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EQUILIBRIUM IN COOPERATIVE GAMES OF POLICY FORMULATION

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Abstract

A major theme of the literature on policy games has been to examine when non-cooperative play of such games can result in "optimal", or "cooperative", outcomes. The literature does not examine what explicitly cooperative play of these games would involve. We propose a formulation of cooperative play based on the "dynamic coalitions" notion of Boyd and Prescott (1986a,1986b). We show that cooperative equilibria of certain kinds of policy games exist and are unique. However, the equilibrium outcomes of cooperative policy games are not the "optimal" outcomes, but rather, the time consistent ones.

A version of the cooperative framework is shown to produce some sharp implications about optimal tax problems: when agents and the government have the same preferences, non-lump-sum taxes are not distorting. Some empirical implications of this result are discussed.

1. Introduction

Applications of game theory to problems of macroeconomic policy have produced many useful insights about how economies can end up in equilibria that are inferior from both a social point of view and from the point of view of all of the actors involved. These models, many of them adapted from the industrial organization literature, have been used to illustrate the emergence of equilibria with high inflation, with inefficient capital taxation, with high unemployment and so on. In some sense the models are too successful; the possibilities of equilibria being such that they are capable of explaining virtually any outcome. Motivating the choice of a particular equilibrium is often a matter of artful storytelling and persuasion. Further, the literature from which many of these models are adapted, duopoly theory, is one where competition is the relevant paradigm, a model that does not fit easily with some notions of the relationship between a democratic government and the public it represents. Our objective in this paper is to suggest an alternative approach to modelling macroeconomic policy, one that embodies several features we view as more representative of the policy process and that produces unique outcomes in a framework of cooperation.

In the standard game theoretic view of macroeconomic policy the actions of the government are viewed as the outcome of a dynamic game in which the government and the public are both behaving strategically. Kydland and Prescott (1977) were among the first to point out that government policies derived from techniques such as dynamic programming can imply future values of optimal policies that will not be thought optimal when the future becomes the present. The government will eventually be inclined to deviate from the policy rule and the private agents in the economy, aware of the problem, will expect them to do so.

More recently, this literature has recognized that the setting depicted by the Kydland-Prescott analysis is too limiting because it neglects the fact that repeated interactions between the government and private agents create an environment where governments can increase their credibility by establishing a reputation for sticking to the optimal plans of earlier periods. In dynamic game models with this feature it is possible to support equilibria that are better than that depicted above. Indeed, it is sometimes possible to support equilibria that achieve a socially optimal outcome. To support such equilibria, however, it is necessary to make some assumption about how players respond to the behavior of their opponents off the equilibrium path. These models assign a central role to the perceived significance of unanticipated moves because the equilibria are supported by suppositions about what the beliefs or probability assessments of the opponents will be in the wake of such moves. Consequently, games with this feature produce a continuum of reputational equilibria.

A major thrust in the literature on dynamic policy games has been to analyze when reputational considerations could lead to optimal outcomes and to characterise such outcomes as "cooperative". Barro and Gordon (1983a) consider what punishment strategy is likely to induce the government to stick to low inflation policies. Cooley and Feldman (1986) argue that, in realistically structured games, very mild responses to disequilibrium behavior are likely to lead to optimal ("cooperative") outcomes. While these arguments are suggestive, there are no compelling grounds for assuming that an economy will end up in a nice equilibrium. Moreover, there is little discussion in this context of what cooperation means when current players cannot commit their "future selves" to any particular course of action.

In the following sections we describe a game of policy formulation that is based on an explicit, and we think natural, notion of cooperation in a dynamic setting. The cooperative concept we employ is closely related to the dynamic coalition models that have been successfully applied by Boyd and Prescott (1986a,b) to the study of financial intermediation and firm growth. In the absence of precommitment, policy makers at different dates are de facto separate players in a policy game. We incorporate this observation into a cooperative context by treating policy makers at each date as separate players in a cooperative game.[1] A government is then defined as a coalition of policy makers at different dates. These policy makers can either play a cooperative game with private agents (sections 2 and 3) or play non-cooperatively against them (section 5). When play is cooperative with private agents we allow the government to select its most preferred point in the core, subject to an individual rationality condition for private agents.

A natural first reaction to this would seem to be that cooperative play of this type should result in the "optimal" or so called cooperative outcome that could be obtained if government pre-commitment were feasible in a standard non-cooperative game. In fact we show that a unique equilibrium to this cooperative game exists, and that this equilibrium corresponds to the time consistent, and not the optimal, solution to the government's problem. Thus, cooperation of a natural type does <u>not</u> lead to what are commonly called cooperative outcomes.

The format of the paper is as follows. Section 2 lays out a cooperative game involving the government and private agents. In Section 3 we show that a unique equilibrium to this cooperative game exists and that the equilibrium coincides with the solution to a simple dynamic programming problem. If the objective functions of the government and of the private agents coincide the equilibrium will be a unique Pareto optimum. When they do not coincide there is still a time inconsistency problem and this is illustrated by example in Section 4. The cooperative game of policy need not be structured such that the government and individuals always cooperate. Rather, the game can be reformulated as one where the policy makers play a cooperative game among themselves and act as a Stackelberg leader with respect to the private agents. This framework is discussed in Section 5.

Any successful theory of economic policy making should be able to address optimal taxation issues. Taxes that are not lump sum generally introduce distortions that can have serious welfare consequences. In Section 6 we use the cooperative model of policy making to analyse an optimal taxation game where the government and the private sector have the same objective function and the government is precluded from using lump sum taxes. The equilibrium allocation that emerges from this game is the same as if the government could raise all of its revenue with lump sum taxes. This result has some interesting empirical implications which we discuss in a concluding section.

2. A Cooperative Policy Game

In this section we lay out a cooperative policy game played by a government and a private agent (or a set of homogeneous private agents). Our formulation, except for the cooperative nature of the game and the equilibrium concept employed, will closely resemble that of Kydland and Prescott (1977). We focus on an environment with a finite time horizon, and do not explicitly incorporate uncertainty into our notation, although the latter simplification is inessential. The solution concept we devise is very similar to the idea of Coalition Proof Nash Equilibria recently introduced independently by Bernheim, Peleg, and Whinston (1987).

Let time be indexed by t; $t=1,\ldots,T$. Let $\pi = (\pi_1,\pi_2,\ldots,\pi_T)$ be a sequence of policy choices for periods $1,\ldots,T$, and let $x =(x_1,x_2,\ldots,x_T)$ be a sequence of actions by private agents. (Policies and private actions can be viewed as chosen from some compact set, which is suppressed in our discussion). The government (or policy maker) has the objective function $S(x_1,\ldots,x_T,\pi_1,\ldots,\pi_T)$, which is assumed to be continuous and quasi-concave. The private agent has an objective function, $V(x_1,\ldots,x_T,\pi_1,\ldots,\pi_T)$, with the same properties.

