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Abstract

This paper developes a theory of the international trade pattern in risky assets by applying the law of comparative advantage to asset trade. According to this law there is a tendency for a country to import assets that have relatively high autarky prices. The autarky price of an asset is high if the autarky real interest rate is low, or if the asset's autarky risk measure (the product of the risk premium and the asset price) is low. It is examined how autarky interest rates and risk measures are affected by international differences in (i) stochastic properties of output/endowments, (ii) the rate of time preference, (iii) the degree of risk aversion, and (iv) subjective beliefs, and how such differences predict overall capital account deficits or surpluses as well as the composition of the capital account into trade in arbitrary risky assets and the special cases of sure indexed bonds, claims to output (equity), and Arrow-Debreu securities.

1. Introduction

This paper develops a simple but general theory of the determinants of the international pattern of trade in risky assets. The importance of international trade in risky assets is obvious, with increased liberalization of international capital movements, and with the observation that in practice all assets are risky in the sense that their real returns are uncertain.[1] Yet it seems that there is much less research done on the pattern of trade in explicitly risky assets than on the pattern of trade in goods.

The theory is developed by borrowing from and synthesizing several strands of literarure. We start from the modern formulations of standard international trade theory, more precisely the general law of comparative advantage as developed by Deardorff (1980) and Dixit and Norman (1980). According to the law of comparative advantage there is a positive correlation between a country's net import of goods and the country's autarky prices relative to world prices (or relative to autarky prices in the rest of the world), such than on average a country is a net importer of goods for which autarky prices are relatively high. With only two goods, the law of comparative advantage provides an exact relation between the trade pattern and relative autarky prices. With more than two goods, it provides only a correlation between the vectors of net import and relative autarky prices, and it does not provide an exact relation for each individual good.

It is well known that the standard trade theory can be extended to an intertemporal theory of international borrowing and lending, by interpreting commodities as dated goods. The law of comparative advantage then implies

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[1] Stocks and equities are obviously risky assets, but so are all nominal bonds in any currency since there is exchange rate and price level risk. Exchange rate risk make even very short-term bank deposits risky. A non-risky asset would be a hypothetical appropriately indexed (to some consumer price index, say) short-term deposit. Even such an asset is not sure in utility terms (see footnote 14).

that a country will on average have a trade surplus in periods for which the autarky present value of goods is relatively high, that is, for which autarky interest rates are relatively low.[2] It is also clear that the standard trade theory can be extended to a the case with uncertainty, where goods are distinguished by the state of the world in which they occur.[3] The princple of comparative advantage then says that a contry will on the average import goods in states for which the autarky prices for Arrow-Debreu securities, that is, state-contingent deliveries, are relatively high.

A special case of trade in risky assets has received considerable interest. This is trade in claims to firms' profits, equity. After pioneering work by Helpman and Razin (1978), a number of papers have recently examined the effects on trade in equities on welfare, resource allocation, and the goods trade pattern.[4]

Here we will reformulate the law of comparative advantage so as to cover the case of trade in any arbitrarily specified set of assets, complete or incomplete.[5] This will allow us to include as special cases trade in sure indexed bonds, trade in Arrow-Debreu securities, and trade in equities, or rather claims to firms' output (we shall make simplifying assumption of exogenous stochastic outputs/endowments and no inputs, so as to be able to disregard the effect of trade in assets on production decisions, in which case

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2 For an explicit statement of the intertemporal extension of the standard trade theory, see Persson and Stockman (1987).

3 See Pomery (1984), Helpman (1985a), and Persson and Stockman (1987).

4 See for instance Pomery (1984) and later work by Helpman (1985a,b) and Grossman and Razin (1984, 1985). Cole (1986) examines the effect of trade in different kinds of assets (ex post securities, Arrow-Debreu state-contingent deliveries, and Helpman-Razin equities) on variance and covariance of key real variables, like output, consumption, and trade balance.

5 The set of assets is complete (incomplete) in the usual sense of having at least as many (fewer) linearly independant assets as (than) the number of states of the world.

claims to claims to profit and claims to output coincide).

In standard trade theory, there are basically two approaches to examine the determinants of the trade pattern. One is to start from the law of comparative advantage and its emphasis on autarky price differences, and then to go behind the autarky price differences and explain how these are caused by underlying differences between countries with respect to technology, endowments, preferences, or other characteristic. The other, the "direct" approach, is to look directly at trade equilibria without any reference to autarky prices, and infere how differences between countries directly determines the trade patter. Whereas the autarky prices approach was common in the early work on the goods trade pattern,[6] the direct approach has more recently been the dominant one, both in standard trade theory and in the literature on trade in equities referred to above.[7]

There is, however, a special reason for basing a theory of the trade pattern for risky assets on relative autarky prices. The reason is that we can borrow from the general-equilibrium asset-pricing theory developed by Lucas (1978), Breeden (1979), and others. It turns out to be very convenient to use this theory in order to express autarky asset prices in terms of autarky real interest rates and risk premia. Our work is hence closely related to international applications of this asset pricing theory, for instance Lucas (1982), Stulz (1981, 1984), Svensson (1985) and Stockman and Svensson (1987). That literature has focused on the determinants of prices on internationally traded risky assets, but not examined the trade pattern in risky assets in itself. In the typical set-up, like in Lucas (1982), there is

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6 See for instance the classic paper by Jones (1956).

7 For examples of use of the direct approach to the determinants of the pattern of trade, see the survey by Ethier (1984), Dixit and Woodland (1982) and Markusen and Svensson (1985) for trade in goods, and Svensson (1984) and Ethier and Svensson (1986) for trade in goods and factors.

trade in the outside assets, namely claims to output (equities), currencies and claims to government transfers. Since representative consumers with identical preferences are assumed, there is no trade in other, inside assets (which does not prevent any arbitrary inside asset to be priced, though). Furthermore, the trade pattern in the existing outside assets is trivial, since a perfectly pooled equilibrium is assumed, in which all investors hold the same portfolio.[8] In our analysis, equilibria will generally not be perfectly pooled.

We mentioned that our theory is general in the sense of covering any arbitrary complete or incomplete set of assets, including as special cases sure indexed bonds, equities and claims on output, and Arrow-Debreu securities. Also, our theory includes the determinants of the aggregate current account and capital account, hence aggregate international borrowing and lending, as well as the composition of the capital account, the trade in individual assets (subject to the qualification that when there are many assets results are in the form of correlations and hold on average, but not exactly, for each individual asset).

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[8] That is, relative to autarky each country (in a two-country world) exports half of its assets and imports half of the other country's assets. Still, capital movements, and correlations between key macro variables like investment, the current account, output, etc., can be studied, as in Stockman and Svensson (1987), but any current and capital account movements are due exclusively to revaluation of domestically based assets relative to foreign based assets, not to changes in the ownership of assets.

Dumas (1986) considers a model whith two investors with different degrees of risk aversion where the investors' portfolios are revised over time and asset trade between them occur. Stockman and Dellas (1986) and Stulz (1986) consider international asset pricing models with nontraded goods, where consumers do not have perfectly pooled equilibria but hold a larger share of domestic assets. The focus is exclusively on equilibrium asset price and exchange rate determination and variability. Stockman and Hernandez (1986) utilizes an international asset pricing model to demonstrate that the effect on policy like capital controls depends crucially on whether the private sector can hedge against the policy by trading in risky assets (in their case Arrow-Debreu securities). Gordon and Varian (1986) discuss welfare effects of taxes on internationally traded risky assets in a CAPM model and examine the analog to the optimum tariff result for trade in goods.

The first step in our method is to express the autarky asset price for a given asset in terms of the autarky real interest rate and the autarky risk measure (the risk measure is the product of the risk premium and the asset price). Differences in countries' autarky real interest rates affect the autarky prices of and trade in all assets, and are related to whether a country has an overall capital account deficit or surplus and hence is a net lender or borrower. A country with a relatively low autarky real interest rate has a tendency to have an overall capital account deficit and be a net lender. Differences in autarky risk measures are specific to individual assets and are related to the trade in individual assets. A country with a relatively low autarky risk measure for an asset (that is, for which an asset is relatively less risky) has a tendency to import that asset.

