying target for the nominal interest rate that follows a random walk,

$$(4) \qquad \bar{R}_t = \bar{R}_{t-1} + u_t,$$

where u_t is an independent, white-noise process with moments, $(0, \sigma_u^2)$.

One motivation for equation (4) is that the nominal interest rate is the tax rate on money, and the government sets this tax rate as part of an overall problem of optimal public finance. The desire to smooth taxes intertemporally, as stressed in Barro (1979) and Mankiw (1986), tends also to motivate smoothness in individual components of the tax package, such as the tax rate on money.[1] In this context smoothness means that the government avoids predictable movements up and down of the tax rates. Consequently, tax rates—here the target nominal interest rate, $\bar{R}_t$—would follow a Martingale process. This property holds for the random walk given by equation (4).

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[1]Lucas(1984) views the tax rate on money as a determinant of the relative cost of cash and credit goods. Therefore, if the tax rate on final output is set optimally, it is unclear that the tax rate on money should be positive—that is, that money-using goods should be taxed more heavily than credit- (or barter-) using goods. (For a similar argument, see Kimbrough, 1986.) However, a positive tax rate on money does allow the government to tax some black-market activities where final product is not taxed. Also, if the main existing taxes are on some of the factor inputs, especially labor, then it may be desirable to tax other inputs, such as monetary services.

Because of the lower bound of zero on the nominal interest rate, equation (4) cannot apply universally. However, a random walk may be a satisfactory approximation for a broad range of nominal interest rates, even if not for samples (such as the period from the mid 1930s to the early 1950s in the United States) where the rates get close to zero. In any event, for the present analysis it would make little difference whether $\bar{R}_t$ were non-stationary—as implied by equation (4)—or instead had a long-run tendency to revert to a stationary mean. The present analysis focuses on high-frequency properties of the nominal interest rate, monetary growth rate, and inflation.

The monetary authority controls the quantity of money (the monetary base), m_t, in each period. The assumption below is that the authority sets m_t to further an objective that involves two considerations: first, keeping the nominal interest rate, R_t, close to its contemporaneous target, $\bar{R}_t$, and second, holding down the one-period-ahead forecast variance of the price level. (Recall that the policymaker cannot influence output or the expected real interest rate in this model.) It is further assumed that the policymaker's information at time t allows him to make m_t a function of the contemporaneous variables, R_t, $\bar{R}_t$, and α_t, and of all lagged variables. However, because of information lags, it is impossible to have contemporaneous feedback from p_t to m_t. In other words, information about the general price level takes one period to arrive; so a period would be on the order of 1 to 3 months. Given these assumptions, the policymaker's optimal rule for monetary growth can be written in the linear form,[2]

$$(5)\quad m_t - m_{t-1} = \mu_t + \lambda_1(R_t - \bar{R}_t) - \lambda_2(R_{t-1} - \bar{R}_{t-1}) - \gamma(\bar{R}_t - \bar{R}_{t-1}) + \delta a_t$$

The term, μ_t, represents the permanent component of monetary growth. (A temporary part of monetary growth—possibly representing control errors—could also be added.) Consistency with the target for the nominal interest rate, $\bar{R}_t$, and the permanent part of the expected real interest rate, r_t, requires

$$(6)\quad \mu_t = \bar{R}_t - r_t$$

(Recall that the model abstracts from long-term growth, so that the averages of monetary growth and inflation are equal over the long term.) Since $\bar{R}_t$ and r_t follow independent random walks, the monetary growth rate in equation (5) includes the random-walk component, μ_t. In other words, if the nominal-interest-rate target or the expected real interest rate is non-stationary, the monetary growth rate must also be non-stationary. (For a related discussion of trend non-stationarity in money, see Goodfriend, 1986, pp. 10ff.)

Monetary growth in equation (5) depends on the current and lagged gap between the actual and target nominal interest rate. The term, $\lambda_1(R_t - \bar{R}_t)$

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[2]Given the monetary authority's objective and the assumption that private agents have the same information as the policymaker, additional lagged variables—such as p_{t-1}—would not appear in equation (5).

with $\lambda_1 > 0$ (which turns out to be the optimal sign), allows for the standard positive response of current monetary growth to an excess of the nominal interest rate above target. But, in the present model, the expected real interest rate is exogenous. Therefore, a positive reaction of $m_t - m_{t-1}$ to $R_t - \bar{R}_t$ can work to reduce R_t only if it lowers expected inflation, $E_t p_{t+1} - p_t$.[3] This reduction in expected inflation tends to occur if expected future monetary growth, $E_t m_{t+1} - m_t$, declines. In other words, an excess of R_t over $\bar{R}_t$ must create a tendency for some of today's infusion of money to be taken back in the future; for example, in the next period. This effect follows from the term, $-\lambda_2(R_{t-1} - \bar{R}_{t-1})$ with $\lambda_2 > 0$, in equation (5). In fact, to get a negative relation between $R_t - \bar{R}_t$ and $E_t m_{t+1} - m_t$ (and hence, $E_t p_{t+1} - p_t$) it is necessary only to have $\lambda_2 > 0$. The value of λ_1 is irrelevant in this context because it affects equally the levels of money for periods t and t+1. However, the choice of λ_1 turns out to matter if the monetary authority cares not only about targeting nominal interest rates, but also about the predictability of the price level. This last consideration pins down the desired response of the level of money to an interest-rate gap, which then determines the value of λ_1 (and thereby makes determinate the levels of money and prices at each date).

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[3]Shiller (1980, p. 130) recognizes this possibility but regards it as implausible: "We usually think that increasing high-powered money is, if anything, a signal of higher inflation. It would seem implausible, then, that these lower interest rates are due to lower inflationary expectations. It is conceivable that exogenous increases in the money stock might be a sign of lower inflation over a certain time horizon if the parameters of our model were just right." In the present model the parameters turn out to "just right" as a consequence of the monetary authority's own optimal behavior.

Equation (5) allows also for responses of monetary growth to shifts in the interest-rate target, $\bar{R}_t - \bar{R}_{t-1}$, and in the permanent part of money demand, $a_t = \alpha_t - \alpha_{t-1}$. In both cases the monetary reactions turn out to affect the forecast variance of the price level. In a more general model, the monetary authority would lack full current information about a_t, especially because permanent shifts in money demand would be difficult to distinguish from temporary shifts (η_t). However, the authority would presumably always know its own interest-rate target, $\bar{R}_t$.

The linear model described by equations (1)-(6) can be solved in the usual way by the method of undetermined coefficients (see Lucas, 1973, Barro, 1976, McCallum, 1983, 1986, and Goodfriend, 1986). The main issue is the specification of the information set used to compute the expectation, $E_t p_{t+1}$. I assume that this information set corresponds to that of the monetary authority; namely it includes R_t, m_t, $\bar{R}_t$, α_t, and all lagged variables. Given this specification, the analysis is straightforward (although lengthy), and I present only the final form of the solution:[4]

$$(7) \qquad R_t = \bar{R}_t + \left[\frac{1}{1+\beta+\lambda_2}\right](\eta_t + v_t)$$

$$(8) \qquad p_t = -\alpha_t + \delta a_t + m_{t-1} + (1+\beta)\bar{R}_t - r_t - \gamma(\bar{R}_t - \bar{R}_{t-1}) - \lambda_2(R_{t-1} - \bar{R}_{t-1})$$
$$+ \left[\frac{\beta+\lambda_1}{1+\beta+\lambda_2}\right]v_t - \left[\frac{1+\lambda_2-\lambda_1}{1+\beta+\lambda_2}\right]\eta_t$$

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[4]The derivation uses McCallum's (1983, 1986) procedure for selecting the unique, bubble-free solution.

$$(9) \qquad m_t = m_{t-1} + \delta a_t + \bar{R}_t - r_t - \gamma(\bar{R}_t - \bar{R}_{t-1}) - \lambda_2(R_{t-1} - \bar{R}_{t-1})$$

$$+ \left[\frac{\lambda_1}{1+\beta+\lambda_2}\right](\eta_t + v_t)$$

Note that monetary growth, m_t-m_{t-1} from equation (9), includes the permanent component, $\mu_t = \bar{R}_t-r_t$, and the responses, δa_t, $-\lambda_2(R_{t-1}-\bar{R}_{t-1})$, and $-\gamma(\bar{R}_t-\bar{R}_{t-1})$. The reaction to the current interest-rate gap, $\lambda_1(R_t-\bar{R}_t)$, shows up in the equilibrium solution as accommodations of monetary growth to the temporary shocks to real money demand, η_t, and the expected real interest rate, v_t.