As has been widely noted [see, e.g., Kydland-Prescott (1977), p. 627], if future policy makers cannot be bound to a decision at time one, policy makers at each date are de facto separate players. We capture this feature of policy games using an idea of Myerson (1983). We adopt the view that the policy maker at each date t is a distinct player from policy makers at other dates. Policy formulation can then be viewed as a cooperative game among policy makers at different dates, just as Myerson (1983) views mechanism design as the outcome of a cooperative game among different possible "types" of the same (informed) principal.

We imagine that all policy makers and private agents meet at the beginning of time to choose a sequence of actions. Some such sequence is proposed. If a coalition of policy makers (and private agents) can form and find a preferred subsequence of policy actions, this initial sequence is blocked. Blocked sequences are never implemented. But, since the payoffs of future players depend on actions of current players, it is necessary to specify how potential blocking coalitions view the actions of their complements. Here we adopt the natural formulation (natural since current policy makers preceed future policy makers in time) that blocking coalitions take the choices of their complements as given. This makes it appropriate to assume, as Kydland and Prescott (1977) do, that private agents take the whole future sequence of government policy actions as given in their decision making.

The sequences of actions that are not blocked at the beginning of time are the cooperative equilibria we are interested in. Lest the notion of all future policy makers and private agents meeting at the beginning of time and forming coalitions seems unnatural, we note that it is common in dynamic economic models to view all agents (including possibly unborn agents) as meeting at the beginning of time to trade in Walrasian auction markets. This permits static competitive equilibrium tools to be employed. Here we allow policy makers and private agents to meet at the beginning of time, to propose sequences of actions, and to form blocking coalitions. This allows us to use standard static core concepts. Thus, ours is simply a cooperative analog to standard approaches to dynamic competitive analysis.

Formally, then, the <u>policy maker at t</u>, denoted agent p_t, is a policy maker who faces a partial history of choices, which we denote hereafter by $h_{t-1} = (x_1,\dots,x_{t-1},\pi_1,\dots,\pi_{t-1})$, and can choose a sequence $(\pi_t,\dots,\pi_T)$. The <u>private agent at t</u>, who is also a player in the same cooperative game, is denoted agent a_t. This agent, facing a given history of choices, can choose a sequence of actions $(x_t,\dots,x_T)$.

A <u>coalition at t</u> is a subset of players dated $\tau=t,\dots,T$. Or, in other words, a coalition at t is a subset of players $\{p_\tau\}_{\tau=t}^{T} \cup \{a_\tau\}_{\tau=t}^{T}$. A coalition consisting only of private agents at t; $\{a_\tau\}_{\tau=t}^{T}$, obtains the payoff

$$(1) \qquad \bar{V}_t \equiv \max_{\{x_\tau\}_{\tau=t}^{T}} V(x_t,\dots,x_T,0,\dots,0 \mid h_{t-1}),$$

where h_{t-1} denotes the inherited history of the game. The interpretation of (1) is as follows: if private agents, and private agents alone, defect from a coalition at date t, they inherit the past history of the game $h_{t-1}=(x_1,\dots,x_{t-1},\pi_1,\dots,\pi_{t-1})$. Since no policy makers are included in the coalition, the government has been "shut down" or replaced. We denote "shutting down" of the government at t by setting $\pi_t = \pi_{t+1} = \dots = \pi_T = 0$. However, private agents from t on are free to make arbitrary choices $(x_t,x_{t+1},\dots,x_T)$, and hence a coalition of this type obtains the payoff given in (1).

We will say that a set of actions $(x_1,\dots,x_T,\pi_1,\dots,\pi_T)$ is <u>blocked</u> by $\{a_\tau\}_{\tau=t}^{T}$ (the coalition of private agents at t)[2] if there exist values $(\hat{x}_t,\dots,\hat{x}_T)$ such that

$$(2) \qquad V(\hat{x}_t,\dots,\hat{x}_T,0,\dots,0 \mid h_{t-1}) > V(x_1,\dots,x_T,\pi_1,\dots,\pi_T)$$

and

(3) $V(\hat{x}_t,\ldots,\hat{x}_T,0,\ldots,0 \mid h_{t-1}) \geq \bar{V}_s$; $s=t+1,\ldots,T$,

where $\bar{V}_s$ is defined by (1).

Equation (3) merits some discussion. What (3) says is that the set of choices $(x_1,\ldots,x_{t-1},\hat{x}_t,\ldots,\hat{x}_T,\pi_1,\ldots,\pi_{t-1},0,\ldots,0)$ is not itself blocked by a subset of the coalition $\{a_\tau\}_{\tau=t}^T$. That is, it cannot itself be blocked by private agents at some date later than t if $\{a_\tau\}_{\tau=t}^T$ is to constitute a blocking coalition. Equation (3) can be viewed as a requirement that a set of actions cannot be blocked by making a "threat" that will not actually be carried out by subsequent players. Having said this, it will be noted that private agents can always obtain the payoff $\bar{V}_s$ at s, which trivially satisfies (3). In some sense (3) is an inessential condition, then, but it is discussed here because an analogous condition plays an important role below.

As a further definition, a set of choices $(x_1,\ldots,x_T,\pi_1,\ldots,\pi_T)$ is <u>blocked by the grand coalition at t</u> $(\{p_\tau\}_{\tau=t}^T \cup \{a_\tau\}_{\tau=t}^T)$ if there exist values $(\hat{x}_t,\ldots,\hat{x}_T,\hat{\pi}_t,\ldots,\hat{\pi}_T)$ such that

(4) $$S(\hat{x}_t,\ldots,\hat{x}_T,\hat{\pi}_t,\ldots,\hat{\pi}_T \mid h_{t-1}) > S(x_1,\ldots,x_T,\pi_1,\ldots,\pi_T)$$

(5) $$V(\hat{x}_t,\ldots,\hat{x}_T,\hat{\pi}_t,\ldots,\hat{\pi}_T \mid h_{t-1}) \geq \bar{V}_s;\ s=t,\ldots,T,$$

where $\bar{V}_s$; $s=t,\ldots,T$, is defined by (1), and such that $(x_1,\ldots,x_{t-1},\hat{x}_t,\ldots,\hat{x}_T,\pi_1,\ldots,\pi_{t-1},\hat{\pi}_t,\ldots,\hat{\pi}_T)$ is not itself blocked by a coalition $\{p_\tau\}_{\tau=s}^T \cup \{a_\tau\}_{\tau=s}^T$; $s \geq t+1$.

Some discussion of this definition is in order. Equation (4) says that for the grand coalition at t to block the choices $(x_1,\ldots,x_T,\pi_1,\ldots,\pi_T)$ there must exist a choice of actions that makes the policy maker strictly better off at t. Notice that there is no analogous requirement that private agents be made better off. This allows the government a wide scope for discretion since it can take any actions (and effectively dictate actions of private agents) that do not result in the government being "shut down." In this sense our formulation closely resembles Myerson's (1983), in that all interesting play of the game is among policy makers at different dates.