The second step is to examine what determines the differences between countries' autarky real interest rates and risk measures. We will look at the effect on autarky real interest rates and risk measures of differences between countries with respect to technology, endowments and preferences, more precisely (i) the stochastic properties of output/endowments, (ii) the rate of time preference, (iii) the degree of risk aversion, and (iv) expectations (subjective probability beliefs).

The paper is organized as follows. Section 2 and 3 deal with preliminaries and can be skimmed by readers not interested in the standard derivation of the law of comparative advantage. Section 2 describes the model, the equilibrium for a single country, and demonstrates gains from trade in risky assets. Section 3 describes a world equilibrium with two countries and derives the law of comparative advantage for trade in risky assets. Section 4, the core of the paper, discusses the determination of autarky asset prices, derives the effect of cross-country differences in technology/endowments and preferences on autarky real interest rates and risk measures, and finds the trade pattern for arbitrary assets as well as the special cases of sure bonds, claims to output, and Arrow-Debreu securities. Section 5 concludes. An Appendix presents a proof of a theorem used.

2. Equilibrium in a Single Country and Gains from Asset Trade

We consider a situation with one good and two periods. Let us first look at a single country, called the home country. The home country has given output/endowments of the good in the two periods. Output in period 1 is certain, whereas output in period 2 is uncertain. We represent the uncertainty by S possible states of the world in period 2, denoted s = 1, 2,..., S. The outputs can then be represented by the (1+S)-vector $(y^1, y^2) = (y^1, (y_s^2))$, where y^1 denotes period 1 output, and $y^2 = (y_s^2)$ is an S-vector where compenents y_s^2 denote period 2 output in state s. Goods are perishable and there is no storage or other investment technology.

There is a given set J of J different assets. (We let J denote both the set and the number of elements of the set.) These assets are traded on a world asset market in period 1, before the uncertainty about the state of the world in period 2 is resolved. Each asset $j \in J$ is characterized by a given (total real) return S-vector $R_j = (R_{js})$, whose compenents R_{js} are the (net) returns paid in goods in state s in period 2. Dividends are not necessarily positive in all states.

Let us look at some special assets. First, the sure bond pays one unit of the good in each state. It is identified with j = 0 and is defined by

(2.1a) $\quad R_{0s} = 1$ for s = 1,..., S.

A second special case is trade in claims to output. Let us indentify a claim to home period 2 output as asset j = h, defined by the return vector

(2.1b) $\quad R_{hs} = y_s^2$ for s = 1,..., S.

Third, the Arrow-Debreu securities are the S assets who each pay one unit of the good in one specific state only. We identify the Arrow-Debreu security for state s with $j = s$, $s = 1,\dots, S$. It is defined by

(2.1c) $\quad R_{s\sigma} = 1$ for $\sigma = s$, $R_{s\sigma} = 0$ for $\sigma \neq s$, $\sigma = 1,\dots, S$.

In standard terminology, the asset market is said to be complete if the set J of assets is such that the SxJ-matrix R formed by J columns of return vectors R_j, $j \in J$, is of rank S. Then agents can reach the same consumption bundle across states via trade in the available assets as they can via trade in the S Arrow-Debreu securities. If the rank of the matrix R is less than S, the asset market is incomplete. Our analysis does not presume that the asset market is complete or that trade in Arrow-Debreu securities is feasable, but incorporates these possibilities as special cases.

The home country has a representative consumer who is entitled to home output in the two periods. The consumer has a subjective probability distribution $f = (f_s)$ over the states of the world. The consumer has preferences over consumption (1+S)-vectors $(c^1, c^2) = (c^1, (c^2_s))$ of period 1 consumption, c^1, and period 2 consumption in state s, c^2_s. The preferences can be represented by the additively separable expected utility function

(2.2) $\quad u(c^1, c^2) = U(c^1) + \beta\Sigma_s f_s U(c^2_s)$,

where $U(\cdot)$ is a standard increasing concave sufficiently differentiable von Neumann-Morgenstern utility function and $\beta > 0$ is is the subjective discount factor.[9]

Let m denote (net) import of period 1 goods, and let the J-vector $z = (z_j)_{j\in J}$ denote (net) import of the J assets from the world asset market in period 1. Then period 1 consumption and period 2 consumption in state s is given by

(2.3a) $\quad c^1 = y^1 + m$ and

(2.3b) $\quad c^2_s = y^2_s + \Sigma_{j\in J} R_{js} z_j$.

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[9] As is well known, representing preferences by an additively separable expected utility function does not allow a separation between risk aversion and intertemporal substitution in consumption (see Selden (1978, 1979)).

It is practical to define preferences directly over import of period 1 goods and assets. Substitution of (2.3) into (2.2) allows us to define the trade utility function $\tilde{U}(m,z)$ by

(2.4) $\tilde{U}(m,z) \equiv U(y^1 + m) + \beta\Sigma_s f_s U(y_s^2 + \Sigma_{j\in J}R_{js}z_j)$.

Let p and $q = (q_j)_{j\in J}$ denote the price of period 1 goods and the J-vector of asset prices. It is convenient to define the balance-of-payments (deficit) function B(p,q,u) as the minimum expenditure on import of goods and assets required to reach a given utility level.[10] That is,

(2.5) $B(p,q,u) = \min\{pm + qz \mid \tilde{U}(m,z) \geq u\}$,

where qz denotes the inner product $\Sigma_{j\in J}q_jz_j$. (The balance-of-payments function is simply the standard expenditure function minus the value of period 1 output.)

In the rest of the paper we will take period 1 goods to be the numeraire, p = 1, and hence express asset prices q in terms of period 1 goods.

It is now easy to represent a trade equilibrium for the economy, an equilibrium in which the economy faces a given vector of asset prices q on the world asset market. It is simply given by the equations

(2.6) $B(1,q^t,u^t) = 0$,

(2.7a) $m = B_p(1,q^t,u^t)$ and

(2.7b) $z = B_q(1,q^t,u^t)$.

Equation (2.6) says that the balance of payments are zero in equilibrium, whereas equations (2.7a) and (2.7b) express import of goods and assets as the derivative of the balance-of-payments function with respect to the price of period 1 goods and asset prices respectively, exploiting standard properties of expenditure functions. For given world asset prices q^t, equations (2.6) and (2.7) can be solved for the corresponding home utility level u^t and the

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10 This function occurs in the literature under a variety of names. See Lloyd and Schweinberger (1986) for references to its use in previous literature.

import m and z of goods and assets.

An <u>autarky equilbrium</u>, an equilibrium without access to the world asset market, is given by the equations (2.6) and

(2.8) $\quad B_q(1,q,u) = 0,$

the latter stating that the import of assets is zero. (Import of period 1 goods is then also zero, $B_p(1,q,u) = 0$, but by Walras's Law that equation is redundant.) Equations (2.6) and (2.8) can be solved for the autarky asset prices q and the autarky utility level u.

It follows that the <u>gains-from-trade</u> theorem holds: Let u^t be the utility level associated with a trade equilbrium, and let u be the utility level in an autarky equilibrium. Then we have

(2.9) $\quad u^t \geq u.$

The proof is as in the standard trade model (see for instance Dixit and Norman (1980) or Woodland (1982)). First, we have

(2.10) $\quad B(1,q^t,u^t) = 0 = m^a + q^t z^a \geq B(1,q^t,u).$

The balance of payments in the trade equilibrium is zero (the first equality in (2.10). This trivially equals the value at trade asset prices q^t of the autarky import m^a and z^a of period 1 goods and assets, since these are zero (the second equality in (2.10)). Zero import gives autarky utility level u. The minimum import expenditure at trade prices required to reach utility level u cannot be larger, and will be less if there is some substitution and trade prices differ from autarky prices (the inequality in (2.10)). Second, since the balance-of-payments function is increasing in utility, (2.9) follows from (2.10).

We note that the gains-from-trade theorem implies that trade in complete or incomplete asset markets is better than autarky. However, in analogy with the case with goods trade only, it does not follow that trade in more assets is better than in fewer, unless the prices of all previously traded assets remain unchanged. The usual terms-of-trade qualification applies: if the prices of assets previously imported (exported) increase (decrease) when trade in additional assets is opened up, the negative terms-of-trade effect may outweigh the gains from trade.