The price level in equation (8) depends on m_t (from equation (9)) and on movements in the demand for money. For example, an increase in $\bar{R}_t$ reduces the demand for money and thereby raises the price level by the extra multiple β (above the one-to-one effect implied by the connection between $\bar{R}_t$ and m_t-m_{t-1}).

Equation (7) shows that R_t depends on η_t+v_t, which combines the temporary shocks to money demand and the expected real interest rate. In the absence of any reaction of money supply, these terms would add to R_t. (An increase in η_t raises R_t by lowering today's price level relative to the next period's; an increase in v_t raises R_t directly by the increase in the expected real interest rate.) The reaction of future monetary growth, $m_{t+1}-m_t$, to the term, $-\lambda_2(R_t-\bar{R}_t)$, offsets these forces. In particular, equation (7) shows that a higher value of λ_2 reduces the effect on R_t from the temporary shocks, η_t+v_t.

II. Monetary Policy

I assume that the monetary authority seeks to minimize the magnitude of departures of R_t from $\bar{R}_t$, but also desires to hold down the (one-period-ahead) forecast variance of the price level, $E_t(p_{t+1} - E_t p_{t+1})^2$. (In a larger model the government may dislike a higher variance of prices because it leads to more random redistributions of wealth or to poorer information that agents use to make allocative decisions.) Specifically, the objective is

$$(10) \qquad \text{MIN.} \quad J \equiv A \bullet \text{VAR}(R - \bar{R}) + B \bullet \text{VAR}(p)$$

where A and B are positive coefficients, and the variances are based on information from the previous period. (See Goodfriend, 1986, pp. 5ff., for a discussion of related objectives for the central bank.) Using equations (7) and (8), the two variances are

$$(11) \qquad \text{VAR}(R - \bar{R}) = \left[\frac{1}{1+\beta+\lambda_2}\right]^2 (\sigma_\eta^2 + \sigma_v^2),$$

$$(12) \qquad \text{VAR}(p) = (1-\delta)^2\sigma_a^2 + (1+\beta-\gamma)^2\sigma_u^2 + \left[\frac{\lambda_1+\beta}{1+\beta+\lambda_2}\right]^2 \sigma_v^2 + \left[\frac{1+\lambda_2-\lambda_1}{1+\beta+\lambda_2}\right]^2 \sigma_\eta^2$$

Inspection of equations (11) and (12) indicates that $\delta = 1$ and $\gamma = 1 + \beta$ would minimize VAR(p) for a given value of VAR($R-\bar{R}$). The result $\delta = 1$ means that the monetary authority accommodates fully the (assumed observable) permanent shift in money demand, a_t. However, $0 < \delta < 1$ would tend to arise if a_t were fully observable only with a lag. The response $\gamma = 1 + \beta$ avoids the effect on p_t from an unexpected shift, u_t, in the interest-rate target, $\bar{R}_t$ (and hence, from equation (6), in the permanent part of monetary growth, μ_t). On the other hand, future values of prices, p_{t+1}, ..., still depend on u_t. Therefore, the identification of the coefficient γ from a data set is especially sensitive to the relation between the period of the observations and that in the theory. The longer the observation interval the smaller the reaction coefficient γ appears to be. In any event, I carry along the values γ and δ as parameters.

I assume that the monetary authority chooses the interest-rate reaction parameters, λ_1 and λ_2, to minimize J in equation (10). Note that VAR($R-\bar{R}$) in equation (11) is independent of λ_1. (The contemporaneous reaction of money to the interest rate, which depends on λ_1, affects the levels of money and prices, but not the rates of change that matter for the nominal interest rate.) Hence, λ_1 can be chosen to minimize VAR(p) for a given value of λ_2. In particular, the solution for λ_1 as a function of λ_2 does not depend on the weights, A and B, in equation (10). The resulting condition is

$$(13) \qquad \lambda_1 = \frac{(1+\lambda_2)\sigma_\eta^2 - \beta\sigma_v^2}{\sigma_\eta^2 + \sigma_v^2}$$

Given this choice for λ_1 as a function of λ_2, VAR(p) in equation (12) becomes

$$(14) \qquad \mathrm{VAR}(p) = (1-\delta)^2\sigma_a^2 + (1+\beta-\gamma)^2\sigma_u^2 + \sigma_\eta^2\sigma_v^2/(\sigma_\eta^2 + \sigma_v^2)$$

which is independent of λ_2. Therefore, as long as λ_1 varies along with λ_2 to satisfy equation (13), λ_2 can be chosen (independently of the weights A and B) to minimize $VAR(R-\bar{R})$. It follows immediately from equation (11) that the best choice is $\lambda_2 \to \infty$.[5] Equation (13) then implies $\lambda_1 \to \infty$, but the ratio, λ_1/λ_2, remains finite and is given by

$$(15) \qquad \frac{\lambda_1}{\lambda_2} = \frac{\sigma_\eta^2}{\sigma_\eta^2+\sigma_v^2}$$

Hence $0 \leq \lambda_1/\lambda_2 \leq 1$—the lagged reaction of money to the nominal interest rate is greater in magnitude (and opposite in sign) to the contemporaneous reaction. However, in the limit, each reaction become infinite in order to keep the nominal interest rate, R_t, arbitrarily close to its target, $\bar{R}_t$, in each period.

Using the form of the monetary rule from equation (5) and the optimal choices for λ_1 and λ_2, the equilibrium solutions for R_t, p_t, and m_t in equations (7)-(9) become[6]

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[5]The choice $\lambda_2 \to -\infty$ seems also to work. However, $\lambda_2 \leq -(1+\beta)$ can be ruled out on grounds discussed by McCallum (1986, p. 140, n.7). In particular, if $\lambda_2 \leq -(1+\beta)$, then the realization of a shock—say η_t—causes an unstable dynamic response of the price level.

[6]The terms, $-(\eta_{t-1}+v_{t-1})$, in equations (17) and (18) are the limit of the expression, $-\lambda_2(R_{t-1}-\bar{R}_{t-1})$, as $\lambda_2 \to \infty$. Note that $R_{t-1}-\bar{R}_{t-1} = (\eta_{t-1}+v_{t-1})/(1+\beta+\lambda_2)$ from equation (7).

$$(16) \qquad R_t = \bar{R}_t = R_{t-1} + u_t$$

$$(17) \qquad p_t = -\alpha_t + \delta a_t + m_{t-1} + (1 + \beta)\bar{R}_t - r_t - \gamma(\bar{R}_t - \bar{R}_{t-1}) - (\eta_{t-1} + v_{t-1})$$

$$- \left[\frac{\sigma_v^2}{\sigma_\eta^2 + \sigma_v^2}\right]\eta_t + \left[\frac{\sigma_\eta^2}{\sigma_\eta^2 + \sigma_v^2}\right]v_t$$

$$(18) \qquad m_t = m_{t-1} + \delta a_t + \bar{R}_t - r_t - \gamma(\bar{R}_t - \bar{R}_{t-1}) - (\eta_{t-1} + v_{t-1})$$

$$+ \left[\frac{\sigma_\eta^2}{\sigma_\eta^2 + \sigma_v^2}\right](\eta_t + v_t)$$

Equation (18) shows that monetary growth partially accommodates the current temporary shocks to money demand and the expected real interest rate, $\eta_t + v_t$; that is, the coefficient is $\sigma_\eta^2/(\sigma_\eta^2 + \sigma_v^2)$. Since σ_η^2 is the variance of temporary shocks to money demand, and σ_v^2 is the variance of temporary shocks to the expected real interest rate, the result says that contemporaneous monetary accommodation is greater the larger the variance of money demand relative to that of the expected real interest rate. Interpreting σ_η^2 as the variance of the LM curve and σ_v^2 as the variance of the IS curve, the results are reminiscent of those found by Poole (1970). However, in the present model, the tradeoff is not between targeting nominal interest rates and targeting monetary aggregates. The targeting of the nominal interest rate is complete here independently of the values of σ_η^2 and σ_v^2 (that is, of the relative volatility of the LM and IS curves). In the present model, the tradeoff that determines the extent of current accommodation comes, in equation (17), from the negative response of p_t to the money-demand shock, η_t, and the positive response to the real-interest-rate shock, v_t. (The former reflects the negative effect on prices from an increase in money demand less the positive effect from the monetary response. The latter reflects only the monetary reaction.) The extent of monetary accommodation is determined to make the overall variance of p_t from these two sources of disturbances as small as possible.