Equation (5) is an individual rationality condition for private agents; the government can, _de facto_, force private agents to join a blocking coalition so long as they are not worse off than they would be with no government. Equation (5) also requires that the blocking choices $(\hat{x}_t,\ldots,\hat{x}_T,\hat{\pi}_t,\ldots,\hat{\pi}_T)$ cannot themselves be blocked by a subset of private agents at a date later than t. Obviously, then, the formulation of the game is such that the government and private agents are not at all symmetric. We view this as a translation to a cooperative context of the usual stance adopted in non-cooperative policy games: the government is a Stackelberg leader and takes the reaction function of private agents as given. Here, the government is allowed to select its most preferred point in the core.

Finally, we require that, for $\{p_\tau\}_{\tau=t}^T \cup \{a_\tau\}_{\tau=t}^T$ to constitute a blocking coalition, there must exist choices $(\hat{x}_t,\ldots,\hat{x}_T,\hat{\pi}_t,\ldots,\hat{\pi}_T)$ for this coalition such that $(x_1,\ldots,x_{t-1},\hat{x}_t,\ldots,\hat{x}_T,\pi_1,\ldots,\pi_{t-1},\hat{\pi}_t,\ldots,\hat{\pi}_T)$ will not itself be blocked by a coalition that forms at a later date. Thus we do _not_ allow $(x_1,\ldots,x_T,\pi_1,\ldots,\pi_T)$ to be blocked by a coalition at t that "threatens" to take actions at some date s > t that will not be carried out at s.

It is now necessary to write down formally what we mean by this. $(x_1,\ldots,x_{t-1},\hat{x}_t,\ldots,\hat{x}_T,\pi_1,\ldots,\pi_{t-1},\hat{\pi}_t,\ldots,\hat{\pi}_T)$ is blocked by a coalition $\{p_\tau\}_{\tau=s}^T \cup \{a_\tau\}_{\tau=s}^T$ (s > t) if any of the following conditions are satisfied.

(i) There exist values $(\tilde{x}_T,\tilde{\pi}_T)$ such that

(6) $$S(\hat{x}_t,\ldots,\hat{x}_{T-1},\tilde{x}_T,\hat{\pi}_t,\ldots,\hat{\pi}_{T-1},\tilde{\pi}_T \mid h_{t-1}) > S(\hat{x}_t,\ldots,\hat{x}_T,\hat{\pi}_t,\ldots,\hat{\pi}_T \mid h_{t-1})$$

and $$V(\hat{x}_t,\ldots,\hat{x}_{T-1},\tilde{x}_T,\hat{\pi}_t,\ldots,\hat{\pi}_{T-1},\tilde{\pi}_T \mid h_{t-1}) \geq \bar{V}_T.$$

(ii) There exist values $(\tilde{x}_{T-1},\tilde{x}_T,\tilde{\pi}_{T-1},\tilde{\pi}_T)$ such that:

(7) $$S(\hat{x}_t,\ldots,\hat{x}_{T-2},\tilde{x}_{T-1},\tilde{x}_T,\hat{\pi}_t,\ldots,\hat{\pi}_{T-2},\tilde{\pi}_{T-1},\tilde{\pi}_T \mid h_{t-1}) >$$

$$S(\hat{x}_t,\ldots,\hat{x}_T,\hat{\pi}_t,\ldots,\hat{\pi}_T \mid h_{t-1}),$$

$$V(\hat{x}_t,\ldots,\hat{x}_{T-2},\tilde{x}_{T-1},\tilde{x}_T,\hat{\pi}_t,\ldots,\hat{\pi}_{T-2},\tilde{\pi}_{T-1},\tilde{\pi}_T \mid h_{t-1}) \geq \bar{V}_{T-1},$$

$$V(\hat{x}_t,\ldots,\hat{x}_{T-2},\tilde{x}_{T-1},\tilde{x}_T,\hat{\pi}_t,\ldots,\hat{\pi}_{T-2},\tilde{\pi}_{T-1},\tilde{\pi}_T \mid h_{t-1}) \geq \bar{V}_T,$$

and such that there is no pair (x_T^*,π_T^*) satisfying

(8) $$S(\hat{x}_t,\ldots,\hat{x}_{T-2},\tilde{x}_{T-1},x_T^*,\hat{\pi}_t,\ldots,\hat{\pi}_{T-2},\tilde{\pi}_{T-1},\pi_T^* \mid h_{t-1}) >$$

$$S(\hat{x}_t,\ldots,\hat{x}_{T-2},\tilde{x}_{T-1},\tilde{x}_T,\hat{\pi}_t,\ldots,\hat{\pi}_{T-2},\tilde{\pi}_{T-1},\tilde{\pi}_T \mid h_{t-1}),$$

and

$$V(\hat{x}_t,\ldots,\hat{x}_{T-2},\tilde{x}_{T-1},x_T^*,\hat{\pi}_t,\ldots,\hat{\pi}_{T-2},\tilde{\pi}_{T-1},\pi_T^* \mid h_{t-1}) \geq \bar{V}_T.$$

Similar conditions are defined for $s = t+1,\ldots,T-2$.

3. Equilibrium

There is a unique equilibrium to the cooperative game laid out above. The equilibrium coincides with the solution to a very simple dynamic programming problem implying that dynamic programming techniques are applicable here, despite the presence of a time consistency problem which we illustrate below. In this section, we first describe the dynamic programming problem as a prelude to a constructive proof of the existence of equilibrium. To begin, let $g_T(x_1,\dots,x_{T-1},\pi_1,\dots,\pi_{T-1})$ and $f_T(x_1,\dots,x_{T-1},\pi_1,\dots,\pi_{T-1})$ be the values of π_T and x_T respectively that solve the problem

$$(9) \qquad \max_{x_T,\pi_T} \; S(x_1,\dots,x_{T-1},x_T,\pi_1,\dots,\pi_{T-1},\pi_T)$$

subject to

$$(10) \qquad V(x_1,\dots,x_{T-1},x_T,\pi_1,\dots,\pi_{T-1},\pi_T) \geq \bar{V}_T$$

where $\bar{V}_T$ is as defined by equation (1). Then define recursively, for $t=1,\dots,T-1$, $g_t(x_1,\dots,x_{t-1},\pi_1,\dots,\pi_{t-1})$ and $f_t(x_1,\dots,x_{t-1},\pi_1,\dots,\pi_{t-1})$ to be the values of π_t and x_t that solve the problem

$$(11) \qquad \max_{x_t,\pi_t} \; S[x_1,\dots,x_{t-1},x_t,f_{t+1}(\cdot),\dots,f_T(\cdot),\pi_1,\dots,\pi_{t-1},\pi_t, \\ g_{t+1}(\cdot),\dots,g_T(\cdot)]$$

subject to

$$(12) \qquad V[x_1,\dots,x_{t-1},x_t,f_{t+1}(\cdot),\dots,f_T(\cdot),\pi_1,\dots,\pi_{t-1}, \\ \pi_t,g_{t+1}(\cdot),\dots,g_T(\cdot)] \geq \bar{V}_t.$$

We assume that $\pi_t = 0$ is a feasible choice for the government at each date, so the constraint set in each of these problems is non-empty. Also, under standard assumptions on $S(-)$ and $V(-)$, $g_t(-)$ and $f_t(-)$ are continuous functions $\forall t$, although this is not necessary in our analysis. Thus, $g_t(-)$ and $f_t(-)$ give standard dynamic programming solutions to the government's problem of maximizing the value of its objective function subject to a set of individual rationality conditions for private agents.