3. World Equilibrium and the Law of Comparative Advantage

Next we shall consider a two-country world equilibrium. We let the rest of the world be represented by another country, called the foreign country. It has output $(y*^1, y*^2) = (y*^1, (y*^2_s))$ in the two periods (output is uncertain in period 2), access to a world market with the same set J of assets as the home country, a representative consumer entitled to foreign output in the two periods and with a subjective probability distribution $f* = (f*_s)$, a von Neumann-Morgenstern utility function $U*(\cdot)$, a subjective discount factor $\beta* > 0$, and a trade utility function over period 1 goods (net) import m* and asset (net) import z*, $\tilde{U}*(m*,z*)$, defined by the analog to (2.4). We can then represent the foreign country by a balance-of-payments function $B*(p,q,u*)$ defined by the analog of (2.5). A trade equilibrium for the foreign country is then, for given asset prices q^t relative to period 1 goods, the utility level $u*^t$ and the import m* and z* of period 1 goods and assets that solve the equations analog to (2.6) and (2.7). An autarky equilibrium for the foreign country is an autarky asset price vector q* and a utility level u* that fulfill the analogs of (2.6) and (2.8).

A world equilbrium is a vector $(q^t;m,z,u^t;m*,z*,u*^t)$ such that (q^t,m,z,u^t) and $(q^t,m*,z*,u*^t)$ are trade equilibria for the home and the foreign country, respectively, and such that the world asset market and period 1 goods market is in equilibrium,

(3.1a) $z + z* = 0$, and

(3.1b) $m + m* = 0$.

(The world market for period 1 goods is in equilibrium whenever the asset market is in equilibrium, given the budget constraint (2.6) for the home country and the analog for the foreign country.)

Let m and z be the home country's import of period 1 goods and assets in a world equilibrium, and let q and q* be home and foreign autarky asset prices relative to period 1 goods. Then the law of comparative advantage can be written on the form

(3.2) $(q-q^*)z \geq 0.$

It states that on the average, the home country will import assets whose autarky prices are higher in the home country than in the foreign country. If only one asset is traded we have an exact relation between autarky asset prices and the trade pattern: The asset will be imported (and period 1 goods will be exported) if and only if the autarky price of the asset is higher in the home country than in the foreign country. If more than one asset is traded, the law of comparative advantage provides a "tendency" for a particular asset to be imported if its autarky price is relatively high, rather than an exact relation for import in any individual asset.[11]

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[11] As Deardorff (1980) emphasizes, a positive inner product $xy = \Sigma_j x_j y_j \geq 0$ does not exactly provide a positive correlation between the J-vectors $x = (x_j)$ and $y = (y_j)$, unless either $\Sigma_j x_j = 0$ or $\Sigma_j y_j = 0$. This is so, since the sample correlation coefficient cor(x,y) is proportional to the sample covariance cov(x,y) and the latter fulfills $\mathrm{cov}(x,y) = xy - \Sigma_j x_j \Sigma y_j / J$. Deardorff shows how one can construct correlations in two ways. One way is to exploit the balance-of-payments constraint. Let q^t be the asset prices in terms of goods in the world equilibrium. Then (3.2) implies that the (J+1)-vectors $(1,((q_j-q_j^*)/q_j^t))$ and $(m,(q_j^t z_j))$ are positively correlated, and hence that the J-vector $((q_j-q_j^*)/q_j^t)$ of relative autarky asset prices defined this way and the J-vectors of values of asset imports $(q_j^t z_j)$ are positively correlated. The other way is to restrict the vector of goods and asset prices to be in the unit simplex. Then (3.2) implies that the (J+1)-vectors $((1,q)/(1+\Sigma_j q_j) - (1,q^*)/(1+\Sigma_j q_j^*))$ and (m,z) are positively correlated.

For our purpose it is sufficient to interprete (3.2) as stating that there is tendency for asset j to be imported into the home country ($z_j > 0$)

The proof of the law of comparative advantage is as in the standard trade model (see Deardorff (1980), Dixit and Norman (1980) or Woodland (1982)). We have

(3.3) $m + qz \geq B(1,q,u^t) \geq B(1,q,u) = 0.$

The first inequality follows since import (m,z) gives utility u^t but is not necessarily the combination of net import of goods and assets that minimize expenditure at autarky prices. The second inequality follows since we know from the gains-from-trade theorem that the home country's utility level u^t in any trade equilibrium cannot fall short of the utility level in autarky u, and the balance-of-payments function is increasing in utility. The equality follows from the budget constraint (2.6). An analogous argument for the foreign country gives

(3.4) $m^* + q^*z^* \geq 0,$

which we by (3.1) can write as

(3.5) $-m - q^*z \geq 0.$

Addition of (3.3) and (3.5) gives (3.2).

When discussing the the determinants of the trade pattern, one can either examine the world equilibrium directly, or rely on the law of comparative advantage. In the former case, one discusses how differences between countries directly determine the trade pattern, without looking at the autarky prices. In the latter case, one looks at how differences between countries determine relative autarky prices, and the from that indirectly inferes the determinants of the trade pattern. In recent discussions of the trade pattern of goods and factors in the standard trade model, the former route has usually been chosen (see references mentioned in the Introduction). In our case, it is convenient to shoose the latter route, since we can then directly apply a

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when its home autarky price (measured in goods) is higher than its foreign autarky price (measured in goods) $(q_j > q_j^*)$.

standard theory of asset pricing.

4. The Pattern of Trade in Risky Assets

a. The current account and the capital account

Let us state the balance-of-payments relation for the home country in a trade equilibrium. We can write it as

$$m + q^t z = B(1, q^t, u^t) = 0, \tag{4.1}$$

stating that the sum of the current account deficit (net import of goods m)[12] and the capital account deficit (the value of net import of assets qz) is zero. Hence what is being determined in a trade equilibrium is not only the aggregate current and capital account deficits, that is, whether the home country is a net borrower or lender (the intertemporal trade pattern), but also the components of the capital account, the disaggregate trade pattern in individual assets (the interstate trade pattern).

If we would like to concentrate on the intertemporal trade pattern, we could simplify the model by considering trade in only one asset, and even disregard the effect of uncertainty and incomplete markets by then assuming that there is no uncertainty and only one state in period 2 (S=1). This gives us the simpliest possible model to discuss international borrowing and lending. If we would like to concentrate exclusively on the trade pattern in risky assets, we could eliminate the first period, and assume that assets are traded before uncertainty is resolved. This then abstracts from intertemporal trade and gives us the simpliest possible model of trade in risky assets, "interstate" trade.

As we shall see, in the more general model intertemporal trade and interstate trade are not independent, and, for instance, the available assets affect a country's current account. Therefore, we choose to keep the

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[12] Since there is no initial international debt, the trade balance and the current account coincide.

two-period framework. This also has the advantage that the expressions for asset prices to be derived are similar to those used in the asset-pricing literature.

b. Autarky asset prices

The home autarky asset price q_j of a particular asset j with return vector $R_j = (R_{js})$ is simply given by the marginal rate of substitution between asset j and period 1 goods of the trade utility function (2.4) at zero import of goods and assets, $\tilde{U}_j(0,0)/\tilde{U}_m(0,0)$, where $\tilde{U}_j$ and $\tilde{U}_m$ denote the partial with respect to z_j and m. It follows from (2.4) that the autarky asset price will fulfill

$$(4.2) \qquad q_j = \beta\Sigma_s f_s U_c(y_s^2)R_{js}/U_c(y^1) = \beta E[U_c(y^2)R_j]/U_c(y^1),$$

the familiar expression of the discounted expected utility of period 2 returns over the marginal utility of period 1 counsumption. (We alternately use use the obvious notation $E[\xi] = \Sigma_s f_s \xi_s$ for expected values.)

It is practical to relate the price of an asset to the real interest rate on a sure bond, and to the risk measure for the asset. In order to do that, We rewrite the asset price as

$$(4.3) \qquad q_j = \beta E[U_c(y^2)/U_c(y^1)]E[R_j] + \beta Cov[U_c(y^2)/U_c(y^1),R_j],$$

where we have used that $E[xy] = E[x]E[y] + Cov[x,y]$. The first term on the right-hand side is the value of a claim to a sure delivery in period 2 of the expected returns $E[R_j]$. The second term is related to the asset's risk properties. To be specific, consider first the sure bond, which pays one good in each state in period 2, $R_{0s} = 1$ for $s = 1,\dots, S$. Its autarky price is, by (4.2),

$$(4.4) \qquad q_0 = \beta E[U_c(y^2)]/U_c(y^1),$$

from which we can define the autarky (real) interest rate, r, by

$$(4.5) \qquad q_0 = 1/(1+r).$$

Clearly, the first term on the right-hand side of (4.3) is simply $E[R_j]/(1+r)$.