With a one-period lag, monetary growth has an inverse, one-to-one reaction to the temporary shocks ($\eta_{t-1} + v_{t-1}$ in equation (18)). This response generates the reduction in expected inflation (see equation (17)) that allows the monetary authority to offset an incipient excess of R_t over $\bar{R}_t$. In particular, although the temporary shock, $\eta_t + v_t$, induces an increase in today's monetary growth, it also generates the promise of an even greater reduction in next period's monetary growth.

A permanent shock to money demand, $a_t = \alpha_t - \alpha_{t-1}$, affects m_t by the multiple δ, and therefore affects p_t by the multiple δ-1. As noted before, the value $\delta = 1$ would insulate the price level from this disturbance.

Finally, a shock to the interest-rate target, $u_t = \bar{R}_t - \bar{R}_{t-1}$, affects m_t by the factor, $1-\gamma$, and thereby affects p_t by the factor, $1+\beta-\gamma$. (The extra response β reflects the response of money demand to the change in the permanent part of monetary growth.) Again, the choice $\gamma = 1+\beta$ eliminates the response of p_t (but not p_{t+1}, ...) to this shock.

One of the prime sources of temporary shifts to money demand, η_t, would be temporary fluctuations in output. The results in equations (17) and (18) imply (for a given expected real interest rate) that these temporary, exogenous shifts in output would be contemporaneously negatively correlated with the price level and contemporaneously positively correlated with the money supply. (The same results obtain for permanent shifts in output, as reflected in a_t, if $0 < \delta < 1$). Thus the results are consistent with Fair's (1979) findings about the relation between shocks to output and prices for the United States in the post-World War II period. Also, the results accord with many analyses that report a positive correlation between money and output, although the relation in this model reflects only the endogenous response of the money supply (which has been stressed by King and Plosser, 1984). On the other hand, lagged output (that is, η_{t-1}) would be negatively correlated with current money (and prices). This result means that monetary growth would exhibit a countercyclical reaction to lagged output. This type of relation has been found for M1 growth in the post-World War II United States in Barro (1981b).

III. Implications of the Theory for Monetary-Base Growth and Inflation

Let $\Delta R_t = R_t - R_{t-1}$, $\Delta m_t = m_t - m_{t-1}$ (the growth rate of the monetary base), and $\Delta p_t = p_t - p_{t-1}$ (the inflation rate). Equation (16) implies that ΔR_t is white noise. (If $\bar{R}_t$ were not a random walk, but instead had a mean-reverting tendency in the long run, then the process for R_t would change accordingly.) Equations (17) and (18) prescribe the patterns for Δp_t and Δm_t that are consistent with this process for ΔR_t, given the underlying model in equations (1)-(6). These predictions about inflation and monetary-base growth are the principal empirical content of the theory.

Taking first differences of equation (18) leads to

$$(19)\ \Delta m_t = \Delta m_{t-1} + \delta a_t + (1-\gamma)u_t - w_t + \left[\frac{\sigma_\eta^2}{\sigma_\eta^2+\sigma_v^2}\right](\eta_t + v_t)$$

$$- \delta a_{t-1} + \gamma u_{t-1} - \left[\frac{2\sigma_\eta^2+\sigma_v^2}{\sigma_\eta^2+\sigma_v^2}\right](\eta_{t-1} + v_{t-1}) + (\eta_{t-2} + v_{t-2})$$

$$= \Delta m_{t-1} + E_t = \Delta m_{t-1} + e_t + a_1 e_{t-1} + a_2 e_{t-1}$$

where E_t is a composite error term and e_t is a white-noise disturbance. In other words, the model implies that Δm_t is an ARIMA (0,1,2) process. Furthermore, the theory imposes restrictions on the coefficients of this process. The unitary coefficient on Δm_{t-1} reflects the nonstationarity in monetary growth that is induced mainly by the nonstationarity of the nominal-interest-rate target (equation (4)). (Nonstationarity of the expected real interest rate in equation (3) also matters here.)

The two MA coefficients must be such as to satisfy the conditions,

$$(20)\ a_1(1 + a_2)\sigma_e^2 = \mathrm{COV}(E_t, E_{t-1}) = -\delta^2\sigma_a^2 + \gamma(1-\gamma)\sigma_u^2 - (2\sigma_\eta^2 + \sigma_v^2)^2/(\sigma_\eta^2 + \sigma_v^2)$$

$$(21)\quad a_2\sigma_e^2 = \mathrm{COV}(E_t, E_{t-2}) = \sigma_\eta^2 > 0$$

$$(22)\quad \sigma_e^2[1 + (a_1)^2 + (a_2)^2] = \mathrm{VAR}(E_t)$$

$$= (\text{terms involving } \sigma_a^2, \sigma_u^2, \sigma_w^2, \sigma_\eta^2, \sigma_v^2)$$

where σ_e^2 is the variance of e_t. If $\gamma(1-\gamma)\sigma_u^2 \leq \delta^2\sigma_a^2 + 4\sigma_\eta^2 + (\sigma_v^2)^2/(\sigma_\eta^2 + \sigma_v^2)$, then equations (20) and (21) imply $a_1 \leq 0$ and $a_2 \geq 0$. Moreover, the magnitude of a_1 is much greater than that of a_2—one inequality that holds is $|a_1| \geq 4a_2/(1+a_2)$. As σ_u^2 and σ_w^2 become small, the solution approaches stationarity for $\bar{R}_t$ and r_t, and hence for monetary growth and inflation. In particular, as σ_u^2 and σ_w^2 approach zero, the solution tends toward $a_1+a_2 = -1$.

The equation for the inflation rate comes from first differencing of equation (17). After substituting for Δm_{t-1} on the right side (using equation (18)) and simplifying, the results are

$$\begin{aligned}
\Delta p_t = \Delta p_{t-1} &- (1-\delta)a_t + (1+\beta-\gamma)u_t - w_t - \left[\frac{\sigma_v^2}{\sigma_\eta^2+\sigma_v^2}\right]\eta_t + \left[\frac{\sigma_\eta^2}{\sigma_\eta^2+\sigma_v^2}\right]v_t \\
&+ (1-\delta)a_{t-1} + (\gamma-\beta)u_{t-1} + \left[\frac{\sigma_v^2}{\sigma_\eta^2+\sigma_v^2}\right]\eta_{t-1} - \left[\frac{2\sigma_\eta^2+\sigma_v^2}{\sigma_\eta^2+\sigma_v^2}\right]v_{t-1} + v_{t-2} \qquad (23) \\
&= \Delta p_{t-1} + F_t = \Delta p_{t-1} + f_t + b_1 f_{t-1} + b_2 f_{t-2}
\end{aligned}$$

where F_t is a composite error term and f_t is a white-noise disturbance (which is not generally independent of e_t). As before, Δp_t is an ARIMA (0,1,2) process; the unitary coefficient on Δp_{t-1} again reflects mainly the nonstationarity of the nominal-interest-rate target. The two MA coefficients satisfy

$$(24)\quad b_1(1+b_2)\sigma_f^2 = \mathrm{COV}(F_t,\ F_{t-1}) = -(1-\delta)^2\sigma_a^2 - (1+\beta-\gamma)(\gamma-\beta)\sigma_u^2$$

$$- \left[\frac{\sigma_v^2}{\sigma_\eta^2+\sigma_v^2}\right](4\sigma_\eta^2 + \sigma_v^2)$$

$$(25)\qquad b_2\sigma_f^2 = \mathrm{COV}(F_t,\ F_{t-2}) = \sigma_\eta^2\sigma_v^2/(\sigma_\eta^2 + \sigma_v^2) > 0$$

$$(26)\qquad \begin{aligned} &\sigma_f^2[1 + (b_1)^2 + (b_2)^2] = \\ &\mathrm{VAR}(F_t) = (\text{terms involving } \sigma_a^2,\ \sigma_u^2,\ \sigma_w^2,\ \sigma_\eta^2,\ \sigma_v^2) \end{aligned}$$