In the following theorems we state two results. The first is that the choices $x_t = f_t(x_1,\ldots,x_{t-1},\pi_1,\ldots,\pi_{t-1})$, $\pi_t=g_t(x_1,\ldots,x_{t-1},\pi_1,\ldots,\pi_{t-1})$ $\forall t$ are equilibrium choices. The second is that these are the only equilibrium choices.

<u>Theorem 1</u>. Suppose the sequence of actions $(x_1,\ldots,x_T,\pi_1,\ldots,\pi_T)$ given by $x_t=f_t(-)$, $\pi_t=g_t(-)$ $\forall t$ is chosen. Then this set of actions is not blocked.

To prove the theorem, we begin by noting that these choices cannot be blocked by any coalition consisting only of private agents. This is true trivially since, by construction, $V(x_1,\ldots,x_T,\pi_1,\ldots,\pi_T) \geq \bar{V}_t$ for all t.

Therefore, we need only prove that these choices are not blocked by the grand coalition $(\{p_\tau\}_{\tau=t}^{T} \cup \{a_\tau\}_{\tau=t}^{T})$ at any t. We give a proof by induction. Consider first period T. We claim that for any (given) history $h_{T-1}=(x_1,\ldots,x_{T-1},\pi_1,\ldots,\pi_{T-1})$, the choices $x_T^*=f_T(h_{T-1})$ and $\pi_T^*=g_T(h_{T-1})$ are not blocked at T. This follows trivially from the definition of $f_T(-)$ and $g_T(-)$.

To continue with the proof by induction, we now suppose that, for any (given) history h_q the values $x_s^*=f_s(h_{s-1})$ and $\pi_s^*=g_s(h_{s-1})$; $s=q+1,\ldots,T$ are such that $(x_1,\ldots,x_q,x_{q+1}^*,\ldots,x_T^*,\pi_1,\ldots,\pi_q,\pi_{q+1}^*,\ldots,\pi_T^*)$ is not blocked at date q+1 or later. Then we claim that, if

$$x_q = x_q^* = f_q(h_{q-1}) \text{ and } \pi_q = \pi_q^* = (h_{q-1}),$$

$(x_1,\ldots,x_{q-1},x_q^*,\ldots,x_T^*,\pi_1,\ldots,\pi_{q-1},\pi_q^*,\ldots,\pi_T^*)$ is also not blocked at date q or later.

In particular, by induction $(x_1,\ldots,x_{q-1},x_q^*,\ldots,x_T^*,\pi_1,\ldots,\pi_{q-1},\pi_q^*,\ldots,\pi_T^*)$ is not blocked at date q+1 or later. Then, if it is blocked, it is blocked at date q. Consequently there exist values $(\hat{x}_q,\ldots,\hat{x}_T,\hat{\pi}_q,\ldots,\hat{\pi}_T)$ such that

$$(13) \qquad S(x_1,\ldots,x_{q-1},\hat{x}_q,\ldots,\hat{x}_T,\pi_1,\ldots,\pi_{q-1},\hat{\pi}_q,\ldots,\hat{\pi}_T)$$

$$> S(x_1,\ldots,x_{q-1},x_q^*,\ldots,x_T^*,\pi_1,\ldots,\pi_{q-1},\pi_q^*,\ldots,\pi_T^*) \ ,$$

and such that

$$(14) \qquad V(x_1,\ldots,x_{q-1},\hat{x}_q,\ldots,\hat{x}_T,\pi_1,\ldots,\pi_{q-1},\hat{\pi}_q,\ldots,\hat{\pi}_T)$$

$$\geq \bar{V}_k; \ k \geq q.$$

Moreover, $(x_1,\ldots,x_{q-1},\hat{x}_q,\ldots,\hat{x}_T,\pi_1,\ldots,\pi_{q-1},\hat{\pi}_q,\ldots,\hat{\pi}_T)$ cannot itself be blocked at some date later than q.

Now, for (13) and (14) to hold, it is clearly the case that $\hat{x}_k \neq f_k(\cdot)$ and/or $\hat{\pi}_k \neq g_k(\cdot)$ for some $k > q$. Let $\bar{k}$ be the largest date such that either $\hat{x}_k \neq f_k(\cdot)$ or $\hat{\pi}_k \neq g_k(\cdot)$. We now show that $(x_1,\ldots,x_{q-1},\hat{x}_q,\ldots,\hat{x}_T,\pi_1,\ldots,\pi_{q-1}, \hat{\pi}_q,\ldots,\hat{\pi}_T)$ is blocked at date $\bar{k}$ by the grand coalition. In particular, set $\tilde{x}_{\bar{k}} = f_{\bar{k}}(x_1,\ldots,x_{q-1},\hat{x}_q,\ldots,\hat{x}_{\bar{k}-1},\pi_1,\ldots,\pi_{q-1},\hat{\pi}_q,\ldots,\hat{\pi}_{\bar{k}-1})$, set $\tilde{\pi}_{\bar{k}} = g_{\bar{k}}(x_1,\ldots,x_{q-1},\hat{x}_q,\ldots,\hat{x}_{\bar{k}-1},\pi_1,\ldots,\pi_{q-1},\hat{\pi}_q,\ldots,\hat{\pi}_{\bar{k}-1})$, set $\tilde{x}_{\bar{k}+1} = f_{\bar{k}+1}(x_1,\ldots,x_{q-1},\hat{x}_q,\ldots,\hat{x}_{\bar{k}-1},\tilde{x}_{\bar{k}},\pi_1,\ldots,\pi_{q-1},\hat{\pi}_q,\ldots,\hat{\pi}_{\bar{k}-1},\tilde{\pi}_{\bar{k}})$, etc. Then, by the definition of the functions $f_s(\cdot)$ and $g_s(\cdot)$,

$$(15)\quad S(x_1,\ldots,x_{q-1},\hat{x}_q,\ldots,\hat{x}_{\bar{k}-1},\tilde{x}_{\bar{k}},\ldots,\tilde{x}_T,\pi_1,\ldots,\pi_{q-1},\hat{\pi}_q,\ldots,\hat{\pi}_{\bar{k}-1},\tilde{\pi}_{\bar{k}},\ldots,\tilde{\pi}_T) > S(x_1,\ldots,x_{q-1},\hat{x}_q,\ldots,\hat{x}_T,\pi_1,\ldots,\pi_{q-1},\hat{\pi}_q,\ldots,\hat{\pi}_T),$$

and

$$V(x_1,\ldots,x_{q-1},\hat{x}_q,\ldots,\hat{x}_{\bar{k}-1},\tilde{x}_{\bar{k}},\ldots,\tilde{x}_T,\pi_1,\ldots,\pi_{q-1},\hat{\pi}_q,\ldots,\hat{\pi}_{\bar{k}-1},\tilde{\pi}_{\bar{k}},\ldots,\tilde{\pi}_T) \geq \bar{V}_{\bar{k}}.$$