Next, let us define the autarky risk measure for asset j, Π_j, as

(4.6) $$\Pi_j = -\beta \mathrm{Cov}[U_c(y^2)/U_c(y^1), R_j].$$

The risk measure is equal to the negative of the covariance between the marginal rates of substitution in consumption $\beta U_c(y_s^2)/U_c(y^1)$ and the returns R_{js}.[13] Hence it is positive or negative depending upon whether period 2 marginal utilities and returns are negatively or positively correlated. The risk measure for an asset can be interpreted as a measure of how risky that asset is relative to the sure bond. If the risk measure is positive, the asset is riskier than the sure bond. If it is negative, the asset is less risky than the sure bond.[14]

By (4.4)-(4.6) we can now rewrite (4.3) as

(4.7) $$q_j = E[R_j]/(1+r) - \Pi_j.$$

We see that the asset price can be written as the present value of its expected return minus its risk measure. It is clear from (4.7) that autarky prices for a given asset may differ accross countries because autarky interest rates, autarky risk measures, or both, differ across countries. The analysis below consequently examines the underlying determinants of differences in autarky interest rates and risk measures.

c. <u>Trade in risky assets</u>

We shall examine the difference in the home and foreign countries' autarky asset prices of a given asset $j \in J$. We will look for conditions under

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[13] The risk <u>premium</u> can be defined as Π_j/q_j and fulfills $\Pi_j/q_j = -\beta \mathrm{Cov}[U_c(y^2)/U_c(y^1), R_j/q_j]$ and is hence the negative of the covariance between the marginal rates of substitution and the ex post <u>rates</u> of return R_{js}/q_j.

[14] Note that the sure bond has a sure return, but that the utility value of the return is risky, since marginal utility itself is risky. Hence there is nothing paradoxical with assets that are less risky than the sure bond. A sure-utility bond (in autarky) (j = u) would have returns $R_u = (R_{us})$ fulfilling $U_c(y_s^2)R_{us} = 1$, hence $R_{us} = 1/U_c(y_s^2)$ for s = 1,..., S.

which the home country's autarky asset price exceeds the foreign country's autarky asset price, and hence under which there will be a tendency in a world equilibrium for asset j to be imported by the home country and exported by the foreign country. In the special case where asset j is the only traded asset we will know for sure that asset j will be imported.

The home autarky asset price of asset j is given by expression (4.2), or (4.7). The foreign autarky asset price is given by an analogous expression, with a * denoting foreign output and preferences. Let us assume that the subjective probability distribution is the same in the home and foreign country,

(4.8) $f = f^*$,

so the expected return for a given asset j is the same in both countries

(4.9) $E[R_j] = E^*[R_j]$.

(Below we shall discuss also the case when the subjective probability distribution differ across countries and (4.9) does not hold.) Let us also restrict the discussion to assets with positive expected return,

(4.10) $E[R_j] > 0$.

(If the expected return is negative, we can simply redefine the asset by changing the sign of its returns.) Using expression (4.7) and its foreign analog, we can write the autarky price difference for asset j as

(4.11) $q_j - q_j^* = (r^* - r)E[R_j]/[(1+r)(1+r^*)] + (\Pi_j^* - \Pi_j)$.

Hence the home autarky interest rate and the risk measure for asset j should be relatively low for the asset to have a higher home autarky price and for the home country to have a tendency to import the asset.

If the countries are identical in all respects, the autarky asset prices will be identical, there is no basis for trade, and zero trade will be a trade equilibrium. Hence, trade here arises because of differences between the countries. The countries can differ either with regard to their outputs, or with regard to their preferences, including their subjective probability distributions. Let us first consider a situation when the only difference between the countries is with regard to their outputs.

(i) Differences in output

Thus, we assume that the foreign country is identical to the home country in all respects except the outputs, and we drop the * on the foreign country's preferences.

Let us first look for conditions under which the home autarky interest rate is lower than the foreign one,

(4.12) $r < r*$.

If the home autarky interest rate is lower, this contributes to the home autarky prices being higher for all assets, and hence to a tendency for the home country to import all assets and be a net lender. If the only asset traded is the sure bond, we have an exact result and know for sure that the the home country will import the sure bond and be a net lender.

We can examine this by looking at the difference in autarky prices of the sure bond. The difference is given by

(4.13) $$q_0 - q_0^* = [1/(1+r) - 1/(1+r*)] =$$
$$= \beta E[(U_c(y^2)/U_c(y^1)) - (U_c(y*^2)/U_c(y*^1))].$$

We like to know under what conditions this differense is positive. Let us first assume that the countries differ only with respect to period 1 output. We then have

(4.14) $$q_0 - q_0^* = \beta E[(U_c(y^2)]/[1/U_c(y^1) - 1/U_c(y*^1)].$$

Since the marginal utility of consumption is decreasing, it follows directly that the home autarky price of the sure bond is higher, and the home autarky interest rate lower, if the home country has a higher period 1 output,

(4.15) $y^1 > y*^1$.

This is a standard consumption smoothing result (across countries, though, not across time).[15] The home country has relatively more output in period 1, and it will export goods in period 1 and import goods in period 2, by being a net lender in period 1.

Let us next assume that period 1 output is the same in the two countries, but that period 2 output is different. Then we have

(4.16) $q_0 - q_0^* = \beta E[(U_c(y^2) - (U_c(y*^2)]/U_c(y^1)$.

Since the marginal utility of consumption is decreasing, it follows (see Theorem 1 in Lippman and McCall (1981)) that a sufficient condition for (4.16) to be positive is that home period 2 output is stochastically smaller than foreign period 2 output, that is, home period 2 output is first-order stochastically dominated by foreign period 2 output, denoted

(4.17) $y^2 <_1 y*^2$.

First-order stochastic dominance of home output by foreign output implies that the expected value of home output is smaller,

(4.18) $Ey^2 < Ey*^2$,

and can be understood as a generalization of that property.[16]

This result is also a straight-forward consumption smoothing results. If the home country has lower expected period 2 output than the foreign country, it will export goods in period 1 and import goods in period 2, by being a net lender in period 1.

Under the assumption that preferences exhibit non-increasing absolut risk aversion the third-order derivative U_{ccc} of the von Neumann-Morgenstern

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15 If both countries have less period 1 output than period 2 output (average or for each state of the world), home consumption becomes more unevenly divided over time with trade in the sure bond than in autarky.

16 Let $G(\cdot)$ and $G*(\cdot)$ denote the cumulative distribution functions for the random variables y and y*, respectively. We say that y* is stochastically larger than y, written $y* >_1 y$, or $G* >_1 G$, if and and only if $G(x) - G*(x) \leq 0$ for all x. Equivalently, we say that y* stochastically dominates y to the first order. See Lippman and McCall (1981).

utility function is positive,[17]

(4.19) $U_{ccc} > 0$,

and the marginaly utility of consumption is a convex function of consumption. Then, another sufficient condition for (4.16) to be positive (see Theorem 2 in Lippman and McCall (1981)) is that home period 2 output is more risky than foreign period 2 output, that is, home period 2 output is second-order stochastically dominated by foreign period 2 output, denoted

(4.20) $y^2 <_2 y*^2$.

A special case of this is when home and foreign period 2 output have the same mean but home output has a larger variance,

(4.21) $\mathrm{Var}[y^2] > \mathrm{Var}\ [y*^2]$,

or when home period 2 output is a mean-preserving spread of foreign period 2 output. Second-order stochastic dominance can hence be understood as a generalization of those special cases.[18]

Intuitively we can understand this result the following way. If marginal utility is a convex function of consumption, Jensen's inequality implies that increased variance in consumption increases expected marginal utility, which increases the price of the sure bond and decreases the interest rate. If the third-order derivative is negative, the opposite result holds. This is an example of the ambiguity of the effect on saving on increased riskiness of future income (see the survey by Sandmo (1974)). In the literature there is

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17 The measure of local absolut risk aversion is $-U_{cc}/U_c$. We have $(d/dc)(-U_{cc}(c)/U_c(c)) = -U_{ccc}/U_c + (U_{cc}/U_c)^2 \leq 0$, which implies $U_{ccc} \geq (U_{cc})^2/U_c > 0$.