Unless the term, $(1+\beta-\gamma)(\gamma-\beta)\sigma_u^2$, is negative and very large in magnitude, the coefficients satisfy $b_1 \leq 0$ and $b_2 \geq 0$. The magnitude of b_1 tends again to be much greater than that of b_2—in particular (under the same condition on the σ_u^2 term), $|b_1| \geq 4b_2/(1+b_2)$. Again, $\sigma_u^2 = \sigma_w^2 = 0$ implies $b_1+b_2 = -1$.

IV. Seasonals

So far, the model contains no systematic seasonals, but these are known to be important for money in the post-World War II period, and for nominal interest rates before the founding of the Federal Reserve (see, for example, Kemmerer, 1910, Ch. 2; Macaulay, 1938, Chart 20; Shiller, 1980, pp. 136-137; Clark, 1986; Miron, 1986; and Mankiw, Miron and Weil, 1986). I consider briefly here the implications of systematic seasonals in money demand and in the real interest rate. For simplicity, I now neglect the various stochastic terms considered before. Given the linearity of the model, the new effects would be additive to those from the stochastic terms.

The model with deterministic seasonals and no stochastic shocks is

$$(27) \quad R_t = r + p_{t+1} - p_t + T_t$$

$$(28) \quad m_t - p_t = \alpha - \beta R_t + S_t$$

$$(29) \quad m_t - m_{t-1} = \mu + \Sigma_t$$

where T_t, S_t, and Σ_t are seasonal factors, and $E_t p_{t+1} = p_{t+1}$ applies in this deterministic model. Suppose, as has been argued is true of the Federal Reserve, that the monetary authority sets Σ_t to offset the effects of T_t and S_t on the nominal interest rate. Then, with $R_t = \bar{R}$, equation (27) implies

$$p_{t+1} - p_t = \bar{R} - r - T_t$$

Using $R_t = \bar{R}$, equation (28) implies

$$m_t - m_{t-1} = p_t - p_{t-1} + S_t - S_{t-1}$$

Substituting into this last relation for $p_t - p_{t-1}$ from above (with a one-period lag) and for $m_t - m_{t-1}$ from equation (29) yields (after setting $\mu = \bar{R} - r$)

$$\Sigma_t = -T_{t-1} + S_t - S_{t-1}$$

This seasonal pattern for monetary growth eliminates the seasonal in the nominal interest rate—that is, achieves $R_t = \bar{R}$.

The implied relations for monetary growth and inflation are

$$(30) \quad \Delta m_t = \mu - T_{t-1} + S_t - S_{t-1}$$

$$(31) \quad \Delta p_t = \mu - T_{t-1}$$

Note that, if the seasonal applied to money demand (S_t), but not to the real interest rate (T_t), then the seasonal in monetary growth would eliminate the seasonal in inflation along with that in the nominal interest rate. But, if there is a seasonal in the real interest rate, then a seasonal in inflation remains.

Since the seasonals in money demand and the real interest rate were assumed to be deterministic and understood by the monetary authority, the seasonal in the nominal interest rate could be eliminated by introducing a deterministic seasonal into monetary growth. In practice, the seasonals in money demand and the real interest rate could evolve stochastically, and also be unknown to the monetary authority. But, even in this case, the policymaker could remove the seasonal in the nominal interest rate by pursuing the type of feedback reaction to the nominal interest rate that was considered above. Hence, if the elimination of seasonals in nominal interest rates is deemed to be desirable (on public-finance grounds?), then the possibility of removing them in this way stengthens the case for interest-rate targeting.

V. Empirical Findings

The main empirical results involve seasonally unadjusted data since 1890 on nominal interest rates (4- to 6-month prime commercial paper[7]), the monetary base (unadjusted for changes in reserve requirements), the consumer price index (CPI-U, available since 1913, except that the index without the shelter component was used since 1970), and the producer price index (PPI, all commodities). All variables are monthly but observed at the quarterly intervals of January, April, July, and October. The identification of the period in the theory with quarters is, of course, somewhat arbitrary. (In the theory the period relates especially to the flow of information about prices and to the reaction of money to incipient movements in the nominal interest rate away from its target.)

The underlying data are averages of daily figures for interest rates and the monetary base (except that before August 1917 the available figures on the monetary base are at the end of each month). The price indices are some kind of average of observations during each month, although for the CPI some of the components are sampled only quarterly. The 3-month spacing between each monthly observation should minimize some of the problems related to time-averaged data.

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[7]Results for the nominal interest rate are similar with the 3-month time loan rate used by Mankiw, Miron and Weil (1986).

Figures 1-4 depict the four time series under study.[8] All variables are measured at annual rates. The nominal interest rate, shown from 1860 to 1986 in Figure 1, has much more high frequency movement before 1914 (when the Fed was established) than afterwards. As noted by Macaulay (1938, Chart 20), Shiller (1980, pp. 136-137), Miron (1986), Clark (1986), and Mankiw, Miron, and Weil (1986), among others, this pattern turns out to reflect seasonals and other temporary movements in the nominal interest rate that were much more important during the earlier period than later on. In addition, the nominal interest rate appears to be stationary in the earlier sample and non-stationary in the later one. Note also that the graph shows the extremely low nominal interest rates from the mid 1930s to the early 1950s, which includes the period of explicitly pegged Treasury Bill rates from April 1942 to mid 1947 (and with a moving peg from then to the Fed-Treasury Accord of March 1951 and its confirmation by the Fed in March 1953[9]). However, it is unclear from the graph whether this pegging involves a special policy

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[8]The nominal interest rate applies to 4- to 6-month commercial paper (6-month paper in recent years), as reported since 1890 in U.S. Board of Governors of the Federal Reserve System, *Banking and Monetary Statistics*, *Banking and Monetary Statistics, 1941-1970*, *Annual Statistical Digest, 1970-1979* and later issues, and the *Federal Reserve Bulletin*. Earlier data, from Macaulay (1938, Appendix Table 10), refer to 60-90 day commercial paper. (These were adjusted upward by .014 to merge with the other series in 1890.) The monetary base since 1914 comes from the Federal Reserve sources noted above. Earlier data come from the National Bureau of Economic Research. The CPI since 1913 is from the Bureau of Labor Statistics (CPI-U, with the CPI less shelter used since 1970 to avoid problems with mortgage interest costs). The PPI (all commodities) since 1913 comes from the Bureau of Labor Statistics. Data from 1890 to 1912 are from U.S. Department of Labor, 1928 (kindly provided by Jeff Miron). Data before 1890 are from Warren and Pearson, 1933, Table 1.

[9]For a discussion of the Accord, see Friedman and Schwartz, 1963, pp. 623ff.

or was instead just the consequence of the nominal interest rate coming close to its lower bound of zero. Probably it is no accident that the period of precise pegging coincides with the time of lowest nominal interest rates.