Moreover, by induction , $(x_1,\ldots,x_{q-1},\hat{x}_q,\ldots,\hat{x}_{\bar{k}-1},\tilde{x}_{\bar{k}},\ldots,\tilde{x}_T,\pi_1,\ldots,\pi_{q-1},\hat{\pi}_q,\ldots,\hat{\pi}_{\bar{k}-1},\tilde{\pi}_{\bar{k}},\ldots,\tilde{\pi}_T)$ is not blocked at any date later than $\bar{k}$. This implies that $(x_1,\ldots,x_{q-1},\hat{x}_q,\ldots,\hat{x}_T,\pi_1,\ldots,\pi_{q-1},\hat{\pi}_q,\ldots,\hat{\pi}_T)$ is itself blocked at $\bar{k}$, which contradicts that $(x_1,\ldots,x_{q-1},x^*_q,\ldots,x^*_T,\pi_1,\ldots,\pi_{q-1},\pi^*_q,\ldots,\pi^*_T)$ is blocked at q. Thus, for any history h_{q-1} the choices $(x^*_q,\ldots,x^*_T,\pi^*_q,\ldots,\pi^*_T)$ are not blocked. But then, the choices $x_t = f_t(-)$, $\pi_t = g_t(-)$ $\forall t$ are not blocked, establishing the theorem.

As is apparent, the theorem, in addition to establishing the existence of an equilibrium, gives a characterization of this equilibrium as the solution to a simple dynamic programming problem. Further, it is the case that this equilibrium is unique.

<u>Theorem 2</u>.

Consider a sequence of actions $(x_1,\ldots,x_T,\pi_1,\ldots,\pi_T)$. Suppose that either $\pi_s \neq g_s(-)$ and/or $x_s \neq f_s(-)$ for some date s. Then this choice of actions is blocked.

The method of proof is to fix an arbitrary sequence $(x_1,\ldots,x_T,\pi_1,\ldots,\pi_T)$ with either $x_s \neq f_s(-)$ for some s, or $\pi_s \neq g_s(-)$ for some s. A blocking coalition is then constructed, establishing the theorem.

To begin, let $\bar{s}$ be the largest date for which $\pi_s \neq g_s(-)$ or $x_s \neq f_s(-)$. Then the coalition $\{p_\tau\}_{\tau=\bar{s}}^T \cup \{a_\tau\}_{\tau=\bar{s}}^T$ can choose $\hat{\pi}_{\bar{s}} = g_{\bar{s}}(x_1,\ldots,x_{\bar{s}-1},\pi_1,\ldots,\pi_{\bar{s}-1})$, $\hat{x}_{\bar{s}} = f_{\bar{s}}(x_1,\ldots,x_{\bar{s}-1},\pi_1,\ldots,\pi_{\bar{s}-1})$, and can choose $\hat{\pi}_q = g_q(-)$,

$\hat{x}_q = f_q(\cdot)$ for all $q \geq \bar{s} + 1$. Then, by the definitions of $g_q(\cdot)$ and $f_q(\cdot)$,

$$(16) \qquad S(x_1,\ldots,x_{\bar{s}-1},\hat{x}_{\bar{s}},\ldots,\hat{x}_T,\pi_1,\ldots,\pi_{\bar{s}-1},\hat{\pi}_{\bar{s}},\ldots,\hat{\pi}_T) > S(x_1,\ldots,x_T,\pi_1,\ldots,\pi_T),$$

and

$$(17) \qquad V(x_1,\ldots,x_{\bar{s}-1},\hat{x}_{\bar{s}},\ldots,\hat{x}_T,\pi_1,\ldots,\pi_{\bar{s}-1},\hat{\pi}_{\bar{s}},\ldots,\hat{\pi}_T) \geq \bar{V}_{\bar{s}}.$$

Moreover, by the same argument used in the proof of Theorem 1, the actions $(\hat{x}_{\bar{s}},\ldots,\hat{x}_T,\hat{\pi}_{\bar{s}},\ldots,\hat{\pi}_T)$ are not themselves blocked by a subset of the coalition $\{p_\tau\}_{\tau=\bar{s}}^T \cup \{a_\tau\}_{\tau=\bar{s}}^T$ (that is, are not themselves blocked at some date later than $\bar{s}$). Hence $\{p_\tau\}_{\tau=\bar{s}}^T \cup \{a_\tau\}_{\tau=\bar{s}}^T$ satisfies the definition of a blocking coalition, establishing the theorem.

In summary, we have proved that the cooperative game of policy formulation set out above has a unique equilibrium for arbitrary finite horizons. (Whether the same result can be proved in infinite horizon settings is a topic for future research). Furthermore, the equilibrium of this game can be characterized as the solution of a simple dynamic programming problem.

4. Discussion

The results above have some sharp implications. For instance, if the government objective function coincides with private agents' objective functions, then the equilibrium above is a unique Pareto optimum and is time consistent. This is easy to see, since in this case the equilibrium of the cooperative policy game is the solution to the problem

$$\max V(x_1,\ldots,x_T,\pi_1,\ldots,\pi_T)$$

Clearly $\max V(x_1,\ldots,x_T,\pi_1,\ldots,\pi_T) \geq \bar{V}_t \ \forall t$ (with $\bar{V}_t$ defined by (1)), so that the solution to this problem solves the problem (9) and (10), and is an equilibrium.

This conclusion raises the question of whether there can be time consistency problem in this context. It has already been noted in the literature [see, e.g., Fischer (1980)] that when the government objective function coincides with that of private agents, cooperative behavior eliminates time consistency problems. However, this is not the case when the government's objective function differs from that of private agents. To illustrate this point, we construct an example in which $(x_1,\ldots,x_T,\pi_1,\ldots,\pi_T)$ is chosen cooperatively at $t=1$. We then show that, in this example, these choices are not time consistent even though play between the government and private agents is cooperative.

Our example is a two period problem, with some features that resemble the economy analyzed by Fischer (1980). To make the example more concrete, we will change our notation somewhat. Let g_t denote government expenditures at t, and let c_t denote consumption by the representative private agent at t. At time 1 the representative agent has an endowment of a single good, w, which can be allocated to consumption, government expenditures, or to an investment. We let k_2 denote the quantity of the good set aside at time 1 to be used in production at $t = 2$. Then the technological constraints for this economy are

$$c_1 + g_1 + k_2 \leq w \tag{18}$$

$$c_2 + g_2 \leq Rk_2. \tag{19}$$

Finally, the objective function of the government is $S(c_1,c_2,g_1,g_2) = g_2$, and the objective function of the representative agent is $V(c_1,c_2,g_1,g_2) = c_1 + \beta \min[c_2,\delta g_2]$. It is assumed that $\delta > 0$, and that $\beta R \geq 1$.