18 Let $G(\cdot)$ and $G*(\cdot)$ denote the cumulative distribution functions for the random variables y and y*, respectively. We say that y* is less risky than y, written $y* >_2 y$, or $G* >_2 G$, if and and only if $\int_{-\infty}^{x}[G(z) - G*(z)]dz \geq 0$ for all x. Equivalently, we say that y* stochastically dominates y to the second order. See Lippman and McCall (1981).

general aggreement that non-increasing absolut risk aversion and hence a positive third-order derivative of the von Neumann-Morgenstern utility function is the most relevant case.

Let us next turn to differences in the risk measures. From (4.11) we know that there is a tendency for asset j to be imported if the home autarky risk measure is lower, that is, if the home country perceives asset j as less risky than the foreign country. Risk terms are specific to individual assets and depend on the individual risk characteristics of the asset. Hence differences in risk measures for a given asset give information about trade in that specific asset, whereas differences in autarky interest rates affect autarky asset prices for all assets, and hence give information about the aggregate asset trade (whether a country will be a net lender or not).

Let us look at the condition for the home autarky risk measure for asset j to be lower,

(4.22) $\quad \Pi_j < \Pi^*_j.$

Assume that period 1 output is the same in both countries. From (4.6) we see the home autarky risk measure then is lower if

(4.23) $\quad \mathrm{Cov}[U_c(y^2), R_j] > \mathrm{Cov}[U_c(y*^2), R_j],$

that is, if the return is more positively correlated with home marginal utility of consumption than with foreign marginal utility of consumption.

Since marginal utility of consumption is decreasing in consumption, we might believe that (4.23) implies the simple condition that the return should be more negatively correlated with home period 2 output than with foreign period 2 output,

(4.24) $\quad \mathrm{Cov}[y^2, R_j] < \mathrm{Cov}[y*^2, R_j].$

However, (4.24) follows only under special circumstances. First, assume that (y^2, R_j) is bivariat normal.[19] Then, according to a theorem of Rubinstein

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[19] Note that the assumption of a normal distribution is problematic, since

(1976) we have, under some mild regularity conditions,

(4.25) $\quad \mathrm{Cov}[U_c(y^2),R_j] = E[U_{cc}(y^2)]\mathrm{Cov}[y^2,R_j]$.

If we also assume that $(y*^2,R_j)$ is bivariat normal, it follows that (4.25) and its analog for $(y*^2,R_j)$ imply

(4.26) $\quad |E[U_{cc}(y^2)]|\mathrm{Cov}[y^2,R_j] < |E[U_{cc}(y*^2)]|\mathrm{Cov}[y*^2,R_j]$.

Clearly, $|E[U_{cc}(y^2)]|$ need not equal $|E[U_{cc}(y*^2)]|$, and (4.24) may imply a bias relative to (4.26). Under the assumption that the third-order derivate U_{ccc} is positive, $|U_{cc}(\cdot)|$ $(= -U_{cc}(\cdot))$ is decreasing, and it follows that $y*^2$ stochastically larger than y^2, (4.17), implies that $|E[U_{cc}(y^2)]| >$ $|E[U_{cc}(y*^2)]|$. Even with a positive third-order derivative, $|U_{cc}(\cdot)|$ can be either convex or concave, or neither, and hence $y*^2$ less risky than y^2, (4.20), has no determinate effect on it. (With constant relative risk aversion, $|U_{cc}(\cdot)|$ is convex and (4.20) implies that $|E[U_{cc}(y^2)]| >$ $|E[U_{cc}(y*^2)]|$.)

Second, assume that the bivariate distribution of (y^2,R_j) is symmetrical, but not necessarily normal, and assume that y^2 is has small even higher-order moments (for instance because y^2 is bounded). Then it is easy to show that

(4.27) $\quad \mathrm{Cov}[U_c(y^2),R_j] = U_{cc}(Ey^2)\mathrm{Cov}[y^2,R_j]$

is approximately true. Making the same assumptions for $(y*^2,R_j)$ and $y*^2$, we have that (4.27) implies

(4.28) $\quad |U_{cc}(Ey^2)|\mathrm{Cov}[y^2,R_j] < |U_{cc}(Ey*^2)|\mathrm{Cov}[y*^2,R_j]$.

(This expression differ from (4.26) since the bivariate normal distribution does not fulfill the assumption of sufficently small higher-order moments.) Again, $y*^2$ stochastically larger than y^2, which implies $Ey^2 < Ey*^2$, leads to $|U_{cc}(Ey^2)| > |U_{cc}(Ey*^2)|$, and (4.24) implies a bias relative to (4.28).

The bias is caused by the same circumstances that cause autarky real interest rates to differ. We have already noted that low home autarky

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it implies that y^2 can take negative values with positive probability.

interest rates creates a tendency for all assets to be imported. Let us therefore isolate the effects of the individual risk measures of assets, by assuming that autarky interest rates are the same. That is also likely to remove (most of) the bias in (4.24), and make the latter a good approximation.[20] We then conclude, that under the assumption of equal autarky interest rates, the condition is simply that the return should be more negatively correlated with home relative period 2 output than with foreign relative period 2 output. Then asset j is less risky in the home country, its autarky risk measure is lower, its autarky asset price is higher, and there is a tendency that the asset will be imported by the home country.

Let us next consider trade in claims to home and foreign output. Because of symmetry we need only look at a claim to foreign output. The claim to foreign (period 2) output is represented by the asset j = f, for which $R_{fs} = y_s^{*2}$ for s = 1,..., S. Trade in the claim to foreign output is of course affected by differences in autarky interest rates, since these affect trade in all assets. Suppose now that autarky interest rates are the same. Then the autarky risk measure is the only source of differences in autarky asset prices. The condition for the home autarky risk measure for the claim to foreign output to be low, and thus for a tendency for the home country to import the claim to foreign output, is, from (4.24),

$$(4.29) \qquad \mathrm{Cov}[y^2, y^{*2}] < \mathrm{Cov}[y^{*2}, y^{*2}] = \mathrm{Var}[y^{*2}].$$

We know that $\mathrm{Cov}[y^2, y^{*2}] \leq (\mathrm{Var}[y^2]\mathrm{Var}[y^{*2}])^{1/2}$. If we assume that home and

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[20] An interesting special case is when the von Neumann-Morgenstern utility function has constant absolut risk aversion (that is, when $U(c) = -\exp[-\gamma c]$, $\gamma > 0$, in which case the constant $\gamma = -U_{cc}/U_c$ is Arrow-Pratt's measure of absolut risk aversion). If (y^2, R_j) and (y^{*2}, R_j) are normal (that is, (4.25) and its foreign analog hold), it is easy to show that $\Pi_j = \gamma\mathrm{Cov}[y^2, R_j]/(1+r)$ and $\Pi_j^* = \gamma\mathrm{Cov}[y^{*2}, R_j]/(1+r^*)$. Then, for $r = r^*$, $\Pi_j < \Pi_j^*$ is equivalent to (4.24).

foreign period 2 output has the same variance, which from our previous discussion is in accordance with the assumption of equal autarky interest rates, we get that a sufficient conditon for (4.29) is that home and foreign output are less than perfectly positively correlated. By symmetry, there will be a tendency for the home country to export a claim to home output, if home and foreign output are less than perfectly correlated.

Let us finally consider Arrow-Debreu securities. The home autarky price of Arrow-Debreu security s, s = 1,.., S, will be

(4.30) $q_s = \beta f_s U_c(y_s^2)/U_c(y^1)$.

When the countries' period 1 output are equal, it follows directly that there is a tendency for Arrow-Debreu security s to be imported if home period 2 output in state s is lower than that of the foreign country,

(4.31) $y_s^2 < y_s^{*2}$.