Figure 2 shows the growth rate of the monetary base since 1878. For the entire sample period, the plot indicates a great deal of high frequency variation, which turns out to reflect seasonals and other temporary fluctuations. Unlike for the nominal interest rate, it is less clear visually what sort of break in the monetary process might have occurred around 1914.

Figure 3 shows the growth rate of the consumer price index since 1913, and Figure 4 the growth rate of the producer price index since 1860. The decreasing volatility of each series over time probably reflects, at least in part, the increasing coverage of goods.

Table 1 contains regression results for the recent period, 1954.1 to 1986.4. Starting in 1954 avoids the extremely low nominal interest rates through the early 1950s, for which the lower bound of zero would be significant (so that nominal interest rates could not be approximated as a random walk). Also, this sample excludes the effects on the price indices from the controls during World War II and the Korean War.

The basic format of the empirical results consists of estimated equations for an ARMA representation with systematic seasonals,

$$
\begin{aligned}
Y_t &= c_1 S_{1t} + c_2 S_{2t} + c_3 S_{3t} + c_4 S_{4t} + c_5 Y_{t-1} \\
&\quad + e_t + c_6 e_{t-1} + c_7 e_{t-2} + c_8 e_{t-3} + \cdots
\end{aligned}
\tag{32}
$$

where e_t is a white-noise error and Y_t represents R_t, ΔM_t, ΔP_t, or $\Delta(PPI)_t$. (R is the commercial paper rate, ΔM is the growth rate of the monetary base, ΔP is the growth rate of the CPI, and $\Delta(PPI)$ is the growth rate of the producer price index.[10]) The variable S_{1t} is a seasonal dummy for the first quarter (1 for January, 0 otherwise), and similarly for S_{2t} (for April), S_{3t} (for July), and S_{4t} (for October). For R_t as the dependent variable, the hypothesis under a regime of interest-rate smoothing is $c_1 = c_2 = c_3 = c_4 = 0$ (or possibly a constant), $c_5 = 1$, $c_6 = c_7 = c_8 = \ldots = 0$. For ΔM_t, ΔP_t, and $\Delta(PPI)_t$, the model under interest-rate smoothing suggests nonzero values for c_1, c_2, c_3, and c_4, $c_5 = 1$, $c_6 \leq 0$, $c_7 \geq 0$ (with $|c_6|$ much greater than c_7 and $c_6 + c_7 \geq -1$), and $c_8 = \ldots = 0$. (However, some of these restrictions depend somewhat on identifying the period in the theory with quarters in the data.)

Aside from the estimated coefficients and (asymptotic) standard errors, the table reports the following statistics:

Q(10): Q-statistic for serial correlation of residuals with 10 lags, with degrees of freedom and asymptotic significance level (based on the χ^2 distribution) shown in parentheses,

Seasonals: likelihood-ratio statistic (equal to $-2\bullet\log$ of likelihood ratio) for the equation with seasonals against the null hypothesis of the same equation except for no seasonality ($c_1 = c_2 = c_3 = c_4$), with degrees of freedom (3) and asymptotic significance level (based on the χ^2 distribution) shown in parentheses.

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[10] Schwert (1987b, Table 9) shows that an ARIMA (0,1,4) process works well on seasonally adjusted monthly data for the growth rate of the monetary base, CPI inflation, and PPI inflation.

The random-walk model, $R_t = R_{t-1}$ + constant (where the constant could be set to zero here), is satisfactory for the nominal interest rate in the post-1954 period. Notably, Q(10) from line 1 of Table 1 has a significance level of .18, while the likelihood-ratio statistic for seasonals has a significance level of .67.[11] The unrestricted estimate of R_{t-1} is .934, s.e. = .030. The implied "t-value" relative to unity is 2.2, which is below the .10 critical value of 2.6 from the Dickey-Fuller test (Fuller, 1976, Table 8.5.2, the section for $\hat{\tau}_{\mu}$). Given the random-walk-like behavior of the nominal interest rate, the theory's other predictions should apply to monetary-base growth and inflation.

The estimated equation for the growth rate of the monetary base, shown in line 4 of the table, exhibits strong seasonality, with a likelihood-ratio statistic of 105. The estimated MA(1) coefficient is highly significant, -.80, s.e. = .09, and conforms in sign and rough magnitude with the model's predictions. The estimated MA(2) coefficient, -.05, s.e. = .09, differs insignficantly from zero, but also differs insignficantly from the small positive values suggested by the theory. The ARIMA (0,1,2) specification appears satisfactory according to the Q-statistic, which has a significance level of .29. Moreover, the unrestricted estimate of DM_{t-1}, shown in line 5,

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[11]Weak evidence of seasonality in the nominal interest rate appears in some sub-samples of the post-1954 period—for example, for 1954.1-1959.4 and the 1970s. However, the seasonals look very different for these two periods. The seasonal found for the 1954-1959 period seems to be consistent with the results of Diller (1969, Ch. 3).

is .96, s.e. = .12, which differs insignificantly from one according to the Dickey-Fuller test.[12]

For the CPI inflation rate, shown on line 7, the ARIMA (0,1,2) model has significant seasonals, but at levels much smaller than those for the growth rate of the monetary base. The pattern of the two MA coefficients (-.68, s.e. = .09, and .06, s.e. = .09) also accords with the theory. However, the Q-statistic with 10 lags has a significance level of .04. This problem disappears with the addition of an MA(3) term (line 8 of the table), which has an estimated coefficient of .36, s.e. = .10. If the coefficient of Δp_{t-1} is unrestricted, as in line 9 of the table, its estimated coefficient is .89, s.e. = .07, which differs insignificantly from one at the .10 level according to the Dickey-Fuller test.

Overall, the CPI inflation rate shows somewhat greater "persistence" than predicted by the theory. This outcome may be explicable from a model that includes gradual adjustment of prices or in money demand. However, to fit the data, the model would have to generate extra persistence in prices without simultaneously generating this persistence in the monetary base or the nominal interest rate. It is also possible that the results can be explained on purely mechanical grounds, which include the infrequent sampling of some components of the CPI and the departure between reported and transactions prices.

The underlying theory regarded the nominal interest rate as controllable by the monetary authority, but treated the expected real interest rate as

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[12] Schwert's (1987a, Table 3) Monte Carlo results indicate that the Dickey-Fuller test for a unit root works well if the underlying process is ARIMA (0,1,1). Therefore this test should be appropriate in the present context.

exogenous with respect to monetary variables. Hence monetary policy affected the nominal interest rate only by influencing the expected rate of inflation. Many economists are skeptical about this model because they think of nominal interest rates as highly flexible and of actual and expected inflation rates as sticky in the short run. The results in Table 1 conflict with this view in that the residual standard deviation for quarterly CPI inflation—2.3% per year on line 8—is about double that of the nominal interest rate—1.2% per year on line 1. Although the significance of the estimated MA(3) coefficient for CPI inflation (line 8) suggests some extra persistence, the results also indicate a substantial amount of variability in quarter-to-quarter inflation rates. Given that the standard deviation of inflation is twice that of the nominal interest rate, it is plausible that the monetary influences on the nominal interest rate could operate mainly through effects on expected inflation.

The producer price inflation rate, considered in lines 11-13 of the table, also shows extra persistence. In this case (on line 13) the estimated MA(3) and MA(4) coefficients are each positive and significant (.25, s.e. = .10, and .26, s.e. = .11, respectively). The unrestricted estimate of the Dp_{t-1} coefficient is .76, s.e. = .10. The implied "t-value" of 2.4 is just below the .10 critical value of 2.6 from the Dickey-Fuller test. A possible interpretation of these results is that the producer price index amounts to a bad proxy for the price level that matters in the theory (in the determination of money demand and for the calculation of the expected real interest rate). Then the extra persistence may reflect the characteristics of this measurement error. This viewpoint may also apply, but probably with lesser weight, to the CPI.