To begin, we analyze the solution to the problem of a government that can commit to a set of choices (c_1,c_2,k_2,g_1,g_2) at $t = 1$. In order to solve this problem, we need to derive $\bar{V}_1$ as defined by equation (1). Here, clearly, $\bar{V}_1 = w$. Then at $t = 1$ the government and the representative agent, playing cooperatively, choose (c_1,c_2,k_2,g_1,g_2) to solve the problem of maximizing g_2 subject to (18), (19), and

(20) $$c_1 + \beta \min [c_2,\delta g_2] \geq w.$$

The solution to this problem will satisfy (20) at equality and will also satisfy $c_2 \leq \delta g_2$. To see this, simply notice that if c_1, c_2, k_2, and g_2 have been chosen so that $c_2 > \delta g_2$, then it is possible to raise g_2 and reduce c_2 without violating (19) or (20), and still leave c_1 and k_2 unaltered. Then the solution to the problem above has $g_1 = 0$, and from (18)-(20),

(21) $$w-k_2 + \beta \min[Rk_2-g_2,\delta g_2] = w.$$

Since $Rk_2 - g_2 \leq \delta g_2$, (21) may be rewritten as

(22) $$k_2 = \beta(Rk_2 - g_2),$$

or alternatively as

(23) $$g_2 = (\frac{\beta R-1}{\beta})\ k_2.$$

Then the government must maximize g_2 subject to (23). Clearly the solution to this problem is to set $k_2 = w$, with $c_1 = 0$ and $g_2 =(\beta R-1)w/\beta$.

If at $t = 2$ the government and private agents re-solve their problem, they will take c_1, g_1 and k_2 as given, and choose c_2 and g_2 to solve the problem of maximizing g_2 subject to (19) and

(24) $$\beta \min[c_2, \delta g_2] \geq \bar{V}_2,$$

with $\bar{V}_2$ given by (1). Clearly $\bar{V}_2 = 0$, so the solution to this problem is to set $g_2= Rk_2$ and $c_2 = 0$. We see that even though play is cooperative, a time inconsistency problem can arise.

The cooperative equilibrium derived above for this game sets $c_2= g_2 = k_2 =0$, and sets $c_1 = w$. Again, the "optimal" solution and the cooperative equilibrium for this economy diverge, so cooperative play of the game does not preclude the occurrence of time consistency problems.

5. A Reformulated Game.

In the policy game analyzed above, coalitions could consist of the entire set of policy makers, $\{p_\tau\}_{\tau=1}^{T}$, and the entire set of private agents, $\{a_\tau\}_{\tau=1}^{T}$. There is nothing in the analysis, however, that requires that private agents be able to play cooperatively with the government. Specifically, we can restructure the game to allow policy makers to play a cooperative game among themselves and to (together) be a Stackelberg leader vis-a-vis private agents.

Formally, we follow Kydland and Prescott (1977), and let the policy makers $\{p_\tau\}_{\tau=1}^{T}$ announce a sequence of policies $(\pi_1,\ldots,\pi_T)$ at time 1. Agents' actions x_t at t are given by a reaction function

$$(25) \qquad x_t = X_t(x_1,\ldots,x_{t-1},\pi_1,\ldots,\pi_T);\ t=1,\ldots,T.$$

Then a <u>coalition at t</u> is a set of players $\{p_\tau\}_{\tau=t}^{T}$. A sequence of policy choices $(\pi_1,\ldots,\pi_T)$ is <u>blocked</u> by the coalition $\{p_\tau\}_{\tau=t}^{T}$ if there exist values $(\hat{\pi}_t,\ldots,\hat{\pi}_T)$ such that

$$(26) \qquad S[\tilde{x}_1,\ldots,\tilde{x}_{t-1},X_t(\tilde{x}_1,\ldots,\tilde{x}_{t-1},\pi_1,\ldots,\pi_{t-1},\hat{\pi}_t,\ldots,\hat{\pi}_T),\ldots,$$
$$X_T(\cdot),\pi_1,\ldots,\pi_{t-1},\hat{\pi}_t,\ldots,\hat{\pi}_T] > S(\tilde{x}_1,\ldots,\tilde{x}_T,\pi_1,\ldots,\pi_T),$$

where $\tilde{x}_t = X_t(\tilde{x}_1,\ldots,\tilde{x}_{t-1},\pi_1,\ldots,\pi_T)$ $\forall t$, and such that $(\pi_1,\ldots,\pi_{t-1}, \hat{\pi}_t,\ldots,\hat{\pi}_T)$ is not itself blocked by a coalition $\{p_\tau\}_{\tau=q}^{T}$; $q \geq t+1$.

Now define $f_T(x_1,\ldots,x_{T-1},\pi_1,\ldots,\pi_{T-1})$ as:

$$f_T(\cdot) \equiv \underset{\pi_T}{\operatorname{argmax}}\ S[x_1,\ldots,x_{T-1},X_T(x_1,\ldots,x_{T-1},\pi_1,\ldots,\pi_{T-1},\pi_T),\pi_1,\ldots,\pi_{T-1},\pi_T]$$

where $x_1,\ldots,x_{T-1}$ and $\pi_1,\ldots,\pi_{T-1}$ are taken as parametric. Further, following Kydland and Prescott (1977, p. 622), define $f_t(h_{t-1})$ to be the "best" policy choice at t, given the history h_{t-1}, where these policy choices are determined recursively. Or, in other words, $f_t(h_{t-1})$ is Kydland and Prescott's "consistent policy" choice at t.

We are now prepared to state an analog to Theorem 1.

Theorem 3. The sequence of policy actions $(\pi_1,\ldots,\pi_T)$ given by $\pi_t = f_t(\tilde{x}_1,\ldots,\tilde{x}_{t-1},\pi_1,\ldots,\pi_{t-1})$; $t=1,\ldots,T$, is not blocked, where $\tilde{x}_t = X_t(\tilde{x}_1,\ldots,\tilde{x}_{t-1},\pi_1,\ldots,\pi_T)$ as above.[3]

The proof exactly parallels the proof of Theorem 1, and is therefore omitted. Notice, though, that the theorem yields the result that Kydland and Prescott's "consistent solution" is an equilibrium to the cooperative game considered in this section.

Moreover, no other sequence of choices $(\pi_1,\ldots,\pi_T)$ results in an equilibrium. Again, the argument exactly parallels that in the proof of Theorem 2, and hence is omitted here. However, it bears emphasizing that Kydland and Prescott's consistent solution is the only equilibrium of this cooperative game of policy formulation.

Among the most interesting policy issues are those involving optimal taxation. In this reformulated game, when the government and private agents do not cooperate with each other, these issues are exactly as discussed by Fischer (1980) or Kydland and Prescott (1980). In the cooperative game laid out in Section 2, however, some more interesting conclusions emerge.