That is, trade in Arrow-Debreu securites is simply related to the relative scarcity of period 2 output.

Let us summarize the results on output differences and asset trade (see Table 1, row (i)). First, in general a low home autarky interest rate implies a tendency for the home country to import all assets and be a net lender. If the only traded asset is a sure bond, it will definitely be imported by the home country. The home autarky interest rate is low if home period 1 output is high, or if home period 2 output is stochastically smaller than foreign period 2 output. The home autarky interest is also low if preferences exhibit decreasing absolut risk aversion, and if home period 2 output is riskier than foreign period 2 output. Second, in general a low autarky risk measure for an asset (the product of the risk measure and the asset price) implies a tendency for the home country to import the asset. The autarky risk measure is low if the asset's returns are more positively correlated with home autarky marginal rates of substitution than with foreign autarky marginal rates of substitution. If autarky interest rates are equal, under some restrictions there is a more specific result: If the bivariate distributions between the return and home period 2 output, and between the return and foreign period 2 output, are either normal or symmetric with small higher-order moments, the autarky risk measure is low if the asset's return is more negatively correlated with home output than with foreign output. Third, if autarky interest rates are equal, there is a tendency for the home country to import a claim to foreign output, and export a claim to home output, if home and foreign output are less than perfectly positively correlated. Fourth, there is a tendency to import an Arrow-Debreu security for a particular state if home period 2 output in that state is lower than in the foreign country.

Next, we assume that outputs are identical in the two countries, but that preferences differ.[21] We shall consider differences in the rate of time preference (the subjective discount factor), the degree of risk aversion, and the subjective probability distribution.

(ii) Differences in the rate of time preference

The effect of differences in the rate of time preference is easy to see. Consider the situation when the home country has a lower rate of time preference than the foreign country. That is, the home subjective discount factor is larger,

(4.32) $\beta > \beta*$.

It follows from the definition of the autarky asset price (4.2) that home autarky asset prices will be higher for all assets (with positive asset prices).[22] Everything else equal, there is a tendency for all assets to be

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21 We assume $(y_s^2) = (y_s^{*2})$, that is, home and foreign period 2 output are perfectly correlated. This is of course not equivalent to assuming that home and foreign output are i.i.d. In the former case, claims to home and foreign output are perfect substitutes. In the latter case, they are not.

22 We relize that assuming that expected dividends are positive, (4.10), is

imported into the home country, and the home country to be a net lender.

(iii) Differences in risk aversion

Suppose preferences are characterized by constant relative risk aversion. That is, the home country has the von Neumann-Morgenstern utility function

(4.33) $U(c) = c^{1-\rho}/(1-\rho)$,

with $\rho > 0$ being the degree of (relative) risk aversion. The foreign country has the von Neumann-Morgenstern utility function $U^*(c)$ with degree of risk aversion ρ^*. We assume that the countries differ only with respect to the degree of relative risk aversion, and that the home country is more risk averse,

(4.34) $\rho > \rho^*$.

Since we want preferences to differ only with respect to the degree of risk aversion, we want to restrict their (generalized) autarky rates of time preference to be equal. The simplest way to do this here is to assume that home and foreign period 1 output is equal to expected period 2 output. We even set period 1 output equal to unity, which is not a restriction since preferences are homothetic with (4.33).[23] Hence,

(4.35) $y^1 = Ey^2 = 1$.

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not the same thing as assuming that the asset price is positive, since the risk term may positive and larger than the present value of the expected return.

[23] The rate of time preference δ is usually defined as the subjective consumption interest rate consistent with stationary consumption. That is, with an additively separable utility function it is simply given from the subjective discount factor by $\delta = 1/\beta - 1$. We can define a generalized rate of time preference $\delta(c^1,c^2)$ as the subjective consumption rate of interest consistent with an arbitrary allocation of (sure) consumption c^1 and c^2 in period 1 and 2. This generalized rate of time preference is given by $\delta(c^1,c^2) = U_c(c^1)/\beta U_c(c^2) - 1$ for a two-period additively separable utility function. When the the home and foreign countries have the same subjective discount factor but different relative risk aversion, the generalized rates of time preference are not equal except for $c^1 = c^2$. Assuming (4.35) takes care of that complication.

First, we examine interest rates. Let us as above take the autarky price difference for the sure bond, which under (4.34) and (4.35) simplifies to

$$q_0 - q_0^* = \beta E[(y^2)^{-\rho} - (y^2)^{-\rho*}], \tag{4.36}$$

since under (4.33) and (4.35) $U_c(c) = c^{-\rho}$ and $U_c(y^1) = 1$. In the Appendix it is shown that it is always true for a non-degenerate random variable x that $a > a* > 0$ and $Ex = 1$ implies $E[x^{-a} - x^{-a*}] > 0$. Therefore (4.36) is positive, the home autarky price of the sure bond is higher, the home autarky interest rate is lower, and there is a tendency for the home country to import all asset and be a net lender in period 1.

This result may at first appear paradoxical. The home country is shifting consumption away from the certain period 1 into the uncertain period 2, in spite of being more risk averse then the foreign country. The paradox is easily resolved if we restrict ourself to local considerations and small variances in period 2 output. Then the crucial element is the local convexity of the marginal utility of consumption. Higher constant relative risk aversion means that the marginal utility is locally more convex as a function of consumption, and Jensen's inequality implies that variance in output then increases expected period 2 marginal utility. That makes period 2 consumption more attractive to the home country, and hence induces it to save. The result is in accordance with the literature on saving under uncertainty referred to above (see for instance the survey by Sandmo (1974)).

The result that for small variances in period 2 output the home autarky interest rate is lower when the home country has higher relative risk aversion is straight-forward since the function $h(x) = x^{-a} - x^{-a*}$ is locally convex for $a > a* > 0$ and $x = 1$. That the same result holds for arbitrary non-degenerate distributions is is less obvious, since the function h(x) is not globally convex. The proof in the Appendix uses a variant of the proof in Pratt (1964, Theorem 1) that the risk premium is higher for von Neumann-Morgenstern utility functions with higher absolute risk aversion.

Next, we look at the autarky risk measures for a given asset j. Let us again assume that (y^2, R_j) is bivariate normal, so that Rubinstein's (1976) theorem and thereby (4.25) holds. Then the difference in the autarky risk measures equal

(4.37) $\Pi^*_j - \Pi_j = \beta\{E[U_{cc}(y^2)] - E[U^*_{cc}(y^2)]\}Cov[y^2, R_j]$

(we have used $U_c(y^1) = 1$). We have $U_{cc}(c) = -\rho c^{-\rho-1}$, $U^*_{cc}(c) = -\rho^* c^{-\rho^*-1}$, and $Ey^2 = 1$. From (A.1) in the Appendix it follows that this implies that $E[U_{cc}(y^2)] < E[U^*_{cc}(y^2)]$. (From (A.1) it follows that $E[(y^2)^{-(\rho+1)}] > E[(y^2)^{-(\rho^*+1)}]$, hence $E[-(y^2)^{-(\rho+1)}] < E[-(y^2)^{-(\rho^*+1)}]$ and finally $E[-\rho(y^2)^{-(\rho+1)}] < E[-\rho^*(y^2)^{-(\rho^*+1)}]$.) It follows that the condition for the home autarky risk measure to be lower, and for a tendency for asset j to be imported into the home country, is

(4.38) $Cov[y^2, R_j] < 0.$

The return should be negatively correlated with period 2 output. From (4.6) and (4.25) this also implies that the risk measure should be negative,

(4.39) $\Pi_j < 0.$

Since the sure bond has a zero risk measure, this means that there is a tendency for the home country to import asset j if the asset is less risky than the sure bond.

It is easy to show that our other set of assumptions, namely that the bivariate distribution (y^2, R_j) is symmetric and has sufficiently small higher order moments, give the same conditions (4.38) and (4.39).

Consider also trade in claims to output ($R_h = R_f = y^2$). Since period 2 output is positively correlated with itself, it follows directly from the above analysis that there is a tendency for a claim to output to be exported by the home country, since it has a positive risk measure and is risker than the sure bond.