Table 2 shows comparable results for the interwar period, 1922.1-1940.4. There is now some indication of predictable movements in the nominal interest rate. For example, in line 2 of the table, the estimated MA(1) coefficient is .24, s.e. = .12, and the likelihood-ratio statistic for the seasonals has a significance level of .04. However, the seasonal coefficients are small in magnitude. The unrestricted estimate of R_{t-1} on line 3—.949, s.e. = .029—again differs insignificantly from one. Overall, these results for the interest rate turn out to be a middle ground between those shown in Table 1 for the post-1954 period and those examined below for the pre-1914 period, which reveal substantial predictable movements in the nominal interest rate.

The growth rate of the monetary base, considered in line 5 of Table 2, again exhibits pronounced seasonality, although the pattern differs from that for the post-1954 period. The ARIMA (0,1,2) process appears satisfactory according to the Q-statistic, which has a significance level of .25. Also, the unrestricted estimate of the Δm_{t-1} coefficient (line 6) is .95, s.e. = .25, which differs insignificantly from one. However, the estimated MA(2) coefficient (line 5) of -.28, s.e. = .12, differs significantly from the hypothesized value, which is small and positive. One possible story is that this "error" in the structure of the monetary process for the interwar period explains why some short-term predictability remained in the nominal interest rate.

The ARIMA (0,1,2) processes appear satisfactory for the CPI and PPI inflation rates (lines 8 and 11 of Table 2). The estimated coefficients for ΔP_{t-1} (lines 9 and 12) again differ insignificantly from one. However, the estimated MA(2) coefficient for PPI inflation is significantly negative (line 11).

Table 3 shows results for the period 1890.3-1913.4, which applies to the gold standard and precedes the founding of the Federal Reserve. For this period the nominal interest rate appears to be stationary and a coefficient of zero for R_{t-1} is satisfactory (lines 2 and 3 of the table). However, the estimated coefficient of R_{t-1} on line 3—.19, s.e. = .41—also differs insignificantly from one according to the .10 critical value of the Dickey-Fuller test. There is now substantial short-run predictability of movements in the nominal interest rate; in line 2 the likelihood-ratio statistic for seasonality has a significance level of .001. In addition, the first three MA coefficients are positive and significant (.49, s.e. = .11; .21, s.e. = .11; and .25, s.e. = .11).

Given the absence of interest-rate smoothing, the behavior of the monetary base and the price level before 1914 should differ from that found in the later periods. The results suggest that the growth rate of the monetary base before 1914 (which coincides in this period with currency in circulation) is stationary, and a coefficient of zero for Δm_{t-1} is satisfactory (lines 5 and 6 of Table 3). (The estimated coefficient of Δm_{t-1} is -.23, s.e. = .36, which differs significantly from 1 at about the .01 level according to the Dickey-Fuller test.) There are significant seasonals in monetary-base growth, as shown on line 5 by the significance level of .000 for the likelihood-ratio statistic.[13] However, this seasonal in the monetary

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[13] I have made no adjustment here for the fact that the end-of-month data before August 1917 apply to different days of the week. The finding of significant seasonals in monetary-base growth before 1914 accords in a general way with Kemmerer (1910, Ch. 6), but seems to conflict with results reported by Clark (1986, pp. 106ff.).

base did not eliminate the seasonal in the nominal interest rate. In fact, since the United States was on the gold standard, the behavior of the monetary base (and the U.S. price level) would have been largely constrained to be consistent with the world price level, including its seasonal pattern if it had one. Therefore, it would not generally be possible under this type of monetary system to choose a seasonal in the monetary base that removed the seasonal in the nominal interest rate.

Aside from the seasonals, the results for the growth rate of the monetary base on line 5 indicate a positive MA(1) coefficient, .28, s.e. = .11. The simple specification that monetary-base growth is an MA(1) with seasonals appears satisfactory according to the Q-statistic.

Viewed jointly, the results for the nominal interest rate and the monetary base in Tables 1-3 are consistent with the viewpoint (expressed recently by Mankiw, Miron, and Weil, 1986) that shifts in monetary policy after the founding of the Federal Reserve in 1914 were responsible for the elimination of predictable temporary movements, including seasonals, in the nominal interest rate. The present analysis identifies these shifts in monetary policy with specific changes in the process for monetary-base growth. Namely, the growth rate became non-stationary, a substantially negative MA(1) coefficient appeared, and the seasonal patterns changed. Moreover, the results for the interwar period suggest that the Federal Reserve did not get the monetary process right immediately. Only in the post-1954 period does all the short-term predictability of nominal interest rate movements seem to disappear. On the other hand, the results are also consistent with the idea that the elimination of a serious gold standard—also occurring in 1914—was responsible for the changed behavior of nominal interest rates. The elimination of the gold standard may have been a prerequisite for the implementation of a monetary policy that successfully targeted nominal interest rates.[14]

Results for the PPI inflation rate from 1890 to 1913 appear on lines 7 and 8 of Table 3. This inflation rate exhibits significant seasonality and appears to be stationary (the estimated coefficient of Δp_{t-1} on line 8 of Table 3 is .24, s.e. = .10, which is significanlty below 1). The estimated MA coefficients are insignificant, except for a negative MA(4) (-.38, s.e. = .11), which might reflect stochastic variation in seasonals. The CPI is unavailable for this period, except for rough estimates on an annual basis.

Concluding Observations

Theoretical reasoning suggests that interest-rate targeting is a reasonable guide for monetary policy. In a model where expected real interest rates and output are exogenous with respect to monetary variables, the central bank influences nominal interest rates by altering expected rates of inflation. It turns out that the monetary authority can come arbitrarily close to meeting its (time-varying) target for the nominal interest rate, even while holding down the forecast variance of the price level. The latter

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[14] Clark (1986, pp. 85ff.) points out that the main seasonal in nominal interest rates ended at about the same time—around 1914—in a number of industrialized countries. This outcome accords with the idea that the ending of the gold standard freed up all the central banks simultaneously. However, it would be worthwhile to examine the changes in the monetary processes for the various countries. Also, it is worth considering whether interest-rate targeting by more than one country is feasible under fixed exchange rates (even in the absence of a serious gold standard).

objective pins down the extent of accommodation of the money supply to temporary shifts in the demand for money. The greater the variance of shocks to money demand (i.e., of the LM curve) relative to that of the expected real interest rate (i.e., the IS curve), the greater the degree of accommodation.

Incipient increases in the nominal interest rate (caused by temporary shocks to money demand or the expected real interest rate) lead in the usual way to monetary expansion—e.g., to open-market purchases of bonds. This response lowers expected inflation because the influx of money is temporary. That is, the central bank plans to take back later some of today's infusion of money, and people's expectations of this behavior lowers anticipated growth rates of money and prices. Therefore, the nominal interest rate falls back toward its target value even though the expected real interest rate does not change.

If the target nominal interest rate moves as a random walk, the successful targeting by the central bank implies that the nominal interest rate also follows this process. Given this policy of interest-rate targeting—and the assumed specification for money demand and the expected real interest rate—the growth rate of the monetary base and the price level must follow ARIMA (0,1,2) processes. The unit roots in these processes reflect mainly the non-stationarity of the nominal interest rate. The moving-average terms correspond to the responses to temporary shocks—in particular, the tendency for infusions of money (in response to incipient rises in the nominal interest rate) to be followed by removal of money in the future.

Empirical evidence for the United States since 1890 accords in the main with the theoretical propositions. In particular, the results indicate that shifts in monetary policy after the founding of the Fed in 1914 led to the elimination of predictable temporary movements, including seasonals, in the nominal interest rate (on short-term commercial paper). The results identify the changes in monetary policy with specific changes in the process for monetary-base growth. Namely, the growth rate became non-stationary, a substantially negative moving-average term appeared (indicating the tendency for reversals in monetary growth), and the seasonal patterns changed. The results suggest that it was not until the post-1954 period that the Fed fully smoothed the nominal interest rate in the sense of achieving nearly random-walk like behavior in this rate.