6. Optimal Taxation.

The cooperative view of policy determination discussed in Sections 2 and 3 delivers very sharp implications about the equilibrium outcome of games determining the level of taxation. In particular, our analysis predicts that, even when the government is precluded from the use of lump-sum taxes, equilibrium allocations in a cooperative policy game will be the same as if the government could raise all its revenue via lump-sum taxation. This section illustrates an optimal taxation exercise in the context of a two-period model. As will be clear, the analysis could easily be extended to incorporate an arbitrary time horizon.

The notation we employ here is identical to that of section 4, except that now we let agents allocate some labor effort. Let $\bar{\ell}$ be the time endowment of the representative agent at each date, and let ℓ_t denote labor supply at t. Also, we let τ_t denote an n-vector of date t tax parameters (t=1,2). The game is one of optimal taxation, so that the government objective function and private objective functions coincide. This objective function is denoted $V(c_1,\bar{\ell}-\ell_1,c_2,\bar{\ell}-\ell_2,g_1,g_2)$. V( ) is increasing in each argument, is twice continuously differentiable, and is strictly quasi-concave. The technology of the economy is as follows:

$$c_1 + g_1 + k_2 \le q(\ell_1); \; q' > 0, \; q'' \le 0 \tag{27}$$

$$c_2 + g_2 \le f(\ell_2,k_2) \tag{28}$$

where f is increasing in each argument, is twice continuously differentiable, and is concave. In addition, there is an exogenous constraint on how revenue can be raised, so that the government faces a financing constraint at each date:

$$g_1 = R_1(\tau_1,\ell_1,k_2) \tag{29}$$

$$g_2 = R_2(\tau_2,\ell_2,k_2) \tag{30}$$

$R_t(\cdot)$, t=1,2, is assumed to be continuously differentiable.[4] An example of this setup would be a game where the government chooses a set of proportional taxes on factor incomes.

According to Theorem 1, the unique equilibrium of the cooperative policy game can be determined as follows. At t=2, given the inherited choices $(\bar{c}_1, \bar{\ell}_1, \bar{g}_1, \bar{k}_2, \bar{\tau}_1)$, the government chooses c_2, ℓ_2, g_2, and τ_2 to solve the problem

$$\max V(\bar{c}_1, \bar{\ell}-\bar{\ell}_1, c_2, \bar{\ell}-\ell_2, \bar{g}_1, g_2)$$

subject to $c_2 + g_2 \leq f(\ell_2, \bar{k}_2)$ and (30).[5] Assuming an interior solution, the first order conditions for this problem can be manipulated to obtain

$$(31) \qquad V_3(\cdot)\left[f_1(\ell_2, \bar{k}_2) - \frac{\partial R_2}{\partial \ell_2}\right] - V_4(\cdot) + V_6(\cdot)\frac{\partial R_2}{\partial \ell_2} = 0.$$

and

$$(32) \qquad -V_3(\cdot)\frac{\partial R_2}{\partial \tau_{2i}} + V_6(\cdot)\frac{\partial R_2}{\partial \tau_{2i}} = 0; \qquad i=1,\ldots,n.$$

where V_j and f_j denote the partial derivatives of these functions with respect to their j^{th} arguments.

As is apparent from (32), if $\frac{\partial R_2}{\partial \tau_{2i}} \neq 0$ for some i in equilibrium, then

$$(33) \qquad V_3(\cdot) = V_6(\cdot).$$

Then, using (33) in (31) yields

$$(34) \qquad V_3(\cdot)f_1(\ell_2, \bar{k}_2) = V_4(\cdot)$$

(33), (34), and $c_2 + g_2 = f(\ell_2, \bar{k}_2)$ determine equilibrium values of c_2, ℓ_2, and

g_2. Any choice of tax parameters satisfying (30) is then an equilibrium choice.

We denote the values c_2, ℓ_2, and g_2 determined in this way as:

$$c_2 \equiv h(c_1,\ell_1,g_1,k_2,\tau_1)$$

$$g_2 \equiv m(c_1,\ell_1,g_1,k_2,\tau_1)$$

$$\ell_2 \equiv n(c_1,\ell_1,g_1,k_2,\tau_1).$$

Under our assumptions the above are functions. Also, it should be apparent that these values of c_2, g_2, and ℓ_2 are exactly the values that would be chosen if the government could employ lump-sum taxation at t=2.

At t=1 the government chooses c_1, ℓ_1, k_2, g_1, and τ_1 to solve the problem

$$\max V[c_1,\bar{\ell}-\ell_1,h(c_1,\ell_1,g_1,k_2,\tau_1),\ \bar{\ell}-n(c_1,\ell_1,g_1,k_2,\tau_1),g_1,m(c_1,\ell_1,g_1,k_2,\tau_1)]$$

subject to (27) and (29).[6] If we define

$$\tilde{h}(\ell_1,k_2,\tau_1) \equiv h[q(\ell_1) - R_1(\ell_1,k_2,\tau_1)- k_2,\ell_1,R_1(\ell_1,k_2,\tau_1),k_2,\tau_1]$$

$$\tilde{m}(\ell_1,k_2,\tau_1) \equiv m[q(\ell_1) - R_1(\ell_1,k_2,\tau_1)- k_2,\ell_1,R_1(\ell_1,k_2,\tau_1),k_2,\tau_1]$$

$$\tilde{n}(\ell_1,k_2,\tau_1) \equiv n[q(\ell_1) - R_1(\ell_1,k_2,\tau_1)- k_2,\ell_1,R_1(\ell_1,k_2,\tau_1),k_2,\tau_1]$$

then the first order conditions for this problem (assuming an interior optimum) can be manipulated to obtain

$$(35)\qquad V_1(\cdot)[q'(\ell_1)- \frac{\partial R_1}{\partial \ell_1}] - V_2(\cdot) + V_3(\cdot)\frac{\partial \tilde{h}}{\partial \ell_1}$$

$$- V_4(\cdot)\frac{\partial \tilde{n}}{\partial \ell_1} + V_5(\cdot)\frac{\partial R_1}{\partial \ell_1} + V_6(\cdot)\frac{\partial \tilde{m}}{\partial \ell_1} = 0$$

$$(36) \qquad - V_1(-)[1 + \frac{\partial R_1}{\partial k_2}] + V_3(-) \frac{\partial \tilde{h}}{\partial k_2} - V_4(-) \frac{\partial \tilde{n}}{\partial k_2}$$

$$+ V_5(-) \frac{\partial R_1}{\partial k_2} + V_6(-) \frac{\partial \tilde{m}}{\partial k_2} = 0$$

and

$$(37) \qquad - V_1(-) \frac{\partial R_1}{\partial \tau_{1i}} + V_3(-) \frac{\partial \tilde{h}}{\partial \tau_{1i}} - V_4(-) \frac{\partial \tilde{n}}{\partial \tau_{1i}}$$

$$+ V_5(-) \frac{\partial R_1}{\partial \tau_{1i}} + V_6(-) \frac{\partial \tilde{m}}{\partial \tau_{1i}} = 0; \quad i=1,\ldots,n.$$