Let us finally consider the special case of Arrow-Debreu securities. We get that the difference between the autarky prices for Arrow-Debreu security s is given by

(4.40) $\quad q_s - q^*_s = \beta f_s[U_c(y^2_s) - U^*_{cc}(y^2_s)] = \beta f_s[(y^2_s)^{-\rho} - (y^2_s)^{-\rho^*}].$

It follows that the conditon for a tendency for the home country to import Arrow-Debreu security s is simply that period 2 output in state s should be below its expected value,

(4.41) $\quad y^2_s < Ey^2 = 1.$

When period 2 output is low, marginal utility in the home country is higher since a higher risk aversion means that marginal utility decreases more rapidly with consumption.

Let us summarize the results under the assumption that the home country differs from the foreign country only in that it has a higher constant relative risk aversion (see Table 1, row (iii)). First, the home autarky interest rate is lower, and there is a tendency for the home country to import all assets and be a net lender. Second, under some restrictions, there is a tendency for the home country to import assets with negative risk measures, that is, assets that are negatively correlated with period 2 output and less risky than the sure bond. Third, there is a tendency for the home to export a claim to output, since that is an asset which is more risky than the sure bond. Fourth, there is a tendency for the home country import Arrow-Debreu securities for states with lower than expected period 2 output.

(iv) Differences in subjective probabilites

Finally, we consider the case when countries differ only with respect to their subjective probability distributions, their beliefs. That is,

(4.42) $\quad f = (f_s) \neq f^* = (f^*_s).$

For a given asset j with returns $R_j = (R_{js})$ it is no longer true that that $E[R_j] = \Sigma_s f_s R_{js}$ is equal to $E^*[R_j] = \Sigma_s f^*_s R_{js}$. Therefore, the previous method of expressing the asset price in terms of the interest rate and the risk measure is not applicable. It is no longer true that a low autarky interest rate increases the relative autarky price for all assets. Hence it is no longer true that a low autarky interest rate implies a tendency for all assets to be imported. A low autarky interest rate implies only that there is a tendency for the sure bond to be imported.

The difference between the autarky prices of the sure bond is

(4.43) $$q_0 - q_0^* = \beta\{E[U_c(y_s^2)] - E^*[U_c(y_s^2)]\}/U_c(y^1).$$

We can directly apply our results on the autarky interest rates for differences in period 2 output. First, since marginal utility of consumption is decreasing, as sufficient conditon for a lower home autarky interest rate is that the home subjective induced probability distribution over (both countries') period 2 output, $G(y^2)$, is first-order dominated by the foreign subjective induced probability distribution over (both countries') period 2 output, $G^*(y^2)$, that is,[24]

(4.44) $$G <_1 G^*.$$

Put differently, the home country has more pessimistic beliefs about both countries' period 2 output than the foreign country. Second, if the von Neumann-Morgenstern utility function has decreasing absolut risk aversion, marginal utility is convex, and a sufficient condition for a lower home autarky interest rate is that the home subjective induced probability distribution over (both countries') period 2 output is second-order dominated by the foreign subjective induced probability distribution over (both countries') period 2 output, that is,

(4.45) $$G <_2 G^*.$$

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[24] The cumulative distribution functions $G(y^2)$ and $G^*(y^2)$ are defined by $G(y^2) = \{\Sigma_s f_s$: all s fulfilling $y_s^2 \le y^2\}$, and $G^*(y^2) = \{\Sigma_s f_s^*$: all s fulfilling $y_s^2 \le y^2\}$.

Put differently, the home country believes that both countries' period 2 output is more risky than the foreign country believes.

For an arbitrary asset j, the difference between the home and foreign autarky price of asset j is

(4.46) $$q_j - q^*_j = \beta\Sigma_s(f_s - f^*_s)U_c(y^2_s)R_{js}/U_c(y^1).$$

Expression (4.46) states that there is a tendency for asset j to be imported into the home country if the vector of probability differences, $f - f^* = (f_s - f^*_s)$, is positively correlated with the vector of marginal-utility weighted returns, $(U_c(y^2_s)R_{js})$.[25] Thus, we have the rather obvious result that the home country has a tendency to import an asset when it assigns higher probabilities than the foreign country to the states where the assets pays well (where paying well means that the product of marginal utility of consumption and returns is large).

For a claim to period 2 output, the autarky price difference is

(4.47) $$q_h - q^*_h = \beta\{E[U_c(y^2)y^2] - E^*[U_c(y^2)y^2]\}/U_c(y^1).$$

Let us consider the case with constant relative risk aversion. We have that the product of marginal utility and output is $U_c(y^2_s)y^2_s = (y^2_s)^{1-\rho}$. This product is increasing or depending upon whether the degree of relative risk aversion is below or above unity.

Let us consider the case when the degree of risk aversion is above unity ($\rho > 1$). Then the product of marginal utility and output is decreasing and convex, and we have the same two sufficient conditions for a tendency for the home country to import a claim to period 2 output as we have stated above for the tendency to import the sure bond, namely that the home country has more pessimistic beliefs about both countries' period 2 output than the foreign

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[25] We note that (4.46) being positive is equivalent to a positive correlation between the vectors $f - f^* = (f_s - f^*_s)$ and $(U_c(y^2_s)R_{js})$, since $\Sigma_s(f_s - f^*_s) = 0$ (cf. footnote 11 above).

country ((4.44)), or that home country believes that both countries' period 2 output is more risky than the foreign country believes ((4.45)).

If the degree of relative risk aversion is below unity ($\rho < 1$), the product of marginal utility and output is increasing and concave. Then the two sufficient conditions are reversed. The home country should have more optimistic beliefs about both countries' period 2 output than the foreign country, that is,

(4.48) $\quad G >_1 G*$,

or the home country should believe that both countries' period 2 output is less risky than the foreign country believes, that is,

(4.49) $\quad G >_2 G*$.

For the special case of Arrow-Debreu securities, the difference in autarky prices is simply

(4.50) $\quad q_s - q_s^* = \beta(f_s - f_s^*)U_c(y_s^2)/U_c(y^1)$.

We see that there is a tendency to import Arrow-Debreu securities for states that are assigned larger probability by the home country

(4.51) $\quad f_s > f_s^*$.

Let us summarize the results on differences in subjective beliefs (see Table 1, row (iv)). First, the home autarky interest rate will be low, and there will hence be a tendency for the home country to import the sure bond, if the home country has more pessimistic beliefs about the two countries' period 2 output than the foreign country, or (when preferences in the two countries exhibit decreasing absolut risk aversion) the home country believes that both countries' period 2 output is more risky than the foreign country believes. Counter to previous cases, a low home autarky interest rate does not imply that home autarky prices for other assets are low, and hence does not imply a tendency to import all assets. Second, there is, rather obviously, a tendency for the home country to import an arbitrary asset if the home country assigns higher probabilities than the foreign country to states for which the marginal utility times returns is high. Third, the tendency to import a claim on period 2 output depends on the degree of relative risk aversion. If the degree of relative risk aversion is above (below) unity, there is a tendency for the home country to import a claim to period 2 output if the home country has more pessimistic (optimistic) beliefs about the two countries' period 2 output than the foreign country, or if the home country believes the two countries' period 2 output is more (less) risky than the foreign country. Fourth, there is a tendency to import Arrow-Debreu securities for states that are assigned higher probabilities by the home country than by the foreign country.

5. Conclusions

We have presented a theory of the determinants of the trade pattern in risky assets, by extending the law of comparative advantage according to which trade is correlated with autarky price differences. Hence we have looked at how differences between countries with regard to technology, endowments and preferences determine autarky asset price differences and consequently the trade pattern in risky assets. We have derived results on the effect of differences in (i) output/endowments, (ii) rate of time preferences, (iii) risk aversion, and (iv) subjective beliefs, on the trade pattern in arbitrary risky assets as well as the special cases of sure bonds, equity, claims to output, and Arrow-Debreu securities. The results have been summarized after each subsection in section IV, and they are also summarized in Table 1.

We realize from our results that, when asset markets are incomplete, overall capital account deficits or surpluses depend on what assets are available for international trade. For instance, consider the case when countries differ only with respect to the stochastic properties of their output. If there is trade in claims to one country's output only, whether a country is a net borrower or lender depends on whether it is claims to its output or other countries' output that is traded (as we saw above, a country has a tendency to export claims to its own output and import claims to other countries' output). It follows that in a monetary model with incomplete markets, it will matter for the capital flows what currency avaialable assets are nominated in, since the real return on the assets will be affected by price level risk.