One interesting topic for future research involves applying the model to other countries. At a theoretical level this extension raises questions about the interplay between the exchange-rate regime and the possibilities for independent interest-rate targeting by individual central banks. One specific issue is why the elimination of predictable short-run movements in nominal interest rates appeared to occur simultaneously around World War I in several industrialized countries (see Clark, 1986, and Mankiw, Miron and Weil, 1986). The founding of the Fed and the elimination of the classical gold standard are possible explanations that are worth exploring.

The empirical work for the United States (or other countries) can be usefully extended to consider in more detail the joint determination of the nominal interest rate, monetary base, and the price level. Such a joint treatment would allow testing of the model's prediction concerning the relative variances of these variables and the covariances among them. In addition, it would be possible to estimate coefficients such as the interest-sensitivity of money demand and the reaction of money supply to some disturbances.

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Table 1
Regression Results for 1954.1-1986.4

Dep. Var, Y_t		C	S1	S2	S3	S4	Y_{t-1}	MA(1)	MA(2)	MA(3)	MA(4)	$\hat{\sigma}$	Q(10) (degrees of freedom, significance level)	Seasonals (degrees of freedom, significance level)
R	(1)	.000					1					.0116	13.9	1.6
		(.001)											(10, .18)	(3, .67)
	(2)	.004					.934					.0115	12.2	1.5
		(.002)					(.030)						(9, .20)	(3, .68)
Δm	(3)	.000										.0849	601	190
		(.007)											(10, .000)	(3, .000)
	(4)		.044	-.081	.099	-.059	1	-.799	-.054			.0326	9.7	105
			(.006)	(.006)	(.006)	(.006)		(.089)	(.089)				(8, .29)	(3, .000)
	(5)		.045	-.079	.099	-.056	.963	-.787	-.001			.0325	9.5	127
			(.006)	(.010)	(.006)	(.012)	(.115)	(.148)	(.092)				(7, .22)	(3, .000)
Δp	(6)	.000					1					.0300	39	18.0
		(.003)											(10, .000)	(3, .001)
	(7)		-.006	.015	.005	-.013	1	-.676	.063			.0237	16.6	23
			(.004)	(.004)	(.004)	(.004)		(.089)	(.090)				(8, .04)	(3, .000)
	(8)		-.007	.015	.005	-.013	1	-.721	.077	.355			7.2	15.2
			(.004)	(.004)	(.004)	(.004)		(.090)	(.090)	(.097)		.0228	(7, .42)	(3, .003)
	(9)		-.003	.019	.010	-.008	.886	-.657	.087	.361			4.8	14.0
			(.005)	(.004)	(.005)	(.005)	(.070)	(.114)	(.092)	(.101)		.0223	(6, .57)	(3, .005)
Δ(PPI)	(10)	.000					1					.0453	46	12.0
		(.004)											(10, .000)	(3, .009)
	(11)		.020	-.008	.001	-.014	1	-.484	-.129			.0396	20.1	8.0
			(.007)	(.007)	(.007)	(.007)		(.089)	(.090)				(8, .01)	(3, .05)
	(12)		.020	-.008	.002	-.014	1	-.493	-.106	.197	.141	.0391	13.8	8.4
			(.007)	(.007)	(.007)	(.007)		(.090)	(.091)	(.096)	(.097)		(6, .04)	(3, .04)
	(13)		.026	.002	.011	-.004	.760	-.325	-.048	.250	.262	.0375	6.8	7.9
			(.007)	(.008)	(.008)	(.008)	(.098)	(.134)	(.100)	(.102)	(.109)		(5, .24)	(3, .05)

Table 2
Regression Results for 1922.1-1940.4

Dep. Var. Y_t		C	S1	S2	S3	S4	Y_{t-1}	MA(1)	MA(2)	$\hat{\sigma}$	Q(10) (degrees of freedom, significance level)	Seasonals (degrees of freedom, significance level)
R	(1)	-.0007					1			.0047	16.6	6.0
		(.0005)									(10, .09)	(3, .12)
	(2)		-.0014	.0001	-.0024	.0009	1	.235		.0045	10.7	8.7
			(.0010)	(.0010)	(.0010)	(.0010)		(.119)			(9, .30)	(3, .04)
	(3)		.0001	.0016	-.0010	.0022	.949	.259		.0045	9.3	8.7
			(.0014)	(.0013)	(.0013)	(.0013)	(.029)	(.123)			(8, .32)	(3, .04)
Δm	(4)	.003					1			.1346	21.3	19.4
		(.015)									(10, .02)	(3, .000)
	(5)		-.090	-.011	.075	.039	1	-.659	-.276	.0957	10.4	16.4
			(.022)	(.022)	(.022)	(.022)		(.122)	(.122)		(8, .25)	(3, .001)
	(6)		-.086	-.010	.076	.045	.946	-.740	-.125	.0959	10.1	23
			(.038)	(.023)	(.022)	(.032)	(.250)	(.282)	(.132)		(7, .19)	(3, .000)
Δp	(7)	.000					1			.0669	18.8	12.1
		(.008)									(10, .04)	(3, .009)
	(8)		-.036	.007	.034	-.001	1	-.640	.136	.0536	3.5	12.8
			(.012)	(.012)	(.012)	(.012)		(.120)	(.120)		(8, .90)	(3, .007)
	(9)		-.035	-.002	.027	.001	.745	-.509	.188	.0506	2.3	15.8
			(.012)	(.014)	(.013)	(.012)	(.202)	(.233)	(.129)		(7, .94)	(3, .002)
Δ(PPI)	(10)	.000					1			.1424	16.6	4.8
		(.016)									(10, .09)	(3, .19)
	(11)		-.057	.003	.038	.015	1	-.573	-.316	.1198	5.6	3.0
			(.027)	(.027)	(.027)	(.027)		(.120)	(.120)		(8, .69)	(3, .40)
	(12)		-.054	-.002	-.033	.016	.841	-.503	-.203	.1172	4.9	3.8
			(.032)	(.036)	(.035)	(.027)	(.709)	(.720)	(.269)		(7, .67)	(3, .29)

Table 3
Regression Results for 1890.3-1913.4

Dep. Var. Y_t		C	S1	S2	S3	S4	Y_{t-1}	MA(1)	MA(2)	MA(3)	MA(4)	$\hat{\sigma}$	Q(10) (degrees of freedom, significance level)	Seasonals (degrees of freedom, significance level)
R	(1)	.000					1					.0110	24.5	17.6
		(.001)											(10, .008)	(3, .001)
	(2)		.057	.055	.055	.062	0	.492	.211	.252		.0088	11.8	17.5
			(.002)	(.002)	(.002)	(.002)		(.107)	(.107)	(.108)			(7, .11)	(3, .001)
	(3)		.045	.045	.045	.052	.186	.318	.113	.210		.0088	11.3	17.0
			(.025)	(.023)	(.023)	(.023)	(.408)	(.422)	(.234)	(.138)			(6, .08)	(3, .001)
Δm	(4)	.001					1					.1069	41	48
		(.011)											(10, .000)	(3, .000)
	(5)		.048	.011	-.009	.104	0	.280				.0666	7.2	38
			(.014)	(.014)	(.014)	(.014)		(.106)					(9, .62)	(3, .000)
	(6)		.072	.022	-.007	.101	-.227*	.516				.0666	6.6	37
			(.039)	(.022)	(.014)	(.014)	(.357)	(.373)					(8, .58)	(3, .000)
Δ(PPI)	(7)	.000										.1608	26	24
		(.017)											(10, .006)	(3, .000)
	(8)		-.036	.002	-.024	.088	.238*				-.381	.1058	6.9	27
			(.024)	(.022)	(.022)	(.022)	(.098)				(.107)		(8, .54)	(3, .000)

*Estimated coefficient is significantly below 1.0 at less than .05 level according to Dickey-Fuller test (Fuller, 1976, Table 8.5.2, section for $\hat{\tau}_{\mu}$).