We now note some facts about the partial derivatives of the functions $\tilde{h}(-)$, $\tilde{m}(-)$, and $\tilde{n}(-)$. In particular, since $c_2 + g_2 \equiv \tilde{h}(-) + \tilde{m}(-) \equiv f(\ell_2,\bar{k}_2)$,

$$(38) \qquad \frac{\partial \tilde{h}}{\partial \ell_1} + \frac{\partial \tilde{m}}{\partial \ell_1} = f_1(-) \frac{\partial \tilde{n}}{\partial \ell_1}$$

$$(39) \qquad \frac{\partial \tilde{h}}{\partial k_2} + \frac{\partial \tilde{m}}{\partial k_2} = f_1(-) \frac{\partial \tilde{n}}{\partial k_2} + f_2(-)$$

and

$$(40) \qquad \frac{\partial \tilde{h}}{\partial \tau_{1i}} + \frac{\partial \tilde{m}}{\partial \tau_{1i}} = f_1(-) \frac{\partial \tilde{n}}{\partial \tau_{1i}} \; ; \; i=1,\ldots,n.$$

Now, using (40) in (37), and making note of (33) and (34), we obtain

$$(41) \qquad V_1(-) = V_5(-)$$

if $\frac{\partial R_i}{\partial \tau_{1i}} \neq 0$ for some i. Further, using (41) and (39) in (36), and making use of (33) and (34), we obtain

$$V_1(\cdot) = V_3(\cdot)\, f_2(\ell_2, k_2). \tag{42}$$

Finally, using (41), (38), (33), and (34) in (35) yields

$$V_1(\cdot)q'(\ell_1) = V_2(\cdot). \tag{43}$$

Conditions (41)-(43) and (27) determine c_1, ℓ_1, g_1, and k_2. The tax parameters τ_{1i} can take on any values that satisfy (29) in equilibrium.

This completes the description of equilibrium values in this cooperative game of dynamic optimal taxation. As should be apparent, all equilibrium allocations will be identical to those that would obtain if the government were allowed to employ lump-sum taxation here. Hence, even though the government is formally precluded from the use of lump-sum taxes, there are no "distortions" from the use of non-lump sum taxes.

7. Concluding Comments.

The attempts to isolate particular equilibria among the plethora of possible equilibria in repeated noncooperative game models of economic policy seem to us an enterprise that is unlikely to be entirely successful.[7] It is difficult to understand how private agents and policy makers will coordinate on the same equilibrium path when there are so many sequential equilibria and so many degrees of freedom to the problem. The alternative we have adopted in this paper is to assume from the outset that policy is formed in a framework of cooperation where both the government and the private sector are free to pursue their own objectives and where the government has the characteristics of a Stackelberg leader, a feature that is common in other applications of game theory to problems of policy formulation.

The cooperative setup considered here produces a unique equilibrium. Moreover it is one where, when the private and government objective functions coincide, the equilibrium allocation will be a Pareto optimum and non-lump sum taxes turn out to be non-distorting. The intuition behind this is straightforward; in such an equilibrium the coalition of policy makers at different dates looks just like the coalition of agents and so the taxes act like lump sum transfers.

Most applications of game theory to macroeconomics or international economics seem tailored toward explaining particular empirical observations - episodes of high inflation, competitive devaluations and so on. Here the motivation was to explore the implications of a cooperative model of policy making and to analyze its equilibria. The analytical framework of Sections 2 and 3 has quite provocative empirical implications. Moreover, there appears to be some loose empirical support for the analysis.

If taxes are collected in a framework of cooperation as outlined in Sections 2 and 3 then arbitrary methods of levying taxes are equivalent (in terms of equilibrium allocations) to the use of lump-sum taxation. In our view, this result has bearing on some puzzling empirical claims in various literatures. For instance, Kormendi (1983) and Aschauer (1985) claim to provide evidence supporting a Ricardian equivalence proposition for the postwar U.S. However, it is not possible to derive such a proposition if taxation is distorting. Our results illustrate how Kormendi's and Aschauer's findings are possible in an economy with (apparently) distorting taxation. Similarly, Sargent (1982) and Smith (1985a,b, 1987) claim to provide empirical support for models giving rise to Modigliani-Miller Theorems for open market operations. Such theorems can be derived only when non-distorting taxes are available. Thus, the Sargent-Smith claim requires that the apparent use of distorting taxes be illusory.

Furthermore, there are claims in the empirical public finance literature that the marginal excess burden of public funds could be quite low. Edgar Browning (1976) estimates the marginal excess burden to be as low as 9 cents per dollar, of which 2-2 1/2 cents represents estimated costs of collection and enforcement. Charles Stuart (1984), using a different methodology, obtains estimates as low as 9 cents on the dollar, while Ballard, Shoven, and Whalley (1985) get estimates as low as 15 cents. While in general the range of estimates obtained is quite large [see, e.g., Browning (1987)], and very sensitive to small variations in parameter values, these results can be viewed as suggestive that distortions due to taxation may be quite small.

Finally, the findings of researchers such as Kydland and Prescott (1982) and Hansen (1986) that competitive models, which are distortion free, can readily mimic U.S. economic time series, are at least consistent with the idea that economic distortions due to taxation are not important.

While the evidence just cited is not very direct, it is at least suggestive that there is some support for the empirical implications of a cooperative view of the problem of policy formation, or at a minimum, enough to warrant further investigation.

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Notes

1. This idea is similar to that employed by Myerson(1983) where different possible types of the same principle play a cooperative game of mechanism design. Our notion of cooperation will also turn out to closely resemble that used by Bernheim, et. al. (1987) to define "coalition proof" equilibria.

2.Or, as a shorthand, we will often just say "is blocked at t".

3. Notice that we have not shown that these rules describing the "best" policy at t exist. Thus a more correct statement of the theorem would be that, if rules describing the best policy choice $f_t(x_1,...,x_{t-1},\pi_1,...,\pi_{t-1})$ exist, then the sequence of policy actions described in the theorem is not blocked.

4. The omission of c_t as an argument of the function $R_t(\cdot)$ $(t=1,2)$ amounts to a standard normalization in optimal tax settings.

5. As above, the constraint $V_2(\bar{c}_1,\bar{\ell}-\bar{\ell}_1,c_2,\bar{\ell}-\ell_2,\bar{g}_1,g_2) \geq \bar{V}_2$ does not bind.

6. Again, the constraint $V_1[c_1,\bar{\ell}-\ell_1,h(\cdot),\bar{\ell}-n(\cdot),g_1,m(\cdot)] \geq \bar{V}_1$ does not bind in this problem.

7. As the literature on "refinements" of perfect or sequential equilibrium concepts strongly suggests. See, e.g., Kohlberg and Mertens (1986).

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