The results derived have been interpreted in terms of trade in risky assets between countries. Obviously, the model and its results can also be interpreted in terms of trade in risky assets between individuals.[26]

An important characteristic of our approach is that an assets is defined in terms of an exogenously given vector of next period's <u>total real</u> returns across states of the world. Most assets, however, have total real returns endogenously determined. For instance, the returns on equity, being claims to profits, are clearly endogenously determined when production decisions and goods and factor prices are endogenously determined. Even for an asset with exogenously given returns in terms of a particular good, the appropriate "real" return depends on endogenous relative goods prices when there are many goods. With many periods, the total return in next period on a long-term asset is the sum of next period's endogenous asset price and the "direct" return/dividend (which may or may not also be endogenous). Generally, for most assets the stochastic properties of the total real returns are endogenously determined and part of the equilibrium, and the stochastic properties differ between trade equilibria and autarky equilibria. From the

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[26] Varian (1987) analyzes the effect on the <u>volume</u> of asset trade of differences of opinion between agents in a model with trade in Arrow-Debreu securities, using what we have called in the Introduction the "direct" approach. Our analysis of the effect of differences in subjective believes on the trade <u>pattern</u> in risky assets, using the law of comparative advantage, can hence be seen as complementary to his.

point of view of our approach, if an asset has one total real return vector in a trade equilibrium, and another total real return vector in autarky, it is actually two different assets.

Hence, since most assets have endogenous total returns, it may seem that our approach with exogenously specified total returns should have very restricted applicability. We argue, however, that our approach can be used also to predict the trade pattern for assets with endogenously determined returns. The trick is to identify a particular asset's (endogenously determined) total real return vector across states of the world in a trade equilibrium, and then ask how a hypothetical asset with such a total real return vector (taken to be exogenous and hence held fixed) would be priced in autarky. The home and foreign autarky asset prices of the hypothetical asset will then predict the direction of trade in the particular asset considered.

Taking the above into account, it is possible to extend the analysis to many goods and to more than two periods. As in the standard trade theory, the predictions of the law of comparative advantage are weaker for individual assets and goods, the more assets and goods there are.

The analysis have been restricted to a barter model without any money. It is clearly desirable to include the possibility of nominal assets and to analyze also the trade pattern in such assets. Extending the model to include money and other nominal assets raises several issues, though. One issue, already mentioned above, is that the appropriate total real returns in trade equilibrium on any nominal asset considered have to be identified. We have already mentioned that the real return on nominal assets will depend on price level risk, which in turn will depend on contries' monetary policies. For instance, different exchange rate regimes and corresponding different monetary policies will affect the trade pattern in nominal assets and hence overall capital flows. Another issue is that the law of comparative advantage uses the gains-from-trade theorem, which does not necessarily hold if there are domestic distortions in autarky. Hence it will be crucial for the analysis how money is modelled, more precisely whether money is modelled as having real effects and possibly being distortionary, or whether money is modelled as being neutral. Svensson (1987) will discuss these issues and the international trade pattern for nominal assets within the context of the law of comparative advantage. Persson and Svensson (1987) will examine the effect of different exchange rate regimes and corresponding exchange rate variability on capital movements within the direct approach to the determination of the trade pattern in risky assets.

Appendix

We would like to prove that, for any random variable x with a non-degenerate probability distribution, we have

(A.1) $a > a* > 0$ and $Ex = 1$ implies $E[x^{-a}] > E[x^{-a*}]$.

The result is obvious for probability distributions with small variances. It follows from Jensen's inequality since the function $h(x) = x^{-a} - x^{-a*}$ is locally convex at $x = 1$ for $a > a* > 0$. We have

(A.2) $h_x(x) = -ax^{-a-1} + a*x^{-a*-1}$, and

(A.3) $h_{xx}(x) = a(a+1)x^{-a-2} - a*(a*+1)x^{-a*-2} =$

$= x^{-a*-2}[a(a+1)x^{-(a-a*)} - a*(a*+1)]$,

and hence $h_{xx}(1) = a(a+1) - a*(a*+1) > 0$ for $a > a* > 0$.

The result is not obvious for arbitrary non-degenerate distributions, since the function h(x) is not globally convex. (From (A.3) we see that there exists $b > 1$ such that $h_{xx}(x) < 0$ for $x > b$ ($b = [a*(a*+1)/a(a+1)]^{-1/(a-a*)}$).)

In order to prove that (A.1) is true for any probability distribution, we modify a proof of Pratt (1964, Theorem 1) that the risk premium (defined as π in $u(E[x] - \pi) = E[u(x)]$) is higher for von Neumann-Morgenstern utility functions $u(c)$ with higher absolut risk aversion $-u_{cc}(c)/u_c(c)$. Let us define $g(x) = x^{-a}$ and $g*(x) = x^{-a*}$, and let us assume $a > a* > 0$ and $Ex = 1$ throughout. We define the certainty equivalents $\bar{x}$ and $\bar{x}*$ by

$$g(\bar{x}) = E[g(x)] \text{ and } g*(\bar{x}*) = E[g*(x)]. \tag{A.4}$$

First, we show that

$$\bar{x} < \bar{x}* < 1. \tag{A.5}$$

The second inequality in (A.5) holds for $\bar{x}$ and $\bar{x}*$, since both $g(\cdot)$ and $g*(\cdot)$ are decreasing and convex, hence by Jensen's inequality $g(\bar{x}) > g(E[x])] = g(1) = 1$ and $g*(\bar{x}*) > g*(E[x]) = g*(1) = 1$. To show the first inequality in (A.5), we invert the functions $g(\cdot)$ and $g*(\cdot)$ in (A.4) and form

$$\begin{aligned}\bar{x} - \bar{x}* &= g^{-1}(E[g(x)]) - g*^{-1}(E[g*(x)]) = \\ &= g^{-1}(E[y]) - g*^{-1}(E[g*(g^{-1}(y))]),\end{aligned} \tag{A.6}$$

where we have introduced $y = g(x)$, hence $x = g^{-1}(y)$. We note that $g*(g^{-1}(y)) = (y^{-1/a})^{-a*} = y^{a*/a}$. Hence $g*(g^{-1}(y))$ is strictly concave in y, since $0 < a*/a < 1$. By Jensen's inequality we then have

$$E[g*(g^{-1}(y))] < g*(g^{-1}(E[y])), \tag{A.7}$$

which together with (A.6) implies the first inequality in (A.5).

Second, from (A.5) we have

$$(\bar{x})^{-a} > (\bar{x})^{-a*}, \tag{A.8}$$

since $(\bar{x})^{-a}$ is increasing in a for $\bar{x} < 1$, and

$$(\bar{x})^{-a*} > (\bar{x}^{*})^{-a*}, \tag{A.9}$$

since x^{-a*} is decreasing in x.

Finally, (A.8) and (A.9) together with (A.4) implies (A.1).

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Table 1. Summary of Results

(Import (yes or no (= export), or condition for import) of what asset under what difference.)

Asset:	Sure bond ($r < r*$)	Arbitrary asset ($\Pi_j < \Pi^*_j$)	Output claim	Arrow-Debreu
Differences in:				
(i) Output	$y^1 > y*^1$ $y^2 <_1 y*^2$ $y^2 <_2 y*^2$	$Cov[U_c(y^2),R_j] > Cov[U_c(y*^2),R_j]$ $Cov[y^2,R_j] < Cov[y*^2,R_j]$	h: No f: Yes	$y^2_s < y^{*2}_s$
(ii) Time preference ($\beta > \beta*$)	Yes	Yes	Yes	Yes
(iii) Risk aversion ($\rho > \rho*$)	Yes	$Cov[y^2,R_j] < 0$ $\Pi_j < 0$	No	$y^2_s < Ey^2$
(iv) Subjective beliefs ($f \neq f*$)	$G <_1 G*$ $G <_2 G*$	$\Sigma_s(f_s - f^*_s)U_c(y^2_s)R_{js} > 0$	$\rho > 1$: $G <_1 G*$ or $G <_2 G*$ $\rho < 1$: $G >_1 G*$ or $G >_2 G*$	$f_s > f^*_s$

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