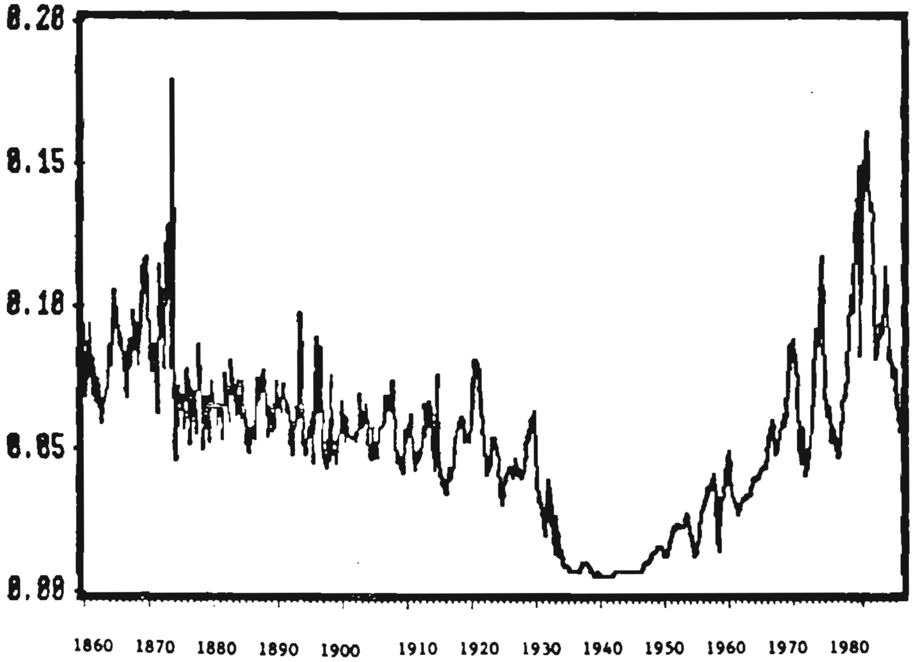

Figure 1

Commercial Paper Rate, 1860.1-1986.4

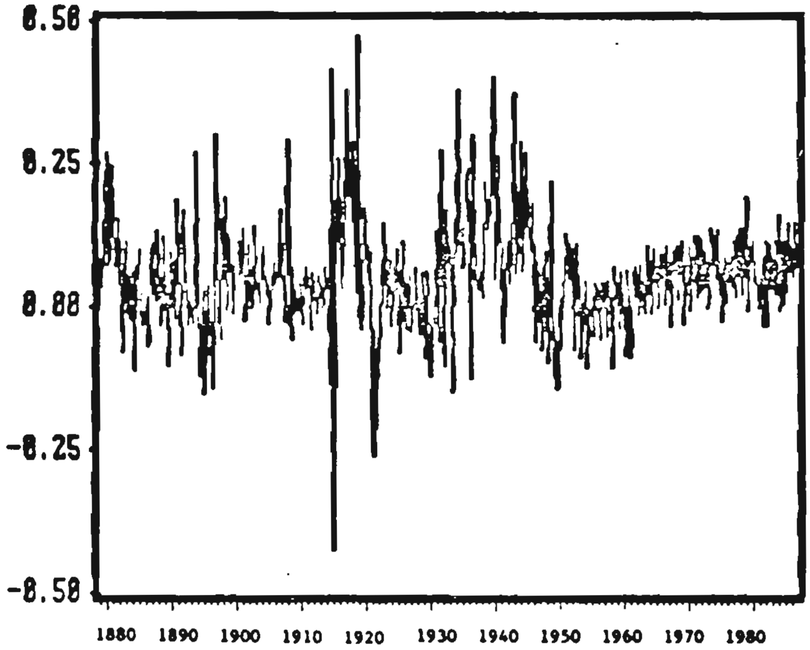

Figure 2

Growth Rate of Monetary Base, 1878.4-1986.4

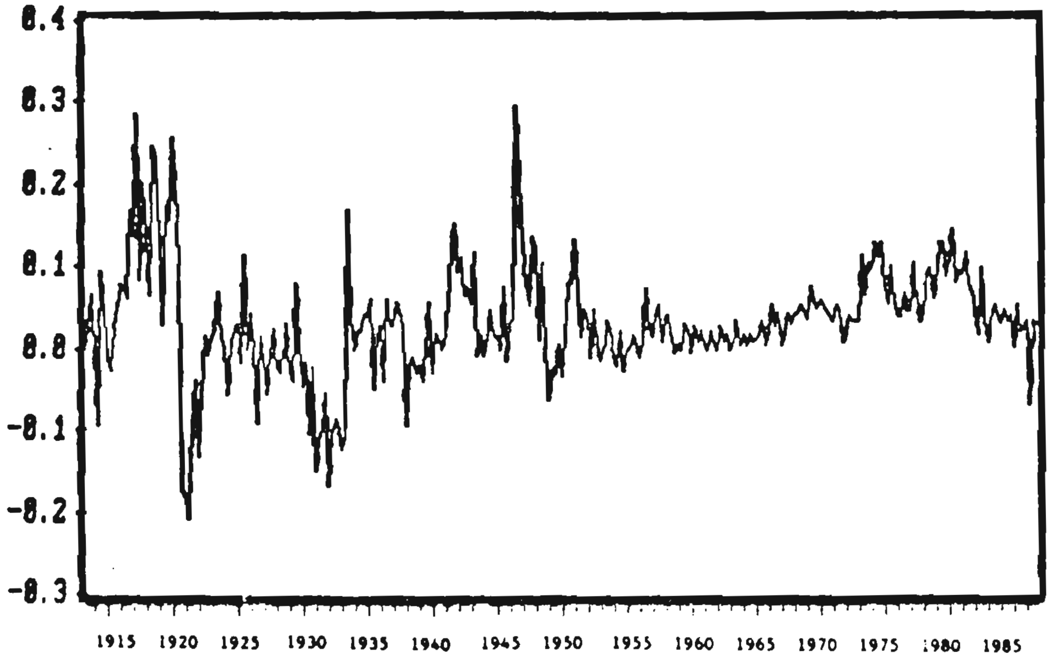

Figure 3

CPI Inflation Rate, 1913.2-1986.4

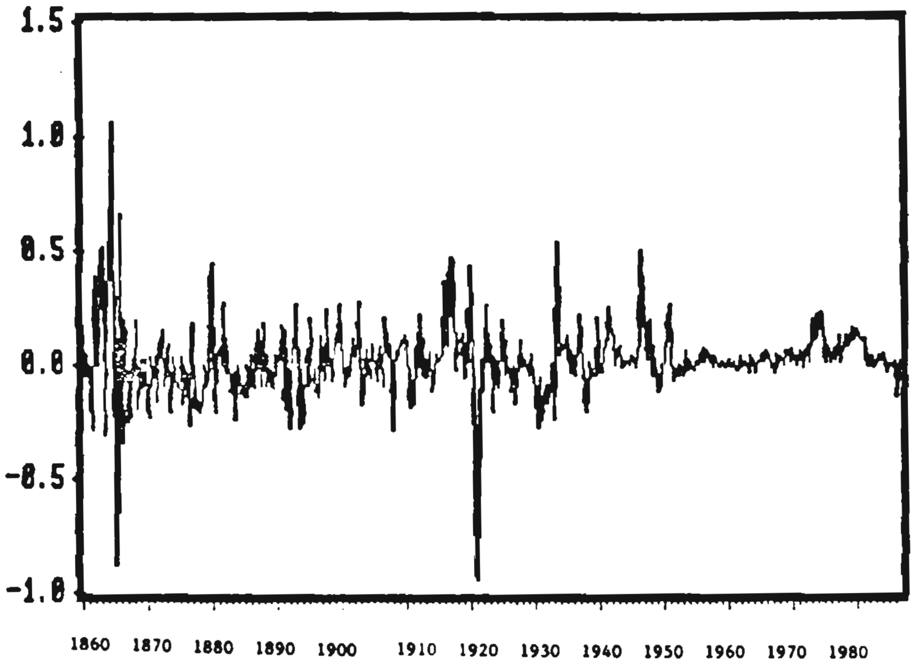

Figure 4

Producer Price Inflation Rate, 1860.1-1986.4

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